INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team BIPOP

Modelling, Simulating, Controlling Non-Regular Dynamical Systems

Rhône-Alpes

THEME 4A

Activity Report

2003

Table of contents

1. Team

This newly created project team gathers researchers from the former teams Bip and Numopt. For 2003, the work had much in common with Numopt, whose activity report may also be consulted.

Head of project-team
Bernard Brogliato [senior research scientist]

administrative assistant
Françoise de Coninck [jointly with IS2, Helix]
Elodie Toihein [since September 2003, jointly with Popart, IS2]

Staff members Inria
Vincent Acary [junior research scientist, since October 2003]
Claude Lemaréchal [senior research scientist]
Pierre-Brice Wieber [junior research scientist]

Ph. D. student
Jean-Mathieu Bourgeot [Ministry fellowship]
Sophie Chareyron [Siconos fellowship]
Jérôme Malick [ENS fellowship]

Post-Doc students
Vincent Acary [from September 2001 to October 2003]
Aris Daniilidis [from October to December 2003]

2. Overall Objectives

Generally speaking, this project deals with non-regular systems, with emphasis on

- dynamic systems, mostly mechanical systems with unilateral constraints and Coulomb friction, but also electrical circuits with ideal diodes, etc;
- biped robots and their connection with human walking, a rich and instructive instance of such systems;
- numerical methods for nonsmooth optimization, and more generally the connection between continuous and combinatorial optimization.

3. Scientific Foundations

3.1. Introduction

As mentioned in the previous section, our activities cover essentially three fields.

3.2. Dynamic non-regular systems

Key words: *mechanical systems*, *impacts*, *unilateral constraints*, *complementarity*, *modelling*, *analysis*, *simulation*, *control*, *convex analysis*.

Dynamical systems (we limit ourselves to finite-dimensional ones) are said to be *non-regular* whenever some nonsmoothness of the state arises. This nonsmoothness may have various roots: for example some outer impulse, entailing so-called *differential equations with measure*. An important class of such systems can be described by the complementarity system

$$\left\{\begin{array}{l} \dot{x} = f(x,u,\lambda)\,, \\ 0 \leq y \perp \lambda \geq 0\,, \\ g(y,\lambda,x,u,t) = 0\,, \\ \text{re-initialization law of the state } x(\cdot)\,, \end{array}\right. \tag{1}$$

where $\perp$ denotes orthogonality; u is a control input. Now (1) can be viewed from different angles.

- Hybrid systems: it is in fact natural to consider that (1) corresponds to different models, depending whether $y_i = 0$ or $y_i > 0$ (y_i being a component of the vector y). In some cases, passing from one mode to the other implies a jump in the state x; then the continuous dynamics in (1) may contain distibutions.
- Differential inclusions: $0 \leq y \perp \lambda \geq 0$ is equivalent to $-\lambda \in \mathrm{N}_K(y)$, where K is the nonnegative orthant and $\mathrm{N}_K(y)$ denotes the normal cone to K at y. Then it is not difficult to reformulate (1) as a differential inclusion.
- Dynamic variational inequalities: such a formalism reads as $\langle \dot{x}(t) + F(x(t),t), v - x(t) \rangle \geq 0$ for all $v \in K$, $x(t) \in K$, where K is a nonempty closed convex set. When K is a polyhedron, then this can also be written as a complementarity system as in (1).

Thus, the 2nd and 3rd lines in (1) define the modes of the hybrid systems, as well as the conditions under which transitions occur from one mode to another. The 4th line defines how transitions are performed by the state x. There are several other formalisms which are quite related to the complementarity one. Two tutorial-survey papers have been published [7][8], whose aim is to introduce the dynamics of complementarity systems and the main available results in the fields of mathematical analysis, analysis for control (controllability, observability, stability), and feedback control.

3.3. Biped robots

Key words: *control*, *sensor-based control*, *mechanics*, *solid mechanics*, *modelling*, *robotics*, *mobile robotics*, *simulation of mechanical systems*.

3.3.1. Modelling

A biped robot can be modelled [27][28] as a tree-like articulated chain of rigid bodies in R^3. Walking is characterized by different phases, mainly (for a given leg): swing (35% of the cycle), support (65%); there is a double-support phase (12%), which does not exist when running. A finer decomposition takes into account the movement of the mass center, and above all of the foot. These different phases are characterized by different contacts between the system and the ground.

As a result, the mechanical model of such a system has three aspects:

- the dynamics of a rigid articulated system, free in the space, representable by Lagrangian equation;
- a set of equality and inequality constraints, depending on the phase, which expresses the existence of contacts without penetration nor sliding; each one of these sets defines an operating mode;
- the selection of impact laws modelling the transitions (assumed instantaneous) between these modes.

This makes up a sophisticated hybrid system, whose study is still little explored, see [19] for a survey on modelling, stability and control of biped robots.

3.3.2. Controlling

The pace naturally adopted by a human walker is regular and symmetric, consuming little energy for a reasonable speed. Some hybrid systems, such as leaping robots or transmissions with slack, present likewise limit cycles corresponding to dynamic equilibria, possibly stable in a certain domain. For the simplest walking robot – a compass on a slope – these cycles correspond to passive periodic trajectories (without external action), in which the transition between kinematic and potential energies is entirely balanced by the energy absorbtion during the impact [25].

As a result of these considerations, our approach of control aims at constructing cyclic trajectories, minimizing energy – whatever this means. Also, it is important to guarantee a global progression while preserving a particular mechanical stability, which is dynamic. Classical approaches – for example following accurately nominal articulated trajectories – are therefore inadequate, except if one is just interested in controlling the attitude. The field is far from being settled, so we have to explore diverse control techniques: nonsmooth optimization, predictive control, adaptive learning, task function control [26]... including sensor-based control, allowing to use local measures of distance, proximity, reaction to the ground etc.

3.4. Nonsmooth optimization

Key words: *optimization*, *numerical algorithm*, *convexity*, *Lagrangian relaxation*, *combinatorial optimization*.

Here we are dealing with the minimization of a function f (say over the whole space R^n), whose derivatives are discontinuous. A typical situation is when f comes from dualization, if the primal problem is not strictly convex – for example a large-scale linear program – or even nonconvex – for example a combinatorial optimization problem. Also important is the case of spectral functions, where $f(x) = F(\lambda(A(x)))$, A being a symmetric matrix and λ its spectrum.

For these types of problems, wer are mainly interested in developing efficient resolution algorithms. Our basic tool is bundling [3] and we act along two directions:

- To explore application areas where nonsmooth optimization algorithms can be applied, possibly after some tayloring. A rich field of such application is combinatorial optimization, with all forms of relaxation [5][4].
- To explore the possibility of designing more sophisticated algorithms. This implies an appropriate generalization of second derivatives when the first derivative does not exist and uses advanced tools of nonsmooth analysis, for example [6].

4. Application Domains

Many systems (either actual or abstract) can be represented by (1). Some typical examples are:

- Mechanical systems with unilateral constraints and dry friction (the biped robot is a typical example), including kinematic chains with slack, phenomena of liquid slosh, etc.
- Electrical circuits with ideal diodes and/or transistors MOS.
- Optimal control with constraints on the state, closed loop of a system controlled by an MPC algorithm, etc.

This class of models is not too large (to allow thorough studies), yet rich enough to include many applications. This goes in contrast to a study of general hybrid systems. Note for example that (1) is a “continuous” hybrid system, in that the continuous variables x and u prevail in the evolution (there is no discrete control to commute from a mode to the other: only the input u can be used).

Walking robots – for example hexapods – possess definite advantages over the rolling ones whenever the ground is not plane or free: clearing obstacles is easier, holding on the ground is lighter, adaptivity is improved. However, if the working environment of the system is adapted to man, the biped technology must be preferred, to preserve good displacement abilities without modifying the environment. This explains the interest displayed by the international community in robotics toward humanoid systems, whose aim is to back man in some of his activities, professional or others. For example, a certain form of help at home to disabled persons could be done by biped robots, as they are able to move without any special adaptation of the environment.

Optimization exists in virtually all economic sectors. Simulation tools can be used to optimize the system they simulate. Another domain is parameter *identification* (Idopt or Estime teams), where the deviation between measurements and theoretical predictions must be minimized. Accordingly, giving an exhaustive list of applications is impossible. Some domains where Inria has been implied in the past, possibly through the former Promath and Numopt teams are: production management, geophysics, finance, molecular modelling, robotics, networks, astrophysics, crystallography,...

5. Software

Two sorts of software are developed within Bipop. For optimization, see the Numopt activity report. We describe here our views on software for nonsmooth dynamics.

In the framework of the European project Siconos, BIPOP is the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. The aim of this work is to provide a common platform for the simulation, the modelling, the analysis and the control of abstract nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modelling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building expert interfaces and end-user front-end through Scilab.

For the design, two majors goals have been outlined. The first one is to offer a set of numerical algorithms already well identified, to solve such problems. The second goal is to implement an object-oriented structure which will be able to model the known-how of the Siconos community in terms of formalization and modelling. An effort is made to realize this software design in using modern tools of object-oriented analysis and design. The specification step is now completed and the phase of conceptual and architectural design is beginning. The first version of the platform will be delivered in September 2004. Details on the operation of software development may be found at http://www.inrialpes.fr/bipop/wp2

6. New Results

6.1. Stability and Feedback Control

6.1.1. Lyapunov stability theory for nonsmooth Lagrangian systems

Participants: Sophie Chareyron, Pierre-Brice Wieber, Bernard Brogliato.

The dynamics of mechanical systems with unilateral constraints involves nondifferentiable and even discontinuous functions on restricted domains. The classical results on system stability need therefore to be reformulated in this specific framework. Following Zubov [29], we rewrite the main definitions and results on Lyapunov stability theory. Then, based on an energy approach, we show that a nonsmooth Lagrangian dynamical system without friction is stable if the generalized forces derive from a potential. These results are a generalization of the classical Lagrange-Dirichlet (or Lejeune-Dirichlet) theorem on the stability of mechanical systems with a strict convex potential function. They habe been published in [16].

6.1.2. Tracking control of complementarity Lagrangian systems

Participants: Jean-Matthieu Bourgeot, Bernard Brogliato.

The tracking control problem for Lagrangian system has been solved in the cases of unconstrained or permanently constrained systems, at the end of the eighties. The case of unilaterally constrained systems requires special care and suitable stability notions that extend the Lyapunov stability. This work [15] continues previous works [23][24]. It clarifies the design and the role of the transition phases (stabilisation on the constraint surface or detachment from this surface) in the closed-loop stability. Roughly speaking, it extends the so-called passivity-based control design to this type of nonsmooth dynamical systems.

6.1.3. Stability of evolution variational inequalities

Participant: Bernard Brogliato.

This work is done in cooperation with Daniel Goeleven of Univ. de la Réunion. It concerns the stability of a class of evolution variational inequalities of the form $\langle \dot{x}(t) + Ax(t), v - x(t) \rangle \geq 0$ for all $v \in K$, $x(t) \in K$; K is a nonempty closed convex set. We derive sufficient conditions for Lyapunov stability and propose several criteria that allow one to test whether or not the matrix A is stable on K [18]. The asymptotic stability requires an extension of the Krakovskii-LaSalle invariance theorem to such nonsmooth and nonlinear systems, which is done in [17].

6.1.4. Absolute stability with maximal monotone mappings

Participant: Bernard Brogliato.

The so-called absolute stability problem is of importance in Systems and Control theory, and consists of studying the stability of a system made of the negative feedback interconnection of a dissipative system with a nonlinear characteristic. In [16] absolute stability is extended to a feedback branch containing a maximal monotone operator. The system is recast in the framework of differential inclusions by a suitable coordinate change. In particular relative degree one dissipative circuits with ideal diodes, fit within the study.

6.1.5. Adaptation of reference trajectories for the control of walking systems

Participant: Pierre-Brice Wieber.

The first task that a walking system has to fulfill is to avoid to fall, which amounts to a *viability* problem. Unfortunately, viability is not an easily computable property; as a result we rather must deal with approximations of it. One approximation is to consider Lyapunov stability, not only with respect to a single reference trajectory, but with respect to a whole set of reference trajectories. This allows then to consider an adaptation of the reference trajectory to the actual state of the system: preliminary simulation results show that it is indeed a promising approach.

6.1.6. Simulation of walking: humanoid robots and electro-stimulated paraplegic people

Participants: Pierre-Brice Wieber, Matthieu Guilbert.

The modelling and simulation software for the BIP robot (written in C/C++/Maple/Scilab) has been reorganized and modularized in order to stick better to the sensing and actuating subsystems of the experimental platform, but also to adapt it to the simulation of electro-stimulated paraplegic people, focusing mainly on the muscles' dynamical model. This software platform is being extensively used now at INRIA Rhône-Alpes, at LMS in Poitiers and at LIRM in Montpellier.

6.1.7. BIP robot

Participant: Pierre-Brice Wieber.

The computing power of the BIP robot has been completely restructured, using now 2 CPUs (one internal and one external to the robot), communicating through a local IP network; this allows the implementation of sophisticated observers and control laws. A task function control law taking care of force allocation has then been implemented and optimized in order to require only 1ms of computation time, allowing a comfortable sampling rate, even through the IP network. First experimentations of 3D standing and walking with this new control law are undertaken.

6.2. Controllability

Participant: Bernard Brogliato.

The controllability studies concern two classes of complementarity systems: juggling systems and planar evolution variational inequalities. In the first case, it is shown that studying controllability amounts to solving a nonlinear system subject to inequality constraints [22]. The second study [14] shows that controllability depends on the form of the convex set K in which the system is constrained to evolve. Interestingly enough, the complementarity conditions that are activated on the boundary of K, may improve the controllability in some cases.

6.3. Modelling

Participants: Vincent Acary, Bernard Brogliato.

This work concerns the modelling of multiple impacts: an impact is said to be multiple when several impacts occur at the same time on the system. The multiple impact mappings which have already been proposed in the literature, are not satisfactory because they often result in post-impact velocities which blatantly disagree with experiments, and/or are not tractable from the numerical point of view. In [9][10][20] we propose the use of so-called impulse correlation ratios, which seem to be suitable physical parameters for the design of multiple impact laws. We fist investigate the classical Netwon's cradle (a chain of balls), the ultimate goal being to apply the results to the circuit breakers of Schneider Electric.

6.4. Optimization

Participants: Claude Lemaréchal, Jerôme Malick.

See the Numopt activity report for the year 2003.

8. Other Grants and Activities

8.1. Actions européennes

The Bipop project coordinates the European project Siconos (modelling, SImulation and COntrol of NOnsmooth dynamical Systems, IST 2001-37172), which is an FP5 project starting September 2002 and ending September 2006. See http://maply.univ-lyon1.fr/siconos/.

8.2. Teaching

– GSI, University Joseph Fourier (J.-M. Bourgeot, tutoring 21)
– Ensimag, Grenoble (V. Acary, C. Lemaréchal, tutoring optimization 18h)
– Institut National Polytechnique, Grenoble (S. Chareyron, tutoring 24h)
– ISTG, Polytech'Grenoble (J.-M. Bourgeot, tutoring electronics and automatic control, 66h).

8.3. Participation to conferences, seminars, invitations

– SICONOS Meetings: Grenoble, January and July; Montpellier, April; Barcelona, November 2003 (several participants and presentations each time).
– Meeting for the ROBEA project ¨Commande pour la marche et la course d'un robot bipède¨: Versailles, January, Grenoble, November 2003 (2 participants, 1 presentation each time).
- School "LMGC90": Montpellier, April (1 participant)
– Meetings for the ROBEA project: ¨Contrôle du mouvement du membre inférieur humain paralysé sous stimulation électrique¨, Montpellier, April and November 2003 (2 participants, 1 presentationn each time).
– JJCR'17: Journées Jeunes Chercheurs en Robotique: Versailles, April 2003 (2 participants, 1 presentation).
– Colloque national de calcul des structures, Giens, May 2003 (1 participant, 1 presentation).
– JDA 2003, Valenciennes, June 2003; [12].
– Workshop on Free Boundary Problems: Applications to the fluid mechanics, friction and impact phenomena: Saint Etienne, September 2003 (3 participant).
– Journée INRIA-SMAI de réflexion sur les logiciels. Valorisation et diffusion de logiciels , les grands challenges applicatifs, Rocquencourt, October 2003.
– School on Nonsmooth Dynamics, Autrans, October 2003 (5 participants, 1 presentation).
– European Control Conference, Cambridge, September 2003 (1 participant: minicourse on complementarity systems)
– ADHS03, IFAC conference on Analysis and design of Hybrid Systems, Saint Malo, June 2003 (1 participant)
– Colloquium in the honour of Jean-Jacques Moreau for his 80th birthday, Montpellier, November 2003 (4 participants, 1 presentation)

10. Bibliography

Major publications by the team in recent years

[1] J. BONNANS, J. GILBERT, C. LEMARÉCHAL, C. SAGASTIZÁBAL. *Optimisation Numérique: aspects théoriques et pratiques.* Springer Verlag, Paris, 1997.

[2] J.-B. HIRIART-URRUTY, C. LEMARÉCHAL. *Fundamentals of Convex Analysis.* Springer Verlag, Heidelberg, 2001.

[3] J.-B. HIRIART-URRUTY, C. LEMARÉCHAL. *Convex Analysis and Minimization Algorithms.* Springer Verlag, Heidelberg, 1993, Two volumes.

[4] C. LEMARÉCHAL. *Lagrangian relaxation.* M. JÜNGER, D. NADDEF, editors, in « Computational Combinatorial Optimization », Springer Verlag, Heidelberg, 2001, pages 115-160.

[5] C. LEMARÉCHAL, F. OUSTRY. *Semi-definite relaxations and Lagrangian duality with application to combinatorial optimization.* Rapport de Recherche, number 3710, Inria, 1999, http://www.inria.fr/rrrt/rr-3710.html.

[6] C. LEMARÉCHAL, F. OUSTRY, C. SAGASTIZÁBAL. *The $\mathcal{U}$-Lagrangian of a convex function.* in « Transactions of the AMS », number 2, volume 352, 2000, pages 711-729.

Articles in referred journals and book chapters

[7] B. BROGLIATO. *Some perspectives on the analysis and control of complementarity systems.* in « IEEE Transactions on Automatic Control », number 6, volume 48, 2003, pages 918-935.

[8] W. HEEMELS, B. BROGLIATO. *The complementarity class of hybrid systems.* in « European Journal of Control », number 2-3, volume 9, 2003, pages 322-360.

Publications in Conferences and Workshops

[9] V. ACARY, B. BROGLIATO. *Concurrent multiple impacts modelling Case-study of a 3-ball chain.* in « Second MIT Conference on Computational Fluid and Solid Mechanics », Elsevier, June, 2003.

[10] V. ACARY, B. BROGLIATO. *Modélisation des impacts multiples simultanés.* in « 6ème Colloque National en Calcul des Structures, Giens », May, 2003.

[11] C. AZEVEDO, P. POIGNET, B. ESPIAU. *Contrôle de la locomotion artificielle: de l'homme aux robots.* in « Workshop "Humanoïdes" », Société Biomécanique de France, pages 53-64, Valenciennes, 2003.

[12] J.-M. BOURGEOT, B. BROGLIATO. *Poursuite de trajectoires dans les systèmes lagrangiens de compl@mentarit@.* in « Actes des Journ@es Doctorales d'Automatique 2003 », Presses Universitaires de Valenciennes, pages 371-376, 2003.

[13] S. CHAREYRON, M. ATHANS. *Nonlinear Filtering for a Nonlinear Vision-Based System for the Relative Tracking of Two Marine Vehicles.* in « IEEE 11th Mediterranean Conf. on Control and Automation MED'03 »,

pages 18-20, Rhodes, Greece, June, 2003.

Internal Reports

[14] B. BROGLIATO. *Some results on the controllability of planar evolution variational inequalities.* Research Report, number 4788, Inria, 2003, http://www.inria.fr/rrrt/rr-4788.html.

Miscellaneous

[15] J. BOURGEOT, B. BROGLIATO. *Tracking control of complementarity Lagrangian systems.* To appear.

[16] B. BROGLIATO. *Absolute stability and the Lagrange-Dirichlet theorem with monotone multivalued mappings.* To appear in Systems and Control Letters.

[17] B. BROGLIATO, D. GOELEVEN. *The Krakovskii-LaSalle invariance principle for a class of unilateral dynamical systems.* Submitted to Mathematics of Control, Signals and Systems.

[18] D. GOELEVEN, B. BROGLIATO. *Stability and instability matrices for linear evolution variational inequalities.* To appear in IEEE Transactions on Automatic Control.

[19] Y. HURMUZLU, F. GÉNOT, B. BROGLIATO. *Modelling, stability and control of biped robots – A general framework.* To appear in Automatica.

[20] C. MERLE, V. ACARY. *Modélisation numérique des impacts multiples dans les mécanismes. Application aux mécanismes de disjoncteurs électriques.* Technical report, Stage Ingénieur DEA, Esm2, université d'Aix–Marseille II, 2003.

[21] P.-B. WIEBER, C. CHEVALLEREAU. *Online adaptation of reference trajectories for the control of walking systems.* Submitted to the International Journal of Robotics Research.

Bibliography in notes

[22] B. BROGLIATO, M. MABROUK, A. ZAVALA-RIO. *A controllability criterion for linear juggling mechanical systems.* in « 3rd Contact Mechanics International Symposium, Peniche, Portugal », 2001.

[23] B. BROGLIATO, S. NICULESCU, M. MONTEIRO-MARQUES. *On tracking control of a class of complementary-slackness hybrid mechanical systems.* in « Systems and Control Letters », number 4, volume 39, 2000, pages 255-266.

[24] B. BROGLIATO, S. NICULESCU, P. ORHANT. *On the control of finite-dimensional mechanical systems with unilateral constraints.* in « IEEE Transactions on Automatic Control », number 2, volume 42, 1997, pages 200-215.

[25] A. GOSWANI, B. THUILOT, B. ESPIAU. *A Study of the Passive Gait of a Compass-like Biped Robot: Symmetry and Chaos.* in « International Journal on Robotic Research », number 12, volume 17, 1998, pages 1282-1301.

[26] C. SAMSON, B. ESPIAU, M. LE BORGNE. *Robot Control : The Task Function Approach.* Clarendon Press, Oxford, 1991.

[27] C. VAUGHAN, B. DAVIS, J. O'CONNOR. *Dynamics of human gaits.* in « Human Kinetic », 1992.

[28] D. WINTER. *Biomechanics and Motor Control of Human Movement.* Wiley-Interscience, 1990.

[29] V. ZUBOV. *Methods of A.M. Lyapunov and their application.* Noordhoff, 1964.