INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team coprin

Constraints solving, OPtimization, Robust INterval analysis

Sophia Antipolis

THEME 2B

Activity Report

2003

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1. Team

COPRIN is a joint project between INRIA, CNRS, University of Nice-Sophia Antipolis (UNSA) and CERMICS.

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2. Overall Objectives

Key words: *constraints programming*, *interval analysis*, *symbolic-numerical calculation*, *numerical robustness*, *systems solving*, *robotics*, *mechanism theory*, *molecular chemistry*.

COPRIN is a joint project between Nice/Sophia-Antipolis University (UNSA), CNRS, ENPC and INRIA. Its scientific objective is to develop and implement systems solving algorithms based on constraints propagation methods, interval analysis and symbolic computation, with interval arithmetics as the primary tool.

We are interested in real-valued constraint satisfaction problems (($f(X) = 0$, $f(X) \leq 0$)), in optimization problems and in the proof of the existence of properties (for example it exists X such that $f(X) = 0$ or it exists two values X_1, X_2 such that $f(X_1) > 0$ and $f(X_2) < 0$)

Solutions will be searched within a finite domain (called a *box*) which may be either continuous or mixed (i.e. for which some variables must belong to a continuous range while other variables may only have value within a discrete set). An important point is that we aim to find all the solutions within the domain as soon as the computer arithmetics will allow it: in other words we are looking for *certified* solutions.

Our research aims to develop algorithms that can be used for any problem or are specific to a given class of problem, especially problems that are issued from application domains for which we have an internal expertise (such as mechanism theory and software engineering).

Implementation of the algorithms will be performed within the frameworks of the generic software tool `IcosAlias`, currently under development, whose purpose is to allow one to design and test solving algorithms obtained as the combination of various software modules. `IcosAlias` will be based on the already existing libraries `ICOS` and `ALIAS`.

As a theoretical complexity analysis of the solving algorithms is usually extremely difficult, the efficiency of the algorithms will be experimentally evaluated through `IcosAlias` on various bench examples.

3. Scientific Foundations

The scientific objective of the COPRIN project is to develop and implement systems solving algorithms based on constraints propagation methods, interval analysis and symbolic computation, with interval arithmetics as the primary tool.

The results obtained with these algorithms are certified in the sense that all solutions will be obtained and can be calculated with an arbitrary accuracy. Furthermore some of our algorithms will allow us to deal with systems involving uncertain coefficients.

A system will be constituted by a set of relations that may use all the usual mathematical operators and functions (hence we may deal, for example, with the relation $\sin(x + y) + \log(\cos(e^x) + y2) \leq 0$).

We are interested in real-valued constraint satisfaction problems (($f(X) = 0$, $f(X) \leq 0$)), in optimization problems and in the proof of the existence of properties (for example it exists X such that $f(X) = 0$ or it exists two values X_1, X_2 such that $f(X_1) > 0$ and $f(X_2) < 0$)

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Our approach is to develop various operators that will be applied in sequence on a box:

1. *exclusion operators*: these operators determine that there is no solution to the problem within a given box
2. *contractors*: these operators may reduce the size of the box i.e. decrease the width of the allowed ranges for the variables
3. *existence operators*: they allow one to determine that there is a unique solution within a given box and are usually associated to a numerical scheme that enables to compute this solution in a safe way

If a given box is not rejected by the exclusion operators and is not modified by the other operators, then we will bisect one of the variables in order to create two new boxes that will be processed later on. Methods for choosing the bisected variable are also clearly within the scope of the project.

Our research aims to develop operators that can be used for any problem or are specific to a given class of problem, especially problems that are issued from application domains for which we have internally an expertise (such as mechanism theory and software engineering). Furthermore we will study symbolic computation based methods:

- to develop a user-friendly interface that will automatically generate an executable program, run it and return the result to the interface
- to analyze the semantics and syntax of the relations involved in a problem in order to generate automatically specific operators or to obtain a better interval evaluation of the relations (as interval arithmetics is very sensitive to the relations syntax)
- to allow for the calculation of the solution with an arbitrary accuracy: certified interval solution will be obtained through a compiled program which is much more efficient than its symbolic computation equivalent and then a symbolic computation procedure will be used to calculate the solution up to the desired accuracy

4. Application Domains

Key words: *molecular chemistry*, *robotics*, *mechanism theory*, *software engineering*.

While the methods developed in the project may be used for a very broad set of application domains, it is clear that the size of the project does not allow all of them to be addressed. Hence we have decided to focus our applicative activities on two domains for which we have already an expertise: mechanism theory (including robotics) and software engineering.

- mechanism theory: our research focusing on optimal design and geometrical modeling of mechanisms, especially for the machine-tool industry, automotive suspensions and medical robotics. As some problems in mechanism theory and molecular chemistry are basically equivalent, they will also be addressed
- software engineering: our research focusing on the automatic generation of test data sets i.e. to generate input data so that a given step in a software module will be executed

5. Software

5.1. ALIAS

Participants: Jean-Pierre Merlet, Yves Papegay, David Daney.

Key words: *constraints solving*, *optimization*, *symbolic computation*.

The `ALIAS` library (*Algorithms Library of Interval Analysis for Systems*) is a collection of procedures based on interval analysis for systems solving and optimization. Its development has started in 1998.

`ALIAS` is constituted of two parts:

- `ALIAS-C++`: the C++ library (300 000 code lines) which is the core of the algorithms
- `ALIAS-Maple`: the Maple interface for `ALIAS-C++` (40 000 code lines). This interface allows one to specify a solving problem within Maple and to get the results within the same Maple session. The role of this interface is not only to generate automatically the C++ code, but also to perform an analysis of the problem in order to improve the efficiency of the solver. Furthermore a distributed implementation of the algorithms is available directly within the interface

Our effort this year has focused on improving the implementation and the documentation while using the library to solve practical problems (for example within the framework of the ROBEA-MAX project [15]). A web page with 54 application examples is now available together with pages describing the basic principles of interval analysis.

The current version of the library is available through the Web page http://www-sop.inria.fr/coprin/logiciels/ALIAS.

5.2. ICOS

Participants: Yahia Lebbah, Claude Michel, Michel Rueher.

Key words: *constraints solving*.

ICOS solves rigorously nonlinear problems modeled with the AMPL mathematical programming language (see http://www.ampl.com). ICOS computes a small and safe box for each solutions of such systems. The constraint-solving algorithm of ICOS is based on a combination of interval analysis methods, constraint programming and linear programming relaxation techniques. ICOS has been evaluated on a variety of benchmarks from kinematics, mechanics and robotics. It outperforms interval methods as well as CSP solvers and it compares well with state-of-the-art optimization solvers. Online implementation of ICOS will be soon available through the web site of the COPRIN project.

5.3. INCOP

Participants: Bertrand Neveu, Gilles Trombettoni.

Key words: *local search*, *meta-heuristics*.

We have designed and implemented a new C++ library INCOP of incomplete methods for solving combinatorial optimization problems. This library offers classical local search methods such as simulated annealing, tabu search, a population based method "Go With the Winners" (GWW). Several problems have been encoded, including Constraint Satisfaction Problems, graph coloring, frequency assignment. The user can easily add new algorithms and encode new problems. The neighborhood management has been carefully studied. First, an original parameterized move selection allows us to easily implement most of the existing meta-heuristics. Second, different levels of incrementality can be specified for the configuration cost computation, which highly improves efficiency.

INCOP has shown great performances on a sample of well-known benchmarks. It outperforms most of the existing concurrent tools. The challenging `flat300_28` graph coloring instance has been colored in 30 colors for the first time by a standard Metropolis algorithm. The first version of this library, named INCOP [32][34][33], is now available at http://www-sop.inria.fr/coprin/neveu/incop.

We have also designed a new hybrid algorithm scheme, a hybridization of the population based "Go With the Winners" algorithm with local search. The randomization step of GWW is replaced by a local search in order to favor better solutions during the search. An instantiation of this schema, named GWW-grw, has four parameters and we proposed a simple way to tune them [32][35]. Good results for frequency assignment problems have been obtained.

6. New Results

6.1. Robotics and mechanism theory

Key words: *robotics*, *calibration*, *robot accuracy*, *mechanism theory*, *parallel robots*.

The core of our activity in robotics and mechanism theory is the optimal design of mechanism and the analysis of parallel robots [17][6]. This year we have focused on:

- the dimensional synthesis of 3-dof positioning device: last year we have solved the problem of dimensional synthesis of 3R mechanism (for which no solution was known): a set of poses that have to be reached by the wrist of the robot are specified and the problem is to determine the geometries of all the robots that can reach all the poses in the set. We have started this year a preliminary study of the same problem for others mechanical architectures (such as RPS, CPS, ...). This study has shown that while we were able to determine some solutions to the problem, it was difficult to find all the solutions. As for the 3R structure it seems that a solving algorithm combining various formulations of the problem must be developed

- exact determination of a robot dexterity over a given workspace
- optimal geometry of modular parallel robots
- the determination of the location of the measurement poses for optimal calibration of robots with a local method
- calibration of parallel robots based either on an algebraic approach or on interval analysis

6.1.1. *Exact calculation on the dexterity of a robot*

Participant: Jean-Pierre Merlet.

The minimal and maximal values of the eigenvalues of the Jacobian matrix of a robot, together with the condition number of this matrix, are the usual indexes to characterize the dexterity of the robot. Although these values may be computed easily for a given pose of the robot, it is much more difficult to determine exactly the minimum and maximum of these values over a given workspace. We have developed a generic algorithm based on interval analysis that allows one to determine these quantities up to an arbitrary accuracy. As an example, the case of the 3-dof Orthoglide robot of IRCCYN (Nantes) has been treated. This work should have been presented at the IFToMM World Congress that has been postponed due to the SARS virus.

6.1.2. *Modular parallel robots*

Participant: Jean-Pierre Merlet.

A modular parallel robot has a geometry that may be modified to adjust the performances of the robot to the task. Although prototypes of such robot exist, there is no known algorithm that allows one to determine the best geometry of the robot from given constraints on the task.

We have considered modular robots for which the location of the anchor points on the base may slide along a given direction. We have shown that it was possible to determine all the locations of the anchor points so that a given trajectory of the robot (defined by arbitrary analytical time functions) will be fully included in the workspace of the robot. Furthermore as the set of possible locations is defined as a set of ranges we are still able to optimize a secondary criteria. For example we have shown that it was possible to maximize the minimal value of the stiffness of the robot along the normal of a planar trajectory: typically a gain of 20% on the minimal stiffness is obtained while the average stiffness on the trajectory may be improved by 40% [29].

6.1.3. *Micro-robot*

Participant: Jean-Pierre Merlet.

In the last years we have developed a parallel 3-dof micro-robot to be used for endoscopy in collaboration with the LMARC laboratory of Besançon University, the Technion of Haifa and the company DG Créations. Two prototypes have been developed with a diameter of respectively 7mm for a length of 28 mm and 8.6 mm with a similar length. Motions have been tested but we have been confronted with a major problem: the small electrical motors that are used in the prototypes (from the companies MicroMotor and RMB) will usually fail after a cumulative time of use of about 10 minutes. It seems that the gear box provided with these motors (necessary as their rotation speed is over 100 000 rpm) are very delicate (and cannot be repaired on site). We are investigating how to solve this problem together with considering the use of alternate actuators. This problem has evidently postponed the first clinical trial that should have taken place this year.

6.1.4. *Determination of measurement configurations for robot calibration by local convergence method associated to meta-heuristic methods*

Participants: David Daney, Blaise Madeline.

Kinematic parameters of a robot are determined in a calibration method by solving an over-constrained system of equations [16]. This system is a function of informations obtained by controlling and measuring the robot state in different poses. An optimal choice of these poses within the robot workspace allows one to improve the numerical quality of the system of equations and increase the robustness of identification with respect to measurement noise. We propose to determine these optimal poses by a numerical optimization algorithm associated to meta-heuristic methods to decrease the sensibility to local minima. We show that our algorithm allows us to divide by twenty the identification error compared to randomly chosen poses when using the simplest method of parallel robot calibration.

6.1.5. Identification of parallel robot kinematic parameters under uncertainties by using algebraic methods

Participants: David Daney, Ioannis Emiris.

This work implements algebraic elimination methods for an original and general calibration method of parallel robots [20]. It focuses on two approaches, namely algebraic variable elimination and monomial linearization, both being based on sparse resultant combined with numerical linear algebra. Several experiments have allowed us to compare these two methods together with a classical numerical optimization method in the presence of measurement errors. Our main conclusion is that elimination methods offer an interesting alternative to more well-established methods for parallel robot calibration by satisfying the goals of accuracy and robustness. Moreover, our methods require no initial estimate and no hypothesis on the noise distribution and allow one to derive a quality index for the solution.

6.1.6. Guaranteed parallel robot calibration with measurement errors

Participants: David Daney, Arnold Neumaier, Yves Papegay.

The identification of robot kinematic parameters is difficult to certify due to the errors in the measurements needed by a calibration process. If these errors are taken into account the kinematic parameters cannot be determined exactly and we propose to bound the possible values of the parameters without assuming any knowledge on measurement distribution. For that purpose, the measurement errors are introduced inside the constraint equations as additional variables with an interval representation. The problem is then to solve a parametric system of equations by interval analysis and constraint programming methods. We have already general purpose algorithms allowing to get an approximation of the set of solutions. But these algorithms must be adapted to deal more efficiently with this specific problem.

6.1.7. Influence of joint tolerances on closed-loop kinematic chains

Participants: David Daney, Yves Papegay, Jean-Pierre Merlet.

The manufacturing tolerances associated to the joints of a kinematic chain modify the theoretical properties of the mechanism (accuracy, number of degrees of freedom ...). The values of theses tolerances are constrained to lie within ranges and we propose to propagate their intervals to verify their influences on the robot properties. This work is difficult for closed-loop kinematic chains as the closure equations become a system of nonlinear equations parameterized by intervals but we have started to develop a generic framework that may allow to analyze the most important robot properties.

6.2. Systems solving in continuous domains

Key words: *constraint programming*, *interval analysis*, *symbolic-numerical calculation*, *numerical robustness*, *systems solving*, *constraint satisfaction problems (CSP)*.

Systems solving and optimization are clearly the core of our research activities. We focus on systems solving as many applications in engineering sciences require finding all isolated solutions to systems of constraints over real numbers. It is difficult to solve as the inherent computational complexity is NP-hard and numerical issues are critical in practice. For example, it is far from being obvious to guarantee correctness and completeness as well as to ensure termination. Overall complexity of our solvers cannot be estimated in general and consequently only extensive experiments allow to estimate their practical complexity.

Our research focus on the following axis:

- developing new algorithms for local and global filtering, exclusion and existence operators. This is one of the main axis of our theoretical work. It involves numerical analysis, symbolic computation, constraints programming.
- developing specific solvers for systems sharing the same type of structure (e.g. systems of distance equations). Here also a theoretical work allows to specialize the mathematical tools we are using according to the problem at hand for a better efficiency. In parallel specific data structure are used in the implementation
- systems decomposition: the objective is to decompose large systems into sub-systems that are independent or loosely connected and are solved in sequence, allowing an important improvement of efficiency compared to general solver
- developing our generic systems solving software `IcosAlias`. Existing solvers exhibit lack of flexibility: our objective is to develop a framework that will allow to modify easily the solving strategy, to test new algorithms and to develop solvers for specific systems

6.2.1. Linearization and global filtering for numerical constraint systems

Participants: Yahia Lebbah, Claude Michel, Michel Rueher.

The purpose of our research is to introduce and to study a new branch and bound algorithm called `QuadSolver`. The essential feature of this algorithm is a global constraint (called `Quad`) that works on a tight and safe linear relaxation of the polynomial relations of the constraint systems. More precisely, `QuadSolver` is a branch and prune algorithm that combines `Quad`, local consistencies and interval methods [25][31][26].

`QuadSolver` has been evaluated on a variety of benchmarks from kinematics, mechanics and robotics. On these benchmarks, it outperforms classical interval methods as well as CSP solvers and it compares well with state-of-the-art optimization solvers.

The relaxation of nonlinear terms is adapted from the classical the "Reformulation-Linearization Technique (RLT)" linearization method. The simplex algorithm is used to narrow the domain of each variable with respect to the subset of the linear set of constraints generated by the relaxation process. The coefficients of these linear constraints are updated with the new values of the bounds of the domains and the process is restarted until no more significant reduction can be done. We have demonstrated that the `Quad` algorithm yields a more effective pruning of the domains than local consistency filtering algorithms (e.g., 2b–consistency or box–consistency). Indeed, the drawback of classical local consistencies comes from the fact that the constraints are handled independently and in a blind way. For example, when dealing with quadratic constraints, classical local consistencies do not exploit the semantic of quadratic term; for reducing the domains of the variables. Conversely, linear programming techniques do capture most of the semantics of nonlinear terms (e.g., convex and concave envelopes of these particular terms). The extension of `Quad` for handling any polynomial constraint system requires to replace non-quadratic terms by new variables and to add the corresponding identities to the initial constraint system. However, a complete quadrification would generate a huge number of linear constraints. We have introduced a heuristics based on a good tradeoff between a tight approximation of the non linear terms and the size of the generated constraints system.

A safe rounding process is a key issue for the `Quad` framework. The simplex algorithm is used to narrow the domain of each variable with respect to the subset of the linear set of constraints generated by the relaxation process but most implementations of the simplex algorithm are numerically unsafe. Moreover, the coefficients of the generated linear constraints are computed with floating point numbers. So, two problems may occur in the `Quad`-filtering process:

1. the whole linearization may become incorrect due to rounding errors when computing the coefficients of the generated linear constraints ;

2. some solutions may be lost when computing the bounds of the domains of the variables with the simplex algorithm.

We propose a safe procedure for computing the coefficients of the generated linear constraints. The second problem has recently been addressed by Neumaier and Shcherbina [8] which have proposed a simple and cheap procedure to get a rigorous upper bound of the objective function. The incorporation of these procedures in the `Quad`-filtering process allows us to call the simplex algorithm without worrying about safety.

6.3. Distance constraints

Key words: *distance equations*, *numerical robustness*.

Systems of distance constraints occur frequently in various application fields such as mechanism theory, CAD, molecular biology. This motivates a large effort of the project to develop specific solvers, combining theoretical works on the analysis of these equations (including taking into account uncertainties in the constraints), effective implementation and extensive testing.

6.3.1. Improved exclusion scheme for distance constraints

Participant: Jean-Pierre Merlet.

An exclusion scheme allows one to calculate a box around a certified solution that is guaranteed to contain only this solution. Propagation of this box on the search space enables to reduce it, thereby improving the global efficiency of the solver.

For distance constraints we have already provided a specific version of the Kantorovitch scheme that produces a larger box than the general version of the theorem. We have then investigated the inflation approach of Neumaier [7] based on the H-matrix theory. In its general version this method determines incrementally the largest exclusion box and is relatively computer intensive. We have shown that for the specific case of distance constraints we may compute directly the width of the exclusion box without relying on an iterative scheme. This calculation has been implemented in the specific distance constraints solver of `ALIAS`.

6.3.2. Semantic decomposition for solving distance constraints

Participants: Heikel Batnini, Michel Rueher.

Most of the solver of numerical CSP are based on a relaxation of local consistencies. These techniques prune the domains of the variables by computing an external approximation of the solution space. However, the resulting domains may still contain a huge number of locally inconsistent values, especially when the solution space is composed of disjoint subspaces. Splitting techniques are often used to isolate solutions. However, splitting is ineffective in the presence of continuous subspaces of solutions. Moreover classical splitting methods do not take advantage of the properties of the constraints, especially and more particularly the properties of geometrical constraints.

We have introduced in [18][19] a domain decomposition, based on the semantic properties of distance constraints. This method splits and reduces the domains of two points w.r.t. their distance relation, using the monotonic and convex parts of the canonical form of the equations. We halve also proposed a new pruning technique, named LDF (Local Decomposition Filtering), propagates this decomposition over the whole set of constraints, following the standard approximation scheme. The disjoint subspaces of solutions are represented by a graph of sub-domains which has the same structural properties as the arc-consistency graph of a CSP on finite domains. Thus, classical search algorithms, such as MAC (Maintaining Arc Consistency) or FC (Forward Checking), may be used to isolate potential solution subspaces. We have demonstrated that LDF achieves a better pruning than 2B-consistency, and is more efficient than a combination of 2B-consistency and splitting, for small but interesting problems.

Further works concern the integration of this semantic decomposition in a general splitting algorithm for solving a larger class of problems, such as classical robotic benchmarks.

6.3.3. Uncertainty management

Participants: Carlos Grandon, Bertrand Neveu.

We have studied how to manage small uncertainties in the parameters of a system of distance equations. The problem has no more isolated solution points, but isolated sub-spaces of solutions and we cannot directly apply classical interval solvers. We have proposed the following method: we first compute the solutions of the system without uncertainties and then we use these solutions to determine a dynamic splitting policy of the domains. Filtering algorithms (for example 3B-consistency) are then applied to each subspace to obtain boxes containing the solutions of the system with uncertainties. We have studied the limitations of this method. It is complete (no region with solutions is lost) but the regions finally found do not always correspond to an extension around the solution points without uncertainties. If new regions with solutions appear when the uncertainties are taken into account, the method is not able to isolate them. When regions overlap, the splitting will separate two regions that will be each smaller than the extensions of solution points. Finally, we have started to extend our method to other types of equations.

6.4. Resolution of geometrical constraints systems by rigidity, consistency and interval analysis

Key words: *rigidity*, *geometric decomposition*, *systems decomposition*, *computer vision*.

6.4.1. Solving geometric constraints with recursive rigidification and interval analysis

Participants: Christophe Jermann, Bertrand Neveu, Gilles Trombettoni.

Geometric constraint satisfaction problems (GCSPs) are ubiquitous in applications like CAD, robotics or molecular biology [5][3][11]. They consist in searching positions, orientations and dimensions of geometric objects bound by geometric constraints. The goal of the thesis [12] was to find an efficient and complete solving method for GCSPs.

We have compared solving methods and decomposition techniques, and we have focused on Hoffmann's decomposition and interval solving methods. We define a general framework for the study of rigidity in GCSPs, a concept used in all the geometric decomposition methods [3].

Hoffmann's method has limits inherent to all the structural geometric approaches. We propose the degree of rigidity concept to overcome some of these limits together with a new decomposition and its combination with interval solving techniques [22][24][23].

6.4.2. Scene Modeling Based on Constraint System Decomposition Techniques

Participants: Gilles Trombettoni, Christophe Jermann.

This work is performed in collaboration with Marta Wilczkowiak from the MOVI project at INRIA Rhônes-Alpes. Marta Wilczkowiak's PhD thesis is supervised by Edmond Boyer and Peter Sturm.

We present a new approach to 3D scene modeling based on geometric constraints [37][38]. Contrary to the existing methods, we can quickly obtain 3D scene models that respect the given constraints *exactly*. Our system can handle a large variety of linear and non-linear constraints in a flexible way.

To deal with the constraints, we decided to exploit the properties of the `GPDOF` algorithm developed by Trombettoni [11]. The approach is based on a dictionary of so-called *r-methods*, relying on geometry theorems, which can solve a subset of geometric constraints in a very efficient way. `GPDOF` is used to find, in polynomial-time, a reduced parameterization of a scene, and to decompose the equation system induced by constraints into a sequence of r-methods. We have validated our approach in reconstructing 3D models of building from images, using linear and quadratic geometric constraints.

6.4.3. Solving decomposable systems by Inter Block Backtracking

Participants: Christophe Jermann, Bertrand Neveu, Gilles Trombettoni.

We have studied a technique, called inter-block backtracking (`IBB`), which improves the solving by interval techniques of decomposed systems of non-linear equations over the reals [23].

This approach, introduced in 1998 by Bliek, Neveu and Trombettoni [1], handles a system of equations previously decomposed into a set of (small) $k \times k$ sub-systems, called blocks. A solution is obtained by combining the solutions computed in the different blocks. The approach seems particularly suitable for improving interval solving techniques.

In this paper, we analyze into the details the different variants of `IBB` which differ in their backtracking and filtering strategies. We also introduce `IBB-GBJ`, a new variant based on Dechter's graph-based back-jumping.

An extensive comparison on a sample of 8 CSPs has allowed us to better understand the behavior of `IBB`. In particular, we clearly show that limiting the scope of the filtering to the current block is very fruitful. For all the tested instances, `IBB` efficiency is larger by several orders of magnitude as compared to global solving.

6.5. Driving language for constraints solving algorithms

Participants: Gilles Chabert, Gilles Trombettoni, David Daney, Yahia Lebbah, Jean-Pierre Merlet, Claude Michel, Bertrand Neveu, Yves Papegay, Michel Rueher.

Key words: *solving framework*, *algorithm specification*.

The subject of Gilles Chabert's internship was to design a driving language for constraint solving algorithms over the reals. We decided to first reduce the field of study to the classic box reduction methods introduced by the constraint programming community, which embraces hull-consistency, box-consistency and stronger properties (3B, Bound,...). We obtained a small language, simple enough to define those methods in a few lines of code. This language is fine-grained enough to build whatever new sequence of the underlying functions (as constraint evaluation, projection, or interval shaving).

We have also implemented a prototype for this language, in order to check its completeness, verify that the execution results match those obtained with existing solvers, and eventually test the benefits of particular programs corresponding to new kinds of reductions.

This work is the first step of a larger project of an open platform for constraint solving, in which Gilles Chabert's PHD work will take part.

6.6. Modal Intervals and Universally Quantified Constraints

Participant: Alexandre Goldsztejn.

Key words: *modal intervals*, *universally quantified constraints*.

Modal intervals and their arithmetic are a generalization of classical intervals which let one change the quantifiers usually appearing in classical interval arithmetic. Hence, they have a great potential applicability to universally quantified constraints. Within this latter class of problems, we begin to study the reliable projection of the solution set of a parametric system of equations:

$$f : E \times X \subset \mathbb{R}^p \times \mathbb{R}^n \to \mathbb{R}^n$$

Reliable projection can be expressed in the following quantified formulation: $B \in \mathbb{IR}^p$ is inside the projection of the solution set $f = 0$ iff,

$$\forall t \in B, \exists x \in X, f(p, x) = 0$$

We first proposed a classical method – not using modal intervals – to solve this problem relying on the parametric Miranda theorem [21] and we plan to build a solver using this method.

In a next step, we will use modal intervals in order to overcome the main restriction of the latter method, i.e. dealing with under-constrained parametric systems.

6.7. Resolution of constraints satisfaction problem in finite fields

Another research axis is systems solving in finite domains. Although our main effort focus on problems with continuous domain variables, finite domains may have some interest for many application fields.

6.7.1. Genetic Algorithms for graph coloring problems

Participants: Blaise Madeline, Bertrand Neveu.

Key words: *genetic algorithm*, *genetic operator*, *graph coloring*.

In his PhD thesis [13], Blaise Madeline has addressed the use of evolutionary algorithms (EA) to solve constraint satisfaction problems in finite domains, without any particular specialization nor hybridization. A comparison between tree search methods and meta-heuristics on over-constrained graph coloring, in a context of minimal tuning of the parameters was proposed. The search landscape was studied for understanding why the various methods have so different efficiency. Finally, a new genetic operators (crossover, mutation, diversification) with a parameter setting simpler than the classical operators was introduced.

6.8. Symbolic tools for modeling and simulation

Participant: Yves Papegay.

Key words: *modeling*, *simulation*, *symbolic computation*, *code generation*, *accuracy*, *reliability*.

Industrial modeling and simulation processes are usually based on scientific theories, using formula for describing physical features and computation algorithms. Based on these formula, numerical methods are developed and implemented for simulation and visualization of these features. Due to the large number of parameters and equations involved in industrial models and to the diversity of physical contexts, it is a huge work to produce and test such numerical codes.

In a joint activity with Airbus aerodynamics department, we are specifying, designing and implementing tools for automatic generation of numerical simulators from symbolic formulas.

The first step in this framework was to solve several edition, communication and documentation problems. We did it in a prototype developed above the Mathematica software. We then formalized a theoretical approach of variables, parameters, and models based on graphs and hyper-graphs theory.

In 2003 we applied CSP decomposition methods for producing evaluation of model in the non-oriented case. We also developed automatic generation procedures of numerical codes (C Language). This allows to use the same model for studying both design and performances analysis problem.

6.9. Web pages

Participant: Jean-Pierre Merlet.

The COPRIN web pages offer now some practical examples of the use of interval analysis, some explanations on the basic methods used for constraints solving and an important set of systems solving examples (among them the well known difficult Katsura problem which is solved, probably for the first time, for n=20).

7. Contracts and Grants with Industry

7.1. Airbus France

Participant: Yves Papegay.

To improve the production of numerical (flight) simulators from models of aerodynamics, Airbus France is interested in tools like those described in 6.8.

In 2003, a new contract has been set up for prototyping some code generation features. For confidentiality reasons, no further details can be given here.

7.2. Alcatel Space Industries

Participants: Jean-Pierre Merlet, Fang Hao, David Daney, Yuan Cheng.

We have started at the end of 2002 a collaboration with Alcatel for the optimal design of a space-based observation telescope. The objectives of this study are to develop an instrument with a low inertia (to reduce the energy necessary for orienting the instrument and to allow fast orientation changes) while being deployable and to allow an active accurate control of the secondary mirror location.

In 2002/2003 we have proposed an innovative structure and we have completed a first optimal design study for a prototype. A small scale prototype is currently being developed in collaboration with the Technical University of Braunschweig and COPRIN is acting as sub-contractor of Alcatel for an ESA contract.

This prototype will be used by Alcatel to test the deployment phase and the accuracy. In parallel the measurement data will be used to develop a calibration method for the full-scale instrument and the optimal design methodology will be improved by taking into account the influence of the shading of the structure on the instrument performances.

7.3. Amadeus

Participants: Bertrand Neveu, Gilles Trombettoni, Michel Rueher.

Amadeus is a company that manages flight fares for several airlines and we have started a collaboration with this company in 1998 to develop new optimization algorithms based on constraint programming and graph methods for fare quote problems. We are also heavily involved in the development of the test suite that is used for evaluation by the developers of Amadeus.

Several experimental prototypes have been developed and many successful ideas have been embedded in other software which are sold by Amadeus to travel agencies

7.4. Constructions mécaniques des Vosges (CMW)

Participants: David Daney, Jean-Pierre Merlet.

Together with the SPACES project of INRIA-Lorraine we have a long-time collaboration with this company for the development of a fast-speed milling machine. Numerous software have been developed during this collaboration [14] and are ready for a transfer. A development plan is currently being investigated by the Lorraine Region, ANVAR and ENSAM and we are currently actively participating to its elaboration.

7.5. Institut Laue Langevin

Participant: Jean-Pierre Merlet.

In 2001/2002 we have collaborated with this Institute for the development of fine positioning device that will be used for a strain imager. A neutron stream issued from a nuclear reactor is used to determine the stress level in mechanical parts. But this level can only be determined for a small area: hence it is necessary to move the part so that the neutronic sensor measurement will cover the whole part. Accuracy, high load capabilities and stiffness are the constraints that have to be satisfied. An optimal design study of a parallel robot has been performed in 2002. But the project has been delayed and the final engineering has been sub-contracted. ILL has however decided that the COPRIN project should evaluate the final design. A full performance analysis has been performed and has exhibited some flaws in the proposed design.

8. Other Grants and Activities

8.1. National initiatives

8.1.1. Project ROBEA "MAX"

Participant: Jean-Pierre Merlet.

This project, that has been funded by the CNRS and has been completed on September 2003, has as objective to design tools for the design of complex mechanical systems. The partners are:

- LIRMM (Montpelier)

- LASMEA, IFMA (Clermont-Ferrand)
- IRCCYN (Nantes)

Our contribution is the use of interval analysis based methods for the determination of robot performances and for systems solving that arise when dealing with the design, control and calibration of such systems. A follow-up of this project (ROBEA MP2) will be funded by CNRS.

8.1.2. Project MathStic "Robot cuspidaux"

Participant: Jean-Pierre Merlet.

The purpose of this project is a detailed study of the cuspidal robots i.e. robots that, while keeping the end-effector pose, can change their joints configuration without crossing a singularity. The partners are:

- IMAR (Rennes)
- LIP6, SPACES (Paris)
- IRCCYN (Nantes)

Our contribution is the determination of all possible design parameters of a specific robot mechanical architecture that leads to cuspidal robots using a complete analysis that has been performed by LIP6, SPACES and IMAR.

8.1.3. ACI V3F: Validation and checking of floating point number computations

Participants: Claude Michel, Michel Rueher, Yahia Lebbah, David Daney.

Key words: *floating point number arithmetic*, *checking*, *validation*, *constraint programming*.

The use of floating point numbers to represent real numbers is the root of an important quantity of failures and potential faults in software for critical systems. The modeling of such systems, combined with model checking techniques, proof and test case generation techniques, enhances the quality of the development process and improves the reliability of systems which integrates pieces of software [2]. Unfortunately, the currently available approaches, notations and techniques do not really take into account floating point numbers although the usual way to do computation over the reals with a computer is to use floating point numbers. The main difficulty to get a correct account of floating point numbers comes from:

- the poor properties of floating point number arithmetic,
- the dependency of floating point number properties to the computer architecture (even if the floating point unit is IEEE 754 compliant).

The aim of the V3F ACI project is to provide tools required to evaluate the representation of reals by means of floating point numbers during the software validation and checking phases. More precisely, our aim is to develop a framework relying on CSP approaches for the validation of program computations with hypothesis coming from the modeling phase. Constraint methods have been successfully used in many applications related to software validation and checking. They already have shown their capabilities in automatic test case generation, in model checking as well as in code analysis. However, CSP techniques are then restricted to integer, rational and real numbers. Thus, the challenge is to provide the solving techniques to handle floating point numbers. We will develop solving techniques adapted to floating point numbers to validate and check critical software. We will also study the use of such a solver in the processes of model checking, of automatic test case generation and of static code checking.

V3F ACI project is a joint research project with:

- LIFC, Laboratoire d'Informatique de l'Université de Franche–Comté (CNRS - INRIA),
- IRISA, Institut de Recherche en Informatique et Systèmes Aléatoires, Rennes,
- CEA, Commissariat à l'Energie Atomique, Saclay, Paris.

9. Dissemination

9.1. Leadership within scientific community

- Y. Lebbah was member of the program committee of “Journées Nationales sur la résolution pratique des problèmes NP-complets” (JNPC conference).
- J-P. Merlet hold the position of CISC (Advisor for Science Information and Communication) of INRIA Sophia, is member of the INRIA Evaluation Board, is a member of the CERT/ONERA Scientific Review Board for the SACSO project, is suppleant member of the "commission de spécialistes" (61th section) of Nice University, is chairman of the IFToMM (International Federation on the Theory of Machines and Mechanisms) Technical Committee on "Computational Kinematics" and has been nominated this year Chairman of the French section of IFToMM. He is an Associate Editor of IEEE Transactions on Robotics and has been member of the Program Committee of the IEEE Int. Conf. on Robotics and Automation conference, of the Scientific Committee of "Congrès français de Mécanique", while being session Chairs for these conferences and of the Program Committee of the RAAD Conference. He has been reviewer for IEEE Transactions on Robotics and Automation ASME J. of Mechanical Design, J. of Intelligent and Robotic Systems, Mechanism and Machine Theory, Robotics and Autonomous System, Robotica, European J. of Mechanics, Robotica, Int. J. for Numerical Methods in Engineering, Journal Européen des Systèmes Automatisés (JESA). He is a member of an informal Advisory Committee involving academics and industrial partners that is promoting nanobiotechnology at Sophia-Antipolis
- B. Neveu was member of program committee of “Journées Nationales sur la résolution pratique des problèmes NP-complets” (JNPC conference) and reviewer for IJCAI 2003 and CP 2003.
- M. Rueher hold the position of chairman of the Project committee of I3S and is a member of the Specif Committee which awards the best French PhD in computer science.
- M. Rueher and J-P. Merlet are members of the `Ensemble` working group that promotes the use of interval analysis in the field of Control Theory

9.2. Conference and workshop committees, invited conferences

9.2.1. National Conferences

- H. Batnini has presented a paper to EJC03 (GdR ALP) and to Student School on Algorithmic Languages and Programming, Marne la Vallée, France, March 31-April 3, to JNPC-JFPLC 2003: “Journées Nationales sur la résolution de problèmes NP-complets”, to the “Journées Francophones de Programmation en logique avec contraintes”, Amiens, France, June 17-19
- G. Chabert has participated to JNPC-JFPLC 2003 (“Journées Nationales sur la résolution de problèmes NP-complets”) and to the “Journées Francophones de Programmation en logique avec contraintes”, Amiens, France, June 17-19. He has presented his work at one of the meeting of the `Ensemble` working group.
- D. Daney has participated to the “Journées Nationales de Calcul Formel”, Luminy, France, January 20-24 and to the “Journées ROBÉA/MAX”, Sophia Antipolis, France, April 7.
- Y. Lebbah has participated to JNPC-JFPLC 2003: “Journées Nationales sur la résolution de problèmes NP-complets”, to the “Journées Francophones de Programmation en logique avec contraintes”, Amiens, France, June 17-19
- J-P. Merlet has presented a paper in the CNRS Algorithmic School of Dijon [27], in the SMF Symbolic Computation School of Marseille [28] and has attended the “Journées Nationales de la Robotique”

- B. Neveu has participated to ROADEF 2003: 5th congress of “La Sociéte Française de Recherche Opérationnelle et d’Aide à la décision”, Avignon, France, February 26-28, to JNPC-JFPLC 2003: “Journées nationales sur la résolution de problèmes NP-complets”, “Journées Francophones de Programmation en logique avec contraintes”, Amiens, France, June 17-19, to National working group (“Action Spécifique du CNRS”) on Geometric constraints, meeting in Dijon, June 25-26
- M. Rueher has participated to National working group (“Action Spécifique du CNRS”) on Geometric constraints, meeting in Dijon, June 25-26

9.2.2. International Conferences

- H. Batnini has presented a paper to CP 2003: 9th International Conference on Principles and Practice of Constraint Programming, Kinsale, Ireland, September 29-October 3.
- Y. Lebbah has participated to the 2nd International Workshop on Global Constrained Optimization and Constraint Satisfaction, Lausanne, Switzerland, November 18-21.
- J-P. Merlet has presented a keynote speech during the Coco’03 conference [30], a paper at the IEEE ICRA Conference [29] and has participated in the Workshop held at Stanford University in honor of Pr. B. Roth.
- C. Michel has participated to CPAIOR 2003 : International workshop on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, Montreal, Canada, May 8-10.
- B. Neveu has participated to Optimization days, Montreal, Canada, May 7, to CPAIOR 2003 : International workshop on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, Montreal, Canada, May 8-10, to IJCAI 2003: International Joint Conference on Artificial Intelligence, Acapulco, Mexico, August 12-15 and to CP 2003: 9th International Conference on principles and practice of Constraint Programming, Kinsale, Ireland, September 29-October 3.
- Y. Papegay attended the 5th International Mathematica Symposium in London, UK, July 7-11 and has presented the work based on the parallel implementation of `ALIAS` at the EuroPVM/MPI 2003 conference in Venice, Italy, September 29-October 2 [36].
- M. Rueher has participated to CPAIOR 2003 : International workshop on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, Montreal, Canada, May 8-10 and to CP 2003: 9th International Conference on Principles and Practice of Constraint Programming, Kinsale, Ireland, September 29-October 3.
- G. Trombettoni has presented a paper to CPAIOR 2003 : International workshop on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, Montreal, Canada, May 8-10, has participated to Optimization days , Montreal, Canada, May 7, and to IJCAI 2003: International Joint Conference on Artificial Intelligence, Acapulco, Mexico, August 12-15.

9.3. Teaching

- J-P Merlet has given robotics teaching at ISIA (10 hours)
- C. Michel took part in the teaching of constraints to Master students (8 h).
- B. Neveu has participed to the IA course at ENTPE in Lyon (6 h).
- M. Rueher, B. Neveu and G. Trombettoni have given lectures on constraint programming in the Computer Science DEA at UNSA and at ESSI (30 h).

- M. Rueher has given teaching in "Database systems", 2rd year of the ESSI engineer school in computer science, "Introduction to logic programming ", 2nd year of the ESSI engineer school in computer science and "Constraint programming", 3rd year of the ESSI engineer school in computer science and DEA of computer sciences.
- G. Trombettoni has given several courses in computer science at the IUT GTR (Telecommunications and networks) of Sophia-Antipolis.

9.3.1. PhD thesis

- J-P. Merlet has been jury member of 10 PhD (3 as Chairman of the jury, one in Austria)
- B. Neveu has been member of jury of 2 PhD defense committees.
- M. Rueher has been member of jury of 2 PhD defense committees.
- J-P. Merlet and D. Daney have acted as advisors for the PhD of O. Kilit within the framework of a collaboration with IAS

9.3.2. Responsability for teaching

- M. Rueher is head of DESS ISI (Master in computer science) and ESSI3, the 3rd year of the ESSI engineer school in computer science.
- G. Trombettoni was in charge of the courses in computer science at the IUT. He also leads a group of students at the IUT, called "année spéciale", who can obtain the diploma in one year only.

9.4. PhD thesis

Current PhD thesis:

1. H. Batnini, *Contraintes globales sur le continu*, University of Nice-Sophia Antipolis.
2. G. Chabert, *Langage de pilotage et de paramétrage d'algorithmes de résolution de contraintes par intervalles*, University of Nice-Sophia Antipolis.
3. L. Rolland, *Algorithmes algébriques pour la commande de robots parallèles de haute précision*, University of Nancy, to be defended on December 10, 2003.

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