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Project-Team corida

Contrôle Robuste Infini-Dimensionnel et Applications

Lorraine

THEME 4A

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1. Team

CORIDA of INRIA of the CNRS and of the University Henri Poincaré, via the Elie Cartan Institute of Nancy[1] *(UMR 7502 CNRS-INRIA-UHP).*

Head of project-team
Marius Tucsnak [Professor at University Henri Poincaré Nancy 1]

Administrative Assistant
Hélène Zganic

Staff members INRIA
Karim Ramdani [Junior research scientist]
Takéo Takahashi [Junior research scientist]

Staff members University Henri Poincaré
Francis Conrad [Professor]
Frédéric Magoulès [Assistant Professor at ESSTIN[2]]
Bruno Pinçon [Assistant Professor at ESIAL[3]]
Lionel Rosier [Professor at ESSTIN]
Jean-François Scheid [Assistant Professor at ESIAL]

Staff member Institut Nantional Polytechnique de Lorraine
Antoine Henrot [Professor]

Staff members University of Metz
Fatiha Alabau [Professor]
Jean-François Couchouron [Assistant Professor]
Jean-Pierre Croisille [Professor]

Ph.D. students
Pierre Le Gall [BDI CNRS-Région]
Patricio Cumsille [CONICYT-INRIA grant]
Stephan Duprey [CIFRE grant with EADS]

2. Overall Objectives

CORIDA is a team labelled by INRIA, by CNRS and by University Henri Poincaré, via the Institut Elie Cartan of Nancy (UMR 7502 CNRS-INRIA-UHP). The main focus of our research is the robust control of systems governed by partial differential equations (called PDE's in the sequel). A special attention is devoted to systems with hybrid dynamics such as the fluid-structure interactions or multi-scale systems. The equations modeling these systems couple either partial differential equations of different types or finite dimensional systems and infinite dimensional systems. We mainly consider inputs acting on the boundary or which are localized in a subset of the domain.

Infinite dimensional systems theory is motivated by the fact that a large number of mathematical models in applied sciences are given by evolution partial differential equations. Typical examples are the transport, heat or wave equations, which are used as mathematical models in a large number of problems in physics, chemistry, biology or finance. In all these cases the corresponding state space is infinite dimensional. The understanding of these systems from the point of view of control theory is an important scientific issue which received a considerable attention during the last decades. Let us mention here that a basic question like the study of the controllability of infinite dimensional linear systems requires sophisticated techniques such as non harmonic analysis (cf. Russsell [44]), multiplier methods (cf. Lions [38]) or micro-local analysis techniques

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[1] *IECN*
[2] Ecole Supérieure des Sciences et Technologies de l'Ingénieur de Nancy
[3] Ecole Supérieure d'Informatique et Applications de Lorraine

(cf. Bardos-Lebeau-Rauch [32]). Like in the case of finite dimensional systems, the study of controllability should be only a starting point of the study of important and more practical issues like feedback optimal control or robust control. It turns out that most of these questions are open in the case of infinite dimensional systems. Consequently, our aim is to develop tools for the robust control of infinite dimensional systems. More precisely, given an infinite dimensional system one should be able to answer two basic questions:

1. The existence of a feedback operator with robustness properties;
2. Find an algorithm allowing the approximate computation of this feedback operator

The answer to question 1 above requires the study of infinite dimensional Riccati operators and it is a difficult theoretical question. The answer to question 2 depends on the sense of the word "approximate". In our meaning "approximate" means "convergence", i.e., that we look for approximate feedback operators converging to the exact one when the discretization step tends to zero. From a practical point of view this means that our control laws have to give good results if we use a large number of state variables. This fact is no longer a practical limitation of such an approach, at least in some important applications where strong computers are now available. We intend to develop a methodology applicable to a large class of applications. Let us mention here only two of them, which received a considerable attention during the last year.

1. **Acoustics and aero-acoustics.** We consider two types of applications :

   - Noise reduction by using active control (in a bounded region like the cockpit of a plane) or by using absorbing materials (in open regions around highways, airports or railway stations).
   - Times reversal techniques for acoustic focusing in medical imaging, non destructive testing or sub-marine communication.

2. **The control of VLT's (Very Large Telescopes).** The operation of the current telescopes is based on the reception of infra-red waves. The reception is inevitably disturbed by the atmosphere, from where a correction of the wavefront is needed. Currently this correction is carried out by a mirror, whose diameter is approximately 20 centimeters, provided by a thousand of piezoelectric actuators. The future telescopes will be characterized by diameters much larger and the fact that the spectrum of the analyzed wavefront lies in the visible field. It is estimated that to correct the image with the same quality, the density of the actuators will have to be a hundred times higher and that it will be necessary to replace the piezoelectric actuators by actuators resulting from micro-technology. It is thus a question of developing tools to model and to control the mirrors, allowing this change of scale.

3. Scientific Foundations

3.1. Analysis and control of fluids and of fluid-structure interactions

Key words: *Navier-Stokes equations*, *Korteweg de Vries equations*.

Participants: Patricio Cumsille, Lionel Rosier, Jean-François Scheid, Takéo Takahashi, Marius Tucsnak.

Glossary

Analysis and control of fluids and of fluid-structure interactions Analysis and control of systems modelling the motion of several solids (rigid or elastic) in a fluid (perfect or viscous).

The problems we consider are modeled by the Navier-Stokes, Euler or Korteweg de Vries equations (for the fluid) coupled to the equations of the solids. One of the main difficulties of this problem comes from the fact that the domain occupied by the fluid is one of the unknowns of the problem. We have thus to tackle a *free boundary problem*.

The control of fluid flows is a major challenge in many applications: aeronautics, pollution issues, regulation of irrigation channels or of the flow in pipelines, $\cdots$. All these problems cannot be easily reduced to finite dimensional models so a methodolology of analysis and control based on PDE's is an essential issue. In a first approximation the motion of fluid and of the solids is decoupled. The most commonly used models for an incompressible fluid are given by the Navier-Stokes or by the Euler's equations.

The optimal open loop control approach of these models has been developed from both the theoretical and numerical points of view. Controllability issues for the equations modeling the fluid motion are by now well understood (see, for instance, [35] and the references therein). The feedback control of fluid motions has also been recently investigated by several research teams (see, for instance [31] and references therein) but this field still contains an important number of open problems (in particular those concerning observers and implementation issues). One of our aims is to develop efficient tools for computing feedback laws for the control of fluid systems.

In real applications the fluid is often surrounded by or surrounds an elastic structure. In the above situation one has to study fluid-structure interactions. This subject has been intensively studied during the last years, in particular for its applications in noise reduction problems, in lubrication issues or in aeronautics. In this type of problems, a PDE's system modelling the fluid in a cavity (Laplace equation, wave equation, Stokes, Navier-Stokes or Euler systems) is coupled to the equations modelling the motion of a part of the boundary. The difficulties of this problem are due to several reasons such as the strong nonlinear coupling and the existence of a free boundary. This partially explains the fact that applied mathematicians have only recently tackled these problems from either the numerical or theoretical point of view. One of the main results obtained in our project concerns the global existence of solutions in the case of a two-dimensional Navier-Stokes fluid (see [6]). On the other hand, it seems that the corresponding problem for a perfect fluid (modelled by the Euler equation) has not yet been investigated.

The numerical methods used for computing the solutions of fluid or fluid structure problems in a direct setting (i.e., with given inputs) considerably progressed during the last years. For the corresponding control problems the literature contains only a small number of effective methods. Our first results in this direction concern a model arising in hydraulics (the linearized Saint-Venant equations).

Another topic of great interest is the control of the interface of two fluids (typically water and air) by using as input the velocity of a moving wall which is a part of the boundary. One of the most popular models for this problem is given by the shallow water equations (Saint Venant equations) which neglect the dispersive effects. The controllability of several important systems governed by this type of equations has received a considerable attention during the last decade. Let us mention here the important work by Coron [33]. If dispersive effects are considered the relevant model is given by the Korteweg de Vries equation. The first work on the control of this equation goes back to Russell and Zhang (see [45]). An important advance in the study of this problem has been achieved in the work [7] where, for the first time, the influence of the length of the channel has been precisely investigated.

3.2. Well–posed linear systems and weak coupling

Participants: Fatiha Alabau, Francis Conrad, Jean-François Couchouron, Marius Tucsnak.

Key words: *linear evolution equations*, *stabilization*, *coupling mechanism*, *boundary control*.

Glossary

Well–posed linear systems and weak coupling Well–posed linear systems and weak coupling

We consider well posed systems coupling two types of PDE's or coupling PDE's and ordinary differential equations. The methods we use combine energy estimates, multiplier techniques and spectral analysis.

Well–posed linear systems form an important class of infinite dimensional systems which has been introduced by Salamon in [47]. Roughly speaking a *well–posed linear system* is a linear time-invariant system such that on any finite time interval, the operator mapping the initial state and the input function to the final state and the output function is bounded. An important subclass of well–posed linear systems is formed by the *conservative systems* which satisfy an energy-balance equation. More precisely, in a conservative system, the energy stored in the system at a given time plus the outgoing power equals the sum of the initial energy stored in the system and of the incoming power. It turns out that a large number of systems governed by partial differential equations are of this type. Moreover, conservative systems have remarkable properties like the fact that their exact controllability is equivalent to their stability. Therefore a systematic functional analytic approach to this system seems important for the infinite dimensional systems community.

We are in particular interested in problems in which two types of PDE's interact such as: a plate equation and a wave equation, a wave or plate type equation coupled to ordinary differential equations, or two wave equations coupled by lower order terms. This type of system is sometimes designed as a "hybrid system" (notice that this term is often used in an another framework in control theory). The main difficulty of these problems is that the inputs act on only one of the equations of the system. The basic question is to know if such a system is stabilizable. A general framework for this type of problem has been given in Alabau [2] and [1] where the use of multipliers method yields promising results. A different way to tackle the same problem is to study first the simultaneous controllability of the uncoupled systems. The case in which one of the systems is finite dimensional has been tackled in Tucsnak and Weiss [8].

3.3. Optimal location of sensors and of actuators

Key words: *decay rate*, *robustness*.

Participant: Antoine Henrot.

We focus here on algorithms for optimizing the location and the shape of actuators and of sensors for the stabilization of systems governed by PDE's. Consider a control problem for a system governed by PDE's with the input acting on the interior of the domain or on a part of the boundary. An important question is to find the location and the form of the control region in order to optimize a criterion imposed by the user. This criterion should take in consideration the energy decay rate and the robustness properties. A priori the topology of the control region is unknown, and thus the first step in such a study should be the application of topological optimization techniques. An important particular case occurs when the actuators and sensors contain smart materials. Generally, the optimal location problems are far from the classical convex optimization problems and they don't have a unique global optimum. To our knowledge, the only problem where the explicit solution is known has been studied in Ammari, Henrot and Tucsnak [3]. Finding numerical methods to approach the optimum location in more general situations is a hard task.

3.4. Frequency domain methods for the analysis and control of systems governed by pde's

Key words: *Helmholtz equation*, *time-reversal*, *control and stabilization*, *numerical approximation of LQR problems*.

Participants: Frédéric Magoulès, Karim Ramdani, Takéo Takahashi, Marius Tucsnak.

We use frequency tools to analyze two different types of problems. The first one concerns the optimal control, the stabilization and the approximation of systems governed by PDE's. The second one concerns time-reversal phenomenon.

A frequency approach can be used to analyze problems arising in two different areas.

3.4.1. Optimal control, stabilization and their numerical approximation

The first area concerns the numerical approximation of the feedback operators arising in optimal control problems for well posed systems. This question is related to the uniform (with respect to discretization parameter) exponential stability of the discretized problems.

Our approach is based on an original characterization of exponentially stable systems recently proposed by K. Liu [39]. The originality of this characterization lies in the fact that it provides, for time-dependent problems, exponential stability criteria **that do not depend on time, but depend on the frequency variable** conjugated to it. Studying the exponential stabilizability of a given system amounts then to establishing uniform estimates (with respect to the frequency). In other words, the problem of stabilizability for the wave equation, for instance, comes down to a high-frequency analysis for Helmholtz operator. Let us emphasize here one further important advantage of this frequency characterization. It lies in the fact that this tool can also be used for the analysis of discretized problems. The estimates to be proved must then be uniform with respect to **both the frequency and the mesh size**. For these applications it is essential to develop numerical methods suitable for spectral calculations on operators obtained by solving Helmholtz type equations in unbounded domains.

3.4.2. Time-reversal

The second area in which we make use of the same frequency tools is the analysis of time-reversal. This phenomenon is a direct consequence of the reversibility of the wave equation in a non dissipative medium. It can be used to **focus an acoustic wave** on a desired target through a complex and/or unknown medium. The procedure followed to achieve this is quite simple. First, **time-reversal mirrors** are used to generate an incident wave that propagates through the medium. Then, the same mirrors measure the acoustic field diffracted by the targets, time-reverse it and back-propagate it in the medium. Iterating the scheme, we observe that the incident wave emitted by the mirrors focuses on the scatterers. Obviously, time-reversal has many applications covering a wide range of fields, among which we can cite **medicine** (kidney stones destruction or medical imaging), **sub-marine communication** and **non destructive testing**. When time-harmonic waves are used, the analysis of time-reversal involves Helmholtz operator.

3.5. Implementation

Key words: *Dicretization*, *Riccati equation*.

Participants: Frédéric Magoulès, Bruno Pinçon.

This is a transverse research axis since all the research directions presented above have to be validated by giving control algorithms which are have to be implemented in real control systems. We stress below some of the main points which are common (from the implementation point of view) to the application of the different methods described in the previous sections. For many infinite dimensional systems the use of co-located actuators and sensors and of simple proportional feedback laws gives satisfying results. However, for a large class of systems of interest it is not clear that these feedbacks are efficient, or the use of co-located actuators and sensors is not possible. This is why a more general approach for the design of the feedbacks has to be considered. Among the techniques in finite dimensional systems theory, those based on the resolution of an infinite dimensional Riccati equation seem to be the most appropriate for a generalization to infinite dimensional systems. The classical approach is to approximate an LQR problem for a given infinite dimensional system by finite dimensional LQR problems. As it has been already pointed out in the literature this approach should be carefully analyzed since, even for some very simple examples, the sequence of feedback operators solving the finite dimensional LQR is not convergent. Roughly speaking, this means that when refining the mesh we obtain a closed loop system which is not exponentially stable (even if the corresponding infinite dimensional system is theoretically stabilized). In order to overcome this difficulty, several methods have been proposed in the literature : filtering of high frequencies, multigrid methods or the introduction of a numerical viscosity term. We intend to apply first the numerical viscosity method introduced in [48], for optimal and robust control problems.

4. Application Domains

4.1. Panorama

Key words: *acoustics*, *aero-acoustics*, *control of VLT's (Very Large Telescopes)*.

As we already stressed in the previous chapters the robust control of infinite dimensional systems is an emerging theory. Our aim is to develop tools applicable to a large class of problems which will be tested on models of increasing complexity. We describe below only the applications in which the members of our team have obtained important achievements in 2003.

4.2. Acoustics

Key words: *Noise reduction*, *Helmholtz equation*, *time-reversal*, *domain decomposition*, *parallel computing*.

We can distinguish two types of problems:

1. **Noise reduction**;
2. **Domain decomposition**

4.2.1. Noise reduction

Stefan Duprey started his thesis in the Research Center of EADS last January with a CIFRE contract. Antoine Henrot is his advisor, and his work is also supervised in Nancy by Frédéric Magoulès, Karim Ramdani and Marius Tucsnak. In EADS, at Suresnes, his work is supervised by Isabelle Terrasse and François Dubois.

This work is concerned with the reduction of noise due to the plane's engines during take-off. It could be decomposed into several steps:

1. Write a code to compute the flow. Starting from the Euler equations in the potential and isentropic case, we are lead to solve a well-known non-linear elliptic problem, studied for example, in classical books like those due to Glowinski or Nečas. To solve numerically this non linear problem, we use a fixed-point algorithm which turns out to be convergent.
2. Assume the acoustic perturbation to be potential and decoupled from the flow. By linearization of Euler equations, we get a linear problem satisfied by the acoustic potential. The coefficients of this equation involve the potential flow computed in step 1. The boundary conditions are either of Neumann or impedance type.
3. We have to write a code to compute the solution of step 2. The fact that the flow can be considered constant at infinity simplifies the equation outside a large domain containing the plane. This leads to two possible ways to solve this problem: using globally a Lorentz transform or using a domain decomposition method.
4. When the two previous codes work satisfactory, we can imagine optimization procedures by acting either on the shape of the engine or on its coating.

During this first year, Stefan Duprey has completely finished point 1 (including a theoretical study of the convergence of the algorithm). Point 2 is now well understood. It remains nevertheless to finish the theoretical study (existence and uniqueness). He has just begun these last months to write the code described in step 3.

4.2.2. Domain decomposition

The limitation of the noise level generated by airplanes or trains is of major interest during the architecture and construction process of new airports and/or railway stations. The analysis of different configurations of the buildings or the uses of new architectural materials can be performed very quickly through numerical simulation. In order to obtain accurate numerical results, realistic mathematical models involving Helmholtz equation are needed. The numerical resolution of such problems requires then large computer memory. The use of parallel computers or PC networks has become very helpful for such purposes. Our aim is to develop new mathematical domain decomposition methods suitable for the fast and accurate simulation of such problems. These methods are based on two steps. First, the global domain is splitted into several sub-domains and some interface boundary conditions are introduced on the interfaces between the sub-domains. Secondly, a sub-structuring formulation of the global problem leads to a condensed interface problem which is solved iteratively. Each iteration involves the solution of an acoustic problem in each sub-domain.

Interface boundary conditions are the key ingredient to design efficient domain decomposition methods. Without a global preconditioner, convergence cannot be obtained for any method in a number of iterations less than the number of sub-domains minus one (in the case of a one-way splitting). This optimal convergence can be obtained with generalized Robin type boundary conditions associated with an operator equal to the Steklov-Poincaré operator in the continuous case and to the Schur complement matrix in the discrete case. However , this optimal condition cannot be used in practice since its excat computation is too expensive.

Our goal is to define new approximations of the Steklov-Poincaré operator and of the Schur complement matrix.

4.3. Control of VLT's (Very Large Telescopes)

Key words: *adaptive optics*, *wavefront*, *turbulence*, *robust control*.

The objective of this work is to use tools of infinite-dimensional system automatics for the control of large telescopes. The future telescopes will be characterized by diameters much larger and the fact that the spectrum of the analyzed wavefront lies in the visible field. It is estimated that to correct the image with the same quality, the density of the actuators will have to be multiplied by one hundred and that it will be necessary to replace the piezoelectric actuators by actuators resulting from micro-technology. In theory, a telescope provided with actuators and sensors can be modeled like a finite-dimensional system. When the number of sensors and actuators becomes very large it is often difficult to use this type of modeling to control the telescope.

Our prime objective is to obtain, by techniques of asymptotic analysis, models based on systems of partial differential equations, with distributed control. According to the structure of the system obtained, we suggest to apply modern techniques resulting from the theory of the control of **infinite**-dimensional systems. The input and the output of the system will remain of finite dimensions, which will allow the direct application of the results to the initial system. The obtained systems will couple equations modeling the structure and the equations modeling the sensors and the actuators (for example equations of electrostatics). One of the difficulties of the problem lies in the fact that control occurs only in one of the equations of the system. A specific attention will be paid to the problems of optimal positioning of the control fields. It is the question of finding the localization and the form to be given to the actuators and the sensors so that the control is as effective as possible.

In a first approximation, which is valid for infra-red waves, we use the geometrical optics to study the system. In this case, by linearizing the equations, we have justified some of the approximations used in engineers literature. Currently, we work directly on the nonlinear equations obtained with the geometrical optics approach, and we look for an approximation valid when the spectrum of the analyzed wavefront lies in the visible field.

5. Software

5.1. SCISPT Scilab toolbox

Participant: Bruno Pinçon [correspondant].

Key words: *Scilab*, *sparse matrices*.

SCISPT is a Scilab toolbox which interfaces the sparse solvers umfpack v4.0 of Tim Davis and taucs snmf of Sivan Toledo.

Our aim is to develop Scilab tools for the control of PDE's. This task requires powerful sparse matrix primitives, which are not currently available in Scilab. We have thus developed the SCISPT Scilab toolbox, which interfaces the sparse solvers umfpack v4.0 of Tim Davis and taucs snmf of Sivan Toledo. It also provides various utilities to deal with sparse matrices (estimate of the condition number, sparse pattern visualization, ...). We intend to extend this work in the framework of collaborations with the Scilab consortium recently created.

5.2. Parallel Computational Acoustic Library

Participant: Frédéric Magoulès.

The Parallel Computational Acoustics Library is a finite element library able to solve huge acoustics problems in parallel.

This library contains three main types of functions - those for pre-processing (mesh data), those for processing (involving numerical and matrix analysis), and those for post-processing (visualization, noise rendering). This work was motivated by the need to integrate the various finite elements, mesh generation, mesh partitioning, domain decomposition methods and parallel solvers, and plotting programs developed by the group.

The Parallel Computational Acoustics Library contains the most recent and powerful developments in finite element methods for acoustics and parallel iterative domain decomposition methods. There are two groups of algorithms in the library: the first one is based on well established methods which are generally used in the industry, while the second one uses results recently obtained. This helps the library to be used at the same time by industrial partners (ONERA, Hutchinson S.A.) and academic researchers. The library is therefore able to solve huge acoustic problems that were not possible to solve so far. The Parallel Computational Acoustics Library is written is FORTRAN 90 and uses the MPI library for parallel data exchange.

Interactive visualization tools using the VTK library have been developed. An additional noise rendering interface is available in order to listen the results issued from the simulation.

6. New Results

6.1. Analysis and control of fluids and of fluid-structure interactions

Key words: *Navier-Stokes equations*, *Korteweg de Vries equations*.

Participants: Patricio Cumsille, Lionel Rosier, Jean-François Scheid, Takéo Takahashi, Marius Tucsnak.

Our research work in 2003 represents the last step of our program concerning welposedness for the PDE's system modeling the motion of rigid bodies in an incompressible fluid. In [22] we give an existence and uniqueness result in the case of a viscous fluid filling the exterior of an infinite cylinder. The generalization of this result to more general geometries is studied in the thesis of Patricio Cumsille. The case of a fluid-rigid body system filling a bounded domain is studied in [21]. The main result asserts global existence of strong solutions (up to contacts) in the 2D case and local existence in the 3D case.

As it is well-known, the motion of a perfect incompressible fluid is described by the Euler equations. In [26] we obtain the global existence in time for the equations modeling the motion of a ball in a perfect incompressible fluid (the two dimensional case). In a work in progress we tackle the case of a rigid body of arbitrary form.

One of the main results we have obtained during the last year is the convergence of the Lagrange-Galerkin problem for the equations modelling the motion of rigid bodies in a viscous incompressible fluid, which has been shown in [25]. As for the existence results, the fact that we have a free boundary considerably complicates the numerical analysis.

The focus of our research in 2004 will be the control of fluids and the analysis of fluid-structure interactions. We will consider two types of problems:

1. Control of the motion of the rigid bodies moving in a fluid by using a velocity or a torque control acting on the rigid only;
2. Control of a free surface of a fluid by using a moving wall.

Similar techniques have been used for phase transition problems. The aim is to study solidification processes of metallic alloys and phase separation phenomena. These physical situations are described by *phase-field* type models where, for instance, the change of phase (e.g. solid/liquid) is taken into account by an order parameter (the phase-field) satisfying a partial differential equation. From a mathematical point of view, these models are described through systems of nonlinear evolution equations. The well-posedness is proved for a fourth order parabolic system related to a phase separation process [20], as well as for a phase-field model leading to degenerate parabolic equations [16]. We have also considered the numerical solution of a phase field model for the solidification of a binary alloy. *A posteriori* error estimates have been obtained for a finite element method and we have developed a related adaptive mesh strategy for an anisotropic problem leading to dendritic growth [17].

For the control of the surface of a fluid by a wavemaker, we have obtained a null-controllability result for the full non-linear system, by using some Carleman estimate together with weighted estimates [18]. In particular, our result tells us that a small soliton moving from the left to the right may be caught up and annihilated by a set of waves generated by the wavemaker. This result is currently investigated from a numerical viewpoint. Another question of interest is the control of a 1-D tank containing a fluid modelled by Boussinesq equations. This question will be investigated in a joint work with J. Ortega (Santiago de Chile) and S. Micu (Craiova).

Let us also mention two other types of related results.

The first one concerns a degenerate parabolic equation proposed by Robert and Sommeria [43] to describe the relaxation towards a statistical equilibrium state for a two-dimensional incompressible perfect fluid with a vortex patch as initial value. We have shown in Rosier and Rosier [19] that a finite speed propagation occurs for the extremal values of the vorticity, due to the degenerated diffusion term. This result means that the value of the vorticity in the core of a vortex patch does not change for small times. This (natural) property does not hold for Navier-Stokes equations. Indeed, in this case the value of the vorticity decreases instantaneously at the core of any vortex patch due to strong diffusion effects.

Finally, J.P.- Croisille works with Pr. Ben-Artzi and Pr. Fishelov from University of Jerusalem, and with Pr. Trachtenberg from University of Tel-Aviv on the numerical simulation of incompressible and non stationary Navier-Stokes equations. To achieve this, a Stephenson compact difference scheme is used. A particular care is devoted to the numerical analysis of such schemes, and to their relation with the box schemes. 2D Numerical results have been obtained.

6.2. Well–posed linear systems and weak coupling

Participants: Fatiha Alabau, Francis Conrad, Jean-François Couchouron, Marius Tucsnak.

In [24] we construct a conservative linear system from two very simple ingredients : a self-adjoint positive and boundedly invertible operator A_0 on a Hilbert space H and a bounded operator C_0 from the domain of $A_0^{\frac{1}{2}}$ into another Hilbert space U. It turns out that our construction appears naturally in mathematical models of vibrating systems with damping. In [23] we show that the systems constructed in [24] are exactly controllable if and only if they are exactly observable, if and only if they are exponentially stable. Moreover, the exponential stability is characterized by frequency domain conditions of Hautus type. Finally, in [13] we consider a class of conservative systems with a "degenerated" damping and we introduce a new method for the study of their stability.

On the other hand, we have completed our work on spectral methods for systems coupling PDE's and ordinary differential equations, initiated in [5]. In one of our previous works (cf. [4]) we got the optimal decay rate for a control acting on the displacement and on the velocity at the boundary. In some particular cases it can be shown (by using Shkalikov's theory, for instance) that the decay rate is given by the spectral abscissa of the closed loop system. In these particular cases we showed by analytical methods that the control on the displacement diminishes the decay rate. In more complicated situations, numerical simulations of the spectra indicate the same effect of the displacement control. We also continued the work on observability and on control of weakly coupled hyperbolic systems. In [10] Alabau considers an abstract system of two hyperbolic equations subject to a one force control. We show that this single information allows us to get back a weaken energy of the full system.

A related question has been tackled by Alabau in [9]. The above quoted paper deals with the stabilization of hyperbolic systems by a nonlinear feedback which can be localized on a part of the boundary or locally distributed. The main result yields a general formula for the energy decay rates in terms of the behavior of the nonlinear feedback close to the origin. These results significantly improve the decay rates given by Lasiecka and Tataru [37] and Martinez [40] and generalize results of Nakao, Komornik and Zuazua.

Recently, J-F. Couchouron worked on stabilization problems for the wave or plate equations using new results on precompactness of trajectories. He also obtained in a joint work with M. Kamenski and R. Precup new principles for Hammerstein type inclusions. These results provide in particular a non linear average principle for periodic solutions of second order PDE's (cf. [11]). Another recent work [12], in collaboration with P. Ligarius, concerns the construction of Luenberger type non linear observers in reflexive Banach spaces.

6.3. Optimal location of sensors and of actuators

Participant: Antoine Henrot.

This topic was the subject of the thesis of Pascal Hébrard who left our team at the end of 2002. He is currently working as a research engineer at Dassault Systems. Nevertheless, we kept in touch during this year since we wanted to understand and write precisely the "spillover phenomenon" that was pointed out last year. Let us explain what it is. When we want to damp a vibrating body, we have in principle to act on all the eigenfrequencies of the body. In practice, it is common to consider that high frequencies are not so penalizing and that we can only take into account the low frequencies. Therefore, we decided to simplify the criterion was introduced in [14] and which involves all the eigenmodes (this criterion corresponds to some rate of decay of the total energy of the system), by introducing a new criterion, say J_N involving only the N first eigenmodes of the damped system. Then, we are led to look for an internal domain which damps the best as possible those N modes, that is which maximizes our criterion J_N. We were able to prove, in one dimension (that is to say for a damped string) that:

- There exists a unique solution ω_N^* to the previous minimization problem. This set ω_N^* represents the optimal location of actuators when we want to damp only the N first modes.
- The set ω_N^* is composed of at most N connected components.
- When the length constraint l goes to zero (i.e. we consider a small zone of control), the set ω_N^* concentrates on the nodes of the $N+1$-th mode. This means that it does not control it at all. In other terms, the best domain for the N-th first eigenmodes is the worse for the $N+1$-th eigenmode!

This work will appear in [41]. We would now like to know if this phenomenon is related to the choice of our model (criterion and state equation) or if it is a more general situation. Moreover, we would like to extend this result to higher dimension. Even in two dimensions, for general domains, the formal proof seems much harder, in particular we need spectral properties which are not known in a general context.

With Steve Cox, we began studying another problem which also concerns the damping of eigenmodes in a string. This problem is related to a model for harmonics on string instruments, and can be set as follows : is it possible to achieve “Correct Touch” in the pointwise damping of a fixed string? By correct touch, we mean the following. When we place a finger lightly at one of the nodes of the low frequency harmonics, it forces the string to play a note that sounds like a superposition of those normal modes with nodes at the location of the finger. Now, the question is to determine what should be the pressure of the finger in order to damp the remaining modes in the best way. From a mathematical point of view, we consider the wave equation with a damping like $b\delta_a u_t$ where u_t is the speed, a the location of the pressure, δ_a is the Dirac distribution at a and b the intensity of the pressure. We want to determine, for each a what is the b that minimizes the spectral abscissa of the modes not vanishing at a. This involves a precise qualitative analysis of the spectrum of the damped operator in the complex plane. This is still a work in progress, but we have already obtained significant results which are given in [46].

6.4. Frequency tools for the analysis of pde's

Participants: Frédéric Magoules, Karim Ramdani, Takéo Takahashi, Marius Tucsnak.

6.4.1. Control, stabilization and their numerical approximation for systems governed by pde's

It is well-known that the solution of LQR optimal control problems is given through a feedback operator involving a Riccati operator P. This operator solves a Riccati equation in infinite dimension. Of course, in practice, one can only determine an approximate solution P_h of this equation, and the natural question that arises is the following : does the approximate solution obtained using this operator P_h (instead of P) converge to the solution of the continuous problem. This question has been so far studied by many authors (see for instance [34][36][30]). In all these papers, one of the main assumptions is that the discretized systems have to be **uniformly stabilizable** with respect to the discretization parameter h. Unfortunately, most of the standard numerical methods (finite element or finite differences) do not fulfill this condition.

Using the frequency characterization of stabilizability proposed by K. Liu in [39], we have given in [29] a general technique ensuring the uniform stabilizability of classical numerical methods (finite element or finite differences). This technique consists in adding to the standard numerical schemes a suitable numerical viscosity. The main novelties brought in by our results lie in their generality (they hold for a wide class of second order evolution equations) and in their easy implementation.

6.4.2. Time-reversal

As already mentioned above, we worked on one aspect of time-reversal, known as the **D.O.R.T method** (see for instance [42] and the references therein), developed at the Laboratoire Ondes et Acoustique (L.O.A.) of the E.S.P.C.I. This technique follows from two experimental observations. Firstly, the number of scatterers contained in a propagation medium is exactly the number of non-zero eigenvalues of the time-reversal operator T. Secondly, each corresponding eigenvector selectively focuses on each scatterer. When we started working on this topic, our goal was to provide for the **first mathematical justification** of these experimental results.

The first model we have considered is a **far field model**, in which the time-reversal mirrors are located at infinity. Their interaction with the scatterers can thus be neglected. We have shown in [15] that the experimental results do not hold in general, but do hold for **small and distant scatterers** (compared to the wavelength). These original results have been obtained thanks to an asymptotic analysis of the scattering amplitude of the problem. The second model we have considered is a **near field model**, in which the mirrors are too close to the scatterers to be neglected. This question is studied in the framework of the PhD thesis of C. Benamar at the laboratory L.A.M.S.I.N (E.N.I.T, Tunis), whose advisor is C. Hazard (S.M.P., CNRS/ENSTA/INRIA).

Several questions are still open. One of them is to understand the relation between the time-reversal in the frequency and the time domains. This question probably requires the use of scattering theory.

7. Contracts and Grants with Industry

The collaboration with EADS described in 4.2 is formalized by a CIFRE contract.

Moreover, F.Magoulès participates to a contract with Hutchinson. The aim is to develop some GUI (Graphics Users Interface) with the VTK (Visualization ToolKit) library able to deal with hudge meshes in a very short time. The VTK library is based on an OpenGL kernel and is programmed in C++ through an object oriented approach. More details can be found in references [28][27].

8. Other Grants and Activities

8.1. National initiatives

- At INRIA : Marius Tucsnak is member of the Evaluation Committee of INRIA, of the Project Committee of INRIA-Lorraine and he participated to the Hiring Committee for junior researchers of IRISA.

- In the Universities and in CNRS committees:
  - Antoine Henrot is the chair of the CNRS GDR entitled "ANOFOR" (New Applications of Shape Optimization).
  - Francis Conrad is the coordinator of the Master program in Mathematics of Henri Poincaré University.
  - Marius Tucsnak was the coordinator of a specific CNRS action on control theoretical tools for adaptive optics systems.

8.2. Bilateral agreements

- INRIA-CONYCIT grant with the University of Chile;
- BRANCUSI grant with the University of Craïova (Romania).

8.3. Visits of foreign researchers

J. Ortega (Santiago), J. San Martén (Santiago), G. Weiss (London), B. Zhang (Cincinnatti).

9. Dissemination

9.1. Participation to International Conferences and Various Invitations

9.1.1. Invited conferences

- A. Henrot
  - Workshop on Homogenization and Optimal Design, Pisa, Italy, october 2003;
  - Calculus of Variations, University of Chambéry, june 2003.
- M. Tucsnak
  - Workshop on Control Theory and its Applications, University of Hong Kong, december 2003;

9.1.2. Participation to international conferences

- F. Alabau
  - *July 2003:* IFIP TC 7 Conference on System Modelling and Optimization, Sophia-Antipolis (France).
- K. Ramdani
  - *July 2003:* WAVES 2003, Jyvaskyla (Finland).
  - *July 2003:* IFIP TC 7 Conference on System Modelling and Optimization, Sophia-Antipolis (France).
- M. Tucsnak
  - *July 2003:* IFIP TC 7 Conference on System Modelling and Optimization, Sophia-Antipolis (France).

9.1.3. Invitations

- L. Rosier
  - Imperial College, London, April 2003;
  - Santiago de Chile, September 2003;
  - Politecnico di Torino (Italy), October 2003
- M. Tucsnak
  - Santiago de Chile, February 2003;
  - Imperial College, London, February 2003
  - Universidad Autonoma de Madrid, October 2003

9.1.4. Editorial activities

L. Rosier is associated editor of "ESAIM COCV". M. Tucsnak is associated editor of "SIAM Journal on Control".

9.2. Teaching activities

Most of the project members are professors or assistant professors so they have an important teaching activity. We mention here only the graduate courses.

- Real and Complex Analysis (F. Conrad)
- Optimization (F. Conrad)
- Scientific Computing (A. Henrot)
- Introduction to Nonlinear Systems (L. Rosier)
- Navier-Stokes equations (M. Tucsnak)
- Distributions and Partial Differential Equations (M. Tucsnak)

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