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Project-Team Fractales

Fractals, Complex Models and Artificial Evolution

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THEME 4A

Activity Report

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1. Team

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2. Overall Objectives

Key words: *2-microlocal analysis*, *evolutionary algorithm*, *fractal*, *fractal compression*, *fractional Brownian motion*, *genetic algorithm*, *Hölder functions*, *IFS*, *image analysis*, *image compression*, *inverse problem*, *iterated functions system*, *large deviations*, *multifractal analysis*, *optimisation*, *signal analysis*, *stable laws*, *texture analysis*, *time-frequency analysis*, *TCP traffic*, *watermarking*, *wavelets*.

The tools developed in the *FRACTALES* team deal with the mathematical, algorithmic and computational aspects of the modelling and analysis of complex signals. Examples include radar images, internet or road traffic data, financial series, speech or musical signals, biomedical signals or robotic data.

Research is centered around two synergetic topics:

- Fractal Analysis and Modelling: multifractal analysis, 2-microlocal analysis, fractal stochastic processes.
- Evolutionary Algorithms.

Evolutionary stochastic optimisation methods have proved efficient in the framework of fractal signals and allowed to address formerly unresolved applications. Conversely, analyzing the fractal irregularity of signals brings up new elements for the theoretical understanding of evolutionary techniques. Interaction between Evolutionary Algorithms and Fractals is central to the team's research topics.

Applications developed in the team concern:

- Image and Signals: denoising, watermarking, stereovision, audio2midi,
- Telecom: analysis and modelling of TCP traffic,
- Interactive systems: Evolutionary Robotics, Text-mining, e-learning.

The FRACTALES team also develops several freewares, most notably FRACLAB (a matlab/scilab toolbox for 1D and 2D signal processing) and EASEA (a specification language for evolutionary algorithms).

The FRACTALES team has strong collaborations with IrCcyn in Nantes, and regularly collaborates with ENSTA, with French universities: Orsay (LRI), Toulouse, and with several foreign universities: St-Andrews (Scottland), Montréal (Quebec), Yale (USA). The team is involved in the European Network of Excellence EVONET.

The FRACTALES team has industrial contracts with Dassault Aviation, Novartis Pharma (Switzerland), and Paraschool.

3. Scientific Foundations

3.1. Pointwise regularity

Participants: Julien Barral, Jacques Lévy Véhel, Stéphane Seuret.

In collaboration with Stéphane Jaffard (Université Paris XII).

Key words: *Hölder exponent*, *pointwise regularity*, *2-microlocal analysis*.

In many applications, the local regularity of a function contains essential information for further processing. Studying the local regularity may be done in several ways. We focus on Hölder exponents and 2-microlocal analysis, which is an extension of the Hölder regularity.

There are many ways to perform the fractal analysis of a signal. Our team deals with two of these: Local regularity and multifractal analysis

In the first case, one associates to a signal $f(t)$ a function $\alpha(t)$, the *Hölder function* of f, which measures the regularity of f at each point t. This quantity may be evaluated in several manners. For instance, the pointwise Hölder exponent α of f at x_0 is defined as:

$$\alpha(x_0) = \lim_{\rho \to 0} \sup \{\alpha \quad : \exists c > 0, |f(x) - f(x_0)| \leq c|x - x_0|^{\alpha}, |x - x_0| < \rho\}$$

(this definition requires that α is not an integer and that f is non differentiable).

One may also define a local exponent $\alpha_l(x_0)$ as:

$$\alpha_l(x_0) = \lim_{\rho \to 0} \sup \{\alpha \quad : \exists c > 0, |f(x) - f(y)| \leq c|x - y|^{\alpha}, |x - x_0| < \rho, |y - x_0| < \rho\}$$

α and α_l are different in general (for $f(x) = |x|^\alpha \sin \frac{1}{|x|^\beta}$, $\alpha(0) = \alpha$, while $\alpha_l(0) = \frac{\alpha}{1+\beta}$). They have very different properties. For instance, α_l is stable through differentiation ($\alpha_l(f', x_0) = \alpha_l(f, x_0) - 1$), as α is not.

As a rule, the smaller $\alpha(t)$ is, the more irregular the function f is at t. A discontinuous bounded function has exponent 0, while $\alpha(t) > 1$ entail that f is at least once differentiable at t. Characterizing signals through their Hölderian regularity has been considered by many authors, both from a theoretical point of view (for instance in relation with wavelets decompositions) and in applications (e.g. turbulence analysis, image segmentation). Such an approach is fruitful as soon as some relevant information is contained in the irregularities of the signal, rather than, for instance, in its amplitude or Fourier content. This occurs in particular when ones tries to detect edges in images, or to analyze non-voiced parts of speech signals. Natural questions in this frame, that we have partially solved, include the characterization of the Hölder functions, the comparison of the different ways to measure the local regularity, and the problem of their estimation on real signals.

A generalization of Hölder regularity is provided by 2-microlocal analysis. This analysis allows to describe in great details the local regularity behaviour. Our work deals with various extensions of 2-microlocal analysis, providing time domain characterization of 2-microlocal spaces, and the estimation of 2-microlocal quantities from sampled data.

3.2. Multifractal analysis

Participants: Julien Barral, Christophe Canus, Jacques Lévy Véhel, Stéphane Seuret, Claude Tricot.

Key words: *Hausdorff spectrum*, *large deviations spectrum*, *multifractal analysis*.

Multifractal analysis provides both a local and a global description of the singularities of a signal: The local description is obtained via the Hölder exponent; The global one is contained in the various multifractal spectra. These multifractal spectra describe geometrically or statistically the distribution of singularities on the support of the signal.

In some situations, the Hölder function of a signal is simple while the signal is irregular. This occurs for instance in the case of the Weierstrass function or the fractional Brownian motion, which are nowhere smooth, but whose Hölder function is constant. There also exist irregular signals for which the Hölder function is even more irregular. For instance, f might be continuous but α_f everywhere discontinuous. The basic example of this situation is the graph of a Fractal Interpolation Function. In such situations, it is more interesting to use multifractal analysis than the raw Hölder function: Basically, instead of recording, for each t, the value of the exponent, one groups all the points with same α in a subset E_α. The irregularity is then characterized in a global way by computing, for each α, the Hausdorff dimension $f_h(\alpha)$ of the set E_α. One thus evaluates, in a geometrical way, the "size" of the subsets of the domain of f where a given singularity occur. Another possibility is to use a statistical description of the distribution of the singularities: More precisely, the *large deviation multifractal spectrum* $f_g(\alpha)$ estimates the exponential speed of decay of the probability to encounter a singularity equal to α at resolution n, when n tends to infinity.

This kind of analysis, which first appeared in the context of turbulence, has developed a lot, both theoretically (analysis of self-similar measures/functions, in a deterministic or stochastic frame, analysis of capacities, higher-order spectra) and in applications (study of DLA, geophysics, signal/image processing, TCP traffic analysis).

Our work in multifractal analysis deals with the theoretical computation of spectra, their comparison (multifractal formalism), and the design of robust estimators in the deterministic and stochastic frames.

3.3. Fractal Process

Participants: Antoine Ayache, Julien Barral, Lotfi Belkacem, Michel Guglielmi, Jacques Lévy Véhel, Stéphane Seuret.

In collaboration with Serge Cohen (Université de Toulouse), Jacques Peyrière (Université d'Orsay).
Key words: *(multi-)fractional Brownian motion*, *Lévy processes*.

Long-memory processes (i.e. those with slowly decaying autocorrelation) and processes with infinite marginal variance display interesting and sometimes counter-intuitive properties. We study certains of these processes such as (multi-)fractional Brownian motion and Lévy processes.

We study processes such as the fractional Brownian motion (fBm) or α-stables processes, which exhibit fractal properties such as auto-affinity ($x(at) \stackrel{d}{=} a^H x(t)$, where $\stackrel{d}{=}$ means equality in distribution), local irregularity, or long range memory (i.e. $E(x(t)x(t+\tau)) \sim |\tau|^\beta$ when $\tau \to \infty$, $-1 < \beta < 0$). These processes have two main features that make them different from « classical » models:

- α–stables processes have, for $\alpha < 2$, infinite variance. This induce discontinuities in the sample paths.
- Long-memory processes exhibit a divergence of the spectral density at 0, which translates into « pseudo-cycles » of all sizes on the paths.

In both cases, most classical tools (central limit theorem, usual estimators) have to be adapted. Our works deal with the description of certain fractals and multifractal properties of these processes. We also develop extensions that make them more fitted to certains applications. For instance, the local regularity of fBm is almost surely the same at each point. This prevents the use of fBm as a model in certain situations (e.g. TCP traffic modelling). We have defined a generalization of fBm, called multifractional Brownian motion (mBm), which allows an independent control of the Hölder exponent at each point.

3.4. Genetic Algorithms, Evolutionary Algorithms

Participants: Jacques Lévy Véhel, Evelyne Lutton, Marc Schoenauer.

Key words: *Evolutionary algorithms*, *genetic algorithms*, *deceptivity analysis*, *stochastic optimisation*, *inverse problems*, *schema theory*.

When using fractal tools for the analysis of complex signals, one often have to deal with large and extremely irregular optimisation problems. Evolutionary algorithms (including Genetic Algorithms) have proven to be powerful tools in this framework, and were able to provide robust solutions, impossible to obtain with other techniques. Conversely, works performed in the team proved also that "fractal" tools were efficient to refine and complement theoretical analysis of simple evolutionary algorithms.

Genetic Algorithms (GA) and more generally evolutionary algorithms (EA) are currently known as efficient stochastic optimisation tools, and are widely used in various application domains. These techniques are based on the evolution of a population of solutions to the problem, the evolution being driven by a "fitness" function that is maximized during the process. Successive populations of solutions are thus built, that fit increasingly well (the values of the fitness function increase). Their evolution is based on stochastic operators: selection (the fitness function is used as a sort of "bias" for a random selection in the population), and the "genetic" operators, mainly crossover (combination of two solutions) and mutation (perturbation of a solution). This technique is based on the assumption that well fitted solutions (also called individuals) can provide better solutions with help of the genetic operators. This assumption can be proven to be connected to some notions of "GA-difficulty" of the function to be optimized: one usually talks about "deception" or sometimes also about "fitness landscape."

Theoretical investigations on GA and EA generally concern convergence analysis (and convergence speed analysis on a locally convex optimum for Evolution Strategies), influence of the parameters, GA-easy or GA-difficulty analysis. For simple GA, these analyses are based on several approaches: Proof of convergence based on Markov chain modelling [52][50], deceptive functions analysis, based on Schema analysis and Holland's original theory [54], and finally modelling as dynamical systems, where fractal-like behaviour have been exhibited [55].

From the theoretical viewpoint some tools, developed in the framework of fractal theory, can be used in order to perform a finer analysis of Genetic Algorithms behaviour (mainly based on the schema theory). Actually, an analysis of how GA optimize some "fractal" functions (Hölder functions) makes it possible to model the influence of some parameters of the GA. Such an analysis can then be generalized and gives clues about how to tune some of the GA parameters in order to improve its efficiency. Finally, a further analysis on the same theoretical basis allows the influence of the coding in a GA to be analyzed[10].

3.5. New evolutionary models

Participants: Pierre Collet, Evelyne Lutton, Marc Schoenauer.

Key words: *interactive evolution*, *Parisian approach*, *multi-objective optimisation*, *co-evolution and social insects colonies*.

The versatility of evolutionary algorithms permits to address optimisation problems that involve non-standard search spaces (lists, graphs, ...). These are very difficult, irregular, impossible to address with other techniques. It is however possible to do "more than optimisation" thanks to artificial Darwinism and population-based methods. This is a major point of our research. We are in particular interested in various evolutionary techniques based on a modified formulation of the problem to be solved: Interactive evolutionary algorithms, evolution and learning, co-evolution and "Parisian" evolution, multi-objective optimisation.

Simulated Darwinist evolution can be exploited in various ways, and recent research tend to prove the interest of new evolutionary models. Our works cover several aspects:

- *Parisian approach*: This technique proposed by the FRACTALES team is related to co-evolution techniques. It consists in formulating a problem no longer as the search for an optimum with a population of points in a search space, but as the search for an equilibrium state for a population of "parts" of solutions, that collectively built the searched solution. Such a formulation is not always possible for optimisation problems (the problem has to be shared into interdependent subproblems). However, when applicable, this approach is beneficial in terms of efficiency and computation time. It has been applied to inverse problem for IFS, stereovision (the quasi-real-time "flies" algorithm) for obstacle detection, fractal compression and text-retrieval.
- *Interactive evolutionary algorithms*: When an evolutionary process involves an interaction with a human user (usually fitness evaluation is partly set by the user), one has to reconsider several important points of the evolutionary loop. This research topic is very active. For example, interaction with humans raises several problems, mainly linked to the "user bottleneck," i.e. the human fatigue. Solutions have to be found in order to avoid systematic and boring interactions. Our work deal with the analysis and development of various user-interaction modes, including learning and Parisian approaches. Current applications include text-retrieval (ELISE), e-learning, over-constrained problems resolution, and artistic design (ArtiE-Fract).
- *Evolution and learning*: the domains of learning and evolution are complementary, and advances in each domain may benefit to the other. Learning can help evolution (learn which are the efficient operators or parameter settings, which are the interesting regions of the search space, or learn an approximation of the fitness function ...). Conversely, the learning itself can be considered as an optimisation problem (even multi-criterion) [59].
- *Multi-objective approach*: most of real world problems involve several objective, sometimes conflicting (example: minimise a cost while maximizing a technical performance). The usual methods to deal with this in conventional optimisation techniques are based either on a weighted aggregation of the various objectives or on the use of objectives as constraints. Evolutionary techniques offer an elegant and efficient alternative: The regular selection mechanism can be updated on order to directly consider the Pareto dominance. This permits to sample a Pareto front by a single run of an evolutionary algorithm. Current work of the team consider the fine tuning of this new selection operator, and adaptation of other components of the algorithm in order to improve the performances.

4. Application Domains

4.1. TCP Traffic

Participants: Julien Barral, Jacques Lévy Véhel, Stéphane Seuret.

Key words: *multifractal analysis*, *multifractional Brownian motion*, *TCP*.

Internet traffic possesses radically different characteristics, as compared to conventional traffic, whose study requires new tools. In particular, the strong sporadicity has important consequences on the queuing behaviour.

Conventional traffic models generally assume that the arrival processes have short-term memory. It appears that Internet traffic usually does not satisfy such an assumption. In particular, many types of traffic on the Internet are strongly sporadic on several times scales. Recent models based on fBm take into account such features. The success of fBm as a traffic model relies partly on the fact that the long term memory is controlled by a single parameter. As long range dependence is an order 2 statistics, it is natural to enquire whether fBm is also a good model for higher-order statistics of real traffic.

Multifractal analysis allows to answer this question. The multifractal spectrum of fBm is trivial, since fBm is monofractal. We have shown through intensive numerical studies that LAN traffic recorded at Berkeley and CNET exhibit to the contrary a strong multifractal behaviour over 3 to 4 time scales.

The observed spectra evidence differences between incoming and outgoing traffics. Furthermore, the shape of the spectrum of the Berkeley traffic provides information on the stationarity of the process. More generally, the multifractal characteristics of traffic traces have consequences on the queuing behaviour.

Our recent work has dealt with the possible sources of multifractality. We have in particular shown that the very mechanism of TCP is a cause of multifractality.

4.2. Inverse problems

Participants: Pierre Collet, Evelyne Lutton.

Key words: *genetic algorithms*, *evolutionary algorithms*, *stochastic optimisation*, *inverse problem*, *genetic programming*.

Several inverse problems connected to fractal analysis of signals can be successfully solved using evolutionary algorithms: inverse problems for IFS with applications to speech signal modelling, inverse problems for finite automata. It is however crucial to efficiently exploit the evolutionary loop: Experiments have proved that a carefully parameter setting and an efficient encoding noticeably increase the algorithms efficiency.

A standard inverse problem can be formulated in the following way: For a given system it is possible to compute an output from input data but reversely it is extremely difficult to estimate the input data that have produced a given output.

A classical strategy, the “black-box” approach, considers the previous problem as an optimisation one: Optimise the input data so that their computed output resembles the given output. In general evolutionary techniques are well adapted to difficult inverse problems. Especially in the domain of fractal analysis, several inverse problems have been successfully addressed, including:

- the inverse problem for IFS, [61][58][56]. Our work deals with the use of complex IFS models (mixed, polar) with genetic programming and Parisian approach. An application to speech signal modelling has also been proposed,
- inverse problems for finite automata.

The resolution of these “academic” fractal inverse problems have led to several applications: image compression [60][53], and fractal antennas optimisation [51].

4.3. Image analysis

Participants: Pierrick Legrand, Jacques Lévy Véhel, Evelyne Lutton, Ina Taralova.

Key words: *multifractal analysis*, *denoising*, *change detection*, *segmentation*.

The multifractal analysis of images is based on the definition of measures from image grey-levels. Multifractal spectra are then computed. Contrarily to more classical approaches, there is no filtering. Segmentation, denoising or change detection are performed on image points using local as well as global informations provided by the spectra.

Image analysis is a fundamental component of computer vision problems, with applications in robotics, medical or satellite imaging ...Segmentation is an important step that provides a description of the image in terms of regions and contours.

Classical approaches in this domain are based on the general assumption that an image is the sampling of a underlying piecewise C^1 process. Filtering yields then signal gradients where extrema roughly correspond to contours. Multi-resolution techniques may then be used to refine these results.

The main drawback of this approach is due to the preliminary smoothing, that produces a loss of precision. The hypothesis of a piecewise C^1 underlying process is not always realistic: Textures for example will in general puzzle these detectors.

An alternate approach is to consider that the image represents a measure known at fixed resolution. The irregularities of this measure can then be studied with the help of multifractal analysis. The general principle is the following: First, various measures and capacities are defined from the image grey-levels. The corresponding multifractal spectra are then computed, providing both local (via α) and global (via $f(\alpha)$) information. No hypothesis is made on the regularity of the signal.

5. Software

5.1. FRACLAB: a Fractal Matlab/Scilab toolbox

Participants: Christophe Canus, Pierrick Legrand, Jacques Lévy Véhel.

FracLab is a general purpose signal and image processing toolbox based on fractal and multifractal methods.

FracLab can be approached from two different perspectives:

- Fractal analysis: A large number of procedures allow to compute various fractal quantities associated with 1D or 2D signals, such as dimensions, Holder exponents or multifractal spectra.
- Signal processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modelling, segmentation,classification, and synthesis.

Note that FracLab is not intended to process "fractal" signals (whatever meaning is given to this word), but rather to apply fractal tools to the study of irregular but otherwise arbitrary signals. A graphical interface makes FracLab easy to use and intuitive. In addition, various wavelet-related tools are available in FracLab.

FracLab is a free software. It mainly consists of routines developed in C-code interfaced with Matlab (Ver. 5) and Scilab (a free scientific software package for numerical computations from INRIA). It runs under Linux and Windows environments

Since version 1.0 (June 2001), FracLab has been downloaded more than 2000 times. A few dozens of laboratories seem to use it. FracLab has been referenced in several research papers.

5.2. EO: Evolving Objects, a evolutionary C++ library

Participants: Pierre Collet, Olga Roudenko, Marc Schoenauer.

Key words: *evolutionary algorithm*, *software components*.

EO (Evolving Objects) is an evolutionary C++ freeware (and distributed via the Source Forge server). Launched at Granada university, EO is now supported by the European Network of Excellence EVONET. Marc Schoenauer is one of the main developers of this library.

The two main original points of EO in comparison to other similar libraries is the independence with respect to the common paradigm (such as genetic algorithms, evolution strategies or genetic programming) and its complete independence between the search space (the objects to be evolved) and the *evolutionary engine*.

EO allows to use various data structure as search space (genotypes): basic types (bit strings, real parameter, GP trees), as well as complex ones.

The evolutionary engine of EO provides all common evolutionary schemes, plus experimental ones. The EASEA language and its graphical interface GUIDE give access in an intuitive way to a large variety of evolutionary engines, including several multi-objective engines. Parallelization primitives are also available (master-slave and island models), thanks to a collaboration with the Lille university.

5.3. EASEA : an evolutionary algorithms specification language

Participants: Pierre Collet, Jean Louchet, Evelyne Lutton, Marc Schoenauer.

Key words: *evolutionary algorithm*, *stochastic optimisation*.

EASEA (EAsy Specification of Evolutionary Algorithms) was initiated inside the EVO-Lab collaborative action (1999-2000). Its aim was to simplify the programmation of an evolutionary algorithm, especially for non-computer scientist. A simple specification of an evolutionary algorithm written in an « .ez » file is used by EASEA. It then produces a C++ source file using the primitives of an underlying evolutionary library. The complex programming tasks are hidden to the user.

The description of an evolutionary algorithms becomes very short, and thanks to the EASEA compiler this specification file can be compiled at any place. The current version (UNIX and Windows) can produce a C++ source file for the GALib or the EO library, or JAVA source files for the DREAM library.

EASEA is now largely used,

- as a teaching support (ENSTA, école Polytechnique, Université du Littoral, Université de Dijon, école Centrale, école des Ponts, CESTI Toulon, University of Massachusetts Dartmouth),
- as research and industrial development tool (projet SINUS, ENSTA, Laboratoire d'Informatique du Littoral, General Electric (France), Université d'Alger, University of Exeter (UK), Napier University (Ecosse), South-Bank University (Londres), Vrije University of Amsterdam, University of Dortmund, Granada University).

A graphical interface, GUIDE is also available. It provides an unified representation of the evolutionary engines (AG, ES, EP, ...), and gives access to unexplored schemes with a versatile presentation.

EASEAv0.7 is available on http://www-rocq.inria.fr/EASEA

6. New Results

6.1. Study of multifractal formalism for functions

Participant: Stéphane Seuret.

Key words: *Hölder spaces*, *multifractal formalism for functions*.

A multifractal formalism is a formula which allows (when it holds) to easily compute the multifractal spectrum of the considered function f. Indeed, when this formalism holds, the spectrum is obtained as the Legendre transform of a free energy function η_f derived from f.

We investigate the relation between the failure of the multifractal formalism and the presence of oscillating singularities. Indeed, they often occur together. We show that when the formalism is broken, i.e. when $\dim\{x :$ the Hölder exponent at x is $h\} > (\eta_f - 1)^*(h)$, there is a set of Hausdorff dimension exactly equal to $\dim\{x :$ the Hölder exponent at x is $h\}$ which contains only oscillating singularities for f.

This gives rise to a surprising result. Indeed, in view of the last result, a natural way to generate oscillations is to force the multifractal formalism to fail. This is achieved by performing a threshold procedure on the wavelet coefficients of the function f. More precisely, if $f = \sum_j \sum_k d_{j,k}\psi_{j,k}$, we set $f^t = \sum_j \sum_k d^t_{j,k}\psi_{j,k}$ where

$$d^t_{j,k} = d_{j,k} 1\!\!1_{|d_{j,k}| \geq 2^{-j\gamma}},$$

for some $\gamma > 0$. We show that this threshold operator may create oscillating singularities: if x has a pointwise Hölder exponent larger than h for f, than either f^t is C^∞ at x, or x is an oscillating singularities for f^t. We exhibit an example of function for which the threshold operator only creates oscillations. Wavelet threshold is a famous method in signal and image processing (used for example in the JPEG and JPEG2000 algorithms), and this result can explain some defaults (different from the Gibbs phenomenon) observed in image compression.

6.2. Multifractal wavelet series

Participants: Julien Barral, Stéphane Seuret.

Key words: *analysis of local regularity*, *multifractal formalism*, *random measures*, *wavelets*.

Modelling physical phenomena is a central problem. One way in this direction is the construction of processes whose regularity can be controlled. Combining wavelet and multifractal measures theories we propose a new model defined as follows. Given a positive finite Borel measure μ on $\mathbb{R}$, we set for every integers j and k,

$$d_{j,k} = 2^{-j(s_0 - \frac{1}{p_0})} \left(\mu([k2^{-j}, (k+1)2^{-j}))\right)^{\frac{1}{p_0}},$$

where s_0 and p_0 are two positive real numbers such that $s_0 - \frac{1}{p_0} > 0$.

We show that if μ satisfies some multifractal formalism for measures, then the function F_μ defined by

$$F_\mu(x) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} d_{j,k}\psi(2^j x - k)$$

inherits the multifractal properties of μ, and it satisfies some multifractal formalism for functions.

6.3. General results for a class of multifractal multiplicative measures

Participant: Julien Barral.

In collaboration with Benoît Mandelbrot (Yale University).

Key words: *Dimension*, *Random measures*, *Martingales*, *Multifractal analysis*, *Statistical self-affinity*, *Statistical self-similarity*, *Poisson point processes*.

We wrote three papers to appear in “A Jubilee of B. Mandelbrot” devoted to a class of random measures generated by multiplicative processes.

Part I surveys the main motivations which led B. Mandelbrot to introduce such statistically self-affine multifractal measures as models for turbulence, from the initial limit lognormal processes to multiplicative cascades of random weights, and finally the multifractal products of pulses. A discussion contrasts the recent class of multifractal products of cylindrical pulses with the well-known canonical cascade measures.

Part II presents the examples of Part I as particular elements of a class of random measures generated by multiplications of functions for which several fundamental problems, namely non-degeneracy, finiteness of moments, dimension of the carrier and multifractal analysis can be studied and solved. The results complete Kahane’s general theory of T-martingales and are applied to new examples.

Part III provides the proofs of results in Part II.

6.4. Mathematical multifractal analysis of a simplified model for TCP Traffic

Participants: Julien Barral, Jacques Lévy Véhel.

Key words: *Multifractal processes*, *Hölder singularities*, *Hausdorff dimension*, *spectrum of singularities*, *Lévy processes*, *Internet Traffic Control Protocol*.

The “explanation” of the multifractality of TCP from basic features of the Internet is a difficult problem. Models investigated so far have been based on the paradigm of multiplicative cascades. However, such models are not convincing because there is no physical evidence that genuine traffic actually behaves as a cascading or *multiplicative* process. As a matter of fact, TCP traffic is rather an *additive* process, where the contributions of individual sources of traffic are merged in a controlled way.

The exact details of TCP are too intricate to allow for a tractable mathematical analysis. We consider a simplified model that captures the main ingredients of the congestion avoidance and flow control mechanisms of TCP for an infinite superposition of sources, in which each “source” i of traffic sends “packets” of data continuously. At time t, it sends $Z_i(t)$ packets. Between two consecutive time instants t and $t+1$, two things may happen: The source i may experience a “loss”, i.e. the flow control mechanisms of TCP detects that a packet sent by the source did not reach its destination. In this case, TCP forces the source to halve the number of packets sent at time $t+1$. If there is no loss, the source increases $Z_i(t)$ by 1. The durations $(\tau_k^{(i)})_{k\geq 1}$ between the time instants k and $k+1$ where a given source i experiences losses are modelled by a sequence of independent exponential random variables with parameter λ_i. The total traffic $Z(t)$ is then the sum of an infinite number of independent sources with varying rates λ_i, where $(\lambda_i)_{i\geq 1}$ is a non-decreasing sequence of positive numbers. We have proved that, under certain conditions, the Hausdorff multifractal spectrum of Z coincides with the one of a Lévy process with Blumenthal–Getoor exponent $\beta = \inf\{\gamma \geq 1;\ \sum_{i\geq 1} \frac{1}{\lambda_i^{\gamma-1}} < \infty\} \in [1,2]\}$: Its graph is linear with slope β between 0 and $1/\beta$.

6.5. Multifractional Brownian Motion

Participants: Antoine Ayache, Erick Herbin, Jacques Lévy Véhel.

Key words: *2-microlocal analysis*, *identification of pointwise Hölder exponent*, *Gaussian processes*, *Generalized multifractional Brownian motion*.

The Multifractional Brownian Motion (mBm) is a Gaussian process that was introduced in the *Fractales* project in 1995. The aim was to generalize the classical fractional Brownian motion in order to account for varying local regularity, a feature commonly encountered in real-world processes. Formally, the mBm may be represented as:

$$Z(t) = \mathrm{Re}\left(\int_{\boldsymbol{R}^d} \frac{(e^{it.\xi}-1)}{|\xi|^{H(t)+d/2}}\, d\tilde{W}(\xi)\right), \tag{1}$$

where $d\tilde{W}$ is the complex-valued stochastic measure

$$d\tilde{W} = dW_1 + idW_2,$$

dW_1 and dW_2 being two independent real-valued Brownian measures. $H(t)$ is a $\beta-$ Hölder continuous function ranging in $(0,1)$. A major interest of mBm is that, provided $\beta > \sup_t H(t)$, the pointwise Hölder exponent of Z at any given t is almost surely equal to $H(t)$. This allows to model a host of real-world phenomena, ranging for Internet traffic traces to natural landscapes. We have pursued the study of this process and have made progress in several directions.

1. mBm with irregular multifractional function.
The case where $\beta < \sup_t H(t)$ was left unanswered so far. Let $\alpha_Z(t)$ (resp. $\alpha_H(t)$) denote the Hölder exponent of Z (resp. H) at t. We have proved that, as soon $\beta > 0$, for all t, $\alpha_Z(t) = \min(H(t), \alpha_H(t))$ almost surely. This allows to obtain "multfractal" mBms: It suffices to choose H to be both multifractal and everywhere smaller than its regularity([44]).

2. Identification of the pointwise Hölder exponent of the gmBm.
Another way to obtain a possibly multifractal mBm is to generalize it as follows : Instead of a single function H, one uses a sequence H_n of smooth functions, and one replaces the integral in (1) by a sum of integrals, each involving one H_n and being taken on a frequency octave (see [1] for details). Then, under certain conditions, we have proved previously that, for any given t, almost surely, $\alpha_Z(t) = \liminf_{n\to\infty} H_n(t)$. A natural question is to enquire wether it is possible to estimate $\alpha_Z(t)$ from a sample path of the process. While it does not seem easy to do so when H is an arbitrary liminf of continuous functions, we have obtained the following *a priori* unexpected result: *as soon as the pointwise Hölder function of the GMBM belongs to the first class of Baire (i.e when $\{\alpha_X(t)\}_t$ is a limit of continuous functions), it may be estimated almost surely at any point t.* A Central Limit Theorem for our estimator has also been derived. Thus, even very irregular variations of the Hölder regularity of the GMBM may be detected and estimated in practice (see figures 1 and 2). This has important consequences in applications of the GMBM to signal and image processing. It may also lead to new methods for the practical computation of multifractal spectra ([14]).

3. 2-microlocal analysis of Gaussian processes.
2-microlocal analysis provides a finer way to measure the local regularity than the sole Hölder exponents. We have shown that, for continuous Gaussian processes, the limiting behaviour of the incremental variance allows to derive almost sure results on the shape of the 2-microlocal frontier at any fixed point t ([45]). This allows to recover previously known results in a more general frame.

6.6. Geometric Harmonics

Participant: Stéphane Lafon.

In collaboration with Ronald Coifman (Yale University).
Key words: *kernels*, *dimension reduction*, *low dimensional embedding*, *diffusion*.

The goal is to efficiently describe functions on a set Γ and to use the analysis of these functions to understand the geometry of Γ.

Understanding the geometry of sets in high dimension and finding ways to efficiently represent these sets is of central importance in many applications ranging from DNA micro-arrays processing to image analysis to information retrieval. Paradoxically, although some work existed in graph theory, only recently have people attempted to develop (practical) tools that are able to handle this kind of data (e.g. see Roweis' LLE algorithm or Tenenbaum's Isomap).

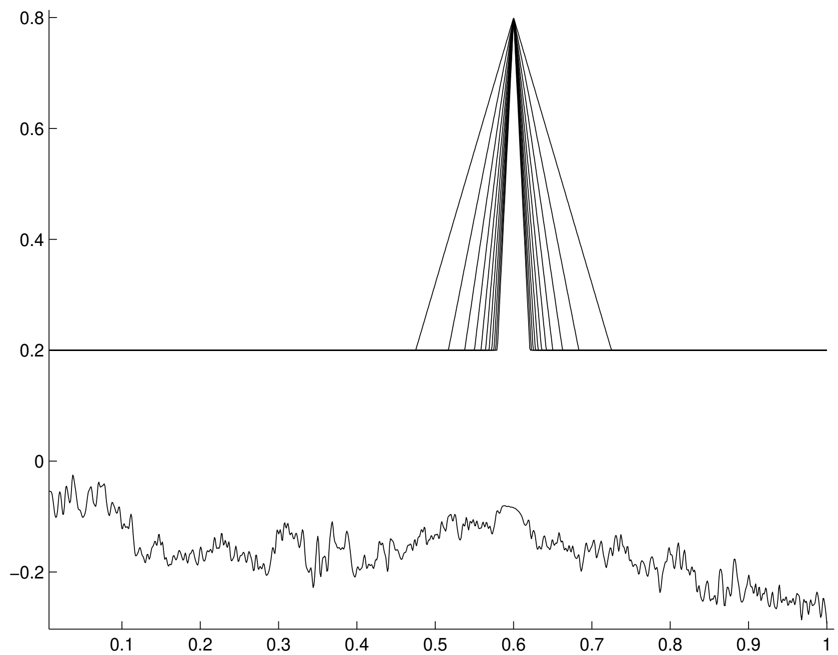

Figure 1. Simulated GMBM and associated sequence H_n converging to $H(t) = 0.2$ for $t \neq 0.6$, $H(0.6) = 0.8$.

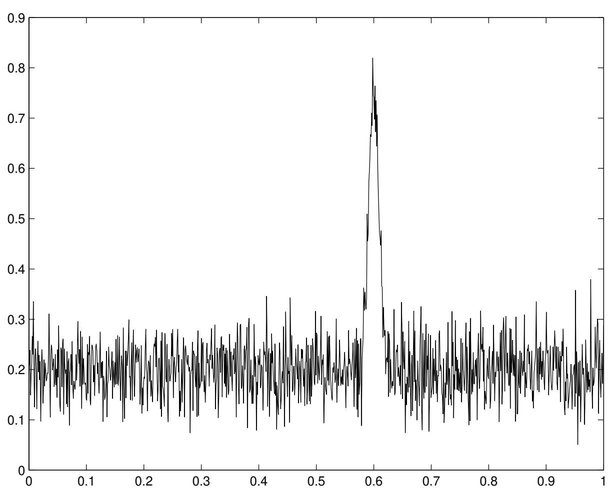

Figure 2. Estimated Hölder function of the GMBM in figure 1.

Our approach consists in considering positive definite kernels on $\Gamma \times \Gamma$. Such a kernel $k(x, y)$, when it only takes on positive values (but we do not restrict the theory to these kernels), defines a diffusion process on the set, and Γ can be endowed with a natural diffusion metric. This metric states that two points of Γ will be very similar if the diffusion is high between them. A remarkable fact is that this diffusion metric can be easily computed as follows:

- the eigenfunctions of k define an embedding of Γ into l^2. These eigenfunctions, viewed as coordinates on Γ, are termed *geometric harmonics*.
- in the embedding space, the diffusion balls at time t are (translation invariant) ellipsoids whose axes are parallel to the coordinate axes.

A simple but powerful corollary of these points is a natural and painless way of recovering the parameterization of a manifold embedded in a Euclidean space. Indeed, the geometric harmonics achieve dimension reduction of the data by providing a low dimensional embedding, i.e. a system of coordinates on Γ that preserves the diffusion metric. In that sense, the geometric harmonics provide great insight on the *intrinsic* geometry of Γ.

We also showed that the embedding provided by the Laplace-Beltrami operator, and that metric corresponding to heat diffusion on Γ, could be efficiently computed using other kernels.

Our research is currently focusing on how to extend the geometric harmonics to the ambient space and its implications on the *extrinsic* geometry of Γ.

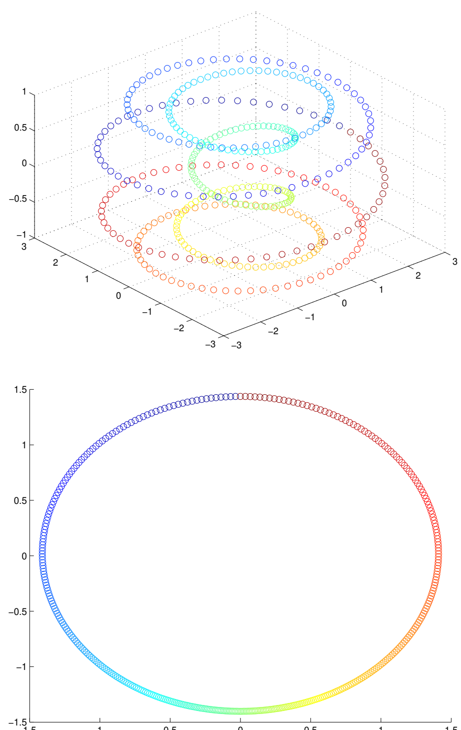

Figure 3. Left: the set Γ is a spring curve in $\mathbb{R}^3$, all points being permuted at random. Right: The set is embedded as a circle in the plane, and the arc length parameterization is recovered.

6.7. Analysis of some simple adaptive Evolution Strategies

Participants: Anne Auger, Marc Schoenauer.

In collaboration with Claude Le Bris (CERMICS, ENPC).
Key words: *Convergence of evolutionary algorithms*, *Evolution Strategies*.

Evolution Strategies are evolutionary algorithms devoted to the parametric optimisation. Since their invention in the mid-sixties, theoretical work have mainly investigated local properties on the sphere function ($f(x) = \sum_{i=1}^{dim} x_i^2$). In this work [40], we consider more general functions ($C^{1+\alpha}$) and investigate on these functions convergence properties for adaptive $(1, \lambda) - ES$ with Gaussian mutations. Moreover geometrical convergence rates are derived. The theory of supermartingale is used to prove these results.
This work has been presented in the GECCO2003 Conference [25].

6.8. Convergence results for the (1,λ) Self Adaptive Evolution Strategies

Participant: Anne Auger.

Key words: *Convergence of evolutionary algorithms*, *Asymptotic convergence rate*, *Self-adaptivity*, *Evolution Strategies*.

Self Adaptive Evolution Strategies (SA-ES) are evolutionary algorithms recommended by the state of the art in practical parametric optimisation. We investigate the global convergence of the (1,λ)-SA-ES on the sphere function ($f(x) = \sum_{i=1}^{dim} x_i^2$) and give an estimation of the asymptotic convergence rate. For this, the theory of Markov chains on a continuous state space is used, more precisely *Harris recurrence* and *geometric ergodicity* properties are demonstrated for an hidden Markov chains flowing from the definition of the (1,λ)-SA-ES algorithm. The asymptotic convergence of the (1,λ)-SA-ES then comes.
This work has been presented at the Evolutionary Algorithms workshop of the Thirtieth International Colloquium on Automata, Languages and Programming Eindhoven, The Nether lands, July 2003 [24].

6.9. Analysis of the GAuGE System

Participant: Anne Auger.

In collaboration with Miguel Nicolau and Conor Ryan from the University of Limerick, Ireland.
Key words: *Genetic algorithm*, *Grammatical evolution*, *Mapping process*.

This work explores the mapping process of the GAuGE system, a recently introduced position-independent genetic algorithm, that encodes both the positions and the values of individuals at the genotypic level. A mathematical formalization of its mapping process has been investigated and has been used to characterize the functional dependency feature of the system. An analysis of the effect of degeneracy in this functional dependency has been then performed, and a mathematical theorem showing that the introduction of degeneracy reduces the position specification bias of individuals has been proved. Experimental results have been preformed, that backup these findings.
This work has been presented at the conference Evolution Artificielle 2003 [37].

6.10. Estimation of Distribution Algorithms

Participants: Anne Auger, Marc Schoenauer, Michael R. Wagner.

Key words: *EDAs*, *Distribution sampling*.

Estimation of Distribution Algorithms (EDAs) are optimisation algorithms that manipulate a distribution over the search space. This distribution is sampled, and from a selection of the sample (according to the target function to optimize) a new distribution is derived. The work achieved during M. Wagner's Summer Internship in the Fractales Project considered real-valued problems, and Gaussian distribution. Different selection operators have been considered, and it has been experimentally shown that the deterministic selection, advocated by most authors, results in a non-convergence of the algorithm whenever the distribution is not centered on the solution. This had been completely missed by previous works because all test functions had their optimum “in the middle” of the search space, and the initial distribution was clearly surrounding that optimum. A simple modification of the eigenvalues of the correlation matrix of the Gaussian distribution has been designed and seems to satisfactorily solve the problem.

On-going work is concerned with replacing the simple Gaussian distribution by a mixture of Gaussian, or other more complex distribution models, and the EM-like *MIXMOD* algorithm developed at INRIA is used to build the distribution from a given sample. This study of the best way to select in a given sample also lead Anne Auger to design a stochastic approximation of the gradient and Hessian of the target function and to hybridize EDAs with deterministic gradient-based methods.

6.11. Local regularity analysis of road profiles

Participants: Pierrick Legrand, Jacques Lévy Véhel.

Key words: *Bayesian multifractal denoising*, *Hölder regularity*, *interpolation*.

This study tries to answer a question raised by the LCPC, Nantes: Is it possible to relate the local regularity of road profiles to the road-tyre friction ? A major problem in this study is to obtain accurate road profiles, especially at fine scales. It appears that both laser and tactile captors yields data which suffer from a strong amount of high-frequency noise. Unfortunately, it seems that the high-frequency information is important in relating the shape of a profile to the road-tyre friction coefficient. We have developed two fractal-based methods to overcome this problem.

1. Bayesian multifractal denoising.

A celebrated wavelet-based method for data denoising is based on thresholding "small" coefficients, since these are usually due to noise. This method and its numerous variants possess excellent theoretical properties. They also are well-documented to provide, in many situations of interest, very good experimental results. However, due to their very assumptions, they are not well fitted to the processing of irregular signals as are road profiles. Such signals exhibit many relatively small wavelet coefficients. Wavelet thresholding will in general result in an oversmoothing of the data. While this may be innocuous in certain applications, in our case it entails a loss of important information pertaining to friction. We have developed an alternative technique, based on the assumption that the profiles are "multifractal", *i.e.* have a wildly varying local regularity. Using the multifractal spectrum as a *prior* information, it is easy to derive a Bayesian estimator of the original profile. This estimator will in general share the regularity properties of the noise-free data [32]. As a validation of this technique, we have shown that certain descriptors computed from the Bayesian-denoised signal show improved correlation with the friction coefficient, as compared to both the noisy profile and its wavelet-threshold denoising [47].

2. Hölder regularity-based interpolation.

Another path for recovering the high-frequency content of noisy profiles is as follows: Start from low-frequency acquired profiles, and interpolate the samples to obtain "clean" high-frequencies. Obviously, any smooth interpolation technique (e.g. spline interpolation) is of no use here, since we are trying to recover an irregular high-frequency content. The principle of the Hölder regularity-based interpolation is to interpolate the data in the wavelet-domain in such a way that the local Hölder exponent is preserved in the process. As a consequence, regions which are smooth, or, to the contrary, irregular in the low-frequencies will remain so after interpolation [34]. Again, this technique is well adapted to the case of irregular signals. As was the case for the Bayesian multifractal denoising, this approach has been validated through a model of road-tyre friction and its correlation to experimental data [47].

6.12. Interactive Evolutionary denoising

Participants: Jacques Lévy Véhel, Evelyne Lutton, Karim Seghouane.

Key words: *Multifractal denoising*, *interactive evolution*, *denoising*.

Multifractal denoising techniques necessitate a delicate parameter setting, which may slightly vary for each image. Theoretical investigations do not always provide solutions to a fully satisfying parameter setting. There remains “free” parameters to be set, which actually depend on a subjective evaluation. In this work, we intend to address the problem of parameter adaptation using an interactive evolutionary scheme. The idea is to evolve a well chosen parameter subset in order to optimise an expert evaluation, who has to compare a set of output images (the “population”) via an interface.

This study, started in October 2003, is the postdoctoral subject of Karim Seghouane.

6.13. The fly algorithm for stereovision

Participants: Amine Boumaza, Jean Louchet.

In collaboration with Michel Parent (projet IMARA).

The Fly algorithm described here is an evolutionary algorithm devised for parameter space exploration in computer vision applications

It is based on the *Parisian* paradigm which considers the whole evolving population as a potential solution and each individual as a part of the solution of the problem rather than as a complete solution. The principle of its application to stereo-vision is to evolve a population of 3-D points (the ‘flies’) and optimise an objective function such that the flies concentrate onto the surfaces of the objects in the scene.

Figure 4. Reconstruction results (red points), shown on the left and the right images.

A matching sensor fusion and trajectory planning algorithm has been developed and tested on a simulator. Undergoing work aims at implementing them on a Cycab experimental vehicle equipped with a stereo camera system and an on-board PC. Our algorithm fuses on-board lidar data with stereo information to build a representation of the scene. Figure 4 shows typical reconstruction results. The results were obtained after 30 to 40 generations of 5000 flies after 1.05s to 1.4s of computation time on a standard commercial PC with a 2GHz CPU. The fly algorithm does not achieve reconstruction results comparable in accuracy with other state-of-the-art stereo techniques but is a cost effective method allowing flexible real-time speed-accuracy trade-off.

Amine Boumaza’s PhD thesis will normally end in early 2004, another PhD student, Olivier Pauplin has begun his PhD in December 2003 and will continue his work on the development and implementation of the fly algorithm in robotic applications with IMARA.

6.14. Multi-objective sensor planning with evolutionary Algorithms

Participants: Enrique Dunn, Jean Louchet, Evelyne Lutton, Marc Schoenauer.

In collaboration with Gustavo Olague (CICESE, Mexique).
Key words: *Multi-objective optimisation*, *3D inspection system*, *evolutionary algorithms*, *robot vision*.

This study takes place in a collaboration with the computer science department of the CICESE of Mexico under a LAFMI grant, and concern the study of an experimental robot vision system for 3D automated inspection. The purpose of this collaboration is to design efficient and adapted evolutionary techniques to tackle the various complex optimisation problems involved in such systems

During the 3 month intern of Enrique Dunn, who is currently PhD student at CICESE under the direction of Gustavo Olague, a multi-objective formulation of the successive positions of the camera was studied.

Experiments on a simulated environment have provided convincing results. An implementation on the experimental system of the CICESE will follow.

6.15. Interactive parametrisation of Cochlear Implants with Evolutionary Algorithms

Participant: Pierre Collet.

In collaboration with Pr. Bruno Frachet, head of ORL service of Hôpital Avicenne and Claire Bourgeois-République, Dijon University.
Key words: *interactive evolutionary algorithms*, *Parisian approach*, *bio-computer-science*.

Cochlear implants are electronic devices which attempt to give back auditive sensations to otherwise incurable deaf patients whose auditive nerve is however still functional. The input is an acoustic signal that is obtained from an external microphone. The signal is processed through a microprocessor which turns it into electric impulses that are sent into an array of electrodes which is inserted into the cochlea, in order to artificially stimulate the auditive nerve.

Many deaf people can benefit from cochlear implants. However, the history of the patient as well as the cause of deafness has a lot of influence on the good parametrisation of the cochlear implant. Many studies show that the brain area responsible for audition is very plastic, meaning that all patients are different.

All in all, it is very difficult to draw conclusions and develop a deterministic fitting methodology when patients with a similar background react very differently. In 2003, around 50 000 deaf people have been implanted with such devices around the world.

Success depends on many different factors (age of implantation, number of years of deafness before implantation, etc) but also implant parametrisation. The aim of this work is to develop an interactive software based on an evolutionary algorithm that would be capable of helping both patients and practitioner in the parametrisation of the cochlear implant [28][29].

Claire Bourgeois-République is now starting the third year of her PhD. A prototype is currently being elaborated with the MXM implant manufacturer. First results are expected by the beginning of January.

6.16. Interactive Genetic Programming for the Text-Retrieval.

Participants: Pierre Collet, Yann Landrin-Schweitzer, Evelyne Lutton.

In collaboration with Thérèse Vachon and Pierre Parisot from IK@N section Knowledge Engineering, Novartis Pharma AG.

Keyword-based text-retrieval systems are generally unable to handle the relevancy of detected keywords within their context. Attempts to solving this hurdle often rely on thesauri. These dictionaries of synonyms build the notion of "sense" by grouping together terms of close signification. However, this methods is only as efficient as the thesauri are pertinent, and does not offer leeway for users' specificities. We attempt to provide context analysis by an alternate method: evolutive individual search profiles. A working prototype, christened Elise, has provided an experimental validation of this approach. It uses text-retrieval and semantic tools in a genetic programming framework, designed for an easy evolution toward an industrial-grade system.

Tests conduced on TREC bases (an international reference for the evaluation of search engines) have shown Elise's unique capabilities. The evolutionary approach let it react extremely quickly and efficiently to knowledge context variations and users' search behaviors. The analysis of the users' profiles semantic contents have further shown high lexical innovation and generalization capabilities.

The two task of bringing Elise up to date with top-notch text-retrieval tools and improving its specific capabilities are the core of a Post-Doc finded by Novartis Pharma.

6.17. Resolution of strongly constrained problems

Participants: Evelyne Lutton, Matthieu Pierres.

Key words: *algorithme évolutionnaire*, *resource allocation problem*, *contraints resolution*.

In collaboration with Francois Fages (projet CONTRAINTES).
Constraint satisfaction problems are in general extremely complex optimisation problems. The problem of interest in this work is the one of office affectation on the INRIA Rocquencourt campus: the demand of research teams exceed the actual resource. This is thus an over-constrained problem. Moreover the constraints and preferences of each team are difficult to represent and tune within standard constraints satisfaction software. Evolutionary techniques have been used as a complement to constraint satisfaction tools with respect to the user interaction. Actually many constraints are difficult to express and the relative importance of each constraint is an important factor for effective usage of constraint satisfaction software.

We have experimented a multi-user interactive evolutionary approach for the management of relative weights of user preferences, based on a Parisian and multi-population paradigm. A prototype will be available at the end of 2003 for testing.

6.18. Artificial Ant Colonies for E-Learning.

Participants: Pierre Collet, Evelyne Lutton, Yann Semet, Grégory Valigiani.

Key words: *Ant Colony Optimisation (ACO)*, *E-Learning*.

We apply the Ant Colony Optimisation (ACO) heuristics [57][48][49] to an E-learning problem: The pedagogical material of an online teaching web site for high school students is modelled as a navigation graph where nodes are exercises or lessons and arcs are hypertext links. The arcs' valuation, representing the pedagogic structure and conditioning the web site's presentation, is gradually modified through the release and evaporation of virtual pheromones that reflect the successes and failures of students roaming around the graph.

A compromise is observed to emerge between the pedagogical structure as originally dictated by professors, the collective experience of the whole pool of students and the particularities of each individual.

The purpose of this study conducted for Paraschool, the leading French e-learning company, is twofold: enhancing the web site by making its presentation intelligently dynamic and providing the pedagogical team with a refined auditing tool that could help it identify the strengths and weaknesses of its pedagogic choices.

This work, initiated by B. Leblanc's 2002 post doctoral work and based on Y. Semet's M. Eng. thesis work, resulted in three publications to date [38][39][18]. A software prototype was also implemented and calibrated on Paraschool's website. This fruitful collaboration is continuing with G. Valigiani's PhD thesis.

6.19. Modelling of Termite Nest formation

Participants: Emmanuel Cayla, Evelyne Lutton.

Key words: *Social insects*, *termites*, *fractal growing model*.

In collaboration with Michel Lepage, Pascal Jouquet (Laboratoire Fonctionnement et Evolution des Systèmes Ecologiques, UMR 7525- Ecole Normale Supérieure) and Yves Le Goff (Ecole Nationale Supérieure des Arts et Métiers, Laboratoire Mécanique des Fluides).

The aim of this collaboration with biologists is to understand the mechanisms of nest construction for a particular specie of termites (macrotermes bellicosus). These termites are living in africa (in dry as well as forest areas), they build specific structures, rather irregular, but with some charateristic towers-like components. Their nest is composed of several internal and external structures, with food strorage area, mushrom plantation (they actually grow and eat a particular specie of mushroom), queen chamber and nursery. The nest is a structure that evolves gradually, with respect to the size and age of the colony, as well with respect to the environment and climatic conditions. The challenge is to understand the connections between nest architechture and climatic conditions (and eventually elaborate behavioural models of it).

In this work, we consider a simplified model of some external nest structure, that are usually built by the colony within a single night. The proposed model is based on a population behaviour with elementar social interactions (an ACO model), that has been derived from biologic observations. 2D simulation proves the capability of such a model to produce fractal structures, similar to the natural ones (see figure 5).

Figure 5. A fractal model of small towers construction.

6.20. Agent-based Computational Economics

Participants: Marc Schoenauer, Yann Semet.

In collaboration with S. Gelly and Michèle Sebag (Orsay University)
Key words: *Agent-based Computational Economics*, *Speculative bubbles*.

Modeling economic or social systems in general and financial markets in particular with distributed networks of evolutionary agents is a very active and growing field known as Agent-based Computational Economics (ACE). It aims at explaining global behaviours and structures of social systems in terms of multiple iterative interactions of simple but adaptive localized agents. Our work studies a community of computationally simulated agents, and concentrates on exchange price fixing under the double auction mechanism.

We work within two distinct frameworks. The first is based on a simple simulation of a goods market where two populations, buyers and sellers, interact to iteratively set an exchange price for a single consumption good (an asset cannot be stored and later resold). The exchange prices as well as the global surplus are both found to consistently converge to the predictable equilibrium. The second framework is an elementary artificial stock market where the focus is on the relationship between individual rationalities and the birth, rise and "panicky" downfall of speculative bubbles.

Originated under the DREAM European project as an exploration of the ACE field [42], this work is today more focused on the particular case of financial markets and aims at explaining speculative phenomena as arising from the interactions of a network of distributed evolutionary agents.

6.21. Anticipation in Evolutionary Robotic Learning

Participants: Nicolas Godzik, Marc Schoenauer.

In collaboration with Michèle Sebag (Orsay University)
Key words: *Robotic*, *Anticipation*.

Nicolas Godzik's thesis is strongly relied to the ROBEA Project (ROBotics and Artificial Entities, a French robotic program launched by CNRS in 2001). The first year was carried out in collaboration with Marc Schoenauer and Michèle Sebag.

This first work was inspired by Dawkins' ideas: natural evolution succeeds thanks to the existence of basis bricks and the capacity of evolution to reorganize these bricks. This work is an intermediate between Brooks' subsumption architecture, in which the user must manually decompose the target task into different basic tasks, and the black-box evolutionary approach, as reported by Nolfi and Floreano in their book, where a controller (neural network) is evolved with very little information. In our approach, symbolic controllers and supervisors (e.g. neural networks) have been developed that use as outputs some elementary behaviors. The results have been experimented on standard benchmark experiments like the battery-recharging experiment, and demonstrated the validity of the approach. This project developed this year with the arrival of a new partner, the team "Situated Perception" at LIMSI. Nicolas Godzik and Jeremie Mary have created a new robot simulator that is much more flexible than the port of EvoRobot from S. Nolfi. This software has also been interfaced with the evolutionary library EO (but can be used as stand-alone too). The graphical interface is based on OpenGL. The main advantage of this software is that it can handle unlimited number of robots, and can be customized to any type of robot.

One goal of the work undertaken this year is to understand the role of the perception cells in the controller of the robot. Another theme of Nicolas Godzik's thesis is inspired by the sensori-motor contingencies theory. In this approach the consequences of the actions are very important to disambiguate a confusing situation. To make this more explicit, consider the following example: when someone is trying to find a chair in a given area, the sensori-motor contingencies result in that the chair is not only viewed as a shape with four legs but also as an object which allows one to sit on it.

To take this point of view into account, an original neural network architecture has been designed, which introduces an anticipation part: some outputs are compared to the actual values of the sensors at the next step of the robot life, and the weights of the whole network are adjusted using a back-propagation-like rule after those values have been compared to the actual sensor values. On-going validating experiments include the Lanzi maze and the Morris swimming pool, both experiments involving ambiguous situations.

6.22. Evolutionary multi-objective optimisation

Participant: Marc Schoenauer.

In collaboration with Olga Roudenko, Hatem Hamda and Mohamed Jebalia (Orsay University)
Key words: *Multi-objective*, *Optimum Design*, *Stopping criterion*, *Pareto crossover*.

Olga Roudenko's PhD dissertation is centered on Evolutionary Multi-objective Optimisation Algorithms (EMOAs). Its main contributions are the definition of a stopping criterion for EMOAs, a multi-objective specific crossover and an application to two real-world problems from car industry. The goal of EMOAs is to sample the Pareto front of the problem at hand, i.e. the set of the best compromises between the contradictory objectives.

In the mean time, Hatem Hamda, after defending his PhD thesis in Tunis in May 2003, is continuing the study of EMOAs applied to Topological Optimum Design, inside some collaboration between INRIA and the Tunisian Universities. A first work involves multi-loading problems: the different loadings can be viewed as different objectives, or can be aggregated into one single objective as a weighted sum. But it is in general difficult to assess how much contradictory different loadings are. The expected outcome of this study is some measure of "contradictoriness" of different loading for the TOD problem, or of objectives in EMOAs in general.

Note that Hatem Hamda is using as a representation for structures the Voronoi representation that he has studied in his PHD thesis, while a newcomer in the team, Mohamed Jebalia, is starting at the master level to study new representations based on tree-like genotype representing VRML programs to describe a structure. This work is also part of the international collaboration INRIA/Tunisia.

6.23. Other Evolutionary Optimisation problems

Participant: Marc Schoenauer.

In collaboration with E. Dupuy, L. Jarlan.
Key words: *Optimisation*, *Launch vehicles*, *remote sensors*.

Some researchers in CESBIO (Toulouse) working on Sahelian surface modeling using radar remote sensing data are using Evolution Strategies to optimize the parameters of their models [20].

EADS Launch Vehicles has started a prospective study on the multi-objective optimisation of the whole launch vehicle using evolutionary algorithms. E. Dupuy, with the help of M. Schoenauer, within a research contract of 6 months with Fractales project, has developed the interface between EO and EADS simulation packages, and has validating the Evolutionary approach on single objective problems: whereas the gradient method that was used up to now requires a precise tuning of its starting point from the expert, the evolutionary method, though more expensive in terms of CPU, can robustly find good solutions ... that can then easily be fine-tuned by a few iterations of the gradient method. The very first result on multi-objective problems are encouraging, and EADS is proposing a continuation of the contract.

7. Contracts and Grants with Industry

The team has contracts with:

- NOVARTIS PHARMA about text-retrieval with evolutionary algorithms, PhD and Post-doctoral position of Yann Landrin-Schweitzer.
- DASSAULT AVIATION on terrain modelling based on mBm (PhD of Erick Herbin).
- PARASCHOOL on evolutionary optimisation of pedagogical path (e-learning, PhD of Gregory Valigiani).
- EADS Launch Vehicles, on multi-objective optimisation.

8. Other Grants and Activities

8.1. National initiatives

Our project a has collaborations with :

- IrCcyn, Institut de Recherche en Cybernétique et communications de Nantes, since 1996. Areas of collaborations: study of $1/f$ noises, Watermarking, regularity analysis of road profiles. For the last three years, the software "FracLab" has mainly been developed at IRCCYN.
- Littoral university (Calais), on e-learning (P. Collet and C. Fonlupt),

- Clermont Ferrand University (A. A. Benassi) and Toulouse Paul Sabatier University (A. Ayache et S. Cohen) on Multifractional Brownian Motion.
- Clermont Ferrand University (C. Tricot) on multifractal analysis.
- Yale University(R. Coiffman) for data analysis in high dimension (S. Lafon Ph.D. thesis).
- Orsay University (J. Peyrière) on multifractal analysis and warping.
- Monastir University (F. Ben Nasr, M. Mensi, H. Tlili) on multifractal analysis and diffusion on fractals.
- ROBEA: CNRS-INRIA ROBEA program (ROBotique et Entites artificielles) coordinated by Marc Schoenauer and Michèle Sebag (LRI-Orsay). Other participant: Laboratoire de Mécanique de l'INSA Rouen.
- l'Université d'Orsay, LRI (M. Sebag) on evolution and learning.

8.2. European initiatives

The team belongs to EvoNet, the European Excellence Network on artificial evolution.

8.3. International initiatives

The FRACTALES team collaborates with a Mexican research institute (CICESE, Física Aplicada, Gustavo Olague) under a LAFMI grant.

9. Dissemination

9.1. Organization committees

Pierre Collet, Evelyne Lutton and Marc Schoenauer have organised the « Evolution Artificielle '2003 » conference (Marseille, Nov 2003), and are member of the streering commitee of the french association for artificial evolution.

Marc Schoenauer is chairman of the AFIA (Association Francaise d'Intelligence Artificielle). He is main node and chairman of the electronic communication committee of EvoNet since its creation in 1995. He is member of the PPSN Steering Committee, of the ISGEC Executive Board (International Society on Genetic and Evolutionary Computation). He is *Tutorial Chair* of CEC'2003 (Congress on Evolutionary Computation).

9.2. Editorial Boards

Jacques Lévy Véhel is associate Editor of the journal « FRACTALS », referee for IEEE Trans. Signal Proc., IEEE Trans. Image Proc., Fractals, ICIP conference.

Evelyne Lutton has been referee for IEEE Transactions on Evolutionary Computation, IEEE Signal Processing Letters, Genetic Programming and Evolvable Machine, et IEEE Computer Graphics and Applications.

Marc Schoenauer is editor in chief of the *Evolutionary Computation* journal, MIT Pres., associate editor of *IEEE Transactions on Evolutionary Computation* and *Applied Soft Computing* (Elsevier), member of the advisory board of *Natural Computing Series*, Springer Verlag, of the editorial board of *Journal of Genetic Programming and Evolvable Machines* (Kluwer), *TCS-C – Theoretical Computer Science, Natural Computing* (Elsevier), and *Journal of Heuristics* (Elsevier). He is referee for the *Mathématiques et Applications* journal of SMAI (Industrial and Applied maths society).

9.3. Teaching at University

- « Wavelets and Fractals » DEA AIA of Ecole Centrale de Nantes (Jacques Lévy Véhel, 10 h).

9.4. Other Teaching

- "Fractals and Wavelets" Ecole Centrale de Paris (Jacques Lévy Véhel, Evelyne Lutton, Julien Barral, Stéphane Seuret, 39 h).
- "Fractals and Wavelets" ENSTA (Evelyne Lutton, Jacques Lévy Véhel, 21 h)
- "Fractals and Time-frequency analysis" Centrale de Nantes (Jacques Lévy Véhel, 15 h).
- "Fractals" ESIEA (Jacques Lévy Véhel, 15 h).
- "Fractal Analysis" INT (Jacques Lévy Véhel, 6 h).
- "Artificial Evolution" ENSTA (Evelyne Lutton, Pierre Collet, Cyril Fonlupt, Michèle Sebag, 21 h).
- "Optimisation" ENPC (Marc Schoenauer).
- Marc Schoenauer is part time teacher at Ecole Polytechnique.

9.5. Ph.D. Theses

- Yann Landrin-Schweitzer: "Algorithmes Génétiques Interatifs pour le Text-Retrieval," September 12, 2003, Orsay university.
- Stéphane Seuret: "Analyse 2-microlocale et quelques applications à l'analyse multifractale", november 5, 2003, Ecole Polytechnique.

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