INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team OMEGA

Méthodes numériques probabilistes pour les équations aux dérivées partielles et les mathématiques financières

Sophia Antipolis - Lorraine

THEME 4B

Activity Report

2003

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1. Team

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On October 26th Axel Grorud suddenly died. Axel was a member of the OMEGA team since its creation in 1994. He brought us his mathematical skills and his thorough knowledge in finance. He also brought us his warm friendship. We all miss him a lot. We dedicate this report to his memory.

2. Overall Objectives

2.1. Introduction

The Inria Research team OMEGA is located both at Inria Sophia-Antipolis and Inria Lorraine. The team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, neurobiology, chemical kinetics.

Our competences cover the mathematics behind stochastic modeling and stochastic numerical methods. We also benefit from a wide experimental experience on calibration and simulation techniques for stochastic models, and on the numerical resolution of deterministic equations by probabilistic methods. We pay a special attention to collaborations with engineers, practitioners, physicists, biologists and numerical analysts.

2.2. Probabilistic numerical methods

Concerning the probabilistic resolution of linear and nonlinear partial differential equations, the OMEGA team studies Monte Carlo methods, stochastic particle methods and ergodic methods. For example, we are interested in fluid mechanics equations (Burgers, Navier-Stokes, etc.), in equations of chemical kinetics and in homogeneization problems for PDEs with random coefficients.

We develop simulation methods which take into account the boundary conditions. We provide non asymptotic error estimates in order to describe the global numerical error corresponding to each choice of numerical parameters: number of particles, discretization step, integration time, number of simulations, etc. The key argument consists in interpreting the algorithm as a discretized probabilistic representation of the solution of the PDE under consideration. Therefore part of our research consists in constructing probabilistic representations which allow us to derive efficient numerical methods. In addition, we validate our theoretical results by numerical experiments.

2.3. Stochastic models: calibration and simulation

In financial mathematics and in actuarial science, OMEGA is concerned by market modelling and specific Monte Carlo methods. In particular we study calibration questions, financial risks connected with modelling errors, and the dynamical control of such risks. We also develop numerical methods of simulation to compute prices and sensitivities of various financial contracts.

In neurobiology we are concerned by stochastic models which describe the neuronal activity. We also develop a stochastic numerical method which will hopefully be useful to the Odyssée project to make more efficient a part of the inverse problem resolution whose aim is to identify magnetic permittivities around brains owing to electro-encephalographic measurements.

3. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and, instead, try to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modeling issues: one has to choose appropriate stochastic models, which requires a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The OMEGA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The OMEGA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The OMEGA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

OMEGA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Fluid Mechanics, Chemical Kinetics, Finance and Biology.

4.1.1. Fluid Mechanics

In Fluid Mechanics OMEGA develops probabilistic methods to solve vanishing vorticity problems and to study complex flows at the boundary, in particular their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our expertise concerns

- The convergence analysis of the stochastic particle methods on theoretical test cases. In particular, we explore speed up methods such as variance reduction techniques and time extrapolation schemes.
- The design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluid (the FENE model). A second one concerns the Lagrangian modeling of turbulent flows and its application in combustion for two–phase flows models (joint collaboration with Électricité de France).
- The Monte Carlo methods for the simulation of fluid particles in a fissured (and thus discontinuous) porous media.

4.1.2. Chemical kinetics

An important part of the work of the OMEGA team concerns the coagulation and fragmentation models.

The areas in which coagulation and fragmentation models appear are numerous : polymerization, aerosols, cement and binding agents industry, copper industry (formation of copper particles), behavior of fuel mixtures in engines, formation of stars and planets, population dynamics, etc.

For all these applications we are led to consider kinetic equations using coagulation and fragmentation kernels (a typical example being the kinetics of polymerization reactions). The OMEGA team aims to analyze and to solve numerically these kinetic equations. By using a probabilistic approach we describe the behavior of the clusters in the model and we develop original numerical methods. Our approach allows to intuitively understand the time evolution of the system and to answer to some open questions raised by physicists and chemists. More precisely, we can compute or estimate characteristic reaction times such as the gelification time (at which there exists an infinite sized cluster) the time after which the degree of advancement of a reaction is reached, etc.

4.1.3. Finance

For a long time now OMEGA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modeling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.4. Biology

For a couple of years OMEGA has studied stochastic models in biology, and developed stochastic methods to analyze stochastic resonance effects and to solve inverse problems. For example, we are concerned by the identification of an elliptic operator involved in the calibration of the magnetic permittivity owing to electro-encephalographic measurements. This elliptic operator has a divergence form and a discontinuous coefficient. The discontinuities make difficult the construction of a probabilistic interpretation allowing us to develop an efficient Monte Carlo method for the numerical resolution of the elliptic problem.

6. New Results

6.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Olivier Bardou, Mireille Bossy, Pierre Étoré, Axel Grorud, Antoine Lejay, Sylvain Maire, Miguel Martinez, Denis Talay.

Key words: *Euler scheme*, *divergence form operator*, *discontinuous coefficient*.

6.1.1. The Euler scheme for a process associated with a divergence form operator, and application to the MEG model

The MEG problem, which consists in estimating the conductivity coefficients of the different brain layers, is an inverse problem that the ODYSSEE project at INRIA solves numerically using an iterative algorithm. At each step of the algorithm, it is necessary to compute a few values of the solution of a particular elliptic PDE defined in the brain [51]. This elliptic PDE involves a Divergence Form Operator L defined as

$$Lu = \nabla \cdot [a \nabla u] \, .$$

The matrix a is discontinuous along the boundaries separating the different layers of the brain and this phenomenon makes it quite hard to solve the elliptic PDE numerically by probabilistic methods [53].

However, in the one-dimensional context, the stochastic process associated to L is the weak solution of a stochastic differential equation that involves the local time of the unknown process [52]. In order to simulate the solution of such a kind of equation and to develop an efficient Monte Carlo method for the above elliptic equation, M. Martinez, D. Talay and A. Lejay propose a Euler scheme with a discontinuous coefficient whose theoretical analysis and numerical study are still in progress. Motivated by part of this theoretical analysis we consider the one-dimensional stochastic differential equation

$$X_t = X_0 + \int_0^t \sigma(X_s) dB_s + \frac{1}{2} \int_{\mathbb{R}} L_t^x(X) \nu(dx) \tag{1}$$

where B is a Brownian Motion and $L^x(X)$ denotes the right local time of the semimartingale X at point x. We have proven that, under appropriate conditions on the function σ, there exists a weak solution to (1).

6.1.2. Stochastic resonance and neuronal systems

This year, we have continued our work on the modeling of stochastic resonance effects in the neuronal activity. We have to face important technical difficulties due to the huge complexity of the analytical formulae describing the probability densities of particular stopping times related to our model for the electric potential along the neurons. We are trying to deal with approximate formulae which would allow us to quantify the level of random noise which should be added to the internal noise in order to improve the efficiency of the neuronal activity, in the sense that the period of an electrical input signal is better recognized by the neuronal system.

6.1.3. Bootstrap for maximum likelihood estimators of diffusion processes

Key words: *statistics of diffusion processes*, *likelihood*, *bootstrap*, *asymptotic expansions*.

The well known bootstrap method belongs to the family of modern statistical tools exploiting Monte Carlo simulations to obtain precise estimators and powerful tests for complex models. It has been used for over twenty years to refine the performances of estimators in various statistical settings. Yet, as far as we know, very little is known on the theory of the bootstrap in the context of Brownian diffusions. Thus, O. Bardou and D. Talay propose a new methodology to construct bootstrap corrections to the maximum likelihood estimators of diffusion processes.

Let B be a standard Brownian motion over some probability space and X, the unique strong solution of the following stochastic differential equation:

$$dX_t = \theta b_t(X)dt + dB_t,\ t \in [0, T].$$

The process X is supposed to be ergodic. It is well known that the maximum likelihood estimator $\hat{\theta}_T$ of the parameter θ can be written as

$$\hat{\theta}_T = \theta + \frac{\int_0^T b_s(X)\,dB_s}{\int_0^T |b_s(X)|^2\,ds}.$$

Exploiting the self normalized martingale structure of this expression, we are able to construct a bootstrap procedure correcting for the bias and the coverage errors of confidence intervals of $\hat{\theta}_T$. The methodology mainly relies on asymptotic expansions of the bias and the distribution function of $\hat{\theta}_T$ in powers of $T^{-\frac{1}{2}}$. During this work, numerous technical questions related to the long run convergence of the process X have been risen and are now under study.

6.1.4. Estimation of the skew coefficient for stochastic processes reflected in a boundary domain

Key words: *skew processes*, *reflected processes*, *statistics of random processes*, *ergodicity*.

Let B be a standard Brownian motion over some probability space and X, the unique strong solution of the following stochastic differential equation:

$$X_t = x + \int_0^t \sigma(X_s)\,dB_s + (2\alpha - 1)L_t^0(X) - K_t^{[a,\,b]},\ x \in [a, b],\ t \in [0, T].$$

The process $L^0(X)$ is the symmetric local time in 0 and $K^{[a,\,b]}$, a process inducing the reflection over the interval $[a,\, b]$. The function σ is known. A typical example of such a process is the so-called skew Brownian motion arising in the modelling of geophysical phenomena.

The coefficient α is called the skew coefficient. In this setting, O. Bardou and M. Martinez construct an estimator of α from a single observation of the process X over the time interval $[0, T]$. They prove the strong consistency and the asymptotic normality of this estimator as T goes to infinity. Numerical simulations assess the efficiency of their approach.

O. Bardou and M. Martinez have also extended their results to the case of several discontinuities, and now aim to introduce a drift term in the dynamics of the process X.

6.1.5. Approximation and integration by Monte Carlo methods

S. Maire has worked on sequential Monte Carlo methods to compute approximations and integrals of multivariate real-valued functions. In [25] he has described properties of his algorithm and applications to the approximation of univariate smooth functions. He has also shown how to use the algorithm to integrate multivariate smooth functions by introducing approximation spaces similar to Korobov ones (see [24][26]). In [43] he has proposed to accelerate the algorithm by using Quasi-Monte Carlo sequences.

In collaboration with Emmanuel Gobet (CMAP, École Polytechnique), S. Maire has developed an adaptive Monte Carlo method to compute a spectral approximation of the solution of the Poisson equation. The variance and the bias due to the simulations of the involved stochastic processes geometrically decrease when the number of steps increases. The method is compared to deterministic methods (see [39]).

6.1.6. Simulation of particles in a fissured network

In [23] A. Lejay proposes an algorithm to simulate a fluid particle in a fissures network based on the simulation of a diffusion process on a graph. A. Lejay and P. Étoré are now trying to improve this algorithm to take a convective term into account, and to test it on real data.

6.1.7. Probabilistic numerical methods for McKean–Vlasov equations

In collaboration with Arturo Kohatsu-Higa (Universitat Pompeu Fabra, Barcelona), M. Bossy and D. Talay work on Romberg extrapolation methods for stochastic particle methods for McKean–Vlasov equations. The aim is to accelerate the convergence rate of these methods with respect to the time discretization step.

In the particular case of the Burgers equation with a smooth initial condition, it is shown that the Romberg extrapolation leads to a discretization scheme of order 2. A paper is in preparation.

6.1.8. Statistical and renewal results for the random sequential adsorption model applied to a unidirectional multicracking problem

Key words: *brittleness*, *coating*, *composite material*, *crack*, *fibre*, *Markov chain*, *Palm measure*, *Poisson point process*, *relaxation of stress*, *renewal process*, *rupture*, *stationary processes*.

In collaboration with P. Calka (Université Paris 5) and A. Mézin (École des Mines, Nancy), P. Vallois works on a stationary process on the real line which models the positions of the multiple cracks that are observed in composite materials submitted to a fixed unidirectional stress ε. This model generalizes to some extent the one-dimensional Random Sequential Adsorption construction. We calculate the intensity of the process and the distribution of the inter-crack distance in the Palm sense. Another point of view is developed, where the positions of the cracks X_i^ε are described from a fixed origin. We prove that the sequence $\{(X_i^\varepsilon, Y_i^\varepsilon), 1 \le i \le n\}$ is a conditioned renewal process, where Y_i^ε is the value of the stress at which X_i^ε forms. The approaches "in the Palm sense" and "fixed origin" merge for $n \to +\infty$. We also investigate the saturation case, i.e. $\varepsilon = +\infty$.

A paper was submitted in June 2003 to "Stochastic Processes and their Applications" [37].

6.2. Financial Mathematics

Participants: Mireille Bossy, Awa Diop, Bernard Roynette, Denis Talay, Etienne Tanré, Pierre Vallois.

6.2.1. Discretization of Cox–Ingersoll–Ross type

Key words: *Euler scheme for SDEs*, *approximation of the Cox-Ingersoll-Ross model and of Bessel processes*.

A. Diop, M. Bossy and D. Talay have finished to study the discretization of generalized Cox–Ingersoll–Ross and Hull–White models for instantaneous interest rates. In these models the drift coefficient has bounded derivatives whereas the diffusion coefficient is of the type $\sigma(x) = x^\alpha$, with $1/2 \le \alpha < 1$. For such models we propose a discretization scheme which guarantees the positivity of the approximated processes and a good convergence rate in the weak sense when the test functions are smooth.

6.2.2. Range of a Brownian motion with drift

According to the Black-Scholes model, stock prices are exponentials of Brownian motions with drifts. The drifts are the instantaneous expected rates of return of the stocks. Thus, it seems possible to construct an estimator of an unknown drift based upon the successive price amplitudes and to provide a rigorous mathematical framework for technical analysis methods used by practitioners in financial institutions. To this end, in [34] É. Tanré and P. Vallois have studied the range (that is, the difference between the running maximum and minimum) of a Brownian motion with drift. Assume that the drift is positive. The process tends to infinity as time increases. In particular, there exists an absolute minimum. Therefore one can decompose the Brownian path backwards in time (from the instant when the process reaches his minimum) through successive amplitudes. This introduces a countable sequence of random times (which are not stopping times) and of processes. The probability distribution of these various objects is described. The authors deduce the probability distribution of the first time $\theta(a)$ when the range process of a Brownian motion with drift reaches the level a.

6.2.3. Modelling of financial techniques

In collaboration with Rajna Gibson (Zürich University) and Christophette Blanchet (Université de Nice Sophia-Antipolis), A. Diop, É. Tanré, D. Talay, M. Martinez and D. Rovanova elaborate an appropriate mathematical framework to develop the analysis of the financial performances of some financial techniques which are often used by the traders; to study the impact of model risk on such strategies and to study the question: is it possible to improve some techniques used in practice by adding mathematical models? We are finishing to prepare a paper on our results. The involved mathematical techniques are issued from statistics of random processes and stochastic control. This research is funded by NCCR FINRISK (Switzerland) and is a part of its project "Conceptual Issues in Financial Risk Management".

In addition, Ouaile El Fetouhi and D. Talay have developed an original and promising model for the VWAP which is a financial index that the traders try to beat. This model may allow one to analyse the performances of the VWAP techniques.

6.2.4. Game theory and market of electricity

In collaboration with Nadia Maizi (CMA, École des Mines de Paris), Geert Jan Olsder (Delft University, the Netherland) and Odile Pourtallier (Comore and Miaou projects, INRIA Sophia Antipolis), M. Bossy and É. Tanré have applied game theory to model the market of electricity.

The deregulation of the market of electricity in European countries, initiated in December 1996, has raised lot of modifications, in particular new spot markets of electricity have emerged. These markets are close to the pollution right markets that start to appear as a consequence of the application of the Kyoto protocol and thus are very peculiar. As, in addition, the electricity cannot be stored, the classical market analysis methods do not apply, and new approaches need to be explored. We have analyzed a simple model with one market and N producers using game theory. In particular we showed that for a simple bid (quantity-price), it is not possible to find a Nash equilibrium. This has raised the necessity to introduce more complex bids, and consequently, in terms of game theory, more complex strategy set. On a theoretical ground, we have analyzed the possibility to use conjectural strategies and inverse Stackelberg equilibrium.

6.2.5. A ruin problem

Key words: *Lévy processes*, *ruin problem*, *hitting time*, *overshoot*, *undershoot*, *asymptotic estimates*, *functional equation*.

In collaboration with A. Volpi (ESSTIN), P. Vallois works on the ruin time of insurance companies. Let X be a Lévy process, right continuous with left limits, started at 0, with Lévy measure ν. We are interested in the first hitting time of level $x > 0 : T_x := \inf\{t \geq 0;\ X_t > x\}$. Setting $Z_t := x - X_t$, then $T_x := \inf\{t \geq 0;\ Z_t < 0\}$ is the ruin time to a company whose fortune is modelled by Z. We also consider the overshoot K_x, respectively the undershoot $L_x : K_x := X_{T_x} - x, L_x := x - X_{T_x-}$. The aim is the study of the joint distribution of (T_x, K_x, L_x). Our approach uses the joint Laplace transform of (T_x, K_x, L_x) defined as

$$F(\theta, \mu, \rho, x) = \mathbb{E}\left(e^{-\theta T_x - \mu K_x - \rho L_x} 1_{\{T_x < +\infty\}}\right)$$

for all $\theta \geq 0$, $\mu \geq 0$, $\rho \geq 0$ and $x \geq 0$. If $\theta = \mu = 0$, $\quad F(0,0,0,x) = \mathbb{P}(T_x < +\infty)$ is the ruin probability.

When $\nu(\mathbb{R}) < +\infty$ we exhibit conditions for which $F(\theta, \mu, \rho, x) \sim_{x \to +\infty} C_0(\theta, \mu, \rho) e^{-\gamma_0(\theta) x}$.

Suppose now that ν has exponential moments. Then $F(\theta,\mu,\rho,\cdot)$ has the following expansion:

$$F(\theta,\mu,\rho,x)=C_0(\theta,\mu,\rho)e^{-\gamma_0(\theta)x}+\sum_{i=1}^{p}\left(C_i(\theta,\mu,\rho)e^{-\gamma_i(\theta)x}+\overline{C_i}(\theta,\mu)e^{-\overline{\gamma_i}(\theta)x}\right)+\mathrm{O}\left(e^{-Bx}\right),$$

where $0\leq\gamma_0(\theta)<Re\gamma_1(\theta)\leq\cdots\leq Re\gamma_p(\theta)$. When the support of ν is included in $[0,+\infty)$, the constants $C_i(\theta,\mu,\rho)$ are explicit.

We also study the rate of decay of the ruin probability and prove that, after a suitable normalization, the triplet (T_x,K_x,L_x) converges in distribution as $x\to\infty$.

Two papers have been submitted to the “Annales de l’Institut Henri Poincaré” [46][45].

6.3. Chemical Kinetics

Participants: Jean-François Collet, Madalina Deaconu, Étienne Tanré.

6.3.1. Non extensive thermodynamics

Key words: *Shannon entropy*, *Gibbs measures*, *Tsallis distribution*.

This work is originally motivated by a desire to understand precisely what, among all possible convex quantities, singles out the Shannon entropy. In the framework of distribution functions with a certain number of prescribed moments, J-F. Collet gives an explicit relation between the structure of the minimizer (say, a Gaussian distribution) and the quantity minimized at equilibrium (say, the Shannon entropy). This correspondance being established, properties of the entropy may be read-off from the expression of the minimizer. We then show that the entropy satisfies a natural homogeneity property if and only if the minimizer is Gaussian. If one then relaxes the homogeneity assumption in a natural way, a new distribution may arise, which turns out to be the Tsallis distribution. Besides providing a new characterization of this ditribution, this establishes a link between classical moment systems and non extensive thermodynamics, which we plan to investigate in the future.

6.3.2. Dissipative systems with no invariant measure

Key words: *parabolic equations*, *fundamental solution*, *intermediate asymptotics*.

In many examples of dissipative systems arising in probability theory (for instance some Kolmogoroff equations associated to stationary Markov processes), the uniqueness of the equilibrium measure together with a dissipation property (e.g. the existence of a Lyapunov functional) may be used to derive trend to equilibrium for large times. We are interested in systems which do not possess any invariant measure. For some of them J-F. Collet has shown that some quantities do decrease as time increases, a fact which may be used to study the large time asymptotics. A typical example is that of linear parabolic PDEs with time-dependent coefficients. In some cases this may be used to yield very quick proofs of the existence of some intermediate asymptotics.

6.3.3. Coagulation-fragmentation equations

The phycisist Smoluchowski introduced in 1917 a mathematical model which describes coagulation phenomena. It has many applications such as polymerization, formation of stars and planets, behavior of fuel mixtures in engines, etc. This system describes a non linear evolution equation of infinite dimension.

The aim of the probabilistic approach to this system is to give new results or to confirm conjectures formulated by analysts or physicists, with the methods of stochastic analysis.

The model

The equation describes the dynamics of an infinite system of particles in which coagulation phenomena occur. The particles are characterized by their mass. From a physical point of view, it is natural to suppose that the *rate of coagulation* of two particles depends on their masses.

Let us denote by $n(k,t)$ the density of particles of mass k at time t in a unit volume.

The Smoluchowski coagulation equation gives the time evolution of $n(k,t)$. It takes the following form :

$$\begin{cases} \dfrac{d}{dt}n(k,t) = \dfrac{1}{2}\displaystyle\sum_{j=1}^{k-1} K(j,k-j)n(j,t)n(k-j,t) - n(k,t)\sum_{j=1}^{\infty} K(j,k)n(j,t) \\ n(k,0) = n_0(k), \; k \geq 1, \end{cases} \quad (SD)$$

where K is the **coagulation kernel**. We assume that K is symmetric and positive.

Due to the presence of the infinite series, this problem is not a classial initial value problem for a system of non linear ordinary differential equations.

Results

In [49], we constructed a non linear process which *represents* the solution of (SD) (and also the continuous version of the Smoluchowski coagulation equation). In [50], we approximate this process with a finite system of particles. We also give the rate of convergence when the number of particles goes to infinity and we describe the error process (Central Limit Theorem).

Thanks to this representation, we study the asymptotic behaviour of the solution in the particular case when the kernel is homogeneous $(K(ax, ay) = a^{\lambda}K(x, y))$. In collaboration with Christophe Giraud (Université de Nice Sophia-Antipolis), we try to prove the convergence of every solution to a self-similar solution which depends on the initial condition through a few moments only.

In a more complicated model, we also allow particles to break into two particles (phenomenon of fragmentation). In this context, during the summer school CEMRACS 2003, Francis Filbet (CNRS-Orléans) and É. Tanré studied the large time behaviour of the solution. When the fragmentation and coagulation kernels are well chosen, every solution converges numerically to a stationary solution. We endeavor to prove this convergence.

This methodology was successfully applied to a problem originating in the modelling of industrial crushers. More precisely, a fundamental problem related to the optimization of the crushing process is to estimate the minimum amount of time required to achieve a prescribed degree of crushing. This question was suggested to us by R. Rebolledo (Pontificia Universidad Católica de Chile) during his visit in the framework of the INRIA-CONICYT collaboration programme. By modelling crushing as a pure fragmentation process, we designed an algorithm which yields a method for the computation of residence tumes in crushers.

Another generalization of Smoluchowski's model is obtained when one takes into account the position of particles as a supplementary variable (spatially non-homogeneous model). Nicolas Fournier (Université Nancy 1), B. Roynette and É. Tanré have proved the almost sure convergence of the position to 0 and of the mass to infinity with a particular choice of diffusion for the position process. [18].

6.4. Stochastic analysis and applications

Participants: Christophe Ackermann, Antoine Lejay, Ivan Nourdin, Bernard Roynette, Pierre Vallois.

In this section we present our results on issues which are more abstract than the preceding ones and, at first glance, might appear decorrelated from our applied studies. However most of them are originally motivated by modelling problems, or technical difficulties to overcome in order to analyse in full generality stochastic numerical methods or properties of stochastic models.

6.4.1. Rough paths theory

The theory of rough paths allows one to define stochastic integrals path by path as continuous functionals of their integrands (see, e.g., [22]). A. Lejay and Laure Coutin (Laboratoire de Statistiques et Probabilités, Université Toulouse III) have shown in [38] that this theory is coherent with the classical conditions allowing one to obtain the limit of stochastic integrals with respect to a sequence of continuous semi-martingales. They are now trying to use the theory of rough paths to study Stochastic Partial Differential Equations.

6.4.2. *Stochastic differential equations driven by a fractional Brownian motion*

Key words: *m-order integral*, *Itô's formula*, *fractional Brownian motion*.

The aim of this study is the development of a generalized stochastic calculus applied to processes which are not semimartingales. We especially focus on extension of Itô's formula to the fractional Brownian motion B^H. This process has been considered intensively in stochastic analysis and in many applications, e.g., in hydrology, telecommunications, fluidodynamics, economics and finance. This is a joint work by P. Vallois, M. Gradinaru (Université Nancy 1) and F. Russo (Université Paris 13). A paper was submitted in October 2003 to the Annales de l'Institut Henri Poincaré.

Recall that if the Hurst exponent H of B^H is different from $1/2$, then B^H is not a semimartingale. If f is of class C^2, and $H > 1/6$, we prove the following Itô formula:

$$f(B_t^H) = f(B_0^H) + \int_0^t f'(B_s^H) d^\circ B_s^H.$$

where $\int_0^t f'(B_s^H) d^\circ B_s^H$ denotes the symmetric (or Stratonovich) integral. This identity is due to the fact that the symmetric 3-order integral still exists (and vanishes) for $H > \frac{1}{6}$. Moreover $H = \frac{1}{6}$ is a barrier for validity. The introduction of a new class of integrals allows us to extend the previous identity to any $0 < H < 1$.

I. Nourdin studies the one-dimensional stochastic differential equation

$$dX_t = \sigma(X_t) dB_t^H + b(X_t) dV_t, \qquad t \in (0,1),$$

where B^H is a fractional Brownian motion of Hurst index $H \in (0,1)$, V is a bounded variation process and σ, b are real functions. By using the Russo-Vallois definition of the stochastic integral with respect to B^H, he has proven the existence and the uniqueness of the solution. He also has studied the convergence of the discretization Euler scheme and determined its rate of convergence.

6.4.3. *On first range times of linear diffusions*

Key words: *Bessel bridges*, *Bessel functions*, *Brownian motion*, *convexity*, *$h-$transforms*, *infimum*, *Ray-Knight theorem*, *supremum*.

P. Vallois and P. Salminen (University of Abo) consider first range times (with randomised range level) of a linear diffusion on $\mathbb{R}$. Inspired by the observation that the exponentially randomised range time has the same law as a similarly randomised first exit time from an interval, we study a large family of non-negative 2-dimensional random variables (X, X') with this property. The defining feature of the family is $F^c(x,y) = F^c(x+y, 0), \quad \forall x \geq 0,\ y \geq 0$, where $F^c(x,y) := P(X > x,\ X' > y)$.

We also explain the Markovian structure of the Brownian local time process when stopped at an exponentially randomised first range time. It is shown that squared Bessel processes with drift are serving hereby as a Markovian element.

This work was submitted on May 2003 to "Bernoulli" [33].

6.4.4. *Limiting laws associated with Brownian motion perturbated by normalized weights*

Key words: *Normalized exponential weights*, *limiting laws*, *Wiener measure*, *Sturm-Liouville equation*, *Ray-Knight's theorems*, *rate of convergence*, *Bessel processes*.

Let B be a one dimensional Brownian motion, with local time process $(L_t^x; t \geq 0, x \in \mathbb{R})$. B. Roynette, P. Vallois and M. Yor (Université Paris 6) determine the rate of decay of $Z_t^V(x) := E_x\left[\exp\{-\frac{1}{2}\int_{\mathbb{R}} L_t^y V(dy)\}\right]$, $t \geq 0, x \in \mathbb{R}$ as t goes to infinity, where $V(dy)$ is a positive Radon measure on $\mathbb{R}$.

If $\int_{\mathbb{R}}(1+|x|)V(dx)<\infty$, we prove that $Z_t^V(x) \sim_\infty \varphi_V(x)t^{-1/2}$, where the function φ_V solves the Sturm-Liouville equation $\varphi''(dx) = \varphi(x)V(dx)$, with some boundary conditions. If $\int_{-\infty}^{0}(1+|x|)V(dx)<\infty$ and $V(dy)$ is "large" at $+\infty$, the equivalent of $Z_t^V(x)$ is of the same type. When $V(dx) = V(x)dx$ and $V(x)$ converges to a finite limit as $x \to \pm\infty$, then $Z_t^V(x)$ is equivalent to $ke^{-\gamma_0 t}, t \to \infty$. If $V(dx) = [\lambda/(\theta+x^2)]dx,\ \lambda,\theta > 0$, the rate of decay is polynomial: $Z_t^V(x) \sim_\infty kC(x/\sqrt{\theta})t^{-n}$, with $n = (1+\sqrt{1+4\lambda})/4$. Taking $V(dx) = [1/(1+|x|^\alpha)]dx, 0<\alpha<2$ we only obtain a logarithmic equivalent : $\ln(Z_t^V(x)) \sim_\infty -kt^{-\frac{\alpha-2}{\alpha+2}}$.

The knowledge of the rate of decay of $Z_t^V(x)$ allows to determine limiting probabilities measures. More precisely let $Q_{x,t}^V$ be the probability measure defined by

$$Q_{x,t}^V = \frac{1}{Z_t^V(x)}\exp\{-\frac{1}{2}\int_{\mathbb{R}} L_t^y V(dy)\}\ W_x,$$

where W_x denotes the Wiener measure. We prove that $Q_{x,t}^V$ converges as $t \to \infty$ to $P_x^{\varphi_V}$ and $P_x^{\varphi_V}$ is the law of the diffusion process X_t^x, solution of the stochastic differential equation: $X_t = x + B_t + \int_0^t \frac{\varphi_V'}{\varphi_V}(X_s)ds;\quad t \geq 0$.

A paper is accepted for publication [27] and two other papers are submitted.

6.4.5. Independence of time and position for a stochastic process

In collaboration with Gerard Lorang (who has a permanent position in Luxembourg), Christophe Ackermann and Bernard Roynette consider a Bernoulli symmetric random walk (S_n) and the stopping times T such that T and S_T are independent. They describe the probability distributions of S_T. They also provide a new characterization of the stopping distributions of the Brownian motion.

7. Contracts and Grants with Industry

7.1. Collaboration with EDF-Chatou: How to model the price of electricity on a spot market

Participants: Mireille Bossy, Madalina Deaconu, Etienne Tanré.

We continue our collaboration with the company Électricité de France on the process of formation of electricity prices on local spot markets directed by several producers. We aim to model the spot prices on the different markets resulting from a production equilibrium between the asks and the producers. We have elaborated simple assumptions on the main components of the problem, that is, the ask and bid functions. We fully describe the possible equilibria.

7.2. Contract with Gaz de France

Participants: Olivier Bardou, Axel Grorud, Antoine Lejay, Love Lindholm, Charlotte Pouderoux, Denis Talay.

The aim of our collaboration with Gaz de France is to simulate the possible future prices of contracts related to exchange rates and gas and oil indices. We are interested in continuous time stochastic differential models of the indices and the exchange rates involved in the contracts.

The first task was to calibrate the model. We thus have estimated the volatility and the drift coefficient using various non-parametric estimators. In addition, since we are interested in the simultaneous evolution of the indices and the exchange rates, we are also interested in finding the correlations between the different stock prices. We therefore had to estimate the number of Brownian motions driving the stochastic evolution of the indices and the exchange rates. We have finally proposed a model to Gaz de France. Our partners have approved this model.

We now focus on variance reduction techniques which may allow us to improve the efficiency of Monte Carlo simulations to compute the prices and sensitivities of the contracts.

8. Other Grants and Activities

A. Lejay is co-responsible with Iraj Mortazavi (Université Bordeaux 1) for the project “Probabilistic models and particles methods for nuclear waste disposal” within the “Groupe de Recherche MOMAS”.

The team Omega participates to the “Groupe de Recherche GRIP” on stochastic interacting particles. D. Talay serves as a member of the scientific committee of this GdR.

9. Dissemination

9.1. Animation of the scientific community

D. Talay serves as Associated Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

M. Bossy is member of the administration committee of the French Society of Applied Mathematics (SMAI). M. Deaconu is member of the scientific committee of the MAS group (Probability and Statistics) within SMAI.

M. Deaconu is member of the “Comité des Projets” of LORIA and INRIA Lorraine and of the “Commission pour les postes d’accueil” of INRIA Lorraine.

M. Deaconu is member of the “Conseil du laboratoire” of the Institut Élie Cartan and of the “Commission de spécialistes” of the Mathematics Department of Université Nancy 1.

M. Deaconu and A. Lejay are responsibles for the organization of the conference *Journées MAS 2004* to be held in Nancy in September 2004.

M. Bossy, M. Deaconu and E. Tanré are the organizing committee of the international conference MC2QMC wich will be held in Juan-les-Pins, June 7-10, 2004 (MC2QMC is the Sixth International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing, twinned with the Second International Conference on Monte Carlo and Probabilistic Methods for Partial Differential Equations). D. Talay and H. Niederreiter (University of Singapore) co–chair the Conference.

M. Deaconu and D. Talay are permanent reviewers for the *Mathematical Reviews*.

A. Lejay is member of the “Commission des moyens informatiques” of INRIA Lorraine.

A. Lejay and Iraj Mortazavi (Université Bordeaux I) are organizing a workshop *Probabilistic models and particles methods for nuclear waste disposal* to be held in Paris in January 2004.

B. Roynette is the head of the Mathematics Department of Université Nancy 1.

P. Vallois is the head of the Probability and Statistics group of Institut Élie Cartan.

P. Vallois is member of the council of the UFR STMIA.

9.2. Teaching

D. Talay has a part time position of Professor at École Polytechnique. He also teaches probabilistic numerical methods at Université Paris 6 (DEA de Probabilités) and within the FAME Ph.D. program (Switzerland).

M. Bossy gives a course on “Stochastic calculus and financial mathematics” in the DESS IMAFA (“Informatique et Mathématiques Appliquées à la Finance et à l’Assurance”, Université de Nice Sophia Antipolis), and a course on “Risk management on energetic financial markets” in the master “Ingénierie et Gestion de l’Energie” (École des Mines de Paris) at Sophia-Antipolis.

M. Bossy and É. Tanré give the course on “Stochastic modeling and financial applications” in the DEA de Mathématiques of Université de Nice Sophia Antipolis.

M. Deaconu gives a 30h course in Mathematical Finance at the IUP Sciences Financières of Université Nancy 2.

P. Vallois gives courses in Mathematical Finance in a DESS programme and at the IUP Sciences Financières. He also gives the DEA course "Introduction to the Brownian motion and continuous martingales" at Université Nancy 1.

9.3. Ph.D. theses

Christophe Ackermann defended his Ph.D. thesis entitled *Processus associés à l'équation de diffusion rapide ; indépendance du temps et de la position pour un processus stochastique* at Université Nancy 1 in December 2003.

Christophe Berthelot defended his Ph.D. thesis entitled *Évaluation d'une architecture vectorielle pour des méthodes de Monte-Carlo. Analyse probabiliste de conditions au bord artificielles pour des inéquations variationnelles* at the Université Paris 6 in September 2003.

Awa Diop defended her Ph.D. thesis entitled *Sur la discrétisation et le comportement à petit bruit d'EDS unidimensionnelles dont les coefficients sont à dérivées singulières* at Université de Nice Sophia-Antipolis in May 2003.

Agnès Volpi defended her Ph.D. thesis entitled *Étude asymptotique de temps de ruine et de l'overshoot* at Université Nancy 1 in June 2003.

Marian Ciuca defended his Ph.D. thesis entitled *Estimation paramétrique sous contraintes. Applications en finance stochastique* at Université de Provence in December 2003.

9.4. Participation to congress, conferences, invitations,...

M. Bossy gave a seminar leacture at the Laboratoire J. A. Dieudonné, Université de Nice Sophia-Antipolis, in February 2003.

A. Lejay has given lectures at the workshop on Numerical homogenization in porous media in December 2003; at the IV IMACS Seminar on Monte Carlo methods in Berlin in September 2003; at the Congresso de Matematicas Capriocaron (COMCA 2003) at Antofagasta (Chile) in August 2003; at the probability seminar of the Institut Élie Cartan in April 2003; at the SIAM Conference on Mathematical and Computational Issues in Geosciences at Austin in March 2003.

A. Lejay gave in mini-lecture at the *Ist Winter School of Stochastic Analysis and Statistics* at Valparaiso in August 2003.

I. Nourdin has given a lecture at the *Journées de Probabilités* at Toulouse in September 2003.

B. Roynette has given lectures at the Journées processus aléatoires et particules at Orléans in March 2003; at the *Journées de Probabilités* at Toulouse in September 2003; at the *Colloque international de mathématiques, analyse et probabilités* at Hammamet (Tunisia) in December 2003.

D. Talay has given two plenary lectures: *21th IFIP TC 7 Conference on System Modeling and Optimization* and *IVth IMACS Seminar on Monte Carlo Methods MCM-2003*. He also gave a lecture at the *MS Durham Symposium on Markov Chains* at the University of Durham (Grande Bretagne), at the workshop 'Stochastic Processes and Applications to Mathematical Finance' at the Ritsumeikan University (Kyoto, Japan) and at the meeting *Journées processus aléatoires et particules* at Université d'Orléans.

P. Vallois has given lectures at the meeting *Journées processus aléatoires et particules* in Orléans in March 2003; at the *International Seminar on stability problems for stochastic models* at Pamplona (Spain) in May 2003; at the *11-th Journées Évry-Nancy-Strasbourg* in May 2003; at the *Journées de Probabilités* at Toulouse in September 2003; at the *Winter school, Analyse stochastique and applications* in Marrakech in December 2003.

A. Lejay spent two weeks in Chile within the INRIA-CONICYT collaboration (July/August). He has also been invited one week by Prof. T. Lyons at the Mathematical Institute of Oxford University.

É. Tanré has participated to the Summer School CEMRACS 2003.

9.4.1. Invitations

The seminar *Théorie et applications numériques des processus stochastiques* organized at Sophia-Antipolis by M. Bossy has received the following speakers: Larbi Alili (ETH Zürich), Vlad Bally (Université de Marne la Vallée), Sylvain Maire (Université de Toulon), Florent Malrieu (Université de Rennes), Carlos M. Mora (Universidad de Concepcion, Chile), Rolando Rebolledo (Universidad Católica de Chile, Chile), Maria-Soledad Torres (Universidad de Valparaiso, Chile), Pierre-Louis Lions (Collège de France).

The seminar *Mathématiques financières* organized at Sophia-Antipolis by M. Bossy has received the following speakers: Christophette Blanchet (Université de Nice Sophia-Antipolis), Sébastien Chaumont (Université Nancy 1), Gaël Giraud (Université de Strasbourg), Benjamin Jourdain (ENPC–CERMICS), Pierre-Louis Lions (Collège de France).

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