INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team SYDOCO

SYstèmes Dynamiques, Optimisation et Commande Optimale

Rocquencourt

THEME 4A

Activity Report

2003

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1. Team

Head of project-team
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Ph. D. student
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Julien Laurent-Varin [bourse CNES]
Stefania Maroso [bourse MESR - depuis octobre]
Elisabeth Ottenwaelter [IUT Paris]
Hector Ramirez-Cabrera [Université du Chili]

Visiting scientist
Jose Aliste [Université du Chili, 3 mois]
Felipe Alvarez [Université du Chili, 2 semaines]
Radia Bessi-Fourati [ENIT - Tunis, 2 semaines]
Abdeslem Boukhtouta [Université le Val - Québec, 3 mois]
Henda El Fekih [ENIT - Tunis, 2 mois]
Alexander Shapiro [Georgie tech., 1 mois]
Mikhail Solodov [IMPA - Rio de Janeiro, 3 semaines]

2. Overall Objectives

Develop new algorithms in deterministic and stochastic optimal control, and deal with associated applications, especially for aerospace trajectories and management for the power industries (hydroelectric resources, storage of gas and petroleum).

In the field of deterministic optimal control, our objective is to develop algorithms combining iterative fast resolution of optimality conditions (of the discretized problem) and refinement of discretization, through the use of interior point algorithms. At the same time we wish to study multiarcs problems (separations, rendez-vous, formation flights) which necessitates the use of decomposition ideas.

In the field of stochastic optimal control, our first objective is to develop fast algorithms for problems of dimension two and three, based on fast computation of consistent approximations as well as splitting methods. The second objective is to link these methods to the stochastic programming approach, in order to deal with problems of dimensions greater than three.

3. Scientific Foundations

For deterministic optimal control problems there are basically three approaches. The so-called direct method consists in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure; see Betts [15]. The indirect approach eliminates control variables using Pontryagin's maximum principle, and solves the resulting two-points boundary value problem by a multiple shooting method. Finally the dynamic programming approach solves the associated Hamilton-Jacobi-Bellman (HJB) equation, which is a partial differential equation of dimension equal to the number

n of state variables. This allows to find the global minimum, whereas the two other approaches are local; however, it suffers from the curse of dimensionality (complexity is exponential with respect to n). There are various additional issues: decomposition of large scale problems, simplification of models (leading to singular perturbation problems), computation of feedback solutions.

For stochastic optimal control problems there are essentially two approaches. The one based on the (stochastic) HJB equation has the same advantages and disadvantages as its deterministic counterpart. The stochastic programming approach is based on a finite approximation of uncertain events called a scenario tree (for problems with no decision this boils down to the Monte Carlo method). Their complexity is polynomial with respect to the number of state variables but exponential with respect to the number of time steps. In addition, various heuristics are proposed for dealing with the case (uncovered by the two other approaches) when both the number of state variables and time steps is large.

4. Application Domains

Aerospace trajectories (rockets, planes), automotive industry (car design), chemical engineering (optimization of transient phases, batch processes). Storage and management, especially of natural and power resources, portfolio optimization.

5. Software

We have presently two research softwares. The first is an implementation of interior point algorithms for trajectory optimization, and the second is an implementation of fast algorithms for bidimensional HJB equations of stochastic control.

6. New Results

6.1. Trajectory optimization

Participants: Sandrine Avril, Frédéric Bonnans, Mounir Haddou, Julien Laurent-Varin.

In collaboration with Nicolas Bérend (Onera-Châtillon) and Christophe Talbot (Direction des Lanceurs, Cnes) In the framework of the thesis of S. Avril, we have studied a two level formulation of the problem of optimal transfer for low-thrust satellites, where the upper level corresponds to the slowly varying orbital parameters. In the framework of the thesis of J. Laurent-Varin, a first version of the interior point algorithm code, based on an Runge-Kutta discretization of arbitrary order, ran for a simplified ascent problem (of a space rocket). This was published in the 5th International Conference on Launcher Technology. We have obtained theoretical results on asymptotically optimal refinement. They will be published in the 16th IFAC Symposium on Automatic Control in Aerospace, St Petersburg, 2004.

6.2. Numerical methods for the HJB equations

Participants: Frédéric Bonnans, Stefania Maroso, Elisabeth Ottenwaelter, Housnaa Zidani.

In the framework of the thesis of E. Ottenwaelter, we have continued the study of fast solvers for two dimensional HJB equations of stochastic control. The relations with a classical concept of arithmetic, namely the Stern-Brocot, have been clarified; they allow a computation of the coefficients of the scheme (at a given point of the state grid and for a given control) in $\mathrm{O(p)}$ iterations, where p is the size of the stencil. The method has been implemented; the numerical experiments confirmed the theory. The corresponding Inria report will appear in January 2004.

During the internship of S. Maroso (April-July 2003) we have studied the possibility of fast solvers for three dimensional HJB equations of stochastic control. This amounts to the computation of the projection onto a cone in R^6, and hence, is more difficult. We concentrated on schemes using only the closest neighbouring points. If consistency holds, we could improve an explicit decomposition scheme due to Arturo Pratt (U. Chile). If a projection is needed we have computed a reduced basis on which coefficients must be (but an iterative procedure is needed in order to compute these coefficients). In the framework of the thesis of S. Maroso, started in September 2003, we are currently studying the litterature on error estimates for the HJB equations of stochastic control.

6.3. Variational problems in mechanics

Participants: Frédéric Bonnans, Radhia Bessi Fourati [Enit-Tunisie], Hichem Smaoui [Enit-Tunisie].

We have ended the study on the minimization of potential energy for static equilibrium problems and performed various numerical tests of a relaxation type decomposition scheme. A more or less complete second order sensitivity analysis was performed. The Inria report has been published this year as well as the corresponding paper in [3]

6.4. Positive semidefinite optimization

Participants: Frédéric Bonnans, Hector Ramirez-Cabrera.

We have obtained some necessary or sufficient conditions for strong regularity (in the sense of Robinson, Math. Oper. Res. 1980) of positive semidefinite optimization problems. We also started a similar study for second order cone problems.

6.5. Optimization with chance constraints

Participants: Frédéric Bonnans, Agnès Sulem [Mathfi Project], René Aïd [EDF R & D].

In the framework of the intership of N. Boulanger (Mastere ENS Ulm and DEA Paris IX Dauphine) and motivated by the problem of management of water resources for hydraulic energy, we have discussed the optimization of Markov chains with constraints of VaR (value at risk) type and the associated duality gap.

6.6. $\mathcal{VU}$-space decomposition theory Bundle methods for constrained problems

Participant: Claudia Sagastizábal.

Consider the problem of minimizing a nonsmooth function that is lower semicontinuous and subdifferentially regular. In [6] we extended the class of functions with "primal dual gradient structure" (`pdg`) to Lipschitz continuous functions. We showed that, under certain assumptions of transversality, such functions have a $\mathcal{U}$-Hessian, which is related to the second-order epiderivative; see also [8]. We also showed that `pdg` functions are "partially smooth", as defined by A. Lewis.

In [13] we show that when strong transversality is satisfied, there exists a C^2 trajectory leading to $\overline{x}$ and an associated subdifferential that is C^1. As a result, there exists a space decomposition mapping that is C^1 and a second order expansion of f on the trajectory. For $\overline{x}$ a minimizer, we give conditions on f to ensure that for any point near $\overline{x}$ its corresponding proximal point is on the trajectory. This purely theoretical result is fundamental for future minimization algorithms and their implementations, since it is known that a sequence of null steps from a bundle mechanism can approximate proximal points with any desired accuracy.

Suppose the function is not `pdg` structured, but has additional properties of prox-regularity and prox-boundedness. In [9] we make use of $\mathcal{VU}$-space decomposition theory to connect three minimization-oriented objects. These objects are $\mathcal{U}$-Lagrangians obtained from minimizing a function over $\mathcal{V}$-space, proximal points depending on minimization over $\Re^n = \mathcal{U} \oplus \mathcal{V}$, and epi-derivatives determined by lower limits associated with epigraphs. We relate second-order epi-derivatives of a function to the Hessian of its associated $\mathcal{U}$-Lagrangian. We also show that the function's proximal points are on a trajectory determined by certain $\mathcal{V}$-space minimizers.

In [14] we showthat the use of a parametrized penalty function in nonsmooth convex optimization can be avoided without using the relatively complex filter methods. In particular, we propose an approach which appears to be more direct and easier to implement, in the sense that it is closer to the well-developed unconstrained bundle methods. Preliminary computational results are also reported. %Joint work with P. Rey (DII, U. de Chile) and M. Solodov (IMPA).

6.7. Industrial applications Energy problems

Participant: Claudia Sagastizábal.

In [7] we consider the inclusion of commitment of thermal generation units in the optimal management of the Brazilian power system. By means of Lagrangian relaxation we decompose the problem and obtain a nondifferentiable dual function that is separable. We solve the dual problem with a bundle method. Our purpose is twofold: first, bundle methods are the methods of choice in nonsmooth optimization when it comes to solve large-scale problems with high precision. Second, they give good starting points for recovering primal solutions. We use an inexact augmented Lagrangian technique to find a near-optimal primal feasible solution. We assess our approach with numerical results.

7. Contracts and Grants with Industry

7.1. Trajectory optimization

We have agreements of cooperation with Onera and CNRS concerning the studies on transfer or orbits for low-thrust satellites, and optimal trajectories for future launchers.

8. Other Grants and Activities

8.1. International collaborations

- Brazil: Collaboration with IMPA. With C. Sagastizábal we have continued the work on bilevel optimization.
- Canada: Collaboration with U. Laval: with A. Boukhtouta, work on the optimization of management of water.
- Chile: Collaboration with DIM: with F. Alvarey and his student J. Aliste we have started to work on the optimal reentry by a HJB approach.
- Tunisia: Collaboration ENIT: work with H. Smaoui and his student R. Bessi Fourati on variational problems in mechanics. One paper published this year, thesis to end by January 2004.

8.2. Scientific responsabilities outside Inria

F. Bonnans is Vice President for Publications of SMAI, the French Applied Mathematics Society. This means supervising three scientific journals, one proceedings series, and a series of books. It was decided this year to launch another series of books for Master studies.

8.3. Visites et invitations de chercheurs

C. Sagastizábal (IMPA - Brazil), A. Boukhtouta (U. Laval - Canada), F. Alvarez and J. Aliste (DIM - Chile), H. Smaoui and R. Bessi Fourati (ENIT - Tunisie).

9. Dissemination

9.1. Animation de la communauté scientifique

- Journées MODE 2003. 27-29 mars - Pau - France. Membre du Comité scientifique : F. Bonnans

9.2. Teaching

- F. Bonnans/ Professeur chargé de cours, Ecole polytechnique and Course on Continuous Optimization, DEA "OJME" Optimisation, Jeux et Modélisation en Economie, Université Paris VI et Paris X.

9.3. Participation à des colloques, congrès

- 5th International Conference on Launcher Technology. 25-28 Novembre - Madrid. Exposé: J. Laurent-Varin "Interior points algorithm for solving re-entry control problem".
- International Symposium on Mathematical Programming 2003. 18-22 Aout - Copenhagen - Denmark. Exposés: R. Bessi "The obstacle problem for water tank" - J. Laurent-Varin "An interior points algorithm for optimal control problems" - H. Ramirez "A Global Algorithm for Nonlinear Semidefinite Programming".
- 5ème congrès de la Société Française de Recherche Opérationnelle et d'Aide à la Décision. 26-28 février - Avignon - France. F. Bonnans "Optimisation de la conception de moteur hybride".
- Numerical methods for HJB equations and applications.11-12 Septembre - ENSTA-Paris. Organizing Commitee : H. Zidani (chair) - J.F. Bonnans.

9.4. Organisation de conférences et séminaires

- Réflexion sur les logiciels : valorisation et diffusion de logiciels, les grands challenges applicatifs (dans le cadre des 20 ans de la SMAI). 9 octobre - Rocquencourt. Co-Organisation : F. Bonnans.
- CEA-EDF-INRIA - Course on "Stochastic Programming". 17-20 mars - Inria-Rocquencourt - France. Organizer : F. Bonnans.
- Applied Mathematics and Applications of Mathematics Conference. 10-13 february - Nice - France. Organiser of a mini symposium : F. Bonnans. Talks : F. Bonnans "Fast interior-point solvers for optimal control problems" - H. Zidani "Discretization of stochastic optimal control problems".

10. Bibliography

Books and Monographs

[1] J.F. Bonnans, J.Ch. Gilbert, C. Lemaréchal, C. Sagastizábal. *Numerical optimization: Theoretical and Practical Aspects.* Springer Verlag, Heidelberg, 2003.

Articles in referred journals and book chapters

[2] J.F. Bonnans, Th. Guilbaud. *Using Logarithmic Penalties in the Shooting Algorithm for Optimal Control Problems.* 2003, in « Optimal Control, Applications and Methods », volume 24, number 5, pages 257-278.

[3] J.F. BONNANS, R. BESSI FOURATI, R. SMAOUI. *The obstacle problem for water tanks.* 2003, in « J. Mathématiques Pures et Appliquées », volume 82, number 11, pages 1527-1553.

[4] J.F. BONNANS, PH. CHARTIER, H. ZIDANI. *Discrete approximations of the Hamilton-Jacobi equation for an optimal control problem of a differential-algebraic system.* 2003, in « Control and Cybernetics », volume 32, number 1, pages 33-55.

[5] J.F. BONNANS, H. ZIDANI. *Consistency of Generalized Finite Difference Schemes for the Stochastic HJB Equation.* 2003, in « SIAM J. Numerical Analysis », volume 41, number 3, pages 1008-1021.

[6] R. MIFFLIN, C. SAGASTIZÁBAL. *Primal-Dual Gradient Structured Functions: second-order results; links to epi-derivatives and partly smooth functions.* in « SIAM Journal on Optimization », 2003, volume 13, pages 1174–1194, number 4.

[7] A. BELLONI, A. DINIZ, M.E. MACEIRA, C. SAGASTIZÁBAL. *Bundle relaxation and primal recovery in unit commitment problems. the Brazilian case.* in « Annals of Operations Research », 2003, volume 120, pages 21–44.

[8] R. MIFFLIN, C. SAGASTIZÁBAL. *On the relation between $\mathcal{U}$-hessians and second-order epiderivatives.* in « European Journal of Operations Research », To appear.

[9] R. MIFFLIN, C. SAGASTIZÁBAL. *$\mathcal{U}$-Lagrangians to Second Order Epiderivatives and Proximal Tracks.* in « Journal of Convex Analysis », 2003, To appear.

Miscellaneous

[10] J.F. BONNANS, J.M. COGNET, S. VOLLE. *Estimation of the local volatility of financial assets by a numerical inversion method.* 2003, submitted.

[11] S. MAROSO. *Résolution numérique de l'équation HJB du contrôle stochastique en dimension 3.* 2003, Rapport de stage du DEA OJME.

[12] N. BOULANGER. *Optimisation sous contrainte de risque.* 2003, Rapport de stage de DEA.

[13] R. MIFFLIN, C. SAGASTIZÁBAL. *$\mathcal{VU}$-Smoothness and Proximal Point Results for Nonconvex Functions* . 2003, Submitted.

[14] R. 14 MIFFLIN, P.A. REY, C. SAGASTIZÁBAL. *An infeasible bundle method for nonsmooth convex constrained optimization without a penalty function or a filter.* 2003, Submitted.

Bibliography in notes

[15] J.T. BETTS. *Practical Methods for Optimal Control using Nonlinear Programming.* SIAM, 2001.