INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team Apics

Analysis and Problems of Inverse type in Control and Signal processing

Sophia Antipolis

THEME NUM

Activity Report

2004

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1. Team

Head of Team
Laurent Baratchart [DR INRIA]

Vice-Head of Team
Jean-Baptiste Pomet [CR INRIA]

Administrative Assistant
France Limouzis [AI INRIA, partial time in the team]

Staff Member
José Grimm [CR INRIA]
Juliette Leblond [CR INRIA]
Martine Olivi [CR INRIA]
Fabien Seyfert [CR INRIA]

Ph. D. Students
David Avanessoff [Fellow, INRIA]
Fehmi Ben Hassen [Co-advised, ENIT Tunis]
Alex Bombrun [Fellow, INRIA]
Imen Fellah [Co-advised, ENIT Tunis]
Moncef Mahjoub [Co-advised, ENIT Tunis)]
Vincent Lunot [Fellow, Inria, since November, 2nd, 2004]

Scientific Advisors
Andrea Gombani [LADSEB-CNR, Padova, Italy]
Mohamed Jaoua [University of Nice-Sophia Antipolis]
Jean-Paul Marmorat [CMA]
Jonathan Partington [University of Leeds, GB]
Edward Saff [Vanderbilt University, Nashville, USA]

Visiting Scientists
Amina Amassad [Delegation from UNSA since September]

Students Interns
Christelle Aziadjonou [UNSA, maîtrise Maths, July-August]
Jonathan Chetboun [UNSA, maîtrise Maths, July-August]
Frank-Olivier Helme [LATP-CMI, Aix-Marseille I, DEA Maths, June-July]
Jean-Michel Guieu [LATP-CMI, Aix-Marseille I, DEA Maths, June-July]

Post-Doctoral Fellows
Bilal Atfeh [Since September, funded by an INRIA grant]
Mario Sigalotti [funded by a Marie Curie grant]
Per Enqvist [Until July]

2. Overall Objectives

The Apics Team was created in January 2004 as a follow-up to the Miaou Team. On November 4th, the "Comité des Projets" of INRIA-Sophia recommended to the head of INRIA upgrading APICS to a Project Team.

The Team develops constructive methods for modeling, identification and control of dynamical systems.

2.1.1. Research Themes

- Meromorphic approximation in the complex domain, with application to frequency identification and design of transfer functions, as well as singularity detection for the 2-D Laplace operator. Development of software for filter identification and the synthesis of microwave devices.

- Inverse potential problems in 3-D and analysis of harmonic fields with applications to source detection and electro-encephalography.
- Control and structure analysis of non-linear systems: continuous stabilization, linearization, and near optimal control with applications to orbit transfer of satellites.

2.1.2. International and industrial partners

- Industrial collaborations with Alcatel Space, Alcatel-R&I, CNES, IRCOM.
- Exchanges with UST (Villeneuve d'Asq), CMI-Université de Provence (Marseille), CWI (the Netherlands), CNR (Italy), SISSA (Italy), the Universities of Illinois (Urbana-Champaign), of South Florida (Tampa), of California (San Diego, Santa Barbara), of Alabama (Mobile), of Minnesota (Minneapolis), of Vanderbilt (Nashville), of Padova (Italy), of Beer Sheva (Israel), of Leeds (GB), of Maastricht and of Amsterdam (The Netherlands), of TU-Wien (Austria), of TFH-Berlin (Germany), of Kingston (Canada), of Szegëd (Hungary), of CINVESTAV (Mexico), ENIT (Tunis), VUB (Belgium).
- The project is involved in a NATO Collaborative Linkage Grant (with Vanderbilt University and ENIT-LAMSIN), in a EMS21-RTG NSF program (with Vanderbilt University), in the ACI "Obs-Cerv" (with the Teams Caiman and Odyssée from Inria-Sophia Antipolis, among others), in a STIC Convention between INRIA and Tunisian Universities, in an EPSRC Grant with Leeds University (UK), in the ERCIM "Working Group Control and Systems Theory", in the ERNSI and TMR-NCN European research networks, and in a Marie-Curie EIF European program.

3. Scientific Foundations

3.1. Identification and deconvolution

Let us first introduce the subject of Identification in some generality.

Abstracting in the form of mathematical equations the behavior of a phenomenon is a step called *modeling*. It typically serves two purposes: the first is to describe the phenomenon with minimal complexity for some specific purpose, the second is to *predict* its outcome. This is general practice in most applied sciences, be it for design, control or prediction, although it is generally thought of as another optimization problem yet.

As a general rule, the user imposes the model to fit a parameterized form that reflects one's own prejudice, knowledge of the underlying physical system, and the algorithmic effort consented. Looking for such a trade-off usually raises the question of approximating the experimental data by the prediction of the model when the latter is subject to external excitations assumed to be the cause of the phenomenon under study. The ability to solve this approximation problem, which is often non-trivial and ill-posed, often conditions the practical usefulness of a given method.

It is when the predictive potential of a model is to be assessed that one is led to *postulate* the existence of a *true* functional correspondence between data and observations, thereby entering the field of *identification* itself. The predictive power of a model can be expressed in various manners all of which are attempts to measure the difference between the true model and the observations. The necessity of taking into account the difference between the observed behavior and the computed behavior induces naturally the notion of *noise* as a corrupting factor of the identification process. This noise incorporates into the model, and can be handled in a deterministic mode, where the quality of an identification algorithm is its robustness to small errors. This notion is that of well-posedness in numerical analysis or stability of motion in mechanics. The noise however is often considered to be random, and then the true model is estimated by averaging the data. This notion allows approximate but reasonably simple descriptions of complex systems whose mechanisms are not well known but plausibly antagonistic. Note that, in any case, some *assumptions* on the noise are required in order to justify the approach (it has to be small in the deterministic case, and must satisfy some independence and ergodicity properties in the stochastic case). These assumptions can hardly be checked in practice, so that the satisfaction of the end-user is the final criterion.

Hypothesizing an exact model also results in the possibility of choosing the data in a manner suited for identifying a specific phenomenon. This often interacts in a complex manner with the *local* character of the model with respect to the data (for instance a linear model is only valid in a neighborhood of a point).

We now turn to the activity of the team proper to identification. Although the subject, on the academic level, has been the realm of the stochastic paradigm for more than twenty years, it is in a deterministic approach to identification of linear dynamical systems (i.e. 1-D convolution processes) based on approximation in the complex domain, that the Team made perhaps its most original contributions. Naturally, the deep links stressed by the spectral theorem between time and frequency domains induce well-known parallels between function theory and probability, and the work of the Apics Team[1] can be partly recast from the stochastic viewpoint. However, the issue was rather tackled by translating the problem of identification into an inverse problem, namely the reconstruction, from boundary data, of an analytic function in a domain of the plane. For convolution equations in dimension one—that is, ordinary differential equations possibly in infinite dimensional spaces —such translation is provided by the Fourier transform. For certain elliptic partial differential equations in dimension two, identification is also connected to analytic continuation, but this time it is the form of the fundamental solution that introduces holomorphy, especially in the case of the Laplacian whose solutions are logarithmic potentials.

The data are considered without postulating an exact model, but we simply look for a convenient approximation to the data in a range of frequency representing the working conditions of the underlying system. A prototypical example that illustrates our approach is the harmonic identification of dynamical systems which is widely used in the engineering practice, where the data are the responses of the system to periodic excitations in its band-width. We look for a stable linear model that describes correctly the behavior in this band-width, although the model can be inaccurate at high frequencies (that can seldom be measured). In most cases, we also want this model to be rational of suitable degree, either because this is imposed by the physical significance of the parameters or because complexity must remain reasonably low to allow the efficient use of the model for control, estimation or simulation. Other structural constraints, arising from the physics of the phenomenon to be modeled, often superimpose on the model. Note that, in this approach, no statistics are used for the errors, which can originate from corrupted measurements or from the limited validity of the linear hypothesis.

We distinguish between an identification step (called non-parametric in a certain terminology) that is provided with an infinite dimensional model, and an approximation step in which the order is reduced subject to certain specific constraints on the considered system. The first step typically consists, mathematically speaking, in reconstructing a function, analytic in the right half-plane, knowing its pointwise values on a portion of the imaginary axis, in other terms, to make the principle of analytic continuation effective on the boundary of the analyticity domain. This is a classical ill-posed issue (the inverse Cauchy problem for the Laplace equation) that we embed into a family of well-posed extremal problems, that may be viewed as a Tikhonov-like regularization scheme related to the spectral theory of analytic operators. This first step could in fact be made in higher dimensions, where analytic functions are replaced by harmonic fields. The second step is typically a rational or meromorphic approximation procedure (although other approximating families may arise as well) in some class of analytic functions in a simply connected domain, say the right half-plane in the case of harmonic identification. To make the best possible use of the allowable number of parameters, or to privilege some specific physical parameters of the system, it is generally important, in the second step, to compute optimal or nearly optimal approximants. Rational approximation in the complex plane is a classical and difficult problem, for which only few effective methods exist. In relation to system theory, mainly two difficulties arise: the necessity of controlling the poles of the approximants (to ensure the stability of the model), and the need to handle matrix-valued functions in the case where the system has several inputs and outputs.

[1] and of the former MIAOU-project

Rational approximation in the L^p sense to a transfer function on the imaginary axis (i.e the boundary of the right half-plane) acquires a particular significance in this context for $p = 2$ and $p = \infty$. If $p = 2$, it corresponds to parametric identification of minimum variance when the system is fed with white noise input (the case of colored noise corresponds to weighted approximation), and it also corresponds to the minimization of the $L^2 \to L^\infty$ error in operator norm in the time domain. If $p = \infty$, the approximation consists in minimizing the power transfer $L^2 \to L^2$ of the error (both in the time and frequency domains since the Fourier transform is an isometry). These problems contribute a generalization (both rational and matrix-valued) of Szegö theory on orthogonal polynomials, that seems the most natural framework for setting out many optimization problems related to linear system identification. Concerning this second step, it is worth pointing out that the analogs to rational functions in higher dimensions are the gradients of Newtonian potentials of discrete measures. Very little is known at present on the approximation-theoretic properties of such objects.

We shall explain in more detail the above two steps in the sub-paragraphs to come. For convenience, we shall approach them on the circle rather than the line, which is the framework for discrete-time rather than continuous-time systems. The two frameworks are mathematically equivalent *via* a Möbius transform.

3.1.1. Analytic approximation of incomplete boundary data

Keywords: *extremal problems*, *frequency-domain identification*, *meromorphic approximation*.

Participants: Laurent Baratchart, José Grimm, Juliette Leblond, Jean-Paul Marmorat [CMA, École des Mines], Jonathan Partington, Fabien Seyfert.

The title refers to the construction of a convolution model of infinite dimension from frequency data in some bandwidth Ω and some reference gauge outside Ω. The class of models consists of stable transfer functions (*i.e.* analytic in the domain of stability, be it the half-plane, the disk, etc), and possibly also transfer functions with finitely many poles in the domain of stability *i.e,* convolution operators corresponding to linear differential or difference equations with finitely many unstable modes. This issue arises in particular for the design and identification of linear dynamical systems, and in certain inverse problems for the Laplacian in dimension two.

Since the question under study may occur on the boundary of planar domains of various shapes when it comes to inverse problems, it is common practice to normalize this boundary once and for all, and to apply in each particular case a conformal transformation to bring back to the normalized situation. The normalized contour chosen here is the unit circle. We denote by D the unit disk, by H^p the Hardy space of exponent p, R_N is the set of all rational functions having at most N poles in D, and $C(X)$ is the set of continuous functions on X. We are looking for a function in $H^p + R_N$, taking on an arc K of the unit circle values that are close to some experimental data, and satisfying on $T \setminus K$ some gauge constraints, so that a prototypical Problem is:

(P) Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p + R_N$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

In order to impose pointwise constraints in the frequency domain (for instance if the considered models are transfer functions of loss-less systems, see section 4.3.2), one may wish to express the gauge constraint on $T \setminus K$ in a more subtle manner, depending on the frequency:

(P') Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $g \in H^p + R_N$ such that $|g - \psi| \leq M$ a.e. on $T \setminus K$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Problem (P) is an extension to the meromorphic case, and to incomplete data, of classical analytic extremal problems (obtained by setting $K = T$ and $N = 0$), that generically go under the name *bounded extremal problems*. These have been introduced and intensively studied by the Team, distinguishing the case $p = \infty$ [36] from the cases $1 \leq p < \infty$, among which the case $p = 2$ presents an unexpected link with the Carleman reconstruction formulas [3].

Deeply linked with Problem *(P)*, and meaningful for assessing the validity of the linear approximation in the considered pass-band, is the following completion Problem:

(P'') Let $p \geq 1$, $N \geq 0$, K an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $h \in L^p(T \setminus K)$ such that $\|h - \psi\|_{L^p(T \setminus K)} \leq M$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

A version of this problem where the constraint depends on the frequency is:

(P''') Let $p \geq 1$, $N \geq 0$, K an arc the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $h \in L^p(T \setminus K)$ such that $|h - \psi| \leq M$ a.e. on $T \setminus K$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

Let us mention that Problem *(P'')* reduces to Problem *(P)* that in turn reduces, although implicitly, to an extremal Problem without constraint, (i.e. a Problem of type *(P)* where $K = T$) that is denoted conventionally by *(P_0)*. In the case where $p = \infty$, Problems *(P')* and *(P''')* can viewed as special cases of *(P)* and *(P'')* respectively, but if $p < \infty$ the situation is different. One can also chose different exponents p on K and $T \setminus K$ (the Problem is then said to be of mixed type), and this comes up naturally when identifying lossless systems where the constraint $|h| \leq 1$ must hold at each point while the data, whose signal-to-noise ratio is small on the ends of the bandwidth, are better approximated in the L^2 sense. Mixed Problems have begun to be studied within the Team. One has to stress the perhaps counter-intuitive fact that these have in general no solution unless the gauge constraint is accounted for, that is, if one sets formally $M = +\infty$. For instance, considering Problem *(P'')*, a function given by its trace on a subset K of positive measure on the unit circle can always be extended in such a manner as to be arbitrarily close, on K, to a function analytic in the disk; however, it goes to infinity in norm on $T \setminus K$ when the approximation error goes to zero, unless we are in the ideal case where the initial data are *exactly* the trace on K of an analytical function. The phenomenon illustrates the ill-posedness of the analytic continuation on the boundary of the analyticity domain.

The solution to *(P_0)* is classical if $p = \infty$: it is given by the Adamjan-Arov-Krein (in short: AAK) theory. If $p = 2$ and $N = 0$, then *(P_0)* reduces to an orthogonal projection. AAK theory plays a great role in showing the existence and uniqueness of the solution to *(P'')* when $p = \infty$, under the assumption that the concatenated function $f \vee \psi$ belongs to $H^\infty + C(T)$, and to compute this solution by solving iteratively a spectral problem relative to a family of Hankel operators whose symbols depend implicitly on the data. The robust convergence of this algorithm in separable Hölder-Zygmund classes has been established [35]. In the Hilbertian case $p = 2$, again for $N = 0$, the solution of *(P)* is obtained by solving a spectral equation, this time for a Toeplitz operator, depending linearly on a parameter λ that plays the role of a Lagrange multiplier and makes the dependence of the solution implicit in M. The ill-posed character of the analytic continuation described above is to the effect that, if the data are not exactly analytic, the approximation error on K tends to 0 if, and only if, the constraint M on $T \setminus K$ goes to infinity [3]. This phenomenon can be quantified in Sobolev or meromorphic classes of functions f, and asymptotic estimates of the behavior of M and of the error respectively can be obtained, based on a constructive diagonalization scheme for Toeplitz operators due to Rosenblum and Rovnyak, that makes the spectral theorem effective [30]. These results indicate that the error decreases much faster, as M increases, if the data have a holomorphic extension to a neighborhood of the unit disk, this being conceptually interesting for discriminating between nearly analytic data and those that are not close to a linear stable model; from the point of view of effective computing arises the problem of representing the functions through expansions that are specifically adapted to the underlying geometry, for instance, rational bases whose poles cluster at the endpoints of K. Research in this direction is in its infancy.

The study of Problem *(P')* has been carried out in the case where $p = 2$, $\psi = 0$, and the function M is in L^∞ of $T \setminus K$ and bounded from below almost everywhere by a strictly positive constant. Together with the existence and uniqueness of the solution, we have proved that the constraint is saturated pointwise, that is $|g| = M$ a.e. on $T \setminus K$, this being perhaps counter-intuitive. We obtained fixed point equations that characterize the solution, involving the resolvant of a Toeplitz operator, but with a multiplier that is here a function [37]. The study of the convergence of an iterative scheme is under examination, the goal being its software implementation. Note that if we approach the multiplier by a step function, we get a string of spectral equations similar to these used for solving Problem *(P)*.

An algorithm that consists in discretizing the modulus constraint and using Lagrange duality-based optimization techniques has been implemented and performs satisfactorily. It has interesting connections with affine Riemann-Hilbert problems *cf.* 6.9.

We emphasize that *(P)* has many analogs, equally interesting, that occur in different contexts connected to conjugate functions. For instance one may consider the following extremal Problem, where the constraint on the approximant is expressed in terms of its real and imaginary parts while the criterion takes only its real part into account:

Let $p \geq 1$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$, and $\alpha, \beta, M > 0$; find a function $g \in H^p$ such that $\alpha \|Re(g-\psi)\|_{L^p(T\setminus K)} + \beta \|Im(g-\psi)\|_{L^p(T\setminus K)} \leq M$ and such that $Re(g-f)$ is of minimal norm in $L^p(K)$ under this constraint.

This is a natural formulation for issues concerning Dirichlet-Neumann problem for the Laplace operator, see sections 4.2 and 6.8, where data and physical prior information concern real (or imaginary) parts of analytic functions.

For $p = 2$, existence and uniqueness of the solution have been established in [55] as well as a constructive solution procedure which, in addition to the Toeplitz operator that characterizes the solution of *(P)* in the case $p = 2$ and $N = 0$, involves a Hankel operator (this extends the results of [53]).

Situations with other values of p will be considered, as well as a suitable general weighted formulation of constrained extremal problems on T.

In the non-Hilbertian case, where $p \neq 2, \infty$, but still $N = 0$, the solution of *(P)* can be deduced from that of *(P_0)* in a manner analogous to the case $p = 2$, though the situation is a bit more tricky concerning duality, because one remains in a convex setup (in infinite dimension, of course), for which local optimization methods can be applied.

If $p < \infty$ and $N > 0$, there is up to now no algorithmic solution to Problem *(P_0)* which is proved convergent. However, the progress that were made allow us to conceive a coherent picture of the main issues and to develop rather efficient numerical schemes whose global convergence has been proved for prototypical classes of functions in Approximation theory. The essential features of the approach are summarized below.

First of all, in the case $p = 2$ and $N > 0$ which is of particular importance, Problem *(P_0)* can be reduced to that of rational approximation which is described in more details in section 3.1.2. Here, the link with classical interpolation theory, orthogonal polynomials, and logarithmic potentials is strong and fruitful. Second, a general AAK theory in L^p has been proposed which is relatively complete for $p \geq 2$ [41]. Although it does not have, for $p \neq \infty$, the computational power of the classical theory, it has better continuity properties and stresses a continuous link between rational approximation in H^2 (see section 3.1.2) and meromorphic approximation in the uniform norm, allowing one to use, in either context, the techniques available from the other. Hence, similar to the case $p = \infty$, the best meromorphic approximation with at most n poles in the disk of a function $f \in L^p(T)$ is obtained from the singular vectors of the Hankel operator of symbol f between the spaces H^s and H^2 with $1/s + 1/p = 1/2$, the error being here again equal to the $(n+1)$st singular number of the operator. This generalization has a strong topological nature and relies on the theory of critical points of Ljusternik-Schnirelman as well as on the particular geometry of the Blaschke products of given degree. Among the common features of this family of problems, the deepest one is perhaps the following: the critical point equations express non-Hermitian orthogonality of the denominator (i.e. the polynomial whose zeroes are the poles of the approximant) against polynomials of lower degree, for a complex measure that depends however on this denominator (because the problem is non-linear). This allows one to extend the index theorem to the case $2 \leq p \leq \infty$ [26] and to tackle the uniqueness problem, to study asymptotic errors, and also, combined with classical techniques of potential theory, to characterize the asymptotic behavior of the poles of the approximants for functions with connected singularities that are of particular interest for inverse problems (cf. section 3.1.3). In the light of these results, and although many questions remain open, one can expect algorithmic progress concerning (P_0) for $N > 0$ and $p \geq 2$ in the forthcoming years. As a consequence, the transition from (P_0) to (P) should follow the same lines as in the analytic case [60].

The case where $1 \leq p < 2$ remains largely open, especially from the constructive point of view, because if the approximation error can still be interpreted in terms of singular values, the Hankel operator takes an abstract form not permitting for a functional identification of its singular vectors. Considering these values for p is not simply an academic exercise: the L^1 criterion induces the operator norm $L^\infty \to L^\infty$ in the frequency domain, which is interesting for damping perturbations. It is possible that some appropriate duality links the case $p < 2$ to the case $2 < p$, but this has not been established yet.

A valuable endeavor would be also to carry over to higher dimensions (in particular in 3-D) the above analysis, where harmonic fields replace analytic functions. On the ball or the half-space, it seems many of the necessary ingredients are available after the progress undergone by harmonic analysis in recent years, with the notable exception of multiplicative techniques. Pushing through some of them would be tantamount to making significant progress in harmonic identification.

3.1.2. Scalar rational approximation

Keywords: *critical point*, *orthogonal polynomials*, *rational approximation*.

Participants: Laurent Baratchart, Juliette Leblond, Martine Olivi, Edward Saff, Herbert Stahl [TFH Berlin].

Rational approximation is the second step mentioned in section 3.1 and we first approach it in the scalar case, for complex-valued functions (as opposed to matrix-valued ones). The Problem can be stated as:

Let $1 \le p \le \infty$, $f \in H^p$ and n an integer; find a rational function without poles in the unit disk, and of degree at most n that is nearest possible to f in H^p.

The most important values of p, as indicated in the introduction, are $p = \infty$ and $p = 2$. In the latter case, the orthogonality between Hardy spaces of the disk and of the complement of the disk (the last one being restricted to functions that vanish at infinity to exclude the constants) makes rational approximation equivalent to meromorphic approximation, i.e. we are back to Problem *(P)* of section 3.1.1 with $p = 2$ and $K = T$. Although no demonstrably convergent algorithm is known for a single value of p, the former Miaou project (the predecessor of APICS) has designed a steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed in theory, and it is the first satisfying this property. Roughly speaking, it is a gradient algorithm, proceeding recursively with respect to the order n of the approximant, that uses the particular geometry of the problem in order to restrict the search to a compact region of the parameter space [1]. This algorithm can generate local *minima* if several exist, thus allowing one to discriminate between them. If there is no local *maximum*, a property which is satisfied when the degree is large enough, every local *minimum* can be obtained from an initial condition of lower order. It is not proved, however, that the absolute *minimum* can always be obtained using the strategy of the hyperion or RARL2 software (*cf.* section 5.2) that consists in choosing the collection of initial points corresponding to critical points of lower degree; note that we do not know of a counter-example either, still assuming that there is no *maximum*, so there is room for a conjecture at this point.

It is only fair to say that the design of a numerically efficient algorithm whose convergence to the best approximant would be proved is the most important problem from a practical perspective. However, the algorithms developed by the team seem rather effective and although their global convergence has not been established. *A contrario*, it is possible to consider an elimination algorithm when the function to approximate is rational, in order to find all critical points, since the problem is algebraic in this case. This method is surely convergent, since it is exhaustive, but one has to compute the roots of an algebraic system with n variables of degree N, where N is the degree of the function to approximate and there can be as many as N^n solutions among which it is necessary to distinguish those that are coefficients of polynomials having all their roots in the unit disk; the latter indeed are the only ones that generate critical points. Despite the increase of computing capacity, such a procedure is still unrealistic granted that realistic values of n and N would be like a ten and a couple of hundreds (cf. section 4.3.2).

To prove or disprove the convergence of the above-described algorithms, and to check them against practical situations, the team has undergone a long-haul study of the number and nature of critical points, depending on the class of functions to be approximated, in which tools from differential topology and operator theory team up with classical approximation theory. The study of transfer functions of relaxation systems (*i.e.* Markov functions) was initiated in [7] and more or less completed in [42], as well as the case of e^z (the prototype of an entire function with convex Taylor coefficients) and the case of meromorphic functions (*à la* Montessus de Ballore) [6]. After these studies, a general principle has emerged that links the nature of the critical points in rational approximation to the regularity of the decrease of the interpolation errors with the degree, and a methodology to analyze the uniqueness issue in the case where the function to be approximated is a Cauchy integral on an open arc (roughly speaking these functions cover the case of singularities of dimension one that are sufficiently regular, *cf.* section 3.1.3) has been developed. This methodology relies on the localization of the singularities *via* the analysis of families of non-Hermitian orthogonal polynomials, to obtain strong estimates of the error that allow one to evaluate its relative decay. Note in this context an analogue of the Gonchar conjecture, that uniqueness ought to hold at least for infinitely many values of the degree, corresponding to a subsequence generating the *liminf* of the errors. This conjecture actually suggests that uniqueness should be linked to the ratio of the to-be-approximated function and its derivative on the circle. When this ratio is pointwise greater than 1 (*i.e.* the logarithmic variation is small), the Schwartz lemma implies uniqueness in degree 1, and the generalization of this elementary fact is an interesting open question.

Another uniqueness criterion has been obtained [41] for rational functions, inspired from the spectral techniques of AAK theory. This result is interesting in that it is not asymptotic and does not require pointwise estimates of the error; however, it assumes a rapid decrease of the errors and the current formulation calls for further investigation.

The introduction of a weight in the optimization criterion is an interesting issue induced by the necessity to balance the information one has at the various frequencies. For instance in the stochastic theory, minimum variance identification leads to weight the error by the inverse of the spectral density of the noise. It is worth noting that most approaches to frequency identification in the engineering practice consists of posing a least-square minimization problem, and to weigh the terms so as to obtain a suitable result using a generic optimization toolbox. In this way we are led to consider minimizing a criterion of the form:

$$\left\| f - \frac{p_m}{q_n} \right\|_{L^2(d\mu)} \tag{1}$$

where, by definition,

$$\|g\|^2_{L^2(d\mu)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |g(e^{i\theta})|^2 d\mu(\theta),$$

and μ is a positive finite measure on T, p_m is a polynomial of degree less or equal to m and q_n a monic polynomial of degree less or equal to n. Such a problem is nicely put when μ is absolutely continuous with respect to the Lebesgue measure and has invertible derivative in L^∞. For instance when μ is the squared modulus of an invertible analytic function, introducing μ-orthogonal polynomials instead of the Fourier basis makes the situation similar to the non-weighted case, at least if $m \geq n-1$ [56]. The corresponding algorithm was implemented in the hyperion software. The analysis of the critical points equations in the weighted case gives various counter-examples to unimodality in maximum likelihood identification [57].

It is worth pointing out that *meromorphic* approximation is better behaved (*i.e.* essentially invariant) with respect to the introduction of a weight, see 6.6. Another kind of rational approximation, that arises in several design problems where only constraints on the modulus are seeked, consists in approximating the module of a function by the module of a rational function, that is, solving for

$$\min \left\| |f| - \left| \frac{p_n}{q_n} \right| \right\|_{L^p(T)}.$$

This problem is strongly related to the previous ones; in fact, it can be reduced to a convergent series of standard rational approximation problems. Note also that if $p = \infty$, and if moduli are squared, *i.e.* if the feasibility of

$$\left\| |f|^2 - \left|\frac{p_n}{q_n}\right|^2 \right\|_{L^\infty(T)} < \varepsilon,$$

is required, one can use the Féjèr-Riesz characterization of positive trigonometric polynomials on the unit as squared moduli of algebraic polynomials to approach this issue as a convex problem in infinite dimension. This constitutes another fundamental direction for dealing with rational approximation in modulus that arises naturally in filter design problems.

3.1.3. Behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Keywords: *discretization of potentials*, *free boundary inverse problems*, *meromorphic approximation*, *orthogonal polynomials*, *rational approximation*, *singularity detection*.

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Reinhold Küstner [CMI Université de Provence, Marseille], Vilmos Totik [univ. Szeged and Scien. Acad., Hungary].

We want here to study the behavior of poles of optimal meromorphic approximants in L^p norm on a closed contour, to functions defined by Cauchy integrals of measures whose support lies inside the contour. If one normalizes the contour to be the unit circle, which is no restriction in principle thanks to conformal mapping but raises of course difficult questions from the constructive point of view for domains whose shape is not standard (*i.e.* polygonal or elliptic), we find ourselves again in the framework of sections 3.1.1 and 3.1.2. The research so far has focused on functions that are analytic on and outside the contour, and have singularities on an open arc inside the contour.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation for the analysis of the error decrease with the degree and for most constructive aspects like uniqueness, so that everything here could take place in section 3.1.1. However, it is the original motivation of APICS to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. This way to tackle a free boundary problem, classical in every respect but still widely open, illustrates the approach of the team to certain inverse problems, and gives rise to an active direction of research at the crossroads of function theory, potential theory and orthogonal polynomials.

As a general rule, critical point equations for these problems express that the polynomial whose roots are the poles of the approximant is a non-Hermitian orthogonal polynomial with respect to some complex measure on the singular set of the function to be approximated. New results were obtained in recent years concerning the location of such zeroes, and the approach to inverse problem for the Laplacian that we outline in this section appears to be attractive when the singularities are one-dimensional, for instance in the case of a cracked domain (see section 4.2). In case the crack is sufficiently smooth, the approach in question is in fact equivalent to meromorphic approximation of a function with two branch points, and we were able to prove [38][33] that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the endpoints of the crack [4]. Moreover the asymptotic density of the poles is nothing but the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are *de facto* well localized if one is able to compute sufficiently many zeros (this is where the method is not fully constructive). It is interesting to note that these results apply also, and even more easily, to the detection of monopolar and dipolar sources, a case where poles as well as logarithmic singularities exist. The case of more general cracks (for instance formed by a finite union of analytic arcs) requires the analysis of the situation where the number of branch points is finite but arbitrary. It is conjectured that the poles tend to the contour $\mathcal{C}$ that links the end points of these analytic arcs while minimizing the capacity of the condenser $(T, \mathcal{C})$, where T is the exterior boundary of the domain (see section 6.7). The conjecture is confirmed numerically and has been actually proved in the case where the locus of minimal capacity is *connected*; this covers a large number of interesting cases, including the case of general polynomial cracks, or of cracks consisting of sufficiently smooth arcs. This breakthrough, we hope, will constitute a substantial progress towards a proof of the general case. It would of course be very interesting to know what happens when the crack is “absolutely non analytic”, a limiting case that can be interpreted as that of an infinite number of branch points, and on which very little is known, although there are grounds to conjecture that the endpoints at least are still accumulation points of the poles. This is an outstanding open question for applications to inverse problems 6.8. Concerning the problem of a general singularity, that may be two dimensional, one can formulate the following conjecture: if f is analytic outside and on the exterior boundary of a domain D and if K is the minimal compact set included in D that minimizes the capacity of the condenser (T,K) under the constraint that f is analytic and single-valued outside K (it exists, it is unique, and we assume it is of positive capacity in order to avoid degenerated cases), then every limit point (in the weak star sense) of the sequence ν_n of probability measures having equal mass at each pole of an optimal meromorphic approximant (with at most n poles) of f in $L^p(T)$ *has its support in K and sweeps out to the boundary of K as the equilibrium measure on K of the condenser* (T,K). Yet this conjecture is far from being solved.

Results of this type open new perspectives in non-destructive control (see section 4.2), in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which a dual approach is proposed: to approximate boundary conditions and not the equation. Note that the problem of finding a crack suggests non-classical variants of rational and meromorphic approximation where the residues of the approximants must satisfy some constraints in order to take into account the boundary conditions, normal or tangential, along the singularity. In fact, the aforementioned results dealing with (unconstrained) meromorphic approximation lead to identify a deformation of the crack (the arc of minimal capacity that links its end points) rather than the crack itself, which is valuable to initialize a heavier direct method but which is not conclusive by itself. In order to limit the deformation which is due to the fact that we did not keep track of the limiting-conditions (especially the fact that the jump across the crack is real), one may consider approximating the complexified solution F of a Neumann problem in a cracked domain D by a meromorphic function of the type $\sum_{j=1}^{n} a_j/(z-z_j)+g(z)$, where g is analytic in D, under the constraint that $\sum_{k\neq j} a_k/(z_j-z_k)+g(z_j)$ is real for each j; in effect, if the poles z_j are distributed along an arc, the above sum is a discrete estimation of the Hilbert transform of the measure defining the function, and enforcing that it is zero should help satisfying the Neumann condition along the arc. Such modifications of the initial problem are only beginning to be considered within the team.

We conclude by mentioning that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is no longer actual, and this makes for a long-term research project with a wide range of applications. It is interesting to mention that the case of sources in dimension three in a spherical geometry, can be attacked with the above 2D techniques as applied to planar sections (see section 6.8).

3.1.4. Matrix-valued rational approximation

Keywords: *inner matrix*, *rational approximation*, *reproducing kernel space realization theory*.

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, José Grimm.

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the degree. Hence the problem reads: *Let $1\leq p\leq\infty$, $\mathcal{F}\in(H^p)^{m\times l}$ and n an integer; find a rational matrix of size $m\times l$ without poles in the unit disk and of McMillan degree at most n nearest possible to $\mathcal{F}$ in $(H^p)^{m\times l}$*. To fix ideas, we may define the L^p norm of a matrix as the p-th root of the sum of the p powers of the norms of its entries.

The main interest of the Apics Team lies in the case $p=2$. Then, the approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [8]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.* matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices using the so-called Douglas-Shapiro-Shields factorization. The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parameterizations valid in a neighborhood of a point), and he must handle changes of chart in the course of the algorithm. The tangential Schur algorithm [22] provides us with such a parameterization and allowed the team to develop two rational approximation codes. The first one is integrated in the hyperion software dealing with transfer matrices while the other, which is developed under the matlab interpreter, goes by the name of RARL2 and works with realizations. Both have been experimented on measurements by the CNES (branch of Toulouse), IRCOM, and Alcatel Space, and they give high quality results [2] in all cases encountered so far. These codes are now of daily use by Alcatel Space and IRCOM, coupled with simulation software like EMXD to design physical coupling parameters for the synthesis of hyperfrequency filters made of resonant cavities, see 7.1.

In the above application, obtaining physical couplings requires the computation of realizations, also called internal representation in system theory. Among the parameterizations obtained via the Schur algorithm, some have a particular interest from this viewpoint [59]. They lead to a simple and robust computation of balanced realizations and form the basis of the RARL2 algorithm.

Problems relative to multiple local minima naturally arise in the matrix-valued case as well, but deriving criteria that guarantee uniqueness is much more difficult than in the scalar case. The case of rational functions of the right degree already uses rather heavy machinery [5], and that of matrix-valued Markov functions, that are the first example beyond rational function has made progress only very recently (*cf.* section 6.6).

In practice, a method similar to the one used in the scalar case has been developed to generate local minima of a given order from those at lower orders. In short, one sets out a matrix of degree n by perturbation of a matrix of degree $n-1$ where the drop in degree is due to a pole-zero cancellation. There is an important difference between polynomial representations of transfer matrices and their realizations: the former lead to an embedding in a ambient space of rational matrices that allows a differentiable extension of the criterion on a neighborhood of the initial manifold, but not the latter (the boundary is strongly singular). Generating initial conditions in a recursive manner is more delicate in terms of realizations, and some basic questions on the boundary behavior of the gradient vector field are still open.

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.1.5. *Linear parametric identification*

Keywords: *critical points*, *parametric identification*, *rational approximation*, *topology of rational matrices*.

Participants: Laurent Baratchart, Martine Olivi.

The asymptotic study of likelihood estimators is a natural companion to the research on rational approximation described above. The context is ultra-classical. Given a discrete process $y(t)$ with values in $\mathbf{R}^p$, and another process with values in $\mathbf{R}^m$, we check for an explanation of y in terms of u as a finite order linear model:

$$\hat{y}(t) = Hu(t) + Le(t),$$

where e is a white noise with p components, uncorrelated to u, assumed to represent the uncertainty in $y(t)$, and where the transfer matrix $[L \quad H]$ that links $(e \quad u)^t$ to $\hat{y}$ is rational and stable of McMillan degree n, the matrix L being also of stable inverse (among all noises with same covariance, and given innovation, we chose those whose spectral factor has minimum phase). The number n is, by definition, the order of the model. If we only suppose that $[H \quad L]$ belongs to the Hardy space H^2 and that L is outer (this means stably invertible in some sense), such a representation is in fact general for *regular* (i.e. purely non-deterministic) stationary processes. Identification in this context appears then as a rational approximation problem for which the classical theory makes a trade-off between two antagonistic factors, namely the bias error on the one hand that decreases when n increases and the variance error on the other hand that increases with n since the dispersion is amplified with the number of parameters. This is the stochastic version of the complexity versus precision alternative which is all-pervasive in modeling.

If one introduces now as a new variable the rational matrix R defined by

$$R = \begin{pmatrix} L & H \\ 0 & I_m \end{pmatrix}^{-1}$$

and if T stands for the first block-row, normalizing the variance of the noise to be the identity matrix, the maximum likelihood estimator is asymptotically equivalent, when the sample size increases, to the minimization of

$$\|T\|_\Lambda^2 = \mathbf{Tr} \left\{ \frac{1}{2\pi} \int_0^{2\pi} T(e^{i\theta})\, d\Lambda(\theta)\, T^*(e^{i\theta}) \right\}, \tag{2}$$

where Λ is the spectral measure of the process $(y \quad u)^t$ (which positive and matrix-valued) and where $\mathbf{Tr}$ indicates the trace. If we further restrict the class of models by assuming that we deal with white noise, that is if $L = I_m$, one obtains a weighted rational approximation problem corresponding to the minimization of the variance on the output error. If moreover u itself is (observed) white noise, the situation becomes that of 3.1.4.

Formulation (2) shows that stochastic identification aims at a twofold generalization, both rational and matrix-valued, of the Szegö theory of orthogonal polynomials on the circle, and this sets up a link with classical function theory.

The consistency problem arises from the fact that the measure Λ is not available, so that one has to estimate (2) from time averages of the observed samples, assuming that the process is ergodic. The question is then to decide whether the argument of the minimum of the estimated functional tends to that of (2) when the sample size increases, and what is the speed of convergence. The most significant result here is perhaps the one asserting that if there exists a functional model linking u to y (*i.e.* u is indeed the cause of the phenomenon), and without assuming compactness of the class of models [52], then consistency holds under weak ergodicity conditions and persistent excitation assumptions. An analogous of the law of large numbers indicates, in this context, that convergence is in the order of $1/\sqrt{N}$, where N is the sample size.

In the preceding result, consistency holds in the sense of pointwise convergence of the estimates on the manifold of transfer functions of given size and order. One contribution of the former Miaou project has been to show that the result holds even if we do not postulate a causal dependency between inputs and outputs, the measure Λ being simply defined as the weak limit of the covariances. A second contribution is that this convergence holds uniformly with all its derivatives on each compact subset of the manifold of models, thereby drawing a path between the algorithmic behavior of the rational approximation problem (number and nature of critical points, decrease of error, behavior of the poles) and that of the minimization of empirical means. This allows one to translate in terms of asymptotic behavior of the estimators virtually all properties that are uniform with respect to the order of the approximants, and without having to assume that the "true" system belongs to the class of models. Let us mention for instance that uniqueness of a critical point in H^2 rational approximation, in the case where the system to approximate is nearly rational of degree n, implies [5] uniqueness of a local minimizer for the output error when the input is a white noise, asymptotically almost surely on every compact, when the density of y with respect to u is nearly rational of degree n. In the case of relaxation systems, with one input-output, that is, if the transfer function is a Markov function, we obtain, in the light of the results exposed in module 3.1.2, the same conclusion when the order of approximation is large enough. This is the first known case of unimodularity where the "true" system does not belong to the class of models. An application to the localization of the poles of rational estimates of the output error of a long memory system was derived from this [31]. Here, we are faced again with the question, already mentioned in the introduction, of how to expand functions in bases that are adapted to the singularities of the spectral density of long memory processes. We believe this research direction would be worth exploring.

3.2. Structure and control of non-linear systems

In order to control a system, one generally relies on a model, obtained from *a priori* knowledge like physical laws or experimental observations. In many applications, one is satisfied with a linear approximation around a nominal point or trajectory. It is however very important to study non-linear systems (or models) for the following reasons. First, some systems have, near interesting working points, a linear approximation that is non-controllable so that linearization is ineffective, even locally. Secondly, even if the linearized model is controllable, one may wish to extend the working domain beyond the validity domain of the linear approximation. Work described in module 3.2.1 dwells on such issues. Finally, certain control problems, such as path planning, are not of a local nature and cannot be answered *via* a linear approximation. The structural study described in module 3.2.2 aims at exhibiting invariants that can be used, either to reduce the study to simpler systems or to make grounds for a non-linear identification theory, that would give informations on model classes to be used in case there is no *a priori* reliable information and still the black-box linear identification is not satisfactory. The success of the linear model, in control or in identification, is due to the deep understanding one has of it; in the same fashion, a refined knowledge of invariants of non-linear systems under basic transformations is a prerequisite for a theory of non-linear identification and control. In what follows, all non-linear systems are supposed to have a state space of finite dimension.

3.2.1. Feedback control and optimal control

Keywords: *control*, *non holonomic mechanical system*, *non-linear control*, *stabilization of non-linear systems*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet, Mario Sigalotti.

Stabilization by continuous state feedback — or output feedback, that is, the partial information case — consists in designing a control law which is a smooth (at least continuous) function of the state making a given point (or trajectory) asymptotically stable for the closed system. One can consider this as a weak version of the optimal control problem: to compute a control that optimizes a given criterion (for instance to reach a prescribed state in minimal time) leads in general to a very irregular dependence on this state; stabilization is a *qualitative* objective (*i.e.* to reach that state asymptotically) which is more flexible and allows one to impose a lot more regularity.

Lyapunov functions are a well-known tool to study stability of non-control dynamic systems. For a control system, a *Control Lyapunov Function* is a Lyapunov function for the closed-loop system where the feedback is chosen appropriately. It can be expressed by a differential inequality called the "Artstein (in)equation [24]", that looks like the Hamilton-Jacobi-Bellmann equation but is largely under-determined. One can easily deduce from the knowledge of a control Lyapunov function a continuous stabilizing feedback.

The team is engaged in obtaining control Lyapunov functions for certain classes of systems. This should be the first step in synthesizing a stabilizing control, but even when such a control is known beforehand, obtaining a control Lyapunov function can still be very useful to study the robustness of the stabilization, or to modify the initial control law into a more robust one. Moreover, if one has to deal with a problem where it is important to optimize a criterion, and if the optimal solution is hard to compute, one can look for a control Lyapunov function which comes "close" (in the sense of the criterion) to the solution of the optimization problem but leads to a control which is easier to work with.

These constructions are exploited in the joint collaborative research conducted with Alcatel Space (see module 7.2), where minimizing a certain cost is very important (fuel consumption / transfer time) while at the same time a feedback law is preferred because of robustness and ease of implementation.

3.2.2. Transformations and equivalences of non-linear systems and models

Keywords: *classification*, *non-linear control*, *non-linear feedback*, *non-linear identification*.

Participants: David Avanessoff, Laurent Baratchart, Monique Chyba [U. of Hawaii (USA)], Jean-Baptiste Pomet.

A *static feedback* transformation of a dynamical control system is a (non-singular) reparametrization of control, depending on the state, and possibly, a change of coordinates in the state space. A *dynamic feedback* transformation of a dynamic control system consists of a dynamic extension (adding new states, and assigning then a new dynamics) followed by a state feedback on the augmented system.

- From the point of view of control, the interest of these transformations is that a command satisfying specific objectives on the transformed system can be used to control the original system including the possibly extended dynamics in the controller. Of course the favorable case is when the transformed system has a structure that can easily be exploited, for instance when it is a linear controllable system.
- From the point of view of identification and modeling, in the non-linear case, the interest is either to derive qualitative invariants to support the choice of a non-linear model given the observations, or to contribute to a classification of non-linear systems which is missing sorely today.

These two aspects will now be developed.

3.2.2.1. Dynamic linearization.

The problem of dynamic linearization, still unsolved, is that of finding explicit conditions on a system for the existence of a dynamical feedback that would make it linear.

These last years [48], the following property of control systems has been emphasized: for some systems (in particular linear ones), there exists a finite number of functions of the state and of the derivatives of the control up to a certain order, that are differentially independent (*i.e.* coupled by no differential equation) and do “parameterize all the trajectories”. This property and its importance in control, has been brought in light in [48], where it is called *differential flatness*, the above mentioned functions being called *flat* or *linearizing functions*, and it was shown, roughly speaking, that a system is differentially flat if, and only if, it can be converted to a linear system by dynamic feedback. On one hand, this property of the set of trajectories has in itself an interest at least as important for control than the equivalence to a linear system, and on the other hand it gives a handle for tackling the problem of dynamic linearization, namely to find linearizing functions.

An important question remains open: how can one algorithmically decide that a given system has this property or not, *i.e.* is dynamically linearizable or not? This problem is both difficult and important for non-linear control. For systems with four states and two controls, whose dynamics is affine in the control (these are the lowest dimensions for which the problem is really non-trivial), necessary and sufficient conditions [10] for the existence of linearizing functions depending on the state and the control (but not the derivatives of the control) can be given explicitly, but they do point to the complexity of the issue.

From the algebraic-differential point of view, the module of differentials of a controllable system is free and of finite dimension over the ring of differential polynomials in d/dt with coefficients in the space of functions of the system, and for which a basis can be explicitly constructed [23]. The question is to find out if it has a basis made of closed forms, that is, locally exact forms. Expressed in this way, it is an extension of the classical integrability theorem of Frobenius to the case where coefficients are differential operators. Together with stability by exterior differentiation (the classical condition), further conditions are required here to ascertain the degree of the solutions is finite, the mid-term goal is to obtain a formal and implementable algorithm, able to decide whether or not a given system is flat around a regular point. One can also consider sub-problems having their own interest, like deciding flatness with a given pre-compensator, or characterizing “formal” flatness that would correspond to a weak interpretation of the differential equation. Such questions can be localized in the neighborhood of an equilibrium point.

3.2.2.2. Topological Equivalence

In what precedes, we have not taken into account the degree of *smoothness* of the transformations under consideration.

In the case of dynamical systems without control, it is well known that, away from degenerate (non hyperbolic) points, if one requires the transformations to be merely continuous, every system is *locally* equivalent to a linear system in a neighborhood of an equilibrium (the Hartman-Grobman theorem). It is thus tempting when classifying *control* systems, to look for such equivalence modulo non-differentiable transformations and to hope bring about some robust “qualitative” invariants and perhaps stable normal forms. A Hartman-Grobman theorem for control systems would say for instance, that outside a “meager” class of models (for instance, those whose linear approximation is non-controllable), and locally around nominal values of the state and the control, no qualitative phenomenon can distinguish a non-linear system from a linear one, all non-linear phenomena being thus either of global nature or singularities. Such a statement is wrong: if a system is locally equivalent to a controllable linear system via a bi-continuous transformation—a local homeomorphism in the state-control space—it is *also* equivalent to this same controllable linear system via a transformation that is as smooth as the system itself, at least in the neighborhood of a regular point (in the sense that the rank of the control system is locally constant), see [28] for details; *a contrario*, under weak regularity conditions, linearization can be done by non-causal transformations (see the same report) whose structure remains unclear, but acquire a concrete meaning when the entries are themselves generated by a finite dimensional dynamics.

The above considerations call for the following question, which is important for modeling control systems: are there local "qualitative" differences between the behavior of a non-linear system and its linear approximation when the latter is controllable?

4. Application Domains

4.1. Introduction

The botton line of the team's activity is twofold, namely optimization in the frequency domain on the one hand, and the control of systems governed by differential equations on the other hand. Therefore one can distinguish between two main families of applications: one dealing with the design and identification of diffusive and resonant systems (these are inverse problems), and one dealing with the control of certain mechanical or optical systems. For applications of the first type, approximation techniques as described in module 3.1.1 allow one to deconvolve linear equations, analyticity being the result of either the use of Fourier transforms or the harmonic character of the equation itself. Applications of the second type mostly concern the control of systems that are "poorly" controllable, for instance low thrust satellites or optical regenerators. We describe all these below in more detail.

4.2. Geometric inverse problems for the Laplacian

Keywords: *Laplace equation*, *inverse problem*, *non destructive control*, *tomography*.

Participants: Amina Amassad, Bilal Atfeh, Laurent Baratchart, Amel Ben Abda [ENIT, Tunis], Fehmi Ben Hassen, Imen Fellah, José Grimm, Mohamed Jaoua, Juliette Leblond, Moncef Mahjoub, Edward Saff.

Localizing cracks, pointwise sources or occlusions in a two-dimensional material, using thermal, electrical, or magnetic measurements on its boundary is a classical inverse problem. It arises when studying fatigue of structures, behavior of conductors, or else magneto-encephalography as well as the detection of buried objects (mines, etc). However, no really efficient algorithm has emerged so far if no initial information on the location or on the geometry is known, because numerical integration of the inverse problem is very unstable. The presence of cracks in a plane conductor, for instance, or of sources in a cortex (modulo a reduction from 3D to 2D, see later on) can be expressed as a lack of analyticity of the (complexified) solution of the associated Dirichlet-Neumann problem that may in principle be approached using techniques of best rational or meromorphic approximation on the boundary of the object (see sections 3.1.1 to 3.1.3 and 6.8,6.3). In this connection, the realistic case where data are available on part of the boundary only is a typical opportunity to apply the analytic and meromorphic extension techniques developed earlier.

The 2D approach proposed here consists in constructing, from measured data on a subset K of the boundary Γ of a plane domain D, the trace on Γ of a function F which is analytic in D except for a possible singularity across some subset $\gamma \subset D$ (typically: a crack). One can then use the approximation techniques described above in order to:

- extend F to all Γ if the data are incomplete (it may happen that $K \neq \Gamma$) if the boundary is not fully accessible to measurements), for instance to identify an unknown Robin coefficient, see [12] where stability properties of the procedure are established;

- detect the presence of a defect γ in a computationally efficient manner, [43];
- obtain information on the location of γ [32], [27].

Thus, inverse problems of geometric type that consist in finding an unknown boundary from incomplete data can be approached this way [4], usually in combination with other techniques [43]. Preliminary numerical experiments have yielded excellent results and it is now important to process real experimental data, that the team is currently busy analysing. In particular, contacts with the Odyssée Team of Inria Sophia Antipolis (within the ACI "Obs-Cerv") has provided us with 3D magneto-encephalographic data from which 2D information was extracted, see section 6.8. The team is also in contact with other laboratories (*e.g.* Vanderbilt Univ. Physics Dept.) in order to work out 2D or 3D data from physical experiments.

In the longer term, we envisage applying this type of methods to problems with variable conductivity, or to the Helmholtz equation. Using convergence properties of approximation algorithms in order to establish stability results for some of these inverse problems is also an appealing direction for future research.

4.3. Identification and design of resonant systems

Keywords: *filtering device*, *hyperfrequency*, *multiplexing*, *surface waves*, *telecommunications*.

One of the best training ground for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption is well-satisfied in the working range of frequency, and whose specifications are made in frequency domain. Resonant systems, acoustic or electromagnetic, are prototypical examples of common use in telecommunications. We shall be more specific on two examples below.

4.3.1. Design of surface acoustic wave filters

Participants: Laurent Baratchart, Andrea Gombani, José Grimm, Martine Olivi.

Surface acoustic waves filters are largely used in modern telecommunications especially for cellular phones. This is mainly due to their small size and low cost. Unidirectional filters, formed of *Single Phase UniDirectional Transducers* (in short: SPUDT) that contain inner reflectors (cf. Figure 1), are increasingly used in this technological area. The design of such filters is more complex than traditional ones.

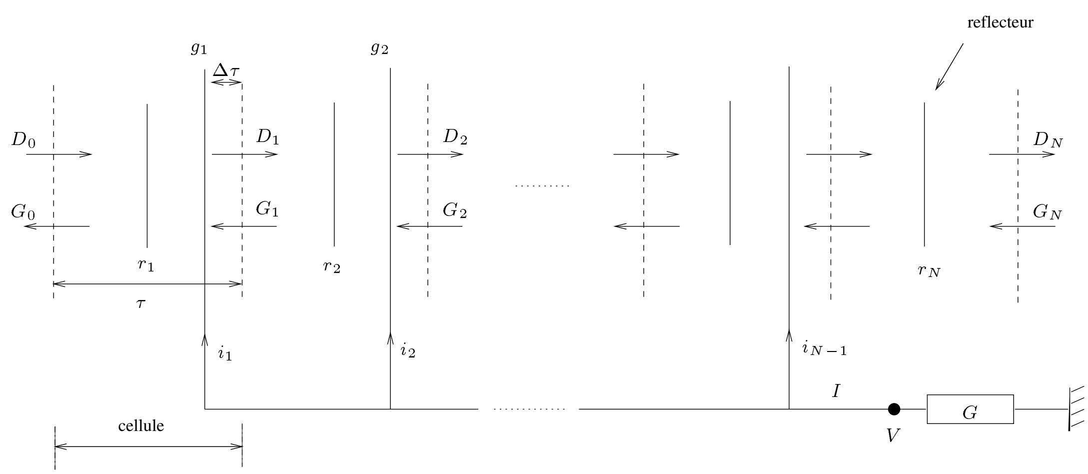

Figure 1. Transducer model.

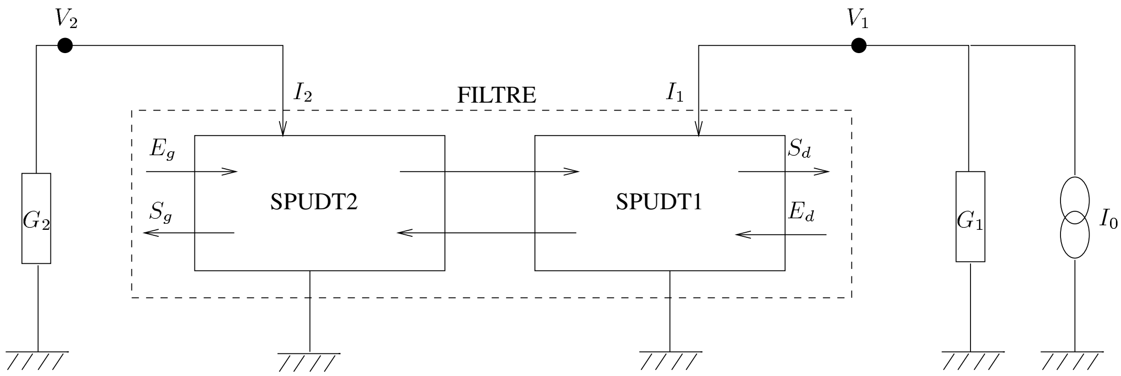

Figure 2. Configuration of the filter

We are interested here in a filter formed of two SPUDT transducers (Figure 2). Each transducer is composed of cells of the same length τ each of which contains a reflector and all but the last one contain a source (Figure 1). These sources are all connected to an electrical circuit, and cause electro-acoustic interactions inside the piezo-electric medium. In the transducer SPUDT2 represented on Figure 2, the reflectors are positioned with respect to the sources in such a way that near the central frequency, almost no wave can emanate from the transducer to the left ($S_g \approx 0$), this being called unidirectionality. In the right transducer SPUDT1, reflectors are positioned in a symmetric fashion so as to obtain unidirectionality to the left.

Specifications are given in the frequency domain on the amplitude and phase of the electrical transfer function. This function expresses the power transfer and can be written as

$$E(r,g) = 2\,\frac{V_2}{I_0} = \frac{2\sqrt{G_1G_2}\quad Y_{12}}{Y_{12}Y_{21} - (Y_{11}+G_1)(Y_{22}+G_2)},$$

where Y is the admittance of the coupling:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}.$$

The design problem consists in finding the reflection coefficients r and the source efficiency in both transducers so as to meet the specifications.

The transducers are described by analytic transfer functions called mixed matrices, that link input waves and currents to output waves and potentials. Physical properties of reciprocity and energy conservation endow these matrices with a rich mathematical structure that allows one to use approximation techniques in the complex domain (see module 7.1) according to the following steps:

- describe the set $\mathcal{E}$ of electrical transfer functions obtainable from the model,
- set out the design problem as a *rational approximation problem* in a normed space of analytic functions:

$$\min_{E\in\mathcal{E}} \|D - E\|,$$

  where D is the desired electrical transfer,
- use a rational approximation software (see module 5.2) to identify the design parameters.

The first item, is the subject of ongoing research. It connects the geometry of the zeroes of a rational matrix to the existence of an inner symmetric extension without increase of the degree (reciprocal Darlington synthesis), see 6.5. Let us mention that the interest of the team for such filters was triggered by a collaboration with Thomson Microsonics.

4.3.2. Hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila, José Grimm, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead to the use of filters with resonant cavities in the hyperfrequency range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far away, and their influence can be neglected).

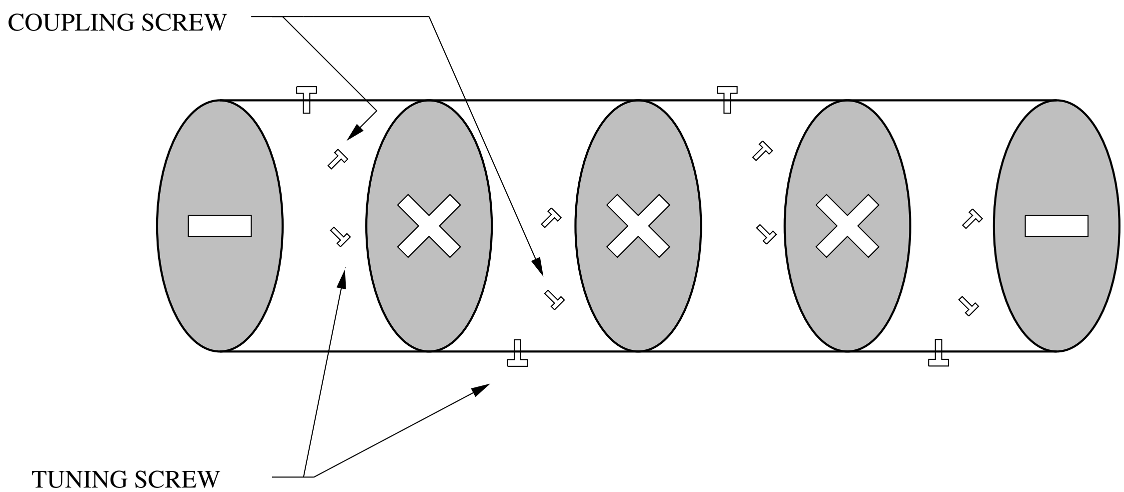

Figure 3. Schematic 4-cavities dual mode filter. Each cavity has 3 screws to couple the modes within the cavity, so that there are 12 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 8 slits are fixed in the design phase.

Each cavity (see Figure 3) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; on the other hand, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero, and hence is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of the rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. After that, the filter can be constructed, and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plug on a resistor at one end and fed with some potential at the other. We are then interested in the power transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas for expressing the length of the screws in terms of parameters of this electrical model). On the other hand, this is also useful for the design of the filter, for analyzing numerical simulations of the Maxwell equations, and for checking the design, particularly the absence of higher resonant modes.

In reality, resonance is not studied via the electrical model, but via a low pass equivalent obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems, in H^2 norm for the transmission and in Sobolev norm for the reflection (the module of he response being respectively close to 0 and 1 outside the interval measurement) cf. module 3.1.1. This gives a scattering matrix of order roughly 1/4 of the number of data points.
- Then one rationally approximate with fixed degree (8 in this example) via the hyperion software cf. module 3.1.4.
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this kind of change of basis).

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 4). Non-physical coupling are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of unsymmetric filters, useful for the synthesis of repeating devices.

The team extends today its investigations, to the design of output multiplexors (OMUX) that couple several filters of the previous type on a manifold. The objective is to establish a global model for the behavior that would take into account:

- within each channel the coupling between the filter and the Tee that connects it to the manifold,
- the coupling between two consecutive channels.

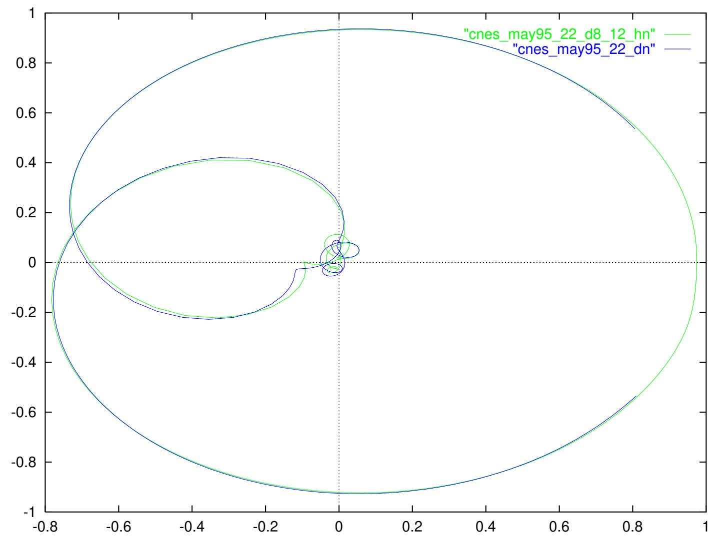

Figure 4. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

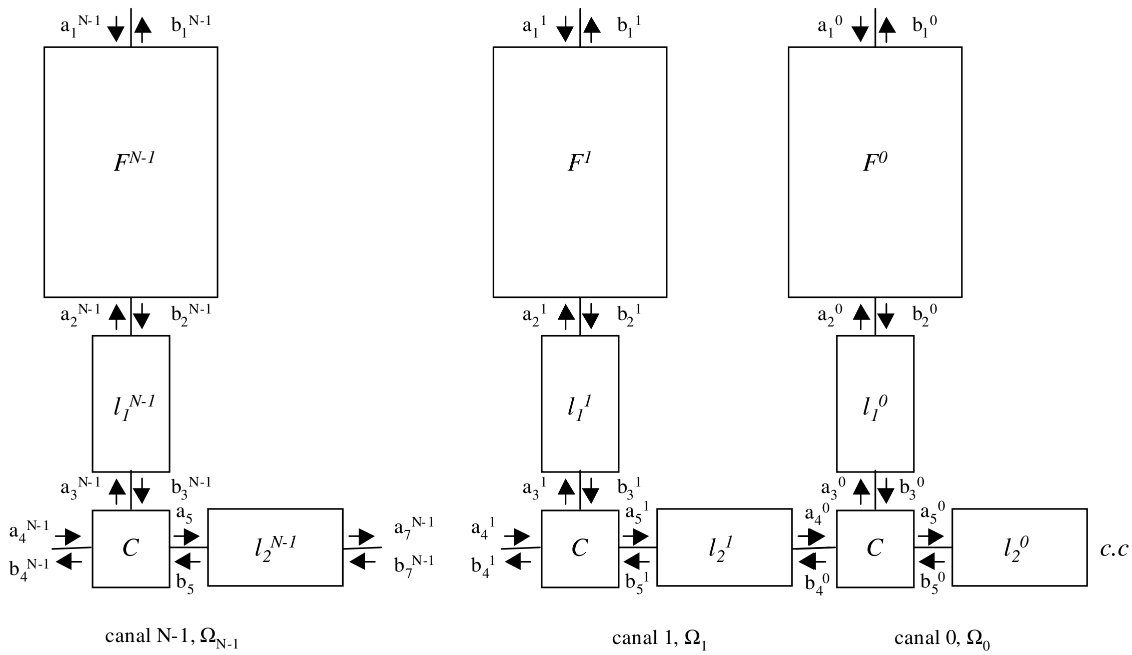

Figure 5. N filters on a manifold. Quantities a and b are incoming and outcoming waves, boxes with an l stand for an adjustable line-length, the 'C' in the square box represents the heart of the Tee (3 by 3 transfer matrix); the big rectangles with an F represent the filters, they are connected to the amplifiers, the bottom row (with the ls and Cs) is the manifold, connected to the antenna on the left, terminated by a cc (short-circuit) on the right.

The model is obtained upon chaining the transfer matrices associated to the scattering matrices. It mixes rational elements and complex exponentials (because of the delays) and constitutes an extension of the previous framework. Under contract with the CNES (see 7.1), the team has started a study of the design with gauge constraints, based on function theoretical tools.

4.4. Spatial mechanics

Keywords: *orbital control*, *satellite*, *spatial mechanics*, *telecommunications*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet, Mario Sigalotti.

The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing; see for instance section 4.3 for an illustration.

Of course, this requires that satellites be adequately located and positioned (correct orientation). This problem and other similar ones continue to motivate research in control from the part of the team. Aerospace engineering in general is a domain that requires sophisticated control techniques, and where optimization is often crucial, due to the extreme conditions.

The team has been working for two years on control problems in orbital transfer with low-thrust engines, under contract with Alcatel Space Cannes, see module 7.2. Technically, the reason for using these (ionic) low thrust engines, rather than chemical engines that deliver a much higher thrust, is that they require much less "fuel"; this is decisive because the total mass is limited by the capacity of the launchers : less fuel means more payload, and fuel represents an impressive part of the total mass.

From the control point of view, the low thrust makes the transfer problem delicate. In principle of course, the control law leading to the right orbit in minimum time exists, but it is quite heavy to obtain numerically and the computation is non-robust against many unmodelled phenomena.

4.5. Non-linear Optics

Keywords: *3R regeneration*, *Optics*, *networks*, *optical fibers*, *telecommunications*.

The increased capacity of numerical channels in information technology is a major industrial challenge. The most performing means nowadays for transporting signals from a server to the user and backwards is via optical fibers. The use of this medium at the limit of its capacity of response causes new control problems in order to maintain a safe signal, both in the fibers and in the routing and regeneration devices.

In a recent past, the team has worked in collaboration with Alcatel R&I (Marcoussis) on the control of "all-optic" regenerators. Although no collaboration is presently active, we consider this a potentially rich domain of applications

4.6. Transformations and equivalence of non-linear systems

Keywords: *identification*, *mobile cybernetics*, *path planning*.

Participants: Laurent Baratchart, Jean-Baptiste Pomet, David Avanessoff.

The works presented in module 3.2.2 lie upstream with respect to applications. However, beyond the fact that deciding whether a given system is linear modulo an adequate compensator is clearly conceptually important, it is fair to say that "flat outputs" are of considerable interest for path planning [58]. Moreover, as indicated in section 3.2, a better understanding of the invariants of non-linear systems under feedback would result in significant progress in identification.

5. Software

5.1. The Tralics software

Participant: José Grimm [manager].

The development of a LaTeX to XML translator, named Tralics was continued. For more details, see module 6.2. TRALICS was sent to the APP in December 2002. Its IDDN number is InterDepositDigitalNumber = IDDN.FR.001.510030.000.S.P.2002.000.31235. Binary versions are available for Linux, Solaris, Windows and Mac-OS X. Its web page is http://www-sop.inria.fr/apics/tralics. It is now licensed under the CeCILL license, see http://www.cecill.info. The latest version is 2.4.

5.2. The RARL2 software

Participants: Jean-Paul Marmorat, Martine Olivi [manager].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see module 3.1.4). Its web page is http://www-sop.inria.fr/miaou/RARL2/rarl2.html. This software takes as input a stable transfer function of a discrete time system represented by

- either its internal realization
- or its first N Fourier coefficients
- or discretized values on the circle

It computes a local best approximant which is *stable, of prescribed McMillan degree*, in the L^2 norm.

It is germane to the arl2 function of hyperion from which it differs mainly in the way systems are represented: a polynomial representation is used in hyperion, while RARL2 uses realizations, this being very interesting in certain cases. It is implemented in MATLAB. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parameterization that has the following advantages

- it incorporates the stability requirement in a buit-in manner,
- it allows the use of differential tools,
- it is well-conditioned, and computationally cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum (see module 6.4) by generating, in a structured manner, several initial conditions. Contrary to the polynomial case, we are in a singular geometry on the boundary of the manifold on which minimization takes place, which forbids the extension of the criterion to the ambient space. We have thus to take into account a singularity on the boundary of the approximation domain, and it is not possible to compute a descent direction as being the gradient of a function defined on a larger domain, although the initial conditions obtained from minima of lower order are on this boundary. Thus, determining a descent direction is nowadays, to a large extent, a heuristic step. This step works well in the cases handled up to now, but research is under way in order to make this step truly algorithmic.

5.3. The RGC software

Participants: Fabien Seyfert, Jean-Paul Marmorat.

The identification of filters modeled by an electrical circuit that was developed inside the team (see module 4.3.2) has led to compute the electrical parameters of the filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called "the arrow form" [44] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. However if the desired realization is not in arrow form, one can show that it can be deduced by an orthogonal change of basis (in general complex). In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and also because the assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. Work is in progress, see section 6.10.

5.4. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for hyperfrequency filters http://www-sop.inria.fr/miaou/Fabien.Seyfert/Presto_web_page/presto_pres.html

In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass hyperfrequency filter parameters. It allows to run the following algorithmic steps, one after the other, or all together in a single sweep:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment);
- automatic determination of an analytic completion, bounded in module for each channel, (see module 6.9);
- rational approximation, of fixed McMillan degree;
- determination of a constrained realization.

For the matrix-valued rational approximation stage Presto-HF relies either on hyperion (Unix or Linux only) or RARL2 (platform independent), both rational approximation engines were developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Alcatel Space in Toulouse.

6. New Results

6.1. Tools for producing the Activity Report (this document)

Keywords: *DTD*, *Perl*, *XML*, *configure*, *make*, *module*.

Participants: José Grimm, Bruno Marmol [DISC], Marie-Pierre Durollet [DISC].

The great novelty in the RAWEB2002 (Scientific Annex to the Annual Activity Report of Inria), was the use of XML as intermediate language, and the possibility of bypassing LaTeX. A working group, formed of M.P. Durollet, J. Grimm, L. Pierron, and I. Vatton (not forgetting A. Benveniste, J.-P. Verjus and J.-C. Le Moal) is in charge of the definition of the tools; in 2003, B. Marmol joined the group, he is in charge of the dissemination of the package.

The construction of the raweb is explained schematically on figure 6. The input is either a LaTeX file, or an XML file. Since 2002, the LaTeX to XML translator is the *Tralics* software, described in module 6.2; it was originally a Perl script, nowadays it is a C++ executable. An XSLT processor (for instance `xsltproc`, from the Gnome tools) is used to convert the XML either into HTML, or into an XSL-FO document, by adding

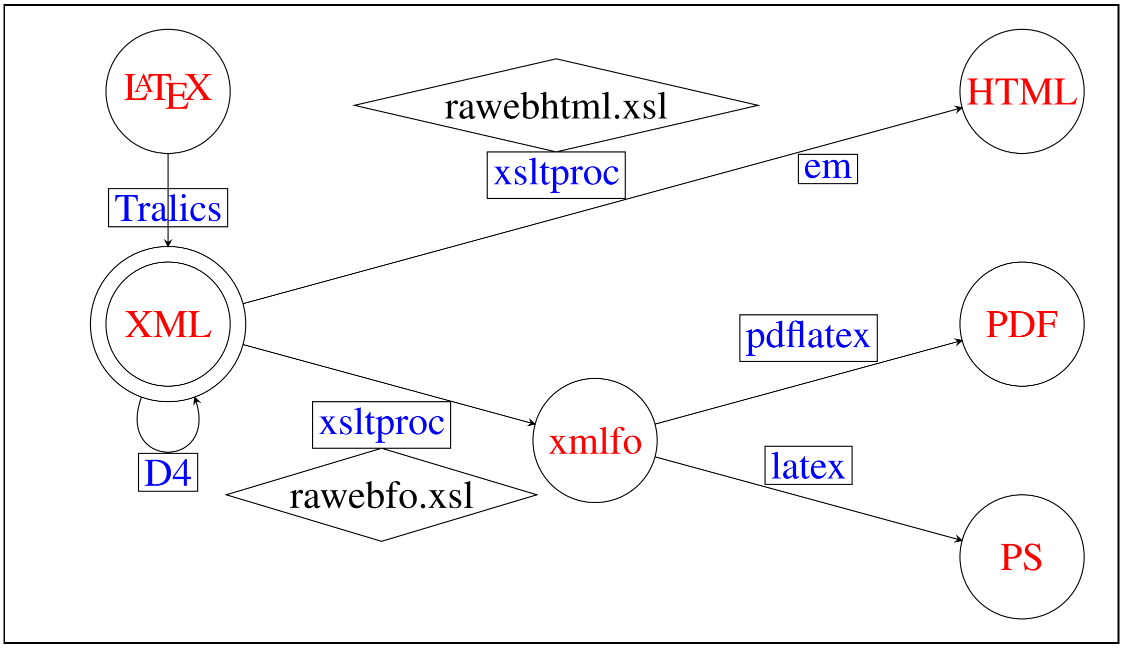

Figure 6. A diagram that explains how the raweb operates. Rectangular boxes contain tools, diamond-shape boxes are style sheets, and circles contain language names. The name ‘XML’ is in a double circle, it is the central object; the arrow labelled ‘D4’ that connects it to itself indicates conversion from one DTD to the other, used in 2004. The box containing ‘em’ represents the Perl script `extract-math.pl` *that handles the math formulas; it uses tools borrowed from* `latex2html`*. This figure was made using the ‘pgf’ package, a new portable graphic format, not yet understood by Tralics.*

some formatting instructions (in this phase, we explain for instance that the text font should be Times). This file is formatted by TeX or pdfTeX, thanks to the `xmltex` package that teaches TeX the subtleties of XML and utf-8 encoding, and two packages for the XSL-FO and MathML commands.

In the original version, one could instruct *Tralics* to produce the XML output, or to convert it also to HTML or Pdf. One could also ask for a direct PostScript version (by-passing the XML phase). This is now governed by a Perl script, called `rahandler.pl`. One can modify this script (for instance, change the name or the pathname of the XSLT processor, or the location of the SGML catalog file); this is now the recommended procedure (of course, it is still possible to specify in the *Tralics* configuration file these names, which are transmitted to the script). The raweb package uses a Makefile to call *Tralics* without options, and then all other tools, (in this case `rahandler.pl` is unused).

As a byproduct, all bibliographical references of years 2000 to 2003 have been translated to XML, sorted by authors, type, year, and put on the web (currently the internal server http://www.inria.fr/interne/disc/).

One important issue was the choice of the DTD (*document type definition*). On one hand, it should follow the pseudo-DTD as defined for the RAWEB six years ago (the Activity Report is a set of modules, with contributors, key-words, etc), and on the other hand, it must be as close as possible to standard DTDs. We have decided to use a variant of the TEI (*text encoding initiative*, see http://www.tei-c.org/) for the text, MathML for the mathematics, and an ad-hoc DTD for the bibliography. This DTD was modified in 2004, independently of *Tralics*. In other words, on Figure 6, a new arrow has to be added: it goes from the old DTD to the new one.

The main difficulty comes from the mathematics: consider a formula like $\mathcal{X}_{\mathtt{y}} = \lim_{x\to 0} \sin^2(x)$. This is translated by *Tralics* into a formula that contains a script X, coded as `<mi>𝒳</mi>`. After conversion to the new DTD, entities are replaced by Unicode characters, so that the X becomes `<mi>𝒳</mi>`. This character seems to be unknown by browsers like Amaya or Mozilla, and is rendered by a question mark or a little box containing the Unicode value (here 01D4B3). This is one of the reasons why math formulas are still replaced by images; in the case `$x+\alpha$`, only the α is converted; this has the advantage to reduce the number of images, but in some cases is not very elegant.

Conversion is done by a dedicated Perl script that extracts from the XML file all formulas, and converts them to a set of pages in a dvi file (we use here the same algorithm for converting the XML to PostScript). Each page is converted to an image via pstoimg, which is a Perl code, part of latex2html. We try to associate each image an Alt field that describes the formula, but this is difficult: for the example we get `${\#119987 _y=lim_{x\#8594 0}sin^2{(x)}}$`.

6.2. Tralics: a Latex to XML Translator

Keywords: *Scanner*, *parsing*, *validation*.

Participant: José Grimm.

The *Tralics* software is a C++ written LaTeX to XML translator, based upon a Perl script that was used for the raweb, and described in [49]. It was presented at the EuroTeX conference in Brest, [50] in 2003. One use of the software is shown on figure 6, and described in module 6.1. Some specific features of the Raweb have been removed from the binary, and put in a configuration file (for instance, the names the Inria research themes, or the sections of the raweb). The default processing mode is no more the raweb; fewer intermediary files are generated.

A second application is the following: when researchers wish to publish an Inria Research Report, they send their PostScript or Pdf document, together with the start of the LaTeX source. This piece of document is converted by *Tralics*, using a special configuration file, that extracts only the title page information (author names, abstract, etc). A perl script removes useless pieces, and produces an HTML notice (see for instance http://www.inria.fr/rrrt/rr-5316.html). As you can see, math formulas like `$(2^n+1)$`, `$n\times n$` are output more or less verbatim, by changing the `\catcode` of some characters, and by redefining all Greek letters and symbols like `\times`.

The main philosophy of *Tralics* is to have the same parser as TeX, but the same semantics as LaTeX. This means that commands like `\chardef`, `\catcode`, `\ifx`, `\expandafter`, `\csname`, etc., that are not described in the LaTeX book and not implemented in translators like latex2html, tth, hévéa, etc., are recognised by Tralics. The program is configurable: the translation depends on some options, and the `\documentclass`. All element names (except p) can be changed by the user.

This year we added constructions like `\newcolumntype` (in fact, we re-implemented completely the array package, although it is still impossible to put arrays into arrays in the Raweb), `\newtheorem` (with all the bells and whistles), and a lot of Unicode characters (there are 1700 commands defined nowadays). We also changed the output of font changing commands: instead of an element named `hi` with an attribute `rend` that could be bold, italic, etc, one can ask *Tralics* to output an element named, for instance, `bold`. We added commands to manipulate MathML objects. Given the following piece of code

```
\providecommand\operatorname[1]{\mathmo{#1}
\mathattribute{form}{prefix}\mathattribute{movablelimits}{true}}
\def\Dmin{\operatorname{dmin}}
```

you can use `\Dmin` like `\min`, for instance $\min_x f(x) > \operatorname*{dmin}_x f(x)$.

There are some unsolved problems: for instance, a figure environment should contain only graphics together with a single caption, commands defined by the picture environment are translated (but refused by the style sheet), non-math material in a math formula is rejected (unless it is formed of characters only).

For more information, see the Tralics web page. It contains a description of each command. A reference manual [51] is in preparation.

6.3. Identification of magnetic dipoles

Participants: Laurent Baratchart, José Grimm, Juliette Leblond, Edward Saff, John Wiskwo [University of Vanderbilt], Eduardo Lima [University of Vanderbilt], Luis Fang [University of Vanderbilt].

The magnetic field produced by a magnetic dipole $\overrightarrow{m}$ located at a point $\overrightarrow{r}'$ is

$$\overrightarrow{B}(\overrightarrow{r}) = \frac{\mu_0}{4\pi}\left\{\frac{3\overrightarrow{m}(\overrightarrow{r}')\cdot(\overrightarrow{r}-\overrightarrow{r}')}{|\overrightarrow{r}-\overrightarrow{r}'|^5}(\overrightarrow{r}-\overrightarrow{r}') - \frac{\overrightarrow{m}(\overrightarrow{r}')}{|\overrightarrow{r}-\overrightarrow{r}'|^3}\right\}. \tag{3}$$

The problem is to identify the location $\overrightarrow{r}' = (x_k, y_k, z_k)$, and the momentum $\overrightarrow{m}_k$ of a sequence of N dipoles indexed by $k = 1, ..., N$, given measurements from a SQUID (superconducting quantum interference device). The assumption that z_k is independent of k (*i.e.* all dipoles lie in a plane) is made, and we assume also that $\overrightarrow{m}_k$ is parallel to the z-axis for all k. In this case the previous formula simplifies to

$$B_z(x,y,z) = \frac{\mu_0}{4\pi}\lambda_k \frac{2z^2-(x-x_k)^2-(y-y_k)^2}{[(x-x_k)^2+(y-y_k)^2+z^2]^{5/2}}$$

The effect of the pick-up coil needed by the SQUID can be modeled by averaging over a small disk, of radius a. Thus we measure a quantity of the form

$$C_z(x,y,z) = \frac{\mu_0}{4\pi^2 a^2}\sum_k \lambda_k \int_{D(0,a)} \frac{2z^2-(x-\alpha-x_k)^2-(y-\beta-y_k)^2}{[(x-\alpha-x_k)^2+(y-\beta-y_k)^2+z^2]^{5/2}} d\alpha d\beta. \tag{4}$$

We have written a simulator for the direct problem; an example is displayed on figure 7. We have written alternate formulas where the double integral is replaced by a simple one. Moreover, integrating by parts, we can replace this integral by a similar one where the exponent on the denominator is $3/2$ instead of $5/2$, thus making the singularities integrable. Writing down an extension of this integral to the complex variable framework ($\xi = x + iy$, the z-plane being fixed) shows that it admits poles and branchpoints as singularities in disks, which account for the location of the sources. This allows us to apply our approximation tools (RARL2 or hyperion, for instance), in order to solve the inverse problem. Our collaborators at Vanderbilt University have measured the magnetic field induced by the ink on a dollar bill; we are testing our methods against these data.

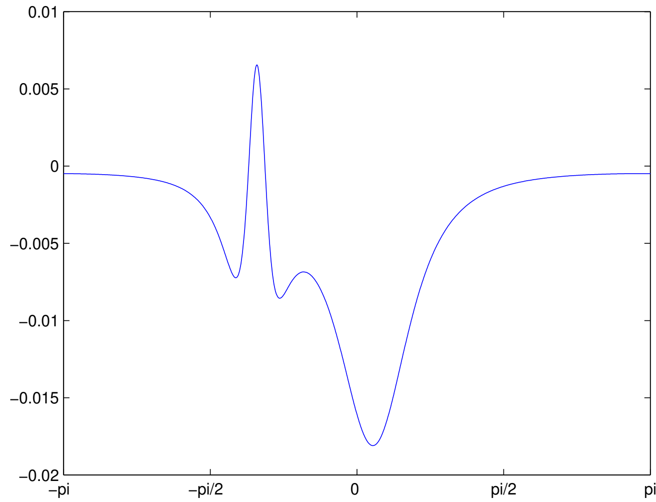

Figure 7. Simulated field measure. On the figure you can see a function $Y = f(X)$, corresponding to $C_z(x, y, z)$ evaluated at 800 points of the unit circle ($x = \cos X, y = \sin X, z$), Computations on done by Matlab, via numeric equation of (4). We have $a = 0.050$, $z = 0.130$, the factor $\mu_0/4\pi^2a^2$ is omitted. There are two sources at $x_1 = 1.7$, $y_1 = 0.3$ with weight 1, and $x_2 = 0.4$, $y_2 = -0.73$ with a weight of 0.05.

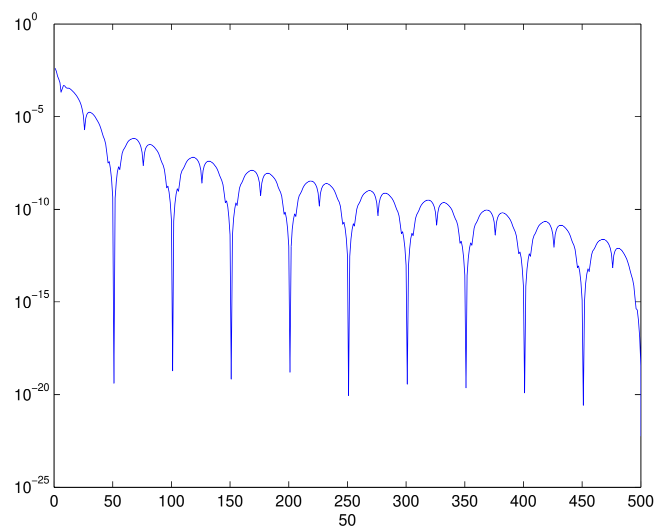

Figure 8. Norm of Fourier coefficients. In this diagram we have shown the norm of the Fourier coefficients of the curve of figure 7 (fifty points of evaluation, linear interpolation between them).

6.4. Parametrizations of matrix-valued lossless functions

Participants: Rémi Drai [CMA], Bernard Hanzon [Univ. Libre (VU) of Amsterdam], Jean-Paul Marmorat, Martine Olivi, Ralf Peeters [Univ. of Maastricht].

These parametrization issues have been studied for several years in the project. The main motivation was to find good optimization parameters for our approximation problems 3.1.4. Atlases of charts have been derived from a matrix Schur algorithm associated with Nevanlinna-Pick interpolation data. In a chart, a lossless function can be represented by a balanced realization computed as a product of unitary matrices from the Schur parameters [20]. Moreover, an adapted chart for a given lossless function can be built from a realization in Schur form. Such a parametrization presents a lot of advantages in view of the approximation problems we have in mind: it ensures identifiability, takes into account the stability constraint, preserves the order and presents a good numerical behavior. This parametrization has been used in the software RARL2 5.1 which deals with rational approximation in L^2 norm.

This year, we studied atlases built from a more general interpolation problem, the contour integral interpolation problem of Nudelman. These atlases were introduced last year to deal with real-valued systems. We paid a particular attention to an atlas which uses a favorable mutual encoding property of lossless functions [16]. It works for both real and complex systems and has been implemented in a new version of the software RARL2 5.1.

Following a different approach based on so-called *nice selections*, balanced canonical forms were constructed which have the property that the corresponding controllability matrix is positive upper-triangular, up to a column permutation. Such canonical forms present, in particular, good truncation properties. It was proved that this atlas can also be obtained from a tangential Schur algorithm by choosing the interpolation points at zero and the directions in a well-specified manner among standard basis vectors [16]. This atlas is minimal in the sense that no chart can be left out without losing the property that the atlas cover the manifold.

The objective of these studies is to have at one's disposal a panel of parametrizations that could be used as optimization parameters and also that could take into account some particular property coming from the physics. This could be symmetry or some other constraint on the realization matrix like for example the structure imposed by the couplings of an hyperfrequency filter 4.3.2.

In view to enlarge the field of application of these parametrizations we started a collaboration with the Delft Center for Systems and Control. The idea was to implement our parametrizations in the software of stochastic identification that they developed so far, and to test the advantages and disadvantages of our different parametrizations on their problems. Another possible field of application is multiobjective control. An approach developed by the robust control community, consists in transforming the multiobjective control problem into a tractable LMI problem whenever the lossless factor of the Youla parameter is known. Then, the search of this lossless factor is limited to a particular form which makes the optimization easier. We think that this approach could be significantly improved using our parametrizations to optimize within the set of all lossless factors of fixed degree.

6.5. The mathematics of Surface Acoustic Wave filters

Participants: Laurent Baratchart, Per Enqvist, Andrea Gombani, Martine Olivi.

Surface Acoustic Waves (in short: SAW) filters consist in a series of transducers which transmit electrical power by means of surface acoustic waves propagating on a piezoelectric medium. They are usually described by a mixed scattering matrix which relates acoustic waves, currents and voltages. By reciprocity and energy conservation, these transfers must be either lossless, contractive or positive real, and symmetric. In the design of SAW filters, the desired electrical power transmission is specified. An important issue is to characterize the functions that can actually be realized for a given type of filter. In any case, these functions are Schur and can be completed into a conservative matrix with an increase of at most 2 of the McMillan degree, this matrix describing the global behavior of the filter. Such a completion problem is known as Darlington synthesis and has always a solution for any higher McMillan degree in the rational case if the symmetry condition is of no concern. However in our case, additional constraints arise from the geometry of the filter as the symmetry and certain interpolation condition. In [29], a complete mathematical description of such devices is given, including realizations for the relevant tranfer-functions, as well as a necessary and sufficient condition for symmetric Darlington synthesis preserving the McMillan degree. More generally, we characterized in [14] the existence of a symmetric Darlington synthesis with specified increase of the McMillan degree: a symmetric extension of a symmetric contractive matrix S of degree n exists in degree $n+k$ if, and only if, $I - SS^*$ has at most k zeros with odd multiplicity. This results tells something about the minimal number of gyrators to be used in circuit synthesis; an article is currently being written to report on these results.

6.6. Rational and Meromorphic Approximation

Participants: Laurent Baratchart, Mihai Putinar [University of California, Santa Barbara], Edward Saff, Pascale Vitse [Université de Franche-Comté, Besançon].

There are natural links between meromorphic approximation of Markov functions and the n-widths of the unit ball of H^p in $L^q(\mu)$, because the extremal functions are essentially the singular vectors of the Hankel operator associated with the approximation problem with exponent s such that $1/s + 2/p = 1$ [39]. Previous work [40] when $p \geq q$ generalizes the asymptotics obtained by Fisher and Stessin for the n-width, and we have taken up this year the study of the case where the support of the measure is a general regular compact set of the disk in an attempt to carry over the relation between embedding operators and rational or meromorphic approximation to 2-D singular sets. This is important for applications to the detection of occlusions in conducting media, see section 4.2. We have shown the accumulation points of the zeros of singular vectors of such operators are carried by the support of the measure, which is an important step to carry out the asymptotics of the singular values. These were obtained when the singular set is a disk, and more general cases are currently under study.

The study of matrix-valued rational approximation to matrix-Markov functions (*i.e.* Cauchy transforms of a positive matrix valued measure) has been pursued, with the aim of proving that every critical point is Markov (former work of the team showed that the best approximant is Markov). There are no new results in this direction so far. We were able, however, to generalize the error rates known in the scalar case under Szegö-type conditions to the matrix case.

6.7. Behavior of poles

Participants: Laurent Baratchart, Reinhold Küstner [CMI Université de Provence, Marseille], Edward Saff, Vilmos Totik [univ. Szeged and Acad. of Sciences, Hungary].

It is known after [41] that the denominators of best rational of meromorphic approximants in the L^p norm on a closed curve (say the unit circle T to fix ideas) satisfy for $p \geq 2$ a non-Hermitian orthogonality relation for functions described as Cauchy transforms of complex measures on a curve γ (locus of singularities) contained in the unit disk D. This has been used to assess the asymptotic behavior of the poles of such an approximant when γ is a hyperbolic geometric arc, that is, under weak conditions on the measure, the counting measure of these poles converges weak-star to the equilibrium distribution of the condenser (T, γ) where T is the unit circle. Non asymptotic bounds have been obtained for the sum of the complement to π of the hyperbolic angles under which the poles "see" γ: the sum of these complements over all the poles (they are n in total if the approximant has degree n) is bounded by the aperture of γ plus twice the variation of the argument of the measure (which is independent of n). This produces "hard" testable inequalities for the location of the poles, that should prove particularly valuable in inverse source problem (because they are not asymptotic in nature), see module 3.1.3. This research has been the object of an article [33].

The more general situation where γ is a so-called "minimal contour" for the Green potential (of which a geodesic arc is an example) has been essentially settled with the same conclusion concerning the convergence of the counting measure of the poles. The writing up of this (rather technical) result is underway, and of particular significance with respect to the determination of $2-D$ sources or piecewise analytic cracks from overdetermined boundary data, see module 3.1.3 and 6.8.

Another issue which especially interesting in connection with crack detection is the behavior of the poles in the case of a more general (not necessarily piecewise analytic) crack. We have shown that, using conformal maps of a ringed domain, one is led to a question similar to that on a geodesic arc but with a less regular measure, and we conjecture that the poles at least accumulate towards the endpoints of the crack. The proof, however, is still far from complete and will require further efforts. A pending issue is also the behavior of poles for 2D singular sets. No results in this direction were obtained so far.

6.8. Inverse Problems for 2D and 3D Laplacian

Keywords: *Laplacian*, *inverse problems*, *non destructive control*, *tomography*.

Participants: Amina Amassad, Bilal Atfeh, Laurent Baratchart, Amel Ben Abda [ENIT, Tunis], Fehmi Ben Hassen, Imen Fellah, Mohamed Jaoua, Juliette Leblond, Moncef Mahjoub, Jean-Paul Marmorat, Jonathan R. Partington, Edward Saff.

The fact that 2D harmonic functions are real parts of analytic functions allows one to tackle issues in singularity detection and geometric reconstruction from boundary data of solutions to Laplace equations using the meromorphic and rational approximation tools developed by the team. Some electrical conductivity defaults can be modeled by pointwise sources inside the considered domain. In dimension 2, the question made significant progress in recent years: the singularities of the function (of the complex variable) which is to be reconstructed from boundary measures are poles (case of dipolar sources) or logarithmic singularities (case of monopolar sources). Hence, the behavior of the poles of the rational or meromorphic approximants, described in modules 3.1.1 to 3.1.3, allows one to efficiently locate their position. This, together with corresponding software implementation, is part of the subject of the Ph.D. thesis of F. Ben Hassen [11] and of an article to appear [27], where the related situation of small inhomogeneities connected to mine detection is also considered.

In 3D, epileptic regions in the cortex are often represented by pointwise sources that have to be localized from potential measures on the scalp of a potential difference, that is the solution of a Laplace equation (EEG, electoencephalography). Note that the head is here modeled as a sequence of spherical layers. This inverse EEG problem is the object of a collaboration between the Apics and Odyssée Teams through the ACI "Obs-Cerv". An interesting breakthrough was made last year which makes it possible now to proceed via best rational approximation on a sequence of 2D disks along the inner sphere [34], [15]. The point here is that, up to an additive function harmonic in the 3D ball, the trace of the potential on each boundary circle coincides with a function having branched singularities in the corresponding disk. The behavior along the family of disks of the poles of their best rational approximants on each circle is strongly linked to the location of the sources, using properties discussed in sections 3.1.3 and 6.7. (in the particular case of a unique source, we end up with a rational function); this is under study as well as a number of important related issues.

First, solving Cauchy problems on an annulus or on a spherical layer in order to treat incomplete experimental data is also a necessary ingredient of the methodology, since it is involved in the propagation of initial conditions from the boundary to the center of the domain, where singularities are seeked, when this domain is formed of several homogeneous layers of different conductivities. On a spherical layer, this is the aim of the post-doctoral trainee of B. Atfeh (it has also been preliminary handled by J. Chetboun and C. Aziadjonou, together with uniqueness issues for constant conductivities). Constructive and numerical aspects of the expected procedures (harmonic 3D projection, Kelvin and Riesz transformation, spherical harmonics) are under study and encouraging results are already available on numerically computed data. This offers an opportunity to state and solve extremal problems for harmonic fields.

We also started to consider more realistic geometries for the 3D domain under consideration. A possibility is to parametrize it in such a way that its planar cross-sections are quadrature domains or R-domains. In this framework, best rational approximation can still be performed in order to recover the singularities of solutions to Laplace equations (F.-O. Helme, summer trainee) but complexity issues have to be examined carefully.

Finally, we begin to consider actual 3D approximation for such inverse problems. Quaternionic analysis seems to be an appropriate tool, but much of the theory (in particular the multiplicative side) remains to be developed.

In the 2D case again, with incomplete data, the geometric problem of finding, in a stable and constructive way, an unknown (insulating) part of the boundary of a domain is considered in the Ph.D. thesis of I. Fellah. Approximation and analytic extension techniques described in section 3.1.1 together with numerical conformal transformations of the disk provide here also interesting algorithms for the inverse problem under consideration. A related result that was obtained this year is an L^p existence and uniqueness result for the Neumann problem on a piecewise $C^{1,\alpha}$ domain with inward pointing cusps (note that the endpoints of a crack are such cusps) when $1 < p < 2$. Although it is reminiscent of classical L^p theorem on Lipschitz domains [54], it seems to be a new result (observe that Lipschitz domains cannot have cusps) exploiting weighted norm inequalities. Moreover, a Cauchy-type representation for the solution was obtained using Smirnov classes representation properties, and the technique generalizes to mixed boundary conditions that occur when the crack is no longer assumed to be a perfect insulator. An article is being written on these aspects.

Finally, solving Cauchy problems on an annulus is the main theme of the PhD thesis of M. Mahjoub. This arises when identifying a crack in a tube or a Robin coefficient on its inner skull. It can be formulated as a best approximation problem on part of the boundary of a doubly connected domain, which allowed both numerical algorithms and stability results to be obtained in this framework [19], thereby generalizing the simply connected situation [12], [46].

6.9. Analytic extension under pointwise constraints

Participants: Laurent Baratchart, Fabien Seyfert.

To carry out identification and design of filters under passivity constraints (such constraints are common since passive devices are ubiquitous, including in particular hyperfrequency filters), it is natural to consider the mixed bounded extremal problem *(P')* stated in section 3.1.1. An algorithm to asymptotically solve this problem in nested spaces of polynomials has been obtained, and its connection to affine Rieman-Hilbert problems has also been carried out. This connection provides a handle to analyze regularity properties of the solution, and gives us an alternative process based on dichotomy. It should be valuable to estimate delays in waveguides, and could complement the existing procedures dealing with this issue in PRESTO-HF.

6.10. Exhaustive determination of constrained realizations corresponding to a transfer function

Participants: Laurent Baratchart, Jean Charles Faugère [project SALSA, Rocquencourt], Fabien Seyfert.

We studied in some generality the case of parameterized linear systems characterized by the following classical state space equation,

$$\begin{aligned} \dot{x}(t) &= A(p)x(t) + B(p)u(t) \\ y(t) &= C(p)x(t) \end{aligned} \tag{5}$$

where $p = \{p_1, \cdots p_r\}$ is a finite set of r parameters and $(A(p), B(p), C(p))$ are matrices whose entries are polynomials (over the field $\mathbb{C}$) of the variables $p_1 \cdots p_r$. For a parameterized system σ and $p \in \mathbb{C}^r$ we call $\pi_\sigma(p)$ the transfer function of the system $\sigma(p)$. Some important questions in filter synthesis concern the determination of the following parameterized sets

$$\begin{aligned} p \in \mathbb{C}^{r_1},\ E_{\sigma_1}(p) &= \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_1}(p)\} \\ p \in \mathbb{C}^{r_2},\ E_{\sigma_1,\sigma_2}(p) &= \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_2}(p)\} \end{aligned} \tag{6}$$

General results were obtained about these sets, in particular a necessary and sufficient condition ensuring their cardinals are finite. In the special case of coupled-resonators an efficient algebraic formulation has been derived which allowed us to compute $E_{\sigma(p)}$ for nearly all common filter geometries. However for a new class of high order filters first presented in [45] the latter procedure breaks down because of the computational complexity of the Gröbner basis computation. This led us to consider homotopic methods based on continuation techniques in order to solve the algebraic system defining $E_{\sigma(p)}$. The usual framework of these methods that is based on the Bezout bound or on mixed volume computations appeared to be intractable in our case mainly because of the degeneracy of our algebraic systems: for example for a 10^{th} order filter, the Bezout bound is about 10^{44} whereas the number of solutions over the ground field $\mathbb{C}$ is known to be only 384. To overcome this difficulty we are currently developing a continuation method which consists of the exploration of the monodromy group of an algebraic variety by following a family of paths that separate the branch points. This method is still under study but preliminary numerical results that yielded the exhaustive computation of $E_{\sigma(p)}$ in the latter 10^{th} order case are very encouraging. Using this method, we envisage to build up a precomputed filter database that would allow a fast computation of high order filters for every specific filtering characteristic.

Results were also obtained about the existence of a "real solution" in the set $E_{\sigma(p)}$ in the case of loss-less characteristics. For the 5^{th} order coupling topology of figure 9 it was shown that one can find an open set U of $\mathbb{C}^r$ for which for all $p \in U$ the set $E_{\sigma(p)}$ contains no "real" element. Conversely it was shown, by an argument based on the Borsuk-Ulam antipodal theorem, that for lossless characteristics and the 6^{th} order coupling topology of figure 10 there generically exists at least one "real" element in $E_{\sigma(p)}$.

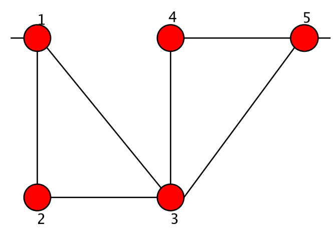

Figure 9. Coupling geometry with no real solution

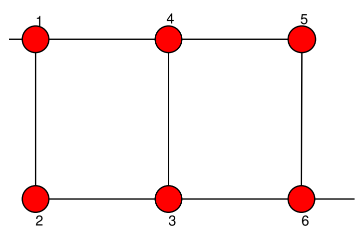

Figure 10. 6^{th} order extended boxed geometry

6.11. Frequency Approximation and OMUX design

Participants: Laurent Baratchart, Vincent Lunot, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

An OMUX (Output MUltipleXor) can be modeled in the frequency domain by chaining of scattering matrices of filters as those described in section 4.3.2, connected in parallel to a common access *via* a wave guide, see figure 5. The problem of designing the OMUX so as to satisfy gauge constraints is then naturally translated into a set of constraints on the values of the scattering matrices and phase shift introduced by the guides in the considered bandwidth.

An OMUX simulator on a matlab platform was designed last year, and we began this year a study of dedicated optimization procedures. The direct approach, as used by the manufacturer, is of course to couple this simulator with an optimizer, in order to reduce transmission and reflection wherever they are too large. This yields unsatisfactory results in cases of high degree and narrow bandwidth, in particular because peaks arise with the dilation of the cavities caused by an increase of the temperature (when the satellite gets exposed to the sun).

The thesis of V. Lunot has just started on the problem of approximation by Schur rational functions (*i.e.* rational functions of modulus bounded by 1 in a half-plane), for this issue arises naturally in connection with the hyperbolic distance induced by chain scattering matrices. We hope that solving iteratively approximations problems of this type with all but one channel being fixed can lead to a tuning of each filter in a diagonal manner. This is an interesting question, both for applications and in itself.

As a result, we expect to be able to produce a multi-phased tuning procedure, first relaxed, channel after channel, then global, using a quasi-Newton method. Note the discretizations in frequency of the integral criterion and the near periodicity of the exponentials (that express the delays) interact in a complex manner, and generate numerous local minima, which is one reason why the optimization problem should be analyzed in depth.

6.12. Low thrust orbital transfer

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet, Mario Sigalotti.

We focus on what we call the controlled Keplerian problem. A satellite is equipped with a low thrust engine and we want to perform an orbital transfer, i.e. to drive the satellite from an initial orbit to a final one. This problem was raised by Alcatel Space. As the old ballistic commands do not work for the low thrust we have to explore new techniques and chose to pursue our ideas on time optimal GTO-GEO transfer.

One of the achievements this year, in the course of the PhD work of A. Bombrun, was to build a family of control Liapunov functions based on the first five integrals of the classical two body problem (this is not new), in such a way that a given transfer (say time-optimal) can be approached very closely by fitting correctly the parameters in the family (this is the novelty). In numerical simulations we show such a Liapunov feedback not only is close to the given time optimal trajectory, but also gives very satisfactory trajectories for different initial conditions. We are trying to show that we can approach any orbital transfer within this family. Of course, this control law enjoys the natural robustness properties of feedback control.

We also proposed a strategy to achieve the full rendez-vous (transfer plus longitude assignment). This is not a full closed-loop strategy, but performs well.

6.13. Local linearization (or flatness) of control systems

Participants: David Avanessoff, Laurent Baratchart, Jean-Baptiste Pomet.

We designed tools for analyzing some over-determined systems of PDEs for which neither the number of independent variables nor the order is *a priori* fixed. It is based on a valuation adapted to the control system [25].

The equations arising when characterizing flatness involve a number of variables which is finite, but not known *a priori*... so it is tempting to take as solutions formal power series in infinitely many variables. The tools above allow us to give a meaning to solutions in such formal power series. A notion of "very" formal integrability was introduced, meaning existence of solutions in this class.

We are not yet able to use this tool for all "equations" of flatness, so that the equations we write in [25] are only *necessary conditions* for flatness. However, if, for some systems, these equations were not very formally integrable, then these systems would not be flat. Unfortunately we did prove that these equations are *always* very formally integrable. Putting a full characterization of flatness in this form is still under course.

This is the topic of David Avanessoff's PhD.

6.14. “Flatness” or parameterizability for systems in small dimension

Participants: David Avanessoff, José Grimm, Jean-Baptiste Pomet.

Here we studied the smallest nontrivial dimensions, i.e. three states and two controls. One can prove that this is the same as studying systems with four states and two controls, whose dynamics are affine in the control. In [10], a sufficient condition for such systems to be flat was given, and this condition was also proved to be necessary for “(x, u)-flatness” (in the language of the above paragraph, a version of flatness where the number of variables to consider is decided in advance). It is conjectured that it is in fact necessary for flatness itself, and even that systems that do not satisfy this condition do not admit a parameterization.

The proof in [10] was very intricate and used computer algebra at one point in the argument. Using a different approach we are able to prove, in a much simpler way, that systems that do not satisfy this condition do not admit a parameterization of order less than 4 (this implies that they are not “(x, u)-flat”; the systems proved to be flat in [10] admit a parameterization of order 1), and in fact we hope to conclude, by this method, that these systems indeed do not admit a parameterization (it was out of question with the method in [10]). This is also the topic of David Avanessoff’s PhD.

6.15. Controllability for a general Dubbins problem

Participant: Mario Sigalotti.

Controllability results for drifted systems are usually obtained by a combination of local and global properties of the system under study. Local controllability properties basically follow from the knowledge of the Lie bracket configuration of the system, while global ones require particular symmetries or some sort of ergodicity. A typical example is the one of a left-invariant control system on a Lie group. Classically, the homogeneity of the manifold given by the group structure is used to obtain global properties out of local ones.

The aim of this research line is to obtain controllability/non-controllability results for special but inhomogeneous drifted systems.

The main object of our research is given by Dubbins-like systems on Riemannian surfaces. The goal is to answer, using control techniques, the following natural question, arising from the works of Dubbins: given a complete, connected, two-dimensional Riemannian manifold M, and (p_1, v_1), (p_2, v_2) in TM, does there exist a curve γ in M, with arbitrary small geodesic curvature, such that γ connects p_1 to p_2 and, for $i = 1, 2$, $\dot\gamma$ is equal to v_i at p_i? The answer clearly depends on the geometric properties of M, and gives a meaning to such properties from a control viewpoint. In [47] we proved that the small-curvature-connectness introduced above holds for compact surfaces, for unbounded surfaces whose Gaussian curvature tends to zero at infinity, and for surfaces which are non-negatively curved outside a compact set.

The case of non-positively curved surfaces was addressed in [21], where necessary and sufficient conditions ensuring such connectness have been established.

A different field of application of the analysis of controllability of inhomogeneous drifted systems is given by non-linear switched systems. More precisely, given a switched system of the type $\dot q = X(q) + uY(q)$, $u \in [-1, 1]$, $q \in \mathbf{R}^2$, where $X + Y$ and $X - Y$ are globally asymptotically stable, we study its stability properties (global uniform asymptotic stability, uniform stability, boundedness, ...) in terms of the topology of the set where X and Y are parallel. All such stability properties can be re-interpreted in terms of the behavior of attainable sets.

7. Contracts and Grants with Industry

7.1. Contracts CNES-IRCOM-INRIA

Contracts n^{o}1 03 E 1034

In the framework of a contract that links CNES, IRCOM and Inria, whose objective is to realize a software package for identification and design of hyperfrequency devices, the work of Inria included:

- modeling of delay, see module 4.3.2,

- exhaustive coupling determination on case studies 6.10),
- OMUX simulator with exact computation of derivatives,

This contract has been renewed for 16 months starting November 2004, in order to develop a generic code for coupling determination and to carry out the optimization of OMUX.

7.2. Contract Alcatel Space (Cannes)

Contract n^{o}1 01 E 0726.

This contract started in 2001, for three years. The topic is the design of control laws for satellites with low-thrust engines,

It finances Alex Bombrun's PhD.

8. Other Grants and Activities

8.1. Scientific Committees

L. Baratchart is member of the editorial board of *Computational Methods in Function Theory*.

8.2. National Actions

Together with project-teams Caiman and Odyssée (INRIA-Sophia Antipolis, ENPC), the University of Nice (J.A. Dieudonné lab.), CEA, CNRS-LENA (Paris), and a few French hospitals, we are part of the national action **ACI Masse de données « OBS-CERV »**, 2003-2006 (inverse problems, EEG). C. Aziadjonou and J. Chetboun were supported by this ACI.

The **region PACA** (Provence Alpes Côte d'Azur) has been partially supporting the post-doctoral stay of Per Enquist until May, 2004. We also obtained a (modest) grant from the region for exchanges with SISSA Trieste (Italy), 2003-2004.

The post-doctoral training of B. Atfeh is funded by **INRIA**.

8.3. Actions Funded by the EC

The team enjoys a **Marie Curie EIF** (Intra European Fellowship) FP6-2002-Mobility-5-502062, for 24 months (2003-2005). This finances Mario Sigalotti's post-doc.

The Team is a member of the **Marie Curie multi-partner training site** *Control Training Site*, number HPMT-CT-2001-00278, 2001-2005. See http://www.supelec.fr/lss/CTS/.

The project is member of Working Group Control and System Theory of the **ERCIM** consortium, see http://www.ladseb.pd.cnr.it/control/ercim/control.html.

8.4. Extra-european International Actions

NATO CLG (Collaborative Linkage Grant), PST.CLG.979703, « Constructive approximation and inverse diffusion problems », with Vanderbilt Univ. (Nashville, USA) and LAMSIN-ENIT (Tunis, Tu.), 2003-2005.

EPSRC grant (EP/C004418) «Constrained approximation in function spaces, with applications», with Leeds Univ. (UK) and Univ. Lyon I.

STIC-INRIA grant with LAMSIN-ENIT (Tunis, Tu.), « Problèmes inverses du Laplacien et approximation constructive des fonctions ».

8.5. Exterior research visitors

The following scientists visited us and gave a seminar:

- Amina Amassad, UNSA, Nice, et équipe APICS, *Contribution à l'analyse et au contrôle des inclusions différentielles en mécanique du contact*.

- Alexander Aptekarev, Keldysh Institute of Applied Mathematics, Moscow, *Family of equilibrium measures and variational principle for Burger's equation*.
- Bilal Atfeh, équipe APICS, *Méthode des lignes de courant appliquée à la modélisation des bassins*.
- Ugo Boscain, SISSA, Trieste, Italy, *Stability of planar switched systems for arbitrary switchings*.
- Imen Fellah, Lamsin-ENIT, Tunisie, *Complétion de données dans les espaces de Hardy et problèmes inverses pour le Laplacien en 2D*.
- Stanislas Kupin, Université de Provence, *Comportement asymptotique de polynômes orthogonaux sur le cercle d'après la régularité des coefficients*.
- Moncef Mahjoub, Lamsin-ENIT, Tunisie, *Complétion de données dans une couronne et ses applications à quelques problèmes inverses*.
- Sylvain Neut, LAIL, Univ de Lille, *Implantation et nouvelles applications de la méthode d'équivalence de Cartan*.
- Laurent Niederman, Département de Mathématiques, Université Paris-Sud, Orsay, *Stabilité Hamiltonienne et théorie de Morse-Sard*.
- Laurent Praly, Centre Automatique et Systèmes, Ecole des mines de Paris, *Diverses stratégies de synthèse de commande stabilisante pour le transfert d'orbite*.
- Witold Respondek, Laboratoire de Mathématiques INSA de Rouen, *Bifurcations des systèmes non linéaires de contrôle sur le plan*.
- Pierre Rouchon, Centre Automatique et Systèmes, Ecole des Mines de Paris, *Invariant observer for mechanical systems*.
- Edward B. Saff, Dept. of Mathematics, Vanderbilt University, *Discretizing manifolds via minimum energy points*.
- Mario Sigalotti, équipe APICS, *Dubbins' problem on surfaces of nonpositive curvature*.
- Nikos Stylianopoulos, Department of Mathematics and Statistics, University of Cyprus, *Conformal mapping of elongated domains with applications to the solution of Laplacian problems*.
- Jan H. Van Schuppen, CWI/VU, *Control and realization of piecewise-affine hybrid systems*.
- Igor Zelenko, SISSA, Trieste, Italy, *Variational approach to problem of equivalence of rank 2 vector distributions*.

9. Dissemination

9.1. Teaching

Courses

- L. Baratchart, DEA Géométrie et Analyse, LATP-CMI, Univ. de Provence (Marseille).

Trainees

- Jonathan Chetboun and Christelle Aziadjonou, *Problème inverse en Electroencéphalographie*
- Frank-Olivier Helme (Mémoire de DEA de Mathématiques pures, Université de Provence, Aix-Marseille I.) *Résolution de problèmes inverses de source dans des domaines paramétrés en dimension 3 par approximation méromorphe*
- Jean-Michel Guieu (Mémoire de DEA de Mathématiques pures, Université de Provence, Aix-Marseille I.) *Comportement asymptotique des pôles dans l'approximation méromorphe des fonctions analytiques en dehors d'un compact du disque*

Ph.D. Students

- David Avanessoff, « Linéarisation dynamique des systèmes non linéares et paramétrage de l'ensemble des solutions » (dynamic linearization of non linear control systems, and parameterization of all trajectories).
- Alex Bombrun, « Commande optimale, feedback, et tranfert orbital de satellites » (optimal control, feedback, and orbital transfert for low thrust satellite orbit transfer)
- Imen Fellah, "Data completion in Hardy classes and applications to inverse problems", co-tutelle with Lamsin-ENIT (Tunis).
- Vincent Lunot, « Optimisation et synthèse d'OMUX »,
- Moncef Mahjoub, "Complétion de données et ses application à la détermination de défauts géométriques." co-tutelle with Lamsin-ENIT (Tunis).

Ph.D. thesis defended

- Fehmi Ben Hassen, «Recovery and identification of pointwise sources and small size inclusions», Lamsin-ENIT, Univ. Tunis II (Tunisie), December 11th 2004.

9.2. Community service

M. Sigalotti is in charge of organizing a seminar on control and identification.

L. Baratchart is a member of the “bureau” of the CP (Comité des Projets) of INRIA-Sophia Antipolis.

J. Grimm is a member of the CUMI (Comité des utilisateurs des moyens informatiques) of the Research Unit of Sophia Antipolis (dissolved in September 2004).

J.-B. Pomet is a representative at the “comité de centre” (until October 2004).

J. Grimm is a representative at the “comité de centre” (Starting October 2004).

J. Leblond is co-directing - together with J.-D. Fournier (CNRS, Obs. Nice) - and J. Grimm is participating in the edition of the proceedings (to appear in 2005) of the CNRS-INRIA summer school “Harmonic analysis and rational approximation: their rôles in signals, control and dynamical systems theory” (Porquerolles, 2003) http://www-sop.inria.fr/apics/anap03/index.en.html.

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Unit of Sophia Antipolis.

F. Seyfert is a member of the CDL (Comité de développement logiciel) of the Research Unit of Sophia Antipolis.

9.3. Conferences and workshops

J.-B. Pomet was an invited researcher at Banach center in Warsaw, for one month in January.

J.-B. Pomet was an invited speaker at the Colloquium on Dynamical Systems, Control and Applications, organized by Univ. Autonoma de Mexico, December, Mexico City.

F. Seyfert was an invited speaker at the “International Workshop on Microwave Filters”, co-organized by the CNES and ESA, September, Toulouse.

David Avanessoff and Mario Sigalotti gave talks at the “2nd Junior European Meeting Control Theory and Stabilization”, Torino, It.

Mario Sigalotti gave a talk at the “First CTS Workshop”, University of Coimbra, Portugal, July 1-3.

Juliette Leblond was invited to give a plenary talk at the Forum des Jeunes Mathématiciennes, IHP, Paris (January), gave a communication at Constructive Functions Tech. 04, Georgia Tech., Atlanta, Georgia, USA (November), and at the annual workshop of the ACI “Obs-Cerv”, ENPC, Champs sur Marne (September).

M. Olivi was invited to give a talk at the Delft Center for systems and control of the Delft University of Technology.

M. Olivi gave two talks at the SSSC 2004, Oaxaca, Mexico, 8-10 December.

A. Bombrun gave a talk at a meeting on spatial mechanics at CNES, Toulouse, in September.

J.-B. Pomet gave a talk at a meeting on geometrical methods and PDEs at CIRM, Luminy, in November.

F. Seyfert gave a talk at the “Journées nationales du calcul formel 2003” about the use of computer algebra based methods for the exhaustive computation of couplings parameters, at “Advances in constructive approximation (Nashville)” about the mixed (L^2, L^∞) bounded extremal problem and at the “IMS 2003 (Philadelphia)” about the determination of a rational stable model from measured scattering data.

L. Baratchart was an invited speaker at the “Funktionentheorie” meeting, Oberwolfach, feb. 2004, at the "Journee Approximation", Lille, march 2004, and at the “Tech ’04 Function Theory” Conference, November 2004, Atlanta (Georgia). He was a regular speaker at the MTNS, Leuwen, July 2004.

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Publications in Conferences and Workshops

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