INRIA

Team corida

Contrôle Robuste des Systèmes Infini-Dimensionnels et Applications

Lorraine

THEME NUM

Activity Report

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1. Team

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2. Overall Objectives

CORIDA is a team labelled by INRIA, by CNRS and by University Henri Poincaré, via the Institut Elie Cartan of Nancy (UMR 7502 CNRS-INRIA-UHP). The main focus of our research is the robust control of systems governed by partial differential equations (called PDE's in the sequel). A special attention is devoted to systems with a hybrid dynamics such as the fluid-structure interactions. The equations modelling these systems couple either partial differential equations of different types or finite dimensional systems and infinite dimensional systems. We mainly consider inputs acting on the boundary or which are localized in a subset of the domain.

Infinite dimensional systems theory is motivated by the fact that a large number of mathematical models in applied sciences are given by evolution partial differential equations. Typical examples are the transport, heat or wave equations, which are used as mathematical models in a large number of problems in physics, chemistry, biology or finance. In all these cases the corresponding state space is infinite dimensional. The understanding of these systems from the point of view of control theory is an important scientific issue which received a considerable attention during the last decades. Let us mention here that a basic question like the

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study of the controllability of infinite dimensional linear systems requires sophisticated techniques such as non harmonic analysis (cf. Russsell [57]), multiplier methods (cf. Lions [50]) or micro-local analysis techniques (cf. Bardos-Lebeau-Rauch [42]). Like in the case of finite dimensional systems, the study of controllability should be only the starting point of the study of important and more practical issues like feedback optimal control or robust control. It turns out that most of these questions are open in the case of infinite dimensional systems. Consequently, our aim is to develop tools for the robust control of infinite dimensional systems. More precisely, given an infinite dimensional system one should be able to answer two basic questions:

1. The existence of a feedback operator with robustness properties;
2. Find an algorithm allowing the approximate computation of this feedback operator

The answer to question 1 above requires the study of infinite dimensional Riccati operators and it is a difficult theoretical question. The answer to question 2 depends on the sense of the word "approximate". In our meaning "approximate" means "convergence", i.e., that we look approximate feedback operators converging to the exact one when the discretization step tends to zero. From the practical point of view this means that our control laws should give good results if we use a large number of state variables. This fact is no longer a practical limitation of such an approach, at least in some important applications where strong computers are now available. We intend to develop a methodology applicable to a large class of applications. Let us mention here only two of them, which received a considerable attention during the last year.

1. **Acoustics and aero-acoustics.** We consider two types of applications :

    – Noise reduction by using active control (in a bounded region such as a plane cockpit) or by using absorbing materials (in open regions around highways, airports or railways).
    – Times reversal techniques for acoustic focusing in medical imaging, non destructive testing or sub-marine communication.

2. **The control of VLT's (Very Large Telescopes).** The operation of the current telescopes is based on the reception of infra-red waves. The reception is inevitably disturbed by the atmosphere, from where a correction of the wavefront is needed. Currently this correction is carried out by a mirror, whose diameter is approximately 20cm, provided by a thousand of piezoelectric actuators. The future telescopes will be characterized by diameters much larger and the fact that the spectrum of the analyzed wavefront lies in the visible field. It is estimated that to correct the image with the same quality, the density of the actuators will have to be a hundred higher and that it will be necessary to replace the piezoelectric actuators by actuators resulting from micro-technology. It is thus a question of developing tools to model and to control the mirrors, allowing this change of scale.

3. Scientific Foundations

3.1. Analysis and control of fluids and of fluid-structure interactions

Keywords: *Analysis and control of fluids and of fluid-structure interactions*, *Analysis and control of systems modelling the motion of several solids (rigid or elastic) in a viscous fluid*, *Korteweg de Vries equations*, *Navier-Stokes equations*.

Participants: Patricio Cumsille, Lionel Rosier, Jean-François Scheid, Takéo Takahashi, Marius Tucsnak.

The problems we consider are modeled by the Navier-Stokes, Euler or Korteweg de Vries equations (for the fluid) coupled to the equations of the solids. One of the main difficulties of this problem comes from the fact that the domain occupied by the fluid is one of the unknowns of the problem. We have thus to tackle a *free boundary problem*.

The control of fluid flows is a major challenge in many applications: aeronautics, pollution issues, regulation of irrigation channels or of the flow in pipelines, $\cdots$. All these problems cannot be easily reduced to finite dimensional models so a methodology of analysis and control based on PDE's is an essential issue. In a first approximation the motion of fluid and of the solids can be decoupled. The most used models for an incompressible fluid are given by the Navier-Stokes or by the Euler's equations.

The optimal open loop control approach of these models has been developed from both the theoretical and numerical points of view. Controllability issues for the equations modeling the fluid motion are by now well understood (see, for instance, [48] and the references therein). The feedback control of fluid motions has also been recently investigated by several research teams (see, for instance [41] and references therein) but this field still contains an important number of open problems (in particular those concerning observers and implementation issues). One of our aims is to develop efficient tools for computing feedback laws for the control of fluid systems.

In real applications the fluid is often surrounded or it surrounds an elastic structure. In the above situation one has to study fluid-structure interactions. This subject has been intensively studied during the last years, in particular for its applications in noise reduction problems, in lubrication issues or in aeronautics. In this type of problems, a PDE's system modelling the fluid in a cavity (Laplace equation, wave equation, Stokes, Navier-Stokes or Euler systems) is coupled to the equations modelling the motion of a part of the boundary. The difficulties of this problem are due to several reasons such as the strong nonlinear coupling and the existence of a free boundary. This partially explains the fact that applied mathematicians have only recently tackled these problems from either the numerical or theoretical point of view. One the main results obtained in our project concerns the global existence of solutions in the case of a two-dimensional Navier-Stokes fluid (see [7]). On the other hand, it seems that the corresponding problem for a perfect fluid (modelled by the Euler equation) has not yet been investigated.

The numerical methods used for computing the solutions of fluid or fluid structure problems in a direct setting (i.e., with given inputs) considerably progressed during the last years. For the corresponding control problems the literature contains only a small number of effective methods. Our first results in this direction concern a model arising in hydraulics (the linearized Saint-Venant equations).

Another topic of great interest is the control of the interface of two fluids (typically water and air) by using as input the velocity of a moving wall which is a part of the boundary. One of the most popular models for this problem is given by the shallow water equations (Saint Venant equations) which neglect the dispersive effects. The controllability of several important systems governed by this type of equations has received a considerable attention during the last decade. Let us mention here the important work by Coron [43]. If dispersive effects are considered the relevant model is given by the Korteweg de Vries equation. The first work on the control of this equation goes back to Russell and Zhang (see [58]). An important advance in the study of this problem has been achieved in the work [8] where, for the first time, the influence of the length of the channel has been precisely investigated.

3.2. Well–posed linear systems and weak coupling

Keywords: *boundary control*, *coupling mechanism*, *linear evolution equations*, *stabilization*.

Participants: Fatiha Alabau, Francis Conrad, Jean-François Couchouron, Marius Tucsnak.

Well–posed linear systems and weak coupling   Well–posed linear systems and weak coupling

We consider well posed systems coupling two types PDE's or coupling PDE's and ordinary differential equations. The methods we use combine energy estimates, multipliers techniques and spectral analysis.

Well–posed linear systems form an important class of infinite dimensional systems which has been introduced by Salamon in [60]. Roughly speaking a *well–posed linear system* is a linear time-invariant system such that on any finite time interval, the operator from the initial state and the input function to the final state and the output function is bounded. An important subclass of well–posed linear systems is formed by the

conservative systems which satisfy an energy-balance equation. More precisely, in a conservative system, the energy stored in the system at time τ plus the outgoing power equals the sum of the initial energy stored in the system and of the incoming power. It turns out that a large number of systems governed by partial differential equations are of this type. Moreover, conservative systems have remarkable properties like the fact that their exact controllability is equivalent to their stability. Therefore a systematic functional analytic approach to this system seems important for the infinite dimensional systems community.

We are in particular interested in problems in which two types of PDE's interact such as: a plate equation and a wave equation, a wave or plate type equation coupled to ordinary differential equations, or two wave equations coupled by lower order terms. This type of system is sometimes designed as a "hybrid system" (notice that this term is often used for a different notion in control theory). The main difficulty of these problems is that the inputs act in only one of the equations of the system. In this case we say that we have a *weak coupling*. The basic question is to know if such a system is stabilizable. A general framework for this type of problem has been given in Alabau [2] and [1] where the use of multipliers method yields promising results. A different way to tackle the same problem is to first study the simultaneous controllability of the uncoupled systems. The case in which one of the systems is finite dimensional has been tackled in Tucsnak and Weiss [9].

3.3. Optimal location of sensors and of actuators

Keywords: *decay rate*, *robustness*.

Participant: Antoine Henrot.

We focus here on algorithms for optimizing the location and the shape of actuators and of sensors for the stabilization of systems governed by PDE's. Consider a control problem for a system governed by PDE's with the input acting at the interior of the domain or on a part of the boundary. An important question is to find the location and the form of the control region in order to optimize a criterion imposed by the user. This criterion should take in consideration the energy decay rate and the robustness properties. A priori the topology of the control region is unknown so the first step in such a study should be the application of topological optimization techniques. An important particular case which occurs if the actuators and sensors contain smart materials. Generally, the optimal location problems are far from the classical convex optimization problems and they don't have a unique global optimum. To our knowledge, the only problem where the explicit solution is known has been studied in Ammari, Henrot and Tucsnak [3]. This is why finding numerical methods to be used in order to approach the optimum location is a hard task.

3.4. Frequency domain methods for the analysis and control of systems governed by pde's

Keywords: *Helmholtz equation*, *control and stabilization*, *numerical approximation of LQR problems*, *time-reversal*.

Participants: Frédéric Magoulès, Bruno Pinçon, Karim Ramdani, Takéo Takahashi, Marius Tucsnak.

We use frequency tools to analyze different types of problems. The first one concerns the control, the optimal control and the stabilization of systems governed by PDE's, and their numerical approximations. The second one concerns time-reversal phenomena, while the last one deals with the numerical approximation of Helmholtz equation using domain decompositions techniques.

3.4.1. Control and stabilization for skew-adjoint systems

The first area concerns theoretical and numerical aspects in the control of a class of PDE's. More precisely, in a semigroup setting, the systems we consider have a skew-adjoint generator. Classical examples are the wave, the Bernoulli-Euler or the Schrödinger equations. Our approach is based on an original characterization of exact controllability of second order conservative systems proposed by K. Liu [51]. This characterization

can be related to the Hautus criterion in the theory of finite dimensional systems (cf. [46]). It provides for time-dependent problems exact controllability criteria **that do not depend on time, but depend on the frequency variable** conjugated to time. Studying the controllability of a given system amounts then to establishing uniform (with respect to frequency) estimates. In other words, the problem of exact controllability for the wave equation, for instance, comes down to a high-frequency analysis for the Helmholtz operator. This frequency approach has been proposed first by K. Liu for bounded control operators (corresponding to internal control problems), and has been recently extended to the case of unbounded control operators (and thus including boundary control problems) by L. Miller [54]. Let us emphasize here that one further important advantage of this frequency approach lies in the fact that it can also be used for the analysis of space semi-discretized control problems (by finite element or finite differences). The estimates to be proved must then be uniform with respect to **both the frequency and the mesh size**.

3.4.2. *Time-reversal*

The second area in which we make use of frequency tools is the analysis of time-reversal for harmonic acoustic waves. This phenomenon [44] is a direct consequence of the reversibility of the wave equation in a non dissipative medium. It can be used to **focus an acoustic wave** on a target through a complex and/or unknown medium. To achieve this, the procedure followed is quite simple. First, time-reversal mirrors are used to generate an incident wave that propagates through the medium. Then, the mirrors measure the acoustic field diffracted by the targets, time-reverse it and back-propagate it in the medium. Iterating the scheme, we observe that the incident wave emitted by the mirrors focuses on the scatterers. An alternative and more original focusing technique is based on the so-called D.O.R.T. method [45]. According to this experimental method, the eigenelements of the time-reversal operator contain important information on the propagation medium and on the scatterers contained in it. More precisely, the number of nonzero eigenvalues is exactly the number of scatterers, while each eigenvector corresponds to an incident wave that selectively focuses on each scatterer.

Time-reversal has many applications covering a wide range of fields, among which we can cite **medicine** (kidney stones destruction or medical imaging), **sub-marine communication** and **non destructive testing**. Let us emphasize that in the case of time-harmonic acoustic waves, time-reversal is equivalent to phase conjugation and involves Helmholtz operator.

3.4.3. *Domain decomposition*

The limitation of the noise level generated by airplanes or trains is of major interest during the architecture and construction process of new airports and/or railway stations. The analysis of different configurations of the buildings or the uses of new architectural materials can be performed very quickly through numerical simulation. In order to obtain accurate numerical results, realistic mathematical models involving Helmholtz equation are needed. The numerical resolution of such problems requires then large computer memory. The use of parallel computers or PC networks has become very helpful for such purposes. Our research is to develop new mathematical domain decomposition methods suitable for the fast and accurate simulation of such problems. These methods are based on two steps. First, the global domain is split into several sub-domains and some interface boundary conditions are introduced on the interfaces between the sub-domains. Secondly, a sub-structuring formulation of the global problem leads to a condensed interface problem which is solved iteratively. Each iteration involves the solution of an acoustic problem in each sub-domain.

Interface boundary conditions are the key ingredient to design efficient domain decomposition methods. Without a global preconditioner, convergence cannot be obtained by any method in a number of iterations less than the number of sub-domains minus one in the case of a one-way splitting. This optimal convergence can be obtained with generalized Robin type boundary conditions associated with an operator equal to the Steklov-Poincaré operator in the continuous case and to the Schur complement matrix in the discrete case. However, in practice, this optimal condition cannot be implemented since it is too expensive to be computed exactly. Our goal is to define new approximations of the Steklov-Poincaré operator and of the Schur complement matrix.

3.5. Implementation

Keywords: *Dicretization*, *Riccati equation*.

Participants: Frédéric Magoulès, Bruno Pinçon, Karim Ramdani.

This is a transverse research axis since all the research directions presented above have to be validated by giving control algorithms which are aimed to be implemented in real control systems. We stress below some of the main points which are common (from the implementation point of view) to the application of the different methods described in the previous sections. For many infinite dimensional systems the use of co-located actuators and sensors and of simple proportional feed-back laws gives satisfying results. However, for a large class of systems of interest it is not clear that these feedbacks are efficient, or the use of co-located actuators and sensors is not possible. This is why a more general approach for the design of the feedbacks has to be considered. Among the techniques in finite dimensional systems theory those based on the solutions of infinite dimensional Riccati equation seem the most appropriate for a generalization to infinite dimensional systems. The classical approach is to approximate an LQR problem for a give infinite dimensional system by finite dimensional LQR problems. As it has been already pointed out in the literature this approach should be carefully analyzed since, even for some very simple examples, the sequence of feedbacks operators solving the finite dimensional LQR is not convergent. Roughly speaking this means that by refining the mesh we obtain a closed loop system which is not exponentially stable (even if the corresponding infinite dimensional system is theoretically stabilized). In order to overcome this difficulty, several methods have been proposed in the literature : filtering of high frequencies, multigrid methods or the introduction of a numerical viscosity term. We intend to first apply the numerical viscosity method introduced in [61], for optimal and robust control problems.

4. Application Domains

4.1. Panorama

Keywords: *acoustics*, *aero-acoustics*, *control of VLT's (Very Large Telescopes)*.

As we already stressed in the previous chapters the robust control of infinite dimensional systems is an emmerging theory. Our aim is to develop tools applicable to a large class of problems which will be tested on models of increasing complexity. We describe below only the applications in which the members of our team have important achievements in 2003.

4.2. Acoustics

Keywords: *Helmholtz equation*, *Noise reduction*.

Stefan Duprey is a PhD student doing his thesis in the Research Center of EADS with a CIFRE contract. Antoine Henrot is his advisor, but his work is also supervised in Nancy by Frédéric Magoulès, Karim Ramdani and Marius Tucsnak. In EADS, at Suresnes, his work is supervised by Isabelle Terrasse and François Dubois.

This work is concerned with the reduction of noise due to the plane's engines during take-off. It could be decomposed in several steps:

1. We write a code to compute the flow. Starting from the Euler equations in the potential and isentropic case, we are lead to solve a well-known non-linear elliptic problem, studied for example, in classical books like Glowinski or Nečas. To solve numerically this non linear problem, we use a fixed-point algorithm which turns out to be convergent.

2. We assume the acoustic perturbation to be potential and decoupled from the flow. By linearization of Euler equations, we get a linear problem satisfied by the acoustic potential. The coefficients of this equation involve the potential flow computed at step 1. The boundary conditions are either of Neumann or impedance type.

3. We have to write a code to compute the solution of step 2. The fact that the flow can be considered constant at infinity simplifies the equation outside a large domain containing the plane. This leads to two possible ways to solve this problem: using globally a Lorentz transform or using a domain decomposition method.

4. When the two previous codes work satisfactory, we can imagine optimization procedures by acting either on the shape of the engine or on its coating.

Stefan Duprey has completely finished point 1 (including a theoretic study of the convergence of the algorithm). Point 2 is now well understood. The theoretical questions of existence and uniqueness are currently studied and the work is in progress.

4.3. Control of VLT's (Very Large Telescopes)

Keywords: *adaptive optics*, *robust control*, *turbulence*, *wavefront*.

The objective of this work is to use of the tools of infinite-dimensional system automatics for the control of large telescopes. The future telescopes will be characterized by diameters much larger and the fact that the spectrum of the analyzed wavefront lies in the visible field. It is estimated that to correct the image with the same quality, the density of the actuators will have to be multiplied by one hundred and that it will be necessary to replace the piezoelectric actuators by actuators resulting from micro-technology. In theory, a telescope provided with actuators and sensors can be modeled like a finite-dimensional system. When the number of sensors and actuators becomes very large it is often difficult to use this type of modeling to control the telescope.

Our prime objective is to obtain, by techniques of asymptotic analysis, models based on systems of partial differential equations, with distributed control. According to the structure of the obtained system we suggest to apply modern techniques resulting from the theory of the control of **infinite**-dimensional systems. The input and the output of the system will remain of finite dimensions, which will allow the direct application of the results to the initial system. The obtained systems will couple equations modeling the structure and the equations modeling the sensors and the actuators (for example equations of electrostatics). One of the difficulties of the problem lies in the fact that control occurs only in one of the equations of the system. A detailed attention will be given to the problems of optimal positioning of the control fields. It is the question of finding the localization and the form to be given to the actuators and the sensors so that the control is as effective as possible.

In a first approximation, which is valid for infra-red waves, we use the geometrical optics to study the system. In this case, by linearizing the equations, we have justified some of the approximations used in engineers literature. Currently, we work directly on the nonlinear equations obtained with the geometrical optics approach, and we look for an approximation valid when the spectrum of the analyzed wavefront lies in the visible field.

5. Software

5.1. SCISPT Scilab toolbox

Keywords: *Scilab*, *sparse matrices*.

Participant: Bruno Pinçon [correspondant].

SCISPT is a Scilab toolbox which interfaces the sparse solvers umfpack v4.3 of Tim Davis and taucs snmf of Sivan Toledo.

Our aim is to develop Scilab tools for the numerical approximation of PDE's. This task requires powerful sparse matrix primitives, which are not currently available in Scilab. We have thus developed the SCISPT Scilab toolbox, which interfaces the sparse solvers umfpack v4.3 of Tim Davis and taucs snmf of Sivan Toledo. It also provides various utilities to deal with sparse matrices (estimate of the condition number, sparse pattern

visualization,...). We intend to extend this work in the framework of collaborations with the Scilab consortium recently created.

5.2. Parallel Computational Acoustic Library

Participant: Frédéric Magoulès.

The Parallel Computational Acoustics Library is a finite element based library able to solve huge acoustics problems in parallel.

This library contains three main types of functions - those for pre-processing (mesh data), those for processing (involving numerical and matrix analysis), and those for post-processing (visualization, noise rendering). This work was motivated by the need to integrate the various finite elements, mesh generation, mesh partitioning, domain decomposition methods and parallel solvers, and plotting programs developed by the group.

The Parallel Computational Acoustics Library contains currently the most recent and powerful developments in finite element methods for acoustics and parallel iterative domain decomposition methods. There are two groups of algorithms in the library: the first one is based on well established methods which are generally used in the industry, while the second one uses the current result of research. This helps the library to be used at the same time by industrial partners (ONERA, Hutchinson S.A.) and academics researchers. The library is therefore able to solve huge acoustic problems that it was not possible to solve so far. The Parallel Computational Acoustics Library is written is FORTRAN 90 and uses the MPI library for parallel data exchange.

Interactive visualization tools using the VTK library have been developed. An additional noise rendering interface is available in order to listen the results issue from the simulation.

6. New Results

6.1. Analysis and control of fluids and of fluid-structure interactions

Keywords: *Korteweg de Vries equations*, *Navier-Stokes equations*.

Participants: Patricio Cumsille, Lionel Rosier, Jean-François Scheid, Takéo Takahashi, Marius Tucsnak.

A part of our activity in this field in 2004 was devoted to the the study of welposedness and to the numerical analysis of the equations modelling the motion of rigid bodies in an incompressible fluid. Concerning the welposedness results, the main achievements are reported in the published paper [33], in the article [25] (accepted for publication) and in the technical report [34]. In [33] we gave an existence and uniqueness result in the case of a viscous fluid filling the exterior of an infinite cylinder. The generalization of this result to more general geometries is studied in [34] and in a forthcoming work of Cumsille and Takahashi. The main result in [25] asserts the global in time existence for the equations modelling the motion of a ball in a perfect incompressible fluid (the two dimensional case). In a work in progress Rosier and Takahashi tackle the case of a rigid body of an arbitrary form, moving in a perfect incompressible fluid. A new problem which we began to tackle in 2004 is the motion of articulated bodies (i.e., formed by several rigid bodies) in a fluid. These models are aimed to contribute to the understanding of an important question in fluid mechanics: giving mathematical models of the motion of aquatic organisms.

Concerning the numerical analysis of the system modelling the motion of rigid bodies in a viscous incompressible fluid, our main results have been announced in [31] and their detailed proofs have been given in the article [22]. These results give sufficient conditions for the convergence of the Lagrange-Galerkin method applied to the equations modelling the motion of rigid bodies in a viscous incompressible fluid. As in the proof of the existence results, the fact that we have free boundaries considerably complicates the numerical analysis.

A new research direction we considered in 2004 is the control of fluid-structure interactions. We tackled three types of problems:

1. Control of the motion of the rigid bodies moving in a fluid by using a velocity or a torque control acting on the rigid only. In a forthcoming paper of Takahashi and Tucsnak (with Jorge San Martin) we give several reachability results at low Reynolds number. These results seem to give a new insight of the very interesting propulsions mechanism used by ocean micro-organisms (like ciliata).

2. Control of the motion of both the rigid bodies and of the fluid by using an input given by the velocity field of the fluid on a part of the exterior boundary of the domain. This question has a more theoretical motivation: we want to show that the presence of the rigid bodies doesn't change in an essential way the controllability properties of the system. In forthcoming joint work with O. Imanuvilov, T.Takahashi has obtained a controllability result for the linearized problem.

3. Control of the motion of an articulated body moving in a fluid. In this case the input function is the torque acting at the joint. This problem can be considered as a first step in the understanding of the control mechanisms guiding fish motion.

A slightly different activity in the same research field concerns the study of phase-field models. More precisely, we consider phase-field models for solidification processes and phase separation phenomena. These models are described through systems of nonlinear evolution equations. The well-posedness is proved for a fourth order parabolic system related to a phase separation process [18], as well as for a phase-field model leading to degenerate parabolic equations [32].

6.2. Optimal location of sensors and of actuators

Participant: Antoine Henrot.

This topic was the subject of the thesis of Pascal Hébrard who left our team at the end of 2002. He is currently working as a research engineer at Dassault Systems. Nevertheless, we kept in touch during this year since we wanted to understand and write precisely the "spillover phenomenon" that we pointed out last year. Let us explain what it is. When we want to damp a vibrating body, we have in principle to act on all the eigenfrequencies of the body. In practice, it is common to consider that high frequencies are not so penalizing and that we can only take into account the low frequencies. Therefore, we decided to simplify the criterion we introduced in [6] and which involves all the eigenmodes (this criterion corresponds to some rate of decay of the total energy of the system), by introducing a new criterion, say J_N involving only the N-the first eigenmodes of the damped system. Then, we are led to look for an internal domain which damps the best as possible those N modes, that is which maximizes our criterion J_N. We were able to prove, in one dimension (that is to say for a damped string) that:

- There exists a unique solution ω_N^* to the previous minimization problem. This set ω_N^* represents the optimal location of actuators when we want to damp only the N first modes.

- The set ω_N^* is composed of at most N connected components.

- When the length constraint l goes to zero (i.e. we consider a small zone of control), the set ω_N^* concentrates on the nodes of the $N + 1$-th mode. It means that it does not control it at all. In other terms, the best domain for the N-th first eigenmodes is the worse for the $N + 1$-th eigenmode!

This work will appear in [17]. We would now like to know if this phenomenon is related to the choice of our model (criterion and state equation) or if it is a more general situation. Moreover, we would like to extend this result to higher dimension. Even in two dimensions, for general domains, the formal proof seems much harder, in particular we need spectral properties which are not known in a general context.

With Steve Cox, we began studying another problem also related to the damping of eigenmodes in a string. It is a question relating to a model for harmonics on stringed instruments which could be set as: is it possible to achieve "Correct Touch" in the pointwise damping of a fixed string? By correct touch, we consider the following. When we place a finger lightly at one of the nodes of the low frequency harmonics, it forces the string to play a note that sounds like a superposition of those normal modes with nodes at the location of the

finger. Now, the question is to determine what should be the pressure of the finger in order to best damps the remaining modes. From a mathematical point of view, we consider the wave equation with a damping as $b\delta_a u_t$ where u_t is the speed, a the location of the pressure, δ_a is the Dirac distribution at a and b the intensity of the pressure. We want to determine, for each a what is the b which minimizes the spectral abscissa of the modes not vanishing at a. This involves a precise qualitative analysis of the spectrum of the damped operator in the complex plane. This is still a work in progress, but we have already obtained significant results which are given in [59].

6.3. Frequency tools for the analysis of pde's

Participants: Frédéric Magoulès, Bruno Pinçon, Karim Ramdani, Takéo Takahashi, Marius Tucsnak.

6.3.1. *Control and stabilization of systems governed by pde's and their numerical approximation*

It is well-known that the solution of LQR optimal control problems is given through a feedback operator involving a Riccati operator P. This operator solves a Riccati equation in infinite dimension. Of course, in practice, one can only determine an approximate solution P_h of this equation, and the natural question that arises is the following : does the approximate solution obtained using this operator P_h (instead of P) converge to the solution of the continuous problem. This question has been so far studied by many authors (see for instance [39][49][40]). In all these papers, on of the main assumptions is that the discretized systems should be **uniformly stabilizable** with respect to the discretization parameter h. Unfortunately, most of the standard numerical methods (finite element or finite differences) do not fulfill this condition. Using the frequency characterization of stabilizability proposed by Liu and Zheng in [53], we have given in [56] a general technique ensuring the uniform stabilizability of classical numerical methods (finite element or finite differences). This technique consists in adding to the standard numerical schemes a suitable numerical viscosity. Compared to the results of [61], the main novelties brought in by our results lie in their generality, since they hold for the finite element space semi-discretization of a wide class of second order evolution equations.

In [37], we have obtained a new spectral formulation of the criterion of Liu [52], which is valid for boundary control problems. This frequency test can be seen as an observability condition for packets of eigenvectors of the operator. This frequency test has been succesfully applied in [37] to study the exact controllability of the Schrödinger equation, the plate equation and the wave equation in a square.

In a recent work [55], we study the internal stabilization of the Bernoulli-Euler plate equation in a square. The continuous and the space semi-discretizated problems are successively considered and analyzed using a frequency domain approach. For the infinite dimensional problem, we provide a new proof of the exponential stability result, based on a two dimensional Ingham's type result. In the second and main part of the paper, we propose a finite difference space semi-discretization scheme and we prove that this scheme yields a uniform exponential decay rate (with respect to the mesh size).

6.3.2. *Time-reversal*

As already mentioned above, we worked on one aspect of time-reversal, known as the **D.O.R.T method** (see for instance [45] and the references therein), developed at the Laboratoire Ondes et Acoustique (L.O.A.) of the E.S.P.C.I. This technique follows from two experimental observations. Firstly, the number of scatterers contained in a propagation medium is exactly the number of non-zero eigenvalues of the time-reversal operator T. Secondly, each corresponding eigenvector selectively focuses on each scatterer. When we started working on this topic, our goal was to provide for the **first mathematical justification** of these experimental results.

The first model we have considered is a **far field model**, in which the time-reversal mirrors are located at infinity. Their interaction with the scatterers can thus be neglected. We have shown in [15] that the experimental results do not hold in general, but do hold for **small and distant scatterers** (compared to the wavelength). These original results have been obtained thanks to an asymptotic analysis of the scattering amplitude of the problem. The second model we have considered is a **near field model**, in which the mirrors a re too close to

the scatterers to be neglected. This question is studied in the framework of the PhD thesis of C. Benamar at the laboratory L.A.M.S.I.N (E.N.I.T, Tunis), whose advisor is C. Hazard (S.M.P., CNRS/ENSTA/INRIA).

Several questions are still opened. One of them is to understand the relation between the time-reversal in the frequency and the time domains. This question requires the use of scattering theory. Another interesting problem, we just started to work on, concerns the acoustic focusing in waveguides using time-reversal. The first numerical experiments realized are very promising and the theoretical analysis is still in progress.

6.3.3. Domain decomposition

Optimized interface conditions have been developed for the non-overlapping Schwarz method in acoustics (cf. [20][19]). These interface conditions are obtained through a continuous analysis. MoreoverAlgebraic interface conditions have been developed for a domain decomposition method with two-Lagrange multipliers ([21][30]). These interface conditions are derived from an algebraic approximation of the Dirichlet-to-Neumann operator. In [14], stabilized finite elements methods have been analyzed for computational acoustics problems. These methods allow to achieve a given accuracy of the solution with half the number of degree of freedom required by classical finite element methods.

6.4. Miscellaneous

Participants: Lionel Rosier, Antoine Henrot.

In this section we briefly describe several important achievements obtained during the last year and which cannot be easily included in one of the themes above.

6.4.1. Lionel Rosier

Reference [10] is a second edition of a book devoted to the Liapunov theory and its applications to nonlinear control theory. A special emphasis is put on the proof of Liapunov converse theorems for nonsmooth systems. The control of the free surface of water in a long canal by means of a wavemaker is investigated in [27]. The fluid motion is described by a Korteweg-de Vries equation in Lagrangian coordinates. The null-controllability is proved thanks to some Carleman estimate.

In [11], we ask whether the null-controllability holds for the Korteweg-de Vries equation on the half-line with a boundary control applied at the origin.

In [24], the authors show the existence and uniqueness of a classical solution for the problem of the motion of a rigid ball surrounded by an incompressible perfect fluid in two space dimensions.

The evolution of density and temperature of a cloud of self-gravitating particles confined to a ball in the three dimensional space is investigated in [29] (see also [28]). The authors prove the global in time existence, the uniqueness, and the convergence of the solution towards an equilibrium state when the initial density profile behaves like $1/r^2$ at the origin.

In [23] the Input-to-State Stability introduced by E. Sontag is used to derive an explicit bound of the state reconstruction error for a class of discrete-time systems with time-varying unknown parameters.

In [38] (see also [26]) a class of chaotic dynamical systems on the $N-$ dimensional torus is proposed for masking some information in secure communications. The information is then recovered thanks to a chaos synchronization process.

6.4.2. Antoine Henrot

In the book [47], we make a detailed study of theoretical questions about shape variations and optimization like continuity with respect to the domain, existence of solutions, differentiability with respect to the domain, geometric properties of optimal shapes and relaxation.

The main result of the paper [12] concerns existence of classical solutions to the multi-layer Bernoulli free boundary problem with nonlinear joining condition and the p-Laplacian as governing operator.

In [16], we study the positivity of the second shape derivative around an equilibrium for a 2-dimensional functional involving the perimeter of the shape and its the Dirichlet energy under volume constraint. We prove that, generally, convex equilibria lead to strictly positive second derivatives.

In paper [13], we prove an isoperimetric inequality for the twisted Dirichlet eigenvalue which was introduced by Barbosa and Bérard in the context of constant mean curvature surfaces. More precisely, we show that in the Euclidean case this eigenvalue is minimized by the union of two equal balls.

7. Contracts and Grants with Industry

The collaboration with EADS described in Subsection 4.2 is formalized by a CIFRE contract.

Moreover, F.Magoulès particpates to a contract with Hutchinson. The aim is to developp some GUI (Graphics Users Interface) with the VTK (Visualization ToolKit) library able to deal with hudge meshes in a very short time. The VTK library is based on an OpenGL kernel and is programmed in C++ through an object oriented approach. More details can be found in references [36][35].

8. Other Grants and Activities
8.1. Introduction
8.1.1. National initiatives

- At INRIA : Marius Tucsnak is member of the Evaluation Committee of INRIA and of the Project Committee of INRIA-Lorraine.
- In the Universities and in CNRS committees:

  - Antoine Henrot is the chair of the CNRS GDR entitled "ANOFOR" (New Applications of Shape Optimization).
  - Our team is part of the newly created GDR entitled "Fluid-Structure Interactions".

8.2. Bilateral agreements

- PICASSO grant with the University of Sevilla;
- BRANCUSI grant with the University of Craïova (Romania).

8.3. Visits of foreign researchers

J. Ortega (Santiago), G. Weiss (London), O. Imanuvilov (Ames), Sorin Micu (Craiova).

9. Dissemination
9.1. Participation to International Conferences and Various Invitations
9.1.1. Invited conferences

- A. Henrot

  - *Metz*: Journées de Metz sur les Problèmes à frontière libre, april 2004;
  - *Obernai*: 36th Congrès National d'Analyse Numérique, June 2004;
  - *Copenhague (Danemark)*: 3rd Colloquium on Applied Analysis, August 2004;
  - *Cagliari (Italie)*: Conference on Elliptic and Parabolic Problems dedicated to Prof. G. Philippin for his 60th birthday, September 2004;
  - *Oberwolfach (Allemagne)*: Oberwolfach Seminars on Shape Optimization, November 2004.

- M. Tucsnak:

  - *El Jadida (Maroc)* Colloque International sur les Mathématiques Appliquées à l'Industrie et la Physique, avril 2004;
  - *Rehovoot (Israel)*: Workshop on dynamical systems, June 2004;
  - *Metz:* Deuxième Colloque sur la Commande des systèmes non linéaires, November 2004.

9.1.2. Participation to international conferences

- B. Pinçon:

    – *September 2004:* IEEE Conference on Computer Aided Control Systems Design, Taipei (Taiwan).
    – *December 2004:* Scilab First International Conference, Rocquencourt (France).

9.1.3. Invitations

- L. Rosier:

    – University of Cincinnati, October 2004;
    – University of Chili, Santiago de Chile, November 2004.

- M. Tucsnak:

    – Santiago de Chile, February 2004;
    – Imperial College,London, December 2004;
    – Universidad Autonoma de Madrid, April 2004

- F. Magoulès

    – Greenwich University, Greenwich, United Kingdom, November 2004
    – Heriot Watt University, Edinburgh, Scotland, May 2004
    – Tokyo Institute of Technology, Tokyo, Japan, March 2004

9.1.4. Editorial activities and scientific committee's memberships

- L. Rosier is associated editor of "ESAIM COCV";
- M.Tucsnak is associated editor of "SIAM Journal on Control";
- M. Tucsnak is a member of the program Committee for the 13th Mediterranean Conference on Control and Automation(MED05) and of the scientific committee of the ECCOMAS Conference on Computational Fluid Dynamics (ECCOMAS CFD 2006)

9.2. Teaching activities

Most of the project members are professors or assistant professors so they have an important teaching activity. We mention here only the graduate courses.

- Scientific Computing (A. Henrot);
- Introduction to Nonlinear Systems (L. Rosier);
- Control of systems governed by PDE's, Finite elements (K. Ramdani)
- Distributions and Partial Differential Equations (M. Tucsnak);

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