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# *Team Mathfi*

# *Financial Mathematics*

*Rocquencourt*

THEME NUM

Activity Report

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# 1. Team

**Head of project-team**
Agnès Sulem [DR, INRIA]

**Administrative assistant**
Martine Verneuille [AI, INRIA]

**Research scientists**
Vlad Bally [Professor, University of Marne la Vallée]
Benjamin Jourdain [Professor, ENPC]
Arturo Kohatsu Higa [DR INRIA, (since October 2004)]
Damien Lamberton [Professor, University of Marne la Vallée]
Bernard Lapeyre [Director of CERMICS, Professor ENPC]
Marie-Claire Quenez [Assistant professor, University of Marne la Vallée]

**Research scientists (partners)**
Emmanuelle Clément [Assistant professor, University of de Marne la Vallée]
José Da Fonséca [PULV]
Gilles Pagès [Professor, University Paris 6]
Jacques Printems [Assistant professor, Université Paris XII]
Emmanuel Temam [Assistant professor, Université Paris VII]

**Visiting scientists**
Arturo Kohatsu Higa [Professor, Universitat Pompeu Fabra, Barcelone (2 months)]
Antonino Zanette [Assistant professor, University of Udine - Italy (4 months)]

**Postdoctoral fellow**
Peter Tankov [(since October)]

**Project engineers**
Adel Ben Haj Yedder
Marc Barton-Smith

**PhD students**
Aurélien Alfonsi [ENPC]
Bouarhi Arouna [MENRT grant, University of Marne la Vallée]
Marie-Pierre Bavouzet [MENRT grant, University Paris 9 Dauphine and teaching assistant, University of Marne la Vallée]
Etienne Chevalier [MENRT grant, University of Marne la Vallée]
Youssef Elouerkhaoui [Citibank, London and University Paris 9 Dauphine]
Julien Guyon [ENPC]
Sandrine Hénon [Cifre agreement CAI and University of Marne la Vallée]
Ahmed Kbaier [MENRT grant, University of Marne la Vallée]
Ralf Laviolette [ENS Cachan]
David Lefèvre [Assistant Professor, ENSTA (since June 2004)]
Vincent Lemaire [MENRT grant, University of Marne la Vallée]
Jerome Lelong [MENRT grant, UMLV and ENPC]
Stephan Menozzi [Ecole Polytechnique]
Marouen Messaoud [grant from INRIA and Tunisia, Université Paris 9 Dauphine]
Nicolas Moreni [MENRT grant, University Paris 6]
David Pommier [University Paris 6]
Cyril Strugarek [Cifre agreement EDF and ENPC]

**Student interns**
Guillaume Dangles [(April to June) ENPC]

Jerome Lelong [Universite de Marne La Vallée, DEA]

# 2. Overall Objectives

MATHFI is a joint project-team with INRIA-Rocquencourt, ENPC (CERMICS) and the University of Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques that pose challenging problems for mathematicians. In this field, the scientific skills of the MATHFI research team are focused on

- the modeling of the price of assets by stochastic processes (diffusions with jumps, stable processes, fractional Brownian processes, stochastic volatility models),
- probabilistic and deterministic numerical methods and their implementation (Monte-Carlo methods, Malliavin calculus, numerical analysis of linear and nonlinear parabolic partial differential equations),
- Stochastic control, with applications to calibration of financial assets, pricing and hedging of derivative products, dynamic portfolio optimization in incomplete markets (transaction costs, discrete time hedging, portfolio constraints, partial observation, asymmetry of information).

Moreover, the MATHFI team is developping a software called PREMIA dedicated to pricing and hedging options and calibration of financial models, in collaboration with a financial consortium. Web Site : http://www-rocq.inria.fr/mathfi/Premia/index.html.

### *2.1.1. International cooperations, Tutorials and industrial relationship*

- International collaborations:
  - Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef). This network has received approval from the European Science Foundation (ESF).
  - Collaborations with the Universities of Oslo, Bath, Chicago, Rome II and III.
  - Part of the cooperation STIC-INRIA-Tunisian Universites.
- Teaching:
  - Teaching in the doctoral programs of Paris and the surrounding region (Paris VI, Paris IX, UMLV, University of Evry, Ecole Polytechnique, ENPC, ENSTA).
- Industrial developments :
  - The PREMIA consortium: A consortium of financial institutions is created centered on the PREMIA option computation software. The consortium is currently composed of : IXIS CIB (Corporate & Investment Bank), CALYON, the Crédit Industriel et Commercial, EDF, GDF, Société générale and Summit.
    Web Site :http://www-rocq.inria.fr/mathfi/Premia/index.html.
  - CIFRE agreements with CALYON on "interest rate models with stochastic volatility" and with CIC on "sparse grids for large dimensional financial issues" (on approval of ANRT)
  - Industrial contract INRIA/EDF on the "identication of Lévy copula for correlation between price and consuption stochastics" (from January 2005)

# 3. Scientific Foundations

## 3.1. Numerical methods for option pricing and hedging and calibration of financial assets models

**Keywords:** *Euler schemes*, *Malliavin calculus*, *Monte-Carlo*, *approximation of SDE*, *calibration*, *finite difference*, *quantization*, *tree methods*.

**Participants:** V. Bally, E. Clément, B. Jourdain, A. Kohatsu Higa, D. Lamberton, B. Lapeyre, J. Printems, D. Pommier, A. Sulem, E. Temam, A. Zanette.

Efficient computations of prices and hedges for derivative products is a major issue for financial institutions. Although this research activity exists for more that fifteen years at both academy and bank levels, it remains a lot of challenging questions especially for exotic products pricing on interest rates and portfolio optimization with constraints.

This activity in the Mathfi team is strongly related to the development of the Premia software. It also motivates theoretical researches both on Monte–Carlo methods and numerical analysis of (integro) partial differential equations : Kolmogorov equation, Hamilton-Jacobi-Bellman equations, variational and quasi–variational inequalities (see [75].

### *3.1.1. MonteCarlo methods.*

The main issues concern numerical pricing and hedging of European and American derivatives and sensibility analysis. Financial modelling is generally based on diffusion processes of large dimension (greater than 10), often degenerate or on Lévy processes. Therefore, efficient numerical methods are required. Monte-Carlo simulations are widely used because of their implementation simplicity. Nevertheless, efficiency issues rely on tricky mathematical problems such as accurate approximation of functionals of Brownian motion (e.g. for exotic options), use of low discrepancy sequences for nonsmooth functions .... Speeding up the algorithms is a major issue in the developement of MonteCarlo simulation (see the thesis of A. Kbaier). We develop Montecarlo algorithms based on quantization trees and Malliavin calculus. V. Bally, G. Pagès and J. Printems have developed quantization methods especially for the computation of American options [53][52], [2], [16][26]. Recently, G. Pagès and J. Printems showed that functional quantization may be efficient for path-dependent options (such that Asian options) and for European options in some stochastic volatility models [32].

### *3.1.2. Approximation of stochastic differential equations.*

In the diffusion models, the implementation of Monte-Carlo methods generally requires the approximation of a stochastic differential equation, the most common being the Euler scheme. The error can then be controlled either by the $L_P$-norm or the probability transitions.

### *3.1.3. PDE-based methods.*

We are concerned with the numerical analysis of degenerate parabolic partial differential equations, variational and quasivariational inequalities, Hamilton-Jacobi-Bellman equations especially in the case when the discrete maximum principle is not valid and in the case of an integral term coming from possible jumps in the dynamics of the underlying processes. In large dimension, we start to investigate sparse grid methods.

### *3.1.4. Model calibration.*

While option pricing theory deals with valuation of derivative instruments given a stochastic process for the underlying asset, model calibration is about identifying the (unknown) stochastic process of the underlying asset given information about prices of options. It is generally an ill-posed inverse problem which leads to optimisation under constraints.

## 3.2. Application of Malliavin calculus in finance

**Keywords:** *Malliavin calculus*, *greek computations*, *sensibility calculus*, *stochastic variations calculus*.

**Participants:** V. Bally, M.P. Bavouzet, B. Jourdain, A. Kohatsu Higa, D. Lamberton, B. Lapeyre, M. Messaoud, A. Sulem, E. Temam, A. Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [79]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [87], D. Bell [54] D. Ocone [89], B. Øksendal [44]) give expositions of the subject. See also V. Bally [48] for an introduction to Malliavin calculus.

>From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [88].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found, such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods. We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here $X$ is a random variable which is supposed to be “smooth” in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where $\Delta$ is a standard normally distributed random variable and $\sigma$ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take $X$ to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals. An important feature is that one has a relatively explicit expression for the weight $Q$ which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators. Let us now look at one of the main consequenses of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where $H$ is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable $X$. We thus obtain an integral representation of the density of the law of $X$. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of $X$ is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of $X$. As $X$ has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no $X^i$ hits exactly $x$. But if you are able to simulate the weight $Q$ as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [61] and [60] and the papers by Bally et al, Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [72].

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modelling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P.Imkeller in [72]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [72]).

## 3.3. Stochastic Control

**Keywords:** *Hamilton-Jacobi-Bellman*, *Stochastic Control*, *free boudary*, *risk-sensitive control*, *singular and impulse control*, *variational and quasi-variational inequalities*.

**Participants:** J.-Ph. Chancelier (ENPC), D. Lefèvre, M. Mnif, M. Messaoud, B. Øksendal (Oslo University), A. Sulem.

Stochastic control consists in the study of dynamical systems subject to random perturbations and which can be controlled in order to optimize some performance criterion function.

We consider systems following controlled diffusion dynamics possibly with jumps. The objective is to optimize a criterion over all admissible strategies on a finite or infinite planning horizon. The criterion can also be of ergodic or risk-sensitive type. Dynamic programming approach leads to Hamilton-Jacobi-Bellman (HJB) equations for the value function. This equation is integrodifferential in the case of underlying jump processes (see [12]). The limit conditions depend on the behaviour of the underlying process on the boundary of the domain (stopped or reflected).

Optimal stopping problems such as American pricing lead to variational inequalities of obstacle type. In the case of singular control, the dynamic programming equation is a variational inequality, that is a system of partial differential inequalities. Singular control are used for example to model proportional transaction costs in portfolio optimisation. The control process may also be of impulse type: in this case the state of the system jumps at some intervention times. The impulse control consist in the sequence of instants and sizes of the impulses. The associated dynamic programming equation is then a Quasivariational inequality (QVI). These models are used for example in the case of portfolio optimisation with fixed transaction costs. Variational and quasivariational inequalities are free boundary problems. The theory of viscosity solutions offers a rigourous framework for the study of dynamic programming equations. An alternative approach to dynamic programming is the study of optimality conditions which lead to backward stochastic differential equations for the adjoint state. See [21] for a maximum principle for the optimal control of jump diffusion processes.

We also study partial observation optimal control problems [45], [11].

## 3.4. Backward Stochastic Differential equations

**Keywords:** *BSDE*.

**Participants:** M.C. Quenez, M. Kobylanski (University of Marne la Vallée).

Backward Stochastic Differential equations (DSDE) are related to the stochastic maximum principle for stochastic control problems. They also provide the prices of contingent claims in complete and incomplete markets.

The solution of a BSDE is a pair of adapted processes $(Y, Z)$ which satisfy

$$-dY_t = f(t, Y_t, Z_t)dt - Z_t' dW_t; \qquad Y_T = \xi, \tag{1}$$

where $f$ is the driver and $\xi$ is the terminal condition [91].

M.C. Quenez, N.El Karoui and S.Peng have established various properties of BSDEs, in particular the links with stochastic control (cf. [86], [76]). There are numerous applications in finance. For example in the case of a complete market, the price of a contingeant claim $B$ satisfies a BSDE with a linear driver and a terminal condition equal to $B$. This is a dynamic way of pricing which provides the price of $B$ at all time and not only at 0. In incomplete markets, the price process as defined by Föllmer and Schweizer (1990) in [63] corresponds to the solution of a linear BSDE. The selling price process can be approximated by penalised prices which satisfy nonlinear BSDEs. Moreover nonlinear BSDEs appear in the case of big investors whose strategies affect market prices. Another application in finance concerns recursive utilities as introduced by Duffie and Epstein (1992) [59]. Such a utility function associated with a consumption rate $(c_t, 0 \leq t \leq T)$ corresponds to the solution of a BSDE with terminal condition $\xi$ which can be a function of the terminal wealth, and a driver $f(t, c_t, y)$ depending on the consumption $c_t$. The standard utility problem corresponds to a linear driver $f$ of the type $f(t, c, y) = u(c) - \beta_t y$, where $u$ is a deterministic, non decreasing, concave function and $\beta$ is the discount factor.

In the case of reflected BSDEs, introduced in [80], the solution $Y$ is forced to remain above some obstacle process. It satisfies

$$-dY_t = f(t, Y_t, Z_t)dt + dK_t - Z_t' dW_t; \qquad Y_T = \xi. \tag{2}$$

where $K$ is a nondecreasing process.

For example the price of an American option satisfies a reflected BSDE where the obstacle is the payoff. The optimal stopping time is the first time when the prices reaches the payoff. ( [81] et [84]).

# 4. Application Domains

## 4.1. Modelisation of financial assets

**Keywords:** *fractional Brownian motion*, *stable law*, *stochastic volatility*.

**Participants:** V. Genon-Catalot, T. Jeantheau, A. Sulem.

### *4.1.1. Statistics of stochastic volatility models.*

It is well known that the Black-Scholes model, which assumes a constant volatility, doesn't completly fit with empirical observations. Several authors have thus proposed a stochastic modelisation for the volatility, either in discrete time (ARCH models) or in continuous time (see Hull and White [70]). The price formula for derivative products depend then of the parameters which appear in the associated stochastic equations. The estimation of these parameters requires specific methods. It has been done in several asymptotic approaches, e.g. high frequency [65], [66], [64].

### *4.1.2. Application of stable laws in finance.*

Statistical studies show that market prices do not follow diffusion prices but rather discontinuous dynamics. Stable laws seem appropriate to model cracks, differences between ask and bid prices, interventions of big investors. Moreover pricing options in the framework of geometric $\alpha$-stable processes lead to a significant improvement in terms of volatility smile. Statistic analysis of exchange rates lead to a value of $\alpha$ around $1.65$. A. Tisseyre has developped analytical methods in order to compute the density, the repartition function and the partial Laplace transform for $\alpha$-stable laws. These results are applied for option pricing in "stable" markets. (see [93]).

### *4.1.3. Fractional Brownian Motion (FBM).*

The Fractional Brownian Motion $B_H(t)$ with Hurst parameter $H$ has originally been introduced by Kolmogorov for the study of turbulence. Since then many other applications have been found.

If $H = \frac{1}{2}$ then $B_H(t)$ coincides with the standard Brownian motion, which has independent increments. If $H > \frac{1}{2}$ then $B_H(t)$ has a *long memory* or *strong aftereffect*. On the other hand, if $0 < H < \frac{1}{2}$, then $\rho_H(n) < 0$ and $B_H(t)$ is *anti-persistent*: positive values of an increment is usually followed by negative ones and conversely. The strong aftereffect is often observed in the logarithmic returns $\log \frac{Y_n}{Y_{n-1}}$ for financial quantities $Y_n$ while the anti-persistence appears in turbulence and in the behavior of volatilities in finance.

For all $H \in (0, 1)$ the process $B_H(t)$ is *self-similar*, in the sense that $B_H(\alpha t)$ has the same law as $\alpha^H B_H(t)$, for all $\alpha > 0$.

Nevertheless, if $H \neq \frac{1}{2}$, $B_H(t)$ is not a semi-martingale nor a Markov process [68][55], [69], [19], and integration with respect to a FBM requires a specific stochastic integration theory.

Consider the classical Merton problem of finding the optimal consumption rate and the optimal portfolio in a Black-Scholes market, but now driven by fractional Brownian motion $B_H(t)$ with Hurst parameter $H \in \left(\frac{1}{2}, 1\right)$. The interpretation of the integrals with respect to $B_H(t)$ is in the sense of Itô (Skorohod-Wick), not pathwise (which are known to lead to arbitrage). This problem can be solved explicitly by proving that the martingale method for classical Brownian motion can be adapted to work for fractional Brownian motion as well [68]. When $H \to \frac{1}{2}+$ the results converge to the corresponding (known) results for standard Brownian motion [69]. Moreover, a stochastic maximum principle holds for the stochastic control of FBMs [55].

## 4.2. Pricing contingent claims in incomplete markets

**Keywords:** *incomplete markets*.

**Participants:** M.C. Quenez, D. Lamberton.

In incomplete markets, the available information may not be restricted to the underlying assets prices. Perfect hedge may not be possible for some contingent claims and pricing can not be done by arbitrage techniques. Moreover, there exists several probabilities, equivalent to the initial one $P$, under which the discounted prices are martingales. By duality, these probabilities are associated to various prices (see [82] et [83]). The upper bound of these prices is characterised as the smallest supermartingale which is equal to $B$ at time $T$. This $Q$-supermartingale can be written as the difference between a $Q$-martingale corresponding to the discounted value of a superhedging portfolio and an optional nondecreasing process. This decomposition implies that this upper bound corresponds to the selling price defined as the lowest price for which there exists a superhedging strategy.

## 4.3. Portfolio optimisation

**Keywords:** *Portfolio optimisation*, *transaction costs*.

**Participants:** T. Bielecki (Northeastern Illinois University, Chicago), J.-Ph. Chancelier, B. Øksendal (University of Oslo), S. Pliska (University of Illinois at Chicago), A. Sulem, M. Taksar (Stony Brook University New York).

We consider a model of $n$ risky assets (called *Stocks*) whose prices are governed by logarithmic Brownian motions, which can eventually depend on economic factors and one riskfree asset (called *Bank*). Consider an investor who has an initial wealth invested in Stocks and Bank and who has ability to transfer funds between the assets. When these transfers involve transaction costs, this problem can be formulated as a singular or impulse stochastic control problem.

In one type of models, the objective is to maximize the cumulative expected utility of consumption over a planning horizon [43] . Another type of problem is to consider a model without consumption and to maximize a utility function of the growth of wealth over a finite time horizon [41]. Finally, a third class of problem consists in maximizing a long-run average growth of wealth [42].

Dynamic programming lead to variational and quasivariational inequalities which are studied theoretically by using the theory of viscosity solutions and numerically by finite difference approximations and policy iteration type algorithms.

The case of fixed costs is studied in [46] and [71]. In [22] we develop methods of risk sensitive impulsive control theory in order to solve an optimal asset allocation problem with transaction costs and a stochastic interest rate.

In the case of jump diffusion markets, dynamic programming lead to integrodifferential equations. In the absence of transaction costs, the problem can be solved explicitly [62]: the optimal portfolio is to keep the fraction invested in the risky assets constantly equal to some optimal value. In the case of proportional transaction costs, there exists a *no transaction region* $D$ with the shape of a cone with vertex at the origin, such that it is optimal to make no transactions as long as the position is in $D$ and to sell stocks at the rate of local time (of the reflected process) at the upper/left boundary of $D$ and purchase stocks at the rate of local time at the lower/right boundary [85]. These results generalize the results obtained in the no jump case. The case of portfolio optimisation with partial observation is studied in [90][45][58], [11].

# 5. Software

## 5.1. Development of the software PREMIA for financial option computations

**Keywords:** *calibration*, *hedging*, *options*, *pricer*, *pricing*.

**Participants:** V. Bally, Adel Ben Haj Yedder, L. Caramellino, J. Da Fonseca, B. Jourdain, A. Kohatsu Higa, B. Lapeyre, M. Messaoud, N. Privault (University of La Rochelle, A. Sulem, E. Temam, A. Zanette.

We develop a software called PREMIA designed for pricing and hedging options on assets and interest rates and for calibration of financial models.

This software [74] contains the most recent algorithms published in the mathematical finance literature with their detailed description and comments of the numerical methods in this field. The target is to reach on the first hand the market makers who want to be informed on this field, and on the other hand the PHD students in finance or mathematical finance.

Premia is thus concentrated on derivatives with rigorous numerical treatment and didactic inclination.

Premia is developed in collaboration with a consortium of financial institutions or departments: It is presently composed of: IXIS CIB (Corporate & Investment Bank), CALYON, the Crédit Industriel et Commercial, EDF, GDF, Société générale and Summit.

History of PREMIA:

The development of Premia started in 1999. There exists now 6 releases.

Premia1, 2 and 4 contain finite difference algorithms, tree methods and Monte Carlo methods for pricing and hedging European and American options on stocks in the Black-Scholes model in one and two dimension.

Premia3 is dedicated to Monte Carlo methods for American options in large dimension. Moreover, it has an interface with the software Scilab [57].

Premia5 and 6 contain more sophisticated algorithms such as quantization methods for American options [2], [52] and methods based on Malliavin calculus both for European and American options [61][60]. Pricing and hedging algorithms for some models with jumps, local volatility and stochastic volatility are implemented. These versions contains also some calibration algorithms.

In 2004, the main development in the release Premia7 has consisted in implementing routines for pricing derivatives in interest rate models: (Vasicek, Hull-White, CIR, CIR++, Black-Karasinsky, Squared-Gaussian, Li, Ritchken, Sankarasubramanian HJM, Bhar Chiarella HJM, BGM). Moreover new algorithms for calibrating in various models (stochastic volatility, jumps, ..) have been implemented and numerical methods based on Malliavin calculus for jump processes have been further explored.

Premia1, 2, 3, 4, 5, 6 have been delivered to the members of the bank consortium in May 1999, December 1999, February 2001, February 2002, February 2003, February 2004 respectively. Premia7 will be delivered in February 2005.

The next release, Premia8, under development in 2005, will be dedicated to the pricing of credit risk derivatives and pricing and calibration for interest rate derivatives.

Premia3 and Premia4 can be downloaded from the web site:
http://www-rocq.inria.fr/mathfi/Premia/index.html.

# 6. New Results

## 6.1. Monte Carlo methods and stochastic algorithms

### *6.1.1. Variance reduction methods in Monte Carlo simulations*

**Participants:** B. Arouna, B. Lapeyre, N. Moreni.

Under the supervision of Bernard Lapeyre, B. Arouna has defended his PHD thesis in which he shown that stochastics algorithms can be efficiently used in order to decrease variance. He provides tractable methods of variance reduction in Monte Carlo estimation of expectations (integrals) and proves associated theoretical results. His work has been published in [13] and [14].

Nicola Moreni is studying variance reduction techniques for option pricing based on Monte Carlo simulation. In particular, in a joint project with the University of Pavia (Italy), he applies path integral techniques to the pricing of path-dependent European options. He has also deals with a variance reduction technique for the Longstaff-Schwartz algorithm for American option pricing. This technique is based on extension of the work of B. Arouna to the case of American options.

### *6.1.2. Monte Carlo methods for American options in high dimension.*

**Participants:** L. Caramellino (University of Rome II), A. Zanette.

We have done numerical comparaison between some recent Monte Carlo algorithms for pricing and hedging American options in high dimension, in particular between the quantization method of Barraquand-Martineau and an algorithm based on Malliavin calculus [30].

## 6.2. Discretization of stochastic differential equations

**Participants:** E. Clément, A. Kohatsu Higa, D. Lamberton, J. Guyon, A. Alfonsi, B. Jourdain.

E.Clément, A. Kohatsu Higa and D.Lamberton develop a new approach for the error analysis of weak convergence of the Euler scheme, which enables them to obtain new results on the approximation of stochastic differential equations with memory. Their approach uses the properties of the linear equation satisfied by the error process instead of the partial differential equation derived from the Markov property of the process. It seems to be more general than the usual approach and gives results for the weak approximation of stochastic delay equations. A paper has been submitted and extensions are studied.

In his thesis, A. Kbaier develops a "statistic Romberg method" for weak approximation of stochastic differential equations. This method is especially efficient for the computation of Asian options.

J. Guyon, PhD student of B. Lapeyre and J.F. Delmas has studied how fast the Euler scheme $X_T^n$ with time-step $\Delta t = T/n$ converges in law to the original random variable $X_T$. More precisely, he has looked for which class of functions $f$ the approximate expectation $\mathbb{E}[f(X_T^n)]$ converges to $\mathbb{E}[f(X_T)]$ with speed $\Delta t$. So far, $(X_t, t \geq 0)$ has been a smooth $\mathbb{R}^d$-valued diffusion. When $f$ is smooth, it is known from D. TALAY and L. TUBARO that

$$\mathbb{E}[f(X_T^n)] - \mathbb{E}[f(X_T)] = C(f)\Delta t + O\left(\Delta t^2\right). \tag{3}$$

Using Malliavin calculus, V. BALLY and D. TALAY have shown that this development remains true when $f$ is only mesurable and bounded, in the case when the diffusion $X$ is uniformly hypoelliptic. When $X$ is uniformly elliptic, J. Guyon has extended this result to the general class of tempered distributions. When $f$ is a tempered distribution, $\mathbb{E}[f(X_T)]$ (resp. $\mathbb{E}[f(X_T^n)]$) has to be understood as $\langle f, p\rangle$ (resp. $\langle f, p_n\rangle$) where $p$ (resp. $p_n$) is the density of $X_T$ (resp. $X_T^n$). In particular, (3) is valid when $f$ is a measurable function with polynomial growth, a Dirac distribution or a derivative of a Dirac distribution. The proof consists in controlling the linear mapping $f \mapsto C(f)$ and the remainder. It can be used to show that (3) remains valid when $f$ is a measurable function with exponential growth, or when the tempered distribution $f$ acts on the deterministic initial value $x$ of the diffusion $X$. An article is being written, underlying applications to option pricing and hedging.

Under the supervision of Benjamin Jourdain, Aurélien Alfonsi is studying the weak and strong rates of convergence of various explicit and implicit discretization schemes for Cox-Ingersoll-Ross processes, both from a theoretical point of view and by numerical experiments.

## 6.3. Approximation of the invariant measure of a diffusion

**Keywords:** *invariant measure*.

**Participants:** E. Clément, D. Lamberton, V. Lemaire, G. Pagès.

Our work focused on the characterization of invariant measures and on processes with jumps. We are working on the approximation of the invariant probability measure for SDEs with locally lipschitz coefficients and for SDEs driven by Levy processes.

We also investigate the numerical analysis of the long run behaviour of dynamical systems (invariant measure of diffusions, recursive learning algorithm) with F. Panloup, PhD student of G. Pagès (see [73]) .

## 6.4. Malliavin calculus for jump diffusions

**Keywords:** *Malliavin calculus*, *jump diffusions*.

**Participants:** V. Bally, M.P. Bavouzet, M. Messaoud.

One of the financial numerical applications of the Malliavin calculus is the computation of the sensitivities (the Greeks) and the conditional expectations.

In the Wiener case (when the asset follows a log-normal type diffusion for example), Fournié, Lasry, Lebouchoux, Lions and Touzi have developped a methodology based on the Malliavin calculus. The main tool is an integration by parts formula which is strongly related to the Gaussian law (since the diffusion process is a functional of the Brownian motion).

In a first stage, V. Bally has worked in collaboration with Lucia Caramellino (University Rome 2) and Antonino Zanette (University of Udine, Italy) on pricing and hedging American options in a local Black Scholes model driven by a Brownian motion, by using classical Malliavin calculus [50]. The results of this work gave rise to algorithms which have been implemented in PREMIA.

V. Bally, M.P .Bavouzet and M. Messaoud use the Malliavin calculus for Poisson processes in order to compute the Greeks (the Delta for example) of European options with underlying following a jump type diffusion. Imitating the methodology of the Wiener case, the key point is to settle, under some appropriate non-degenerency condition, an integration by parts formula for general random variables. Actually, the random variables on which the calculus is based may be the amplitudes of the jumps, the jump times and the Brownian increments.

On the one hand, M.P. Bavouzet and M. Messaoud deal with pure jump diffusion models and Merton model, where the law of the jump amplitudes has smooth density. One differentiates with respect to the amplitudes of the jumps only (pure jump diffusion) or to both jump amplitudes and Wiener increments (Merton model). Under some non-degenerency condition, one defines all the differential operators involved in the integration by parts formula.

Numerical results show that using Malliavin approach becomes extremely efficient for a discontinuous payoff. Moreover, some localization techniques may be used to reduce the variance of the Malliavin estimator. In the case of the Merton model, it is better to use the two sources of randomness, especially when there are more jumps.

On the other hand, V. Bally, MP. Bavouzet and M. Messaoud deal with pure jump diffusion models but differentiate with respect to the jump times. This case is more difficult because the law of the jump times has not smooth density, so that some border terms appear in the integration by parts formula. Thus, one introduces some weight functions in the definition of the differential operators in order to cancel these border terms. But, in this case, the non-degenerency condition is more difficult to obtain.

Another application of the Malliavin's integration by parts formula is to prove that, under appropriate hypothesis, a large variety of functionals on the Wiener space (like solutions of Stochastic Partial Differential Equations) have absolute continuous laws with smooth density. Under uniform ellipticity assumption, A. Kohatsu-Higa developped a methodology which permits to compute lower bounds of the density. Then, V. Bally relaxed this hypothesis, replacing the uniform ellipticity by only local ellipticity around a deterministic curve. Following the work of V. Bally, M.P. Bavouzet is working on an extension of his results to jump diffusion case (driven by a Brownian motion and a Poisson process).

## 6.5. Lower bounds for the density of a functional

**Keywords:** *Lower bounds of density*.

**Participants:** V. Bally, L. Caramellino.

V. Bally worked on lower bounds for the density of functionals on Wiener spaces with applications to locally elliptic diffusion processes. V. Bally obtained a result which gave rise to a preprint submited for publication and is working on further developments. V. Bally and Lucia Caramellino are looking for a lower bound for the density of the law of the portfolio value. In collaboration with Begonia Fernandez and Ana Meda (University of Mexico), V. Bally tries to obtain tubes evaluations for locally elliptic diffusion processes. (see [49][51]).

## 6.6. Quantization methods

**Keywords:** *quantization*.

**Participants:** G. Pagès, J. Printems.

The quantization method applied to mathematical finance or more generally to systems of coupled stochastic differentials equations (Forward/Backward) as introduced in [52] consists in the approximation of the solution of the Backward Kolmogorov equation by means of piecewise constant functions defined on an appropriate Voronoï tesselation of the state space ($\mathbf{R}^d$). The numerical aspects of such a method are to compute such tesselations adapted to the underlying diffusion and to estimate theirs transition probabilities between different cells of two successive meshes (after a time discretization procedure). Hence, it allows the computation of a great number of conditional expectations along the diffusions paths.

For these reasons, such a method seems to work well with the problem of valuation and hedging of financial products. More generally, its applications are concerned with the American options [52],[2], [16], stochastic control [25], nonlinear filtering and related problems (Zakai and McKean-Vlasov type stochastic partial differential equation [67]). See also [26] for a review on the subject.

There exists an “infinite dimensional” version of the quantization method. In particular, when a stochastic process is viewed as an Hilbert value random variable. It is the purpose of the functional quantization to study such quantization (see [78]). This new point of view allows us to treat the problem of valuation of Asiatic options or more generally of “path dependant” options in an $L^2$ framework. See [32] for a numerical study for Asiatic options and European options in the Heston stochastic volatility model. In this paper, log-Romberg extrapolation in space seems to provide very good results. A theoretical justification of such numerical results should be a promising field of study. Note that this should be implemented in the software PREMIA (release 8) (see Premia).

As concerned nonlinear filtering and quantization, let us note a different way of study. In a work in collaboration with Bruno Saussereau (Université de Franche-Comté, Besançon, France), we study the filtering of nonlinear systems from a numerical point of view: we want to compute the conditional expectations of signals when the observation process and/or the dynamic of the signal is not linear by means of a spectral approximation of the Zakai equation. In cite [77], the authors have proposed a spectral approach of nonlinear filtering by means of the Chaos expansion of the Wiener process. Numerical experiments based on this approach together with a quantization method provide promising results (see [40]).

## 6.7. Numerical methods for PDEs in Finance

**Keywords:** *adaptive finite elements*, *finite element*, *lattice-based methods*, *sparse grids*.

**Participants:** Y. Achdou (Prof. Paris 6 University), D. Pommier, J. Printems, A. Zanette.

### 6.7.1. Sparse grids methods for PDEs in Mathematical Finance

Participants: Y.Achdou, D. Pommier, J.Printems

Sparse grids methods are special variants of the Finite Element Method and are useful when the dimension (e.g. number of assets) increases. It relies on the approximation of functions defined on a domain of $\mathbf{R}^d$ by means of special tensor products of $1 - d$ finite elements. These special products are those for which the indices are in the unit simplex. These methods are known to be efficient when the solutions are enough regular (see [56][94][92]). Our work consists in generalize the ideas of "sparse grids" to the regions where we can not control the regularity of the solution (typically near the maturity in Finance).

A Cifre agreement on this subject between Inria and CIC is engaged involving the PhD student David Pommier.

### 6.7.2. *Adaptive finite element discretizations*

Participants: A. Zanette, A. Ern (ENPC), S. Villeneuve (Toulouse university):

We perform numerical studies on adaptive finite element discretizations for pricing and hedging European options with local volatility Black-Scholes models. [20]

### 6.7.3. *Lattice based methods for American options*

Participants: A. Zanette (in collaboration with M.Gaudenzi, F.Pressacco and L.Ziani):

With reference to the evaluation of the speed/precision efficiency of pricing and hedging of American Put options, we present and discuss numerical results in [29][28]. A comparison of the best lattice based numerical methods known in literature is offered along with some key methodological remarks.

## 6.8. Policy iteration algorithms for fixed point problems with nonexpansive operators

**Keywords:** *Howard algorithm*, *dynamic programming*, *nonexpansive maps*, *policy iteration algorithm*.

**Participants:** J.Ph. Chancelier, M. Messaoud, A. Sulem.

The case of Bellman equations associated to optimal control of Markov chains on an infinite horizon with discount factor $\lambda > 0$ has been studied for a long time by many authors (see e.g the monographs by Bertsekas and Puterman). Typically these equations are of the form $v = \sup_{w \in \mathcal{W}} \{\frac{1}{1+\lambda} M^w v + c^w\}$ where $M^w$ is the transition matrix of the Markov chain, $c^w$ is the running utility and $w$ is the control variable with values in some control set $\mathcal{W}$. We know that the iteration policy algorithm converges to the solution of the Bellman equation since the operator $\frac{1}{1+\lambda} M^w + c^w$ is contractive and satisfies a discrete maximum principle.

The problem addressed here concerns more general fixed point problems on a finite state space. Typically the operator we consider is the maximum of a contractive operator and a nonexpansive one which satisfy some appropriate properties. Shortest path problems also lead to some fixed point problems with nonexpansive operators but in a rather different context whereas reflecting boundaries lead to nonexpansive operators on the boundary. This last problem appears to be a special case of ours. We prove the convergence of an iteration policy algorithm and give illustrating examples in optimal control of Markov chains in [31]

These results can be applied to the numerical analysis of quasi variational inequalities (QVIs) associated to combined impulse/stochastic optimal controls.

## 6.9. Stochastic control of Jump diffusions

**Keywords:** *jump diffusions*, *stochastic control*.

**Participants:** B. Øksendal (Oslo University), A. Sulem.

We have completed a book on this subject [12]. The main purpose of the book is to give a rigorous, yet mostly nontechnical, introduction to the most important and useful solution methods of various types of stochastic control problems for jump diffusions and its applications. The types of control problems covered include classical stochastic control, optimal stopping, impulse control and singular control. Both the dynamic programming method and the maximum principle method are discussed, as well as the relation between them. Corresponding verification theorems involving the Hamilton-Jacobi Bellman equation and/or (quasi-)variational inequalities are formulated. There are also chapters on the viscosity solution formulation and numerical methods. The text emphasises applications, mostly to finance. All the main results are illustrated by examples and exercises appear at the end of each chapter with complete solutions. This will help the reader understand the theory and see how to apply it. The book assumes some basic knowledge of stochastic analysis, measure theory and partial differential equations.

## 6.10. Anticipative stochastic optimal control

**Keywords:** *control*, *information*.

**Participant:** D. Lefèvre.

The stochastic maximum principle is a powerful tool to characterize optimal control processes for a controlled system of stochastic differential equations. Control processes represent the policy of the decision-makers and are usually assumed nonanticipative. In this work, we waive this adaptation constraint and allow the policies to be adapted to a larger filtration that the one generated by the Brownian motion which drives the controlled stochastic differential equation. First, we show how to correctly define a unique solution for our resulting anticipative controlled system. Next, we derive a Pontryagin-type maximum principle for possibly anticipative optimal stochastic controls where the definition of the Hamiltonian is substantially modified.

## 6.11. Utility maximization in an insider influenced market

**Keywords:** *antipative calculus*, *asymmetry information*, *forward integrals*, *insider*.

**Participants:** A. Kohatsu Higa, A. Sulem.

We study a controlled stochastic system whose state is described by a stochastic differential equation where the coefficients are anticipating. This setting is used to interpret markets where insiders have some influence on the dynamics of prices.This point of view was already partly studied in Øksendal-Sulem [47] although the financial consequences and modelling possibilities for models of markets with insiders were not exploited there. We give a characterization theorem for the optimal logarithmic portfolio of an investor with a different information flow from that of the insider. As examples, we provide explicit results in the partial information case which we extend in order to incorporate the enlargement of filtration techniques for markets with insiders.

Finally, we consider a market with an insider which influences the drift of the asset process. This last example, which does not seem to fit into the enlargement of filtration set-up, gives a situation where it makes a difference for a small agent to acknowledge the existence of the insider in the market [23].

## 6.12. Credit risk

**Keywords:** *Marshall-Olkin copula*, *credit derivatives*.

**Participants:** B. Jottreau, Y. Elouerkhaoui.

Modelisation of credit default risk has been one of the main themes of the weekly seminar in the University of Marne la Vallée. B. Jottreau has started a thesis on this subject. The next release for Premia, Premia8, will include algorithms for pricing credit risk derivatives.

Y. Elouerkhaoui has continued to work on the valuation and hedging of basket credit derivatives in the Marshall-Olkin copula framework. The two main research themes are: the modelling of default correlation in the context of credit derivatives pricing, and the study of correlation market incompleteness and hedging.

### *6.12.1. Local risk minimization hedging in multi-credit markets*

We have tackled the issue of hedging basket default swaps with their underlying single-name instruments. The payoff of basket products such as first-to-default swaps and CDOs is dependent on the multivariate default behaviour of the underlying credits. Clearly, default correlation risk introduces a market incompleteness, which cannot be hedged with plain vanilla instruments. To handle market incompleteness, we have used a local-risk minimization approach. We have highlighted the various components of the credit-hedging problem: spread risk, default risk and carry; and we have developed hedging strategies corresponding to the minimization of each type of risk. [35]

### *6.12.2. Modelling of default correlation in complex credit derivatives*

We have studied default correlation risk in a new generation of basket products, known as CDOs of CDOs (or CDO squared) and baskets of baskets. The valuation of these products depends on a compounded type of default correlation risk. Similarly to compound options, which depend on "volatility of volatility", the value of CDOs of CDOs depends on the marginal loss distributions of each underlying CDO and their joint dependence, which is referred to as "correlation of correlation". To analyse this type of risk, we have developed the "Equivalent Single-Name Process" approximation, where the characterization of basket securities is simplified and their joint dependence is analytically tractable. We have considered each basket security in the underlying portfolio as a single-name security whose default time is driven by a Poisson process, and we have derived the properties of this process in the MO model. We have used a technique similar to our "Homogeneous Portfolio" approximation. [33]

# 7. Contracts and Grants with Industry

## 7.1. Consortium Premia

**Participants:** V. Bally, M. Barton Smith, A. Ben Haj Yedder, L. Caramellino, J Da Fonseca, B. Jourdain, B. Lapeyre, A. Sulem, E. Temam, A. Zanette.

The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions or departments: IXIS CIB (Corporate & Investment Bank), CALYON, the Crédit Industriel et Commercial, EDF, GDF, Société générale and Summit.

http://www-rocq.inria.fr/mathfi/Premia/index.html.

## 7.2. EDF

- C. Strugarek, P. Carpentier, A. Sulem.
  CIFRE agreement EDF-ENPC on "Optimisation of portfolio of energy and financial assets in the electricity market".
- P. Tankov, A. Alfonsi, A. Sulem.
  Industrial contract INRIA/EDF on the "identication of Lévy copula for correlation between price and consuption stochastics" (from January 2005)
- Project-team Mathfi, Laboratory Cermics
  Discussions for a general industrial convention between EDF and Mathfi and research teams of CERMICS on risk issues in electricity markets.

## 7.3. CIC: "sparse grids for large dimensional financial issues"

**Participants:** D. Pommier, J. Printems, Y. Achdou, A. Sulem.

Cifre agreement on approval of ANRT

## 7.4. CALYON: Interest rate models with stochastic volatility

**Participants:** S. Hénon, D. Lamberton, M.C. Quenez.

Cifre agreement between University of Marne la Vallée and CAI.

# 8. Other Grants and Activities

## 8.1. International cooperations

- Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef). This network has received approval from the European Science Foundation (ESF).
- Collaborations with the Universities of Oslo, Bath, Chicago, Rome II and III.
- Part of the cooperation STIC-INRIA-Tunisian Universites.

# 9. Dissemination

## 9.1. Seminar organisation

- V. Bally : Organiser of a session on "Reduction of Variance", in the conference $MC2QMC$ on MonteCarlo methods (Juan-les-pins, 7-10 2004)
- B. Jourdain, M.C. Kammerer-Quenez : organization of the seminar on stochastic methods and finance, University of Marne-la-Vallée
- M.C. Kammerer-Quenez and A. Kohatsu Higa : members of the organization committee of the Seminaire Bachelier de Mathematiques financieres, Institut Henri Poincaré, Paris.
- D. Lamberton and D. Pagès: Organisers of a session on "Numerical methods for the long range behaviour of markovian models", $MC2QMC$ (Juan-les-pins, 7-10 2004)
- B. Lapeyre:
  - Participation to the organisation of the Conference in the honor of Nicole El Karoui for her 60th birthday, June 2-4 2004 .
  - Participation to the scientific committee of "ACI Nouvelles Interfaces des Mathématiques", May 2004.
- A. Sulem
  - Organisation of a IliaTech meeting on Financial Mathematics and the PREMIA Consortium, June 2004, INRIA-Rocquencourt.
  - Organisation of a session on "Stochastic control and Backward stochastic differential equations", MAS days, SMAI, Nancy, September 2004.

## 9.2. Teaching

- V. Bally, B. Jourdain, M.C.Kammerer-Quenez
  Course on"Mathematical methods for finance", 2nd year ENPC.
- M.P. Bavouzet
  - January to February 2004: Assistant teaching at the engineer school of the university of Marne-la-Vallée, Mathematics, first year
  - March to June 2004: Assistant teaching at the university of Marne-la-Vallée, Mathematics, DEUG Science de la Matière, first year
  - October to December 2004: Assistant teaching at the university of Marne-la-Vallée, Mathematics, Introduction to Mathematics Reasonning, Licence, first year.
- B. Jourdain :
  - course on "Probability theory", first year ENPC
  - course on "Probabilistic tools in finance", 2nd year ENPC
- J.F. Delmas, B. Jourdain :
  course on "Random models", 2nd year ENPC
- M. Delasnerie, B. Jourdain, H. Regnier :
  course on "Monte-Carlo methods in finance", DEA Probabilités et Applications, university Paris VI
- J.P. Chancelier, B. Jourdain, R. Eymard:
  Course "Numerical methods for financial models", DEA doctoral program "Analyse et Systemes Aléatoires", University of Marne-la-Vallée.

- J. Guyon

  - Assistant professor for the course “Probability and applications” (nov. 2004 - jan. 2005), ENPC.
  - Assistant professor for the course “Mathematical methods for finance” (apr.-june 2004), ENPC
  - Assistant professor for the course “Introduction to probability and statistics” (sep. 2003 - jan. 2004), ENSTA

- A. Kohatsu Higa:
  Course on "Insider models with finite utility", Cours Bachelier, Institut Henri Poincaré, Paris.
- D. Lamberton :
  - Course undergraduate students in economy and mathematics, University of Marne-la-Vallée.
  - Course on “Stochastic calculus and applications in finance", graduate program, University of Marne-la-Vallée.
- B. Lapeyre
  - Course on Modelisation and Simulation, ENPC.
  - Projects and courses in Finance, Majeure de Mathématiques Appliquées, École Polytechnique.
  - Course on Monte-Carlo methods and stochastic algorithms, doctoral program in Random analysis and systems, University of Marne la Vallée.
  - EPFL, Cycle d'Etude Postgrade en Ingenierie Mathematiques : " Numerical methods for pricing and hedging options", (15 hours).
- D. Lefèvre

  - “Simulation and Applied Stochastic Processes in Discrete Time”, Master of Science (1st year), Spring 2004, University of Evry Val d'Essonne,
  - “Financial Mathematics”, Licence in Economics (3rd year), Spring 2004, University of Evry Val d'Essonne,
  - “Markov chains”, 2nd year ENSTA, Winter 2004.

- J. Lelong
  Introductive course on the C language numerical methods in Finance, doctoral program in Random analysis and systems, University of Marne la Vallée.
- M. Messaoud

  - Linear Algebra at Dauphine university(Paris).
  - Mathematical finance and introduction to C++ at the Leonard De Vinci university(Paris).

- M.C. Quenez
  - Courses for undergraduate students in mathematics, Université Marne la Vallée
  - Course on recent mathematical developments in finance, graduate program, University of Marne-la-Vallée,
  - Introductary course on financial mathematics, ENPC.
- A. Sulem
  - Course on numerical methods in finance, DEA MASE and EDPA doctoral program, University Paris 9 Dauphine

- E. Temam
  - "Numerical finance", ENSTA
  - "Stochastic process applied to Continuous finance", exercices, University Paris 7.
- P. Tankov
  - Seminars on C++ with applications to numerical analysis and finance at the University of Evry (since October)

## 9.3. Internship advising

- B. Jourdain
  - Guillaume Dangles,"Lattice approximation in square gaussian models of interest rates", scientific training period ENPC (April to June)
- B. Lapeyre
  Jerome Lelong, DEA University of Marne La Vallee,
  "Calibrating with stochastic algorithms" (June to October)
- E. Temam
  - Mamadou Kobar, Master University Paris 7 : "Non recombinant tree for HJM models" : - Marwan Younes, Master University Paris 7 : "Tree methods for LRS model" : - Bertrand Pelletier, Master University Paris 7: "Monte Carlo methods for HJM models"

## 9.4. PhD defences

- Bouhari Arouna
  Thesis defended on December 16th 2004, ENPC.
  Title: "Algorithmes Stochastiques et Méthodes de Monte-Carlo".
  Adviser: B. Lapeyre
- Stéphane Menozzi
  Thesis defended on December 15th 2004, University Paris VI.
  "Discrétisations associées à un processus dans un domaine et schémas numériques probabilistes pour les EDP paraboliques quasi-linéaires"
  Advisers: V. Bally and E. Gobet (Ecole Polythechnique, CMAP)

## 9.5. PhD advising

- V. Bally and D. Lamberton
  - Ahmed Kbaier (3rd year), Grant of Université de Marne-la-Vallée.
  " Approximation of SDE"
- V. Bally and A. Sulem
  M.P. Bavouzet (3rd year), Grant Université Paris Dauphine and INRIA.
  "Malliavin calcul with jumps and application in Finance".
- B. Jourdain
  - Aurélien Alfonsi: (2nd year), ENPC
  "Stable calibration of asset models"
- A. Kohatsu-Higa
  - Salvador Ortiz (University of Barcelona. Main thesis advisor : David Nualart)
  - Karl Larsson (Lund University. Department of Economics)
- D. Lamberton
  - Etienne Chevalier (4th year), Grant Université de Marne-la-Vallée.
  "Exercice boundaries for American options"
- D. Lamberton and M.C. Quenez
  - Sandrine Hénon (4th year), Cifre agreement with Crédit Agricole Indosuez, Université Marne la Vallée.
  "Modelling interest rates with stochastic volatility"
- D. Lamberton and Gilles Pagès
  - Vincent Lemaire (4th year), Grant Université de Marne-la-Vallée.
  "Approximation of the invariant measure of a diffusion by a Euler scheme with decreasing steps"
- B. Lapeyre
  - Nicola Moreni, ENPC, 2nd year :
  Intégrales de chemin et méthodes de Monte-Carlo en finance
  - Ralf Laviolette (3rd year), ENS Cachan.
  "Convergence rate of approximation schemes of SDE for trajectory functionals
  - Julien Guyon, ENPC (2nd year): Convergence rate in Euler schemes for stochastic differential equations with jumps.
- A. Sulem
  - David Lefèvre (4th year), Université Paris-Dauphine
  “Utility maximisation in partial observation”
  - Marouen Messaoud (3rd year), Université Paris-Dauphine
  "Stochastic control, Calibration and Malliavin calculus with jumps"
  - Youssef Elouerkhaoui (3rd year) : (UBS Londres, Citibank from November)(2nd year)
  "Incomplete issues in credit markets"
- A. Sulem and P. Carpentier (ENSTA)
  Cyrille Strugarek, Cifre agreement ENPC–EDF, 2nd year.
  "Optimisation of portfolio of energy and financial assets in the electricity market"
- J. Printems and Y. Achdou (Paris 6)
  David Pommier : began in november 2004.
  Cifre agreement INRIA–CIC in process.
  “Sparse grid for large dimensional financial issues”.

## 9.6. Participation to workshops, conferences and invitations

- B. Arouna
  An Adaptative Monte Carlo Method, Application to Variance Reduction in Finance, MC2QMC, Juan-les-Pins, Juin 2004
- V. Bally:
  - Talk "Lower bounds for the density of locally elliptic Ito processes" in the Franco -Roumain colloquim, Craiova, Romania, August 28-30, 2004.
  - Invited by Begonia Fenrandez in the university of Mexico, October 1-18, 2004.
  - talk on "Lower bounds for the density of locally elliptic Ito processes" in the colloquium "Mexican congress on PDE's", October 2004
  - talk on "Lower bounds for the density of locally elliptic Ito processes" in the workshop in the memory of Axel Gorund in Marseilles, October 22-23, 2004.
  - invitation by Lucia Caramellino in the university Roma III, October 25 - December 6 2004
- M.P. Bavouzet
  - Talk on ‘Monte Carlo method using Malliavin calculus on the Poisson space for the computation of Greeks’, Third world congress of Bachelier finance society, July 21-24 2004, Chicago.
  - Talk on ‘Monte Carlo method using Malliavin calculus on the Poisson space for the computation of Greeks’, Workshop on Lévy processes in finance, May 2004, INRIA
- B. Jourdain
  - MC2MC04, June 7-10 2004, Juan-les-Pins, "Probabilistic approximation of some nonlinear parabolic PDEs" (Plenary session)
- A. Kohatsu-Higa
  - Seminar on "The Euler scheme and parameter estimation", Technische Universität, Chemnitz, October 2004.
  - Visit to the project Complex Systems and Nonlinear Dynamics, Prof. Dr. Günter Radons at Institut für Physik Komplexe Systeme und Nichtlineare Dynamik, Technische Universität, Chemnitz, (Oct. 20-23) and visit to Abdelhadi Benabdallah at Max Planck Institute for the Physics of Complex Systems to incorporate some recent results on stochastic filtering for the simulation of stochastic models in physics.
- D. Lamberton
  - Two armed bandits and finance. Lectures on Mathematical Finance. Rome, June 2004.
  - Approximation de mesures invariants de diffusions. Colloque Contrôle stochastique et applications. Rouen, June 2004.
  - Exercise boundaries of American Options near Maturity. Workshop on Advanced Mathematical Methods for Finance. Munich, October 2004.
- B. Lapeyre
  - "Utilisation d'Agorithmes Stochastiques pour la Reduction de Variance", Conférence en l'honneur du 60ème anniversaire de Nicole El Karoui, 2-4 juin 2004, Institut Henri Poincaré
  - Invitation to the week on "Risk Management and Model Specifications Issues in Finance", April 12-16, 2004, IMA in IMA Thematic Year on Probability and Statistics in Complex Systems: Genomics, Networks, and Financial Engineering, Minneapolis, USA.
  - Invited by the finance group of the “Operations Research and Financial Engineering" department of Princeton University, November 15-19 2004.
  3 talks on “Variance reduction methods for European and American options”.

- D. Lefèvre
  - “Anticipative stochastic control : Application to the value of information in controlled stochastic systems”, Journées MAS de la SMAI, Nancy, September 7, 2004.
  - invited for 6 weeks (June-July) in the university of Pompeu Fabra in Barcelona by A. Kohatsu-Higa
- Nicola Moreni
  - Pricing American Options: a variance reduction technique for the longstaff-schwartz algorithm, Third world congress of Bachelier finance society, July 21-24 2004, Chicago.
  - American Options on High Dimensional Assets, MC2QMC, Juan-les-Pins, Juin 2004
- M. Messaoud

  – " Calibration of local volatility model using entropy minimization", Premia seminar, ENPC, February 2004.

  – Talk on ‘Monte Carlo method using Malliavin calculus on the Poisson space for the computation of Greeks’, Workshop on Lévy processes in finance, May 2004, INRIA Rocquencourt.

  – "Computation of Greeks using Malliavin’s calculus in jump type market models", $6^{th}$ World Congress of the Bernoulli Society for Mathematical Statistics and Probability, 26-31 2004.

- M.C. Quenez
  - "problème d’optimisation d’utilité dans le cas d’incertitude sur le modèle et sur le taux de préférence pour le temps", congrès en l’honneur de Nicole El Karoui (juin 2004), Institut Henri Poincaré.
  - talks on defalut risk models, Seminar, University of Marne-La-Vallée, March-April 2004.
- J. Printems
  - invited speaker in the International congress “Monte Carlo & Quasi Monte Carlo Methods”, 7–10 June 2004, Juan-les-Pins (France). Title : “Optimal quantization methods for Multidimensional American Options” in the mini-symposium “From the Pricing of American Option on Baskets to the Discretization of RBSDEs.” (see Omega).
- A. Sulem

  – Invited speaker in the workshop ""Computational Finance and Physics", Centre of Mathematics for Applications (CMA), University of Oslo, March 2004.

  – talk on “Utility maximisation in an influenced insider market" in the probabilty seminar of the University of Barcelona, May 2004.

  – Invited talk on "A policy iteration algorithm for fixed point problems with nonexpansive operators", 7th RMR on "Stochastic Optimal Control and Applications, University of Rouen, June 11 2004

  – Invited speaker in the workshop "New Techniques in Applied Stochastics", August 2004, Helsinki, Finland

  – Invited speaker in the "Workshop on Mathematical Finance and Insurance", May 2004, Yellow Mountain, China

  – Invited for a plenay conference on "Impulse control of jump diffusion processes: Theoretical and numerical aspects", MAS days of SMAI, September 8th, Nancy

  – seminar on "Maximisation d’utilité dans un marché influencé par un initié", University of Marne la Vallée, November 5th, 2004

  - talk on "Risk sensitive impulse control and application to optimal asset allocation with transaction costs", seminar Instituto per le Applicazioni del Calcolo "M. Picone", Consiglio Nazionale delle Ricerche, Rome, December 20th.
  - invited by R. Natalini in the Instituto per le Applicazioni del Calcolo, Rome, December 18-24.

- P. Tankov
  - Two invited lectures at the 11th workshop on Mathematics and Economics, University of Oslo, 15 October 2004
  - Talk at the financial mathematics and applied probability seminar, King's college, London, 16 November 2004
- A. Zanette
  - *Pricing and Hedging American Options by Monte Carlo methods using a Malliavin calculus approach* MC2QMC 2004 Conferences Juan Les Pins France
  - *Monte Carlo Methods for Pricing and Hedging American Options in High Dimension* **Italian Conference AMASES Modena 2004**

## 9.7. Miscellaneus

- A. Kohatsu Higa :
  - Report on the thesis "Quelques contributions aux Problèmes d'évaluation des options et choix optimal de portefeuille" by A. Bouzguenda Zeghal, University Paris-Dauphine, Decemver 2004.
- D. Lamberton
  -"Associate Editor" of the journal *Mathematical Finance*.
  - Head of the doctoral program DEA "Stochastic Analysis and Systems", Universities of Marne-la-Vallée, Créteil, Evry and ENPC).
- G. Pagès
  Associate Editor of the journal *Stoch. Proc. and their Appl.*
- A. Sulem
  - Report on the thesis of I. Turpin: "Sur l'interprétation probabiliste de solutions faibles d'EDP: Contrôle Stochastique Optimal sous Observations partielles et Equations Différentielles Stochastiques Rétrogrades", Université de Valenciennes, September 2004.
  - First opponent of the PhD thesis of Ariel Almendral Vasquez on "Financial Derivatives Under Generalized Black-Scholes Models; the PDE approach" (December 2004, Université d'Oslo)

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[2] V. BALLY, G. PAGÈS, J. PRINTEMS. *First order schemes in the numerical quantization method*, in "Mathematical Finance", vol. 13, n$^o$ 1, 2003, p. 1–16.

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