INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team OMEGA

Méthodes numériques probabilistes pour les équations aux dérivées partielles et les mathématiques financières

Sophia Antipolis - Lorraine

THEME NUM

Activity Report

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1. Team

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2. Overall Objectives

2.1. Introduction

The Inria Research team OMEGA is located both at Inria Sophia-Antipolis and Inria Lorraine. The team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, neurobiology, chemical kinetics.

Our competences cover the mathematics behind stochastic modeling and stochastic numerical methods. We also benefit from a wide experimental experience on calibration and simulation techniques for stochastic models, and on the numerical resolution of deterministic equations by probabilistic methods. We pay a special attention to collaborations with engineers, practitioners, physicists, biologists and numerical analysts.

2.2. Probabilistic numerical methods

Concerning the probabilistic resolution of linear and nonlinear partial differential equations, the OMEGA team studies Monte Carlo methods, stochastic particle methods and ergodic methods. For example, we are interested in fluid mechanics equations (Burgers, Navier-Stokes, etc.), in equations of chemical kinetics and in homogeneization problems for PDEs with random coefficients.

We develop simulation methods which take into account the boundary conditions. We provide non asymptotic error estimates in order to describe the global numerical error corresponding to each choice of numerical parameters: number of particles, discretization step, integration time, number of simulations, etc. The key argument consists in interpreting the algorithm as a discretized probabilistic representation of the solution of the PDE under consideration. Therefore part of our research consists in constructing probabilistic representations which allow us to derive efficient numerical methods. In addition, we validate our theoretical results by numerical experiments.

2.3. Stochastic models: calibration and simulation

In financial mathematics and in actuarial science, OMEGA is concerned by market modelling and specific Monte Carlo methods. In particular we study calibration questions, financial risks connected with modelling errors, and the dynamical control of such risks. We also develop numerical methods of simulation to compute prices and sensitivities of various financial contracts.

In neurobiology we are concerned by stochastic models which describe the neuronal activity. We also develop a stochastic numerical method which will hopefully be useful to the Odyssée project to make more efficient a part of the inverse problem resolution whose aim is to identify magnetic permittivities around brains owing to electro-encephalographic measurements.

3. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and, instead, try to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modeling issues: one has to choose appropriate stochastic models, which requires a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The OMEGA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The OMEGA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The OMEGA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

OMEGA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Fluid Mechanics, Chemical Kinetics, Finance and Biology.

4.1.1. Fluid Mechanics

In Fluid Mechanics OMEGA develops probabilistic methods to solve vanishing vorticity problems and to study complex flows at the boundary, in particular their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our expertise concerns

- The convergence analysis of the stochastic particle methods on theoretical test cases. In particular, we explore speed up methods such as variance reduction techniques and time extrapolation schemes.
- The design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluid (the FENE model). A second one concerns the Lagrangian modeling of turbulent flows and its application in combustion for two–phase flows models (joint collaboration with Électricité de France).
- The Monte Carlo methods for the simulation of fluid particles in a fissured (and thus discontinuous) porous media.

4.1.2. Chemical kinetics

An important part of the work of the OMEGA team concerns the coagulation and fragmentation models.

The areas in which coagulation and fragmentation models appear are numerous : polymerization, aerosols, cement and binding agents industry, copper industry (formation of copper particles), behavior of fuel mixtures in engines, formation of stars and planets, population dynamics, etc.

For all these applications we are led to consider kinetic equations using coagulation and fragmentation kernels (a typical example being the kinetics of polymerization reactions). The OMEGA team aims to analyze and to solve numerically these kinetic equations. By using a probabilistic approach we describe the behavior of the clusters in the model and we develop original numerical methods. Our approach allows to intuitively understand the time evolution of the system and to answer to some open questions raised by physicists and chemists. More precisely, we can compute or estimate characteristic reaction times such as the gelification time (at which there exists an infinite sized cluster) the time after which the degree of advancement of a reaction is reached, etc.

4.1.3. Finance

For a long time now OMEGA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modeling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.4. Biology

For a couple of years OMEGA has studied stochastic models in biology, and developed stochastic methods to analyze stochastic resonance effects and to solve inverse problems. For example, we are concerned by the identification of an elliptic operator involved in the calibration of the magnetic permittivity owing to electro-encephalographic measurements. This elliptic operator has a divergence form and a discontinuous coefficient. The discontinuities make difficult the construction of a probabilistic interpretation allowing us to develop an efficient Monte Carlo method for the numerical resolution of the elliptic problem.

5. New Results

5.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Olivier Bardou, Mireille Bossy, Mamadou Cissé, Jean-François Collet, Madalina Deaconu, Pierre Étoré, Jean François Jabir, Adil Jarrah, Antoine Lejay, Sylvain Maire, Miguel Martinez, Mohamed El Otmani, Bernard Roynette, Denis Talay, Étienne Tanré, Pierre Vallois.

5.1.1. Monte Carlo methods for elliptic operators with divergence form and discontinuous coefficients. Applications to discontinuous media and a MEG model

Keywords: *Euler scheme*, *Monte Carlo methods*, *Skew Brownian motion*, *diffusion in discontinuous media*, *discontinuous coefficient*, *divergence form operator*, *random walk*.

We have continued to develop approximation methods for stochastic processes related to elliptic operators with divergence form and discontinuous coefficients. Our motivations come from geophysics and biology. In geophysics, we are interested in simulating diffusions in random media. In biology, we are interested by the MEG problem, which consists in estimating the conductivity coefficients of the different brain layers. This is an inverse problem that the ODYSSÉE project at INRIA solves numerically by using an iterative algorithm. At each step of the algorithm, one needs to know the solution of a particular elliptic PDE defined in the brain at a few points (the locations of the sensors) [28]. The elliptic PDE involves a divergence form operator L defined as

$$Lu = \nabla \cdot [a \nabla u] \, .$$

The matrix a is discontinuous along the boundaries separating the different layers of the brain.

When the space is one dimensional, one can represent the solution $u(x)$ by means of the weak solution of a stochastic differential equation which involves the local time of the unknown process [30]. In order to simulate this stochastic process and to develop efficient Monte Carlo methods for the above elliptic equation, A. Lejay, M. Martinez, D. Talay have proposed two different numerical methods [1][19]. One of them consists in discretizing the one-dimensional stochastic differential equation

$$X_t = X_0 + \int_0^t \sigma(X_s) dB_s + \frac{a(0+) - a(0-)}{2a(0+)} L_t^0(X) \qquad (1)$$

where B is a Brownian Motion and $L^0(X)$ denotes the right local time of the semimartingale X at point 0. We have proved [1] that, under appropriate conditions on the function σ, the weak error of the Euler scheme for the equation (1) is of order $1/2$. An approximation using random walks is also currently developed by P. Etoré.

Although all these approaches are specific to one-dimensional media (but the case of diffusions on a graph may also be coveres [32]), they are first steps to solve the open problem of the simulation of stochastic processes in discontinuous multidimensional media.

Collaborations with P. Adler (Institut de Physique du Globe de Paris) and V. Cvetkovic (KTH, Sweden) have been launched this year in order to test and promote our approach outside the mathematical academic world.

5.1.2. Artificial boundary conditions for nonlinear PDEs

In many applications, e.g., in Finance, one needs to solve PDEs in unbounded domains. It is the case for option pricing problems or optimal portfolio management problems in Finance. The probabilistic interpretation of the PDE in term of backward stochastic differential equation is a useful tool to localize the problem in view of its numerical resolution: see [10]. M. Cissé has just started his Ph.D. thesis in this direction, and is studying localization techniques by Dirichlet artificial boundary conditions for American pricing problems. A part of his research is done jointly with M. El Otmani, a Ph.D. student from the university Cadi Ayyad of Marrakech.

With Soledad Torres (Universidad de Valparaiso), M. Bossy and D. Talay study the Hamilton Jacobi Bellman (HJB) equation associated to a stochastic control problem arising from an electricity stock model (see the section 6.2). This control problem involves a state constraint. Consequently, the boundary conditions of the HJB equation are determined by an implicit relation. To solve the PDE numerically, we have introduced approximate boundary conditions by predicting the optimal control close to the boundary, and we have tested the numerical results (see [14]).

5.1.3. Stochastic resonance and neuronal systems

M. Bossy, D. Talay and É. Tanré have continued their work on the modeling of stochastic resonance effects in the neuronal activity. We have to face important technical difficulties due to the huge complexity of the analytical formulae describing the probability densities of particular stopping times: these random times are the bounds of the spike intervals of the electrical potential along the neurons. We are trying to develop accurate approximate formulae which would allow us to quantify the level of random noise which should be added to the internal noise in order to improve the efficiency of the neuronal activity, in the sense that the period of a periodic electrical input signal is better recognized by the neuronal system.

5.1.4. Stochastic modelling of climatic variables

Almost all the team is involved in a just starting joint collaboration with the Laboratoire de Météorologie Dynamique (université Paris 6, École Polytechnique, École Normale Supérieure). This collaboration (the “Meteo-Stoch” project) is funded by the French ministry of Research within the ACI “Nouvelles Interfaces des Mathématiques”. J-F. Jabir’s Ph.D. thesis is funded by a fellowship attached to this project. Our aims are the following ones:

To model and simulate : we aim to construct seasonal and local models by using stochastic processes, to qualitatively analyze these models (do they fit typical behaviors of atmospheric variables?), to develop calibration and simulation methods, to analyse the convergence rates of these methods, etc.

To price financial products : we aim to study the effects of our stochastic models on various pricings of financial assets and various risk measures for financial institutions submitted to climatic risks.

To solve stochastic analysis problems : we aim to solve theoretical questions issued from our modelling; these questions, which concern stochastic particle systems and laws of particular Markov processes, are also motivated by our probabilistic analyses of partial differential equations, and our applications to other fields than Meteorology and Finance (e.g., Biology).

5.1.5. Improvements of the efficiency of probabilistic numerical methods

Keywords: *Dirichlet problem*, *Monte Carlo methods*, *Robbins-Monro algorithms*, *computation of the first eigenvalue*, *entropy minimization*, *ergodic simulations*, *random walk on spheres*, *variance reduction*.

Using B. Arouna's results on adaptive Monte Carlo methods, B. Arouna (Ecole Nationale des Ponts et Chaussées) and O. Bardou have studied to which extent the coupling of a Robbins-Monro algorithm and a Monte-Carlo method lead to efficient variance reductions for Monte Carlo simulations. They have introduced a minimization criterion based on the Kullback-Leibler distance which avoids the projection step involved in the initial algorithm suggested by B. Arouna. They also have shown that their new method enables the Monte Carlo computation of the delta of a European option with a good accuracy.

O. Bardou has studied the dynamical control of the statistical error during the computation of the invariant measure of a random dynamical system by an ergodic simulation. An adaptive estimator of the variance of the simulation is deduced from an almost sure central limit theorem for empirical means. He has also obtained the speed of convergence of the estimator, and extended the result to the local time estimator of the density of a diffusion process.

The random walk on spheres method provides a fast and efficient way to compute the first exit time from a domain of a Brownian motion (or any diffusion process with constant coefficients), and its position at this exit time. This method can be useful to solve Dirichlet problems, or to compute the first eigenvalue of the Laplace operator [34]. The basic idea is the simulate where and when a Brownian particle exits from a sequence of spheres until the exit position is close enough to the boundary of the domain. The spheres are chosen so large as possible. A few years ago, A. Lejay replaced spheres by squares to treat geophysical applications [25]. M. Deaconu and A. Lejay are now interested in the simulation of first exit times and exit locations from rectangles. The difficulty is to understand how to simulate these random variables, and how to choose the rectangles in terms of the geometry of the domain in order to get so a low number of steps, or so a low error variance, as possible. An algorithm is being tested.

S. Maire and C. De Luigi (université de Toulon et du Var) work on quasi-Monte Carlo methods to compute approximations and integrals of multivariate real valued functions. They study the acceleration of a previously developed Monte Carlo algorithm by using quasi-Monte Carlo sequences. In order to make an optimal use of the information given by these sequences, they have developed a least square method which provides very accurate quadratures for smooth functions.

S. Maire and D. Talay have developed a Monte Carlo method to compute the principal eigenvalue of a neutron transport operator. It is based on the combination of the eigenfunction expansion of the solution of the Cauchy problem driven by the transport operator, and its stochastic representation provided by the Feynman-Kac formula. The authors have conisidered numerical and theoretical aspects. Extensions to other operators seem possible.

S. Maire and E. Gobet (École Polytechnique) have developed an adaptive Monte Carlo method to compute a spectral approximation of the solutions of Poisson equations. The variance and the bias due to the simulation of the stochastic processes both decrease geometrically with the number of steps. A global solution is obtained, as accurately as when using deterministic methods (cf. [15]). The theoretical proofs of convergence and the extension of the method to general situations are provided in [16]. Further extensions to domain decomposition methods are in progress.

5.1.6. Gradual deformation method for Poissonian fields

Keywords: *Geostatistics*, *Poissonian and Gaussian random fields*, *inverse problems*.

The gradual deformation method is used in order to construct a Gaussian random field appearing in the coefficients of a partial differential equation (for example, the field represents the permeability of rocks) in order to match measurements (for example, the pressure or flux of the fluid). This problem is very important in geostatistics. The gradual deformation method [29], introduced by L. Hu *et al.* (Institut Français du Pétrole), relies on the fact that the sum of two Gaussian independent random variables has a normal distribution, and consists in solving a series of one-dimensional optimization problems where the statistical structure of the field is kept. As a Poissonian field has also the same kind of properties (it is possible to combine two Poisson point processes with the same intensity measure and obtain a Poisson point process with the same intensity measure), A. Jarrah and A. Lejay adapt the method of gradual deformation of Gaussian fields to the case of Poissonian fields.

5.1.7. *Statistical and renewal results for the random sequential adsorption model applied to a unidirectional multicracking problem*

Keywords: *Markov chain*, *Palm measure*, *Poisson point process*, *brittleness*, *coating*, *composite material*, *crack*, *fibre*, *relaxation of stress*, *renewal process*, *rupture*, *stationary processes*.

In collaboration with P. Calka (Université Paris 5) and A. Mézin (École des Mines, Nancy), P. Vallois works on a stationary process on the real line which models the positions of the multiple cracks that are observed in composite materials submitted to a fixed unidirectional stress ε. This model generalizes the one-dimensional Random Sequential Adsorption construction. We calculate the intensity of the process and the distribution of the inter-crack distance in the Palm sense. Another point of view is developed, where the positions of the cracks X_i^ε are described from a fixed origin. We prove that the sequence $\{(X_i^\varepsilon, Y_i^\varepsilon), 1 \le i \le n\}$ is a conditioned renewal process, where Y_i^ε is the value of the stress at which X_i^ε forms. The approaches "in the Palm sense" and "fixed origin" merge for $n \to +\infty$. We also investigate the saturation case, i.e. $\varepsilon = +\infty$.

5.1.8. *Statistics of random processes*

Keywords: *Edgeworth expansions*, *Poisson equation*, *bootstrap*, *ergodic diffusions*, *statistics of random processes*.

Pursuing their study of the bootstrap method for diffusion processes, O. Bardou and D. Talay establish first order Edgeworth expansions for three kind of functionals of a diffusion process X: the martingales $\int_0^T f(X_s)\,dW_s$, the normalized martingales $\frac{\int_0^T f(X_s)\,dW_s}{\int_0^T f(X_s)^2\,ds}$ and the empirical means $\frac{1}{T}\int_0^T f(X_s)\,ds$.

Pursuing their investigations on the estimation of skew processes, M. Martinez and O. Bardou propose a statistical setting for the estimation of the skew coefficient α and the asymmetry locus γ of the one dimensional process X solution to

$$X_t = x + \int_0^t b(X_s)\,ds + \int_0^t \sigma(X_s)\,dB_s + (2\alpha - 1)L_t^\gamma(X) - K_t^{[a,\,b]}(X),\ \gamma \in [a,b],\ \alpha \in]0,\,1[,$$

where the process K makes X reflect at points a and b, and L^γ is a local time process.

5.1.9. *Coagulation-fragmentation equations*

Keywords: *Chemical Kinetics*, *Smoluchowski equation*.

The physicist Smoluchowski introduced in 1917 a mathematical model which describes coagulation phenomena. It has many applications such as polymerization, formation of stars and planets, behavior of fuel mixtures in engines, etc. This system describes a non linear evolution equation of infinite dimension. The aim of the probabilistic approach to this system is to give new results or to confirm conjectures formulated by analysts or physicists, with the methods of stochastic analysis.

The Smoluchowski equation describes the dynamics of an infinite system of particles with coagulation phenomena. The particles are characterized by their mass. From a physical point of view, it is natural to suppose that the *rate of coagulation* of two particles depends on their masses. Denote by $n(k,t)$ the density of particles of mass k at time t in a unit volume. The Smoluchowski coagulation equation describes the time evolution of $n(k,t)$:

$$\begin{cases} \dfrac{d}{dt}n(k,t) = \dfrac{1}{2}\displaystyle\sum_{j=1}^{k-1} K(j,k-j)n(j,t)n(k-j,t) - n(k,t)\sum_{j=1}^{\infty} K(j,k)n(j,t) \\ n(k,0) = n_0(k),\ k \ge 1, \end{cases} \qquad (SD)$$

where K is the **coagulation kernel**. We assume that K is symmetric and positive.

Due to the presence of the infinite series, this problem is not a classial initial value problem for a system of non linear ordinary differential equations.

In [26], we constructed a non linear process which *represents* the solution of (SD) (and also the continuous version of the Smoluchowski coagulation equation). In [27], M. Deaconu, Nicolas Fournier (université Nancy 1) and E. Tanré have approximated this process with a finite system of particles. We also give the rate of convergence when the number of particles goes to infinity and we describe the error process (Central Limit Theorem).

This methodology was successfully applied to a problem originating in the modelling of industrial crushers. More precisely, a fundamental problem related to the optimization of the crushing process is to estimate the minimum amount of time and energy required to achieve a prescribed degree of crushing. In collaboration with R. Rebolledo (Pontificia Universidad Católica de Chile) within the framework of the INRIA-CONICYT collaboration programme, E. Tanré develop a model which take account the geometry of the crusher, movement of steel's balls, etc. By modelling crushing as a pure fragmentation process, we designed an algorithm which yields a method for the computation of residence times in crushers.

Another generalization of Smoluchowski's model is obtained when one takes into account the position of particles as a supplementary variable (spatially non-homogeneous model). Nicolas Fournier (université Nancy 1), B. Roynette and É. Tanré have proved the almost sure convergence of the position to 0 and of the mass to infinity with a particular choice of diffusion for the position process. [5].

5.2. Financial Mathematics

Participants: Mireille Bossy, Kathryn Kasminski, Miguel Martinez, Denis Talay, Etienne Tanré.

5.2.1. Modelling of financial techniques

In collaboration with Rajna Gibson (Zürich University) and Christophette Blanchet (université de Nice Sophia-Antipolis), M. Martinez, D. Talay and É. Tanré elaborate an appropriate mathematical framework to develop the analysis of the financial performances of financial techniques which are often used by the traders. This research is funded by NCCR FINRISK (Switzerland) and is a part of its project "Conceptual Issues in Financial Risk Management".

In the financial industry, there are three main approaches to investment: the fundamental approach, where strategies are based on fundamental economic principles, the technical analysis approach, where strategies are based on past prices behavior, and the mathematical approach where strategies are based on mathematical models and studies. The main advantage of technical analysis is that it avoids model specification, and thus calibration problems, misspecification risks, etc. On the other hand, technical analysis techniques have limited theoretical justifications, and therefore noone can assert that they are risk-less, or even efficient.

Consider an unstable financial economy. It is impossible to specify and calibrate models which can capture all the sources of instability during a long time interval. Thus it is natural to compare the performances obtained by using erroneously calibrated mathematical models and the performances obtained by technical analysis techniques. To our knowledge, this question has not been investigated in the literature. A paper presenting our first series of results will appear in the proceedings of MC2QMC 2004 [12]. A more complete version of this work will soon be available in the Working Paper Series of NCCR FINRISK. The involved mathematical techniques are issued from statistics of random processes and stochastic control. A Ph.D. student at MIT, Kathryn Kasminski, made interesting simulations to validate our theoretical studies.

In collaboration with Sylvain Rubenthaler (université de Nice Sophia-Antipolis) we now follow another direction. We deal with the following model for a financial market, in which two assets are traded continuously. The first one is an asset without systematic risk, called *bond* (or bank account) whose price at time t evolves according to equation

$$dS_t^0 = S_t^0 r dt; \quad S_0^0 = 1.$$

The remaining asset is subject to systematic risk; we shall refer to it as a *stock* and model the evolution of its price at time t with the linear stochastic differential equation

$$dS_t = \sigma\, S_t\, dB_t \,+\, \mu(t)\, S_t\, dt; \quad S_0 = s_0. \tag{2}$$

$(B_t)_{t\geq 0}$ is a standard one-dimensional Brownian motion on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The process $(\mu(t))_{t\geq 0}$ is random and we shall call it the "drift process". We make the assumption that it is a process taking values $\{\mu_1, \mu_2\}$ and that it is driven by a compound standard Poisson point process $(\tau_n)_{n\geq 0}$ of intensities $\lambda_1 \ell$ and $\lambda_2 \ell$ (here ℓ is just the Lebesgue measure in $\mathbb{R}$) : $(\tau_{2n+1} - \tau_{2n})_{n\in\mathbb{N}}$ is an i.i.d sequence of exponential random variables of parameter λ_1, and $(\tau_{2n+2} - \tau_{2n+1})_{n\in\mathbb{N}}$ is an i.i.d sequence of exponential random variables of parameter λ_2. The difficulty arises here because we suppose that the trader does not observe the jumps of the drift process. Thus, the drift process can be seen as a hidden process. We try to retrieve information on the jumps of the drift process by using the information of the stock.

In this framework, our aim is still to compare typical technical analysis strategies, which are not based on the model, with mathematical strategies that use the knowledge of the model in force.

When the set of parameters $\Theta := (\lambda_1, \lambda_2, \mu_1, \mu_2, \sigma)$ that governs the behavior of the stock $(S_t)_{t\geq 0}$ is given, we derive a general formula for the optimal strategy. As a consequence, at least theoretically, the mathematical point of view is always better than that of the analyst. However, when the set of parameters $\Theta := (\lambda_1, \lambda_2, \mu_1, \mu_2, \sigma)$ that governs the behavior of the stock $(S_t)_{t\geq 0}$ is unknown, the problem becomes very complex. In particular, it appears that the knowledge of (λ_1, λ_2) is very important. This means that the strategies derived from the model are very sensitive w.r.t. λ_1 and λ_2. We try to quantify this sensitivity. Our theoretical study is driven by our numerical experiments

5.2.2. *Game theory and market of electricity*

In collaboration with Nadia Maizi (CMA, École des Mines de Paris), Geert Jan Olsder (Delft University, the Netherland) and Odile Pourtallier (Comore and Miaou projects, INRIA Sophia Antipolis), M. Bossy and É. Tanré have applied game theory to model the market of electricity.

The deregulation of the market of electricity in European countries, initiated in December 1996, has raised lot of modifications, in particular new spot markets of electricity have emerged. These markets are close to the pollution right markets that start to appear as a consequence of the application of the Kyoto protocol and thus are very peculiar. As, in addition, the electricity cannot be stored, the classical market analysis methods do not apply, and new approaches need to be explored. We have analyzed a simple model with one market and N producers using game theory. In particular, we have shown that it is impossible to find a Nash equilibrium for a simple bid (quantity-price). This has raised the necessity to introduce more complex bids, and consequently, in terms of game theory, a more complex strategy set. On a theoretical ground, we have analyzed the possibility to use conjectural strategies and inverse Stackelberg equilibrium. A paper ([4]) will appear in 2005. A second one is in preparation.

5.3. Stochastic analysis and applications

Participants: Christophe Ackermann, Mireille Bossy, Jean-François Collet, Antoine Lejay, Sylvain Maire, Ivan Nourdin, Bernard Roynette, Denis Talay, Étienne Tanré, Pierre Vallois.

In this section we present our results on issues which are more abstract than the preceding ones and, at first glance, might appear decorrelated from our applied studies. However most of them are originally motivated by modelling problems, or technical difficulties to overcome in order to analyse in full generality stochastic numerical methods or properties of stochastic models.

5.3.1. *Non extensive thermodynamics*

Keywords: *Gibbs measures*, *Shannon entropy*, *Tsallis distribution*.

This work is originally motivated by a desire to understand precisely what, among all possible convex quantities, singles out the Shannon entropy. In the framework of distribution functions with a certain number of prescribed moments, J-F. Collet gives an explicit relation between the structure of the minimizer (say, a Gaussian distribution) and the quantity minimized at equilibrium (say, the Shannon entropy). This correspondance being established, properties of the entropy may be read-off from the expression of the minimizer. We then show that the entropy satisfies a natural homogeneity property if and only if the minimizer is Gaussian. If one then relaxes the homogeneity assumption in a natural way, a new distribution may arise, which turns out to be the Tsallis distribution. Besides providing a new characterization of this ditribution, this establishes a link between classical moment systems and non extensive thermodynamics, which we plan to investigate in the future.

5.3.2. Dissipative systems with no invariant measure

Keywords: *fundamental solution*, *intermediate asymptotics*, *parabolic equations*.

In many examples of dissipative systems arising in probability theory (for instance some Kolmogoroff equations associated to stationary Markov processes), the uniqueness of the equilibrium measure together with a dissipation property (e.g. the existence of a Lyapunov functional) may be used to derive trend to equilibrium for large times. We are interested in systems which do not possess any invariant measure. For some of them J-F. Collet has shown that some quantities do decrease as time increases, a fact which may be used to study the large time asymptotics. A typical example is that of linear parabolic PDEs with time-dependent coefficients. In some cases this may be used to yield very quick proofs of the existence of some intermediate asymptotics.

5.3.3. (p, q)-rough paths

Keywords: *differential equations*, *integration of a differential form*, *irregular paths*, *rough paths*.

The theory of rough paths allows one to define differential equations of type

$$y_t = y_0 + \int_0^t f(y_s)dx_s$$

when x is a continuous path which is not smooth (for example, x is α-Hölder continuous, which is the usual case for trajectories of stochastic processes) [31][33] In addition this construction provides the continuity of the map $x \mapsto y$.

The price to pay in order to use this theory is to construct a trajectory X, called a *rough path* lying above x, living in a non-commutative space, and whose first coordinates gives x. The other coordinates of X represents (in some sense) the iterated integrals of x, that is, for example, $\int_0^t (x_r^i - x_0^i)dx_r^j$. The more irregular is x, the higher is the dimension of the space in which X lives. A. Lejay and N. Victoir (Oxford University) [20] extend the rough path theory in order to differential equations of the type

$$\begin{cases} y_t = y_0 + \int_0^t f(y_s, z_s)dx_s + \int_0^t g(y_s, z_s)d\tilde{x}_s, \\ z_t = z_0 + \int_0^t h(y_s, z_s)d\tilde{x}_s, \end{cases}$$

where x is of finite p-variation (for example, x is $1/p$-Hölder continuous) with $p \in [2, 3)$ and $\tilde{x}$ is of finite q-variation with $q < 2$. In particular, they have shown that there is no need to compute the equivalent of the iterated integrals for $\tilde{x}$. Thus, it is unnecessary to construct a rough path lying above the couple $(x, \tilde{x})$, since above x suffices (hence the expression (p, q)-rough path). The regularity assumptions on g and h may then be weakened. As an important byproduct, any rough paths is a geometric rough paths, which means that it can be approximated by a family of rough paths canonically constructed above a smooth path.

5.3.4. Stochastic partial differential equations

Keywords: *Stochastic partial differential equations*, *analytical operator*, *fractional noise*, *mild solutions*, *semi-groups*.

In collaboration with P-L. Lions (Collège de France), M. Bossy, S. Maire, D. Talay and É. Tanré study the long time behaviour of viscosity solutions of fully nonlinear stochastic partial differential equations.

For particular equations and particular initial conditions, the asymptotic law can be fully explicited. Another direction of research concerns numerical methods to approximate these solutions.

M. Gubinelli (Université Paris 13), A. Lejay and S. Tindel (Université Nancy I) are interested in a pathwise definition of Stochastic Partial Differential Equations (SPDE) of the type

$$Y_t = y + AY_t + \int_0^t \sigma(Y_s)dW_s, \tag{3}$$

where A is the infinitesimal generator of an analytical semi-group $(S_t)_{t>0}$, and W is a continuous path which is irregular. In particular, they have shown that (3) admits a *mild solution* given by the perturbation formula

$$Y_t = S_t y + \int_0^t S_{t-s}\sigma(Y_s)dW_s,$$

and that the map $W \mapsto Y$ is continuous. This work could be seen as a first step toward the resolution of SPDE using some ideas coming from the theory of rough paths.

In a joint work [17] by I. Nourdin, P. Vallois, M. Gradinaru (Université Nancy 1) and F. Russo (Université Paris 13), which is accepted for publication in the Annales de l'Institut Henri Poincaré, a generalized stochastic calculus is developed and applied to processes which are not semimartingales. We especially focus on extension of Itô's formula to the fractional Brownian motion B^H. This process has been considered intensively in stochastic analysis and in many applications, e.g., in hydrology, telecommunications, fluidodynamics, economics and finance. Recall that if the Hurst exponent H of B^H is different from $1/2$, then B^H is not a semimartingale. If f is of class C^2, and $H > 1/6$, we prove the following Itô formula:

$$f(B_t^H) = f(B_0^H) + \int_0^t f'(B_s^H)d^\circ B_s^H.$$

where $\int_0^t f'(B_s^H)d^\circ B_s^H$ denotes the symmetric (or Stratonovich) integral. This identity is due to the fact that the symmetric 3-order integral still exists (and vanishes) for $H > \frac{1}{6}$. Moreover $H = \frac{1}{6}$ is a barrier for validity. The introduction of a new class of integrals allows us to extend the previous identity to any $0 < H < 1$.

I. Nourdin studies the one-dimensional stochastic differential equation

$$dX_t = \sigma(X_t)dB_t^H + b(X_t)dV_t, \qquad t \in (0,1),$$

where B^H is a fractional Brownian motion of Hurst index $H \in (0,1)$, V is a bounded variation process and σ, b are real functions. By using the Russo-Vallois definition of the stochastic integral with respect to B^H, he has proven the existence and the uniqueness of the solution. He also has studied the convergence of the discretization Euler scheme and determined its rate of convergence.

5.3.5. *Sharp studies of the laws of particular stochastic processes*

Keywords: *Bessel bridges*, *Bessel functions*, *Brownian motion*, *Ray-Knight theorem*, *h-transforms*.

P. Vallois and P. Salminen (University of Abo) continue their joint work on first range times (with randomized range levels). During P. Vallois'visit at Turku last summer they investigated the case of a Brownian motion stopped at an independent and exponential time.

In collaboration with M. Yor (université Paris 6), B. Roynette and P. Vallois have continued their preceding works [35] on the limit when time goes to infinity of weighted potential measures obtained from the Wiener measure. Let $(P_x)_{x\in\mathbb{R}}$ be the family of Wiener measures defined on the canonical space $(X_t)_{t\geq 0}$, $(\mathcal{F}_t)_{t\geq 0})$. To $(F_t)_{t\geq 0}$ an $(\mathcal{F}_t)$-adapted, non negative process such that $0 < E_x(F_t) < \infty$, for any $t \geq 0$, $x \in \mathbb{R}$, we associate $Q_{x,t}^F$ the probability measure defined on $(\Omega,\ \mathcal{F}_t)$ as follows :

$$Q_{x,t}^F(\Gamma_t) = \frac{1}{E_x[F_t]}\, E_x[1_{\Gamma_t} F_t], \quad \Gamma_t \in \mathcal{F}_t.$$

In[22]) the authors had considered $F_t := \exp\{-\frac{1}{2}\int_0^t V(X_s)ds\}$, where $V : \mathbb{R} \longrightarrow \mathbb{R}_+$ is a Borel function. Their main result was the following : under some suitable assumptions on V, for any given $s \geq 0$ and Γ_s in $\mathcal{F}_s$, $Q^F_{x,t}(\Gamma_s)$ converges as $t \to \infty$, to $Q^{\tilde{F}}_x(\Gamma_s)$ where $\tilde{F}$ is the (P_x)-martingale :

$$\tilde{F}_t = \frac{\phi_V(X_t)}{\phi_V(X_0)} F_t = \frac{\phi_V(X_t)}{\phi_V(X_0)} \exp\{-\frac{1}{2}\int_0^t V(X_s)ds\},$$

and ϕ_V is a "good" positive solution of the Sturm-Liouville equation $\phi'' = V\phi$. These results have recently been extended in [23] and [7]) by considering $F_t = f(X_t, A_t)$, where $f : \mathbb{R} \times \mathbb{R}^d \longrightarrow]0, +\infty[$ is a Borel function, $(A_t \; ; \; t \geq 0)$ is $(\mathcal{F}_t)$ adapted and $\mathbb{R}^d$- valued, and

$$\left(Y_t \stackrel{(\mathrm{def})}{=} (X_t, A_t) \; ; \; t \geq 0\right) \text{ is a } \left((P_x)_{x\in\mathbb{R}} \; ; \; (\mathcal{F}_t)_{t\geq 0}\right)\text{-Markov process.}$$

Jointly with M. Yor (université Paris 6), B. Roynette associate any positive measure c such that $\int_0^\infty x \wedge x^2 c(dx) < \infty$ to an infinitely divisible couple (X, H) of r.v.'s (a Wald couple) such that X and H are infinitely divisible, $H \geq 0$ and for any $\lambda \geq 0$, $E[e^{\lambda X}]E[e^{-\lambda^2 H/2}] = 1$. More generally if the measure c satisfies $\int_0^\infty e^{-\alpha x} x^2 c(dx) < \infty$, for any $\alpha > \alpha_0$, they construct an Esscher family of infinitely divisible Wald couples. We give many examples of such Esscher families and we prove that the particular ones which are associated with gamma and zeta functions enjoy remarkable properties.

This paper is going to appear in the Annales de l'Institut Henri Poincaré.

Jointly with J. Bertoin and M. Yor (université Paris 6), B. Roynette describe connections between two functional spaces : the space of (sub)critical branching mechanisms and the space of Bernstein functions.

6. Contracts and Grants with Industry

6.1. Collaboration with EDF-Chatou: How to model the price of electricity on a spot market

Participants: Mireille Bossy, Madalina Deaconu, Etienne Tanré.

We continue our collaboration with the company Électricité de France on the process of formation of electricity prices on local spot markets directed by several producers. We aim to model the spot prices on the different markets resulting from a production equilibrium between the asks and the producers. We have elaborated simple assumptions on the main components of the problem, that is, the ask and bid functions. We fully describe the possible equilibria [13].

6.2. Collaboration avec EDF-Chatou : Numerical experiments on a stochastic control problem for electricity stock

Participants: Mireille Bossy, Jean François Jabir, Denis Talay.

At the EDF's request, we have done numerical experiments to solve a stochastic control problem. The benchmark concerned the management of a reservoir in upstream of an hydro-electric power station. In the proposed model, the electricity demand is described by a stochastic differential equation. As it is a low cost energy, the water in the reservoir must be used in an optimal way to minimize the global production cost needed to answer the random supply during a given time period. The numerical experiments (resolution of a 2D-Hamilton Jacobi Bellman equation) give satisfactory results that we have compared with more naive strategies (see [14]). When we consider the situation where extra water is supplied in the reservoir during the period, the boundary conditions of the HJB equation become implicit. We need to introduce an optional choice of production close to the boundary.

6.3. Contract with Gaz de France

Participants: Olivier Bardou, Antoine Lejay, Love Lindholm, Denis Talay.

The aim of our collaboration with Gaz de France is to simulate the possible future prices of contracts related to exchange rates and gas and oil indices. We are interested in continuous time stochastic differential models of the indices and the exchange rates involved in the contracts.

The first task was to calibrate the model. We thus have estimated the volatility and the drift coefficient using various non-parametric estimators. In addition, since we are interested in the simultaneous evolution of the indices and the exchange rates, we are also interested in finding the correlations between the different stock prices. We therefore had to estimate the number of Brownian motions driving the stochastic evolution of the indices and the exchange rates. We have finally proposed a model to Gaz de France. Our partners have approved this model.

We then have worked on variance reduction techniques which allow one to improve the efficiency of Monte Carlo simulations to compute the prices and sensitivities of the contracts.

7. Other Grants and Activities

A. Lejay is responsible for the project “Monte Carlo methods for fissured media” within the “Groupe de Recherche MOMAS” funded by ANDRA, C.E.A, E.D.F. and C.N.R.S.

D. Talay is responsible for the Meteo-Stoch project within the ACI “Nouvelles Interfaces des Mathématiques” funded by the French ministry of Research.

The team Omega participates to the “Groupe de Recherche GRIP” on stochastic interacting particles. D. Talay serves as a member of the scientific committee of this GdR.

8. Dissemination

8.1. Animation of the scientific community

D. Talay serves as Associated Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

M. Deaconu and D. Talay are permanent reviewers for the *Mathematical Reviews*.

M. Bossy is member of the administration committee of the French Society of Applied Mathematics (SMAI).

M. Deaconu is member of the scientific committee of the MAS group (Probability and Statistics) within SMAI.

D. Talay and H. Niederreiter (University of Singapore) co–chaired the international conference MC2QMC, Juan-les-Pins, June 7-10, 2004. M. Bossy, M. Deaconu and E. Tanré were the organizing committee. MC2QMC is the Sixth International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing, twinned with the Second International Conference on Monte Carlo and Probabilistic Methods for Partial Differential Equations.

D. Talay, Monique Pontier (université Paul Sabatier, Toulouse) and Étienne Pardoux (université de Provence) organized the meeting “Calcul de Malliavin et Finance” at the Université de Provence. This meeting was dedicated to our colleague Axel Grorud’s memory.

D. Talay was a member of the Scientific Committee of the Septièmes Rencontres Mathématiques de Rouen.

M. Deaconu and A. Lejay organized *Journées MAS de la SMAI 2004* held in Nancy in September 2004. D. Talay was a member of the Scientific Committee.

A. Lejay and I. Mortazavi (Université Bordeaux 1) have organized a workshop *Probabilistic models and particles methods for nuclear waste disposal* in Paris in January 2004, within the *Groupe de Recherche MOMAS*.

P. Vallois organized the 13th Évry-Nancy-Strasbourg meeting in Probability, held in Nancy on 13-14 May 2004.

M. Deaconu is member of the “Comité des Projets” and of the “Commission Ingénieur” of INRIA Lorraine.

M. Deaconu is member of the “Conseil du laboratoire” of the Institut Élie Cartan and of the “Commission de spécialistes” of the Mathematics Department of Université Nancy 1.

A. Lejay is member of a “commission de spécialistes” of the Université Louis Pasteur in Strasbourg. D. Talay is member if a “commission de spécialistes” of the Université de Nice Sophia-Antipolis.

A. Lejay is member of the “Commission des moyens informatiques” of INRIA Lorraine.

B. Roynette is the head of the Mathematics Department of Université Nancy 1.

P. Vallois is the head of the Probability and Statistics group of Institut Élie Cartan.

P. Vallois is member of the council of the UFR STMIA, the ‘Conseil du laboratoire” and the "commission de spécialistes" of the Mathematics Department of Université Nancy 1.

D. Talay chairs the selection committee for junior permanent research positions at Inria Sophia-Antipolis.

D. Talay reported on the researches done within the Research Center of the company Électricité de France and involving stochastic models or probabilistic numerical methods. He presented his report to the Scientific Council, chaired by P-L. Lions (Collège de France), of this company.

8.2. Teaching

D. Talay has a part time position of Professor at École Polytechnique. He also teaches probabilistic numerical methods at Université Paris 6 (DEA de Probabilités) and within the FAME Ph.D. program (Switzerland).

M. Bossy gives a course on “Risk management on energetic financial markets” in the master “Ingénierie et Gestion de l’Energie” (École des Mines de Paris) at Sophia-Antipolis.

M. Bossy gives a 30h course on “Stochastic calculus and financial mathematics” in the Master IMAFA (“Informatique et Mathématiques Appliquées à la Finance et à l’Assurance”, Université de Nice Sophia Antipolis), and a 15h course on “Risk management on energetic financial markets” in the Master “Ingénierie et Gestion de l’Energie” (École des Mines de Paris) at Sophia-Antipolis.

M. Bossy gives a 10h course on “Credit Risk Modelling” in the Master in Mathematics at Université de Nice Sophia Antipolis.

E. Tanré gives a course in the Master IMAFA

M. Deaconu gives a 30h course in Mathematical Finance at the IUP Sciences Financières of Université Nancy 2.

P. Vallois gives courses in Mathematical Finance in a Master at Université Nancy 1.. P. Vallois is the head of the DEA in Mathematics at Université Nancy 1.

8.3. Ph.D. theses

Miguel Martinez defended his PhD thesis entitled *Interprétations probabilistes d’opérateurs sous forme divergence et analyse de méthodes numériques probabilistes associées* at Université de Provence in June 04.

Ivan Nourdin defended his Ph.D. thesis entitled *Calcul stochastique généralisé et applications au mouvement brownien fractionnaire. Estimation non-paramétrique de la volatilité et test d’adéquation* at Université Nancy 1 in June 2004.

8.4. Participation to congress, conferences, invitations,...

O. Bardou gave a lecture at Europlace Institute of Finance in March 2004, at the seminar J.A.Dieudonné in Université de Nice Sophia Antipolis in April 2004, at the MC2QMC Conference in Juan-les-Pins in June 2004, and at the Journées MAS-SMAI in Nancy in September 2004.

M. Martinez has given seminars at the MC2QMC Conference in Juan-les-Pins in June 2004, at the Journées MAS-SMAI in Nancy in September 2004.

M. Deaconu has given a plenary lecture in the Workshop *Stochastic Methods in Coagulation and Fragmentation* in Cambridge in December 2003, and a seminar lecture at the Laboratoire de statistique et probabilités, Université de Toulouse, in January 2004.

P. Etoré talked in the *Groupe de travail de l'équipe EDP de l'Université de Provence* in Porquerolles in June 2004, at the *Journées MAS de la SMAI 2004* in Nancy in September 2004 and at the *Journées Évry-Nancy-Strasbourg* in Évry in November 2004.

A. Lejay gave seminars at CERMICS, INRA (Avignon) U.T.C. de Compiègne, Ponticia Universidad Catolica in Santiago (Chile), Department of Mathematical Engineering of the University of Concepcion (Chile), during the Journées ENS XIV in the Université d'Évry, and the *Groupe de travail de l'équipe EDP de l'Université de Provence* in Porquerolles. He also gave a conference at the *Midnight Sun Narvik conference* in Norway in June 2004

A. Lejay spent two weeks in Chile with a grant INRIA/CONYCIT to work with C. Mora, R. Rebolledo and S. Torrès.

E. Tanré spent two weeks in Chile within the INRIA-CONICYT collaboration and gave a seminar in the Pontificia Universidad Catòlica de Chile.

P. Vallois has given lectures at the meeting *Karlsruher Stochastik Tage 2004 6th German Open Conference on Probability and Statistics* in Karlsruhe in March 2004; at the *Local time-space calculus with applications* at Oberwolfach in May 2004; at the *New techniques in Applied Stochastics* in Helsinki in August 2004.

P. Vallois was invited by Professor P. Salminen at Turku in August 2004.

D. Talay gave a course at the Pluri-thematic School on Ergodic Theory (CIRM, Luminy), a lecture in the Conference in N. El Karoui's honor (Institut Henri Poincaré), and a Colloqium lecture at the Center for Applied Mathematics at Cornell University.

8.4.1. Invitations

The seminar *Théorie et applications numériques des processus stochastiques* organized at Sophia-Antipolis by M. Bossy has received the following speakers: B. Arouna (CERMICS-ENPC), R. Temam (Indiana University, Bloomington), N. El Karoui (Ecole Polytechnique), P-L. Lions (Collège de France), S. Hamadene (Universite du Maine), S. Rubenthaler (UNSA), M. Soledad Torres (Universidad de Valparaiso), F. Bernardin (Labo MAPLY, Université Lyon 1), V. Konakov (Institut d'Economie Mathematique de l'Academie des Sciences de Russie, Moscou).

The seminar *Mathématiques financières* organized at Sophia-Antipolis by M. Bossy has received the following speakers: M. Jeanblanc (Université d'Evry), M. Royer (ISFA Lyon).

The seminar *Probabilités* organized at Nancy by S. Tindel has received the following speakers: P. Imkeller (Berlin), L. Zambotti (Milan), P.Y. Louis (Berlin), C. Tudor (Paris VI), P. Calka (Paris V), M. Mellouk (Montpellier), N. Privault (La Rochelle), M. Martinez (Sophia Antipolis), P. Biane (ENS), L. Gallardo (Tours), B. Bergé (Neuchâtel), F. Comets (Paris VII), J. Mémin (Rennes), A. Guillin (Paris-Dauphine), T. Duquesne (Orsay), G. Miermont (ENS), L. Quer (Barcelone), A. Rouault (Versailles), K. Alexander (South-California), A. de Bouard (Orsay).

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