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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Alice

Geometry and Lighting

Lorraine

THEME COG

Activity Report

2005

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1. Team

ALICE is a new research project, spin-off of the former ISA project.

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2. Overall Objectives

2.1. Overall Objectives

2.1.1. History

ALICE is one of the three INRIA projects proposed by former ISA members. This section summarizes the scientific evolution of the research groups within ISA that yielded those three project proposals. More specifically, since VEGAS and ALICE both do research in geometry, this section explains the two different visions of geometry developed by these two project proposals.

One of the principal research orientations of the ISA project was computer vision and augmented reality. A clearly identified group headed by Marie-Odile Berger developed this approach, so it was natural for them to propose the creation of the MAGRITE project.

The other principal research orientation of the ISA project was physically-based light simulation. The main challenges of this domain are both geometrical (visibility complex, surface intersections) and computational (numerical resolution of an integral equation). To deal with the geometrical problems, a 'geometry research group' was created within ISA. The missions of this group were the following three ones:

1. To generate the surfacic geometry of the scene from volumic Constructive Solid Geometry descriptions, design new *intersection algorithms*. More precisely, given the equation of two surfaces, the goal is to obtain a parameterization of the intersection;

2. Find ways of *attaching photometric properties to the geometric objects* in the scene. For the numerical simulation of light (i.e., energy transfers), it is necessary to find parameterizations with a constant Jacobian (i.e., energy-preserving parameterizations);
3. Optimize point-to-point *visibility* requests (they are massively issued by light simulation algorithms).

To make the scope of this research as general as possible, the two main classes of surfacic representations were considered, i.e., algebraic surfaces and meshed models. When using algebraic surfaces, to represent complex objects, piecewise defined surfaces are used. The geometric continuity between the charts is the main challenge of the Geometric Design domain of research. In other words, Geometric Design is concerned with G^k continuity (Geometric Continuity), defined as follows: a surface of class G^k is a surface for which a parameterization of class C^k exists. To construct a G^1-continuous object with a set of parametric polynomial surfaces defined over triangles, it was proved that at least degree 4 is required [22]. We considered the main class of algebraic surfaces used to represent geometry in the Geometric Design community, i.e., rational fractions called NURBS (Non-Uniform Rational B-Splines). However, it quickly appeared to us that the solution of the mathematical problems expressed with NURBS (surface intersection and energy-preserving parameterization) do not have a closed form in general. As a consequence, two complementary approaches were developed in parallel by ISA:

- The first approach considered an exact solution of a simplified version of the problems, and limited the study to polynomial surfaces of degree 2 (i.e., quadrics). The main advantage is that closed forms for both the problem of surface intersection (point 1 of the research program on the previous page) and energy-preserving parameterization (point 2) could be derived. Using Galois's group theory and projective geometry, it was possible to derive a general expression of the intersection of two quadrics, with a provable minimum number of square roots. This elegant theoretical result answered several open questions and was welcomed by the Computational Geometry community. As a matter of fact, the projective geometry background acquired by the group was also successfully applied to start the study of the visibility complex (point 3). The VEGAS project headed by Sylvain Lazard and Sylvain Petitjean continues to develop this approach;
- The second approach kept the initial specification of the problem: the geometry is represented by a set of high-order surfaces or by meshed models. The group already knows that no closed form can be derived, for this reason, we developed approximated solution mechanisms, based on applied mathematics and numerical analysis. Being able to process industrial-scale models (with millions of primitives) was also one of the major preoccupations of the group. Applied to mesh models, the solutions we developed had different applications in texture mapping (point 2 of the research program). More generally, our solutions were welcomed by the Geometry Processing community, a new discipline of Computer Graphics that recently emerged. In addition, we also started to apply our geometry processing tools to the numerical simulation of light. These two aspects - Geometry Processing and the numerical simulation of light - are the directions of research proposed by the ALICE project.

2.1.2. Overall objectives

ALICE is a new project in Computer Graphics. The fundamental aspects of this domain concern the interaction of *light* with the *geometry* of the objects. The lighting problem consists in designing accurate and efficient *numerical simulation* methods for the light transport equation. The geometrical problem consists in developing new solutions to *transform and optimize geometric representations*. Our original approach to both issues is to restate the problems in terms of *numerical optimization*. We try to develop solutions that are *provably correct*, *numerically stable* and *scalable*.

- By provably correct, we mean that some properties/invariants of the initial object need to be preserved by our solutions.

- By numerically stable, we mean that our solutions need to be resistant to the degeneracies often encountered in industrial data sets.
- By scalable, we mean that our solutions need to be applicable to data sets of industrial size.

To reach these goals, our approach consists in transforming the physical or geometric problem into a numerical optimization problem, studying the properties of the objective function and designing efficient minimization algorithms.
The main applications of our results concern Scientific Visualization. We develop cooperations with researchers and people from the industry, who experiment applications of our general solutions to various domains, comprising CAD, industrial design, oil exploration and plasma physics. Our solutions are distributed in both open-source software (Graphite) and industrial software (Gocad).

3. Scientific Foundations

3.1. Introduction

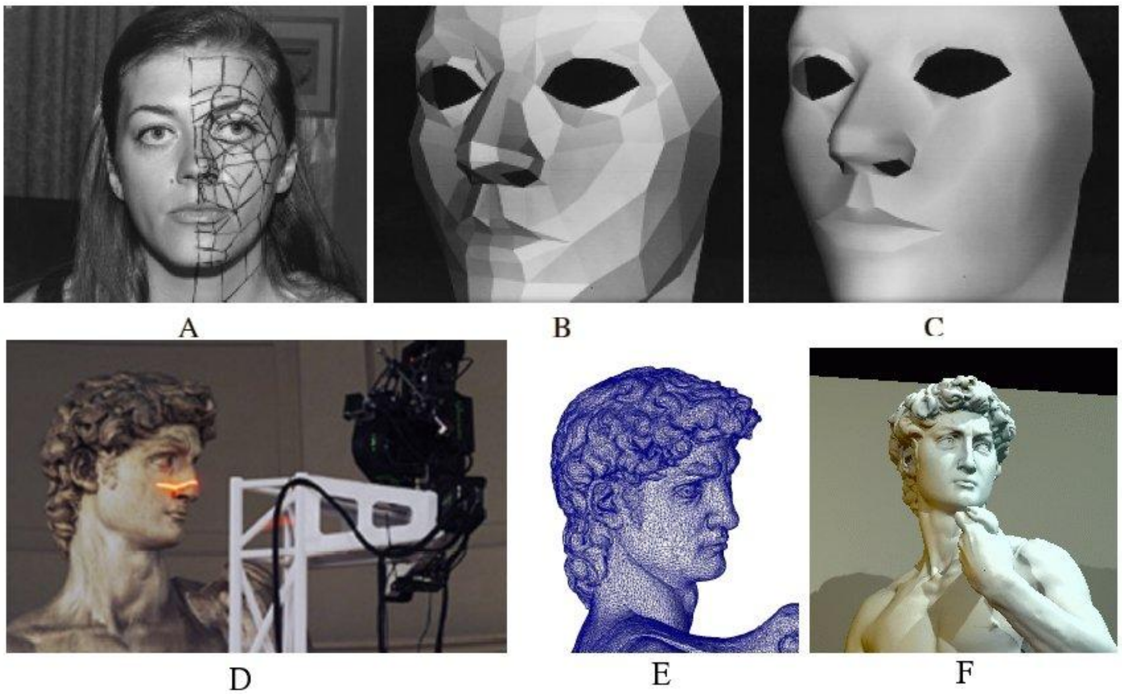

Figure 1. Top: Computer Graphics in the 1970's: Henri Gouraud's shading algorithm. His algorithm is still used today, 30 years after. To obtain the data, Sylvie Gouraud accepted to be manually digitalized (A), this gave this facetted surface (B) which Henri Gouraud did improve with his celebrated smooth shading algorithm (C). Bottom: Computer Graphics in the 2000's: huge advances were made. However, the basic problems still remain unsolved, i.e. finding common representations for data acquisition (D), modeling (E) and image generation (F) (image (D) and 3D model in (E),(F) courtesy of Stanford Digital Michelangelo Project).

Computer Graphics is a quickly evolving domain of research. These last few years, both acquisition techniques (e.g. range laser scanners) and computer graphics hardware (the so-called GPU's, for Graphics

Processing Units) have made considerable advances. However, as shown in Figure 1, despites these advances, fundamental problems still remain open. For instance, a scanned mesh composed of 30 millions of triangles cannot be used directly in real-time visualization or complex numerical simulation. To design efficient solutions for these difficult problems, ALICE studies two fundamental issues in Computer Graphics:

- the representation of the objects, i.e. their geometry and physical properties;
- the interaction between these objects and light.

Historically, these two issues have been studied by independent research communities, in isolation. However, we think that they share a common theoretical basis. For instance, multi-resolution and wavelets were mathematical tools used by both communities [24]. We develop a new approach, that consists in studying the geometry and lighting from the *numerical analysis* point of view. In our approach, Geometry Processing and Light Simulation are systematically restated as a (possibly non-linear and/or constrained) functional optimization problem. Our long-term research goal is to find a formulation that permits a unified treatment of geometry and illumination over this geometry.

3.2. Geometry Processing

Keywords: *Mesh processing, fairing, parameterization, splines, texture mapping*.

Participants: Bruno Lévy, Wan-Chiu Li, Nicolas Ray, Bruno Vallet.

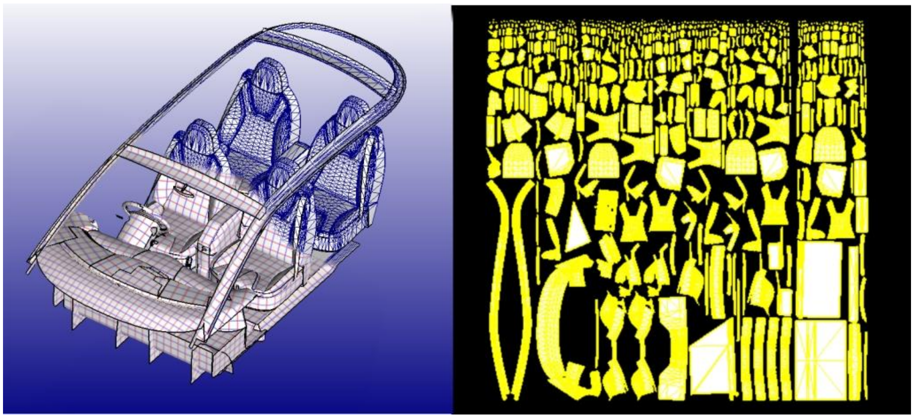

Figure 2. Our Geometry Processing tools applied to a triangulated CAD model of the Avantime car (courtesy of Renault). The constructed 2D parameter space (right) can be used for texturing, real-time visualization and conversion into parametric surfaces.

Geometry Processing recently appeared (in the middle of the 90's) as a promising avenue to solve the geometric modeling problems encountered when manipulating meshes composed of million elements. Since a mesh may be considered to be a *sampling* of a surface - in other words a *signal* - the *digital signal processing* formalism was a natural theoretic background for this discipline (see e.g. [25]). The discipline then studied different aspects of this formalism applied to geometric modeling.

Although many advances have been made in the Geometry Processing area, important problems still remain open. Even if shape acquisition and filtering is much easier than 30 years ago, a scanned mesh composed of 30 millions of triangles cannot be used directly in real-time visualization or complex numerical simulation. For this reason, automatic methods to convert those large meshes into higher level representations are necessary. However, these automatic methods do not exist yet. For instance, the pioneer Henri Gouraud often mentions in his talk that the *data acquisition* problem is still open. Malcolm Sabin, another pioneer of the "Computer Aided Geometric Design" and "Subdivision" approaches, mentioned during several conferences of the domain that constructing the optimum control-mesh of a subdivision surface so as to approximate a given surface is still an open problem. More generally, converting a mesh model into a higher level representation, consisting of a set of equations, is a difficult problem for which no satisfying solution have been proposed. This is one of the long-term goals of international initiatives, such as the AIMShape European network of excellence.

Motivated by gridding application for finite elements modeling for oil and gas exploration, in the frame of the Gocad project, we started studying Geometry Processing in the late 90's [7] and contributed to this area at the early stages of its development. We then developped new algorithms to add interactivity in the method [5]. To improve both the robustness and the flexibility of the method, we then studied a new algorithm to minimize the conformal energy, based on Cauchy-Riemann's equation. As a result, we developped the LSCM method (Least Squares Conformal Maps) in cooperation with Alias Wavefront [8]. We experimented various applications of the method, comprising normal mapping, mesh completion and fairing [6], anisotropic remeshing [1] (in cooperation with P. Alliez and M. Desbrun) and light simulation [9], [16].

3.3. Numerical light simulation

Keywords: *Finite Elements Modeling*, *Monte-Carlo*, *Physically-Based Rendering*, *Radiance*, *Radiosity*.

Participants: Laurent Alonso, Gregory Lecot, Bruno Lévy, Bruno Vallet.

Numerical simulation of light means solving for light intensity in the "Rendering Equation", an integral equation modeling energy transfers (or light *intensity* transfers). The Rendering Equation was first formalized by Kajiya [23], and is given by:

$$\begin{array}{l} I(x,x') = g(x,x')\left[\epsilon(x,x') + \int_S \rho(x,x',x'')I(x',x'')dx''\right] \\ \\ \text{where:} \\ \begin{array}{ll} I(x,x') & \text{denotes the intensity of light passing from point } x' \text{to point } x \\ g(x,x') & \text{is a ``geometric'' term (depends on the distance between } x \text{ and } x', \\ & \quad\quad \text{on the relative direction of their normals,} \\ & \quad\quad \text{and on the visibility between } x \text{ and } x') \\ \epsilon(x,x') & \text{denotes the intensity of emitted light from } x' \text{ to } x \\ \rho(x,x',x'') & \text{denotes the intensity of light scattered} \\ & \text{from the direction of } x'' \text{ to the direction of } x \text{ at point } x' \end{array} \end{array} \tag{1}$$

Computing global illumination (i.e., solving for intensity in Equation 1) in general environments is a challenging task. Global illumination may be considered in terms of computing the interactions between the *lighting signal* and the *geometric signal* (i.e., the scene). These interactions occur at various *scales*. This issue belongs to the same class of problems encountered by Geometry Processing, described in the previous section. As a consequence, the *signal processing* family of approaches is again a well-suited formalism. As such, the *multi-scale* approach is a natural choice, which dramatically improves performances. Environments composed of a large number of primitives, such as highly tessellated models, show a high variability of these scales.

In addition, these methods are challenged with more and more complex materials that need to be taken into account in the simulation. The simple diffuse Lambert law has been replaced with much more complex reflection models. The goal is to create synthetic images that no longer have a synthetic aspect, in particular when human characters are considered.

One of the difficulties is finding efficient ways of evaluating the visibility term. This is typically a Computational Geometry problem, i.e., a matter of finding the right combinatorial data structure (the *visibility complex*), studying its complexity and deriving algorithms to construct it. To deal with this issue, several teams (including VEGAS, ARTIS and REVES) study the visibility complex.

The other terms of the Rendering Equation cannot be solved analytically in general. Many different numerical resolution methods have been used. The main difficulties of the discipline is that each time a new physical effect should be simulated, the numerical resolution methods need to be adapted. In the worst case, it is even necessary to design a new ad-hoc numerical resolution method. For instance, in Monte-Carlo based solvers, several sampling maps are used, one for each effect (a map is used for the diffuse part of lighting, another map is used for caustics ...). As a consequence, the discipline becomes a collection of (sometimes mutually exclusive) techniques, where each of these technique can only simulate a specific lighting effect.

The other difficulty is to satisfy two somewhat antinomic objectives at the same time. On the one hand, we want to simulate complex physical phenomena (subsurface scattering, polarization, interferences, ...), responsible for subtle lighting effects. On the other hand, we want to visualize the result of the simulation in real-time.

We first experimented finite-element methods in parameter space, and developped the *Virtual Mesh* approach [2] and a parallel solution mechanism for the associated hierarchical finite element formulation [3] The initial method was dedicated to scenes composed of quadrics. We combined this method with our Geometry Processing methods to improve the visualization [9]. More recently, we developped a method to directly apply the finite element method on tesselated models [16].

One of our goals is now to design new representations of lighting coupled with the geometric representation. These representations of lighting need to be general enough so as to be easily extended when multiple physical phenomena should be simulated. Moreover, we want to be able to use these representations of lighting in the frame of real-time visualization.

3.4. Guiding principles

Early approaches to Geometry Processing and Light Simulation were driven by a Signal Processing approach. In other words, the solution of the problem is obtained after applying a *filtering scheme* multiple times. This is for instance the case of the mesh smoothing operator defined by Taubin in his pioneering work [25]. Recent approaches still inherit from this background. Even if the general trend moves to Numerical Analysis, much work in Geometry Processing still study the coefficients of the gradient of the objective function *one by one*. This intrinsicly refers to *descent* methods (e.g. Gauss-Seidel), which are not the most efficient, and do not converge in general when applied to meshes larger than a certain size (30000 facets).

In the approach that we develop in the ALICE project, Geometry Processing and Light Simulation are systematically restated as a (possibly non-linear and/or constrained) functional optimization problem. As a consequence, studying the properties of the minimum is easier: the minimizer of a multivariate function can be more easily characterized than the limit of multiple applications of a smoothing operator. This simple remark makes it possible to derive properties (existence and uniqueness of the minimum, injectivity of a parameterization, and independence to the mesh).

Besides helping to characterize the solution, restating the geometric problem as a numerical optimization problem has another benefit. It makes it possible to design efficient numerical optimization methods, instead of the iterative relaxations used in classic methods.

Richard Feynman (Nobel Prize in physics) mentions in his lectures that physical models are a “smoothed” version of reality. The global behavior and interaction of multiple particles is captured by physical entities of a larger scale. According to Feynman, the striking similarities between equations governing various physical phenomena (e.g. Navier-Stokes in fluid dynamics and Maxwell in electromagnetism) is an illusion that comes from the way the phenomena are modeled and represented by “smoothed” larger-scale values (i.e., *fluxes* in the case of fluids and electromagnetism). Note that those larger-scale values do not necessarily directly correspond to a physical intuition, they can reside in a more abstract “computational” space. For instance, representing lighting by the coefficients of a finite element is a first step in this direction. More generally, our approach consists in trying to get rid of the limits imposed by the classic view of the existing solution mechanisms, that come from a physical intuition. Instead of trying to mimic the physical process, we try to restate the problem as an abstract numerical computation problem, on which more sophisticated methods can be applied (a plane flies like a bird, but it does not flap its wings). We try to consider the problem from a computational point of view, and focus on the link between the numerical simulation process and the properties of the solution of the

Rendering Equation. Note also that the numerical computation problems yielded by our approach reside in a high-dimensional space (millions of variables). To ensure that our solutions scale-up to scientific and industrial data from the real world, we develop algorithmic, software and hardware architectures, and distribute these solutions in both open-source software (Graphite) and industrial software (Gocad).

4. Application Domains

4.1. Scientific visualization

Keywords: *Graphic Clusters*, *Large Models*, *Molecular dynamics*, *Numerical mock-ups*, *Oil exploration*, *Plasma*, *Volume Visualization*.

Participants: Fabien Boutantin, Luc Buatois, Laurent Castanié, Xavier Cavin, Christophe Mion, Nicolas Ray, Rodrigo Toledo.

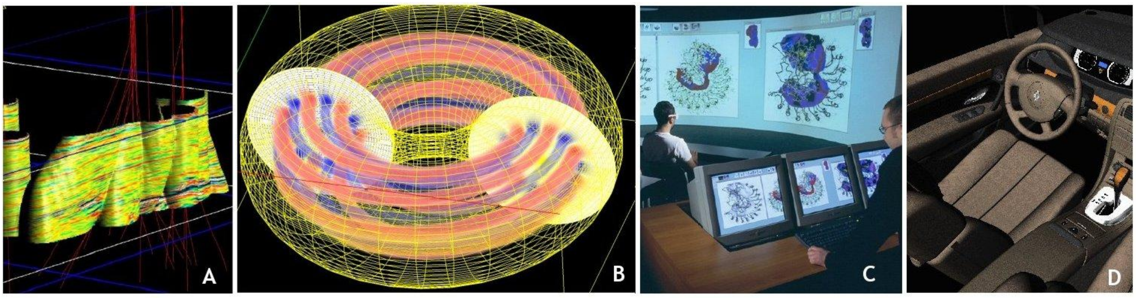

Figure 3. Our applications in oil exploration (A), plasma physics (B), molecular dynamics (C) and design (D)

Besides developping new solutions for Geometry Processing and Numerical Light Simulation, we aim at applying those solutions to real-size scientific and industrial problems. In this context, Scientific Visualization is our main applications domain. With the advances in acquisition techniques, the size of the data sets to be processed increases faster than Moore's law, and represents a scientific and technical challenge. To ensure that our processing and visualization algorithms scale-up, we develop a combination of algorithmic, software and hardware architectures. These developments started initially with our implementation of a parallel solver for the radiosity equation on the multi-processor Origin 3000 [3] and the associated visualization solutions [9]. With the evolution of computer hardware, clusters have appeared as an interesting alternative to multi-processor mainframes. We recently developped a visualization cluster based on this type of architecture [15].

These developments permits our Geometry Processing and Light Simulation solutions to scale-up, and handle real-scale data from other research and industry domains. The following applications are developed within the CRVHP Program (Calcul Réseaux Visualisation Hautes Performances - Computing, Networks, High-Performance Visualization). This program includes more than twenty Research Institutions and Industrial Companies and is supported by the "Contrat de Plan État-Région Lorraine".

4.1.1. Geology

This application domain is lead by the Gocad consortium, created by Prof. Mallet. The consortium involves 48 universities and most of the major Oil and Gas companies. ALICE contributes to Gocad with numerical geometry and visualization algorithms for oil and gas engineering. The currently explored domains are complex and dynamic structural models construction, extremely large sismic volumes exploration, and drilling evaluation and planning. The solutions that we develop are transferred to the industry with Earth Decision Sciences.

4.1.2. Plasma

The computation of turbulent thermal diffusivities in fusion plasmas is of prime importance since the energy confinement time is determined by these transport coefficients. An original approach is developed to study trapped ion instability. A Vlasov code is used to determine the behavior of the instability near the threshold and compare with analytical solutions of the Vlasov equation. Some interesting features which appear in the nonlinear regime are explored thanks to a specialized module of the Graphite library (in cooperation with LPMI-CNRS and CEA).

4.1.3. Molecular dynamics

Protein docking is a fundamental biological process that links two proteins. This link is typically defined by an interaction between two large zones of the protein boundaries. Visualizing such an interface is useful to understand the process thanks to 3D protein structures, to estimate the quality of docking simulation results, and to classify interactions in order to predict docking affinity between classes of interacting zones. Our developments take place in the VMD software (in cooperation with eDAM and the Beckmann Institute at University of Illinois).

4.1.4. Numerical mock-ups

Computed images and immersive visualization systems are used to design and evaluate virtual products in the aircraft and car industry. In this application, the CAD models used are extremely large and the images have to be computed from an accurate physically-based simulation process. Other developments are experimented with the car industry on an application of distant visualization and immersive virtual reality (in cooperation with VSP-Technology).

5. Software

5.1. Graphite

Participants: Fabien Boutantin, Gregory Lecot, Bruno Lévy, Wan-Chiu Li, Nicolas Ray, Rodrigo Toledo, Bruno Vallet, Bin Wang.

Graphite is a research platform for computer graphics, 3D modeling and numerical geometry. It comprises all the main research results of our “Geometry Processing” group. Data structures for cellular complexes, parameterization, multi-resolution analysis and numerical optimization are the main features of the software. Graphite is publically available since October 2003, and is now used by researchers from Geometrica (INRIA Sophia Antipolis), Artis (INRIA Grenoble), LSIIT (Strasbourg), Technion (Israel), Stanford University (United States), Harvard University (United States), University of British Columbia (Canada), MIT (United States). Graphite is one of the common software platforms used in the frame of the European Network of Excellence AIMShape .

5.2. DViz

Participants: Xavier Cavin, Laurent Castanié, Rodrigo Toledo, Christophe Mion.

DViz is a library dedicated to distributed visualization. The development of DViz started in september 2002 and serves as the basis for our works in Scientific Visualization. It allows applications to run on graphics clusters with optimal parallel performance.

5.3. Intersurf

Participants: Xavier Cavin, Nicolas Ray.

Intersurf is a plugin of the VMD (Visual Molecular Dynamics) software. VMD is developped by the Theoretical and Computational Biophysics Group at the Beckmann Institute at University of Illinois. The Intersurf plugin is released with the official version of VMD since the 1.8.3 release. It provides surfaces representing the interaction between two groups of atoms, and colors can be added to represent interaction forces between these groups of atoms.

5.4. Gocad

Participants: Luc Buatois, Laurent Castanié, Bruno Lévy.

Gocad is a 3D modeler dedicated to geosciences. It was developped by a consortium headed by Jean-Laurent Mallet, in the Nancy School of Geology. Gocad is now commercialized by Earth Decision Sciences (formerly T-Surf), a company which was initially a start-up company of the project. Gocad is used by all major oil compagnies (Total-Fina-Elf, ChevronTexaco, Petrobras, ...), and has became a de facto standard in geo-modeling. Laurent Castanié's work (CIFRE Earth Decision Sciences) was successfully integrated in the VolumeExplorer plugin of Gocad.

5.5. Candela

Participants: Laurent Alonso, Gregory Lecot.

Candela is a library dedicated to light simulation. Candela is the property of INRIA-Transfer. Candela-VR is an extension of Candela which makes possible to display the result of a simulation in environments with several CPU's and GPU's. An exclusive user licence was yielded to the start-up company VSP-Technology.

6. New Results

6.1. Geometry Processing

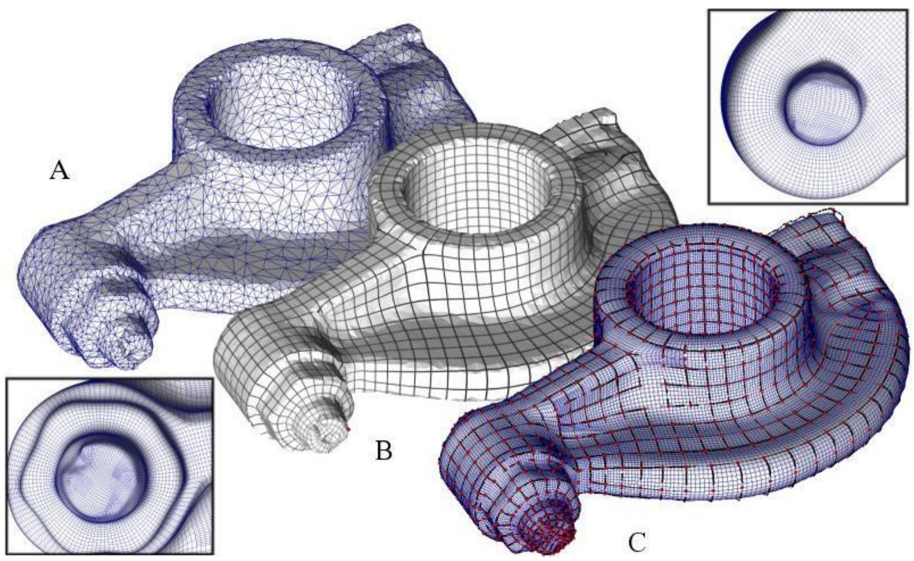

Figure 4. A: triangulated surface (scanned mesh); B: periodic global parameterization; C: automatically constructed Spline surface.

6.1.1. Optimization with dynamic function bases

We have presented in [20] a new framework for numerical optimization with dynamic function bases, and some preliminary results in Geometry Processing (we have also started to study this formulation in the context of light simulation). We consider the problem of the numerical approximation of the solutions of Partial Derivative Equations (or integro-differential equations in the case of global illumination). In its most general form, this class of problem can be expressed by the following equation: $Lf = g$ where L is a linear operator, f is the unknown function, and the function g is the right hand side. The classic Finite Element formulation (Galerkin) projects this equation onto a linear function basis (φ_k). In our setting, the approximation of the solution is also represented in a function basis (φ_k), but all those functions (φ_k) depend on an additional *unknown* vector of parameters p. For instance, we suppose that the function f is a bivariate piecewise linear function, defined on the faces of a Delaunay triangulation. This setting corresponds to exactly one basis function φ_k per vertex k of the triangulation, and the vector of parameters p then corresponds to all the coordinates (x_k, y_k) at all the vertices of the triangulation. The function f is then given by $f(x) = \sum \alpha_i \varphi_i(p, x)$ we will first explore the problem of minimizing the residual $F(\alpha, p) = \|Lf - g\|^2$:

$$F(\alpha, p) = \|Lf - g\|^2 = \alpha^t M \alpha - 2\alpha^t c + < g, g >$$

$$\text{where} \quad m_{i,j} = < L\varphi_i, L\varphi_j > \quad ; \quad c_i = < L\varphi_i, g >$$

The main difficulty comes from the non-linear dependencies introduced by the additional vector of parameters p. The other difficulty is that in the general case, the expression of the energy functional F depends on the value of the parameters p (F is piecewise defined). To compute the fixed points of F, we have designed a general framework, based on Newton's algorithm:

$$\alpha \longleftarrow \alpha_0 \quad ; \quad p \longleftarrow p_0$$

while $\|\nabla F(\alpha, p)\| > \epsilon$

$$\text{solve} \quad \begin{pmatrix} \nabla^2_{\alpha,\alpha} F & \nabla^2_{\alpha,p} F \\ \nabla^2_{p,\alpha} F & \nabla^2_{p,p} F \end{pmatrix} \begin{pmatrix} \delta_\alpha \\ \delta_p \end{pmatrix} = \begin{pmatrix} -\nabla_\alpha F \\ -\nabla_p F \end{pmatrix}$$

$$\alpha \longleftarrow \alpha + \delta_\alpha \quad ; \quad p \longleftarrow p + \delta_p$$

end$//while$

In practice, we will instanciate this general framework into different algorithms. We can now imagine a research program to instanciate our general framework in increasingly complex settings, that we have started studying in cooperation with our partners in AIMShape and ARC Georep:

- 1D: Laplace equation, Poisson equation ;
- 1D + t: Heat equation;
- 2D,$L = Id$: this trivial case corresponds to a function approximation problem. We have started to study the problem and an application for converting bitmap images into vector representations;
- 2D + t: fluid simulation with Navier-stokes;
- 3D, $L = Id$: Mesh-to-Spline conversion, this result is presented below.
- 3D: light simulation; optimizing for the parameters p corresponds to a *numerical approach* to computing the visibility complex;
- 3D + t: various physics simulations.

We have presented in [13] a new method to automatically convert a mesh surface of arbitrary genus into a Spline (corresponding to the $3D, L = Id$ case of our framework). The method is outlined in Figure 4-A,B,C. Figure A shows the initial mesh model; Figure B shows the coordinate system computed by our method; Figure C shows the fitted Spline surface and its control mesh. A quadrilateral chart layout (i.e., the structure of the B-Spline basis functions) and the parameterization emerge simultaneously from a global numerical optimization process. Given the principal directions of curvature on the surface, our method computes two piecewise linear periodic functions aligned with these directions, by minimizing an objective function. The possible applications of our method comprise quad-dominant remeshing, texture mapping and T-spline surface fitting.

6.1.2. Interactive mesh editing

We have proposed in [17] a new topological data structure for representing a set of polygonal curves embedded in a meshed surface. In our representation, the vertices of the curve do not necessarily correspond to the vertices of the surface. The partition of the surface yielded by the intersecting curves is efficiently represented as a "cut-graph". The cut-graph stores combinatorial information of the network of curves. In our approach, the combinatorial form of information is systematically preferred to geometrical information since it improves both robustness and efficiency. Thanks to the topological data structure and algorithms, the cut-graph can be sketched through iterations of designing and erasing curves on the mesh surface in a "nondestructive" way, i.e. without modifying the mesh until the cutting operation is committed. We also demonstrate several prototype curve design tools inspired by 2D vector and bitmap graphics paradigms. We show how to sketch the cut-graph and how these tools can be combined.

6.2. Light Simulation

Figure 5. Left: results of our Discrete Master Element method for applying Finite Element Modeling methods to finely tesselated models; Right: construction of the wavelet basis attached to the object (mesh partition and extrapolation).

We have proposed in [16] a new global light simulation method for diffuse (or moderatly glossy) scenes comprising highly tesselated models with simple topology (e.g., scanned meshes). By using the topological coherence of the surface, we show how to extend a classic Finite Element method called the Master Element: We generalize this method to efficiently handle tessellated models by using mesh parameterization and mesh extrapolation techniques. In addition, we propose a high-order and hierarchical extension of the Master Element method. Our method computes a compact representation of vector irradiance, represented by high-order wavelet bases. For totally diffuse scenes, the so-computed vector irradiance maps can be transformed into light maps. For moderatly glossy scenes, approximated view-dependent lighting can be computed and displayed in real-time by the GPU from the vector irradiance maps. Using our methods, view-dependent solutions for scenes with over one million polygons are computed in minutes and displayed in real time. As with clustering methods, the time complexity of the method is independent on the number of polygons. By efficiently capturing the lighting signal at a suitable scale, the method is made independent of the geometric discretization and solely depends on the lighting complexity. We have demonstrated our method in various settings, with both sharp and soft shadows accurately represented by our hierarchical function basis.

6.3. Scientific Visualization

6.3.1. Out-of core visualization of structured grids

We have presented in [14] and [11] a volume roaming system dedicated to oil and gas exploration. Our system combines probe-based volume rendering with data processing and computing. The daily oil production and the estimation of the world proven-reserves directly affect the barrel price and have a strong impact on the economy. Among others, production and correct estimation are linked to the accuracy of the subsurface model used for predicting oil reservoirs shape and size. Geoscientists build this model from the interpretation of seismic data, i.e. 3D images of the subsurface obtained from geophysical surveys. Our system couples visualization and data processing for the interpretation of seismic data. It is based on volume roaming along with efficient volume paging to manipulate the multi-gigabyte data sets commonly acquired during seismic surveys. Our volume rendering lenses implement high quality pre-integrated volume rendering with accurate lighting. They use a generic multimodal volume rendering system that blends several volumes in the spirit of the “stencil” paradigm used in 2D painting programs. In addition, our system can interactively display non-polygonal isosurfaces painted with an attribute. Beside the visualization algorithms, automatic extraction of local features of the subsurface model also take full advantage of the volume paging.

6.3.2. Visualization clusters

Figure 6. A: our visualization cluster; B: the “power-plant” data base, with more than 13 million polygons, visualized with 15 frames per second on the visualization cluster

Sort-last parallel rendering is an efficient technique to visualize huge datasets on COTS clusters. The dataset is subdivided and distributed across the cluster nodes. For every frame, each node renders a full resolution image of its data using its local GPU, and the images are composited together using a parallel image compositing algorithm. In this paper, we present a performance evaluation of standard sort-last parallel rendering methods and of the different improvements proposed in the literature. This evaluation is based on a detailed analysis of the different hardware and software components. We have presented in [15] a new implementation of sort-last rendering that fully overlaps CPU(s), GPU and network usage all along the algorithm. We present experiments on a 3 years old 32-nodes PC cluster (see Figure 6-A) and on a 1.5 years old 5-nodes PC cluster, both with Gigabit interconnect, showing volume rendering at respectively 13 and 31 frames per second and polygon rendering at respectively 8 and 17 frames per second on a 1024×768 render area, and we have shown that our implementation outperforms or equals many other implementations and specialized visualization clusters. We have also experimented applications to CAD/CAM visualization (Figure 6-B), integrating results from R. Toledo's Ph.D thesis on efficient visualization of higher-order primitives on the GPU.

6.3.3. Molecular docking

Figure 7. Interactive simulation and immersive visualisation applied to molecular docking.

Protein docking is a fundamental biological process that links two proteins. This link is typically defined by an interaction between two large zones of the protein boundaries. Visualizing such an interface is useful to understand the process thanks to 3D protein structures, to estimate the quality of docking simulation results, and to classify interactions in order to predict docking affinity between classes of interacting zones. Since the interface may be defined by a surface that separates the two proteins, it is possible to create a map of interaction that allows comparisons to be performed in 2D. We have presented in [12] a highly efficient algorithm that extracts an interface surface and creates a valid and low-distorted interaction map. Another benefit of our approach is that a pre-computed part of the algorithm enables the surface to be updated in real-time while residues are moved. Applications of this method to biological problems have been published in [18], [19]. Figure 7 shows our tools used in the reality center.

7. Contracts and Grants with Industry

7.1. Start-up company creation

7.1.1. Earth Decision Sciences

The company Earth Decision Sciences (formerly T-Surf) develops and commercializes the modeler Gocad. Gocad is a 3D modeler dedicated to geosciences. This company was initially created as a start-up company of the National School of Geology and members of ISA project. It has now 100 employees in 7 countries (France, United States, Brazil, Dubai, Canada, ...).

7.1.2. VSP-Technology

VSP-Technology was created in December 2001. Its objective is to provide services for a realistic and interactive visualization of complex 3D models. The Candelux and X-Mesh products are based on industrialized research results from the research on global light simulation and on numerical geometry done in ALICE.

7.2. Partnerships

7.2.1. Microsoft Research Cambridge

Our proposal *Geometric Intelligence* on Geometry Processing was selected by Microsoft Research Cambridge (in the frame of the Microsoft call for proposals *tools for advancing science*).

7.2.2. Gocad Consortium

The Gocad software is developed in the context of a consortium that encloses some forty universities and thirty oil and gas companies around the world. This software is dedicated to modeling and visualizing the underground.

7.2.3. Eden Games

Eden Games (group ATARI-Infogrammes) has chosen X-Mesh (from our VSP-Technology start-up) for their next “Alone in the Dark” game. *"X-Mesh showed the best performance, guaranteeing the most efficient unwrapping and proved to be really able to automate the whole process"* (David Nadal, co-founder and Game director of Eden Games).

7.2.4. NVidia

We have signed a Non Disclosure Agreement with NVidia that allows us to have access to their new graphics board before they are commercially available. We have participated in the certification of their drivers for the Quadro 4500 and G-Sync hardware. NVidia has chosen our laboratory to make the public announcement of this hardware in Europe (December, 2). We have ported the NVidia Software Developper Kit to Linux and started to cooperate with them on this topic (with I. Castano and K. Ashido from NVidia).

7.2.5. 3DLabs

We have signed a Non Disclosure Agreement with 3DLabs, and we got a Wildcat Realizm graphics board before it is commercially available. We have developped on this card a new approach for vector texture mapping. We met Randy Rost (3DLabs) at Siggraph about these developments.

8. Other Grants and Activities

8.1. Regional initiatives

8.1.1. The “High Performance Computing Network and Visualization” program (2000-2006)

Participant: Xavier Cavin.

The CRVHP program, led by Xavier Cavin and Jean-Claude Paul is supported by the “Contrat de Plan État-Région Lorraine”. It includes more than twenty Academic Institutions and Industrial Companies. Ressources allocated to the program include two SGI Origin3000, a “Reality Center” immersive environment, a computing cluster and three graphics clusters, all interconnected by high bandwidth networks.

8.2. National initiatives

8.2.1. ACI Data Masses (ministry grant)

Participants: Bruno Lévy, Wan Chiu Li, Nicolas Ray.

The Data Masses ACI (2004-2006, 36 monthes) studies new ways of handling large geometrical databases. ALICE's contribution concerns the parameterization of large triangulated surfaces and the segmentation of geometric objects based on an approximation of the curvature tensor.

8.2.2. ARC Georep (INRIA new investigation grant)

Participants: Xavier Cavin, Bruno Lévy, Wan Chiu Li, Nicolas Ray, Bruno Vallet.

We coordinate the ARC Georep (2005-2006, 24 monthes), that aims at designing new solutions to convert a raw representation of a 3D object into a higher-level representation. The main objective is to show the feasability of this approach by applying it to a real Computer Graphics problem, i.e. to demonstrate the full pipeline: acquisition → geometry processing → application. To reach this goal, this ARC connects participants having skills in various disciplines (3D acquisition: LSIIT, 3D reconstruction: MOVI, Geometry Processing: ALICE, UBC, Numerical Analysis: GRAAL and Computer Graphics: ARTIS, ALICE). We recently submitted a proposal for extending this ARC to new participants in MPII (Hans-Peter Seidel and Alexander Belyaev).

8.2.3. ODL Speech-To-Graphite (INRIA software development grant)

Participants: Fabien Boutantin, Bruno Lévy.

This project (2004-2005, 24 monthes) in cooperation with the Parole project (INRIA Lorraine) and the Datha association develops new tools based on our Graphite software to help deaf persons learn how to read on lips. The software animates a realistic 3D model of a face, driven by Parole's speech recognition techniques. The software was mentioned in an article in the French journal *Sciences et Vie Junior*.

8.2.4. Cooperation with project CALVI (Inria Lorraine)

Participants: Bruno Lévy, Xavier Cavin.

As a followup to our previous ARC Plasma, we work in cooperation with the Calvi project in order to improve visualization methods used by Calvi in their scientific computing applications (B. Lévy is an external collaborator of project Calvi). One of our images showing the results of a plasma simulation developped by Calvi will be used in the French journal *La Recherche*.

8.2.5. Cooperation with EDAM (biochemistry, Nancy)

Participants: Xavier Cavin, Nicolas Ray.

As a followup to our previous ARC Docking, we work in cooperation with the eDAM team. We developped the Intersurf plugin (modelisation of surfaces of interaction between two groups of atoms) for the VMD software [12]. We currently develop the VSM platform (Virtual Screening Manager) [21] to detect matching pairs (from a molecular docking point of view) within large molecular data bases. X. Cavin co-advises with B. Maigret (DR CNRS - eDAM) two Ph.D. theses (M. Chavent and A. Beautrait).

8.2.6. Cooperation with Gocad (Nancy school of geology)

Participants: Laurent Castanié, Bruno Lévy, Luc Buatois.

As a followup to previous cooperation projects (including our ACI Geogrid, coordinated by J.-C. Paul), we work in cooperation with the Gocad group. The Ph.D. theses of L. Castanié and L. Buatois are co-advised by J.-L. Mallet (Prof. Nancy school of geology) and B. Lévy. The recent results of L. Castanié [14], [11] were integrated in the Volume Explorer plugin of the Gocad software, tested by different industrial partners (including ARAMCO).

8.2.7. Associations

- L. Alonso is secretary of the national AGOS association of INRIA.

8.3. European initiatives

8.3.1. AIM at Shape European project

Participants: Bruno Levy, Ben Wan Chiu Li, Nicolas Ray.

The AIMShape European project intends to design geometrical modeling techniques improving the management of semantic information. The 3D modeling and computer graphics research domains require more and more expertise in various areas (differential geometry, numerical algorithms, combinatorial data structure, computer graphics hardware, ...). Achieving significant advances requires to master all these fundamental domains, which requires at least 10 men years for each aspect. In other words, reinventing the wheel can be a dramatic waste of time. This Network of Excellence (NoE) aims at sharing the expertise of European research groups in this area. To better share the knowledge and know-how, we proposed to develop within the network the notion of DSW (Digital Shape Workbench), i.e. a set of common integrated research platforms (CGAL: computational geometry library, Graphite: numerical geometry workbench, Synapse: numerical algorithms). We expect significant new fundamental results as the outcome of this strategy.

8.4. International initiatives

8.4.1. Conferences, meetings and tutorial organization

- B. Lévy organized the workshop of the ARC GEOREP, with participants from University of British Columbia and MPII Saarebruck (Nancy, 25 November)

8.5. Visiting scientists

- March, 25: CEA - LIST (Lab. for Integration of Systems and Technologies) team
- May, 26: Wenping Wang (Hong-Kong University)
- June, 6-12: Alla Sheffer (University of British Columbia, Canada)
- July, 20-21: Ilja Friedel (Caltech, USA),
- October, 10-13: Jiaxin Wang (Tsinghua University)
- October, 24: Pascal Montrocher, Pierre-Etienne Guyaut (Renault, Comp. Sciences innovation dept.)
- November, 21: Olivier Melisson, Alain Rieger (Eikon)
- November, 22: Fournier pharmaceutical laboratory (Paris)
- December, 2-7, Brian Barsky (U.C. Berkeley, USA, on sabbatical in LIFL - Lille)
- December, 19, Amaury de Baynast (AMD France)

9. Dissemination

9.1. Teaching

- G. Lecot teaches “Computer Systems Architecture" (IUT Verdun) ;
- B. Lévy teaches “Numerical algorithms for computer vision and computer graphics” with M.-O. Berger (Computer Science Master, Nancy);
- N. Ray teaches “Computer Systems Architecture”, (IUT Nancy); “Software Design” (IUT Nancy) and “Networking” (IUT Verdun).

9.2. Animation of the scientific community

9.2.1. Conference programs committees

- B. Lévy was a member of the program committee of SMI'05, SPM'05, SGP'05

9.2.2. INRIA committees

- B. Lévy was a member of the "concours CR" committee in Nancy.
- X. Cavin is a member of the "comité des utilisateurs des moyens informatiques" (COMIN)

9.2.3. Other committees

- B. Lévy is a member of the "commission de spécialistes" of the Computer Sciences departement (section 27) in Strasbourg.

9.3. Participation to conferences and workshops, invitations, scientific visits

- Members of the team attended SIGGRAPH 05, Visualization 05, SMI 05, SPM 05, GRAPHITE 05;
- B. Lévy gave a talk at the Inria-Industry meeting (February, 1)
- B. Lévy visited A. Herbert and S. Emmot in Microsoft Research (Cambridge, February, 23-25);
- X. Cavin visited Nicéphore Cité (Chalon-sur-Saône, March, 15);
- B. Lévy visited T. Xin in Microsoft Research China (Beijing, March, 21-25)
- B. Lévy visited J.-H. Yong, S.-H. Hu and J.-C. Paul in Tsinghua University, gave two seminars in Tsinghua University and LIAMA - Academy of Sciences (Beijing, March, 21-25)
- L. Alonso visited J.-H. Yong, S.-H. Hu and J.-C. Paul in Tsinghua University (Beijing, April, 1-10)
- B. Lévy was one of the three Inria researchers invited at the signature of the Memorandum of Understanding between Inria and Microsoft (Paris, April, 26);
- B. Lévy gave an invited talk at the Intech seminar (Sophia, April, 28), visited G. Drettakis, visited P. Alliez, gave a talk at the Geometrica seminar;
- Members of the team attended the Gocad Meeting (Nancy, June, 6-9) and gave three technical presentations;
- B. Lévy gave a talk at the "journées de l'IECN" (Nancy, June, 21);
- B. Lévy attended the Inria-Microsoft workshop (Cambridge, September, 1);
- X. Cavin visited N. Moitessier in Mc Gill University (Montreal, September 1-12);
- B. Lévy gave an invited talk at the IK conference (Israel, November, 7-9);
- X. Cavin and B. Lévy participated to the meeting "Simulation, Visualization, Interaction and Mixed Reality" (Montbonneau, December, 14)

9.4. Software dissemination and patents

- Article about our ODL Speech-2-Graphite in “Science et vie Junior”, October 2005 (joint project with project Parole and association Datha);
- Our Least Squares Conformal Maps parameterization algorithm and his implementation in Graphite have been incorporated in the Blender open-source 3D modeler (see also [20]), with more than 10000 users. LSCM is displayed as one of the important features of the new version;
- Eden Games (group ATARI-Infogrammes) has chosen X-Mesh (from our VSP-Technology start-up) for their next “Alone in the Dark” game, see Gammasutra Column;
- We developped a parameterization package in cooperation with Geometrica and it will be submitted to the advisory board of CGAL;
- An implementation of our texture packing algorithm [8] developped by Microsoft Research China will be integrated in the next release of Microsoft Direct X;
- Our algorithm to reconstruct a triangulation from a set of angles, described in [10] and its applications to texture mapping were patented in cooperation with F. Cuny, VSP-Technology (French patent 04/11154 20/10/2005, co-property INRIA/VSP, extended to international by the INRIA).

9.5. Scientific and Technological Demonstrations

- Members of the team animated the INRIA booth at Siggraph, and did demonstrations of our software Graphite (July, 31-August 4);
- “Fête de la science” (science fair), Nancy, demos and presentations (October, 14-15);
- P. Montrocher and P.E. Guyot visit us (Renault, Computer Science Innovation Dept.), demos in the Reality Center (October, 24);
- “Conseil Général de Meurthe-et-Moselle” visits us, demos in the Reality Center (November, 4);
- “Contrat de Plan État-Région” scientific committee: demos in the Reality Center (December, 1);
- Organization with NVidia of the European announce for their newest NVIDIA Quadro 4500 and G-Sync hardware in the Reality Center, announcements in local and specialized press (December, 2);
- “Journée de la création des entreprises de technologie” (day of the startup companies), demos and presentations (December, 8);
- “Bioinformatique” scientific day, demos in the Reality Center (December, 16);

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