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Team ASPI

Applications statistiques des systèmes de particules en interaction

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2. Overall Objectives

2.1. Overall Objectives

Keywords: *hidden Markov model (HMM), localisation, navigation and tracking, particle filtering, rare event simulation, risk evaluation.*

The scientific objectives of ASPI are the design, analysis and implementation of interacting Monte Carlo methods, also known as particle methods, with focus on

- statistical inference in hidden Markov models, e.g. state or parameter estimation, including particle filtering,
- risk evaluation, including simulation of rare events.

The whole problematic is multidisciplinary, not only because of the many scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have already contributed to establish the foundations of the field : target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods, etc. Intuitively speaking, interacting Monte Carlo methods are sequential simulation methods, in which particles

- *explore* the state space by mimicking the evolution of an underlying random process,
- *learn* the environment by evaluating a fitness function,
- and *interact* so that only the most successful particles (in view of the value of the fitness function) are allowed to survive and to get offsprings at the next generation.

The effect of this mutation / selection mechanism is to automatically concentrate particles (i.e. the available computing power) in regions of interest of the state space. In the special case of particle filtering, which has numerous applications under the generic heading of positioning, navigation and tracking, in target tracking, computer vision, mobile robotics, ubiquitous computing and ambient intelligence, sensor networks, etc. each particle represents a possible hidden state, and is multiplied or terminated at the next generation on the basis of its consistency with the current observation, as quantified by the likelihood function. These genetic–type algorithms are particularly adapted to situations which combine a prior model of the mobile displacement, sensor-based measurements, and a base of reference measurements, for example in the form of a digital map (digital elevation map, attenuation map, etc.). In the most general case, particle methods provide approximations of probability distributions associated with a Feynman–Kac flow, by means of the weighted empirical probability distribution associated with an interacting particle system, with applications that go far beyond filtering, in simulation of rare events, simulation of conditioned or constrained random variables, molecular simulation, etc.

ASPI essentially carries methodological research activities, rather than activities oriented towards a single application area, with the objective of obtaining generic results with high potential for applications, and of bringing these results (and other results found in the literature) until implementation on a few appropriate examples, through collaboration with industrial partners.

The main applications currently considered are geolocalisation and tracking of mobile terminals, calibration of models for electricity price, and risk assessment for complex hybrid systems such as those used in air traffic management.

3. Scientific Foundations

3.1. Monte Carlo methods

Monte Carlo methods are numerical methods that are widely used in situations where (i) a stochastic (usually Markovian) model is given for some underlying process, and (ii) some quantity of interest should be evaluated, that can be expressed in terms of the expected value of a functional of the process trajectory, or the probability that a given event has occurred. Numerous examples can be found, e.g. in financial engineering (pricing of options and derivative securities) [37], in performance evaluation of communication networks (probability of buffer overflow), in statistics of hidden Markov models (state estimation, evaluation of contrast and score functions), etc. Very often in practice, no analytical expression is available for the quantity of interest, but it is possible to simulate trajectories of the underlying process. The idea behind Monte Carlo methods is to generate independent trajectories of the underlying process, or of an alternate instrumental process, and to build an approximation (estimator) of the quantity of interest in terms of the weighted empirical probability distribution associated with the resulting independent sample. By the law of large numbers, the above estimator converges as the size N of the sample goes to infinity, with rate $1/\sqrt{N}$ and the asymptotic variance can be estimated using an appropriate central limit theorem. To reduce the asymptotic variance of the estimator, many variance reduction techniques have been proposed. However, running independent Monte Carlo simulations can lead to very poor results, because trajectories are generated *blindly*, and only afterwards are the corresponding weights evaluated, which can happen to be negligible in which case the corresponding trajectory is not going to contribute to the estimator, i.e. computing power has been wasted.

A recent and major breakthrough, a brief mathematical presentation of which is given in 3.2, has been the introduction of interacting Monte Carlo methods, also known as sequential Monte Carlo (SMC) methods, in which a whole (possibly weighted) sample, called *system of particles*, is propagated in time, where the particles

- *explore* the state space under the effect of a *mutation* mechanism which mimics the evolution of the underlying process,

- and are *replicated* or *terminated*, under the effect of a *selection* mechanism which automatically concentrates the particles, i.e. the available computing power, into regions of interest of the state space.

In full generality, the underlying process is a Markov chain, whose state space can be finite, continuous (Euclidean), hybrid (continuous / discrete), graphical, constrained, time varying, pathwise, etc., the only condition being that it can easily be *simulated*. The very important case of a sampled continuous–time Markov process, e.g. the solution of a stochastic differential equation driven by a Wiener process or a more general Lévy process, is also covered.

In the special case of particle filtering, originally developed within the tracking community, the algorithms yield a numerical approximation of the optimal filter, i.e. of the conditional probability distribution of the hidden state given the past observations, as a (possibly weighted) empirical probability distribution of the system of particles. In its simplest version, introduced in several different scientific communities under the name of *interacting particle filter* [30], *bootstrap filter* [39], *Monte Carlo filter* [46] or *condensation* (conditional density propagation) algorithm [43], and which historically has been the first algorithm to include a redistribution step, the selection mechanism is governed by the likelihood function : at each time step, a particle is more likely to survive and to replicate at the next generation if it is consistent with the current observation. The algorithms also provide as a by–product a numerical approximation of the likelihood function, and of many other contrast functions for parameter estimation in hidden Markov models, such as the prediction error or the conditional least–squares criterion.

Particle methods are currently being used in many scientific and engineering areas : positioning, navigation, and tracking [40], visual tracking [43], mobile robotics [36], ubiquitous computing and ambient intelligence [41], sensor networks [42], risk evaluation and simulation of rare events [38], genetics, molecular dynamics, etc. Other examples of the many applications of particle filtering can be found in the contributed volume [22] and in the special issue of *IEEE Transactions on Signal Processing* devoted to *Monte Carlo Methods for Statistical Signal Processing* in February 2002, which contains in particular the tutorial paper [23], and in the textbook [56] devoted to applications in target tracking. Applications of sequential Monte Carlo methods to other areas, beyond signal and image processing, e.g. to genetics, and molecular dynamics, can be found in [53].

Particle methods are very easy to implement, since it is sufficient in principle to simulate independent trajectories of the underlying process. The whole problematic is multidisciplinary, not only because of the already mentioned diversity of the scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have contributed to establish the foundations of the field : target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods.

3.2. General framework : Particle approximations of Feynman–Kac flows

The following abstract point of view, developed and extensively studied by Pierre Del Moral [28], [25], has proved to be extremely fruitful in providing a very general framework to the design and analysis of numerical approximation schemes, based on systems of branching and / or interacting particles, for nonlinear dynamical systems with values in the space of probability distributions, associated with Feynman–Kac flows of the form

$$\langle \mu_n, f \rangle = \frac{\langle \gamma_n, f \rangle}{\langle \gamma_n, 1 \rangle} \qquad \text{where} \qquad \langle \gamma_n, f \rangle = \mathbb{E}[f(X_n) \prod_{k=0}^{n} g_k(X_k)] \ ,$$

where X_n denotes a Markov chain with (possibly) time dependent state spaces E_n and with transition kernels Q_n, and where the nonnegative potential functions g_n play the role of selection functions. Feynman–Kac flows (FK) naturally arise whenever importance sampling is used, as seen from (IS) above : this applies for instance to simulation of rare events, to filtering, i.e. to state estimation in hidden Markov models (HMM), etc. Clearly, the unnormalized linear flow satisfies the dynamical system

$$\langle \gamma_n, f \rangle = \langle \gamma_{n-1}, Q_n(g_n\, f) \rangle = \langle \gamma_{n-1}, R_n\, f \rangle \quad ,$$

with the nonnegative kernel $R_n(x, dx') = Q_n(x, dx')\, g_n(x')$, and the associated normalized nonlinear flow of probability distributions satisfies the dynamical system

$$\langle \mu_n, f \rangle = \frac{\langle \mu_{n-1}, Q_n(g_n\, f) \rangle}{\langle \mu_{n-1}, Q_n\, g_n \rangle} = \langle \overline{R}_n(\mu_{n-1}), f \rangle \qquad \text{where} \qquad \overline{R}_n(\mu) = \frac{\mu\, R_n}{\langle \mu\, R_n, 1 \rangle} \quad ,$$

which can be decomposed in the following two steps

$$\mu_{n-1} \longmapsto \eta_n = \mu_{n-1}\, Q_n \longmapsto \mu_n = g_n \cdot \eta_n \quad .$$

Conversely, the normalizing constant $\langle \gamma_n, 1 \rangle$, hence the unnormalized (linear) flow as well, can be expressed in terms of the normalized (nonlinear) flow : indeed $\langle \gamma_n, 1 \rangle = \langle \eta_0, g_0 \rangle \cdots \langle \eta_n, g_n \rangle$. To solve these equations *numerically*, and in view of the basic assumption that it is easy to *simulate* r.v.'s according to the probability distributions $Q_n(x, dx')$, i.e. to mimic the evolution of the Markov chain, the original idea behind particle methods consists of looking for an approximation of the probability distribution μ_n in the form of a (possibly weighted) empirical probability distribution associated with a system of particles :

$$\mu_n \approx \mu_n^N = \sum_{i=1}^N w_n^i\; \delta_{\xi_n^i} \qquad \text{with} \qquad \sum_{i=1}^N w_n^i = 1 \quad .$$

The approximation is completely characterized by the set $\Sigma_n = (\xi_n^i, w_n^i\,,\, i = 1, \cdots, N)$ of particle positions and weights, and the algorithm is completely described by the mechanism which builds Σ_k from Σ_{k-1}. In practice, in the simplest version of the algorithm, known as the *bootstrap* algorithm, particles

- are selected according to their respective weights (selection step),
- move according to the Markov kernel Q_k (mutation step),
- are weighted by evaluating the fitness function g_k (weighting step).

The algorithm yields a numerical approximation of the probability distribution μ_n as the weighted empirical probability distribution μ_n^N associated with a system of particles, and many asymptotic results have been proved as the number N of particles (sample size) goes to infinity, using techniques coming from applied probability (interacting particle systems, empirical processes [59]), see e.g. the survey article [28] or the recent textbook [25], and references therein : convergence in $\mathbb{L}^p$, convergence as empirical processes indexed by classes of functions, uniform convergence in time (see also [9], [50]), central limit theorem (see also [48]), propagation of chaos, large deviations principle, moderate deviations principle (see [31]), etc. Beyond the simplest *bootstrap* version of the algorithm, many algorithmic variations have been proposed [33], and are commonly used in practice :

- in the redistribution step, sampling with replacement could be replaced with other redistribution schemes so as to reduce the variance (this issue has also been addressed in genetic algorithms),
- to reduce the variance and to save computational effort, it is often a good idea not to redistribute the particles at each time step, but only when the weights $(w_k^i\,,\, i = 1, \cdots, N)$ are too much uneven.

Most of the results proved in the literature assume that particles are redistributed (i) at each time step, and (ii) using sampling with replacement. Studying systematically the impact of these algorithmic variations on the convergence results is still to be done. Even with interacting Monte Carlo methods, it could happen that some particle ξ_k^i generated in one time step has a negligible weight $g_k(\xi_k^i)$: if this happens for too many particles in the sample $(\xi_k^i\ , \ i = 1, \cdots, N)$, then computer power has been wasted, and it has been suggested to use importance sampling again in the mutation step, i.e. to let particles explore the state space under the action of an alternate wrong mutation kernel, and to weight the particles according to their likelihood for the true model, so as to compensate for the wrong modeling. More specifically, using an arbitrary importance decomposition

$$R_k(x, dx') = Q_k(x, dx')\, g_k(x') = W_k(x,x')\, P_k(x, dx') \ ,$$

results in the following general algorithm, known as the *sampling with importance resampling* (SIR) algorithm, in which particles

- are selected according to their respective weights (selection step),
- move according to the importance Markov kernel P_k (mutation step),
- are weighted by evaluating the importance weight function W_k on the resulting transition (weighting step).

3.3. Statistics of HMM

Keywords: *asymptotic statistics*, *exponential forgetting*, *exponential stability*, *hidden Markov model (HMM)*, *local asymptotic normality (LAN)*.

Hidden Markov models (HMM) form a special case of partially observed stochastic dynamical systems, in which the state of a Markov process (in discrete or continuous time, with finite or continuous state space) should be estimated from noisy observations. The conditional probability distribution of the hidden state given past observations is a well–known example of a normalized (nonlinear) Feynman–Kac flow, see 3.2. These models are very flexible, because of the introduction of latent variables (non observed) which allows to model complex time dependent structures, to take constraints into account, etc. In addition, the underlying Markovian structure makes it possible to use numerical algorithms (particle filtering, Markov chain Monte Carlo methods (MCMC), etc.) which are computationally intensive but whose complexity is rather small. Hidden Markov models are widely used in various applied areas, such as speech recognition, alignment of biological sequences, tracking in complex environment, modeling and control of networks, digital communications, etc.

Beyond the recursive estimation of an hidden state from noisy observations, the problem arises of statistical inference of HMM with general state space, including estimation of model parameters, early monitoring and diagnosis of small changes in model parameters, etc.

Large time asymptotics A fruitful approach is the asymptotic study, when the observation time increases to infinity, of an extended Markov chain, whose state includes (i) the hidden state, (ii) the observation, (iii) the prediction filter (i.e. the conditional probability distribution of the hidden state given observations at all previous time instants), and possibly (iv) the derivative of the prediction filter with respect to the parameter. Indeed, it is easy to express the log–likelihood function, the conditional least–squares criterion, and many other clasical contrast processes, as well as their derivatives with respect to the parameter, as additive functionals of the extended Markov chain.

The following general approach has been proposed :

- first, prove an exponential stability property (i.e. an exponential forgetting property of the initial condition) of the prediction filter and its derivative, for a misspecified model,

- from this, deduce a geometric ergodicity property and the existence of a unique invariant probability distribution for the extended Markov chain, hence a law of large numbers and a central limit theorem for a large class of contrast processes and their derivatives, and a local asymptotic normality property,
- finally, obtain the consistency (i.e. the convergence to the set of minima of the associated contrast function), and the asymptotic normality of a large class of minimum contrast estimators.

This programme has been completed in the case of a finite state space [7], and has been generalized in [32] under a uniform minoration assumption for the Markov transition kernel, which typically does only hold when the state space is compact. Clearly, the whole approach relies on the existence of exponential stability property of the prediction filter, and the main challenge currently is to get rid of this uniform minoration assumption for the Markov transition kernel [26], [9], so as to be able to consider more interesting situations, where the state space is noncompact.

Small noise asymptotics Another asymptotic approach can also be used, where it is rather easy to obtain interesting explicit results, in terms close to the language of nonlinear deterministic control theory [47]. Taking the simple example where the hidden state is the solution of an ordinary differential equation, or a nonlinear state model, and where the observations are subject to additive Gaussian white noise, this approach consists in assuming that covariances matrices of the state noise and of the observation noise go simultaneously to zero. If it is reasonable in many applications to consider that noise covariances are small, this asymptotic approach is less natural than the large time asymptotics, where it is enough (provided a suitable ergodicity assumption holds) to accumulate observations and to see the expected limit laws (law of large numbers, central limit theorem, etc.). In opposition, the expressions obtained in the limit (Kullback–Leibler divergence, Fisher information matrix, asymptotic covariance matrix, etc.) take here a much more explicit form than in the large time asymptotics.

The following results have been obtained using this approach :

- the consistency of the maximum likelihood estimator (i.e. the convergence to the set M of global minima of the Kullback–Leibler divergence), has been obtained using large deviations techniques, with an analytical approach [44],
- if the abovementioned set M does not reduce to the true parameter value, i.e. if the model is not identifiable, it is still possible to describe precisely the asymptotic behavior of the estimators [45] : in the simple case where the state equation is a noise–free ordinary differential equation and using a Bayesian framework, it has been shown that (i) if the rank r of the Fisher information matrix $\mathcal{I}$ is constant in a neighborhood of the set M, then this set is a differentiable submanifold of codimension r, (ii) the posterior probability distribution of the parameter converges to a random probability distribution in the limit, supported by the manifold M, absolutely continuous w.r.t. the Lebesgue measure on M, with an explicit expression for the density, and (iii) the posterior probability distribution of the suitably normalized difference between the parameter and its projection on the manifold M, converges to a mixture of Gaussian probability distributions on the normal spaces to the manifold M, which generalized the usual asymptotic normality property,
- it has been shown in [51] that (i) the parameter dependent probability distributions of the observations are locally asymptotically normal (LAN) [49], from which the asymptotic normality of the maximum likelihood estimator follows, with an explicit expression for the asymptotic covariance matrix, i.e. for the Fisher information matrix $\mathcal{I}$, in terms of the Kalman filter associated with the linear tangent linear Gaussian model, and (ii) the score function (i.e. the derivative of the log–likelihood function w.r.t. the parameter), evaluated at the true value of the parameter and suitably normalized, converges to a Gaussian r.v. with zero mean and covariance matrix $\mathcal{I}$.

4. Application Domains

4.1. Localisation, navigation and tracking

Keywords: *localisation*, *navigation*, *tracking*.

See 5.1.

Among the many application domains of particle methods, or interacting Monte Carlo methods, ASPI has decided to focus on applications in localisation (or positioning), navigation and tracking [40], which already covers a very broad spectrum of application domains. The objective here is to estimate the position (and also velocity, attitude, etc.) of a mobile object, from the combination of different sources of information, including

- a prior dynamical model of typical evolutions of the mobile,
- measurements provided by sensors,
- and possibly a digital map providing some useful feature (altitude, gravity, power attenuation, etc.) at each possible position,

see 5.1. This Bayesian dynamical estimation problem is also called filtering, and its numerical implementation using particle methods, known as particle filtering, has found applications in target tracking, integrated navigation, points and / or objects tracking in video sequences, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc. Particle filtering was definitely invented by the target tracking community [39], [56], which has already contributed to many of the most interesting algorithmic improvements and is still very active. Beyond target tracking, ASPI is also considering various possible applications of particle filtering in positioning, navigation and tracking, see 7.3.

5. Software

5.1. Demos

Participant: Fabien Campillo [corresponding person].

See 4.1.

To illustrate that particle filtering algorithms are efficient, easy to implement, and extremely visual and intuitive by nature, several demos have been programmed by Fabien Campillo, with the corresponding MATLAB scripts available on the site http://www.irisa.fr/aspi/campillo/site-pf/. This material has proved very useful in training sessions and seminars that have been organized in response to demand from industrial partners (SAGEM, CNES and EDF), and this effort will be continued. At the moment, the following four demos are available :

- Navigation of an aircraft using altimeter measurements and elevation map of the terrain : a noisy measurement of the terrain height below the aircraft is obtained as the difference between (i) the aircraft altitude above the sea level (provided by a pression sensor) and (ii) the aircraft altitude above the terrain (provided by an altimetric radar), and is compared with the terrain height in any possible point (read on the elevation map). In this demo, a cloud (swarm) of particles explores multiple possible trajectories according to some raw model, and are replicated or discarded depending on whether the terrain height below the particle (i.e. at the same horizontal position) matches or not the available noisy measurement of the terrain height below the aircraft.
- Tracking a dim point target in a sequence of noisy images. In this *track–before–detect* demo, a point, which cannot be detected in a single image of the sequence, can be automatically tracked in a sequence of noisy images.

- Positioning and tracking in the presence of obstacles. In this interactive demo, presented by Simon Maskell (QinetiQ and CUED, Cambridge University Engineering Department) at a GDR ISIS event co–organized by François Le Gland and Jean–Pierre Le Cadre in December 2002, several stations (the number and locations of which are chosen interactively) try to position and track a mobile from noisy angle measurements, in the presence of obstacles (walls, tunnels, etc., the number, locations and orientations of which are also chosen interactively), which make the mobile temporarily invisible from one or several stations. This nonlinear filtering problem in a complex environment, with many constraints, would be practically impossible to implement using Kalman filters.
- Positioning and tracking of a mobile in a urban area. In this interactive demo, power attenuation maps associated with several base stations (the number and locations of which are chosen interactively) are combined with power measurements of the signal received from the base stations, and with a random walk prior model for the motion of the mobile user, in order to position and track a user in a urban Manhattan–like environment. The user is allowed to enter buildings, where no signal at all is received, and the particle filter is able in principle to lock quickly to the user position whenever he / she leaves the building.

6. New Results

6.1. Adaptive splitting for rare event analysis

Keywords: *asymptotic normality*, *multilevel splitting*, *quantile*, *rare event*.

Participants: Frédéric Cérou, Pierre Del Moral, Arnaud Guyader, Hélène Topart.

The estimation of rare event probabilities is a crucial issue in areas such as reliability, telecommunication networks, air traffic management, etc. In complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately probabilities of order less than 10^{-9}, say. Besides importance sampling, a widespread technique is multilevel splitting, which requires at least some knowledge of the system, to decide where to place the intermediate level sets. This is not always possible, and an adaptive algorithm has been proposed to cope with this problem, and has been analyzed thoroughly in the one–dimensional case.

To be specific, let $X \equiv (X_t\ ,\ t \geq 0)$ denote a strong Markov process with values in $\mathbb{R}$ and with continuous trajectories. Assume that the origin is a recurrent set in the sense that the hitting time

$$T_0 = \inf\{t \geq 0\ :\ X_t = 0\}\quad ,$$

has finite expectation. The problem is to estimate the probability α that the process reaches the level $M \gg 1$ before returning to the origin, i.e. before time T_0, starting from the deterministic initial condition $0 < X_0 = x_0 < M$.

The proposed algorithm is described as follows : At generation $l = 1$:

- independently for $j = 1, \cdots, n$, let the excursion $X^j \equiv (X_t^j\ ,\ 0 \leq t \leq T_0^j)$ be simulated until $T_0^j = \inf\{t \geq 0\ :\ X_t^j = 0\}$, according to the Markov kernel of X and starting from the deterministic initial condition $X_0^j = x_0$, and let

  $$S_{n,j}^1 = \sup_{0 \leq t \leq T_0^j} X_t^j \geq x_0\quad ,$$

  denote the maximum of the excursion X^j, i.e. the closest point to the level M,

- sort the sample $(S^1_{n,1}, \cdots, S^1_{n,n})$ in increasing order

$$S^1_{n,(1)} \leq \cdots \leq S^1_{n,(n-k)} \leq \cdots \leq S^1_{n,(n)} \quad ,$$

  let $\hat{q}_1 = S^1_{n,(n-k)} \geq x_0$ play the role of the (virtual) level at the generation 1, and keep unchanged the k excursions approaching the level M most closely, i.e. let $S^2_{n,j} = S^1_{n,(j)}$ for any $j = n-k+1, \cdots, n$.

At generation l :

- independently for $j = 1, \cdots, n-k$, let the excursion $X^j \equiv (X^j_t \, , \, 0 \leq t \leq T^j_0)$ be simulated until $T^j_0 = \inf\{t \geq 0 \, : \, X^j_t = 0\}$, according to the Markov kernel of X and starting from the initial condition $X^j_0 = \hat{q}_{l-1}$, and let

$$S^l_{n,j} = \sup_{0 \leq t \leq T^j_0} X^j_t \geq \hat{q}_{l-1} \quad ,$$

  denote the maximum of the excursion X^j, i.e. the closest point to the level M,
- sort the sample $(S^l_{n,1}, \cdots, S^l_{n,n})$ in increasing order

$$S^l_{n,(1)} \leq \cdots \leq S^l_{n,(n-k)} \leq \cdots \leq S^l_{n,(n)} \quad ,$$

  let $\hat{q}_l = S^l_{n,(n-k)} \geq \hat{q}_{l-1}$ play the role of the (virtual) level at the generation l, and keep unchanged the k excursions approaching the level M most closely, i.e. let $S^{l+1}_{n,j} = S^l_{n,(j)}$ for any $j = n-k+1, \cdots, n$.

At each generation l, the k excursions approaching the level M most closely are kept unchanged, i.e. the fraction of those excursions which reach the (virtual) level $\hat{q}_l$ is $\dfrac{k}{n}$ exactly, and $(n-k)$ new excursions are simulated, starting from the current (virtual) level $\hat{q}_l$. The algorithm continues until generation $l = L$ where the current (virtual) level $\hat{q}_{L+1} \geq M$ exceeds the level M, i.e. L denotes the first generation where at least one excursion reaches the level M : let k_{L+1} denote the exact number of such excursions, i.e. the number of elements in the sample $(S^L_{n,1}, \cdots, S^L_{n,n})$ that exceed the level M. Then, the rare event probability is estimated as

$$\alpha \approx \hat{\alpha}_n = \frac{k_{L+1}}{n} \, (\frac{k}{n})^L \quad ,$$

and the number of (virtual) intermediate levels is L.

In this one–dimensional framework, almost sure convergence and asymptotic normality of the estimator has been proved, with the same variance as other algorithms that use given intermediate levels. It has also been showed on numerical examples that this method can be used for multidimensional problems as well, even if there is still no convergence result in this case.

6.2. Particle filter without likelihood

Keywords: *kernel method*, *regularization*.

Participants: Vivien Rossi, François Le Gland.

It is commonly assumed that implementing particle filter algorithms requires being able to

- simulate independent realizations of the hidden state,
- and evaluate a likelihood function, which quantifies the consistency of each simulated state with the current observation.

It is possible however to get rid of this assumption, using kernel–based regularization methods that have already been considered in [27], so that ultimately it is enough to

- jointly simulate independent realizations of the hidden state and the observation,
- and validate each simulated pair (hidden state, observation) against the current observation.

If a binary validation rule is used, it may very well happen that all the simulated pairs are rejected, in which case the sequential particle algorithm introduced in [9] and further studied in [17], [14] should be used, which automatically keeps the particle system alive. With this class of algorithms, it is no longer necessary to evaluate a likelihood function, which opens the possibility to approximate the Bayesian filter in situations where

- no explicit expression is available for the likelihood function,
- there is no likelihood function (for instance because the observation noise does not appear in an additive way),
- there is no noise on (some components of) the observation, which can be interpreted as a posterior constraint on the hidden state, etc.

Preliminary results have been obtained by Vivien Rossi in his PhD thesis [57], and the objective of the post–doctoral project is to design new algorithms based on the generalized particle filtering idea, and to study their asymptotic properties.

6.3. Particle approximations of Feynman–Kac flows depending on a parameter

Keywords: *Monte Carlo maximum likelihood (MCML)*, *hidden Markov model (HMM)*.

Participants: Natacha Caylus, François Le Gland.

This is a collaboration with Nadia Oudjane, from the OSIRIS (Optimisation, simulation, risque et statistiques) department of Électricité de France R&D, see 7.2.

In full generality, given nonnegative kernels R_n and a nonnegative measure γ_0, we consider the unnormalized (linear) Feynman–Kac flow

$$\langle \gamma_n, f \rangle = \int_{E_n} \cdots \int_{E_0} f(x_n) \prod_{k=1}^{n} R_k(x_{k-1}, dx_k)\, \gamma_0(dx_0) \ .$$

A well–known example is provided by the unnormalized conditional probability distribution of the hidden state given past observations, when the hidden state and the observation form jointly a Markov chain : this includes HMM and switching AR models as special cases, with the decomposition $R_k(x, dx') = Q_k(x, dx')\, g_k(x')$ where Q_k is the Markov transition kernel and where the selection function g_k is the likelihood function.

If the nonnegative kernels depends smoothly (continuously or differentially) on a parameter, in such a way that the Feynman–Kac flow depends smoothly on the parameter, we would like to design a particle approximation to would depend smoothly on the parameter as well. The need for such a regularity property arises for instance

- in sensitivity analysis, e.g. in the computation of Greeks, in option pricing,

- in statistics of HMM, see 3.3, e.g. in the evaluation of the derivative w.r.t. the parameter of any contrast function that can be expressed in terms of the conditional probability distribution of the hidden state given past observations.

The smooth particle approximation introduced last year has been further studied, where a unique interacting particle system is propagated for a given (pivot) value of the parameter, and where secondary weights are computed separately for each value of the parameter in a neighborhood of the pivot value. Differentiating these secondary weights w.r.t. the parameter yields a particle approximation of the linear tangent Feynman–Kac flow, which coincides with an earlier approach followed in the team, where a particle approximation is derived direxctly from the equation satisfied by the linear tangent Feynman–Kac flow. The new results obtained this year for the joint particle approximation of the Feynman–Kac flow and the linear tangent Feynman–Kac flow are

- a central limit theorem,
- uniform estimates over a neighborhood of the pivot value of the parameter.

6.4. Recursive maximum likelihood estimation for structural health monitoring : tangent filter implementations

Keywords: *linear tangent filter, maximum likelihood estimation, recursive estimation, structural health monitoring*.

Participant: Fabien Campillo.

Tracking varying parameters using likelihood–based recursive algorithms has been shown to adapt well to parameter change on a sample–wise level. The objective of the current work is to extend previous work on high order models from realistic civil aeronautic structures. This work is jointly done with Laurent Mevel from SISTHEM project–team. Work has progressed utilizing Kalman based recursive estimation algorithms and it has been shown that these kind of algorithms can be computed effectively using particular filtering techniques [15]. The main idea for both approaches is to write the recursive score, derived from the likelihood function and based on the recursive computation of the prediction filter and its derivative.

The implementation and simulation studies have been handled by Nimish Sharmah during his internship, under the joint supervision of Laurent Mevel and Fabien Campillo. The work has focused on the evaluation of finding the best parameterization between the complex poles and the frequency/damping coefficients. It has been shown by Monte Carlo studies that the pole estimation is much simpler than finding good estimates for the damping, even if the parameterizations seem to be a priori equal. Monte Carlo studies have shown that the confidence intervals on damping coefficients can be quite high even if the corresponding poles are well estimated.

6.5. Bayesian methods for natural resources management

Keywords: *Bayesian estimation, Markov chain Monte Carlo (MCMC), Metropolis–Hastings, natural resource management*.

Participants: Fabien Campillo, Vivien Rossi, Rivo Rakotozafy.

Markov chain Monte Carlo (MCMC) methods are now widely used to study the evolution of natural resources, such as biomass in fisheries or the dynamics of forests. Although flexible, these methods have however low convergence speeds. By contrast, particle filtering methods are fast but are not well adapted to these models, where measurements are sampled daily, monthly or yearly. Therefore there is no need to treat these measurements in a recursive way, and it seems more appropriate to call upon Monte Carlo methods in interaction, such as population Monte Carlo [24].

In the first case study particle filtering has been applied to calibrate tree–growth models, which are complex and highly nonlinear. MCMC (Metropolis–Hastings) techniques have been compared with particle filtering methods for fitting and identifying such model. The first results showed that MCMC methods are more consuming in terms of computational time. By comparison, particle filtering is faster and less sensitive to prior knowledge, but it also presents a greater variance than the MCMC methods.

This is a collaboration with Cédric Gaucherel, from CEREVE (centre européen de recherche et d'enseignement de géosciences de l'environnement, in Aix–en–Provence), Étienne Rivot (Agrocampus de Rennes) and Rivo Rakotozafy (University of Fianarantsao, in visit within ASPI).

6.6. Nearest neighbor classification in infinite dimension

Keywords: *classification, consistency, non parametric statistics*.

Participants: Frédéric Cérou, Arnaud Guyader.

Given an n–sample $D_n = ((X_1, Y_1), \cdots, (X_n, Y_n))$ of training data, where the 0-1 valued random variable Y_i is the class, or label, of the random element X_i with values in the metric space $(\mathcal{F}, d)$, the problem of classification is to predict the label Y of a new random element X. The joint probability distribution of (X, Y) on $\mathcal{F} \times \{0, 1\}$ is characterized by

- the probability distribution $\mu(dx)$ of X on $\mathcal{F}$,
- and by the *regression* function defined by $\eta(x) = \mathbb{P}[Y = 1 \mid X = x]$ for any x in $\mathcal{F}$.

It is easy to prove that the Bayes classifier

$$g^*(x) = 1_{\{\eta(x) \geq \frac{1}{2}\}} \ ,$$

minimizes the probability of error over all 0-1 valued classifiers g_n defined on $\mathcal{F}$, i.e.

$$\mathbb{P}[g_n(X) \neq Y] \geq L^* \qquad \text{where} \qquad L^* = \mathbb{P}[g^*(X) \neq Y] \ ,$$

is the Bayes probability of error, or Bayes risk.

The k-nearest neighbor classifier g_n consists in the simple following rule : look at the k nearest neighbors of X in the n–sample, and choose 0 or 1 for its label according to the majority rule. In the case where $(\mathcal{F}, d) = (\mathbb{R}^m, \|\cdot\|)$, this classifier is universally consistent [58] : its probability of error converges to the Bayes risk as n goes to infinity, whatever the joint probability distribution of (X, Y), provided that k/n goes to 0. Unfortunately, this result is no longer valid in general metric spaces, and the objective is to find out reasonable sufficient conditions for the weak consistency to hold. Even in finite dimension, there are exotic distances such that the nearest neighbor does not even gets closer (in the sense of the distance) to the point of interest, and the state space $\mathcal{F}$ needs to be complete for the metric d, which is the first condition. Some regularity on η is required next : clearly continuity is too strong because it is not required in finite dimension, and a weaker form of regularity is assumed.

The following consistency result has been obtained [20] : If the metric space $(\mathcal{F}, d)$ is separable and if the following *Besicovich condition*

$$\mu\{x \in \mathcal{F} \, : \, \frac{1}{\mu(B_{x,\delta})} \int_{B_{x,\delta}} |\eta - \eta(x)| \, d\mu > \varepsilon\} \to 0 \ ,$$

holds for every $\varepsilon > 0$ as $\delta \downarrow 0$, or equivalently if

$$\frac{1}{\mu(B_{X,\delta})} \int_{B_{X,\delta}} |\eta - \eta(X)| \, d\mu \to 0 \ ,$$

in probability as $\delta \downarrow 0$, where $B_{x,\delta} = \{x' \in \mathcal{F} \, : \, d(x,x') \leq \delta\}$ denotes the closed ball of radius δ centered at x, then the nearest neighbor classifier is weakly consistent, i.e.

$$\mathbb{P}[g_n(X) \neq Y] \rightarrow L^* \quad ,$$

as $n \uparrow 0$. Note that the Besicovich condition is always fulfilled in finite dimensional vector spaces (this result is called the Besicovich theorem), and that a counterexample can be given [55] in an infinite dimensional space with a Gaussian measure (in this case, the nearest neighbor classifier is clearly nonconsistent). Finally, a simple example has been found which verifies the Besicovich condition with a noncontinuous function η.

7. Contracts and Grants with Industry

7.1. Conditional Monte Carlo methods for risk assessment — IST project HYBRIDGE

Participants: Frédéric Cérou, François Le Gland.

Contract INRIA 1 02 C 0037 — January 2002/April 2005

In view of the undergoing evolution in management and control of large complex real–time systems towards an increasing distribution of sensors, decisions, etc., and an increasing concern for safety criticality, the IST project HYBRIDGE addresses methodological issues in stochastic analysis and distributed control of hybrid systems, with conflict management in air trafic as its target application area. It is coordinated by National Aerospace Laboratory (NLR, Netherlands) and its partners are Cambridge University (United Kingdom), Universita di Brescia and Universita dell'Aquila (Italy), Twente University (Netherlands), National Technical University of Athens (NTUA, Greece), Centre d'Études de la Navigation Aérienne (CENA), Eurocontrol Experimental Center (EEC), AEA Technology and BAe Systems (United Kingdom), and INRIA.

The contribution of ASPI to this project concerns the work package on modeling accident risks with hybrid stochastic systems, and the workpackage on risk decomposition and risk assessment methods, and their implementation using conditional Monte Carlo methods [13], [14]. This problem has motivated our work on the *importance splitting* approach to the simulation of rare events [16].

7.2. Calibration of models for electricity spot and futures price — contract with EDF

Participant: François Le Gland.

See 3.3 and 6.3 .

Contract INRIA 1 04 C 0862 — October 2004/September 2005

This is a collaboration with Nadia Oudjane, from the OSIRIS (Optimisation, simulation, risque et statistiques) department of Électricité de France R&D.

The objective is to estimate parameters in various multi–factor models for electricity spot price, from the observation of futures contracts prices that are traded in the market. This problems fits within the general framework of parameter estimation in hidden Markov models, and we propose to rely on joint particle approximation schemes for the optimal filter and the linear tangent filter, so as to maximize the likelihood function, or other suitable contrast functions, w.r.t. the parameters. In the simple case where the futures contracts are written for electricity delivery over a single period of time, and if multi–factor models are based on Ornstein–Uhlenbeck processes driven by a Brownian motion, then the problem is linear Gaussian and explicit expressions provided by the Kalman filter can be used to assess the performance of the proposed approach. In practice however futures contracts are usually written for electricity delivery over a long period of time, and realistic multi–factor models should be used, based on Ornstein–Uhlenbeck processes driven by a Lévy process, which make the problem non–linear with non Gaussian noise structure.

A two factor model proposed by the industrial partner, with a long–term factor modeled as a Brownian motion with drift and a short–term factor modeled as an Ornstein–Uhlenbeck process driven by an independent Brownian motion, has been considered first. It appears that the log–price of a future contract for delivery over a multiple of the unit delivery period, depends linearly on the long–term factor, and a Rao–Blackwellized form has been derived for the particle approximation of the likelihood function. To locate the global maxima of the likelihood function, particle methods over the parameter space, such as annealed Feynman–Kac models [29] or interacting particle implementation of Bayesian optimal design [54], are currently under investigation.

7.3. Localization and tracking of mobile terminals — contract with FTRD

Participants: Fabien Campillo, Frédéric Cérou, François Le Gland, Julien Guillet.

See 3.3 and 6.3 .

Contract ...— May 2005/April 2006

The objective is to implement and assess the performance of particle filtering in localisation and tracking of mobile terminals in a wireless network, using network measurements (received power level and possibly TDOA (time difference of arrival)) and a database of reference measurements of the power level, available in a few points or in the form of a digital map (power attenuation map). Generic algorithms will be proposed and specialized to the indoor context (wireless local area network, e.g. WiFi) and to the outdoor context (cellular network, e.g. GSM) when necessary. Constraints and obstacles such as building walls in an indoor environment, street, road or railway networks in an outdoor environment, will be represented in a simplified manner, using a prior model on a graph, e.g. a Voronoï graph as in similar experiments in mobile robotics [52]. To assess the performance of the proposed localisation and tracking algorithms, posterior Cramèr–Rao bounds for a Markov process on a graph will be derived. Another objective is to update and enrich the initial database of reference measurements, using network measurements collected on–the–fly.

8. Other Grants and Activities

8.1. Data assimilation for air quality (ADOQA) — ARC INRIA

Participants: Fabien Campillo, Frédéric Cérou, François Le Gland, Vu Duc Tran.

January 2005/December 2006.

This ARC is coordinated by the project–team CLIME from INRIA Rocquencourt and CEREA / ENPC, and its partners are the project–team IDOPT from INRIA Rhône–Alpes, and INERIS.

One objective of this ARC is to investigate advanced sequential methods (as opposed to variational methods) for data assimilation of intrinsically nonlinear models, i.e. coupling of numerical models and measured data. In principle, a data assimilation algorithm should propagate uncertainties through the probability distribution of the state variables, whereas current sequential algorithms, such as the Kalman filter and its simplest extensions, only propagate the first two moments. For large–scale systems (physical state of the atmosphere, of the ocean, chemical composition of the atmosphere, etc.), the direct implementation of sequential Monte Carlo methods seems impractical, and simplified, reduced–order models should be used.

A preliminary investigation has been to better understand and compare on simple examples the qualitative behaviour and the performance, in terms of the sample size, of the ensemble Kalman filter [34], [35] and other similar algorithms, and of sequential Monte Carlo methods.

9. Dissemination

9.1. Scientific animation

F. Campillo has reported on the PhD thesis of David Bourgeois (ONERA and université de Cergy-Pontoise, advisors : Marc Flécheux and Inbar Fijalkow). He is a member of the committee for the PhD thesis of Thomas Bréhard (université de Rennes 1 and IRISA, advisor : Jean–Pierre Le Cadre). He is a member of the «conseil de laboratoire» of IRISA (UMR 6074) and of the «conseil de l'école doctorale de physique, modélisation et sciences pour l'ingénieur de Marseille». He is the INRIA representative for scientific relations with Madagascar, within the SARIMA (support to research activities in computer science and mathematics in Africa) project supported by INRIA and MAE (ministère des affaires étrangères). In relation with this activity, he has spent one week in Antananarivo in August 2005.

F. Le Gland has reported on the PhD thesis of Karim Dahia (ONERA and université Joseph Fourier, advisors : Christian Musso and Dinh–Tuan Pham) and Afef Sellami (université Paris–Dauphine, advisors : Gilles Pagès and Huyên Pham). He is a member of the committe for the HDR (habilitation à diriger les recherches) of James Ledoux (université de Rennes 1) and of Bruno Tuffin (université de Rennes 1).

A. Guyader and F. Le Gland are members of the «commission de spécialistes» in applied mathematics (section 26) of université de Rennes 2.

V. Rossi assembled the research project MIP (Modélisation Intégrée en Dynamique des Populations : Applications à la Gestion et à la Conservation) in partnership with the Centre d'Écologie Fonctionnelle et Évolutive, UMR CNRS 5175 and the UMR ENSAM-INRA Analyse des Systèmes et Biométrie. The main objective is to use Bayesian statistics to circumvent problems generally encountered in population dynamics models. The project funds are grants of the région Languedoc–Roussillon.

9.2. Teaching

F. Campillo gives a course on Markov models, hidden Markov models, filtering and particle filtering at université de Sud Toulon–Var, within the Master «Mathématiques (Filtrage et traitement des données)» and the Master «Sciences et Technologies (Sciences de la mer, environnement, systèmes)».

F. Le Gland gives a course on Kalman filtering, particle filtering and hidden Markov models, at université de Rennes 1, within the Master STI (école doctorale MATISSE), a course on Bayesian filtering and particle approximation, at ENSTA, Paris, within the quantitative finance track, and a course on hidden Markov models, at ENST Bretagne, Brest.

9.3. Participation in workshops, seminars, lectures, etc.

In addition to presentations with a publication in the proceedings, and which are listed at the end of the document, members of ASPI have also given the following presentations.

F. Campillo has given a talk on particle filters and sequential data assimilation at the ARC ADOQA (Data assimilation for air quality) meeting at ENPC (Marne–la–Vallée) in June 2005.

A. Guyader has given a talk on nearest neighbor classification in infinite dimension at the seminar ENSA-M/INRA/UM 2 in Montpellier in January 2005, and at the «Rencontre de Statistiques Mathématiques» at CIRM in Marseilles in December 2005. He has also given a talk on adaptive multilevel splitting for rare event analysis at the «Journées STAR» in Rennes in November 2005. He has organized with Magalie Fromont and Éric Matzner-Løber the «Journées de Statistique et Applications en Biologie» at université de Rennes 2 in September 2005.

F. Le Gland has given a talk on statistical inference of HMM using Monte Carlo methods with interaction, at the 5th workshop on Statistical Inference for Continuous Time Stochastic Processes, organized by Yurii Kutoyants in Le Mans in January 2005, and at the «Journées STAR» in Rennes in November 2005. He has given several talks on a sequential particle algorithm that keeps the particle system alive, at the workshop on «Statistique des Modèles à Données Manquantes», held at université de Marne–la–Vallée in January 2005, at the «Probabilités et Statistiques» seminar of université de Nice–Sophia Antipolis in April 2005, and at the Control seminar of the Department of Engineering of the University of Cambridge in October 2005. He has also given a talk on filtering a diffusion process observed in singular noise, at the joint seminar on Stochastic Analysis, Random Fields and Applications and minisymposium on Stochastic Methods in Financial Models, organized in Ascona in May 2005.

V. Rossi has given a talk on nonlinear filtering using convolution kernels at the «Premières Rencontres des Jeunes Statisticiens» in Aussois in September 2005. He has also given a talk on applications of particle filtering in biological field at the «Journées de Statistique et Applications en Biologie» organized at université de

Rennes 2 in September 2005. He has also given a talk on the introduction of a selection step to avoid the degeneracy of particle filter algorithms at the colloquium «Statistique des Processus. Applications au Traitement du Signal et de l'Image» held at Institut de Mathématiques Appliquées in Angers in September 2005.

9.4. Visits and invitations

F. Le Gland has been invited in October 2005 by Jan Maciejowski and Andréa Lecchini Visentini in the Department of Engineering of the University of Cambridge, and has given there a talk in the Control seminar on a sequential particle algorithm that keeps the particle system alive.

Antoine Lejay, from the OMEGA project at INRIA Lorraine, has visited our group for three days in February 2005, and has given a talk on Monte Carlo methods for discountinuous and fractured media.

Agnès Lagnoux, a PhD student at the Laboratoire de Statistique et Probabilités of université Paul Sabatier, and Pascal Lezaud, from the joint CENA / ENAC department of applied mathematics have visited our group for two days in June 2005, and have given a talk each, on the estimation of a rare event probability, and on an introduction to the theory of extreme values, respectively.

Mathias Rousset, a PhD student at the Laboratoire Jean–Alexandre Dieudonné of université de Nice–Sophia Antipolis, under the supervision of Pierre Del Moral, has visited our group in November 2005, and has given a talk about particle systems and out–of–equilibrium molecular dynamics.

Rivo Rakotozafy, assistant professor at the University of Fianaranstoa, has been awarded by the French embassy in Antananarivo, Madagascar a grant to support three visits (one per year, each stay of three months duration) to prepare a Madagascar habilitation thesis (HDR) under the supervision of Fabien Campillo. A related objective is to set up a collaboration between the University of Fianaranstoa and INRIA.

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