



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team Bang

*Biophysique, Analyse Numérique et
Géophysique*

Rocquencourt

THEME NUM

Activity
R *eport*

2005

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1. Team

BANG (Biophysique, Analyse Numérique et Géophysique) is a continuation of the former project M3N.

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2. Overall Objectives

2.1. Overall Objectives

BANG (Biophysique, Analyse Numérique et Géophysique) is a continuation of the former project M3N. It aims at developing models and numerical methods for two kinds of problems involving Partial Differential Equations. Firstly problems from life sciences (cell movement, cancer modeling...) are considered. Secondly models for complex fluid flows are studied (flows with a free surface, flows of holes and electrons in semiconductors).

The common scientific features behind these applications come from models involving coupled systems of PDEs (as Keller-Segel or Saint-Venant systems) that are solved (simulated) on computers involving new algorithms.

3. Scientific Foundations

3.1. Introduction

Partial Differential Equations are mathematical tools that allow to represent efficiently the evolution of complex physical phenomena. The most classical PDE is certainly the Navier-Stokes system which describes

the evolution of the density $\rho(t, x)$ the velocity $\vec{u}(t, x)$ and the temperature $T(t, x)$ of a fluid parametrized by time t and space position x .

Since the XIXth century this formalism has shown its efficiency and ability to explain both qualitative and quantitative behaviors of fluids. The knowledge that has been gathered on such physical models, on algorithms for solving them on computers, on industrial implementation, opens the hope for success when dealing with life sciences also. This is one of the main goals of BANG.

3.2. Mathematical Modeling

What are the relevant physical or biological variables, what are the possible dominant effects ruling their dynamics, how to analyse the information coming out from a mathematical model and interpret them in the real situations under consideration ? These are the questions leading to select a mathematical model, generally also to couple several of them in order to render all physical or biomedical features which are selected by specialist partners (engineers, physicists, medical doctors). These are usually based on Navier-Stokes system for fluids (as in free surface fluid flows), on parabolic-hyperbolic equations (Saint-Venant system for shallow water, flows of electrons/holes in semiconductors, Keller-Segel model of chemotaxis).

3.3. Multiscale analysis

The complete physical or biomedical description is usually complex and requires very small scales. Efficiency of computer resolution leads to simplifications using averages of quantities. Methods allowing to achieve that goal are numerous and mathematically deep. Some examples studied in BANG are

- Reduction of full 3d Navier-Stokes system to 2d or 1d hyperbolic equations by a section average (derivation of Saint-Venant system for shallow water).
- Coupled multiscale modelling (degenerate semi-conductors, description of tumor from the cell level to the organ scale).
- Description of cell movement from the individual to the collective scales.

3.4. Numerical Algorithms

Numerical methods used in BANG are mostly based on finite elements or finite volume methods. Algorithmic improvements are needed in order to take into account the specificity of each model, of their coupling, or their 3D features. Among them we can mention

- Well-balanced schemes for shallow water system.
- Free-surface Navier-Stokes solvers based on a multilayer St-Venant approach.
- Mixed finite elements for problems with large density variations (semi-conductors, chemotaxis).

4. Application Domains

4.1. Panorama

BANG has decided to develop new biomedical applications and focusses its know-how in these directions, while keeping more classical industrial relations. These are developed in relation with other INRIA projects: GAMMA, REO.

4.2. Cancer modeling

This research activity aims at studying mathematical models related to tumors developments and the control of therapy. Among the many biological aspects let us mention

- cell movements (chemotaxis, vasculogenesis, angiogenesis),
- cell cycle, immune reaction and adaptive dynamics (structured population dynamics),
- modelling and optimization of chemotherapy through differential systems.

4.3. Free surface flows

Several industrial applications require to solve fluid flows with a free surface. BANG develops algorithms in two directions. Firstly flows in rivers and coastal areas using Saint-Venant model with applications to dam break and pollution problems in averaged shallow water systems. Secondly, 3D hydrostatic flows by a multilayer Saint-Venant approach and 3D Navier-Stokes flows.

4.4. Semiconductors

Mathematical models based on drift-diffusion systems or energy transport systems are solved using mixed finite elements methods. BANG has developed a highly sophisticated code which is able to simulate very stiff semiconductor devices.

5. Software

5.1. Introduction

Softwares initiated and developed within former projects (Ménus, M3N) and currently in use in the present project.

5.2. OPTMTR

Generation of metric maps for use with *adapted meshes* generator (with Gamma project)

5.3. EMC2

Interactive 2D mesh generator (with Gamma project)

5.4. HET_2D

Participants: Americo Marrocco [correspondant], Philippe Montarnal [Former PhD student M3N], Abderazzak El Boukili [Former PhD student M3N], Frédéric Hecht [LAN, Université Paris 6.], Jean-Christophe Rioual [Former PhD student, CERFACS].

Research software for the numerical simulation of semiconductor devices. Drift-Diffusion and Energy-Transport models are implemented. The mathematical formulation is described using as unknowns the electrostatic potential, the quasi Fermi levels and additionally the electron temperature. The approximation is carried out via mixed finite elements (Raviart-Thomas element RT_0). Parallel computation via domain decomposition is available for some modules and an interface with the **Bang** software (Gamma project) has been developed for automatic mesh adaption.

6. New Results

6.1. Cancer modeling

Keywords: *cancer chronotherapy, cancer modeling, cell population, differential equations, numerical algorithm, optimal control.*

6.1.1. Dynamics of structured cell populations; applications to cell cycle modelling:

Participants: Fadia Bekkal-Brikci, Jean Clairenbault, Philippe Michel, Benoit Perthame, Melina Rapacioli [CONICET], Edmundo Rofman [CONICET].

- **Circadian rhythm and tumour growth.** A model of the cell cycle of the Von Foerster-McKendrick type (first developed in 2003, INRIA Res. Report # 4892) has been used in a simplified version (2 phases) to test in a theoretical setting the influence of circadian rhythms on tumour growth,

following experimental observations by E. Filipinski et al. at F. Lévi's INSERM lab in Villejuif. The first eigenvalue of a linear PDE system is evaluated with respect to different types of control, constant or periodic, on apoptosis and phase transition rates [21]. The theoretical results show that, contrarily to what was expected on the basis of experimental evidence, periodic control on the sole apoptosis rates enhance tumour growth, whereas periodic control on phase transition rates has variable effects, slowing down or accelerating tumour growth according to phase durations.

- **4-phase cell cycle model.** This model is currently being refined to account for actual physiological phase transitions G1/S and G2/M on the one hand, and to represent recruitment from a quiescent (G0) to a proliferative compartment on the other hand.
- **Modelling neural tube development in the Chicken embryo.** Stimulated by observations coming from a team of Argentinian biologists, pointed to Benoit Perthame and Jean CLairambault by Edmundo Rofman, we are currently developing a space-and-age-structured model of the spatial growth of a population of neuroepithelial cells along the neural tube of the Chicken embryo. This is the main theme of a beginning collaboration between INRIA and CONICET, in which are involved Argentinian biologists and mathematicians and which will see the visit at INRIA of Melina Rapacioli, an Argentinian biologist, in the first weeks of January 2006 (other themes involve modelling of colorectal cancer proliferation and invasion).

6.1.2. Pharmacokinetic-pharmacodynamic (PK-PD) modelling for anticancer therapy

Participants: Jean Clairambault, Francis Levi [INSERM E 0354, Hospital Paul-Brousse, Villejuif].

An ODE model of the action of cytotoxic drugs at the molecular level is currently being developed in order to take into account drugs which are of everyday clinical use in the treatment of colorectal cancer, namely oxaliplatin and 5-fluorouracil, in association. Oxaliplatin (an alkylating agent) is not cell cycle phase-specific, whereas 5-fluorouracil is S-phase specific. The aim of this approach is to provide clinicians with a rationale to optimise the combination of such anti-cancer drugs; this makes it necessary not only to develop the abovementioned model of the cell cycle, but also to represent at the molecular level the action of drugs and their activation or inhibition by enzymes which are regulated with circadian rhythmicity and show genetic polymorphism. Together with the following topic (optimisation), these modelling pharmacological approaches are intended to help clinicians design optimised patient-tailored therapeutics.

6.1.3. Optimisation of anticancer drug infusion by using chronobiology concepts

Participants: Jean Clairambault, Francis Levi [INSERM E 0354, Hospital Paul-Brousse, Villejuif], Claude Basdevant [Université Paris 13], Jean-Charles Gilbert [Estime project], Houssein Eddine Miled, Souad Mezouar.

M2 Master or last year engineering school research stay students: Houssein Eddine Miled (EPT Tunis for Bang); Souad Mezouar (M2 Paris VI for Estime and Bang) have developed and tested, in the existing frame of an ODE system dedicated to modelling anticancer drug therapeutics by continuous infusion [29],[20], other optimisation methods than the Uzawa-like algorithm firstly designed and implemented by C. Basdevant: augmented lagrangian (HEM) and SQPAL (SM). The results confirm the suboptimal solutions found by the Uzawa algorithm, but with faster convergence, especially for SQPAL.

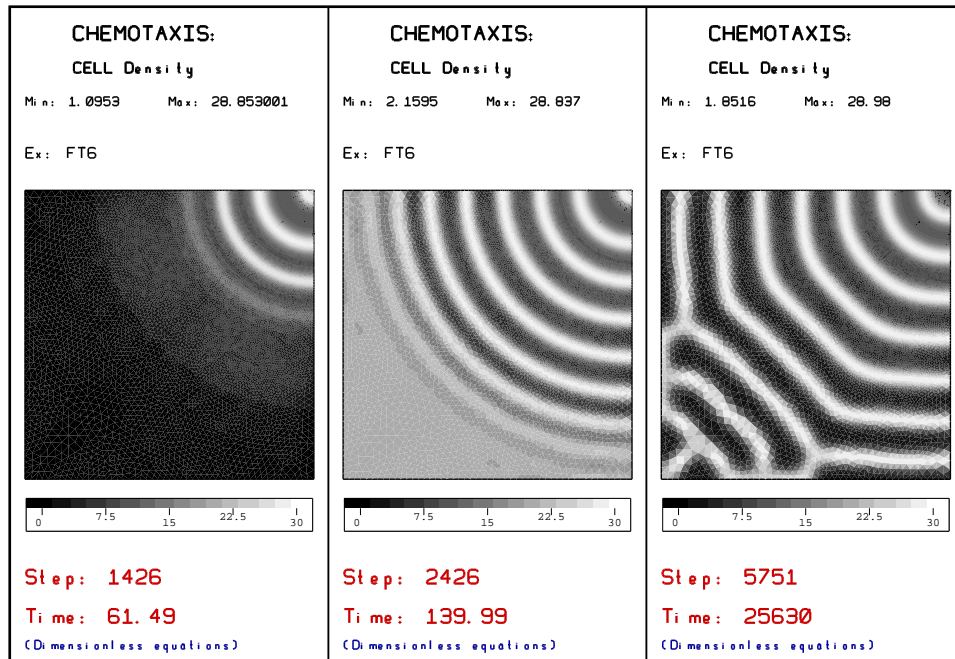
6.1.4. Chemotaxis, angiogenesis and cell motion

Keywords: *biophysics, cancer modeling, chemotaxis, finite element, numerical algorithm, numerical software.*

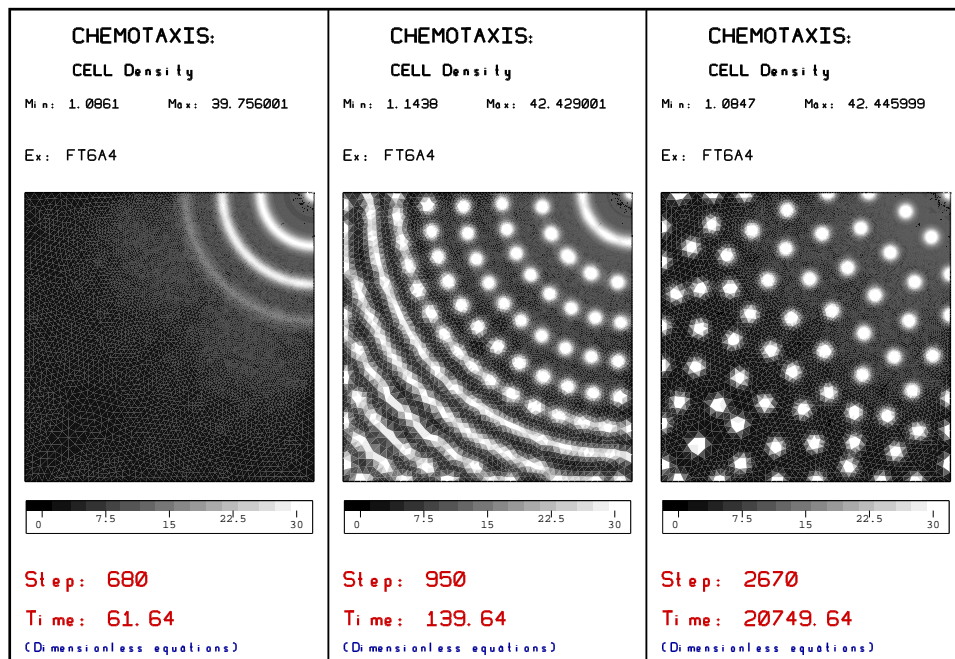
Participants: Vincent Calvez, Americo Marrocco, Benoit Perthame, Hatem Zaag.

Movement of cells are important in the process of cancer development. We have developed some activity in the understanding of mathematical models of chemotaxis and angiogenesis.

Angiogenesis describes the development of capillary blood vessels triggered by a substance emitted by a tumor. Models for the vasculature have been proposed by several authors (Chaplain, Levine, Sleeman) as



(a) Chemotactic sensitivity $\alpha = 2.25$, Solution at $t \sim 60, \sim 140$ and at "stationary state"



(b) Chemotactic sensitivity $\alpha = 2.5$, Solution at $t \sim 60, \sim 140$ and at "stationary state"

Figure 1. NUMERICAL SIMULATION OF BACTERIA AGGREGATION (CHEMOTAXIS).

a coupled parabolic/hyperbolic system. We have investigated existence of weak or strong solutions for this system [34].

Models of cell movement were proposed by Keller-Segel several years ago, as coupled parabolic/elliptic systems. They describe the collective motion of bacteria taking into account the underlying biochemistry (chemotaxis). These models are defined by two partial differential equations, one for the bacterial density and one for the attractant concentration. This system of equations is very similar to the drift-diffusion model for the (unipolar) semiconductor devices, so for the numerical simulation of bacteria aggregation, we derive a new formulation by introducing an unknown variable which is called quasi-Fermi level in semiconductor framework. By this way we can use and extend discretization approach and numerical schemes developed for the semiconductors.

This year we have focused our attention in the numerical simulation of an extended model, in which a third equation is introduced in order to take into account the stimulants (food) concentration. This work was initiated last year but extensions in the implementation have been realized in order to be able to consider the experiments presented in the J.D. Murray's book ([37]). As an example (figure 1-a) we present the solution obtained for a bacterial density evolution in a semi-solid medium (concentric rings of high bacterial density are obtained as in figure 5.17 pp 299 of the Murray's book). By symmetry reasons, the numerical simulation can be carried out on a 1/8 of the initial square domain Ω . By modifying the chemotactic sensitivity from 2.25 to 2.5 (adimensioned values) a totally different distribution is obtained (spots) (fig 1-b). Until the development of the first three high density rings (until approximately $t=60$), the evolutions of the bacterial densities are very similar in the two experiences, nevertheless the dynamics is higher in the second case. Few time later high density "spots" begin to appear and the structure of the first rings is also progressively altered. One can remark for the "stationary case", that the "inoculum point" (center of the square), reach a low level of density for the first experience and maintain a high level of density in the second experience.

The theoretical study of this extended Keller-Segel system has also been initiated. The main technical difficulty is that the extended system does not have an energy structure. Several scales of initial data can be treated using modified energy functional.

This is a long term program because several levels of modelling lead to different energy structures.

6.1.5. Structured population dynamics

Participants: Piotr Gwiazda, Philippe Michel, Stéphane Mischler, Benoit Perthame, Lenya Rizhik.

In biology, it is usual to consider population models structured by a significant parameter (age structured, size structured, maturity structured). These models which can describe mitosis (cell division), growth using a nutrient, death of cells for instance, are not conservation laws. Nevertheless we show that they admit a common relative entropy structure which uses the first eigenelements of the problem. In case of scattering, it is more general than the usual "detailed balance principle". Four types of consequences are deduced from this entropy structure : a priori bounds, large time convergence to the steady state, in some cases, exponential rates of convergence and attraction of orbits by a periodic external forcing (this last is motivated by chronotherapy)

Analysis of Age or Size Stable Dynamics

We consider the renewal equation (also called McKendrick-VonFoerster) equation that arises as a simple model for structured population dynamics and the size structured population equation which does not have representation formula. We prove the exponential convergence in long time to the steady state, after renormalization by a damping factor to compensate for the system growth.

Our approach, by opposition with the original method of Feller based on Laplace transform, uses the direct variable and the specific structure of the equations under consideration. In the case of age structure it uses an entropy approach and new invariants of the equation, to which we systematically associate a condition for the exponential convergence. See [25].

6.2. Free surface geophysical flows

Keywords: 3D Navier-Stokes, Geophysical flows, Saint-Venant equations, debris avalanches, free surface, multilayer system.

Participants: Emmanuel Audusse [Post-doctoral position, Freie Universitat Berlin], François Bouchut, Marie-Odile Bristeau, Astrid Decoene, Jean-Frédéric Gerbeau [Reo project], Benoit Perthame.

We are involved in research concerning the numerical simulation of free surface geophysical flows such as rivers, lakes, coastal areas and also avalanches. Many applications related to environmental problems are concerned : floodings, dam breaks, transport and diffusion of pollutants, debris avalanches ...

In many cases, the shallow water hypothesis is satisfied and these phenomena can be simulated by the Saint-Venant equations, for other cases we have considered a multilayer Saint-Venant system and also the 3D free surface Navier-Stokes equations.

6.2.1. Saint-Venant equations

The Saint-Venant equations are the basic tool of the following developments concerning the multilayer system and the debris avalanches. So we recall the main features of the solvers developed the previous years and which continue to be validated through different applications.

We have developed 1D and 2D solvers for the Saint-Venant equations with source terms, the aim is to obtain robust and efficient numerical tools based on theoretical results ensuring the accuracy and the preservation of physical properties of the flow (conservation, positivity of water depth, equilibrium states...). Our method is based on a *kinetic solver* and a *hydrostatic reconstruction* procedure.

First considering the homogeneous Saint-Venant equations we introduce their kinetic interpretation and deduce a macroscopic finite volume *kinetic solver*. The solver has good stability properties as the inherent preservation of the water depth positivity even when applications with dry areas are considered.

Second we consider the Saint-Venant system with source terms and a *hydrostatic reconstruction* strategy [33] allows us to extend any positivity preserving homogeneous scheme to a positivity preserving *well-balanced* scheme. The idea is to use the relation associated to the equilibrium to define new interface values that are used in the definition of the finite volume fluxes and of the source terms.

Finally we use a conservative and positivity preserving *formally second-order extension* based on linear reconstruction procedures. By introducing an enriched discretization of the source terms we construct a stable and well-balanced “second-order” scheme [18]. This solver is one of the options of the Telemac-2D code of EDF/LNHE.

6.2.2. Multilayer Saint-Venant system

For some applications like the transport of tracers of different densities or the effect of wind on a lake, we need to know the vertical profile of the velocity and by definition, we cannot get relevant information from Saint-Venant equations. These questions are also fundamental in ocean models and atmospheric sciences.

In these cases, and in order to avoid the 3D Navier-Stokes system when large scale problems are considered, we introduce a new multilayer Saint-Venant system [17],[27]. Thanks to a precise analysis of the shallow water assumption we propose an approximation of the *hydrostatic Navier-Stokes equations* which consists in a set of coupled Saint-Venant systems. It extends the range of validity and gives a precise description of the *vertical profile of the horizontal velocity* while preserving the computational efficiency of the classical Saint-Venant system. The fluid is divided in a given number of layers, each layer satisfies the Saint-Venant equations and is linked to the others by pressure terms (water height coupling) and viscosity (velocity coupling). This approach is a transition step towards 3D simulations with the Navier-Stokes system with two main advantages: we have not to deal with a moving mesh and the vertical velocity is only an output variable, it is not a variable of the system since the kinematic boundary condition is applied at each interface.

The Saint-Venant kinetic solver has been extended to this multilayer model and we have performed a numerical implementation which preserves the stability properties of the kinetic scheme. The pressure source terms of the multilayer system appears in a non conservative form, then this term is divided in two parts, a conservative part introduced in the left hand side of the system and a non conservative part treated as a source term and which has to be approximated carefully to obtain a stable scheme. For the bottom topography term we apply the *hydrostatic reconstruction* to the total water depth and the reconstructed local water depths are deduced.

The 1D [17] and 2D [27] multilayer solvers have been developed and validated. The Figures 2-3 show the good agreement of the results obtained with the 2D multilayer solver and with the 3D hydrostatic Navier-Stokes solver (presented in the next section) for a transcritical flow over a bump. These comparisons have been done for different values of the friction and vertical viscosity coefficients.

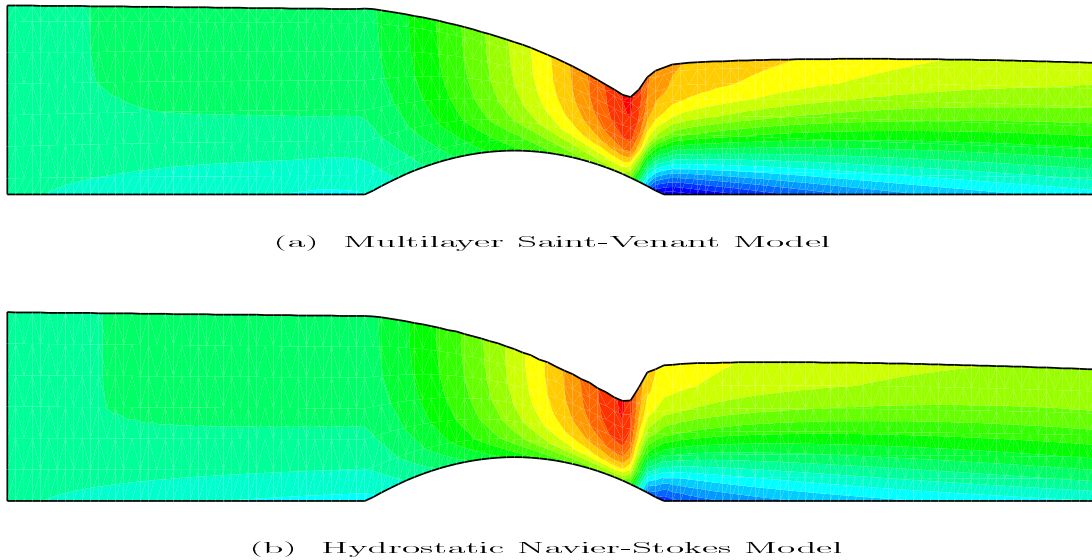
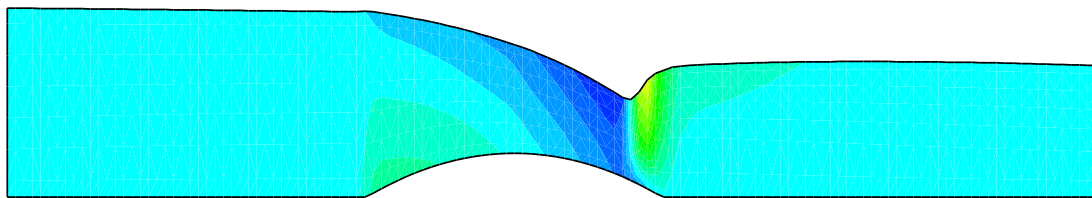


Figure 2. Transcritical flow over a 2D bump. Longitudinal cross-section of horizontal velocities

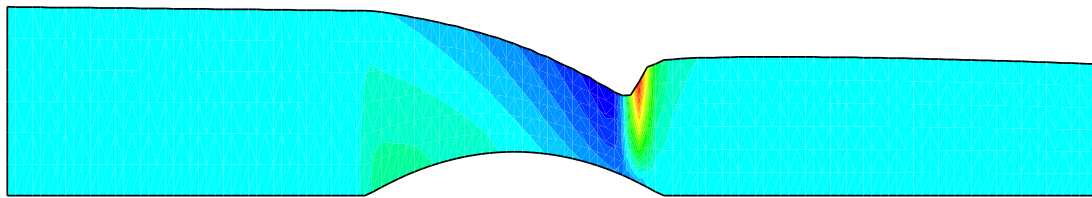
6.2.3. 3D Free surface Navier-Stokes

Besides this multilayer Saint-Venant approach, we are also studying the 3D incompressible Navier-Stokes equations with free surface. These equations are solved by the Telemac-3D software developed by EDF/LNHE, and our aim is to apply our results to this software in order to improve it.

We have considered the model with *hydrostatic approximation* [35], which can be viewed either as an independent model or as an intermediate step in the solution of the non-hydrostatic problem. After time discretization of the equations, the fractional step method can be applied in order to split the non-linear advection terms. The system coupling the 3D horizontal velocity and the 2D free surface function is studied. It is composed by the resulting momentum equation and the depth-integrated continuity equation. We have established a variational formulation, leading to a *mixed and symmetric problem*. We have analyzed the semi-discrete problem and its approximation, and proved that they are generally well posed, provided the time step is not too big and the viscosity coefficient not negligible. In the other case, a particular "inf-sup condition" must be satisfied. This condition is automatically satisfied at the space-continuous level, but not at the discrete level. We have therefore proposed a couple of stable finite elements to approximate the 3D horizontal velocity and the 2D free surface. Finally, we have implemented the resolution of this system into the Telemac-3D system, and the method has been validated through many numerical examples. For instance, we have simulated the three-dimensional flow in an estuary whose geometrical characteristics are very close to those of the Sado estuary in Portugal. Our aim was to verify that the scheme reproduces the vertical recirculation phenomenon



(a) Multilayer Saint-Venant Model



(b) Hydrostatic Navier-Stokes Model

Figure 3. Transcritical flow over a 2D bump. Longitudinal cross-section of vertical velocities

in the region of steep bathymetry making the transition between the estuary and the coastal waters. Figure 4 shows the velocity of the fluid obtained after 2000 seconds with a hydrostatic (upper figures) and a non-hydrostatic simulation (lower figures).

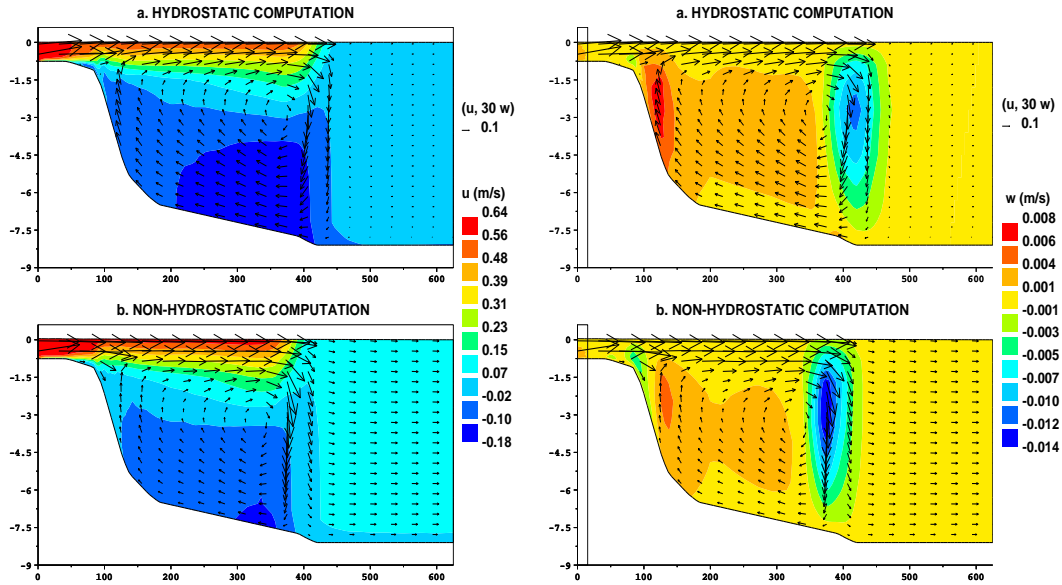


Figure 4. Vertical cross-section at $t = 2000$ s.

We have also worked on improving the treatment of the moving surface in 3D domains. In the Telemac-3D system, the classical *sigma-transformation* technique is applied, which is widely used in the atmospheric and oceanographic communities. The link between this technique and the *ALE approach* was still not clear. Therefore, we have shown that the sigma-transformation is equivalent to the ALE approach for a particular type of ALE mappings. This transformation technique presents many advantages; essentially, the accurate assimilation of the bed and surface boundaries and the possibility of easily incorporating boundary layers. But the classical variants of the sigma-transformation – which are still the ones mostly used – present important drawbacks. Indeed, they limit the possibilities of vertical discretization and therefore prevent from correctly adapting the mesh to the particular needs of the simulation. Especially, they prevent from using horizontal levels, which is essential to the accurate representation of horizontal density-stratifications. Taking advantage of the reinterpretation of the sigma-transformation as a particular ALE formulation, we propose a very general transformation [31], allowing for a great adaptability of the vertical discretization and therefore overcoming some drawbacks of the classical sigma-transformation. Figures 5 and 6 illustrate the improvement obtained using the generalized sigma-transformation for the simulation of a standing basin with a horizontal density-stratification. Figure 5 shows the results obtained using a classical sigma mesh : they reveal a perturbation in the stratification and the creation of spurious velocities.

On the contrary, the results in Figure 6 show an undisturbed stratification and no spurious velocities. They are obtained using the generalized sigma-transformation, which allows to impose a fixed plane at the height of the stratification.

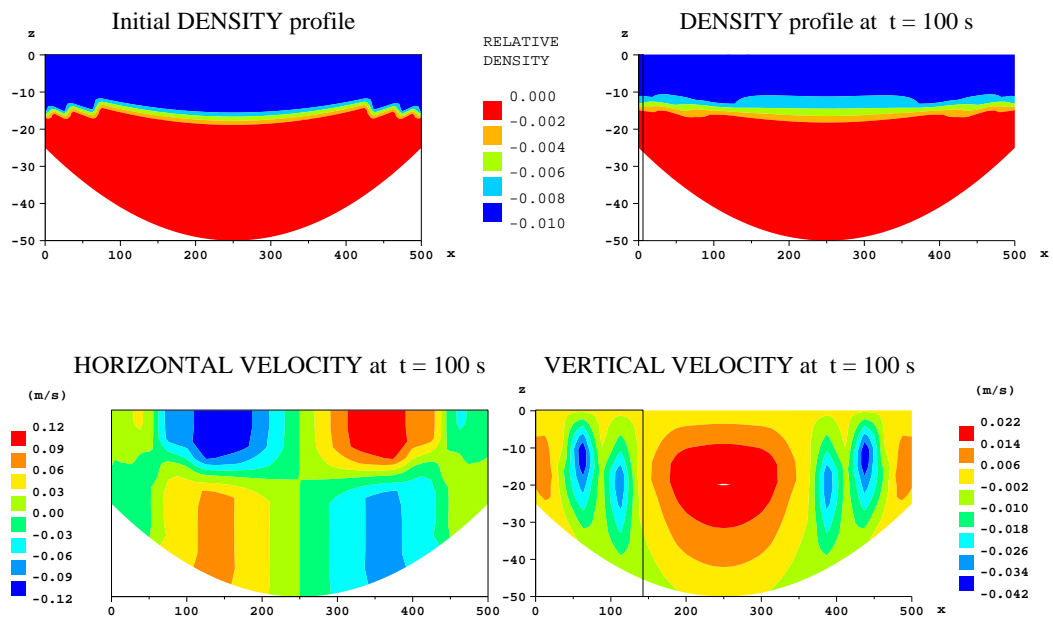


Figure 5. Simulation using a classical sigma mesh. Vertical cross-section at $t = 100$ s

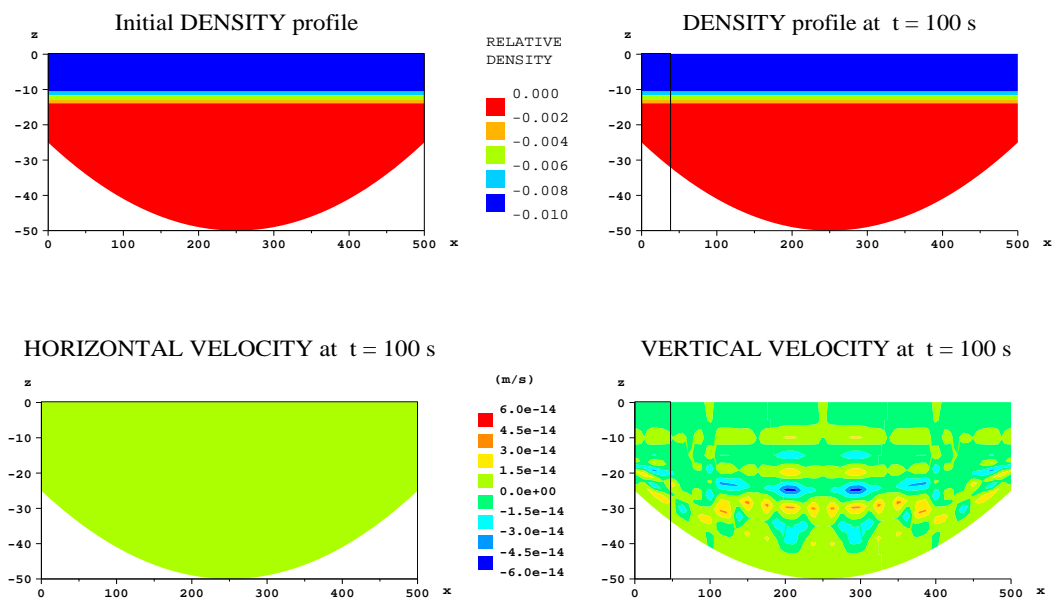


Figure 6. Simulation using a mesh with a fixed horizontal plane at the level of the stratification. Vertical cross-section at $t = 100$ s

Finally, we have presented a *monotonic and conservative* numerical scheme for the resolution of the linear advection problem set on a moving domain [30]. The main idea is to use the *Multidimensional Upwind Residual Distributive* (MURD) approach, well-known for its ability to satisfy the properties just mentioned, as well as the high accuracy and stability of the solution. However, the schemes based on this approach have been developed for problems defined on fixed domains, and their extension to *moving domains* is not yet well-established. It has been proven by several authors that the domain movement can affect numerical schemes in a negative way, leading to a loss of their accuracy, stability or conservation properties. In certain cases, the satisfaction of a particular condition – the so-called *Geometric Conservation Law* (GCL) – ensures the preservation of these properties. We have derived a generic form of MURD schemes for the non-conservative ALE formulation of the continuous linear advection problem with divergence-free advection velocity set on a moving domain. We have then shown that the properties ensured by the residual distribution schemes set on fixed domains are not altered by the domain movement, except from the mass preservation of the advected quantity. Thus, we have formulated the additional condition an *ALE-MURD scheme* must satisfy to be mass-conservative when the domain moves. This conservation constraint is strongly linked to the GCL. Finally, we have applied this conservative advective ALE-MURD scheme to the particular framework of three-dimensional free surface flows.

6.2.4. Granular flows, debris avalanches

The hypothesis of shallow flow is generally well satisfied by debris avalanches. In collaboration with the Laboratoire de Modélisation et Tomographie Géophysique (IPGP, Paris 7) and the Department of Structural and Geotechnical Engineering, Politecnico di Torino, Italy, we have considered the simulation of debris avalanches using the Saint-Venant model in which the behaviour of granular media is taken into account by a Coulomb type friction term [36].

We have recently studied the case where the earth pressure coefficient is not constant but depends whether the downslope flow is expanding or contracting. To simulate this effect, we assume that the earth pressure coefficient depends on the sign of the divergence. In [24], we show the influence of this varying coefficient on some numerical results.

6.3. Parameter identification for a one-dimensional blood flow problem

Keywords: *adjoint equation, blood flow model, inverse problem, multiscale model coarsening, parameter estimation.*

Participants: Astrid Decoene, François Clément [Estime project], Jean-Frédéric Gerbeau [Reo project], Vincent Martin [Reo project].

The purpose of this work is to use a variational method to identify some of the parameters of one-dimensional models for blood flow in arteries. These parameters can be fit to approach as much as possible some data coming from experimental measurements or from numerical simulations performed using more complex models. A nonlinear least squares approach to parameter estimation was chosen, based on the optimization of a cost function. The resolution of such an optimization problem generally requires the efficient and accurate computation of the gradient of the cost function with respect to the parameters. This gradient is computed analytically for a second order Taylor-Galerkin discretization of the one-dimensional hyperbolic, using an adjoint approach.

We have obtained some preliminary numerical tests. In these simulations, we mainly focused on determining a parameter that is linked to the mechanical properties of the arterial walls, the compliance. The synthetic data used to estimate the parameter were obtained from a numerical computation performed with a more accurate model: a three-dimensional fluid-structure interaction model. The first results seem to be promising. In particular, it is worth noticing that the estimated compliance which gives the best fit is quite different from the values that are commonly used in practice.

7. Contracts and Grants with Industry

7.1. Free surface flows

Participants: Emmanuel Audusse [Freie Universitat Berlin], Marie-Odile Bristeau, Astrid Decoene, Jean-Frédéric Gerbeau [Reo project], Benoit Perthame.

The studies concerning the kinetic scheme for Saint-Venant equations and the improvement of the 3D Navier-Stokes solver are supported by a contract with the LNHE of EDF. The results are included in the Telemac-2D and Telemac-3D softwares of EDF.

8. Other Grants and Activities

8.1. Actions at region level

Participation to the GDR-CNRS “CHANT”(équations Cinétiques et Hyperboliques : Aspects Numériques, Théoriques, et de modélisation).(url <http://chant.univ-rennes1.fr>)

Participation to a working group on the debris avalanches with the Laboratoire de Modélisation et Tomographie Géophysique (IPGP, Paris 7) and the Department of Structural and Geotechnical Engineering, Politecnico di Torino, Italy.

For the development of mathematics and scientific computing in life sciences, the project (Bang) has received in year 2005 a grant (tailored with ENS-DMA) from the *Louis D. Foundation* of the french *Académie des Sciences*.

ENS-DMA and Bang project take part to the ANR project *MACBAC* (Analyse multidisciplinaire du processus de colonisation de surface par les bactéries: surfactine, migration, formation des profils) managed by S. Seror at the *Institut de génétique et microbiologie* -Université de Paris Sud.

8.2. European actions

8.2.1. RTN network HYKE

Participation to the european network HYKE (Hyperbolic and Kinetic equations). (url <http://www.hyke.org>).

8.2.2. RTN network M3CS-TuTh

Participation to the european network M3CS-TuTh (Modelling, Mathematical Methods and Computer Simulation of Tumour Growth and Therapy). (url <http://calvino.polito.it/~mcrtn>).

8.3. Visit and invitations

- Invitation of G. Kobelkov (University of Moscow) -1 week-

9. Dissemination

9.1. Scientific community

Benoit Perthame is Editor-in-chief of M2AN and editor in various journals (CALCOLO, CPDE, SIAM J. Math. Analysis, DCDS(B))

9.2. Participation to congresses, workshops,...

- Enumath 2005 “Sixth European Conference on Numerical Mathematics and Advanced Applications”, Santiago de Compostela, Spain, July 2005 (M.O. Bristeau)
- Colloquium INRIA Calcul scientifique, december 2004 (B. Perthame)
- Conference in the honor of B. Engquist -60th Birthday- (B. Perthame)
- **ACM-SFC** Association de Chronobiologie Médicale (Aussois, March) and Société Francophone de Chronobiologie Strasbourg, April 2005 (J. Clairambault)
[J. Clairambault is a member of the bureau of both these chronobiology societies].
- **Argenteuil** J. Clairambault has given a conference for lycée students and teachers (Lycée G. Braque, Argenteuil, in April 2005) on the theme: “Mathematical modelling in oncology: cell cycle, circadian clock, and optimisation of the action of drugs”.
- **ECMTB** European Conference on Mathematical and Theoretical Biology ECMTB 2005, Dresden, July (B. Perthame, J. Clairambault)
[J. Clairambault has co-organised with D. Drasdo (Leipzig) in the ECMTB 2005 Conference a minisymposium dedicated to “Growth regulation in tissues”].
- **MSV** International workshop *Mathématiques en Sciences du Vivant*, organised by Régis Ferrière, Benoit Perthame, and Sylvain Sorin, Paris, September 2005 (B. Perthame, J. Clairambault)
[talk given by J. Clairambault: “Cancer growth and therapy, and the use of mathematical models”]

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Major publications by the team in recent years

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