INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team CONGE

Geometric control for nonlinear systems

Lorraine

THEME NUM

Activity Report

2005

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1. Team

CONGE is a joint team with the LMAM (Laboratoire de Mathématiques et Applications de Metz) which is a laboratory recognized by the CNRS (UMR 7122).

Head of project-team

Jean-Claude Vivalda [Research manager (DR) Inria]

Administrative assistant

Christel Wiemert [Secretary (AI)]

Staff member INRIA

Abderrahman Iggidr [Research officer (CR) Inria]

Staff member Metz University

Philippe Adda [Assistant professor]
Rachid Chabour [Assistant professor]
Gauthier Sallet [Professor]

Staff member Nancy University

Edouard Richard [Assistant d professor]

Visiting scientist

Boris Kalitine [Minsk University – Bielorussia (1 month)]

PhD students

Fehd Ben Aïcha [Thèse CIFRE Renault]
Jean-Luc Dimi [Joint PhD with Brazzaville University (Congo)]
Ismaïl Gourragui [INRIA-LGIPM scholarship, Joint PhD with LGIPM]
Mohamed Mabrouk [Tunisian scholarship]
Papa Ibrahima N'Diaye [regional scholarship, Joint PhD with Saint-Louis University (Senegal), INRIA and INRA]

Research scientist (partner)

Gabriel Abba [Professor (Metz University), LGIPM]
François Léonard [Assistant d professor (ENIM), LGIPM]

2. Overall Objectives

2.1. Overall Objectives

The CONGE project is a joint team with INRIA and Metz University through the LMAM (Laboratoire de Mathématiques et Applications de Metz) which is recognized by the CNRS (UMR 7122).

The research topics are in the framework of the nonlinear systems theory. More specifically, the team deals with the study of the stabilization and the theory of observers and observability. Moreover, modeling and simulation are parts of the issues of the team.

Stabilizing a system about an equilibrium point consists in finding a static or dynamic feedback which makes the equilibrium asymptotically stable. Several tools are used to achieve this task: backstepping, feedforwarding, positive semi-definite functions...

An observer is an additional dynamical system which aims at providing a state estimation. The observer makes use of the known variables of the system, i.e. the inputs and the outputs. The design of observers is a well mastered technique in the case of linear systems but is a more delicate task for nonlinear systems.

3. Scientific Foundations

3.1. Feedback stabilization

We consider finite-dimensional systems written as:

$$\left\{ \begin{array}{lcl} \dot{x} & = & f(x,u) \\ y & = & h(x) \end{array} \right.$$

The problem of stabilization consists in finding a command law $x \mapsto u(x)$ depending on the state x such that the closed-loop system $\dot{x} = f(x, u(x))$ is asymptotically stable about an equilibrium point of interest. One can also find a feedback law which depends only on the output y.

3.2. Positive semi-definite functions

Participants: Boris Kalitine, Rachid Chabour.

A well known theorem due to Lyapunov allows us to conclude to the asymptotic stability of an equilibrium: consider a system of differential equations $\dot{x} = f(x)$ (with $f(0) = 0$), if there exists a positive definite function V such that $\nabla V \cdot f(x)$ is negative definite, then we can assert that 0 is an asymptotically stable equilibrium point. The knowledge of such a Lyapunov function is often necessary for the design of a stabilizing feedback but it is quite difficult to find such a function.

A little-known result in Occident, due to Kalitine and Bulgakov, allows us to make use of *semi-definite* positive functions in the investigation of the stability of an equilibrium. Searching such functions is obviously easier and their use simplifies the design of stabilizing feedback laws. Notice that, on this subject, there exists a great amount of works in the literature of the countries of Eastern Europe: the original result has been extended to discrete-time systems, to PDE, to non autonomous periodic systems, etc. The team is working to extend these results to non-autonomous systems and stochastic systems.

3.3. Observers

Participants: Ismaïl Gourragui, Abderrahman Iggidr, Mohamed Mabrouk, Gauthier Sallet, Jean-Claude Vivalda.

Consider a real system modeled by the differential system:

$$\left\{ \begin{array}{lcr} \dot{x} & = & f(x,u) \\ y & = & h(x) \end{array} \right. \tag{1}$$

where the observation function h represents the set of measures made on the physical system. An observer is an auxiliary dynamical system:

$$\left\{ \begin{array}{l} \dot{z} = \Phi(z,y,u) \\ \hat{x} = \theta(z) \end{array} \right. \tag{2}$$

which provides at any time t an estimation $\hat{x}(t)$ of the real state $x(t)$. More specifically, we have:

$$\lim_{t \to \infty} \parallel \hat{x}(t) - x(t) \parallel = 0.$$

If every parameter of system (1) is known with enough precision and if it is possible to design an observer, a differential equation solver can give an estimation of the state of system (1).

The team investigates the theory of observability and observers for finite dimensional systems. More specifically, a current subject of interest is the design of observers for some mechanical systems (Ph.D. thesis of M. Mabrouk) or for the switched reluctance motor (Ph.D. thesis of I. Gourragui).

Another subject is the design of observers for biological systems, more specifically for systems which model the evolution of fishes populations submitted to a fishing effort. In this case, our aim is to stabilize the size of the population around an equilibrium point but, since it is not possible to measure all the state variables, it is necessary to design an observer which gets an estimation of the sizes of the different age classes. Before designing an observer, it is necessary to investigate the observability property. One definition of this last concept is as follows: system (1) is said *observable* if, given two initial conditions $x_0 \neq \overline{x}_0$, there exists an input u such that the solutions $x(t)$ and $\overline{x}(t)$ starting from x_0 and $\overline{x}_0$ satisfy the inequality $h(x(t)) \neq h(\overline{x}(t))$ for all t in a set of nonzero measure. If we are concerned with the construction of observers that converge with an arbitrary speed, the observability condition is necessary. From a theoretical point of view, it is then important to know "how many" systems are observable, that's why we study the problem of the genericity of the observability for discrete-time

4. Application Domains

4.1. Panorama

Here are some fields of application of the team:

- modeling and control of fluidpower systems;
- modeling and control of switched reluctance motors;
- epidemiology and biological systems.

4.2. Fluidpower

Keywords: *Fluidpower systems*.

Participants: Edouard Richard, Jean-Claude Vivalda.

In this field, our aim is to apply the techniques of nonlinear control to fluidpower systems. We carry out an activity of modeling and analysis of the mathematical properties of the model. The main problems that we plan to study are:

- the modeling and the improvement of the waterjet cutting machine;
- the design of command laws for the automatic control of hydraulic presses.

These works are made within the HYDRAULYCA resource center for fluidpower industry.

4.3. Switched reluctance motor

Participants: Gabriel Abba, Ismaïl Gourragui, François Léonard, Jean-Claude Vivalda.

High-speed Machining experienced a considerable development these last years. Under the impulse of a keen demand of the aircraft industry, high-speed machining centers have been developed to manufacture parts of high degree of accuracy out of aluminium, titanium, and their alloys. This technology makes it possible to machine very hard materials (glass, ceramics). Moreover, specific cutting conditions lead to obtain a better surface quality at lower cost. The mechanical engineering industry then largely attempted to develop the high-speed machining for the manufacture of more conventional parts. In collaboration with the LGIPM (Laboratoire de Génie Industriel et Production Mécanique), a thesis on the subject of the modeling and the automatic control of switched reluctance motor is in preparation. As far as we are concerned with this question, there are several problems of interest:

- there does not exist any precise mathematical model for all the operating range of these motors;
- at very high speed, we have to deal with the problem of command saturation;
- the measure of the angular position can be made only through an observer.

4.4. Epidemiology

Participants: Philippe Adda, Abderrahman Iggidr, Gauthier Sallet, Jean-Luc Dimi.

Epidemiology is a new research topic in our team. Emergent diseases led to a renewed interest for the study of the infectious diseases, so that the mathematical models become significant tools in the analysis of the propagation and the control of infectious pathologies. The understanding of the characteristics of the transmission of an infectious disease in a community or a country can lead to better approaches to decrease the transmission of this disease. The mathematical models can be used to compare, plan, set up, evaluate and optimize various programs of detection, prevention, therapy and control of a disease. We hope to obtain some significant results concerning the intra-host models for paludism and tuberculosis, and more particularly the understanding of the feedbacks implying two classes of auxiliary lymphocites Th_1 and Th_2.

5. New Results

5.1. Semi-definite positive functions

Participants: Boris Kalitine, Rachid Chabour.

A generalization of the works of B. Kalitine to the case of non autonomous differential equations has been made by introducing semi-definite time-varying functions.

5.2. Observers

Participants: Jean-Claude Vivalda, Sabeur Ammar, Mohamed Mabrouk.

We studied the genericity of the observability for discrete-time systems written as:

$$\left\{ \begin{array}{rcl} x_{k+1} & = & f(x_k, u_k) \\ y_k & = & h(x_k, u_k) \end{array} \right.$$

where x_k belongs to a compact manifold and y_k is in $\mathbb{R}^p$. Consider the following mapping:

$$\begin{array}{rccl} \Theta : & M & \longrightarrow & (\mathbb{R}^p)^{2n+1} \\ & x & \longmapsto & (h(x,u_0), h(f^1(x,\underline{u_1}),u_1), \cdots, h(f^{2n}(x,\underline{u_{2n}}),u_{2n}) \end{array}$$

where

$$\begin{array}{rcl} f^1(x,\underline{u_1}) & = & f(x,u_0) \\ f^{k+1}(x,\underline{u_{k+1}}) & = & f(f^k(x,\underline{u_k}),u_k) \\ \underline{u_k} & = & (u_0, \cdots, u_{k-1}). \end{array}$$

We already prove that mapping Θ is, generically, one to one; we recently proved that, generically, it is also an embedding, an article on this subject is in preparation. Another subject of interest is the design of observers for mechanical systems: for some systems with two degrees of freedom, we are able, under some conditions, to exhibit a change of variables which makes the system triangular and allows the construction of an observer which converges exponentially for bounded feedback.

5.3. Epidemiology

Participants: Gauthier Sallet, Jean-Claude Kamgang.

For some class of epidemiological models, a result of global asymptotic stability about the disease free equilibrium has been proved under some assumptions related to the basic reproductive ratio R_0.

6. Contracts and Grants with Industry

6.1. Renault

A “Thèse CIFRE” with Renault about the modelization and the control of the fuel cell has been started in collaboration with SOSSO2 project.

7. Other Grants and Activities

7.1. National initiatives

The team CONGE is one of the founder members of the GDR 1107 (Methods and models of automatic control in the study of the dynamics of ecosystems and renewable resources) of the program environment program and takes part regularly in its work.

7.2. International relations

7.2.1. Algeria

Together with the laboratory of mathematics of the university of Metz the project is working on the creation of a scientific collaboration network between some laboratories of mathematics of the East of Algeria and french universities. The theme of this collaboration is the dynamical systems.

7.2.2. Tunisia

A PAI (project of co-operation) between CONGE and the University of Sfax was accepted.

7.2.3. Europe

The members of the team are also members of the Control Training Site (CTS). Gauthier Sallet is a member of the editorial board of the CTS.

7.3. Visiting scientists

- B. Kalitine (University of Minsk, Bielorussia), May 2005 (1 month);
- A. Benabdallah (University of Sfax, Tunisia), November 2005 (1 week);
- H. Jerbi and M. Ammami (University of Sfax, Tunisia), December 2005 (1 week).

8. Dissemination

8.1. Leadership within scientific community

The team is one of the founders of the ‘ GDR CNRS 1107” : Methods and models of automatic control in the study of the dynamics of ecosystems and renewable resources.

8.2. Conferences and workshops committees, invited conferences

Participants: Rachid Chabour, Gauthier Sallet, Jean-Claude Vivalda.

- R. Chabour and J.-C. Vivalda were invited to the “Journées mathématiques algéro-françaises” (JMAF’05, Constantine).
- A. Iggidr is a member of the scientific and organization committees of the CIMPA summer school “*Modèles et outils mathématiques pour l’analyse et la régulation des systèmes halieutiques*” which held in 2005 in Mauritania. There were 50 participants, 45 hours of lectures have been given by specialists from University of California at Berkeley, IRD, INRIA, IFREMER, University of Marrakech. There were also 14 talks from IMROP researchers and some other participants.
  The school has been supported by CIMPA, INRIA, IRD, French Embassy, IMU, University of Nouakchott and the EDP-CONTROL Network.
- G. Sallet participated to the workshop “International workshop on differential equations in mathematical biology” (11-13 July 2005) and to a conference of the EPIMATH network held in Yaounde (7-12 November 2005).

9. Bibliography

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[3] J. KAMGANG, G. SALLET. *Global asymptotic stability for the disease free equilibrium for epidemiological models*, in "CRAS Série I", vol. 341, 2005, p. 433–438.

Publications in Conferences and Workshops

[4] M. MABROUK, J.-C. VIVALDA. *State transformation for Euler-Lagrange systems*, in "Proceedings of INC-INCO 2005, Barcelone", 2005.

[5] G. SALLET. *Théorie du contrôle et équations différentielles de Riccati*, in "Contrôle non linéaire et applications. École d'été du CIMPA (2003), Tlemcen", 2005.