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Project-Team galaad

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Activity Report

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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable resolution of geometric and algebraic problems, which are encountered in fields such as CAGD, robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry, computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical calculations, usually opposed to symbolic calculations, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: geometry, resolution of algebraic systems of equations, and symbolic-numeric links.

3.2. Geometry

We are interested in geometric modeling problems, based on non-discrete models, mainly of semi-algebraic type. Our activities focus in particular on the following points:

3.2.1. Geometry of algebraic varieties

In order to solve effectively an algebraic problem, a preprocessing analysing step is often mandatory. From such study, we will be able to deduce the method of resolution that is best suited to and thus produce an efficient solver, dedicated to a certain class of systems. The effective algebraic geometry provides us tools for analysis and makes it possible to exploit the geometric properties of these algebraic varieties. For this purpose, we focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials.

3.2.2. Geometric algorithms for curved arcs and surface patches

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately the algebraic varieties. On the other hand, traditional algorithmic geometry deals with problems whose data are linear objects (points, segments, lines) but in very great numbers. Combining these two points of view, we concentrate on problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well known, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

3.2.3. Geometry of singularities and topology

The analysis of singularities for a (semi)-algebraic set provides a better understanding of their structure. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

3.2.4. Geometry, groups, and invariants

The objects in geometrical problems are points, lines, planes, spheres, quadrics, ... Their properties are, by nature, independent from the reference one chooses for performing analytic computations. Which leads us to methods from invariant theory. In addition to the development of symbolic geometric computations that exploit these invariants, we are also interested in developing more synthetic representations for handling those expressions.

3.3. Resolution of algebraic systems

The underlying representation behind a geometric model is often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems. Here are the particular approaches that we develop to handle such questions.

3.3.1. Algebraic methods and quotient structure

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and calculating normal forms in such structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

3.3.2. Duality, residues, interpolation

We are interested in the “effective” use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

3.3.3. Structured linear algebra and polynomials

The preceding work lead naturally to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and the implications in solving polynomial equations [38].

3.3.4. Decomposition and factorisation

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.3.5. Deformation and homotopy

The behavior of a problem in the vicinity of a data can be interpreted in terms of deformations. Accordingly, the methods of homotopy consist in introducing a new parameter and in following the evolution of the solutions between a known position and the configuration one seeks to solve. This parameter can also be introduced in a symbolic manner, as in the techniques of perturbation of non-generic situations. We are interested in these methods, in order to use them in the resolution of polynomial equations as well as for new algorithms of approximate factorization.

3.4. Symbolic-numeric computation

Either in geometric or algebraic problems, symbolic and numeric computation are closely intertwined. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods, as explained now.

3.4.1. Certification

The numerical problems are often approached locally. However in many situations, it is significant to give global answers, making it possible to certify calculations. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures [34].

3.4.2. Approximation

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities. This approach combines geometrical, algebraic and algorithmic aspects.

3.4.3. Degeneracies and arithmetic

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics. For example, we work on the computation of the sign of expressions, and on approaches combining modular and approximate computations, which speed up the exact answer [29].

4. Application Domains

4.1. CAGD

Keywords: *engineering computer-assisted*, *geometric modeling*.

3D modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). The involved mathematical objects have often an algebraic nature, which are then discretized for easy handling. The treatment of such objects can sometimes be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts of CAGD we have.

4.2. Computer vision and robotics

Keywords: *calibration*, *engineering*, *reconstruction*.

Robotics and computer vision come with specific applications of the methods for solving polynomial equations. That is the case for instance, for the calibration of cameras, robots, computations of configurations and workspace. The resolution of algebraic problems with approximate coefficients is omnipresent.

4.3. Molecular biology and geometrical structures

Keywords: *biology*, *health*.

The chemical properties of molecules intervening in certain drugs are related to the configurations (or conformations) which they can take. These molecules are seen as mechanisms of bars and spheres, authorizing rotations around certain connections, similar to robots series. *Distance geometry* thus plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. The methods we develop are well suited for solving such a problem.

5. Software

5.1. synaps, a module for symbolic and numeric computations

Keywords: *C++*, *algebraic number*, *bezoutian*, *effective algebraic geometry*, *eigenvalues*, *genericity*, *geometry*, *iterative methods*, *linear algebra*, *links symbolic-numeric*, *polynomials*, *resultant*, *solving*, *sparse matrices*, *stability*, *structured matrices*.

Participants: Guillaume Chèze, Ioannis Emiris, Grégory Gatellier, Bernard Mourrain [contact person], Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt, Philippe Trébuchet, Elias Tsigaridas, Julien Wintz.

See SYNAPS *web site: http://www-sop.inria.fr/galaad/logiciels/synaps/.*

We consider problems handling algebraic data structures such as polynomials, ideals, ring quotients, ..., as well as numerical computations on vectors, matrices, iterative processes, ...Until recently, these domains were separated: software for manipulating formulas is often not effective for numerical linear algebra; while the numerically stable and efficient tools in linear algebra are usually not adapted to the computations with polynomials.

We design the software SYNAPS (SYmbolic Numeric APplicationS) for symbolic and numerical computations with polynomials. In this library, a list of structures and functions makes it possible to operate on vectors, matrices, and polynomials in one or more variables. Specialized tools such as LAPACK, GMP, SUPERLU, RS, GB, … are also connected and can be imported in a transparent way. These developments are based on C++, and attention is paid to the generic structures so that effectiveness would be maintained. Thanks to the parameterization of the code (*template*) and to the control of their instantiations (*traits*, *template expression*), they offer generic programming without losing effectiveness. This powerful kernel contains univariate and multivariate algebraic solvers as well as subdivision solvers and several resultant-based methods for projection operations.

Many functionalities of the library are now also available through the computer algebra system MATHEMAGIX, as dynamic binary libraries.

5.2. axel, an extension package of synaps for curves and surfaces

Keywords: *computational algebraic geometry*, *curve*, *implicit equation*, *intersection*, *parameterisation*, *resolution*, *singularity*, *surface*, *topology*.

Participants: Bernard Mourrain [contact person], Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt, Julien Wintz.

See AXEL *web site: http://www-sop.inria.fr/galaad/logiciels/axel/.*

We are developing a module called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The library contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This package is now distributed with the library SYNAPS. Some components of the library are connected to industrial CAGD software.

5.3. multires, a maple package for multivariate resolution problems

Keywords: *Polynomial algorithmic*, *eigenvalues*, *interpolation*, *linear algebra*, *residue*, *resultant*.

Participants: Laurent Busé [contact person], Ioannis Emiris, Bernard Mourrain, Olivier Ruatta, Philippe Trébuchet.

See MULTIRES *web site: http://www-sop.inria.fr/galaad/logiciels/multires/.*

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, and is not effectiveness, which is achieved in a more adapted environment as SYNAPS. It provides methods to build matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay [37] for projective resultant, Jouanolou [36] combining matrices of Macaulay type, and Bezout and (sparse) resultant on a toric variety [33], [32]. Also being added are a new construction proposed for the residual resultant of a complete intersection [30], functions for computing the degree of residual resultant illustrated in [31], and the geometric algorithm for decomposing an algebraic variety [35]. The Weierstrass method generalized for several variables (presented in [39]) and a method of resolution by homotopy derived from such generalization are implemented as well. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

6. New Results

6.1. Algebra

6.1.1. Solving algebraic equations

Participants: Mohamed Elkadi, Bernard Mourrain.

In [15], we give an introductory presentation of algebraic and geometric methods for solving a polynomial system $f_1 = \cdots = f_m = 0$. The algebraic methods are based on the study of the quotient algebra $\mathcal{A}$ of the polynomial ring modulo the ideal $I = (f_1, ..., f_m)$. We show how to deduce the geometry of solutions from the structure of $\mathcal{A}$ and in particular, how solving polynomial equations reduces to eigenvalue and eigenvector computation of multiplication operators in $\mathcal{A}$. We give two approaches for computing the normal form of elements in $\mathcal{A}$, used to obtain a representation of multiplication operators. We also present the duality theory and its application to solving systems of algebraic equations. The geometric methods are based on projection operations which are closely related to resultant theory. We present different constructions of resultants and different methods for solving systems of polynomial equations based on these formulations. Finally, we illustrate these tools on problems coming from applications in computer-aided geometric design, computer vision, robotics, computational biology and signal processing.

6.1.2. Computation of normal forms in a quotient algebra

Participants: Bernard Mourrain, Philippe Trébuchet [SALSA].

In the paper [23], we describe a new method for computing the normal form of a polynomial modulo a zero-dimensional ideal I. We give a detailed description of the algorithm, a proof of its correctness, and finally experimentations on classical benchmark polynomial systems. The method that we propose can be thought as an extension of both the Gröbner basis method and the Macaulay construction. We have weaken the monomial ordering requirement for Gröbner bases computations, which allows us to construct new type of representations for the quotient algebra. This approach yields more freedom in the linear algebra steps involved, which allows us to take into account numerical criteria while performing the symbolic steps. This is a new feature for a symbolic algorithm, which has an important impact on its practical efficiency, as it is illustrated by the experiments. This paper received a "Distinguished Paper Award" at ISSAC'05, the annual conference of our community.

6.1.3. Univariate polynomial solvers

Participants: Ioannis Emiris [Univ. Athens], Bernard Mourrain, Fabrice Rouillier [SALSA], Marie-Francoise Roy [IRMAR, Rennes], Elias Tsigaridas [Univ. Athens].

In [17], we explain how the Bernstein's basis, widely used in Computer Aided Geometric Design, provides an efficient method for real root isolation, using De Casteljau's algorithm. The link between this approach and more classical methods for real root isolation such as Uspensky's method is discussed and we analyse these methods, from a complexity point of view.

Solving univariate polynomial equations is a key ingredient of geometric computation on curves and surfaces. With E. Tsigaridas and I. Emiris, we continue this investigation of subdivision solvers, based either on Sturm Sequence computation or on Bernstein polynomial representation. New bounds on the complexity of both approaches are proposed, improving significantly the previously known bounds. Implementations in the library SYNAPS and comparisons with other tools illustrate these improvements. This work is submitted for publication.

6.1.4. *Multivariate polynomial solvers in Bernstein basis*

Participants: Bernard Mourrain, Jean-Pascal Pavone.

The work on a new algorithm for solving a system of polynomials in Bernstein form, has been finalized [25] and submitted for a journal publication. In particular, we analyze its complexity in terms of the degree of equations and of differential invariants of the system. The approach that we develop can be seen as an improvement of the *Interval Projected Polyhedron* algorithm proposed by Sherbrooke and Patrikalakis. It uses a powerful reduction strategy thanks to an univariate root finder based on *bezier clipping* and *Descartes rule*. In the context of the network Aim@Shape, we made some comparisons with other multivariate subdivision solvers, which show the impact of our preconditioning and reductions techniques. This approach is described in the chapter Shape Interrogation of the “State of the Art” book of the network, submitted for publication.

6.1.5. *Discriminants of homogeneous polynomials*

Participant: Laurent Busé.

Roughly speaking, discriminants of homogeneous polynomials give a necessary and sufficient condition so that a given variety in a projective space has singularities. These objects are thus of interest in CAGD since it is useful to detect singularities of a curve or algebraic surfaces which are supposed to represent real objects. We started a work which aimed to give a computational theory of these discriminants. Surprisingly, even if these objects have been extensively studied in algebraic geometry, most of the known results give some invariant properties of these discriminants and only very few explicit formulas are known to compute them. At that time, this work is still in progress.

6.2. Geometry

6.2.1. *Classification of parameterised surfaces*

Participants: Mohamed Elkadi, André Galligo, Thi-Ha Le.

Parametrized surfaces of low degrees are very useful in applications, especially in Computer Aided Geometric Design and Geometric Modeling. The precise description of their geometry is not easy in general. Here we study some of the corresponding projective complex surfaces of low implicit degree (i.e. smaller than 12). We show that, generically up to linear changes of coordinates, they are classified by a few number of continuous parameters (called moduli). We present normal forms and provide compact implicit equations for these surfaces and their singular locus together with a geometric interpretation.

We have also continued our work on the classification and the geometric study of parametric surfaces of bidegree (1,2) over the complex field and over the real field. Patches of these surfaces are used as models in Computer Aided Geometric Design. We recall classical results (for the complex case) which go back to Cremona and to Cayley in the 19th century and were reviewed by Edge in 1931. We provide another point of view by considering a dual scroll similar to the one introduced by Piene and Sacchiero in 1984. This allows a more algebraic complex classification, which eases the obtention of the classification of the real generic cases. We consider a list of non generic cases, classify and further study them geometrically and computationally. This work has been accepted for publication in the proceedings of the last AGGM conference.

6.2.2. *A computational study of ruled surfaces*

Participants: Laurent Busé, Marc Dohm, Mohamed Elkadi, André Galligo.

We studied rational ruled surfaces and the associated so-called μ-bases which were recently considered in a series of articles by F. Chen and coworkers. We gave shorter and more conceptual proofs with geometric insights and more efficient algorithms. In particular, we provided a method to reparameterize an improper parameterization, we briefly explained how to deal with approximate input data and we also provided an algorithmic description of the self-intersection loci. Marc Dohm worked and developed these results during its internship. He is now a Ph.D. in our team and will develop further this topic.

6.2.3. Plane curves intersection problems

Participants: Laurent Busé, Houssam Khalil, Bernard Mourrain.

In this work, in the continuation of the internship of Houssam Khalil in 2004 in our team, we developed an algorithm for solving polynomial equations which uses generalized eigenvalues and eigenvectors of resultant matrices. We give a special attention to the case of two bivariate polynomials and the Sylvester or Bezout resultant constructions which are of interest for application in CAGD. We proposed a new method to treat multiple roots, detailed its numerical aspects and described experiments on tangential problems which show the efficiency of our approach. We also experimented it on an industrial application consisting in recovering cylinders from a large cloud of points and which requires intensive resolution of polynomial equations. This work has been published in [20].

6.2.4. Geometric processing for curves and surfaces

Participants: Laurent Busé, Bernard Mourrain, Jean-Pascal Pavone, Olivier Ruatta.

In the context of the european network Aim@Shape, we review computational problems related to geometric processing for shapes. Indeed, structuring shapes for their analysis usually requires the computation of new geometric features on these shapes, such as the construction of characteristic points or curves on a surface, or the decomposition of this surface, represented as a continuous model, into patches. Such problems occur for instance, when computing points or curves of intersection of several surface, when performing CSG operations, in the visualization of shapes, in the analysis of their structures, their similarities. Intersection is a fundamental operation in Geometric Modeling, but it is also a difficult one. A small perturbation of the input shapes may induces a drastic change in the output, in the case of non-transversal intersection. Another difficulty is the effective representation of an object defined as the intersection of two others. It is not generally possible to represent it exactly and approximation are needed in their treatment (visualization, transmission, ...). In a report delivered for this European network, we describe algorithms which handle such problems. In collaboration, with other partners of the network, we are currently investigating further these critical questions, in order to complete a review on this topic.

A book describing the state of the art for geometric computation on shapes, by the partners of the Aim@Shape network, has been submitted for publication. We contributed to the chapter on Shape Interrogation, by describing methods for solving polynomial equations, such as normal form, resultant-based and subdivision methods.

6.2.5. Self-Intersection

Participants: André Galligo, Jean-Pascal Pavone.

The self-intersection of surfaces is in the CAGD context a real limitation: CAGD systems generally produces surfaces without regards to the rise of self-intersection while they do not provide any way to detect or suppress it. So, in practice it is generally a difficult and time consuming task for the end user to remove "by hand" the unwanted components of the surface. This problem was studied in the GAIA (7.1.2) project. The industrial partner of GAIA, think3, used our algorithm for the automatic suppression (or trimming) of self-intersection in their system. A paper describing this sampling algorithm for the self-intersection of parametrized surfaces has been accepted for publication in the AGGM'04 proceedings. We are now searching for improvements of this approach using polynomials interpolants rather than piecewise linear approximations. We think that the studies of parametrized patches of "small" degrees like biquadratic and bicubic patches can be useful in self-intersection as in intersection (6.2.11). Around the specific problem of bicubic patches self-intersection a paper [21] based on a double point-of-view (resultant and subdivision) has been written and accepted to the ISSAC'05 conference. This work as well as in 6.2.11 strongly depend on the development of the SYNAPS polynomial solvers (6.1.4).

6.2.6. Modelisation with quadrics

Participants: Emmanuel Briand, Bernard Mourrain, Wenping Wang.

Computing the intersection curve of two quadric surfaces is an important problem in geometric computation, ranging from shape modeling in computer graphics and CAD/CAM, collision detection in robotics and computational physics, to arrangement computation in computational geometry.

During the visit of Wenping Wang, we investigated the problem of classification of morphologies of the intersection curve of two quadrics (QSIC) in the 3D real projective space $\mathbb{PR}^3$. We show that there are in total 35 different QSIC morphologies with non-degenerate quadric pencils. For each of these 35 QSIC morphologies, through a detailed study of the eigenvalue curve and the index function jumps we establish a characterizing algebraic condition expressed in terms of the Segre characteristics and the signature sequence of a quadric pencil. We show how to compute a signature sequence with rational arithmetic so as to exactly classify the morphology of the intersection curve of any two given quadrics. This is the content of a paper submitted for publication.

Analysing the relative configurations of two conics and quadrics is a related problem, which Emmanuel Briand considered during his stay in the project. A complete analysis of the possible cases in 2D has been detailled and submitted for publication. Based on tools related to the signature function and its connection with the Uhlig/Williamson normal form, we extend the work of classification of configurations of quadrics to higher dimension, and analyse the stratification of this set of configurations and how the different strata are connected.

6.2.7. The computation of an arrangement of quadrics in 3D

Participants: Julien Wintz, Jean-Pierre Técourt, Bernard Mourrain.

A sweeping algorithm for computing the arrangement of a set of quadrics in $\mathbb{R}^3$ is currently implemented. It defines a "trapezoidal" decomposition in the sweeping plane and maintains its evolution during the sweep. A key point of this algorithm is the manipulation of algebraic numbers. Our aim is to investigate Constructive Solid Geometry (CSG) modeling using implicit geometric objects such as quadratic surfaces, implicit curves and so on.

This implementation makes extensive use of the SYNAPS library especially for the manipulation of algebraic numbers and the use of algebraic tools such as Sturm sequences and Rational Univariate Representation of the roots of a multivariate polynomial system.

6.2.8. Topology of curves and surfaces

Participants: Bernard Mourrain, Jean-Pierre Técourt.

We have developed two new algorithms for meshing curves and surfaces in $\mathbb{R}^3$. In the first algorithm, we deal with algebraic curves defined as the intersection of two implicit surfaces. It is a sweeping algorithm, which assumes that the curve is in generic position with respect to the sweeping direction (here the x-direction). We project the curve onto the xy and xz planes and compute the x-critical points (singular points or points with tangent space orthogonal to the x-direction). We deduce from this computation, the x-coordinates where the topology of the sweeping section can change. We connect the points in the different sections, with an appropriate algorithm, in order to get a graph of points connected by segments, which is isotopic to the tridimensional curve (see [22]). The other algorithm for algebraic surfaces is related to this previous work. It's the first algorithm to provide an isotopic certificated triangulation of a singular algebraic surface. It proceeds as folows. First we compute the polar variety of the surface for a generic direction of projection using the algorithm for 3D curves. Next the topology of specific sections of the surface are computed through the a classical sweeping algorithm. Finally, the different sections are connected using the topology of the polar variety and the faces are triangulated. The justification of the whole algorithm is based on stratification theory and the effective computation of a Whitney stratification of the surface. This result [27] accepted for presentation at the MEGA'05 conference [24], is submitted for a journal publication.

6.2.9. *Meshing real algebraic surfaces*

Participants: Lionel Alberti, George Comte [UNSA], Bernard Mourrain.

We develop a new method to mesh surfaces defined by an algebraic equation, which is able to triangulate the zero set of a polynomial in three variables. The triangulation we obtain is isomorphic to the zero set of the polynomial. This is a development of a previous work for smooth surfaces. We use Bernstein bases to represent the function in a given box and we subdivide this representation according to a generalization of Descartes' rule, until we know the function is monotone in the box, hence assuring that the implicit object is homeomorph to its linear approximation in the cell. In that case, the complexity of the algorithm is related to the Vitushkin variation of the surface. This is published in [19].

In the singular case the surface is no longer homeomorph to a hyperplane in the boxes containing singularities, no matter how small is the box. Therefore Descartes's rule is replaced by a criterion testing whether the surface is homeomorph to a cone which base is the part of the surface lying on the border of the box. The balls satisfying this criterion are called Milnor balls and one can write an effective test thanks to Thom's isotopy lemma which allows to deduce the topological triviality (in this case the conic structure) from transversality conditions that can effectively be tested. This theorem relies on a particular stratification of the algebraic variety called Whitney stratification. It is a very difficult problem to produce efficiently such stratifications in any dimension, but in three dimensional space a result by Speder allows us to compute such stratifications efficiently using projective resultants. This is described in the proceedings of the conference “Singularities” 2005, Marseille, France.

6.2.10. *Approximate implicitization*

Participants: Stéphane Chau, André Galligo.

The methods for finding the exact algebraic representation of rational parametric curves and surfaces are called implicitization and a number of methods for this problem exists such as resultants and Groebner basis. However, for the surfaces, the computation of an exact implicit equation is very time consuming (since it has a high degree) and even worse, it gives an unusable result. Indeed, the implicitization gives a global object (an algebraic surface) whereas the parametrization is local, and then it may be that the algebraic representation has some "phantom components" (i.e. components in excess). Thus, knowing how well a given algebraic surface approximates the corresponding parametric surface in a region of interest is more important than finding the exact representation. This is the approximate implicitization problem. We study this problem for the polynomial parameterized surfaces of bi-degree (2,2). Our approach consists in the construction of some judicious specific geometric constraints to give a linear system of equations.

6.2.11. *Intersection of a family of bi-degree (2,2) Bézier patches*

Participants: Stéphane Chau, André Galligo.

The intersection of procedural surfaces (i.e. surfaces given by evaluation) is one of the most important problem in CAGD. Traditional methods to deal with this problem use triangular meshes, but if the surfaces are complicated, then the number of triangles may be important and it may be inefficient. Moreover, the triangles are not adapted for some complex shapes. So we can try to represent these surfaces by a grid of polynomial parameterized surfaces of bi-degree (2,2) for example. Thus, it is important to have an efficient method to compute the intersection of a family of bi-degree (2,2) Bezier patches. For that, we use a combined subdivision and approximate implicitization method. The general idea is to rely on subdivisions to exclude domains where there is no intersection (eg. using bounded boxes) and then concentrate the complex computation in relevant areas (using approximate implicitization).

6.2.12. *Towards a framework for algebraic modeling*

Participants: Julien Wintz, Jean-Pascal Pavone, Bernard Mourrain.

SYNAPS provides tools for algebraic computations. Its use requires technical skills and a good knowledge of software engineering. So, a framework is beeing studied in order to facilitate the use of such tools and to exploit them in a so-called algebraic modeler, for an interactive use of the algorithms that we develop.

This framework is composed of a toolkit, an exhaustive API (Application Programming Interface) currently studied by the participants, allowing the manipulation of geometric objects towards a rigorous classification between implicit and parametric objects. Several outputs streams are already avalaible but a real time graphical interaction with provided objects and tools is considered in order to make the algorithms accessibility more efficient.

Another aspect of this framework is the sharing of data, in particular among the partners of the european project the AIM@Shape. A formalism to describe algebraic objects has been proposed. Using the metalanguage of terminology description XML, it puts the emphasis on the duality between "mathematical shapes" and "geometric shapes". So, the main concept is the map one, based on the concepts of number, polynomial, bezier, bspline and composition. Using its Document Type Definition (DTD), one can easily provide a statement composed of mathematical notions in order to design geometric shapes computed by our tools.

6.3. Symbolic numeric computation

6.3.1. Solving Toeplitz-block linear systems

Participants: Houssam Khalil, Bernard Mourrain, Michelle Schatzmann [univ. Lyon I].

Structured matrices appear in many applications, such as iterative methods (finite elements, polynomial solvers, ...) in which fast solvers of linear systems are required. We are interested here in fast algorithms to solve a system of equations $Tv = b$ where T is a Toeplitz block Toeplitz matrix, of size $mn \times mn$ with m is the $T's$ size by blocks and n is the size of each block.

We transformed this problem to a rational interpolation problem in two variables, and by using a fast algorithm to solve this rational interpolation problem we device a new algorithm which resolve the problem $Tv = b$ in $\mathcal{O}(nm(\log^2(n) + \log^2(m)))$.

Moreover, we give a relationship between generators of Toeplitz matrix (in the scalar and block case) and bases of module of syzygies of an ideal of polynomials (univariate in scalar case and bivariate in the blocks case). Now we are working to complete this work, especially for the block case. A paper describing this new algorithm is in preparation.

6.3.2. Algorithms for bivariate polynomial absolute factorization

Participants: Guillaume Chèze, André Galligo, Grégoire Lecerf [CNRS, Univ. Versailles].

Absolute factorization stands for factorization over the the algebraic closure of the ground field. This is interesting for the applications; e.g. for a multivariate polynomial with rational coefficients it allows us to decompose further than the rational factorization, for instance into polynomials with real coefficients. This is a long standing problem in Computer algebra.

G. Chèze, in collaboration with A. Galligo, gave the best current algorithm using numerical approximation: it can decompose a bivariate polynomial of degree 200. This work has been accepted for publication in the Journal of Symbolic Computation and will appear in 2006.

G. Chèze, in collaboration with G. Lecerf improved strongly an approach initiated by Gao using exact modular and p-adic computations. They obtained nearly optimal complexity bounds. Their algorithm, implemented in Magma, is able to deal with very large polynomials. This work has also been submitted for publication.

6.3.3. Geometry, approximation and neural networks

Participants: Nikos Pavlidis [University of Patras], Bernard Mourrain, Michael Vrahatis [University of Patras].

In a work developped in the context of our associate team CALAMATA with Greece, we investigated the ability of feedforward neural networks to identify the number of real roots of univariate polynomials, and more generally the ability to approximate semi-algebraic sets. We examined their ability to determine whether a system of multivariate polynomial equations has real solutions on a problem of determining the structure of a molecule. The obtained experimental results indicate that neural networks are capable of performing this task with high accuracy even when the training set is very small compared to the test set. This work is accepted for publication in the journal “Computers and Mathematics with Applications”.

7. Other Grants and Activities

7.1. European actions

7.1.1. acs

Participants: Lionel Alberti, Laurent Busé, Stéphane Chau, André Galligo, Bernard Mourrain [contact person], Jean-Pascal Pavone, Jean-Pierre Técourt, Julien Wintz.

See the ACS *project web site.*
- Acronym: ACS, number FP6-006413
- Title: Algorithms for Complex Shapes.
- Specific Programme: IST
- RTD (FET Open)
- Start date: started 1^{st} May, 2005 - Duration: 3 years
- Participants:
  Univ. Groningen (Netherlands) [coordinating site]
  ETH Zürich (Switzerland),
  Freie Universität Berlin (Germany),
  INRIA Sophia Antipolis (Galaad & Geometrica),
  MPI Saarbrücken (Germany),
  National Kapodistrian University of Athens (Greece),
  Tel Aviv University (Israel)
  GeometryFactory Sarl
- Abstract: computing with complex shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension. Certified topology and numerics, applications to shape approximation, shape learning, robust modeling.

7.1.2. gaia

Participants: Laurent Busé, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo [contact person], Thi Ha Lê, Bernard Mourrain, Jean-Pascal Pavone, Olivier Ruatta.

See the GAIA II *project web site*

In collaboration with the University of Nice UNSA, the GALAAD team is involved in the European project GAIA:
- Acronyme: GAIA II, number IST-2001-35512
- Title: Intersection algorithms for geometry based IT-applications using approximate algebraic methods
- Specific program of the project: IST
- Type of project: FET-Open
- Beginning date: 1st of july 2002 - During: 3 years
- Participation mode of INRIA: participant via the UNSA
- Partners list:
  SINTEF Applied Mathematics, Norvegia,
  Johannes Kepler University, Austria,
  UNSA, France,
  Université de Cantabria, Spain,
  Think3 SPA, Italy and France,
  University of Oslo, Norvegia.

- Abstract of the project: Detection and treatment of intersections and self-intersections, singularity analysis, classification, approximate algebraic geometry and applications to CAGD.

7.1.3. aim@ shape

Participants: Laurent Busé, Emmanuel Briand, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo, Thi Ha Lê, Bernard Mourrain [contact person], Olivier Ruatta.

See the AIM@SHAPE *project web site*

- Acronym: aim@shape, number NoE 50766
- Title: AIM@SHAPE, Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing knowledge Embedded in multidimensional digital objects.
- Type of project: network of excellence
- Beginning date: 1st of january 2004 - During: 4 years
- Partners list:

CNR - Consiglio Nazionale delle Ricerche,
DISI - Universita di Genova,
EPFL - Swiss Federal Institute of Technology,
IGD - Fraunhofer,
INPG - Institut National Polytechnique de Grenoble,
INRIA
CERTH - Center for Research and Technology Hellas,
UNIGE - Université de Genève,
MPII - Max-Planck-Institut für Informatik,
SINTEF,
Technion CGGC,
TUD - Darmstadt University of Technology,
UU - Utrecht University,
WIS - Weizmann Institute of Science.

- Abstract of the project: it is aimed at coordinating research on representing, modeling and processing knowledge related to digital shapes, where by shape it is meant any individual object having a visual appearance which exists in some (two-, three- or higher- dimensional) space (e.g., pictures, sketches, images, 3D objects, videos, 4D animations, etc.).

7.1.4. ecg: Effective Computational Geometry for Curves and Surfaces

Participants: Laurent Busé, Ioannis Emiris, André Galligo, Bernard Mourrain, Jean-Pierre Técourt, Elias Tsigaridas.

See the ECG *project web site.*

INRIA (GEOMETRICA and GALAAD) were coordinating the European project:

- Acronym: ECG, number IST-2000-26473
- Title: Effective Computational Geometry for Curves and Surfaces.
- Specific Programme: IST
- RTD (FET Open)
- Started: May 1st 2001 - Ended: April 30, 2005
- Other participants:

INRIA Sophia Antipolis (Galaad & Geometrica) [coordinating site]
ETH Zürich (Switzerland),
Freie Universität Berlin (Germany),
Rijksuniversiteit Groningen (Netherlands),
MPI Saarbrücken (Germany),
Tel Aviv University (Israel)

- Abstract: Efficient handling of curved objects in Computational Geometry. Geometric algorithms for curves and surfaces, algebraic issues, robustness issues, approximation.

Jean-Daniel Boissonnat was the project manager, and Monique Teillaud was the technical project manager.

7.2. Bilateral actions

7.2.1. Associated team CALAMATA

Participants: Ioannis Emiris, Athanasios Kakargias, Bernard Mourrain [contact person], Nikos Pavlidis, Jean-Pierre Técourt, Elias Tsigaridas, Michael Vrahatis.

The team of Geometric and Algebraic Algorithms at the National University of Athens, Greece, has been associated with GALAAD since 2003. See its web site.

This bilateral collaboration is entitled CALAMATA (CALculs Algebriques, MATriciels et Applications). The Greek team (http://www.di.uoa.gr/~erga/) is headed by Ioannis Emiris. The focus of this project is the solution of polynomial systems by matrix methods. Our approach leads naturally to problems in structured and sparse matrices. Real root isolation, either of one univariate polynomial or of a polynomial system, is of special interest, especially in applications in geometric modeling, CAGD or nonlinear computational geometry. We are interested in computational geometry, actually, in what concerns curves and surfaces.

In 2005, we had the visit of J.P. Pavone and B. Mourrain to Athens, to work on univariate polynomial subdivision solvers, the visit of H. Khalil and B. Mourrain to Kalamata for the workshop CALAMATA, and a presentation of their works on the resolution of polynomial equations, by eigenvector computation, at the conference CASC'05 (B. Mourrain was an invited speaker of CASC'05), the visit of G.Tzoumas and E. Fritzilas for the summer school on open software for algebra and geometry at Sophia Antipolis, the visit of E. Tsigaridas at Sophia Antipolis, for completing the work on the algebraic numbers package in SYNAPS.

7.2.2. NSF-INRIA collaboration

Participants: Laurent Busé, André Galligo, Mohamed Elkadi, Bernard Mourrain [contact person], Jean-Pierre Técourt.

The objective of this collaboration between GALAAD and the Geometric Modeling group at Rice University in Houston, Texas (USA) is to investigate techniques from Effective Algebraic Geometry in order to solve some of the key problems in Geometric Modeling and Computational Biology. The two groups have similar interests and complementary strengths. Effective Algebraic Geometry is the branch of Algebraic Geometry that pursues concrete algorithms rather than abstract proofs. It deals mainly with practical methods for representing polynomial curves and surfaces along with robust techniques for solving systems of polynomial equations. Many applications in Geometric Modeling and Computational Biology require fast robust methods for solving systems of polynomial equations. Here we concentrate our collective efforts on solving standard problems such as implicitization, inversion, intersection, and detection of singularities for rational curves and surfaces. To aid in modeling, we shall also investigate some novel approaches to representing shape. In contemporary Computational Biology, many problems can be reduced to solving large systems of low degree polynomial equations. We plan to apply our polynomials solvers together with new tools for analysing complex shapes to help study these currently computationally intractable problems.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We continued to organize a bi-weekly seminar called "Formes & Formules". The list of talks is available at http://www-sop.inria.fr/galaad/seminaires/.

8.1.2. Comittee participations

- B. Mourrain was a member of the programm comittee of the conference SNC'05 (Symbolic-Numeric Computation), Xi'an, China, 19-21 July; SMI'05 (International conf. on Shape Modeling and Applications), Matsushima, Japan, 13-17 June; CASC'05 (Computer Algebra in Scientific Computing) Kalamata, Greece, 13-17 September; JNCF'05 (Journées Nationales de Calcul Formel) Luminy, France, 21-25 November.

8.1.3. Editorial committees

- M. Elkadi, B. Mourrain, R. Piene [Univ. Oslo] are editors of the book "Algebraic Geometry and Geometric Modeling", which follows the conference held in Nice in 2004.

8.1.4. Organisation of conferences and schools

- B. Mourrain organised the "Quadric Day", the 1st April at Sophia Antipolis, devoted to computational algebraic geometry with quadric surfaces. People from Avignon, Nancy, Spain, Honk Kong (Wenping Wang in visit at that time) attended this informal but interesting meeting.
- M. Attene, J.G. Dumas, B. Mourrain, F. Rouillier, O. Ruatta organised the summer school on open software for Geometric and Algebraic computation at Sophia Antipolis 5-9th September. This school received the support of INRIA, Région PACA, Aim@Shape NoE 50766. 10 tutorials were presented and 30 participants attended it. For more details, see http://www-sop.inria.fr/galaad/conf/05Ecole.

8.1.5. PHD thesis commitees

- A. Galligo and B. Mourrain were members of the jury of the PHD. thesis of J.P. Técourt.
- A. Galligo was a member of the jury of the PHD. thesis of J.G. Ramspacher.

8.1.6. Other comittees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- B. Mourrain (until March 2005) was in charge, with Thierry Vieville, of the "Formation par la recherche" at INRIA Sophia-Antipolis.
- B. Mourrain is a member of the scientific council of SARIMA.
- J.P. Técourt (until fall 2005) was a member of the "Comité de centre" at INRIA Sophia-Antipolis.

8.1.7. WWW server

- http://www-sop.inria.fr/galaad/.

8.2. Participation to conferences and invitations

- L. Busé: invited talk at “Second Latin American School and Workshop on Polynomial Systems” conference in Angra dos Reis, Brasil, 28 February-4 March; visit to J.-P. Jouanolou, Université de Strasbourg, 6-8 April; participation to the MEGA conference in Alghero, Sardigna, Italia, 27 May-2 June; invited course (3 hours) at the CIMPA school on “Grobner basis and applications” in Zanjan, Iran, 14 July-23 July.
- S. Chau: Participation to "Journées de géométrie algorithmique", Grenoble, 24 - 28 January 2005; presentation at "Quadric day", Sophia-Antipolis, 1st April 2005; participation to "Open software for algebraic and geometric computation", Sophia-Antipolis, 05 - 09 September 2005; presentation at "Computation methods for algebraic spline surfaces II", Oslo, 14 - 16 September 2005; participation to "Journées nationales de calcul formel", Marseille, 21 - 25 November 2005.
- G. Chèze: talks at the University of Toulouse and Limoges, May 2005.
- M. Elkadi: participation to the MEGA conference in Alghero, Sardigna, Italia, 27 May-2 June; participation to the International Conference on Constructive and Computational Mathematics, Marrakech, Morrocco, December 14-17, 2005.
- A. Galligo:participation to MEGA’05 (Effective Methods in Algebraic Geometry) Alghero, Sardigna, Italia, 27 May-2 June; ISSAC’05 (International Symposium on Symbolic and Algebraic Computation) Beijing, China, 24-27 July; SNC’05 (Symbolic-Numeric Computation) Xi’an, China, 19-21 July; COMPASS workshop, Oslo, Norway, 14-17 September; TERA’05 in Buenos Aires, Argentina, October 24-28 2005; visit in Israel at Weismann Institute and Technion, November 14-25 2005; ICCCM, Marrakech, Morrocco, December 14-17, 2005.
- H. Khalil: presentation at CASC’05 (Computer Algebra in Scientific Computing) Kalamata, Greece, 13-14 September; JNCF’05 (Journées Nationales de Calcul Formel) Luminy, France, 21-25 November.
- T.H. Lê: presentation at "Quadric day", Sophia-Antipolis, 1st april 2005; participation to the summer school "Open software for algebraic and geometric computation", Sophia-Antipolis, 05 - 09 September 2005; presentation at "Computation methods for algebraic spline surfaces II", Oslo, 14 - 16 September 2005; participation to the school and the conference on "Commutative Algebra", Hanoi, Vietnam, 26 December 2005 - 6 January 2006.
- B. Mourrain: participation to the meeting of the AS Contraintes, St Ouen, France 6-7 January; the workshop on Singularities and Applications, Luminy, France, 7-11 February; the review and management board meeting of Aim@Shape Network, Genova, Italy, 13-15 March; MEGA’05 (Effective Methods in Algebraic Geometry) Alghero, Sardigna, Italia, 27 may-2 june; FOCM’05 (FOundation of Computational Mathematics) Santander, Spain, 29 June-1 July. SNC’05 (Symbolic-Numeric Computation) Xi’an, China, 19-21 July; ISSAC’05 (International Symposium on Symbolic and Algebraic Computation) Beijing, China, 24-27 July; invited speaker at CASC’05 (Computer Algebra in Scientific Computing) Kalamata, Greece, 13-14 September; the GAIA review and the COMPASS workshop, Oslo, Norway, 14-17 September; the ACS workshop Zurich, Swiss, 20-24 September; JNCF’05 (Journées Nationales de Calcul Formel) Luminy, France, 21-25 November.
- Jean-Pierre Técourt: presentation at "Journées de géométrie algorithmique", Grenoble, 24 - 28 january 2005; at the MEGA conference in Alghero, Sardigna, Italia, 27 may-2 june; at the Workshop "Algorithms in real algebraic geometry and applications", Ouessant June 27th - July 1rst, 2005; Invited talk at IHP during the Workshop “Real Algebra, Quadratic Forms and Model Theory; Algorithms and Applications”, Paris, 02-09 November 2005.

8.3. Formation

8.3.1. Teaching at universities

- L. Busé: Course in Master, 2nd year, on “algebraic curves and resultant for CAGD” at the university of Nice (12 hours).
- G. Chèze: Courses in mathematics: analysis (fonctions in several variables) and scientific reasoning, first year of "licence MASS". Courses of commutative algebra (rings and polynomials) in the second year of the "licence Math-Info".
- M. Elkadi: Course and exercises of “Arithmetic” in L2MI. Course and exercices of “Effective Algebraic Geometry”, Master 1. Courses in Master 2 of Mathematics.
- A. Galligo: Courses in "Licence Mass", first year : algebra, scientific reasoning (with Maple); "Maitrise Math", first year: curves and surfaces; "Maitrise Math", second year: nmerical solvers (6 hours).
- H. Khalil: Courses in Analysis, second and fourth years at the university of Lyon.
- B. Mourrain: Courses in Master 2 of Mathemathics, Algorithms for curves and surfaces (11 hours).
- Jean-Pierre Técourt: TD, TP of algorithmic in Maple, L1 MASS. (42h)

8.3.2. PhD theses in progress

- Lionel Alberti, *Vers une théorie quantitative des singularités*, ED SFA, UNSA.
- Stéphane Chau, *Study of singularities used in CAGD*, UNSA.
- Marc Dohm, *Algorithmique des courbes et surfaces algébriques*, UNSA.
- Houssam Khalil, *Matrices structurées en calcul symbolique et numérique*, Univ. Lyon I.
- Thi Ha Lê, *Classification and intersections of some parametrized surfaces and applications to CAGD*, UNSA.
- Julien Wintz, *Méthodes algébriques pour la modélisation géométrique*, INRIA Sophia-Antipolis, ED STIC.

8.3.3. Defended PhD thesis

- Jean-Pierre Técourt, *Sur le calcul effectif de la topologie de courbes et surfaces implicites*, UNSA.

8.3.4. Internships

See the web page of our interships.

- Chen Liang, *Subdivision methods for the topolopy of implicit 3D curves*, 4 November-30 January.
- Rahobisoa Herimalala, *Méthode d'interpolation pour le calcul de déterminant de matrices polynomiales*, 1st April-30 June.

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Doctoral dissertations and Habilitation theses

[11] J.-P. TÉCOURT. *Sur le calcul effectif de la topologie de courbes et surfaces implicites*, Ph. D. Thesis, Université de Nice Sophia-Antipolis, 2005.

Articles in refereed journals and book chapters

[12] L. BUSÉ, M. CHARDIN. *Implicitizing rational hypersurfaces using approximation complexes*, in "J. Symbolic Comput.", vol. 40, n° 4-5, 2005, p. 1150–1168.

[13] L. BUSÉ, A. GALLIGO. *Semi-implicit representations of surfaces in $\mathbb{P}^3$, resultants and applications*, in "J. Symbolic Comput.", vol. 39, n° 3-4, 2005, p. 317–329.

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