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Project-Team geometrica

Geometric Computing

Sophia Antipolis

THEME SYM

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1. Team

Head of project
Jean-Daniel Boissonnat [DR]

Vice-head of project
Olivier Devillers [DR]

Administrative Assistant
Agnès Bessière [TR, assistant of GEOMETRICA and of the projects committee]

Research scientists
Pierre Alliez [CR]
Frédéric Cazals [CR]
David Cohen-Steiner [CR]
Sylvain Pion [CR]
Mariette Yvinec [CR]

Software development staff
Naceur Meskini [Engineer ODL, since September 2005]
Laurent Saboret [Engineer]

Ph.D. Students
Luca Castelli Aleardi [in "cotutelle" with LIX (ÉCOLE POLYTECHNIQUE)]
Christophe Delage [ENS]
Thomas Lewiner [AMX fellow]
Steve Oudot [MENRT monitor fellow until September 10th]
Marc Pouget [detached from the ministry of Education]
Laurent Rineau [ENS]
Marie Samozino [MENRT monitor fellow]
Abdelkrim Mebarki [grant Algerian governement]
Camille Wormser [ENS]
Sébastien Loriot [MENRT monitor fellow since October 2005]
Trung Nguyen [Alcatel, since October 2005]

Post-doctoral fellow
Jean-Philippe Pons since October 2005

Student interns
Pooran Memari [Ecole Polytechnique, April-July 2005]
Julien Hazebrouck [ESSI, July-September 2005]
Nicolas Carrez [SUPELEC Metz, June-August 2005]
Sébastien Loriot [Université de Bourgogne, April-September 2005]
Trung Nguyen [ENS, April-October 2005]
Chinmay Karande [ITT Bombay - India, May-July 2005]

2. Overall Objectives

2.1. Overall Objectives

Geometric computing plays a central role in most engineering activities: geometric modelling, computer aided design and manufacturing, computer graphics and virtual reality, scientific visualization, geographic information systems, molecular biology, fluid mechanics, and robotics are just a few well-known examples. The rapid advances in visualization systems, networking facilities and 3D sensing and imaging make geometric computing both dominant and more demanding concerning effective algorithmic solutions.

Computational geometry emerged as a discipline in the seventies and has met with considerable success in resolving the asymptotic complexity of basic geometric problems including data structures, convex hulls, triangulations, Voronoi diagrams, geometric arrangements and geometric optimisation. However, in the mid-nineties, it was recognized that the applicability in practice of the computational geometry techniques was far from satisfactory and a vigorous effort has been undertaken to make computational geometry more effective. The PRISME project together with several partners in Europe took a prominent role in this research and in the development of a large library of computational geometry algorithms, CGAL.

GEOMETRICA aims at pursuing further the effort in this direction and at building upon the initial success. Its focus is on effective computational geometry with special emphasis on *curves and surfaces*. This is a challenging research area with a huge number of potential applications in almost all application domains involving geometric computing.

The overall objective of the project is to give effective computational geometry for curves and surfaces solid mathematical and algorithmic foundations, to provide solutions to key problems and to validate our theoretical advances through extensive experimental research and the development of software packages that could serve as steps towards a standard for safe and effective geometric computing.

3. Scientific Foundations

3.1. Introduction

The research conducted by GEOMETRICA focuses on three main directions:

- fundamental geometric data structures and algorithms
- robust computation and advanced programming,
- shape approximation and mesh generation.

3.2. Fundamental geometric data structures and algorithms

GEOMETRICA is pursuing long standing research on fundamental geometric data structures and algorithms. GEOMETRICA has a large expertise in Voronoi diagrams and Delaunay triangulations, randomized algorithms, combinatorial geometry and related fields. Recently, we devoted efforts to developing the field of computational geometry beyond linear objects. We are especially interested in developing a theory of curved Voronoi diagrams. Such diagrams allow to model growing processes and have important applications in biology, ecology, chemistry and other fields. They also play a role in some optimization problems and in anisotropic meshing. Euclidean Voronoi diagrams of non punctual objects are also non affine diagrams. They are of particular interest in robotics, CAD and molecular biology. Even for the simplest diagrams, e.g. Euclidean Voronoi diagrams of lines, triangles or spheres in 3-space, obtaining tight combinatorial bounds and efficient algorithms are difficult research questions. In addition, effective implementations require to face specific algebraic and arithmetic questions. Working out carefully the robustness issues is a central objective of GEOMETRICA (see below).

In past years, the main objective of computational geometry has been the design of time efficient algorithms, either from the theoretical point of view of the asymptotic complexity or the more practical of running efficient benchmarks. Surprisingly less interest has been devoted to improve the space behavior of such algorithms although the problem may become of importance when the main memory is not enough and the system has to swap to find the extra memory; this may happen either for massive data or for small memory devices such as PDA. GEOMETRICA intends to attack this problem from several aspects:

— the compression of geometrical objects for storage or network transmission purposes
— the design of algorithms accessing locally the memory to reduce (but not remove) the swapping in memory
— the design of new data-structures to represent geometrical objects using less memory.

For all these aspects we are interested in both the theoretical asymptotic sizes involved and the practical aspect for reasonable input size. Such a distinction between asymptotic behavior and practical interest may appear strange since we are interested in massive data, but even for a big mesh of several millions points which can be called “massive data” we are still far from the asymptotic behavior of some theoretical structures.

3.3. Robust computation and advanced programming

An implementation of a geometric algorithm is called *robust* if it produces a valid output for all inputs. Geometric programs are notorious for their non-robustness due to two reasons: (1) Geometric algorithms are designed for a model of computation where real numbers are dealt with exactly and (2) geometric algorithms are frequently only formulated for inputs in general position. As a result, implementations may crash or produce nonsensical output. This is observed in all commercial CAD-systems.

The importance of robustness in geometric computations has been recognized for a long time, but significant progress was made only in recent years. GEOMETRICA held a central role in this process, including advances regarding the *exact computation paradigm*. In this paradigm, robustness is achieved by a combination of three methods: *exact arithmetic*, *dedicated arithmetic* and *controlled rounding*.

In addition to pursuing research on robust geometric computation, GEOMETRICA is an active member of a European consortium that develops a large library named CGAL. This library makes extensive use of generic programming techniques and is both a unique tool to perform experimental research in Computational Geometry and a comprehensive library for Geometric Computing. The startup company GeometryFactory has been started in January 2003 to commercialize components from CGAL and to offer services for geometric applications.

3.4. Shape approximation and mesh generation

Complex shapes are ubiquitous in robotics (configuration spaces), computer graphics (animation models) or physical simulations (fluid models, molecular systems). In all these cases, no natural *shape space* is available or when such spaces exist they are not easily dealt with. When it comes to performing calculations, the objects under study must be discretized. On the other hand, several application areas such as Computer Aided Geometric Design or medical imaging require reconstructing $3D$ or $4D$ shapes from samples.

The questions afore-mentioned fall in the realm of *geometric approximation theory*, a topic GEOMETRICA is actively involved in. More precisely, the generation of samples, the definition of differential quantities (e.g. curvatures) in a discrete setting, the geometric and topological control of approximations, as well as multi-scale representations are investigated. Connected topics of interest are also the progressive transmission of models over networks and their compression.

Surface mesh generation and surface reconstruction have received a great deal of attention by researchers in various areas ranging from computer graphics through numerical analysis to computational geometry. However, work in these areas has been mostly heuristic and the first theoretical foundations have been established only recently. Quality mesh generation amounts to finding a partition of a domain into linear elements (mostly triangles or quadrilaterals) with topological and geometric properties. Typically, one aims at constructing a piecewise linear (PL) approximation with the “same” topology as the original surface (same topology may have several meanings). In some contexts, one wants to simplify the topology in a controlled way. Regarding the geometric distance between the surface and its PL approximation, different measures must be considered: Hausdorff distance, errors on normals, curvatures, areas etc. In addition, the shape, angles or size of the elements must match certain criteria. We call *remeshing* the techniques involved when the input domain to be discretized is itself discrete. The input mesh is often highly irregular and non-uniform, since it typically comes as the output of a surface reconstruction algorithm applied to a point cloud obtained from a scanning device. Many geometry processing algorithms (e.g. smoothing, compression) benefit from remeshing, combined with uniform or curvature-adapted sampling. GEOMETRICA intends to contribute to all aspects of this matter, both in theory and in practice.

Volumetric mesh generation consists in triangulating a given three-dimensional domain with tetrahedra having a prescribed size and shape. For instance, the tetrahedra in a general purpose mesh should be as regular as possible (isotropic case), whereas they should be elongated in certain directions for problem specific meshes.

Volumetric mesh generation often makes use of surface mesh generation techniques (*e.g.* to approximate the boundary of the domain or interfaces between two media). Thanks to its strong experience with Delaunay triangulations, GEOMETRICA recently made several contributions to the generation of volumetric meshes, and intends to pursue in this direction.

4. Application Domains

4.1. Geometric modeling and shape reconstruction

Keywords: *Geometric modeling, geology, medical imaging, reverse engineering, surface reconstruction*.

Modeling 3D shapes is required for all visualization applications where interactivity is key since the observer can change the viewpoint and get an immediate feedback. This interactivity enhances the descriptive power of the medium significantly. For example, visualization of complex molecules helps drug designers to understand their structure. Multimedia applications also involve interactive visualization and include e-commerce (companies can present their product realistically), 3D games, animation and special effects in motion pictures. The uses of geometric modeling also cover the spectrum of engineering, computer-aided design and manufacture applications (CAD/CAM). More and more stages of the industrial development and production pipeline are now performed by simulation, due to the increased performance of numerical simulation packages. Geometric modeling therefore plays an increasingly important role in this area. Another emerging application of geometric modeling with high impact is medical visualization and simulation.

In a broad sense, shape reconstruction consists of creating digital models of real objects from points. Example application areas where such a process is involved are Computer Aided Geometric Design (making a car model from a clay mockup), medical imaging (reconstructing an organ from medical data), geology (modeling underground strata from seismic data), or cultural heritage projects (making models of ancient and or fragile models or places). The availability of precise and fast scanning devices has also made the reproduction of real objects more effective such that additional fields of applications are coming into reach. The members of GEOMETRICA have a long experience in shape reconstruction and contributed several original methods based upon the Delaunay and Voronoi diagrams.

4.2. Algorithmic issues in Structural Biology

Keywords: *Molecules, docking*.

Two of the most prominent challenges of the post-genomic era are to understand the molecular machinery of the cell and to develop new drug design strategies. These key challenges require the determination, understanding and exploitation of the three-dimensional structure of several classes of molecules (nucleic acids, proteins, drugs), as well as the elucidation of the interaction mechanisms between these molecules.

These challenges clearly involve aspects from biology, chemistry, physics, mathematics and computer science. For this latter discipline, while the historical focus has been on text and pattern matching related algorithms, the amount of structural data now available calls for geometric methods. At a macro-scopic scale, the classification of protein shapes, as well as the analysis of molecular complexes requires shape description and matching algorithms. At a finer scale, molecular dynamics and force fields require efficient data-structures to represent solvent models, as well as reliable meshes so as to solve the Poisson-Boltzmann equation.

4.3. Scientific computing

Keywords: *Unstructured meshes, finite element method, finite volume method*.

Meshes are the basic tools for scientific computation. Unstructured meshes allow to mesh complex shapes and to refine locally the mesh, which may be required because of the geometry or in order to increase the precision of the computation. GEOMETRICA contributes to 2D and 3D meshes, and also to surface meshes. The methods are mostly based on Delaunay triangulations, Voronoi diagrams and their variants. Affine diagrams

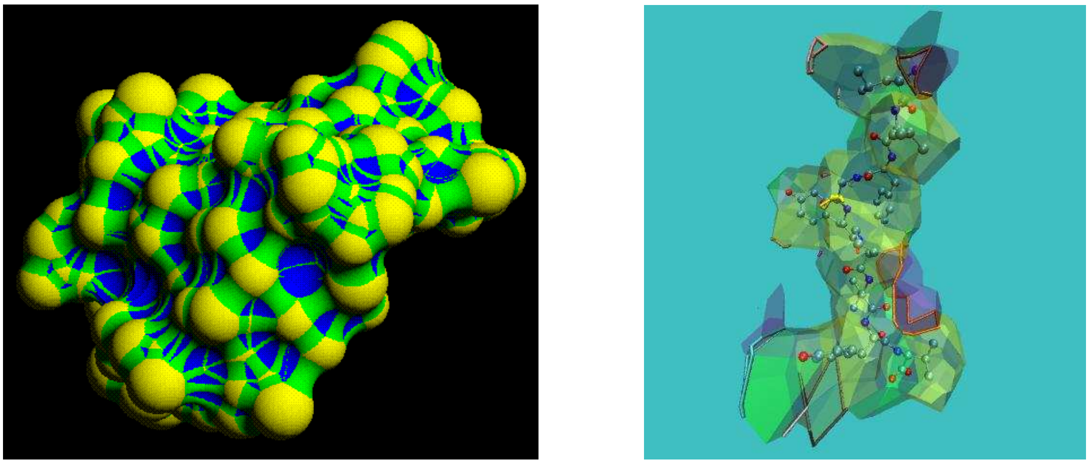

Figure 1. (a) Molecular surface (b) A small peptide surrounded by its interface [52]

are well-suited for volume element methods. Non affine diagrams are especially important in the context of anisotropic meshes. Anisotropic quadrilateral meshes are also of interest.

4.4. Telecommunications

Keywords: *Compression*.

The emerging demand for visualizing and simulating 3D geometric data in networked environments has motivated research on representations for such data. Slow networks require data compression to reduce the latency, and progressive representations to transform 3D objects into streams manageable by the networks. The members of GEOMETRICA have contributed several original compression methods for surface and volume meshes [11]. We investigate both single-rate and progressive compression depending on whether the model is intended to be decoded during, or only after, the transmission. Progressive compression is closely related to both approximation and information theory, aimed at trading off data size for approximation accuracy (so-called rate-distortion tradeoff). We cast this problem into that of *shape compression*.

5. Software

5.1. cgal, the library of geometric algorithms

Participants: Jean-Daniel Boissonnat, Hervé Brönnimann, Frédéric Cazals, Frank Da, Olivier Devillers, Andreas Fabri, Julia Flötotto, Philippe Guigue, Menelaos Karavelas, Abdelkrim Mebarki, Naceur Meskini, Sylvain Pion [contact person], François Rebufat, Laurent Saboret, Monique Teillaud, Radu Ursu, Mariette Yvinec.

CGAL is a C++ library of geometric algorithms developed initially within two European projects (project ESPRIT IV LTR CGAL december 97 - june 98, project ESPRIT IV LTR GALIA november 99 - august 00) by a consortium of eight research teams from the following institutes: Universiteit Utrecht, Max-Planck Institut Saarbrücken, INRIA Sophia Antipolis, ETH Zürich, Tel Aviv University, Freie Universität Berlin, Universität Halle, RISC Linz. The goal of CGAL is to make the solutions offered by the computational geometry community available to the industrial world and applied domains.

The CGAL library consists in a kernel, a basic library and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes : we can choose between Cartesian or homogeneous coordinates, use different types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

The basic library provides a number of geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d-dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface (concept).

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (Ascii files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview, SCILAB...).

GEOMETRICA is particularly involved in the arithmetic issues that arise in the treatment of robustness issues, the kernel, triangulation packages and their close applications such as alpha shapes, general maintainance...

CGAL is about 500,000 lines of code and supports various platforms: GCC (Linux, Mac OS, Solaris, Irix, Cygwin...), MipsPro (IRIX), SunPro (Solaris), Visual C++ (Windows), Intel C++... Version 3.1 has been released on december 21st, 2004. The previous release, CGAL 3.0, has been downloaded 14000 times from our web site, during the 11 months period where it was the main version.

5.2. A web service for surface reconstruction

Participant: David Cohen-Steiner.

In collaboration with Frank Da and Andreas Fabri. http://cgal.inria.fr/Reconstruction/.

The surface reconstruction algorithm developed by D. Cohen-Steiner and F. Da using CGAL is available as a *web service*. Via the web, the user uploads the point cloud data set to the server and obtains a VRML file of the reconstructed surface, which gets visualized in the browser of the user. This allows the user to check if the algorithm fits his/her needs, to learn how to adjust the parameters, before contacting INRIA to obtain an executable. It also provides us with the opportunity to collect real-world data sets used for testing and improving our reconstruction algorithms.

6. New Results

6.1. Introduction

The presentation of our new results follows the three main directions recalled in Section 3.1

- fundamental geometric data structures and algorithms
- robust computation and advanced programming,
- shape approximation and mesh generation.

Contributions on the first item are the construction of Voronoi diagrams of spheres, several results related to visibility and graph algorithms for reporting maximal cliques.

Work on the second research direction includes the design and a preliminary implementation of a certified curved kernel for CGAL. In addition, to further extend the robustness and the efficiency of CGAL, geometric constructions are now filtered in a way similar to predicates.

Work on the third axis gained momentum and is subdivided in Surface Approximation, Mesh generation, Computation and Stability of Geometric Features, Shape Coding and Compression.

Lastly, we report on some applications in Molecular Biology, Visualization and Signal Processing.

6.2. Combinatorics, data structures and algorithms

Keywords: *Voronoi diagrams, combinatorial pattern matching, visibility complex*.

6.2.1. *Voronoi diagrams of spheres*

Participants: Jean-Daniel Boissonnat, Christophe Delage.

In this paper [26], we provide a complete description of dynamic algorithms for constructing convex hulls and Voronoi diagrams of additively weighted points of $\mathbb{R}^d$. The algorithms have been implemented in $\mathbb{R}^3$ and experimental results are reported.

Our motivation comes from the fact that weighted points can be considered as hyperspheres when the weights are positive and is twofold. On the one hand, spheres are non linear objects and, besides the combinatorial and algorithmic questions, numerical and robustness issues deserve a careful investigation, which has not been fully done yet. On the other hand, spheres are objects of major concern in various fields, most notably structural biology, and effective implementations of basic geometric algorithms for spheres are needed.

We first revisit the problem of computing the convex hull of n weighted points of $\mathbb{R}^d$. This problem has already been solved optimally. We present a simpler fully dynamic algorithm and provide a complete description of all the predicates for any dimension d.

We apply our result on the construction of the convex hull of additively weighted points to the construction of a Voronoi cell in the Voronoi diagram of n additively weighted points. The main contribution of this work is to provide a full analysis of the predicates involved, a thorough treatment of the degenerate cases, and a CGAL implementation. Our predicates, when specialized to the planar case ($d = 2$), are simpler and of lower degree than the best predicates known so far. Our implementation follows the CGAL design and, in particular, is made both robust and efficient through the use of a filtered exact arithmetic.

6.2.2. *Visibility issues*

Participant: Olivier Devillers.

In collaboration with H. Brönnimann (Polytechnic Univ. Brooklyn), V. Dujmović and S. Whitesides (Univ. Mac Gill), H. Everett, M. Glisse, X. Goaoc and S. Lazard (INRIA-Vegas), S. Hornus (INRIA-Artis), H.-S. Na (Soongsil Univ., Seoul), F. Sottile (Texas A&M Univ.) and S. Wismath (Univ. Lethbridge).

We investigate several problems related to the computation of the visibility complex between three dimensional objects or dynamic updates of the visibility in two dimensions.

A set of objects in three dimensional space is given, we look at the different kind of lines with respect to these objects. With respect to a single object a line can intersect the object, be tangent or miss the object. In general position, a line is tangent to at most four objects, such a line is called *free* if it intersects no other objects, it is called *free line segment* if it intersects no objects between the points where the line touch the four objects and *occluded* otherwise.

First, we prove that the lines tangent to four (possibly intersecting) convex polyhedra with n edges in total, form $\Theta(n^2)$ connected components in the worst case. In the generic case, each connected component is a single line, but our result still holds for arbitrary degenerate scenes. More generally, we show that a set of k possibly intersecting convex polyhedra with a total of n edges admits, in the worst case, $\Theta(n^2k^2)$ connected components of maximal free line segments tangent to any four of the polytopes. This bound also holds for the number of connected components of possibly occluded lines tangent to any four of the polytopes [44].

In another paper, we investigate the special case of lines tangent to four triangles in three-dimensional space. This give an idea of the constant involved in algorithms on n triangles where finding such tangents is considered as a constant time problem. We establish a lower bound by giving an example having 62 such tangents. The upper bound on the number of connected components of tangents is 162 for possibly intersecting triangles and decreases to 156 if the triangles are disjoint. In addition, if the triangles are in (algebraic) general position, then the number of tangents is finite and it is always even [43].

Finally given a set of n points in the plane, we consider the problem of computing the circular ordering of the points about a viewpoint q and efficiently maintaining this ordering information as q moves [34]. In usual circular ordering an event occur when the viewpoint and two data-point becomes collinear. Here the main difference is that we consider this as an event only if the view point is not in between the two data points. Our main result is that after a $O(n \log n)$ preprocessing, we allow moving point queries along a specified line segment on $O(k \log n + s)$ amortized time where k is the number of different views and s is the output size.

6.2.3. Graph with one bend per edge

Participant: Olivier Devillers.

In collaboration with H. Everett, S. Lazard, M. Pentcheva (INRIA-Vegas) and S. Wismath (U. Lethbridge).

One of the challenge of graph drawing is to draw graphs using a minimal amount of space. One instance of this problem is to allow the graph vertices to lie in 3D at point locations with integer coordinates, and to draw graph edges as **non intersecting** polylines with bends allowed at point location with integer coordinates as well. In our work we propose a construction using one bend per edge to draw the complete graph K_n inside a box of volume $O(n^{2.5})$. The lower bound for non intersecting 3D drawing of K_n is known to be $\Omega(n^2)$, our result represents a significant improvement over previous result of $O(\frac{n^3}{\log n})$. The result trivially applies to any graph with n vertices that is a subgraph of K_n [35], [54].

6.2.4. Reporting maximal cliques

Participant: Frédéric Cazals.

In collaboration with Chinmay Karande, IIT Bombay (3rd year student).

Reporting maximal cliques of a graph is a fundamental problem arising in many areas. To the best of our knowledge, this problem is tackled resorting to the old Bron-Kerbosch algorithm (1973), or its recent variant by I. Koch (2001). Both algorithms suffer from a poor output sensitivity and from *pathological* worst-cases.

In this context, this paper [50] makes three contributions. First, we show that a slight modification of the greedy pivoting strategy used by I. Koch allows one to get rid of the *pathological* worst-cases, also improving overall performances. Second, exploiting the recursive structure of non maximal cliques, we show that the pivoting strategy developed by I. Koch is a particular case of a more general optimization strategy based on the concept of *dominated* nodes. Using different instantiations of this concept, we design four modified Bron-Kerbosch algorithms, with better output-sensitivity. Third, we discuss implementation issues and provide a detailed experimental study on random graphs.

The bottom-line of this study is the investigation of the trade-off between output sensitivity and the overhead associated to the identification of dominated nodes.

6.2.5. Reporting maximal c-connected cliques

Participant: Frédéric Cazals.

In collaboration with Chinmay Karande, IIT Bombay (3rd year student).

Given two graphs, a fundamental task faced by (graph, shape) matching algorithms consists of computing either the (Connected) Maximal Common Induced Subgraphs ((C)MCIS) or the (Connected) Maximal Common Edge Subgraphs ((C)MCES) [1]. In particular, computing the CMCIS or CMCES reduces to reporting so-called *c-connected cliques* in product graphs, a problem for which an algorithm has been presented in a recent paper *I. Koch, TCS 250 (1-2), 2001*. This algorithm suffers from two problems which are corrected in this note [49].

6.3. Geometric computing

Keywords: *algebraic degree of algorithms, computational geometry, degeneracies, perturbation, predicates, robustness*.

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[1] Consider two graphs $F = (V[F], E[F]), G = (V[G], E[G])$. A CIS consists of two subsets of vertices $V^{'}[F] \subseteq V[F]$, $V^{'}[G] \subseteq V[G]$, such that the subgraphs induced by these vertex sets are isomorphic. A CES consists of subsets $V^{'}[F] \subseteq V[F]$,$E^{'}[F] \subseteq E[F]$, $V^{'}[G] \subseteq V[G]$,$E^{'}[G] \subseteq E[G]$, such that $(V^{'}[F]$,$E^{'}[F])$ and $(V^{'}[G]$,$E^{'}[G])$ are isomorphic.

6.3.1. Curved kernel

Participants: Sylvain Pion, Julien Hazebrouck.

In collaboration with Monique Teillaud, Constantinos Tsirogiannis and Ilya Suslov (latter two are student interns).

Work on the CGAL curved kernel has continued, following the past year work on the overall design. We have implemented more primitives dealing with circular arcs, as well as mixed operations with line segments. This allows to compute arrangements of objects commonly used in VLSI (Very Large Scale Integration, see Figure 2). Some contacts with industry (Mania Barco, Dassault Systèmes, GeometryFactory) show that there are needs for robust and efficient computations in this area.

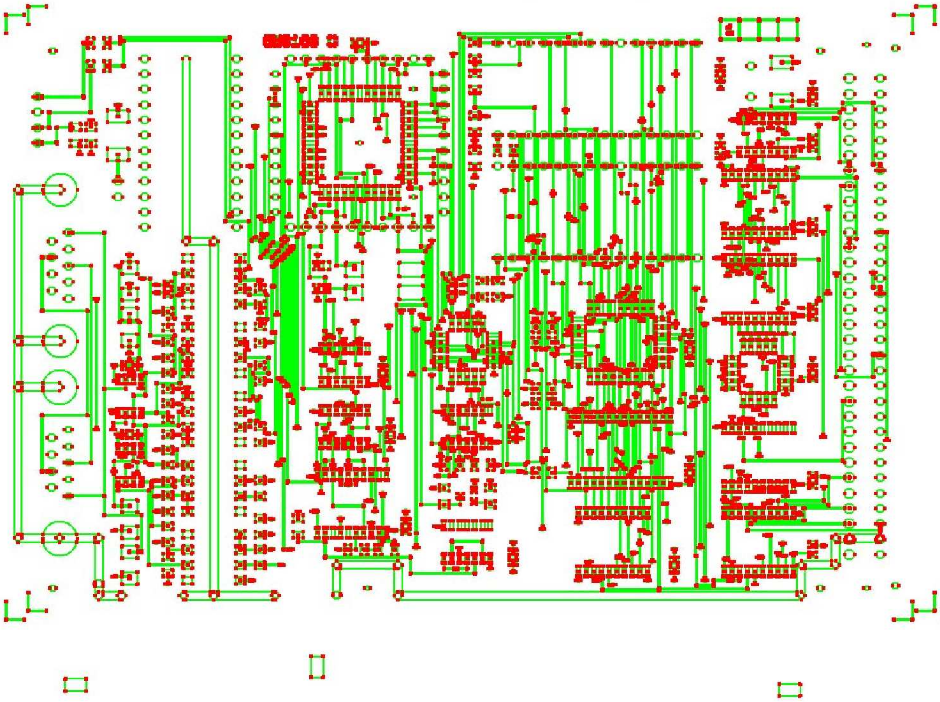

Figure 2. Arrangement of VLSI data composed of circle arcs and line segments.

Some optimization work has also been performed on this initial part of the curved kernel. As usual, the computations are done exactly and efficiently through the use of filtering at various stages. Most notably, filtering with bounding boxes and bounding hexagons around circular arcs already shows a positive impact on running time. This is expected to be more important for higher degree curves such as conic arcs. Some extensions to 3D have also been started for handling sphere patches, as well as for benchmarking the existing methods.

6.3.2. Filtered constructions kernel

Participant: Sylvain Pion.

In collaboration with Andreas Fabri (GeometryFactory).

Basic geometric primitives are usually classified into predicates, functions which return boolean or enumerated results, and constructions, which return new geometric objects. An example of the former is the orientation predicate, and an example of the latter is the computation of the intersection of two line segments. While very efficient methods are now available to compute exact predicates, geometric constructions are also very important.

We have started implementing a generic mechanism for filtering geometric constructions, and providing a full CGAL kernel this way. The 2D version is almost entirely implemented, and it already shows important performance improvements compared to previous solutions based on filtered number types. Many implemented algorithms use geometric constructions and need some guarantees on them, such as the CGAL arrangements, Voronoi diagrams and meshes.

6.4. Surface approximation

Keywords: *Computational topology*, *geometric probing*, *implicit surfaces*, *point set surfaces*, *surface learning*, *surface meshing*.

6.4.1. Surface learning by probing

Participants: Jean-Daniel Boissonnat, Steve Oudot.

In collaboration with Leo Guibas (Stanford University).

We consider the problem of discovering a C^2 unknown surface S using tactile probes [27]. We show that S can be approximated by a triangulated surface W within any desired accuracy. We also bound the number of probes and the number of elementary moves of the probing device. Our solution is an extension of our previous work on Delaunay refinement techniques for certified surface meshing [18]. Our approximating surface enjoys the many nice properties of the meshes obtained by those techniques, e.g. exact topological type, facets with bounded aspect ratio.

Similar results were only known for the case of a single planar convex object, a much easier case which reduces to blind approximation of the support function. The non convex case is different in several respects. First, there is no global parametrization of the boundary and, moreover, we cannot simply probe from infinity and need to determine positions outside the object where to place the probing device and to determine paths along which the probing device can be safely moved without colliding with the object.

Following the perception-action-cognition paradigm, we distinguish between the probing cost, the displacement cost and the combinatorial cost. The information cost measures the number of probes. The displacement cost accounts for moving the probing device. The combinatorial cost accounts for the arithmetic operations and comparisons, and the maintenance of the data structures. We show that it is not possible, in general, to simultaneously optimize all costs and analyze them separately.

6.4.2. Provably good sampling and meshing of Lipschitz surfaces

Participants: Jean-Daniel Boissonnat, Steve Oudot.

In the last decade, a great deal of work has been devoted to the elaboration of a sampling theory for smooth surfaces. The goal was to work out sampling conditions that ensure a good reconstruction of a given smooth surface S from a finite subset E of S. Among these conditions, a prominent one is the ε-sampling condition of Amenta and Bern, which states that every point p of S is closer to E than $\varepsilon\ \mathrm{lfs}(p)$, where lfs is the distance from p to the medial axis of S.

Amenta and Bern have proved that it is possible to extract from the Delaunay triangulation of E a PL surface that approximates S both topologically and geometrically (surface reconstruction problem). Last year, we introduced a weaker notion called loose ε-sample and proposed an algorithm to actually sample and mesh a smooth surface (surface mesh generation) with guarantees.

The sampling conditions proposed so far offer guarantees only in the smooth ($C^{1,1}$) setting. Indeed, the lfs of S vanishes at points where S is not smooth. In [13], we introduce a new measurable quantity, called the Lipschitz radius, which plays a role similar to that of lfs in the smooth setting, but turns out to be well-defined and positive on a large class of (smooth and non smooth) shapes. Given a surface S, the k-Lipschitz radius at a point p, noted $\mathrm{lr}_k(p)$, is the radius of the largest ball B centered at p such that $S \cap B$ is the graph of a k-Lipschitz bivariate function. The class of surfaces with positive k-Lipschitz radius includes piecewise smooth surfaces provided that the normal variation around singular points is not too large.

Our main result is that, if S has a positive k-Lipschitz radius (for some small enough k) and E is a sample of S such that any point $p \in$S is at distance less than a fraction of $\mathrm{lr}_k(p)$, then we obtain similar guarantees as in the smooth setting. More precisely, we show that the Delaunay triangulation of E restricted to S is a 2-manifold isotopic to S lying at Hausdorff distance $O(\varepsilon)$ from S, provided that its facets are not too skinny.

We further extend our results to loose ε-samples. Loose ε-sample have been introduced to analyze smooth surface meshing algorithms [18]. This notion is weaker than the notion of ε-sample. Nevertheless, we show that the previous results still hold if the sample E is only a loose ε-sample. As a straightforward application, the Delaunay refinement algorithm we proved correct for smooth surfaces [18] works fine and offers the same topological and geometric guarantees for Lipschitz surfaces. Specifically, the output of the algorithm is a PL surface with an optimal number of vertices that is isotopic to S, and lies at Hausdorff distance $O(\varepsilon)$ from S.

To the best of our knowledge, this is the first provably correct algorithm for meshing nonsmooth surfaces.

6.4.3. Conformal α-shapes

Participant: Frédéric Cazals.

In collaboration with J. Giesen (ETH Zurich), M. Pauly (ETH Zurich), A. Zomorodian (Stanford University).

We define [31] a new filtration of the Delaunay triangulation of a finite set of points in $\mathbb{R}^d$, similar to the alpha shape filtration. The new filtration is parameterized by a local scale parameter instead of the global scale parameter in alpha shapes. Since our approach shares many properties with the alpha shape filtration and the local scale parameter conforms to the local geometry we call it *conformal alpha shape filtration*. The local scale parameter is motivated from applications and previous algorithms in surface reconstruction. We show how conformal alpha shapes can be used for surface reconstruction of non-uniformly sampled surfaces, which is not possible with alpha shapes.

6.4.4. Extraction and simplification of iso-surfaces in tandem

Participant: David Cohen-Steiner.

In collaboration with Dominique Attali (LIS, Grenoble) and Herbert Edelsbrunner (Duke University).

Isosurfaces of 3D images are usually extracted using the marching cube algorithm. For high resolution images, the resulting surfaces can contain a very large number of triangles, which makes their processing difficult and slow. In this work [25], we combine the marching cube algorithm for surface extraction and the edge contraction algorithm for surface simplification in lock step, which avoids the costly step of storing the entire triangulation. Beyond this basic strategy, we introduce refinements to prevent artifacts in the resulting triangulation, first, by carefully monitoring the amount of simplification during the process and, second, by driving the simplification toward a compromise between shape approximation and mesh quality. We have implemented the algorithm and used extensive computational experiments to document the effects of various design options and to further fine-tune the algorithm.

6.5. Mesh generation

Keywords: *Isotropic meshing, anistropic meshing, tetrahedral meshing, triangle meshing.*

6.5.1. Anisotropic meshes

Participants: Jean-Daniel Boissonnat, Camille Wormser, Mariette Yvinec.

F. Labelle and J. Shewchuk have proposed a discrete definition of anisotropic Voronoi diagrams. These diagrams are parametrized by a metric field. Under mild hypotheses on the metric field, such Voronoi diagrams can be refined so that their dual is a triangulation, with elements shaped according to the specified anisotropic metric field.

We propose an alternative view of the construction of these diagrams and a variant of Labelle and Shewchuk's meshing algorithm [28]. This variant computes the Voronoi vertices using a higher dimensional

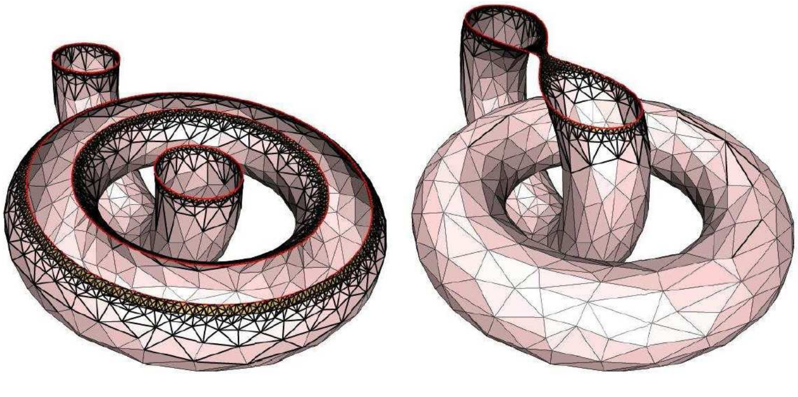

Figure 3. Simultaneous extraction and simplification of an iso-surface.

power diagram and refines the diagram as long as dual triangles overlap. We see this variant as a first step toward a 3-dimensional anisotropic meshing algorithm.

6.5.2. Variational tetrahedral meshing

Participants: Pierre Alliez, David Cohen-Steiner, Mariette Yvinec.

In collaboration with M.Desbrun (California Institute of Technology).

We introduce a novel Delaunay-based variational approach to isotropic tetrahedral meshing [15]. To achieve both robustness and efficiency, we minimize a simple mesh-dependent energy through global updates of both vertex positions and connectivity. Building upon the function approximation approach, we optimize the mesh so as to minimize the L1 distance between an isotropic quadratic function and its linear interpolation on the mesh. Mesh design is controlled through a gradation smoothness parameter and selection of the desired number of vertices. We provide the foundations of our approach by explaining both the underlying variational principle and its geometric interpretation. We demonstrate the quality of the resulting meshes (see Figure 4).

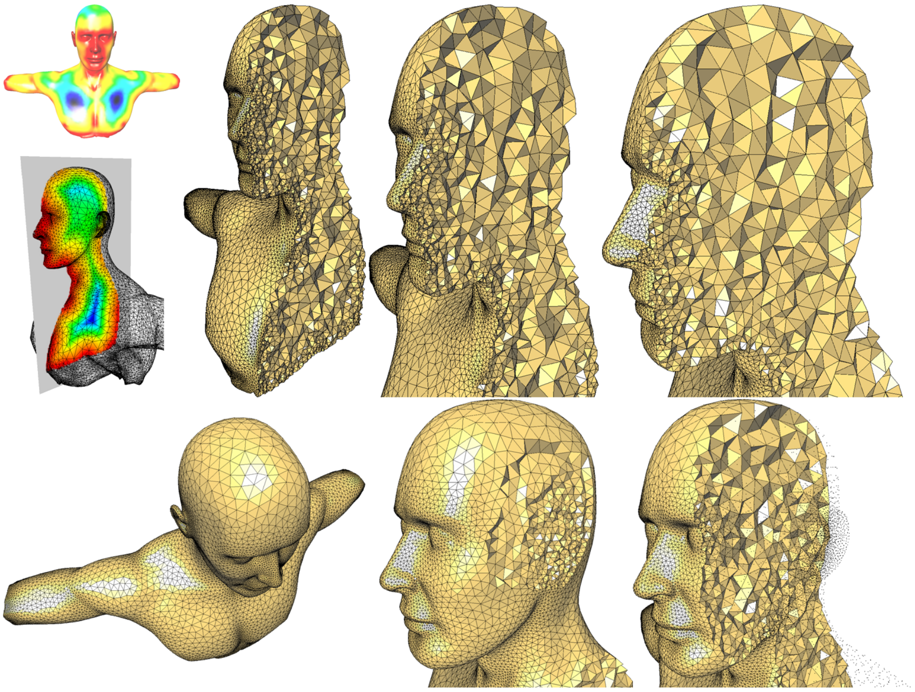

Figure 4. Variational Tetrahedral Meshing: Given the boundary of a domain (here, a human torso), we automatically compute the local feature size of this boundary as well as an interior sizing field (left, cross-section), before constructing a mesh with a prescribed number of vertices (here $65K$) and a smooth gradation conforming to the sizing field (right, cutaway view). The resulting tetrahedra are all well-shaped (i.e., nearly regular).

6.5.3. Meshing volumes bounded by smooth surfaces

Participants: Steve Oudot, Laurent Rineau, Mariette Yvinec.

This work introduces a three-dimensional mesh generation algorithm for domains bounded by smooth surfaces [57], [40]. The algorithm combines a Delaunay-based surface mesher [18] with a Ruppert-like volume mesher, to get a greedy algorithm that samples the interior and the boundary of the domain at once. The algorithm constructs provably-good meshes, it gives control on the size of the mesh elements through a user-defined sizing field, and it guarantees the accuracy of the approximation of the domain boundary. A noticeable feature is that the domain boundary has to be known only through an oracle that can tell whether a given point lies inside the object and whether a given line segment intersects the boundary. This makes the algorithm generic enough to be applied to a wide variety of objects, ranging from domains defined by implicit surfaces to domains defined by level-sets in 3D grey-scaled images or by point-set surfaces (see Fig. 5).

6.6. Computation and stability of geometric features

Keywords: *Ridges, curvature, persistence*.

6.6.1. The implicit structure of ridges of a smooth parametric surface

Participants: Frédéric Cazals, Marc Pouget.

In collaboration with F. Rouillier (INRIA-SALSA, Rocquencourt) and J-C. Faugère (INRIA-SALSA, Rocquencourt).

Given a smooth surface, a blue (red) ridge is a curve such that at each of its points, the maximum (minimum) principal curvature has an extremum along its curvature line. Ridges are curves of *extremal* curvature and therefore encode important informations used for segmentation, registration, matching and surface analysis. State of the art methods for ridge extraction either report red and blue ridges simultaneously or separately —in which case a local orientation procedure of principal directions is needed, but no method developed so far certifies the topology of the curves reported.

On the way to developing certified algorithms independent from local orientation procedures, we make the following fundamental contribution [47]. For any smooth parametric surface, we exhibit the implicit equation $P = 0$ of the singular curve P encoding all ridges of the surface (blue and red), and show how to recover the colors from factors of $\mathcal{P}$. Exploiting $P = 0$, we also derive a zero dimensional system coding the so-called turning points, from which elliptic and hyperbolic ridge sections of the two colors can be derived. (Elliptic sections correspond to a maximum (minimum) of the maximum (minimum) principal curvature, while hyperbolic sections correspond to a minimum (maximum) of the maximum (minimum) principal curvature.) Both contributions exploit properties of the Weingarten map of the surface and require computer algebra. Algorithms exploiting the structure of $\mathcal{P}$ for algebraic surfaces are developed in a companion paper [48].

6.6.2. Topologically certified approximation of umbilics and ridges on polynomial parametric surface

Participants: Frédéric Cazals, Marc Pouget.

In collaboration with F. Rouillier (INRIA-SALSA) and J-C. Faugère (INRIA-SALSA Rocquencourt).

Ridges of a smooth surface are curves of *extremal* curvature and encode important informations used in surface analysis or segmentation. But reporting them requires manipulating third and fourth order derivatives —whence numerical difficulties. Additionally, ridges have self-intersections and complex interactions with the umbilics of the surface —whence topological difficulties.

In this context, we make two contributions for the computation of ridges of polynomial parametric surfaces [48]. First, by instantiating to the polynomial setting a global structure theorem of ridge curves proved in a companion paper [47], we develop the first algorithm to produce an approximation with certified topology of the curve $\mathcal{P}$ encoding all the ridges of the surface. The algorithm exploits the singular structure of $\mathcal{P}$ umbilics and intersection points between red and blue ridges, and reduces the problem to solving zero dimensional systems using Gröbner basis. Second, for cases where the zero-dimensional systems cannot be practically solved, we develop a certified plot algorithm at any fixed resolution —a pixel being lit iff its closure is intersected by the curve $\mathcal{P}$.

These contributions are respectively illustrated for Bézier surfaces of degree four and five. See Figs. 6. and 7.

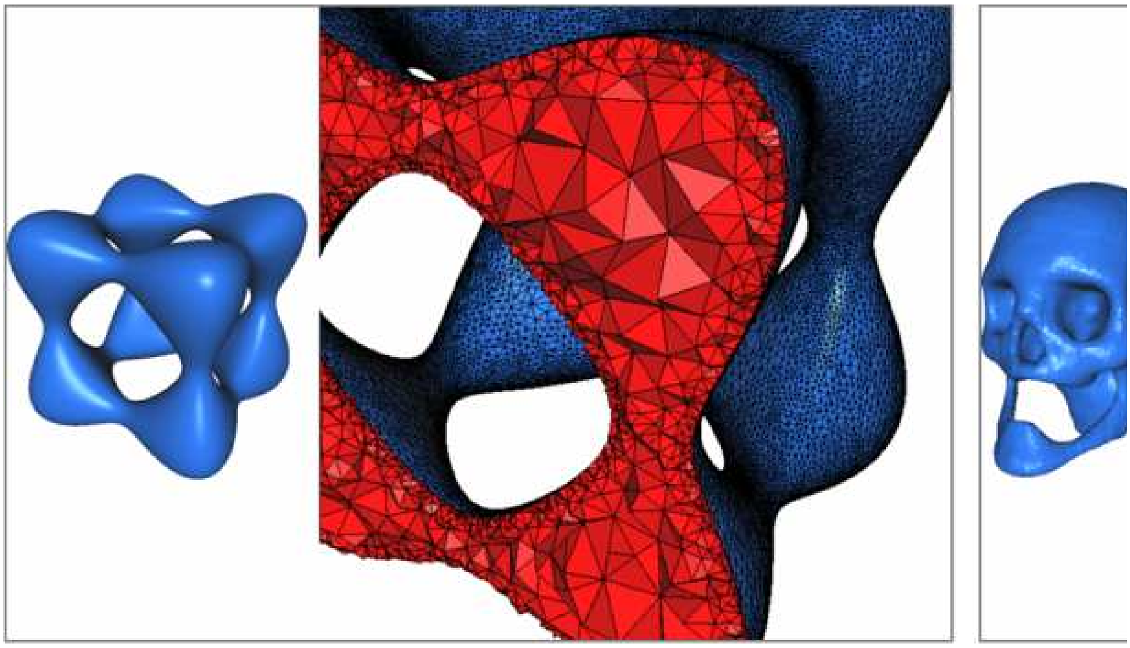

Figure 5. Meshing the volume bounded by an implicit surface (left) or meshing the interior of a skull extracted by contouring a medical image (right).

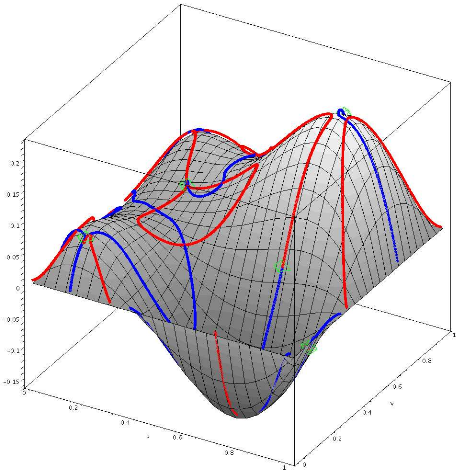

Figure 6. Plot of a degree 4 bivariate Bézier surface with ridges and umbilics.

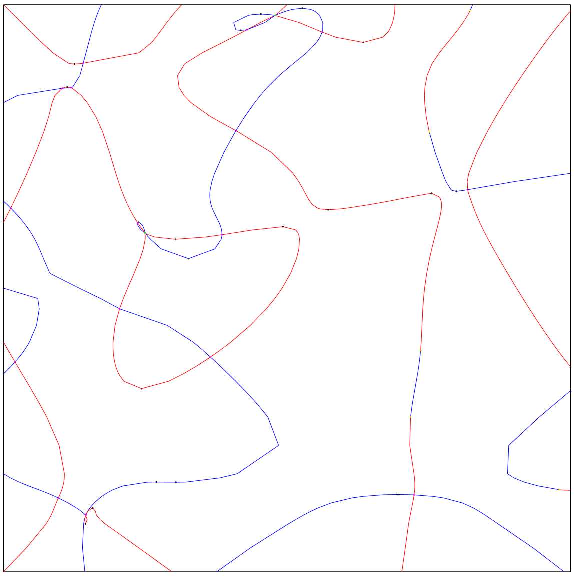

Figure 7. Isotopic approximation of the ridge curve in the parametric domain, with 3-ridge umbilics (green), 1-ridge umbilics (yellow), purple points (pink) and critical points (black).

6.6.3. Stability of persistence diagrams

Participant: David Cohen-Steiner.

In collaboration with Herbert Edelsbrunner and John Harer (Duke University).

The idea of topological persistence is to study the evolution of the topology of the sublevel-sets of a real function defined on a topological space as the threshold increases. It turns out that this information can be encoded as a certain set of intervals called persistence intervals. Each interval represents the life-time of a topological feature —for instance a handle or a connected component— in the evolution of sublevel-sets. Associating with each persistence interval a point in the plane whose coordinates are the bounds of the interval, we obtain the *persistence diagram* of the function. In this work, we prove that the persistence diagrams is stable when the function is corrupted by noise [33]. We apply this result to the comparison and classification of shapes, and to the estimation of the homology of a subset of a metric space from sample points.

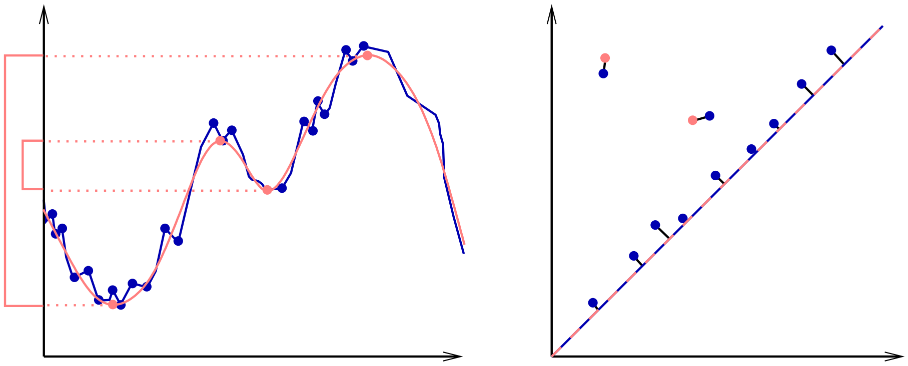

Figure 8. Persistence diagrams of two close functions.

6.6.4. Inequalities for the curvature of curves and surfaces

Participant: David Cohen-Steiner.

In collaboration with Herbert Edelsbrunner (Duke University).

In this paper [32], we bound the difference between the total mean curvatures of two closed surfaces in $\mathbb{R}^3$ in terms of their total absolute curvatures and the Fréchet distance between the volumes they enclose. Unlike previous results, and maybe surprisingly, the closeness between the normals to the two surfaces is not explicitly required. The proof combines the stability property for persistence diagrams [33] with integral-geometric methods. We also bound the difference between the lengths of two curves using the same methods, which gives a generalization of Fàry's theorem.

6.7. Shape coding and compression

Keywords: *Watermarking, graph coding.*

6.7.1. Dynamic coding of triangulations

Participants: Luca Castelli Aleardi, Olivier Devillers.

In collaboration with Gilles Schaeffer (Laboratoire d'Informatique de l'École Polytechnique, LIX)

Our work on the design of compact structures for representing a triangulation in main memory has been extended to allow dynamic updates of the triangulation [30]. The main idea remains a hierarchical structure, splitting a triangulation of n triangles in $O(\log^2 n)$ small pieces, each of them being split again in $O(\log n)$ tiny pieces of size $< \log n$. The difference mainly involves the use of various dynamic arrays to store connectivity information between these pieces and new levels of indirection to be able to manage the variable memory size used to store these pieces. The result is that a triangulation can be stored with a cost of $2.17 \times n$ bits plus a sublinear term $O(\frac{n}{\sqrt{\log n}})$ and allowing dynamic update of the triangulation in $O(\log^2 n)$ per insertion, flip or deletion of a degree 3 vertex [29], [46].

6.7.2. *Compact storage of triangulations in practice*

Participants: Abdelkrim Mebarki, Olivier Devillers, Sylvain Pion.

In the spirit of the mostly theoretical work above, we investigate more practical solution that could be used in CGAL. Classical storage of triangulation usually involve 12 pointers per triangle that is $12 \times 32 \times n$ bits. From a theoretical point of view, going to $12 \times \log n \times n$ bits is trivial, but in practice it is much more difficult because reading and writing memory by words of variable size is not really what is expected by the operating system. We have implemented and experimented a new triangulation data structure in CGAL which allows this kind of operation. We save about 50% of the memory (depending on the triangulation size) with a big increase of the CPU time.

6.8. Applications

6.8.1. *Protein-Protein interfaces revisited. Part II: experimental study*

Keywords: *Macro-molecular complexes and interfaces, Van der Waals models, Voronoï diagrams, docking, proteins.*

Participant: Frédéric Cazals.

In collaboration with F. Proust and J. Janin (CNRS, Orsay).

This paper [52] provides a detailed experimental study of an interface model developed in the companion article F. Cazals and F. Proust, *Revisiting the description of Protein-Protein interfaces. Part I: algorithms*. Our experimental study is concerned with the usual database of protein-protein complexes, split into five families (Proteases, Immune system, Enzyme Complexes, Signal transduction, Misc.) Our findings, which bear some contradictions with usual statements are the following: (i) Connectivity properties incur important variations across families. These properties are sensitive to water molecules and their variations upon consideration of structural water quantify the filling of packing defects by water molecules. (ii) The model of interfaces as a hydrophilic rim and a hydrophobic core is not general. (iii) At the interface scale, curvature properties correlate with the interface surface area, while locally, the absolute mean curvature follows a bimodal distribution. (iv) About 10% of interfaces consists of several connected components, and this multi-patch structure is independent from structural water. Almost all interfaces feature holes of significant size filled by structural water. Stable crystallographic water molecules play a prominent in reconnecting disconnected interfaces. (v) The chemical composition of interfaces in terms or pairwise contacts features a constant ratio of undetermined interactions, with subtle inter-families variations of determined interactions.

Overall, these conclusions shed some light on which structural parameters are most relevant to describe protein-protein interactions.

6.8.2. *Accurate interactive specular reflections on curved objects*

Participants: Frédéric Cazals, Olivier Devillers.

In collaboration with Pau Estella, Ignacio Martin (U. Girona) George Drettakis (INRIA-REVES) and Dani Tost (UPC Barcelona)

We present a new method [36] to compute interactive reflections on curved objects. The approach creates virtual reflected objects which are blended into the scene. We use a property of the reflection geometry which allows us to efficiently and accurately find the point of reflection for every reflected vertex, using only reflector geometry and normal information. This reflector information is stored in a pair of appropriate cube-maps, thus making it available during rendering. The implementation presented achieves interactive rates on reasonably sized scenes. In addition,we introduce an interpolation method to control the accuracy of our solution depending on the required frame rate.

6.8.3. *Farthest point seeding for placement of streamlines*

Participants: Abdelkrim Mebarki, Pierre Alliez, Olivier Devillers.

We propose an algorithm for placement of streamlines from two-dimensional steady vector or direction fields [37]. Our method consists of placing one streamline at a time by numerical integration starting at the furthest away from all previously placed streamlines. Such a farthest point seeding strategy leads to high quality placements by favoring long streamlines, while retaining uniformity with the increasing density. Our greedy approach generates placements of comparable quality with respect to the optimization approach from Turk and Banks, while being 200 times faster. Simplicity, robustness as well as efficiency is achieved through the use of a Delaunay triangulation to model the streamlines, address proximity queries and determine the biggest voids by exploiting the empty circle property. Our method handles variable density and extends to multiresolution.

6.8.4. *Geometric optimization for the conception of telescopes*

Participants: Jean-Daniel Boissonnat, Trung Nguyen.

The goal of this study is to optimize pupil configurations for extended source imaging based on optical interferometry.

The underlying problem can be formally stated as follows: given an objective O supposed to be a disk, how to place a system of circular pupils $\mathcal{P} = \{P_1, ..., P_n\}$ such that its auto-correlation support $\mathcal{C}$ covers entirely the objective where $\mathcal{C} = \{t \in \mathbb{R}^2 \mid \mathcal{P} + t \cap \mathcal{P} \neq \varnothing\}$ and $\mathcal{P} + t = \{p + t \mid p \in \mathcal{P}\}$?

In [59], we show that power diagrams can help solving special cases of the general optimization problem. In particular, when the centers of the pupils are given, we provide efficient algorithms to minimize the total area of the pupils under the constraint that $\mathcal{C}$ covers the objective. Heuristics are also provided for the general problem where the positions of the pupils are unknown.

6.9. Software

Keywords: *C++ standardization, CGAL packages, SCILAB.*

6.9.1. *C++ standardization of interval arithmetic*

Participant: Sylvain Pion.

In collaboration with H. Brönnimann (Polytechnic University Brooklyn), and G. Melquiond (ARENAIRE).

We have submitted a proposal of specification of interval arithmetic to the C++ standardization committee [45]. The goal of this submission are several : have compiler writers aware of the wide use of interval arithmetic (by numerous INRIA projects including GEOMETRICA) and so we hope that they will provide specific optimizations for it, and have a standard interface for interval computations. Some industry members of the C++ standardization committee like Sun are also very interested in getting interval arithmetic standardized.

Our proposal is a basic template class parameterized by a floating point type, and provide some functions around it. Due to implementation cost constraints, we had to make choices between the simplicity of the implementation, and functionality. This appears for example in the choice we made for the specification of comparison operators between intervals. We are working on a revision of our proposal for the next standardization meeting in 2006, integrating the comments we got from the committee members.

6.9.2. *CG-LAB : An interface between cgal and scilab*

Participants: Naceur Meskini, Sylvain Pion.

We have started implementing a SCILAB toolbox dedicated to geometric computing, based on CGAL. A first beta version of this toolbox, to be released in 2005, provides the basic Delaunay triangulations in any dimension, as well as various 2D meshing algorithms.

6.9.3. New CGAL Packages

- **Surface Mesher** (participants: Laurent Rineau, Steve Oudot, Mariette Yvinec, in collaboration with David Rey (from DREAM staff) and Andreas Fabri (from GeometryFactory)). This new CGAL package provides an implementation of the algorithm developped by Boissonnat and Oudot [18] to mesh smooth surfaces. The current version of the package handles implicit surfaces and level-sets in 3D grey-scaled images. The interface offers a few parameters to achieve a fine tuning according to the user needs with respect to the quality of the mesh elements and the accuracy of the surface approximation. The output mesh can also conform to a user defined sizing field. The code follows a C++ generic software design for Delaunay refinement processes proposed by Laurent Rineau.
- **Planar Parameterization of Triangulated Surfaces** (participants: Laurent Saboret and Pierre Alliez, in collaboration with B. Lévy (INRIA-ALICE project)). We are elaborating upon a CGAL package for planar parameterization of triangulated surface meshes. The package implements some of the state-of-the-art parameterization methods, such as LSCM (Lévy et al. 02), fixed or free boundary conformal maps (Eck et al. 95, Meyer et al. 02), Authalic (Meyer et al. 02), mean value coordinates (Floater 03) or Tutte barycentric mapping with uniform weights (Tutte 63). It supports two mesh CGAL data structures and three sparse linear solvers (OpenNL, TAUCS, SuperLU). A first submission of the package to the CGAL editorial board is planned for end of 2005.
- **Placement of Streamlines** (participant: Abdelkrim Mebarki). We are elaborating upon a CGAL package for placement of streamlines in 2D which implements the algorithm presented at Visualization 05 [37]. The input flow field is given as a vector or direction field defined onto a closed planar domain. The algorithm saturates the domain with a set of streamlines which are everywhere tangential to the input flow field. The streamline density is in accordance with either a user-defined input parameter or with a density field. The package comes with an interactive demo available on both Linux and Windows platforms.

7. Contracts and Grants with Industry

7.1. GeometryFactory

The initial development phase of the CGAL library has been made by a European consortium. In order to achieve the transfer and diffusion of CGAL in the industry, a company called GeometryFactory has been founded in January 2003 by Andreas Fabri (http://www.GeometryFactory.com).

The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GeometryFactory is a link between the researchers from the computational geometry community and the industrial users.

It offers licenses to interested companies, and provides support. There are contracts in various domains such as CAD/CAM, medical applications, GIS, computer vision...

GeometryFactory is keeping close contacts with the original consortium members, and in particular with GEOMETRICA.

7.2. Alcatel Alenia Space

Participants: Jean-Daniel Boissonnat, Trung Nguyen.

In collaboration with P. Blanc and F. Falzon (Alcatel Alenia Space).

The goal of this study is to optimize pupil configurations for extended source imaging based on optical interferometry.

The motivation for this work comes from the observation of the Earth from a geostationary orbit (i.e. at a distance of ~ 36000 km) with a resolution of 1 m. A simple calculus shows us that we would need a telescope having a diameter of approximately 20 m for an optical wavelength of ~ 500 nm. Needless to say such an instrument dimension is not adapted to the observation from space and the use of interferometric telescopes (Optical Aperture Synthesis, OAS) is to be considered in this case.

Initial results based on Laguerre diagrams have proven useful in this context [59]. They will be further extended as part of T. Nguyen's PhD thesis.

8. Other Grants and Activities

8.1. National initiatives

8.1.1. ACI

Participants: Pierre Alliez, Luca Castelli Aleardi, Olivier Devillers, Abdelkrim Mebarki.

We are a member of GEOCOMP an initiative from the national «Action Concertée Incitative Masses de Données». The project involves

— the «Laboratoire d'Informatique de l'Ecole Polytechnique (LIX), CNRS, Ecole Polytechnique»,
— the «Laboratoire Bordelais de Recherche en Informatique (LaBRI), CNRS, Université Bordeaux I»,
— the «Service de Physique Théorique, CEA Saclay» and the GEOMETRICA team.

The project investigates compression schemes and compact data structures devoted specifically to geometrical objects.

Project web page: http://www.lix.polytechnique.fr/Labo/Gilles.Schaeffer/GeoComp/

8.1.2. Associated team Genepi

Participant: Sylvain Pion.

Also involved: Monique Teillaud.

We have started an INRIA associated team with Chee Yap (New York University) and Hervé Brönnimann (Polytechnic University Brooklyn), around the subjects of generic programming and robustness of geometric algorithms. This work includes the specification of algorithms in terms of concepts of geometries. It also includes the interface between algorithms and data structures, as well as collaborations on robustness issues around curved objects.

8.2. European initiatives

8.2.1. FET Open ACS

INRIA (teams GALAAD and GEOMETRICA) participates to the IST project ACS.

– Acronym : ACS, numéro IST-006413
– Title : Algorithms for complex shapes with certified topology and numerics.
– Specific program : IST
– RTD (FET Open)
– Starting date : may 1st, 2005 - Duration : 3 years
– Participation mode of Inria : Participant
– Other participants : Rijksuniversiteit Groningen, Eidgenössische Technische Hochschule Zürich, Freie Universität Berlin, Institut National de Recherche en Informatique et Automatique, Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V., National Kapodistrian University of Athens, Tel Aviv University, GeometryFactory Sarl

The ACS project aims at advancing the state of the art in computing with complex shapes. Current technolgy can cope well with curves in the plane and smooth surfaces in three-dimensional space. We want to address a larger class of shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension.

Increasingly demanding applications require efficient and robust algorithms for complex shapes. Topics that arise and that we address are shape approximation (including meshing and simplification), shape learning (including reconstruction and feature extraction), as well as robust modeling (including boolean operations). Our work on these topics will be closely intertwined with basic research on shape representations.

A unique and ambitious feature of our approach is the guaranteed quality of all data structures and algorithms we plan to develop. Through certified topology and numerics, we will be able to prove that the output is topologically and numerically consistent, according to prespecified criteria. A software prototype, dealing with a restricted class of complex shapes, will demonstrate the feasibility of our techniques in practice.

The web site of the project includes a detailled description of the objectives and all results http://acs.cs.rug.nl.

8.2.2. Network of Excellence aim@shape

INRIA is part of the Network of Excellence:

– Acronym : AIM@SHAPE

– Title: Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing Knowledge Embedded in multidimensional digital objects).

– Reference: 506766

– Start Date: 2004-01-01

– Duration: 48 months

– Contract Type: Network of Excellence

– Action Line: Semantic-based knowledge systems

– Project Funding: 5.74 Million Euros.

– Other participants :

– IMATI, Genova, Italy.

– University of Genova, Italy.

– EPFL, Lausanne, Switzerland.

– Fraunhofer Institute, Germany.

– INPG, France.

– Center for Research and Technology, Greece.

– University of Geneva, Switzerland.

– SINTEF, Norway.

– TECHNION, Israel.

– Weizmann Institute, Israel.

– Utrecht University, Netherlands.

The mission of AIM@SHAPE is to advance research in the direction of semantic-based shape representations and semantic-oriented tools to acquire, build, transmit, and process shapes with their associated knowledge. We foresee a generation of shapes in which knowledge is explicitly represented and, therefore, can be retrieved, processed, shared, and exploited to construct new knowledge. The attainment of a new vision of shape knowledge is achieved by: the formalisation of shape knowledge and the definition of shape ontologies in specific contexts; the definition of shape behaviours which formalise the interoperability between shapes; the delineation of methods for knowledge-based design of shapes and the definition of tools for semantics-dependent mapping of shapes. The web site of the network includes a detailled description of the objectives and some results http://www.aim-at-shape.net.

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Editorial boards of scientific journals

- J-D. Boissonnat is a member of the editorial board of *Theoretical Computer Science*, *Algorithmica*, *International Journal of Computational Geometry and Applications*, *Computational Geometry : Theory and Applications*, and *The Visual Computer*.

- M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.
- S. Pion and M. Yvinec are members of the CGAL editorial board.

9.1.2. Conference programs committees

- Jean-Daniel Boissonnat was a member of the program committees of Solid Modelling International 2005, of the ACM Symposium on Solid and Physical Modeling 2005 and of the Eurographics Symposium on Geometry Processing 2005.
- Jean-Daniel Boissonnat was a member of the program committee of the Journée Recherche Industrie «Des données et modèles 3D aux calculs et à l'exploitation de résultats», INRIA Rocquencourt, février 2005.
- Pierre Alliez was member of the paper committee of the third Eurographics Symposium on Geometry Processing 2005, Eurographics 2005 and Pacific Graphics 2005. He was also member of the Video and Multimedia presentation program committee for the 21st Annual ACM Symposium on Computational Geometry (SOCG) 2005.
- Frédéric Cazals was member of the paper committee of the Eurographics Symposium on Geometry Processing 2005, of the Symposium on Point Based Graphics 2005, and of the ACM Symposium on Solid and Physical Modeling 2005.

9.1.3. Ph.D. thesis and HDR committees

- Pierre Alliez was a referee of the Ph.D. thesis committee of Raphaèle Balter (Université de Rennes) and Christian Rössl (MPII Saarbrucken). He was a member of the thesis committee of Marie-Claude Frasson (Université de Nice Sophia-Antipolis), Gabriel Peyré (Ecole Polytechnique) and Guillaume Lavoué (Université de Lyon).
- Jean-Daniel Boissonnat was a referee of the Habilitation dissertations of F. Chazal (Université de Bourgogne) and J. Giesen (ETH Zurich). He was a referee of the Ph.D. thesis committees of M. Flandrin (IFP) and K. Nouioua (Université de Méditerranée).
- Olivier Devillers was a referee of the Ph.D. thesis committee of Geoffroy Lauvaux (Université de Reims Champagne Ardennes) and Ali Asghar Khanban (Imperial college, London). He was a member of the thesis committee of Thomas Lewiner (Paris VI) and referee of the HDR committee of Gilles Schaeffer (Université de Bordeaux).
- David Cohen-Steiner was a referee of the Ph.D. thesis committee of Dmitriy Morozov (Duke University).

9.1.4. INRIA committees

-Jean-Daniel Boissonnat was chairman of the Projects Committee of INRIA Sophia-Antipolis till august 2005. He is chairman of the Evaluation Board of INRIA since november 2005.
- Frédéric Cazals is member of the INRIA Scientific Steering Committee of INRIA (COST).
- Mariette Yvinec is member of the « Comité des utilisateurs des moyens informatiques de recherche de l'INRIA Sophia-Antipolis » (CUMIR)
- Agnès Bessière is member of the « Comité des utilisateurs des moyens informatiques des services de l'INRIA Sophia-Antipolis » (CUMIS)
- Olivier Devillers is member of the committee for « détachements » at INRIA Sophia-Antipolis.
- Agnès Bessière, Olivier Devillers and Pierre Alliez are members of the « comité de centre » at INRIA Sophia-Antipolis.
- Sylvain Pion is a member of the « Commission du Développement Logiciel » (CDL) at INRIA Sophia-Antipolis.
- Pierre Alliez is member of the « Comité des cours et colloques » at INRIA Sophia-Antipolis.

9.1.5. Other committees

- Jean-Daniel Boissonnat is member of the « Commission de spécialistes » of the Ecole Normale Supérieure de Paris.
- Frédéric Cazals is member of the « Commission de spécialistes » of the Mathematics Department of the University of Bourgogne, Dijon, France.

- Olivier Devillers is member of the «Conseil scientifique de l'école doctorale STIC» at Nice University.
- Sylvain Pion is member of the « groupe d'experts de l'AFNOR pour le language C++ ».

9.1.6. *WWW server*

http://www-sop.inria.fr/geometrica/
The GEOMETRICA project maintains on its web site a collection of comprehensive sheets about the subjects presented in this report, as well as downloadable softwares. A surface reconstruction service is also available (see section 5.2).

9.2. Teaching

9.2.1. *Teaching responsibilities*

- In the «Master STIC» of Nice Sophia-Antipolis University Olivier Devillers chairs the second year research speciality : «Image et géométrie pour le multimédia et la modélisation du vivant»
- Olivier Devillers is professor «Chargé de cours» at École Polytechnique.

9.2.2. *Teaching at universities*

- Tsinghua University, Beijing, Advanced Computational Geometry, Jean-Daniel Boissonnat (16h)
- Ecole Centrale Paris, Introduction to Computational Structural Biology (F. Cazals - 9h, M. Nilges (Inst. Pasteur) - 6h)
- MPRI (Master Parisien de Recherches Informatiques) (2005-2006), Cours de 2ième annee , Géométrie algorithmique : Triangulations, maillages et modélisation géométrique, Jean-Daniel Boissonnat and Mariette Yvinec (24h).
- École Polytechnique (Palaiseau), Computational Geometry (2005-2006), 40h (O. Devillers).
- École Polytechnique (Palaiseau), Java programming (2005-2006), exam (O. Devillers).
- ISIA (Sophia-Antipolis), Computational Geometry (2005-2006), 10h (O. Devillers).
- Maîtrise Informatique (Nice), Computational Geometry, 24h (O. Devillers 12h, M. Pouget 12h).
- Master de Mathématiques, options Formes (Nice) (2004-2005) Triangulations, maillages et modélisation géométrique, Jean-Daniel Boissonnat (15h), Mariette Yvinec (15h).
- Master STIC-IGMMV-ISI (Sophia-Antipolis), Algorithmic frameworks for geometry (2005-2006), 15h (O. Devillers).
- Master STIC-IGMMV-ISI (Sophia-Antipolis), Implementing computational geometry (2005-2006), 20h (P. Alliez).
- Master STIC-IGMMV (Sophia-Antipolis), Surfaces and meshes (2004-2005), (F. Cazals - 9h, P. Alliez - 6h).
- Master IVR - Grenoble, Maillages et Surfaces, (F. Cazals - 6h; D. Attali (CNRS) - 6h).
- Master MIGS - Dijon, Maillages et surfaces (P. Alliez - 6h), Robustesse des algorithmes géométriques (S. Pion - 6h), Algorithmes en biologie structurale (F. Cazals - 6h).

9.2.3. *Internships*

Internship proposals can be seen on the web at http://www-sop.inria.fr/prisme/Stages/
- Daniel Loreto, *Delaunay triangulation using swap*. MIT Internship.
- Chinmay Karande, *Partial geometric shape matching using product graphs* IIT Bombay.
- Sébastien Loriot, *Exact arrangement of circles on a sphere. Applications to molecular modeling*, Univ. of Dijon.
- Julien Hazebrouck, *Curved kernel for* CGAL, *application to VLSI*, ESSI.
- Nicolas Carrez, *Utilisation de* CGAL *dans le logiciel de dessin Ipe*, Supelec Metz.
- Pooran Memari, *Calcul d'axes médians stables*, Ecole Polytechnique.
- Trung Nguyen, *Geometric optimization for the conception of telescopes*, ENS Cachan.

9.2.4. *Ongoing Ph.D. theses*

- Luca Castelli, *Compression et entropie d'objets pour la synthèse d'images* en cotutelle avec l'École Polytechnique.
- Christophe Delage, *Non affine Voronoi diagrams*, ENS-Lyon.
- Sébastien Loriot, *Modélisation mathématique, calcul et classification de poches d'arrimage de médicament sur les protéines*, université de Bourgogne.
- Abdelkrim Mebarki, *Structures de données compactes pour la géométrie*, université de Nice-Sophia Antipolis.
- Thanh-Trung Nguyen, *Geometric Optimization for the Conception of Telescopes*.
- Laurent Rineau, *Maillages tétraédriques*, Université de Paris VI.
- Marie Samozino, *Filtrage, simplification et représentation multirésolution d'objets géométriques reconstruits*, Université de Nice-Sophia Antipolis.
- Camille Wormser, *Maillages et diagrammes anisotropes*.

9.2.5. *Ph.D. defenses*

- Thomas Lewiner, *Computational Topology and Applications in Molecular Modeling*, École Polytechnique.
- Steve Oudot, *Sampling and Meshing Surfaces wit Guarantees*, École Polytechnique.
- Marc Pouget, *Geometry of surfaces: from the estimation of local differential quantities to the robust extraction of global differential features*, université de Nice-Sophia Antipolis. Defense date: 12/02/05.

9.3. Participation to conferences, seminars, invitations

9.3.1. *Talks*

Members of the project have presented all published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars or summer schools.

- «Geometry and topology for structural molecular modeling: erroneous judgments and (moderately) high hopes», Workshop *Bioinformatics: algorithms, structures and statistics*, LIX, Ecole Polytechnique, F. Cazals.
- «Contacts in macro-molecular assemblies», Workshop *Visualization of Large Biomolecular Complexes*, La Jolla, USA, F. Cazals.
- «Ridges and umbilics of polynomial parametric surfaces», *Real Algebra, Quadratic Forms and Model Theory; Algorithms and Applications*, Paris, France, M.Pouget.
- «Codage et représentations compactes en géométrie», *séminaire LORIA*, Nancy, O. Devillers.
- «Géométrie algorithmique, arithmétique et algèbre», *Journées Nationales de Calcul Formel*, CIRM, Luminy, S. Pion.
- «CGAL, a computational geometry algorithms library», Summer School on Open Software for Algebraic and Geometric Computations, Sophia Antipolis, September 2005, M. Yvinec.
- «20 ans de recherche sur la reconstruction de surfaces», *Journée Recherche Industrie*, INRIA Rocquencourt, J.-D. Boissonnat.
-«Meshing and reconstructing surfaces», Institut Français du Pétrole, March 2005, J.-D. Boissonnat.
-«Blind approximation of surfaces», *Congrès de la SMAI*, Evian, May 2005, J.-D. Boissonnat.
- «Surface Learning by Probing», *Dagstuhl Seminar on Computational Geometry*, Dagstuhl, March 2005, J.-D. Boissonnat.
-«Voronoi Diagrams and Surface Approximation», Laboratoire d'Informatique Fondamentale, Université de Méditerranée, September 2005. J.-D. Boissonnat.
-«Surface mesh generation by Delaunay refinement», LIAMA, Pékin, October 2005. J.-D. Boissonnat.
- «Centroidal Voronoi Tessellations», SIAM annual meeting, New Orleans, July 2005. P. Alliez.
- «Optimization Techniques for Meshing», SIAM geometric design, Phoenix, October 2005. P. Alliez.
- «Optimization Techniques for Meshing», University of Aachen, Germany, April 2005. P. Alliez.
- «Geometry Compression, a Survey», EUSIPCO, Antalya, Turkey, September 2005. P. Alliez.

- «Persistance topologique», Journées de Géométrie algorithmique, Saint-Pierre de Chartreuse, Janvier 2005. D. Cohen-Steiner.

9.3.2. *The geometrica seminar*

http://www-sop.inria.fr/prisme/seminaire/
The GEOMETRICA seminar featured presentations from the following visiting scientists:
M. Nilges, Institut Pasteur, France
B. Lévy, LORIA, France
Y. Babenko, Vanderbilt University, USA
J-P. Pons, INRIA Sophia, France
T. Lewiner, INRIA Sophia and PUC - Rio de Janeiro, Brazil
J-M. Schlenker, Univ. Paul Sabatier, Toulouse, France
M. Sanner, The Scripps Research Institute, La Jolla, USA
R. Marroquim, A.L. Coimbra Institute, Rio de Janeiro, Brazil
A. Ressayre, Univ. Paris-Sud / INRA

9.3.3. *Scientific visits*

- D. Cohen-Steiner visited Duke University (USA) for one month.
- S. Pion visited the New-York University for four weeks.
- J.-D. Boissonnat visited Tsinghua University (Beijing) for two weeks.

GEOMETRICA has hosted three scientists:

- K. Saetzler, University of Ulster (August-September 2005).
- G. Vegter, University of Groningen (May 2005).
- C. Rössl, MPII Saarbrucken, Germany (December 2005).

10. Bibliography

Major publications by the team in recent years

[1] J.-D. BOISSONNAT, F. CAZALS. *Smooth Surface Reconstruction via Natural Neighbour Interpolation of Distance Functions*, in "Comp. Geometry Theory and Applications", 2002, p. 185–203.

[2] J.-D. BOISSONNAT, M. YVINEC. *Algorithmic geometry*, traduit de la version française (Ediscience international) par Hervé Brönnimann, Cambridge University Press, UK, 1998, http://www-sop.inria.fr/prisme/personnel/yvinec/livre.html.

[3] H. BRÖNNIMANN, C. BURNIKEL, S. PION. *Interval arithmetic yields efficient dynamic filters for computational geometry*, in "Discrete Applied Mathematics", vol. 109, 2001, p. 25–47.

[4] H. BRÖNNIMANN, I. EMIRIS, V. PAN, S. PION. *Sign Determination in Residue Number Systems*, in "Theoret. Comput. Sci.", Special Issue on Real Numbers and Computers, vol. 210, n^o 1, 1999, p. 173–197, ftp://ftp-sop.inria.fr/geometrica/pion/publis/TCS.pdf.

[5] D. COHEN-STEINER, P. ALLIEZ, M. DESBRUN. *Variational Shape Approximation*, in "ACM Transactions on Graphics. Special issue for SIGGRAPH conference", 2004, p. 905–914.

[6] D. COHEN-STEINER. *Quelques problèmes liés à la discrétisation des surfaces*, Thèse de doctorat en sciences, École Polytechnique, France, 2004, http://www.inria.fr/rrrt/tu-1137.html.

[7] O. DEVILLERS. *The Delaunay hierarchy*, in "Internat. J. Found. Comput. Sci.", vol. 13, 2002, p. 163–180, ftp://ftp-sop.inria.fr/geometrica/publis/d-dh-02.pdf.

[8] O. DEVILLERS. *Computational geometry and discrete computations*, in "Proc. 6th Discrete Geometry for Computer Imagery conf.", Lecture Notes in Computer Science, (invited paper), vol. 1176, Springer-Verlag, 1996, p. 317–324, ftp://ftp-sop.inria.fr/geometrica/publis/d-cgdc-96.pdf.

[9] A. KHODAKOVSKY, P. ALLIEZ, M. DESBRUN, P. SCHRÖDER. *Near-Optimal Connectivity Encoding of 2-Manifold Polygon Meshes*, in "Graphical Models", vol. 64, 2002, p. 147–168, http://www.multires.caltech.edu/pubs/ircomp.pdf.

[10] S. PION. *De la géométrie algorithmique au calcul géométrique*, Thèse de doctorat en sciences, université de Nice-Sophia Antipolis, France, 1999, http://www.inria.fr/rrrt/tu-0619.html.

Books and Monographs

[11] P. ALLIEZ, C. GOTSMAN. , N. DODGSON, M. FLOATER, M. SABIN (editors). *Recent Advances in Compression of 3D Meshes*, 2005.

Doctoral dissertations and Habilitation theses

[12] T. LEWINER. *Mesh Compression from Geometry*, Thèse de doctorat en sciences, Université Pierre et Marie Curie, France, 2006.

[13] S. OUDOT. *Sampling and Meshing Surfaces with Guarantees*, Thèse de doctorat en sciences, Ecole Polytechnique, France, 2005.

[14] M. POUGET. *Geometry of surfaces: from the estimation of local differential quantities to the robust extraction of global differential features.*, Thèse de doctorat en sciences, université Nice Sophia-Antipolis, Nice, France, 2005.

Articles in refereed journals and book chapters

[15] P. ALLIEZ, D. COHEN-STEINER, M. YVINEC, M. DESBRUN. *Variational tetrahedral meshing*, in "ACM Transactions on Graphics", SIGGRAPH '2005 Conference Proceedings, vol. 24, 2005, p. 617–625.

[16] J.-D. BOISSONNAT. *Voronoi diagrams, triangulations and surfaces*, in "Differential Geometry and Topology, Computational Geometry", J.-M. MORVAN, M. BOUCETTA (editors). , Science Series III : Computer and Systems Sciences, vol. 197, NATO, 2005, p. 340–368.

[17] J. BOISSONNAT, R. CHAINE, P. FREY, G. MALANDAIN, S. SALMON, E. SALTEL, M. THIRIET. *From arteriographies to computational flow in saccular aneurisms: the INRIA experience*, in "Medical Image Analysis", vol. 9, 2005, p. 101–177.

[18] J.-D. BOISSONNAT, S. OUDOT. *Provably good sampling and meshing of surfaces*, in "Graphical Models", vol. 67, 2005, p. 405–451.

[19] F. CAZALS, C. KARANDE. *An algorithm for reporting maximal c-cliques*, in "Theoret. Comput. Sci.", vol. 349, 2005, p. 484–490.

[20] F. CAZALS, M. POUGET. *Differential topology and geometry of smooth embedded surfaces: selected topics*, in "Internat. J. Comput. Geom. Appl.", vol. 15, 2005, p. 511–536.

[21] F. CAZALS, M. POUGET. *Estimating Differential Quantities using Polynomial fitting of Osculating Jets*, in "Computer Aided Geometric Design", vol. 22, 2005, p. 121–146.

[22] D. COHEN-STEINER, É. COLIN DE VERDIÈRE, M. YVINEC. *Conforming Delaunay Triangulations in 3D*, in "Computational Geometry: Theory and Applications", vol. 28, 2004, p. 217-233.

[23] O. DEVILLERS, P. GUIGUE. *Inner and Outer Rounding of Set Operations on Lattice Polygonal Regions*, in "Comp. Geometry Theory and Applications", vol. 33, 2005, p. 3–17.

[24] C. LI, S. PION, C. YAP. *Recent Progress in Exact Geometric Computation*, in "Journal of Logic and Algebraic Programming", Special issue on the practical development of exact real number computation, vol. 64, 2005, p. 85–111.

Publications in Conferences and Workshops

[25] D. ATTALI, D. COHEN-STEINER, H. EDELSBRUNNER. *Extraction and Simplification of Iso-surfaces in Tandem*, in "Proc. 3rd Annu. Sympos. Geometry Processing", 2005, p. 139–148.

[26] J.-D. BOISSONNAT, C. DELAGE. *Convex hulls and Voronoi diagrams of additively weighted points*, in "Proc. 13th European Symposium on Algorithms", Lecture Notes Comput. Sci., vol. 3669, Springer-Verlag, 2005, p. 367–378.

[27] J. BOISSONNAT, L. GUIBAS, S. OUDOT. *Surface learning by probing*, in "Proceedings of the 21st ACM Symposium on Computational Geometry", 2005, p. 198–207.

[28] J.-D. BOISSONNAT, C. WORMSER, M. YVINEC. *Anisotropic Diagrams: Labelle Shewchuk approach revisited*, in "Proc. 17th Canad. Conf. Comput. Geom.", 2005, p. 266–269.

[29] L. CASTELLI ALEARDI, O. DEVILLERS, G. SCHAEFFER. *Dynamic updates of succinct triangulations*, in "Proc. 17th Canad. Conf. Comput. Geom.", 2005, p. 135–138.

[30] L. CASTELLI ALEARDI, O. DEVILLERS, G. SCHAEFFER. *Succinct representation of triangulations with a boundary*, in "Proc. 9th Workshop on Algorithms and Data Structures", 2005.

[31] F. CAZALS, J. GIESEN, M. PAULY, A. ZOMORODIAN. *Conformal Alpha Shapes*, in "Symposium on Point Based Graphics, Stony Brook, NY, USA", 2005, p. 55–62.

[32] D. COHEN-STEINER, H. EDELSBRUNNER. *Inequalities for the curvature of curves and surfaces*, in "Proc. 21th Annu. ACM Sympos. Comput. Geom.", 2005, p. 272–277, http://doi.acm.org/10.1145/1064092.1064134.

[33] D. COHEN-STEINER, H. EDELSBRUNNER, J. HARER. *Stability of persistence diagrams*, in "Proc. 21th Annu. ACM Sympos. Comput. Geom.", 2005, p. 263–271, http://doi.acm.org/10.1145/1064092.1064133.

[34] O. DEVILLERS, V. DUJMOVIĆ, H. EVERETT, S. HORNUS, S. WHITESIDES, S. WISMATH. *Maintaining Visibility Information of Planar Point Sets with a Moving Viewpoint*, in "Proc. 17th Canad. Conf. Comput. Geom.", 2005, p. 302–305.

[35] O. DEVILLERS, H. EVERETT, S. LAZARD, M. PENTCHEVA, S. WISMATH. *Drawing K_n in Three Dimensions with One Bend per Edge*, in "Proc. 13th Int. Symp. on Graph Drawing", 2005.

[36] P. ESTALELLA, I. MARTIN, G. DRETTAKIS, D. TOST, O. DEVILLERS, F. CAZALS. *Accurate Interactive Specular Reflections on Curved Objects*, in "Vision, Modeling, and Visualization 2005", 2005.

[37] A. MEBARKI, P. ALLIEZ, O. DEVILLERS. *Farthest Point Seeding for Efficient Placement of Streamlines*, in "Proc. of IEEE Visualization", 2005.

[38] G. MELQUIOND, S. PION. *Formal certification of arithmetic filters for geometric predicates*, in "Proc. 17th IMACS World Congress on Scientific , Applied Mathematics and Simulation", 2005.

[39] T. NGUYEN. *A linear algorithm for polygonal approximations of thick curves*, in "Proc. Int. Conf. on Computer Analysis of Images and Patterns", Lecture Notes Comput. Sci., vol. 3691, 2005, p. 17–25.

[40] S. OUDOT, L. RINEAU, M. YVINEC. *Meshing Volumes Bounded by Smooth Surfaces*, in "Proc. 14th International Meshing Roundtable", 2005, p. 203-219.

[41] O. SHALL, M. SAMOZINO. *Surface from Scattered Points: A Brief Survey of Recent Developments*, in "Workshop towards Semantic Virtual Environments", 2005.

Internal Reports

[42] P. ALLIEZ, G. UCELLI, C. GOTSMAN, M. ATTENE. *Recent Advances in Remeshing of Surfaces*, Technical report, AIM@SHAPE Network of Excellence, 2005.

[43] H. BRÖNNIMANN, O. DEVILLERS, V. DUJMOVIC, H. EVERETT, M. GLISSE, X. GOAOC, S. LAZARD, H.-S. NA, S. WHITESIDES. *On the Number of Maximal Free Line Segments Tangent to Arbitrary Three-dimensional Convex Polyhedra*, Rapport de recherche, n^o 5671, INRIA, 2005, http://www.inria.fr/rrrt/rr-5671.html.

[44] H. BRÖNNIMANN, O. DEVILLERS, S. LAZARD, F. SOTTILE. *Lines tangent to four triangles in three-dimensional space*, Rapport de recherche, n^o 5693, INRIA, 2005, http://www.inria.fr/rrrt/rr-5693.html.

[45] H. BRÖNNIMANN, G. MELQUIOND, S. PION. *A Proposal to add Interval Arithmetic to the C++ Standard Library*, Rapport de recherche, n^o 5646, INRIA, 2005, http://www.inria.fr/rrrt/rr-5646.html.

[46] L. CASTELLI ALEARDI, O. DEVILLERS, G. SCHAEFFER. *Dynamic updates of succinct triangulations*, Rapport de recherche, n^o 5709, INRIA, 2005, http://www.inria.fr/rrrt/rr-5709.html.

[47] F. CAZALS, J.-C. FAUGERE, M. POUGET, F. ROUILLIER. *The implicit structure of ridges of a smooth parametric surface*, Technical report, n° 5608, INRIA, 2005, http://www.inria.fr/rrrt/rr-5608.html.

[48] F. CAZALS, J.-C. FAUGERE, M. POUGET, F. ROUILLIER. *Topologically certified approximation of umbilics and ridges on polynomial parametric surface*, Technical report, n° 5674, INRIA, 2005, http://www.inria.fr/rrrt/rr-5674.html.

[49] F. CAZALS, C. KARANDE. *An algorithm for reporting maximal c-cliques*, Rapport de recherche, n° 5642, INRIA, 2005, http://www.inria.fr/rrrt/rr-5642.html.

[50] F. CAZALS, C. KARANDE. *Reporting maximal cliques: new insights into an old problem*, Rapport de recherche, n° 5615, INRIA, 2005, http://www.inria.fr/rrrt/rr-5615.html.

[51] F. CAZALS, M. POUGET. *Topology driven algorithms for ridge extraction on meshes*, Rapport de recherche, n° 5526, INRIA, 2005, http://www.inria.fr/rrrt/rr-5526.html.

[52] F. CAZALS, F. PROUST. *Revisiting the description of Protein-Protein interfaces. Part II: Experimental study*, Technical report, n° 5501, INRIA, 2005, http://www.inria.fr/rrrt/rr-5501.html.

[53] O. DEVILLERS, V. DUJMOVIĆ, H. EVERETT, S. HORNUS, S. WHITESIDES, S. WISMATH. *Maintaining Visibility Information of Planar Point Sets with a Moving Viewpoint*, Rapport de recherche, INRIA, 2005, http://www.inria.fr/rrrt/rr-5742.html.

[54] O. DEVILLERS, H. EVERETT, S. LAZARD, M. PENTCHEVA, S. WISMATH. *Drawing K_n in Three Dimensions with One Bend per Edge*, Rapport de recherche, n° 5708, INRIA, 2005, http://www.inria.fr/rrrt/rr-5708.html.

[55] A. MEBARKI, P. ALLIEZ, O. DEVILLERS. *Farthest Point Seeding for Placement of Streamlines*, Rapport de recherche, n° 5524, INRIA, 2005, http://www.inria.fr/rrrt/rr-5524.html.

[56] G. MELQUIOND, S. PION. *Formally certified floating-point filters for homogeneous geometric predicates*, Rapport de recherche, n° 5644, INRIA, 2005, http://www.inria.fr/rrrt/rr-5644.html.

[57] S. OUDOT, L. RINEAU, M. YVINEC. *Meshing Volumes Bounded by Smooth Surfaces*, Rapport de recherche, n° 5626, INRIA, 2005, http://www.inria.fr/rrrt/rr-5626.html.

Miscellaneous

[58] P. MEMARI. *Calcul d'axes médians stables*, Rapport de stage d'option scientifique, École Polytechnique, France, 2005.

[59] T. NGUYEN. *Geometric optimization for the conception of telescopes*, Rapport de DEA Mathématiques Vision Apprentissage, ENS Cachan, France, 2005.