INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team Mathfi

Financial Mathematics

Rocquencourt

THEME NUM

Activity Report

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2. Overall Objectives

2.1. Overall Objectives

MathFi is a joint project-team with INRIA-Rocquencourt, ENPC (CERMICS) and the University of Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques. The scientific skills of the MathFi research team are focused on probabilistic and deterministic numerical methods and their implementation, stochastic analysis, stochastic control. Main applications concern evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, calibration of financial models. Special attention is paid to models with jumps, stochastic volatility models, asymmetry of information. An important part of the activity is related to the development of the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions.
Premia web Site: http://www.premia.fr .

3. Scientific Foundations

3.1. Numerical methods for option pricing and hedging

Keywords: *Euler schemes*, *Malliavin calculus*, *Monte-Carlo*, *approximation of SDE*, *finite difference*, *quantization*, *tree methods*.

Participants: A. Alfonsi, V. Bally, E. Clément, J. Guyon, B. Jourdain, A. Kbaier, A. Kohatsu-Higa, D. Lamberton, B. Lapeyre, J. Lelong, V. Lemaire, G. Pagès, J. Printems, D. Pommier, A. Sulem, P. Tankov, E. Voltchkova, A. Zanette.

Efficient computations of prices and hedges for derivative products is a major issue for financial institutions.

Monte-Carlo simulations are widely used because of their implementation simplicity and because closed formulas are usually not available. Nevertheless, efficiency relies on difficult mathematical problems such as accurate approximation of functionals of Brownian motion (e.g. for exotic options), use of low discrepancy sequences for nonsmooth functions, quantization methods etc. Speeding up the algorithms is a constant preoccupation in the development of Monte-Carlo simulations. Another approach is the numerical analysis of the (integro) partial differential equations which arise in finance: parabolic degenerate Kolmogorov equation, Hamilton-Jacobi-Bellman equations, variational and quasi–variational inequalities (see [11]).

This activity in the MathFi team is strongly related to the development of the Premia software.

3.2. Model calibration

One of the most important research directions in mathematical finance after Merton, Black and Scholes is the modeling of the so called *implied volatility smile*, that is, the fact that different traded options on the same underlying have different Black-Scholes implied volatilities. The smile phenomenon clearly indicates that the Black-Scholes model with constant volatility does not provide a satisfactory explanation of the prices observed in the market and has led to the appearance of a large variety of extensions of this model aiming to overcome the above difficulty. Some popular model classes are: the local volatility models (where the stock price volatility is a deterministic function of price level and time), diffusions with stochastic volatility, jump-diffusions, and so on. An essential step in using any such approach is the *model calibration*, that is, the reconstruction of model parameters from the prices of traded options. The main difficulty of the calibration problem comes from the fact that it is an inverse problem to that of option pricing and as such, typically ill-posed.

The calibration problem is yet more complex in the interest rate markets since in this case the empirical data that can be used include a wider variety of financial products from standard obligations to swaptions (options on swaps). The underlying model may belong to the class of short rate models like Hull-White [87], [69], CIR [77], Vasicek [106], etc. or to the popular class of LIBOR (London Interbank Offered Rates) market models like BGM [70].

The choice of a particular model depends on the financial products available for calibration as well as on the problems in which the result of the calibration will be used.

The calibration problem is of particular interest for MathFi project because due to its high numerical complexity, it is one of the domains of mathematical finance where efficient computational algorithms are most needed.

3.3. Application of Malliavin calculus in finance

Keywords: *Malliavin calculus, greeks computation, sensibility calculus, stochastic variations calculus*.

Participants: V. Bally, M.P. Bavouzet, J. Da Fonseca, B. Jourdain, A. Kohatsu-Higa, D. Lamberton, B. Lapeyre, M. Messaoud, A. Sulem, E. Temam, A. Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [94]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [97], D. Bell [66] D. Ocone [99], B. Øksendal [110]) give expositions of the subject. See also V. Bally [63] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [98].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [95], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods. We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequenses of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i - x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [83] and [82] and the papers by Bally et al, Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [89].

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P.Imkeller in [89]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [89]).

3.4. Stochastic Control and Backward Stochastic Differential equations

Keywords: *BSDE*, *Hamilton-Jacobi-Bellman*, *Stochastic Control*, *free boundary*, *risk-sensitive control*, *singular and impulse control*, *variational and quasi-variational inequalities*.

Participants: V. Bally, J.-Ph. Chancelier (ENPC), D. Lefèvre, M. Mnif, M. Messaoud, M.C. Kammerer-Quenez, A. Sulem.

Stochastic control consists in the study of dynamical systems subject to random perturbations and which can be controlled in order to optimize some performance criterion. Dynamic programming approach leads to Hamilton-Jacobi-Bellman (HJB) equations for the value function. This equation is of integrodifferential type when the underlying processes admit jumps (see [12]). The theory of viscosity solutions offers a rigourous framework for the study of dynamic programming equations. An alternative approach to dynamic programming is the study of optimality conditions (stochastic maximum principle) which leads to backward stochastic differential equations (BSDE). Typical financial applications arise in portfolio optimization, hedging and pricing in incomplete markets, calibration. BSDE's also provide the prices of contingent claims in complete and incomplete markets and are an efficient tool to study recursive utilities as introduced by Duffie and Epstein [78].

3.5. Anticipative stochastic calculus and insider trading

We study controlled stochastic systems whose state is described by anticipative stochastic differential equations. These SDEs can interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals [102]. This methodology is applied for utility maximization with insiders.

3.6. Fractional Brownian Motion

The Fractional Brownian Motion $B_H(t)$ with Hurst parameter H has originally been introduced by Kolmogorov for the study of turbulence. Since then many other applications have been found. If $H = \frac{1}{2}$

then $B_H(t)$ coincides with the standard Brownian motion, which has independent increments. If $H > \frac{1}{2}$ then $B_H(t)$ has a *long memory* or *strong aftereffect*. On the other hand, if $0 < H < \frac{1}{2}$, then $B_H(t)$ is *anti-persistent*: positive increments are usually followed by negative ones and vice versa. The strong aftereffect is often observed in the logarithmic returns $\log \frac{Y_n}{Y_{n-1}}$ for financial quantities Y_n while the anti-persistence appears in turbulence and in the behavior of volatilities in finance.

For all $H \in (0, 1)$ the process $B_H(t)$ is *self-similar*, in the sense that $B_H(\alpha t)$ has the same law as $\alpha^H B_H(t)$, for all $\alpha > 0$. Nevertheless, if $H \neq \frac{1}{2}$, $B_H(t)$ is not a semi-martingale nor a Markov process [86], [67], [68], and integration with respect to a FBM requires a specific stochastic integration theory.

4. Application Domains

4.1. Application Domains

- Option pricing and hedging
- Calibration of financial models
- Modeling of financial asset prices
- Portfolio optimization
- Insurance-reinsurance optimization policy
- Insider modeling, asymmetry of information

5. Software

5.1. Development of the software PREMIA for financial option computations

Keywords: *calibration*, *hedging*, *options*, *pricer*, *pricing*.

Participants: A. Alfonsi, V. Bally, S. Blunck, A. Cisse, M. Ciuca, B. Jourdain, A. Kohatsu Higa, J. Lelong, B. Lapeyre, M. Messaoud, A. Sulem, P. Tankov, E. Voltchkova, A. Zanette.

The development of Premia software is a joint activity of INRIA and CERMICS, undertaken within the MathFi project. Its main goal is to provide C/C++ routines and scientific documentation for the pricing of financial derivative products with a particular emphasis on the implementation of numerical analysis techniques rather than on the financial context. It is an attempt to keep track of the most recent advances in the field from a numerical point of view in a well-documented manner. The aim of the Premia project is threefold: first, to assist the R&D professional teams in their day-to-day duty, second, to help the academics who wish to perform tests of a new algorithm or pricing method without starting from scratch, and finally, to provide the graduate students in the field of numerical methods for finance with open-source examples of implementation of many of the algorithms found in the literature.

5.1.1. Consortium Premia.

Premia is developed in interaction with a consortium of financial institutions or departments presently composed of: IXIS CIB (Corporate & Investment Bank), CALYON, the Crédit Industriel et Commercial, EDF, Société générale and Summit Systems. The participants of the consortium finance the development of Premia (by contributing to the salaries of expert engineers hired by the MathFi project every year to develop the software) and help to determine the directions in which the project evolves. Every year, during a “delivery meeting”, a new version of Premia is presented to the consortium by the members of the MathFi project working on the software. This presentation is followed by the discussion of the features to be incorporated in the next release. In addition, between delivery meetings, MathFi project members meet individual consortium participants to further clarify their needs and interests. After the release of each new version of Premia, the old versions become available on Premia web site http://www.premia.fr and can be downloaded freely for academic and evaluation purposes. At present, this is the case for releases 3 and 4.

5.1.2. Content of Premia.

The development of Premia started in 1999 and 8 are released up to now.

Releases 1, 2 and 4 contain finite difference algorithms, tree methods and Monte Carlo methods for pricing and hedging European and American options on stocks in the Black-Scholes model in one and two dimensions.

Release 3 is dedicated to Monte Carlo methods for American options in high dimension and is interfaced with the Scilab software [73].

Releases 5 and 6 contain more sophisticated algorithms such as quantization methods for American options [4], [65] and methods based on Malliavin calculus for both European and American options as well as pricing, hedging and calibration algorithms for some models with jumps, local volatility and stochastic volatility.

Premia7 implements routines for option pricing in interest rate models: (Vasicek, Hull-White, CIR, CIR++, Black-Karasinsky, Squared-Gaussian, Li, Ritchken, Sankarasubramanian HJM, Bhar Chiarella HJM, BGM). New calibration algorithms for various models (including stochastic volatility and jumps) were implemented and numerical methods based on Malliavin calculus for jump processes were further explored.

Premia8, the last release of the software developed in 2005 will be presented to the consortium members in February 2006. It is dedicated to the pricing of credit risk derivatives, pricing and calibration for interest rate derivatives and pricing and calibration algorithms in jump models (see detailed description below).

This year, J. Lelong has translated all the Premia documentation in the html format, translated some parts from C to C++ with A. Zanette, and settled a cvs protocol at ENPC to allow joint development. Credit risk algorithms have been developed by M. Ciuca and A. Cisse under the supervision of P. Tankov and A. Alfonsi. E. Voltchkova has implemented pricing methods for European and barrier options in discontinuous models. Two methods have been implemented and tested for all exponential Lévy models used in financial literature: Fast Fourier Transform method of Carr & Madan [72] and finite difference method developed in her PhD thesis. [76], [107]. E. Voltchkova and P. Tankov proposed and implemented several algorithms for hedging options in models with jumps. The pricing part of the algorithms uses the finite difference method mentioned above. This work gave rise to the publications: [43], [54].

5.1.3. Detailed content of Premia Release 8 developped in 2005

Pricing Interest Rate Derivatives Affine models, CEV and Jump Diffusion Libor Market Model, Markov Functional Libor Market model.

- Markov Functional Libor Market model Hunt-Kennedy-Pelsser [88]
- Affine Models
- Collin-Dufresne Goldstein Algorithm [74]
- BGM
  - Andersen Monte Carlo Methods [60]
  - Jump Diffusion Libor Market Model [84]
  - BGM-CEV: Closed Formula, Monte Carlo, Finite Difference [59]

Pricing Credit Risk Derivatives (Defaultable Bonds, Options on Defaultable Bonds, Credit Default Swaps, Credit linked notes, Credit spread options) The credit derivatives market has been growing rapidly and a credit derivative toolbox meets the need to value and analyze more credit derivative instruments. Reduced-form models can be used to price contingent claims subject to default risk, focusing on numerical methods for pricing and hedging most instruments in the credit derivatives asset class (Defaultable Bonds, Options on Defaultable Bonds, Credit Default Swaps, Credit linked notes, Credit spread options, CDS options, CDO). In particular we focus on numerical methods (tree methods, finite difference, Monte Carlo) for SSRD stochastic intensity model and on the valutation of large basket derivatives (Fourier transform, Quasi-Monte Carlo Integration)([103], [105], [28]).

- HW, CIR++

  * HW Tree, Monte Carlo methods [103], [105]
  * CIR++ Monte Carlo Method, Derivatives pricing with the SSRD stochastic intensity model [28]
  * Basket Default Swaps, CDO's and Factor Copulas [92]

Pricing Equity in Black-Scholes and Heston models

- Finite Difference and tree methods for Discrete Monitoring Barrier Options [61]
- Functional quantization algorithms for Asian options in the Heston model.
- Kusuoka Scheme approximation of SDE for Asian options in the Heston model. [96].
- A Moments and Strike Matching Binomial Algorithm.[50].

Pricing and Hedging Equity in Jump models. Pricing and hedging European vanilla and Barrier options on stocks in exponential Lévy models with Fourier transform and finite differentce methods [72], [75], [76], [93]

- Models

  * Exponential Lévy models (Merton's model and more generally other finite intensity Lévy processes with Brownian component (Kou), Tempered stable process, Variance gamma, Normal inverse Gaussian)

- Products

  * European options on stocks
  * Barrier options on stocks

- Methods

  * Fourier transform
  * Finite difference methods

Calibration in Jump models [104]

6. New Results

6.1. Discretization of stochastic differential equations

Participants: A. Alfonsi, E. Clément, T. Ershova, A. Kohatsu Higa, V. Lemaire, D. Lamberton, J. Guyon, B. Jourdain, V. Lemaire, G. Pagès, P. Tankov.

In order to compute options prices and hedges by Monte Carlo simulations, it is necessary to discretize the SDE giving the model with respect to time. Usually this is done by using the standard explicit Euler scheme since schemes with higher order of strong convergence involve multiple stochastic integrals which are difficult to simulate.

D. Lamberton and G. Pagès have studied the approximation of the invariant measure for SDEs with a return condition using the Euler scheme with decreasing time-steps [90]. Their student V. Lemaire has investigated the same scheme when the SDE admits several invariant measures [15], [38]. J. Guyon [33] has proved that for uniformly elliptic SDEs with smooth coefficients, a tempered distribution applied to the density of the Euler scheme converges to the same distribution applied to the density of the solution of the SDE at rate given by the size of the time-step. This allows him to give the exact rate of convergence of the Euler scheme for the deltas and gammas of a european option. E. Clément, A. Kohatsu Higa and D. Lamberton have developed a new approach for the error analysis of weak convergence of the Euler scheme based on the linear equation satisfied by the error process [31]. This method is more general than the usual approach introduced by Talay and Tubaro and provides the means of the weak approximation of stochastic delay equations. In their theses, A. Kbaier [35] has developed a "statistic Romberg method" for the weak approximation of SDEs and A. Alfonsi has studied various explicit and implicit schemes for the discretization of Cox-Ingersoll-Ross processes for which the standard Euler scheme is not well-posed [18]. V. Lemaire [15] has proposed a new explicit scheme with stochastic time-steps to deal with SDEs when the coefficients are locally Lipschitz continuous but fail to be globally Lipschitz continuous.

6.2. Weak approximation of stochastic partial differential equations

Participant: J. Printems.

It is well known that SPDE appear in interest rate models (e.g. HJM-Musiela equations). Our aim here is to study a fully discrete approximation by means of finite elements in space and an implicite scheme in time of a parabolic stochastic partial differential equation in order to approximate the expectation of a functional of the solution (weak approximation). In [55], we show that optimal rates of convergence can be obtained both in time and in space without stability conditions on the time and space steps.

6.3. Monte Carlo simulations and variance reduction techniques

Keywords: *Monte-Carlo*, *variance reduction*.

Participants: N. Moreni, B. Lapeyre.

Nicola Moreni studies variance reduction techniques for option pricing based on Monte Carlo simulation. In particular, in a joint project with the University of Pavia (Italy), he applies path integral techniques to the pricing of path-dependent European options [27]. He has also investigated a variance reduction technique for the Longstaff-Schwartz algorithm for American option pricing [17]. This technique is based on an extension of B. Arouna's work [2], [62] to the case of American options.

6.4. Functional quantization for option pricing in a non Markovian setting

Keywords: *quantization*.

Participants: G. Pagès, J. Printems.

The quantization method applied to mathematical finance or more generally to systems of coupled stochastic differentials equations (Forward/Backward) as introduced in [64] consists in the approximation of the solution of the Backward Kolmogorov equation by means of piecewise constant functions defined on an appropriate Voronoï tesselation of the state space $\mathbf{R}^d$. The numerical procedure consists in computing such tesselations adapted to the underlying diffusion and estimating the transition probabilities between different cells of two successive meshes (after a time discretization procedure). Hence, it allows the computation of a great number of conditional expectations along the diffusions paths.

For these reasons, such a method is efficient for pricing and hedging financial products. More generally, it can be applied to American options [64], [65], [24], stochastic control [100], nonlinear filtering and related problems (Zakai and McKean-Vlasov type stochastic partial differential equation [85]). See also [101] for a review on the subject.

The aim of optimal quantization is to study the best L^2 approximation of Hilbert valued random variables taking at most N values. This approach allows us to study the numerical quantization of the Brownian motion from a functional point of view by considering the Brownian motion as a random variable taking values in $L^2([0,T])$. Similar approach can be considered for other Gaussian and non Gaussian processes. Unfortunatly, the resulting quantization error has a bad rate with respect to N, namely $1/\log(N)$. Nevertheless, numerical computations tell us that things behave better than expected.

In a financial framework, functional quantization is helpful when dealing with options with “non Markovian” payoffs, that is payoffs depending on the whole trajectory of the asset price process, such as time average (Asian) options or maximum (lookback) options. Functional quantization is also useful in the case of “Markovian” payoffs in a stochastic volatility framework since the values of the asset at the maturity may depend on the whole path of the volatility. In these cases, we can approximate the value of the option by the usual numerical integration in a functional space considering the asset price process as some H-valued random variable rather than as a R^d-valued process. Numerical study in the framework of the Heston model can be found in [41].

6.5. Computation of sensitivities (Greeks) and conditional expectations using Malliavin calculus

Keywords: *Malliavin calculus, greecks, jump diffusions.*

Participants: V. Bally, M.P. Bavouzet, L. Caramellino, J. Da Fonseca, M. Messaoud, A. Zanette.

Malliavin calculus for jump diffusions. V. Bally, L. Caramellino (University Rome 2) and A. Zanette worked on pricing and hedging American options in a local Black Scholes model driven by a Brownian motion by using the classical Malliavin calculus [23]. The results of this work gave rise to algorithms which have been implemented in PREMIA. Moreover V. Bally, M. Massoud and M.P. Bavouzet are working on the application of the Malliavin calculus for jump type processes for computing greeks in jump type models as Merton’s model. Two papers have been written, one on differentiation with respect to the jump amplitude and one on differentiation with respect to the jump times. The second subject is much more subtile and needs some theoretical developments which are not standard up to now [48], [47].

Malliavin calculus for the Libor Market Model. M. Messaoud and J. Da Fonseca study an application of Malliavin Calculus for the computation of the Greeks for European interest rate derivative products [58]. Within the Libor market model framework, which is widely used in practice, they can still apply integration by parts even if the Malliavin covariance matrix of the diffusion does not satisfy the non degeneracy condition. They find various non degeneracy conditions on some functional that aggregates the multidimensional diffusion which allow them to integrate by parts. They provide the Malliavin estimators for the delta, the gamma and the global vega for an European swaption. The results can be easily extended to other products. They use localization techniques for variance reduction purpose. Numerical results show that Malliavin estimators outperform substantially finite difference for a discontinuous payoff.

6.6. Lower bounds for the density of a functional

Keywords: *Lower bounds of density.*

Participants: V. Bally, L. Caramellino.

V. Bally obtained a result on lower bounds for the density of a functional on Wiener spaces with special applications to locally elliptic diffusion processes [21]. In collaboration with Lucia Caramellino he is working on lower bounds for the density of the law of the portfolio value and in collaboration with Begonia Fernandez and Ana Meda (University of Mexico) he tries to get tubes evaluations for locally elliptic diffusion processes.

6.7. American Options

Participants: A. Alfonsi, E. Chevalier, D. lamberton, B. Jourdain, A. Zanette.

The exercise boundary of American options near maturity has been studied, in the classical Black-Scholes setting with dividends, by D. Lamberton and S. Villeneuve [91]. Their results have been extended to local volatility models by E. Chevalier in his thesis [30]. Chevalier has results for multi-dimensional models as well and has obtained error estimates for the approximation of the free boundary when an American option is approximated by a Bermudean option (i.e. with a finite number of exercise dates) [29].

Lamberton has started a collaboration with Michalis Zervos (King's College, London) on optimal stopping problems for one dimensional diffusions.

B.Jourdain and A. Zanette [50] have developped a new binomial lattice method (MSM) consistent with the Black-Scholes model in the limit of an infinite step number and such that the strike K is equal to one of the final nodes of the tree. They have obtained asymptotic expansions for the MSM European Put price and delta which motivate the use of Richardson extrapolation. Moreover they have made a numerical comparison between the MSM approach and the best lattice based numerical methods known in literature.

It is well-known that in models with time-homogeneous local volatility functions $\sigma(x)$ and constant interest and dividend rates, the European Put prices are transformed into European Call prices by the simultaneous exchanges of the interest and dividend rates and of the strike and the spot price of the underlying. Aurélien Alfonsi and Benjamin Jourdain have investigated such a Call Put duality for perpetual American options. It turns out that the perpetual American Put price $P(x, y)$ for the spot price x and the strike y is equal to the perpetual American Call price in a model where, in addition to the previous exchanges between the spot price and the strike and between the interest and dividend rates, the local volatility function is modified. The equality of the dual volatility functions only holds in the standard Black-Scholes model with constant volatility. These duality results lead to a theoretical calibration procedure of the local volatility function from the perpetual Call and Put prices for a fixed spot price x_0. The knowledge of the Put (resp. Call) prices for all strikes enables us to recover the local volatility function on the interval $(0,x_0)$ (resp. $(x_0, +\infty)$).

6.8. Calibration

Keywords: *calibration*.

Participant: P. Tankov.

Calibration of models with jumps was one of the principal directions of the PhD research of P. Tankov in CMAP (Ecole Polytechnique), who has continued working in this direction as a post-doctoral fellow in the MathFi project. In winter 2004-2005 he gave a series of lectures on the topic of model calibration and hedging at Ecole Internationale des Sciences de Traitement d'Information. Moreover, he implemented a non-parametric calibration algorithm for exponential Lévy models in Premia.

6.9. Sparse grids methods for PDEs in Mathematical Finance

Keywords: *adaptive finite elements*, *finite element*, *lattice-based methods*, *sparse grids*.

Participants: Y. Achdou, D. Pommier, J. Printems.

In some applications in finance for example the pricing of option on baskets of d assets, the efficient numerical solutions to elliptic partial differential equations (PDEs) in high dimension is necessary. The application of standard numerical schemes fails due to the 'curse of dimension' which means the exponential dependence on the dimension of the number of degrees of freedom. To cope with the 'curse of dimension', so called sparse grids have been proposed by Zenger [108] (see [71]). The sparse grids approach is based on a d-dimensional tensor product basis, which is derived from a one-dimensional hierarchical basis. We use a Galerkin method with wavelets as hierarchical basis (see [109]). Our work consists in reducing time computing by adapting the wavelet basis to sparse tensor product.

A Cifre agreement on this subject between Inria and CIC is engaged involving the PhD student David Pommier.

6.10. Stochastic control - Application in finance and assurance

Keywords: *jump diffusions*, *stochastic control*.

Participants: B. Øksendal (Oslo University), D. Hernandez-Hernandez, M. Mnif, A. Ngo, P. Tankov, A. Sulem.

B. Øksendal (Oslo University) and A.Sulem have written a book on Stochastic control of Jump diffusions [12]. The main purpose was to give a rigorous, yet mostly nontechnical, introduction to the most important and useful solution methods of various types of stochastic control problems for jump diffusions and its applications. The types of control problems covered include classical stochastic control, optimal stopping, impulse control and singular control. Both the dynamic programming method and the maximum principle method are discussed, as well as the relation between them. Corresponding verification theorems involving the Hamilton-Jacobi Bellman equation and/or (quasi-)variational inequalities are formulated. There are also chapters on the viscosity solution formulation and numerical methods. The text emphasizes applications, mostly to finance. All the main results are illustrated by examples and exercises appear at the end of each chapter with complete solutions.

In [39], M.Mnif and A.Sulem study the optimal reinsurance policy and dividends distribution of an insurance company under excess of loss reinsurance. The objective of the insurer is to maximize the expected discounted dividends. They suppose that in the absence of dividend distribution, the reserve process of the insurance company follows a compound Poisson process. Existence and uniqueness results for the associated integro-differential variational inequality in the viscosity sense are given. The optimal strategy of reinsurance, the optimal strategy of dividends pay-out and the value function are then computed numerically.

A. Ngo, P. Tankov and A. Sulem are studing pricing and hedging in markets with jumps using utility maximization and indifference pricing. D. Hernandez-Hernandez has solved the problem of characterization of the indifference price of derivatives for stochastic volatility models.

D. Lamberton and Gilles Pagès have studied the rate of convergence of the classical two-armed bandit algorithm in [57]. They have also investigated another algorithm with a penalization procedure [56].

6.11. Utility maximization in an insider influenced market

Keywords: *anticipative calculus*, *asymmetry information*, *forward integrals*, *insider*.

Participants: A. Kohatsu Higa, A. Sulem.

In [36], A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation where the coefficients are anticipating. This setting is used to interpret markets where insiders have some influence on the dynamics of prices. An insider is an agent who has access to larger information than the one given by the development of the market events and who takes advantage of this in optimizing his position in the market. In [45], A. Kohatsu Higa and A. Sulem give some remarks on the anticipating approach to insider modeling introduced by the authors recently. In particular, they define forward integrals by using limits of Riemmann sums. This definition is well adapted to financial applications. As an application, they consider a portfolio maximization problem of a large trader with insider information. They show that the forward integral is a natural tool to handle such problems and compute the optimal portfolios for an insider and a small trader.

In a joint work with mathematicians from the university of Oslo, A. Kohatsu Higa and A. Sulem consider in [49] the optimization problem of an insider who is so influential in the market to affect the price dynamics: in this sense he is called a “large” insider. The optimal portfolio problem for a general utility function is studied for a financial market driven by a Lévy process in the framework of forward anticipating calculus.

6.12. Backward stochastic differential equations

Participants: D. Hernandez-Hernandez, M.C. Kammerer-Quenez.

Since October 2005, D. Hernandez-Hernandez and Marie-Claire Kammerer-Quenez are working on some problems related with the smoothness of solutions of nonlinear parabolic partial differential equations using representations of backward stochastic differential equations. With Magdalena Kobylanski (UMLV), they are studing in particular the connections between the solutions of BSDEs with quadratic growth and PDEs (in a classical sense). The idea is to use an approximation of the PDE and of the associated BSDE.

6.13. Default risk modeling

Keywords: *Marshall-Olkin copula, credit derivatives*.

Participants: B. Jottreau, Y. Elouerkhaoui.

The modeling of default often leads to market incompleteness and requires specific tools such as utility maximization or martingale measures selection, in order to price and hedge defautable claims. B. Jottreau’s thesis deals with default risk modeling and portfolio optimization. He has extended results of Lukas’ for exponential utility to general utility functions. Future research will include transactions costs.

Y. Elouerkhaoui finishes his PhD thesis on the valuation and hedging of basket credit derivatives in the Marshall-Olkin copula framework, especially on the modeling of default correlation in the context of credit derivatives pricing and the study of correlation market incompleteness and hedging. [81], [79], [80]

The release Premia8 includes algorithms for pricing credit risk derivatives.

6.14. Interest rate modeling

Keywords: *interest rate*.

Participant: S. Henon.

In her thesis, Sandrine Hénon introduces a one factor LGM (Linear Gauss Markov) interest rate model with stochastic volatility [13]. She gives a rather complete treatment of the model, including caplet and swaption pricing.

7. Contracts and Grants with Industry

7.1. Consortium Premia

Participants: A. Alfonsi, V. Bally, S. Blunck, A. Cisse, M. Ciuca, J. Guyon, B. Jourdain, B. Lapeyre, J. Lelong, A. Sulem, P. Tankov, E. Voltchkova, A. Zanette.

The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions or departments: IXIS CIB (Corporate & Investment Bank), CALYON, the Crédit Industriel et Commercial, EDF, Société générale and Summit. http://www.premia.fr

7.2. EDF

Participants: C. Strugarek, P. Carpentier, A. Sulem, P. Tankov, A. Alfonsi, project-team MATHFI, Laboratory CERMICS.

- CIFRE agreement EDF-ENPC on "Optimisation of portfolio of energy and financial assets in the electricity market".
- Industrial contract INRIA/EDF on the "Modeling dependence between electricity price and consumption stochastics".
- General industrial convention between EDF and Mathfi and research teams of CERMICS on risk issues in electricity markets.

7.3. Cédit Industriel et Commercial

Participants: D. Pommier, J. Printems, Y. Achdou, A. Sulem.

Cifre agreement CIC/INRIA on : "sparse grids for large dimensional financial issues”.

7.4. CALYON

Participants: S. Hénon, D. Lamberton, M.C. Kammerer-Quenez.

Cifre agreement between University of Marne la Vallée and CALYON on “ Interest rate models with stochastic volatility “.

7.5. International cooperations

- Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef) supported by the European Science Foundation (ESF).
- Collaborations with the Universities of Oslo, Bath, Chicago, Mexico, Osaka, Rome II and III, Tokyo Institute of Technology.
- Part of the cooperation STIC-INRIA-Tunisian Universities.

8. Dissemination

8.1. Seminar organisation

- B. Jourdain, M.C. Kammerer-Quenez and J. Guyon: organization of the seminar on stochastic methods and finance, University of Marne-la-Vallée
- M.C. Kammerer-Quenez and A. Kohatsu Higa : members of the organization committee of the Seminar Bachelier de Mathématiques financières, Institut Henri Poincaré, Paris.
- B. Lapeyre: Presentation of Mathematical Finance for professors and high schools managers, December 2005.

8.2. Teaching

- A. Alfonsi
  - course on probability theory, ENPC (1st year) (42h)
  - projects on financial mathematics, ENPC (2nd year) (10h)
  - Course on Statistics (10h)
  - Adviser for students in architecture
- V. Bally
  Course "Malliavin calculus and applications in finance" , Master program "Random analysis and systems", University of Marne la Vallée.
- V. Bally, B. Jourdain, M.C.Kammerer-Quenez
  Course on "Mathematical methods for finance", 2nd year ENPC.
- M.P. Bavouzet
  - January to February 2004: Assistant teaching at the engineer school of the university of Marne-la-Vallée, Mathematics, first year.
  - March to June 2004: Assistant teaching at the university of Marne-la-Vallée, Mathematics, DEUG Science de la Matière, first year.
  - October to December 2004: Assistant teaching at the university of Marne-la-Vallée, Mathematics, Introduction to Mathematics Reasonning, Undergraduate program, 3rd year.

- E. Clément
  L3 maths : Differential Calculus - Hilbert Spaces
  M1 maths : Functional Analysis - Probability.
- B. Jourdain :
  - course on "Probability theory and statistics", first year ENPC.
  - course on "Mathematical methods for finance", 2nd year ENPC (with José da Fonseca).
  - Projects and courses in finance, Majeure de Mathématiques Appliquées, Ecole Polytechnique.
- J.F. Delmas, B. Jourdain :
  course on "Random models", 2nd year ENPC
- M. Delasnerie, B. Jourdain, H. Regnier :
  course on "Monte-Carlo methods in finance", Master Probability and Applications, University Paris VI
- J.P. Chancelier, B. Jourdain:
  Course "Numerical methods for financial models", Master program "Analyse et Systemes Aléatoires", University of Marne-la-Vallée.
- B. Jourdain, B. Lapeyre :
  course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, University of Marne-la-Vallée
- J. Guyon
  Course “Probability and Statistics”, teacher (42h, Oct. 2005 - Feb. 2006), ENPC.
  Course “Statistics and data analysis”, teacher (8h, May-June 2005), ENPC.
  Course “Finance: mathematical and numerical aspects”, teacher assistant (9h, April-June 2005), ENPC.
  Course “Introduction to probability and statistics”, teacher assistant (29h, Jan.-June 2005), ENSTA.
- A. Kohatsu-Higa:
  - Lower bound estimates by Malliavin Calculus. Harnack inequalities and positivity for solutions of partial differential equations. Cortona, Italy. June 12-18, 2005 (4hours).
  - Unesco Courses. Insider problems with finite utility. Tunis, November 2005 (12 hours).
  - Numerical Methods in Finance. ENSTA (10 hours).
- D. Lamberton :
  - Second year undergraduate program in economics , 3rd year (linear algebra)
  - Master course “Calcul stochastique et applications en finance", University of Marne-la-Vallée.
  - Course on American options, chaire UNESCO, Tunis, November 21-25.
- B. Lapeyre
  - Course on Modelisation and Simulation, ENPC.
  - Projects and courses in Finance, Majeure de Mathématiques Appliquées, Ecole Polytechnique.
  - Course on Monte-Carlo methods and stochastic algorithms, doctoral program in Random analysis and systems, University of Marne la Vallée.
  - EPFL, Cycle d’Etude Postgrade en Ingenierie Mathematiques : " Numerical methods for pricing and hedging options", (15 hours).
- D. Lefèvre
  - Assistant professor at ENSTA, in charge of the mathematical finance program.
- J. Lelong
  - Introduction to Probability theory and Statistics, ENSTA (24 hours)
  - Statistics projects, ENPC (6 hours)
  - Applied courses on Programming in C, UMLV (18 hours) , Doctoral program ASA.

- M. Messaoud
  Numerical methods in Finance, ENSTA 2nd year program
- M.C. Kammerer-Quenez
  - Courses for undergraduate students in mathematics, University of Marne la Vallée
  - Course on recent mathematical developments in finance, graduate program, University of Marne-la-Vallée,
  - Introductary course on financial mathematics, ENPC.
- A. Sulem
  - Course on numerical methods in finance, Master MASEF and EDPMAD, University Paris-Dauphine (21 hours)
  - Doctoral Course in the framework of the Chaire Unesco program in Tunis on numerical methods in Finance (15 hours) (October 2005) http://www.tn.refer.org/unesco/semestre4/semestre4-fr.htm.
- P. Tankov
  - Seminars on C++ with applications to numerical analysis and finance at the University of Evry
  - Course on Model calibration and option hedging, EISTI (Cergy Pontoise), 18 hours.
- E. Voltchkova
  Assistant teaching at the University of Evry:
  - “Numerical analysis”, 2nd year IUP.
  - “Mathematical finance”, under graduate program in Economics. 3rd year.

8.3. Internship advising

- A. Alfonsi
  Advising a student (PFE, Paris 13 University) on the model of Heston.
- J. Guyon
  Advising of the intership of Majd Cheikh-Ali (ENSTA, 2nd year) on : "Pricing and Hedging within the Fouque-Papanicolaou-Sircar stochastic volatility model". This has lead to a routine implemented in the Premia software.
- B. Jourdain
  Jérôme Elfassy, "Pricing of options in a LIBOR market model with volatility skews", scientific training period ENPC (April to June).
- A. Kohatsu-Higa and P. Tankov.
  Tatiana Ershova (Ecole Polytechnique) : Discretisation of SDE with Kusuoka schemes.
- A. Sulem
  Anh Tuan Ngo (Ecole Polytechnique) : Risk sensitive ergodic control and Hedging in markets with jumps.

8.4. PhD defences

- Sandrine Hénon
  Thesis defended on September 2005, Universty of Marne la Vallée.
  Title: modélisation de la courbe des taux en marché incomplet et calibration de modèles
  Adviser: D. Lamberton and M.C. Kammerer-Quenez
- Ahmed Kbaier
  Thesis defended on December 13th 2005, Universty of Marne la Vallée
  Title: Réduction de variance et discrétisation d'équations différentielles stochastiques. Théorèmes limites presque sures pour les martingales quasi-continues à gauche.
  Advisers: V. Bally and D. Lamberton
- Vincent Lemaire
  Thesis defended on December 8th 2005, University of Marne la Vallée
  Title: Estimation récursive de la mesure invariante d'un processus de diffusion
  Advisers: D. Lamberton and G. Pagès
- Marouen Messaoud
  Thesis defended on January 27th 2006, University Paris-Dauphine.
  Title: Optimal stochastic control and Malliavin calculus. Applications in finance.
  Adviser : A. Sulem.
- Nicolas Moreni
  Thesis defended on December 2th 2005, University Paris VI
  Title: Méthodes de Monte Carlo et pricing d'options
  Adviser: B. Lapeyre

8.5. PhD advising

- V. Bally and D. Lamberton
  - Ahmed Kbaier (3rd year), Grant of University of Marne-la-Vallée.
  " Approximation of SDE"
  (defended December 2005).
- V. Bally and A. Sulem
  M.P. Bavouzet (3rd year), Grant University of Paris Dauphine and INRIA (1/2 ater since September 2005).
  "Malliavin calcul with jumps and application in Finance".
- B. Jourdain
  - Aurélien Alfonsi (2nd year), ENPC
  "Credit risk models. Discretization and calibration of financial models."
- A. Kohatsu-Higa
  - Salvador Ortiz (University of Barcelona. Main thesis advisor : David Nualart)
  - Karl Larsson (Lund University. Department of Economics)
- D. Lamberton and M.C. Kammerer-Quenez
  - Sandrine Hénon (defended, September 2005); CIFRE agreement with CALYON.
  "Modeling interest rates with stochastic volatility"
- D. Lamberton and Gilles Pagès
  - Vincent Lemaire (4th year), Grant of the University of Marne-la-Vallée.
  "Approximation of the invariant measure of a diffusion by a Euler scheme with decreasing steps"
  (defended December 2005)

- B. Lapeyre
  - Nicola Moreni, ENPC, MESR grant Paris 6 (3rd year)
  Intégrales de chemin et méthodes de Monte-Carlo en finance
  defended December 2nd, University Paris 6
  - Ralf Laviolette , ENS Cachan (2nd year)
  "Calcul d'options pour des dérivées énergétiques dans des modèles avec sauts".
  - Jérôme Lelong, ENPC grant, UMLV (2nd year)
  "Stochastic algorithms and calibration problems in Finance"
  - Julien Guyon, ENPC
  "Fine properties of the Euler scheme for Stochastic Differential equations".
- M.C. Kammerer-Quenez
  - B. Jottreau, UMLV
  "modélisation du risque de défaut"
- A. Sulem
  - David Lefèvre, Université Paris-Dauphine
  "Utility maximisation in partial observation"
  - Marouen Messaoud (3rd year), Université Paris-Dauphine
  "Stochastic optimal control, Malliavin calculus and applications in finance" (defended Janruary 27, 2006).
  - Youssef Elouerkhaoui : (Citibank, London)
  "Incomplete issues in credit markets"
- A. Sulem and P. Carpentier (ENSTA)
  Cyrille Strugarek, Cifre agreement ENPC–EDF, 2nd year.
  "Optimisation of portfolio of energy and financial assets in the electricity market"
- J. Printems, Y. Achdou and A. Sulem (Paris 6)
  David Pommier (2nd year)
  Cifre agreement INRIA–CIC
  "Sparse grid for large dimensional financial issues".

8.6. Participation to workshops, conferences and invitations

- A. Alfonsi
  Journées de Probabilités 2005, Nancy: "Schémas de discrétisation pour les processus de Cox-Ingersoll-Ross et carrés de Bessel".
- V. Bally
  - March 2005 : conference in Cambridge (workshop on numerical methods in finance) on "Integration by parts formula for locally smooht laws and applications to sensitivity computations".
  - June 2005 : participation to the conference on "Harnack inequalities" in Cortona. Talk on "Lawer bounds for the density of Ito processes".
  - September 2005 : talk on "lawer bounds for the density of a diffusion process under Hormander condition" in the workshop organised in Nancy in the honor of Professor Malliavin.
  - November 2005 : talk on "Integration by parts formula for locally smooht laws and applications to sensitivity computations" in the workshop organised by AMaMeF in London.
- E. Clément
  Talks have been given on weak approximation results in the Bachelier seminar (IHP), in the study group "Probabilités Numériques, Statistique des Processus et Finance" (Paris 6-Paris 7 universities) and in the study group "méthodes stochastiques et finance" (ENPC-INRIA-UMLV).
- J. Guyon
  - *Institut Elie Cartan, Nancy*, Journées de probabilités, September 2005. Euler scheme and tempered distributions.
  - *University of Technology, Sydney*, Quantitative Methods in Finance 2005 Conference, December 2005. Rate of convergence for the deltas and gammas.
  - *Université de Marne-la-Vallée*, MATHFI seminar, February 2005. Euler scheme and tempered distributions.
  - *Université Paris VI*, seminar of numerical probability, statistics of processes and finance, April 2005. Euler scheme and tempered distributions.
  - *IHP*, Bachelier seminar, June 2005. Rate of convergence for the deltas and gammas.
  - *INRIA Sophia-Antipolis*, OMEGA seminar, September 2005. Euler scheme and tempered distributions.
  - *HSBC, Capital Markets Research, Paris*, November 2005. Rate of convergence for the deltas and gammas.
- D. Hernandez-Hernandez
  On the relation between risk sensitive control and indifference pricing, Bachelier seminar, IHP Paris October 28th.
- A. Kohatsu-Higa

- Congresses and Workshops
  - Kyle-Back type models of insider trading. Probability Workshop. University of the Ryukyus. Okinawa, Japan, September 2005 and AMaMeF Conference. Imperial College, London, October 2005.
  - A variance reduction method for the simulation of densities. Presented at Monte Carlo Methods.University of Cambridge. Isaac Newton Institute for Mathematical Sciences. Developments in Quantitative Finance, May 2005 and Stochastic Numerics Conference. RIMS, Kyoto June 2005.
  - Insider models with finite utility. International Symposium on Stochastic Processes and Mathematical Finance, Kusatsu, Japan, March 2005.
- Seminars
  Insider trading in anticipating markets. Dipartimento di Matematica per le Decisioni. Florence University. May 2005.

- D. Lamberton
  - Méthodes de dualité et discrétisation d'EDS. Ecole Polytechnique, February 2005, and University of Rennes, February 2005.
  - Duality methods and weak convergence of discretizations schemes for SDEs, Isaac Newton Institute, Cambridge, April 2005.
  - three months at the Isaac Newton Institute (April-June 2005) on the programme "Developments in Quantitative Finance".
  -Une introduction à l'évaluation et la couverture des options, Monastir, November 2005.
- B. Lapeyre
  - invited at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK for a week dedicated to MonteCarlo methods in Finance , May 16-20 in the framework of a semester on Developments in Quantitative Finance.
- Nicola Moreni
  September 2004-April 2005: internship in the Financial Engineering group, Banca IMI, Milano
- N. Privault
  Research invitations
  - Matsuyama University, Japan, November 15-21.
  - Wuhan University, China, October 23-29.

  Invited conferences
  - Third conference on stochastic analysis and probability, 13-17 December 2005, Marrakech. "A probabilistic interpretation to the symmetries of a partial differential-difference equation".
  - ESF Workshop on Entropy methods in PDE theory and stochastics, 23-25 November 2005, Johannes Gutenberg-Universität, Mainz. "On convex concentration inequalities".
  - AMaMeF Workshop on stochastic analysis and computational finance, 9-12 November 2005, Imperial College, London. "A computation of Theta by integration by parts in a jump-diffusion model".
  - International conference on mathematical analysis of random phenomena, 12-17 September 2005, Hammamet, Tunisia. "The Skorohod isometry on compact Lie groups via quantum stochastic calculus".
  - Journées de Probabilités, 5-9 September 2005, University of Nancy. "Intégration par parties pour certains processus à sauts et applications".

Seminars

- Université d'Evry:
  8/12/05: "Densités de Wigner sur les algèbres de Lie et calcul de Malliavin".
- Université de Wuhan:
  28/10/05: "Density estimation via integration by parts for point processes".
- Université de Wuhan et Université de Huagong, Wuhan, 25/10/05 et 27/10/05: "Blow up and stability of semilinear PDEs".
- Université de Versailles:
  13/10/05: "Inégalités de concentration convexe et calcul stochastique forward/backward".

- A. Sulem

  - Invited Conference on the PREMIA Software for the SMAI event on Financial Mathematics, November 2005, IHP, Paris.
  - Invited Conference, Stochastic Analysis Conference in Ascona, Swizerland, May 2005.
  - Invited Conference for the conference in the honor of Professor Bernt Øksendal for his 60th birthday, Oslo, June 2005.

- P. Tankov
  - Conferences:
  IV Conference on Lévy processes and Applications, Manchester, UK, 20 January 2005
  Developments in Quantitative Finance, Cambridge, UK, May 2005
  II Abel symposium on Stochastic Analysis and Applications, Oslo, Norway, July 2005
  - Seminars:
  Weierstrass Institute, Berlin, June 2005
  University of Oslo, Norway, September 2005

8.7. Miscellaneous

- A. Alfonsi and J. Lelong
  have written the mathematics exam for the “ concours des Ingénieurs du Service Technique pour la ville de Paris “
- D. Lamberton
  - "Associate Editor" of *Mathematical Finance* and *ESAIM PS*.
  - in charge of the master programme “Mathématiques et Applications" (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).
  - Member of the Steering Committee of the ESF European Network "AMaMeF" (http://www.iac.rm.cnr.it/amamef/) co-organizing (with Agnès Sulem and Arturo Kohatsu-Higa) a workshop on numerical methods in finance within this network (1-3 February 2006).
  - Coordinator of an "ACI" "Méthodes d’équations aux dérivées partielles en finance de marché".
- G. Pagès
  Associate Editor of the journal *Stoch. Proc. and their Appl.*
- N. Privault
  - Referee for the PhD thesis “Calcul d’erreur par formes de Dirichlet: liens avec l’information de Fisher et les théorèmes limites” of Christophe Chorro, defended at Université Paris I on 09/12/05.
  - Member of the jury for the Habilitation “Calcul stochastique gaussien et applications à l’inférence statistique” of Ciprian Tudor, University Paris I, 01/12/05.

9. Bibliography

Major publications by the team in recent years

[1] M. Akian, A. Sulem, M. Taksar. *Dynamic optimisation of long term growth rate for a portfolio with transaction costs - The logarithmic utility case*, in "Mathematical Finance", vol. 11, n° 2, Avril 2001, p. 153–188.

[2] B. Arouna. *Adaptative Monte Carlo Method, A Variance Reduction technique*, in "Monte Carlo Methods and Applications", vol. 10, n° 1, 2004.

[3] V. Bally. *An elementary introduction to Malliavin calculus*, Research Report, n° 4718, Inria, Rocquencourt, February 2003, http://www.inria.fr/rrrt/rr-4718.html.

[4] V. Bally, G. Pagès, J. Printems. *First order schemes in the numerical quantization method*, in "Mathematical Finance", vol. 13, n° 1, 2003, p. 1–16.

[5] E. Clément, D. Lamberton, P. Protter. *An analysis of a least squares regression method for american option pricing*, in "Finance and Stochastics", vol. 6, 2002, p. 449–471.

[6] B. JOURDAIN, C. MARTINI. *American prices embedded in European prices*, in "Annales de l'IHP, analyse non linéaire", vol. 18, n° 1, 2001, p. 1-17.

[7] D. LAMBERTON, B. LAPEYRE. *Une introduction au calcul stochastique appliqué à la finance*, traduction anglaise: An introduction to stochastic calculus applied to finance, Chapman and Hall, 1996, Collection Mathématiques et Applications, Ellipses, 1992.

[8] B. LAPEYRE, E. TEMAM. *Competitive Monte-Carlo Methods for the Pricing of Asian Options*, in "Journal of Computational Finance", vol. 5, n° 1, 2001, p. 39-57.

[9] D. LEFÈVRE. *An introduction to Utility Maximization with Partial Observation*, in "Finance", vol. 23, 2002, http://www.inria.fr/rrrt/rr-4183.html.

[10] B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio with both fixed and proportional transaction costs*, in "SIAM J. Control and Optim", vol. 40, n° 6, 2002, p. 1765–1790.

Books and Monographs

[11] B. LAPEYRE, A. SULEM, D. TALAY. *Simulation of Financial Models: Mathematical Foundations and Applications*, Cambridge University Press, in final form.

[12] B. ØKSENDAL, A. SULEM. *Applied Stochastic Control of Jump Diffusions*, Universitext, Springer Verlag, Berlin, Heidelberg, New York, 2005, http://www.springer.com/sgw/cda/frontpage/0,11855,5-40109-22-34667289-0,00.html.

Doctoral dissertations and Habilitation theses

[13] S. HÉNON. *Modélisation de la courbe des taux en marché incomplet et calibration de modèles*, Ph. D. Thesis, Université Marne la Vallée, September 2005.

[14] A. KBAIER. *Réduction de variance et discrétisation d'équations différentielles stochastiques. Théorèmes limites presque sures pour les martingales quasi-continues à gauche.*, Ph. D. Thesis, Université Marne la Vallée, December 2005.

[15] V. LEMAIRE. *Estimation récursive de la mesure invariante d'un processus de diffusion*, Phd Thesis, Université Marne la Vallée, December 2005.

[16] M. MESSAOUD. *Contrôle optimal stochastique et calcul de Malliavin appliqués à la finance.*, Ph. D. Thesis, Université Paris Dauphine, January 2006.

[17] N. MORENI. *Méthodes de Monte Carlo et pricing d'options*, Ph. D. Thesis, Université Paris VI, December 2005.

Articles in refereed journals and book chapters

[18] A. ALFONSI. *On the discretization schemes for the CIR (and Bessel squared) processes*, in "Monte Carlo Methods and Applications", 2005, p. 355-384.

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