



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*Team mc2*

*Modeling, control and computations :  
applications to fluid mechanics and  
laser-plasma interaction.*

*Futurs*

THEME NUM

*Activity*  
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2005



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# 1. Team

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# 2. Overall Objectives

## 2.1. Overall Objectives

The aim of this project is to develop modeling tools in order to explain, control, simulate and predict some phenomena coming from physics, chemistry or engineering.

A) In this context, **modeling** is :

A1) Phenomenological : use of ad-hoc models in order to represent some precise phenomena.

In nonlinear optics, one uses non linear modeling in order to describe interaction between laser and matter like for example the quadratic nonlinearity in order to describe frequency doubling.

Input/output models in control theory are useful to describe the result of a process without describing it precisely.

The wall conditions in microfluidics are often taken heuristically in order to model the slip at the boundary.

Phenomenological modeling is also used in biology and for complex fluids.

A2) Asymptotical : using asymptotic expansions, we derive simpler models containing all the pertinent phenomena.

We use mainly amplitude equations (for high frequency phenomena) and asymptotics on the geometry (when a characteristic length is smaller than the others). Examples of such a process are the derivation of the nonlinear Schrödinger equation in nonlinear optics or for water-waves and internal waves ; the boundary layer equations in fluid mechanics ; the penalization method for the simulation of incompressible flows with obstacles or the analysis of ribbles in microfluidics that are used to control the mixing of the fluids.

A3) Numerical : direct numerical tools are used to represent the physical phenomena. A typical example is the POD (proper orthogonal decomposition) and its use in control theory and to obtain quick simulations.

B) The simulations that we perform are in the following contexts :

B1) 2D and 3D simulations at high Reynolds number.

We develop direct simulation methods : multigrid techniques, vortex methods, detached Eddy Simulation (DES). The applications that we have in mind are the turbulence, the flow around a vehicle, the stress on a pipe-line. The penalization method is used in order to take into account the obstacles.

B2) Multifluid flows and interfaces problems.

We perform 2D and 3D simulations of multifluid flows at low Reynolds number using level-set methods and mixing models. We describe propagation of tumor and interaction with membranes in the same manner.

The applications are microfluidics, propagation of tumor, porous medium and complex fluids.

B3) Computation of dispersive equations.

We use finite difference methods, spectral methods or mixed methods for extensions of Schrödinger, Zakharov, KdV and Boussinesq equations.

We compute the propagation of laser pulses in crystals, laser-plasma interaction and propagation of ultra-short pulses in order to simulate nuclear fusion. The same methods are used for water-waves or internal waves.

C) Analysis, optimization and control.

C1) Analysis.

Modern experiment techniques and direct numerical simulation give large amounts of data in the form of two and three dimensional vector fields. Analysis and objective interpretation of such fields constitute a complicated problem. The velocity vector fields can rarely be interpreted directly. In most cases data are processed in some way and then visualized. This procedure is prone to arbitrary numerical manipulations which may affect the conclusions which one resumes from analysis. A typical example is the determination of scaling factors in turbulence : inappropriate windowing effects may give virtual scaling factors. We develop tools based on solid mathematical backgrounds to interpret fluid flows on more rigorous grounds.

C2) Shape optimization.

In theory, shape optimization in ground vehicle aerodynamics does not present any difference compared to optimization in aeronautics, a field in which the adjoint method is used as an automatic tool of design in industrial realities like Dassault and Boeing. In practice on the other hand, an important difference remains in the level of approximation used to determine the aerodynamic coefficients. Indeed, although these are strongly non stationary detached flows, in car industry one is often satisfied with a result obtained by a solution of the stationary averaged Navier-Stokes equations. Moreover, the industries of the automobile field use mainly commercial codes and they do not have access to the sources. For all these reasons there is obviously an industrial interest to develop powerful numerical methods adapted to the solution of the continuous adjoint equations of the stationary averaged Navier-Stokes equations.

C3) Control and real time computations .

The use of feedback to control flows of the industrial type is accompanied by two strong, crippling constraints from a point of view of practical applications. On the one hand, in order for the control to have a sense, it is necessary that the energy balance of the operation being positive. In other words, energy spent to control the flow must be lower than the discounted profit. In addition, the controller used in the loop of feedback must be able to react in real time with information coming continuously from the sensors to adapt the signal to be sent to the actuators. Various control methods exist to this end. However, the extension of these techniques to more realistic problems proves to be delicate. In particular, when the dimension of the problem increases (to fix the orders of magnitude of a realistic problem, Spalart and Al, 1997 estimate that for an aerofoil of plane in flying conditions, i.e., for  $Re = O(10^7)$ , it is necessary to employ approximately  $10^{11}$  points and to integrate the

equations on approximately  $10^6$  time steps) these approaches may become inapplicable. Other methods must thus be considered. Within the framework of this research program, we propose to couple small-scale models to reduced order models built starting from existing off-line computations and data-bases.

We emphasize that these three points A, B, C are not independent. For example, the asymptotic expansions in nonlinear optics are performed starting from phenomenological models. The POD method can be performed starting from numerical results obtained thanks to a penalization method.

Our methods are applied to three areas of applications :

a) Fluid dynamics :

Turbulence, drag reduction, stress reduction, form optimization. The challenges are :

- i) Find scale factor for turbulent flows cascades in realistic configurations,
- ii) The control of the drag of a vehicle in order to decrease the fuel consumption,
- iii) The control of the stress for a pipe line or an off-shore platform.

b) Interface problems and complex fluids :

microfluidics, bifluid flows, miscible fluids, tumor propagations, complex fluids, environmental problems (coastal flows, porous medium). The challenge are :

- i) To obtain reliable and simple models that can be used by our partner Rhodia.
- ii) The obtaining of tumor growth models including some mechanics.

c) Nuclear fusion :

laser-matter interaction (with a plasma, a crystal). The challenge is mainly to be able to couple several complex phenomena (Raman, Doppler, Landau, ...).

The final objective of our project is to obtain reliable numerical simulations of various phenomena that can be used by our collaborators from other laboratories or from the industrial community. Our main partners on this project will be Renault, IFP, CIRA (Centro italiano ricerche aerospaziali), CPMOH (Laboratory of Physics, Bordeaux 1 University) for a), the CEA (french nuclear agency) and the laboratory CELIA (joint laboratory between Bordeaux 1 university, CEA and CNRS) for c), the LOF (Laboratory Of the Future, joint lab between Rhodia (biggest french company of chemistry) and the CNRS, located near the campus of Bordeaux for b).

## 3. Scientific Foundations

### 3.1. Numerical modeling, flow analysis, optimization and control

#### 3.1.1. Numerical modeling

In this section we present some numerical methods already developed or under development that will be used to get as realistic as possible approximate solutions of the considered flows. The requirement is always to get a good balance between accuracy and stability.

##### 3.1.1.1. Direct numerical simulation

The first method used which is affordable in 2D is to solve directly the genuine Navier-Stokes equations in primitive variables velocity-pressure on Cartesian domains [19]. The bodies around which the flow has to be computed are modeled using the so-called penalization method or Brinckman-Navier-Stokes equations that is an immersed boundary method in which the bodies are considered as porous media with a very small intrinsic permeability [45]. This method is very easy to handle as it consists only in adding a mass term  $U/K$  in the momentum equations. Both unknowns are strongly coupled in the approximation and solved simultaneously. The discretization is achieved by means of Gear second-order scheme in time and centered second-order finite differences in space for linear terms. A special care of the approximation of the convective terms discretized thanks to a new third-order scheme and treated explicitly allows to reach a significant accuracy when coupled to a good approximation of boundary conditions. In particular the boundary conditions imposed on artificial frontiers of the computational domains avoid any reflections when vortices cross the boundary. To make the approximation efficient enough in terms of CPU time, a multigrid solver with a cell by cell Gauss-Seidel smoother is used. The flow over various bodies as a riser pipe (IFP) or a ground vehicle (Renault) were

successfully simulated by the corresponding code. This code will be used as a reliable solver to get the data necessary to construct the modes of the reduced order model (see below).

#### 3.1.1.2. Vortex method

Vortex methods are Lagrangian techniques that have been proposed as an alternative to more conventional grid-based methods. Their main feature is that the inertial nonlinear term in the flow equations is implicitly accounted by the transport of particles. The method thus avoids to a large extent the classical stability/accuracy dilemma of finite-difference or finite-volume methods. This has been demonstrated in the context of computations for high Reynolds number laminar flows and for turbulent flows at moderate Reynolds numbers [53]. This method has recently allowed to obtain new results concerning the three-dimensional dynamics of cylinder wakes. A collaboration with Professor G. H. Cottet (LMC Grenoble) is undertaken to compare the efficiency of available vortex codes (with non-primitive variables) to novel primitive variable vortex algorithms which should avoid the difficulty of these methods related to the implementation of no-slip boundary conditions. Finally, as the resolution of the convection and of the stretching terms needs a filtering (cutoff) procedure that can be assimilated to a sub-grid scale model, an analysis of the effect and behavior of such models is scheduled using the code developed in Grenoble.

#### 3.1.1.3. Detached-eddy simulation

Detached-eddy simulation (DES) is a hybrid technique proposed by Spalart *et al.* in 1997 as a numerically feasible and plausibly accurate approach for predicting massively separated flows. Traditionally, high Reynolds number separated flows have been predicted using Reynolds averaged Navier-Stokes equations (RANS). Although RANS models are considered as the most practical turbulence handling technique for industrial problems and yield acceptable accuracy of a relatively broad range of attached flows, these models are not adapted to massively separated flows. Another growing approach, Large-Eddy Simulation (LES), offers the advantage to directly compute the dominant unsteady structures of the flow. Unfortunately the high computational cost of applying LES to complete configurations such as an airplane, a submarine, or a road vehicle remains prohibitive because of the resolution required in the boundary layers. The aim of Detached-Eddy Simulation (DES) is to combine the most favorable aspects of both techniques, i.e., application of RANS models for predicting the attached boundary layer and LES for time-dependent three-dimensional large eddies [66]. The cost scaling of the method is then affordable since LES is not applied to solve the relatively smaller structures that populate the boundary layer. Advanced industrial developments have increased the demand for numerically feasible and accurate approach for predicting massively separated flows around complex geometries. In this aspect flow field predictions obtained using DES are encouraging see <http://cfd.me.umist.ac.uk/desider/>. The base model employed in the majority of DES applications is the Spalart-Allmaras (S-A) model. To obtain the DES model formulation, the length scale of the S-A destruction term is modified to be the minimum of the distance to the closest wall and a length scale proportional to the local grid spacing. Concurrently with its encouraging results, weaknesses of DES were discovered. Starting from a valid RANS solution, gradually refining the grid alters the solution in obscure ways. The grid is *ambiguous* and the DES equations fail to recognize that pure RANS behavior was intended. Resolving the issue of *ambiguous* grids is a priority (especially if we think about automatic grid adaptation), but as proven to be a resilient difficulty. A better understanding of the coupling mechanisms between the models is needed.

#### 3.1.1.4. Reduced order models

The baseline for feedback control is a fast and reliable on line simulation to drive the devices which act on the flow. One possible strategy to achieve such goal is reduced order models. Reduced order model ambition is to capture the most significant dynamical features of time dependent phenomena by a limited, i.e.,  $O(10^3)$ , number of degrees of freedom. The ways to obtain such models are manifold. A promising approach which found its place in the simulation of confined spatio-temporal chaos is based on empirical eigenfunctions. From numerical or experimental databases, by means of singular value analysis, it is possible to derive spatial and temporal modes which give an optimal representation of the data set. Numerical evidence has shown that such functions can be used, for example, to study bifurcation diagrams of moderately complex flows. Yet, much work is to be devoted to extend the validity of such models to parameter spaces different from those



they were generated from and it is necessary to define an appropriate way to incorporate the control in the low-order model, not an easy task since most of the times the control is on the boundary.

An example of coupled system where low order modeling may highly increase production performance is the chemical vapor deposition (CVD). CVD process uses chemically reacting gases to form a thin solid film with some controllable properties (e.g., film composition and thickness). Reactions occur in both the gas phase and in the region on top of the film. Deposition process is driven thermally by heating the substrate. This process is widely used in the micro electronics industry for the fabrication of transistors and memory chips. The present challenges for such production technique include real time sensing and control to obtain the required wafer characteristics [56], [62], [63].

### 3.1.2. Flow analysis

It is very exciting to model complex phenomena and to develop methods to compute the corresponding approximate solutions. But an important point is to be able to well understand the phenomena. Classical graphic tools give us the possibility to visualize some aspects of the solution at a given time and to even see in some way their evolution. Nevertheless in many situations it is not sufficient to understand the mechanisms that create such a behavior or to find the real properties of the flow. It is then necessary to analyze carefully the flow for instance the vortex dynamics or to identify the coherent structures to better understand their impact on the whole flow behavior.

#### 3.1.2.1. Vortex dynamics

For dominated convection flows it is possible to use simplified models as *Point vortex* that are able to follow the trajectory of the vortices. These models use hydrodynamics equations of multi pole vortex without the diffusion of particles. Some improvements of these methods are in progress.

One of the most helpful techniques to study two and three- dimensional vortex dynamics is the identification of the coherent structures which are convected with the flow and concentrate a large amount of the energy and the enstrophy. Coherent structures generally refer to the organized and concentrated rotational patterns within the flow and are a useful way to characterize flow evolution in time (e.g. [61]). Some works on various strategies have been developed in collaboration with E. Creusé in Valenciennes and with S. Sherwin at Imperial College in London for 2D and 3D flows ([54]). >From this last collaboration should emerge a mathematical and a numerical way to detect coherent structures in an arbitrary domain.

Once the vortices are well identified it is of main interest to follow them to detect their interactions and observe the consequence on the whole vortex dynamics.

#### 3.1.2.2. Analysis tools for turbulent flows

The Kraichnan-Batchelor classical theory is valid on infinite domains. So the physical as well as the numerical experiments on finite domains yield results that have often significant discrepancies to the theory. Besides, the analysis tools such as the Fourier transform, the wavelets or the proper orthogonal decomposition can give various results when used with various parameters. So the aim of this work is on the one hand to determine the range of the parameters giving reliable results and on the other hand to find out the statistical laws observed by the flow in configurations uncovered by the theory.

Another approach to better evaluate the analysis tools is to use a placebo effect. It is achieved for instance by creating an artificial velocity field where a fundamental characteristic of the flow is not present and by using the classical methods able to detect this characteristic. If the method detects the characteristic it means that it is created by the method itself !

To better analyze the flow it is important to capture the main components of the flow. For decades the flows were split into coherent structures and noise or background but it appears that the definition of a coherent structure is not so clear. It seems that a better choice is to distinguish the kernels of the vortices and the vorticity filaments in 2D turbulence. These vorticity filaments are a link between vortical structures but also roll up in spirals inside the kernels of the vortices. A careful analysis of these two different components could reveal a different statistical behavior and help to understand the turbulent flows.

### 3.1.3. Optimization and control

The final application of the simulation and analysis tools developed above is flow optimization and control. A classical approach to achieve such goals is a shape design based on the control theory. This tool is very efficient for steady problems or problems for which the performance does not change much in time but is not robust enough for real time control. The benefits due to the shape improvement, however, reaches soon its limits and therefore it is necessary to add control devices that locally modify the flow to provoke global improvements of the flow characteristics. We will model both passive and active control. Passive control will consist in steady devices which do not evolve in time, whereas active control will have an automatic interaction with the flow, based on certain real time physical measurements.

#### 3.1.3.1. Shape optimization

When the number of design parameters becomes important, the designer intuition is no more sufficient to obtain significant reduction in cost functions. In contrast, an automatic procedure based on the minimization of the cost function may lead to innovative and unconventional architectures. The main idea is to solve the adjoint of the governing equations, either on the continuous or on the discrete level. Compared to existing techniques of optimization based on control theory, we will consider a method using the adjoint of an inverse problem, to easily deal with flow constraints. The applications envisaged are in the domain of turbomachineries, where it exists a collaboration the EC Lyon [60].

#### 3.1.3.2. Passive control

We will consider three approaches. The first is based on the idea of distributing a porous material between some parts of an obstacle and the flow in order to modify the shear forces in the boundary layer. This approach may pose remarkable difficulties in terms of numerical simulation since it would be necessary, a priori, to solve two models : one for the fluid, one for the porous medium. However, by using the penalization method it becomes a feasible task [45]. This approach has been now used in several contexts and in particular in the frame of a collaboration with Renault and the IFP (French Oil Institute).

Another technique that will be considered is that of injecting minimal amounts of polymers in hydrodynamic flows in order to stabilize the mechanisms which enhance hydrodynamic drag. A PhD grant is required for both the modeling and the numerical issues of such problem. The study is performed in collaboration with CPMOH laboratory in Bordeaux (H. Kellay).

A third approach consists in capturing by appropriately designed cavities large detached structures past bluff bodies. The project outcomes will serve the designers of the next-generation thick-wing aircraft. Thick-wing, or blended wing-body, aircraft are identified as prospective for development over the next 50 years in the NASA and FAA commission report « Securing the Future of U.S. Air Transportation : A System in Peril » released in September 2003. The scopes of this project will be mainly to develop a software tool for designing a flow past a thick airfoil with a trapped vortex assuming that this flow is stable, apart from small-scale turbulence. This project is financed by the European Community [47].

#### 3.1.3.3. Active control

This research program is addressed to conceive, implement and test automatic flow control and optimization aiming mainly at two applications : the control of unsteadiness and the control and optimization of coupled systems. Implementation of such ideas relies on several tools. The common challenges are infinite dimensional systems, Dirichlet boundary control, nonlinear tracking control, nonlinear partial state observation.

The bottom-line to obtain industrially relevant control devices is the energy budget. The energy required by the actuators should be less than the energy savings resulting from the control application. In this sense this research team has gained a certain experience in testing several control strategies with a doctoral thesis (E. Creusé) devoted to increase the lift on a dihedral plane. Indeed the extension of these techniques to real world problems may reveal itself very delicate and special care will be devoted to implement numerical methods which permit on-line computing actual practical applications. For instance the method can be successful to reduce the drag forces around a ground vehicle and a coupling with passive control is under consideration to improve the efficiency of each control strategy.

Another idea to stabilize incompressible flows is to minimize a functional which involves the vorticity. It is a non-convex optimization problem that may be studied with tools that are in common with data-assimilation techniques. The control considered is an appropriate body force whose distribution can be optimized. The existence of such solution does not seem to be difficult to show, at least in two dimensions. The objective is to study an efficient algorithm to solve such problems.

Finally, low order models will be used to solve stabilization problems where the gains of the actuators will be simply computed by Riccati equation. If the control devised by low order models can be simply checked a posteriori, one can also attempt to answer the fundamental issues of controllability and observability of the physical problem thanks to the low dimensionality of the models. [46]

## 3.2. Laser plasma interaction

**Participants:** Mathieu Colin, Thierry Colin, Cédric Galusinski.

The interaction with a plasma implies nonlinear coupling effects. The electric field for a laser pulse has to be considered under the form :

$$e^{i(k \cdot z - \omega t)} A(t, x, y, z) + c.c.$$

$$\text{with } \partial_z A \ll kA, \quad \partial_t A \ll \omega A$$

For small  $\varepsilon$ , it is impossible to compute such a solution directly. Therefore, asymptotic expansions must be used to obtain simplified models. The basic equations to describe electromagnetic waves are Maxwell's equations. The difficult point from the modeling point of view is to describe the interaction with the matter. For propagation in crystals, phenomenological models ( $\chi^2$  and  $\chi^3$  nonlinearities) are mainly used. Basically, if quadratic effects are expected, quadratic terms have to be used ! If for symmetry reason, the quadratic terms are equal to zero, then cubic terms have to be used. Of course, the crystal properties enable to limit the number of free parameters. For propagation in gas, Maxwell-Bloch models (derived starting from the Schrödinger of quantum mechanics) can be used. For interaction with a plasma, one can use either kinetic models (Vlasov or Boltzman) or fluid models (bifluid Euler). In all cases, the solutions involved are so complex that it is impossible to use these models. These systems need to be simplified using asymptotic expansion. The more efficient theory in this direction uses WKB expansions. We will present below our project in this direction.

The basic WKB expansions show that the nonlinear Schrödinger equation is a universal model to describe wave packets in physics ; therefore, the propagation of a laser beam in a media is often described by the nonlinear Schrödinger equation. This equation integrates the dispersion phenomena in media as well as nonlinear effects. It can be derived from nonlinear Maxwell system through an asymptotic development of BKW type. It is well adapted for the propagation of long pulses in homogeneous media for a known direction of propagation. This is the fundamental model used in Mirò, which is the CEA code developed for laser propagation.

It is known that the usual nonlinear Schrödinger equation is not valid anymore if very short pulses or very intense laser beams are considered. The aim of our project is to go beyond the nonlinear Schrödinger equation and to develop some models and numerical codes adapted to more complex situations. The models are intermediate between the full Maxwell system and the nonlinear Schrödinger equation. We split this study into two parts. The first one concerns coupled models, the second one concerns short pulses.

### 3.2.1. Coupled models.

In this part, we are concerned by the case where several components of the laser are present. In this case, the usual paraxial approximation that allows the use of Schrödinger-like model instead of Maxwell's system is not valid anymore. Such situations may occur if several laser beams are used or in the case where some unexpected component are created by a nonlinear instability. The latter situation arises for example in the Raman instability.

### 3.2.1.1. Raman amplification.

This is a three waves instability. The laser enters the plasma and behaves like a pump wave. A backscattered Raman wave is created and propagates backward compared to the laser wave. These two waves interact and electronic plasma waves (with 0 group velocity) are then excited. Then, these three waves combine to create ion acoustic waves that of course have a retroaction to the three preceding waves. We are focusing on large time scale propagation of these three waves whose group velocities are different. A classical asymptotic analysis predicts decoupled waves which is not reasonable from a physical and mathematical point of view. We thus have to derive an intermediate model between Maxwell and Schrödinger equations, which contains the appropriate regime. It can be implemented for numerical simulations. This has been done in 1 D starting from Euler-Maxwell system. Systems of Zakharov type are obtained see [50] and [49].

We are concerned with several extensions of these works. First, the extension in multi-D is not obvious. In fact, we have to deal with geometrical aspects. As a matter of fact, the incident beam and the backscattered Raman wave are not necessary coplanar. Moreover, one can find in the physical literature experiments in which backscattered cone are created. One of our approach is to consider that there is one favored direction corresponding to the most important amplification coefficient. If such a direction does not exist, we plan to recover the cone by domains and to describe the impulsion in these domains by large spectrum accurate models like in [22].

These kind of models are far from being justified, it is also a point that we would like to address.

Finally, in order to increase the precision and to handle easily boundary value problems, we would like to consider Klein-Gordon-type equations, and get rid of the paraxial approximation.

### 3.2.1.2. Crossing beam

Let us consider two laser beams that cross in a nonlinear medium. If the angle between both laser is large, then one just has to consider a two-phases WKB expansion starting from nonlinear Maxwell systems. If the angle is small, then we consider that there is only one laser and we use a single-phase expansion. The problem is to obtain a model that is valid in both cases and in intermediate situations. The idea is to find intermediate systems just like in the spirit of [12] that will allow a continuous transition between both situations. We already have such a system that looks like those used in [50]. The geometry is at least 2-D ! We are performing numerical tests (joint work with G. Gallice, thesis of G. Ebrard, 2005). We want to obtain a whole description of such situations from both the mathematical and numerical point of view. The difficulty is that some oscillatory terms remain in the equations. These terms have a small contribution in some cases and a large one in other cases and we want to have a uniform treatment. The model will be extended to the case where the Raman instability takes place.

### 3.2.1.3. Landau damping.

The main weakness of the above models is that they do not take into account kinetic effects. Of course, we do not want to go back to the kinetic models which are too expansive for our framework simulations. The Zakharov system describes a coupling between the variation of the density of ions and the slowly varying envelope of the electric field. When laser-plasmas interaction in under-dense plasma occurs, one observes dissipative effects of Langmuir waves, due to Landau damping. An energy exchange between the plasma wave and the electrons takes place in the media. Formally obtained from the Vlasov equations, a Zakharov system, integrating Landau damping describes the coupling effects between plasma electronic wave and acoustic electronic wave. The Landau damping term makes this model nonclassical. It is a nonlinear coupled system in frequency as well as in space between the electric field and the electronic distribution. Technically speaking, this is a coupling between a Zakharov type system and a diffusion equation concerning the spatial mean value of the distribution function of the electrons. The process is efficient only for those electrons which velocity is equal to the phase velocity the electric field. The system thus obtained is a coupling between a Zakharov system in physical space and a diffusion equation written in the Fourier space. We already have obtained some numerical results on such a system. The energy is brought to the system through a given pump wave. Our project in this direction is twofold :

i) In this context, the pump wave is given from the physical point of view by a Raman instability. We therefore have to couple the Raman system and the Landau one. >From the numerical point of view, this is far from being easy : the Raman process has to be considered as a boundary value problem while the Landau damping is given in Fourier space (we use periodic boundary conditions...). From the mathematical point of view, it seems to be hopeless to construct solutions for the whole system at the moment. Simplified versions of Schrödinger type seems to be more reasonable.

ii) Another problem is the justification of these models starting from Vlasov-Maxwell system. This problem is not only a mathematical challenge, it is also a necessary step in order to be able to consider non-homogeneous plasmas.

#### 3.2.1.4. Coupling to hydrodynamics.

In all the cases described above, the plasma response is modeled by a single wave equation as in the usual Zakharov system. Previously, the acoustic regime was considered. Nevertheless, this is really insufficient for the expected regime. One possibility is to couple electromagnetism system with hydrodynamics. This problem will be developed in collaboration with G. Gallice (CEA-CESTA) who has a great experience in this domain.

#### 3.2.1.5. Extension to magnetized plasmas.

We propose to extend our methods to the case of magnetized plasmas in order to be able to apply them to the project ITER, which consist in creating nuclear fusion by magnetic confinement. We expect to find intermediate models that will give at least a qualitative description of the physical phenomena. It is a collaboration with T. Goudon (Simpaf, Lilles), E. Sonnendrücker (Calvi, Nancy), P. Degond (Toulouse), R. Abgrall (Scallaplix, Bordeaux).

### 3.2.2. Short pulses

Schrödinger type model are not really relevant for ultra-short laser pulses. Therefore, we should use Maxwell's equations, but the costs in term of computations are exorbitant because of high frequency motion and the small wavelength involved in the problem. Then, we can make some numerical experiments only on short distances (a few millimeters) whereas we are interested in propagation on large distances. We then have already developed some intermediate models. Moreover, for such extreme regimes, it is really important to consider very precise and reliable models. Nevertheless, because of large distances of propagation, we can not consider the microscopic level. Semi-classical method describing energy level will be investigate.

#### 3.2.2.1. Phase modulation method.

When describing the propagation of ultra-short pulses through diffraction webs, it is relevant to introduce intermediate models. The models used are those obtained and justified in [22]. For the propagation after diffraction webs, the model is linear. Even in this case, the situation is not clear. Indeed, the spectrum of the pulse becomes very large and the propagation is stretched (at the beginning  $10^{-13}s$ , at the end  $10^{-8}s$  !). During the propagation, as the pulse becomes larger and larger, the wave phase is getting damaged so that we have to consider discretization mesh with millions points in 1-D. Of course, we can not extend this to the multi-D case.

We plan to introduce a non-planar geometric optic method to overcome this difficulty. That is, we first solve an Hamilton-Jacobi equation in order to find the phase, then the equation for the amplitude is solved. Since we are in a defocusing geometry, we use the derivative of the phase as new unknown and we solve the equation using the method of characteristics. The equation for the phase will always be one-dimensional. The first results in one dimension are very promising. We obtain the same level of precision using 4000 point in time than with Miró with 2 millions of points. We plan to extend the method in 3-D as well to obtain error estimates. We also want to be able to consider nonlinear situation like amplifiers.

#### 3.2.2.2. Multi-level models.

This is a collaboration with B. Nkonga (ScALAppliX). We already have studied 2 and 3 level Maxwell-Bloch systems in order to describe propagations in gas [43], [52]. In these articles, we have studied models describing propagation of a pulse in gas with respectively two and three energy levels. In particular, it was shown in these articles, how such systems can describe a Raman instability in a gas. The idea will be to

complete the model by adding levels and to propose an adapted numerical solution. A first step will be to treat the doppler effect : because molecules are moving with different speeds in the gas, they do not receive the same frequency laser. We then assume that the interaction frequency in Bloch system depends on a new parameter describing the velocity of the atoms. The polarization is then obtained by averaging in the velocity space. It is then a coupled system with kinetic equations in the same spirit as for the Landau damping : we do not consider the molecular level but we try to take into account some very specific properties.

### 3.2.3. Coherent structures and nonlinear Schrödinger equations.

In the theory of PDE's, solitary waves play a very special role. They give important informations on evolution equations when these last ones possess only a local existence theory. In fact, these kind of solutions are global in time. They drop the time parameter in evolution equations so that they are solutions of stationary equations. Another important point in the theory of solitary waves is the study of their orbital stability. The problem is the following : starting from an initial Cauchy data near a solitary wave, is the corresponding solution of the evolution equation stays close in a certain sense to this solitary wave ?

Following this direction, we want to investigate the existence and stability of solitary waves solution to the quasi-linear Zakharov equations describing laser-plasma interactions introduced previously. In order to extend this study, we will also consider some general nonlinear Schrödinger systems (in collaboration with M. Ohta from Saitama University, Japan). We are also going to investigate the so-called non derivative Schrödinger equation from the same point of view.

## 3.3. Multifluid flows and application for Complex fluids, microfluidics and biology

**Participants:** Charles-Henri Bruneau, Thierry Colin, Cédric Galusinski, Iraj Mortazavi.

### 3.3.1. Complex fluid

By a complex fluid, we mean a fluid containing some mesoscopic objects, that is to say structures whose size is intermediate between the microscopic size and the macroscopic size of the experiment. Example of such fluids are given below. For the microfluidic part, we are concerned with the study of fluid flows in microchannel. A microchannel is typically 1 cm long with a section of  $50\mu m \times 50\mu m$ . One of our goals is to be able to simulate the flow of a complex fluid in a microchannel. The history of this study is the following one. There was a collaboration between researchers from the MAB (Applied Math. Laboratory, Bordeaux 1 University) and from the Centre de Recherches Paul Pascal (CRPP, <http://www.crpp.u-bordeaux.fr/>) which is a laboratory for physical chemistry. Our contacts there were A. Colin, D. Monin, D. Roux, A.-S. Wunnenburger. The goal is to study complex fluids containing surfactants in large quantities. It modifies the viscosity properties of the fluids and surface-tension phenomena can become predominant. Three different subjects were treated.

- Foams.

This is a collaboration with P. Fabrie (Bordeaux1), A. Colin, D. Monin (CRPP).

Foams are current states occurring in everyday life. The aim of this work was to study the stability of a foam in terms of several parameters. The foam is created experimentally in a Plexiglas column and one measures the quantity of water contained in the Plateau borders (junction between several bubbles). The water starts to flow down under the action of gravity and the foam breaks from the top. We wrote a free boundary model in order to describe this phenomena. It is a Burgers equation with nonlinear viscosity and a free boundary. A numerical scheme has been implemented and the result is coherent with the experiments done in the thesis of D. Monin. In [51] we proved that the system is globally well-posed.

- Instability of lamellar phases.

Collaboration with A. Colin, D. Roux, A.-S. Wunnenburger (CRPP), thesis of V. Torri (math).

Lamellar phases consist of bilayers of surfactants in a solution. When submitted to a shear stress, these layers become unstable and break. A phase transition occurs and spherulites appear. They have the structure of concentric spheres. It is a typical example of complex fluids. The CRPP is the more famous lab in the world for

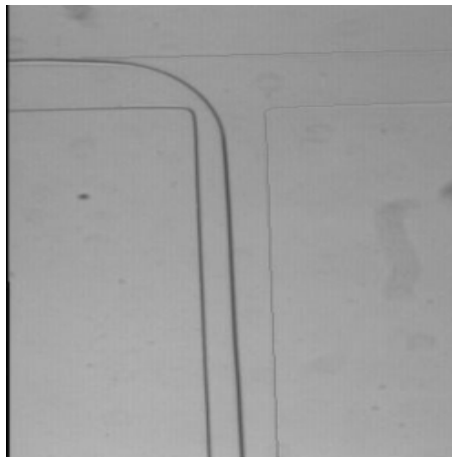
the study of such objects. Even if the apparition of spherulites is well controlled from the experimental point of view, the transition is far from being understood. However if the shear is strong enough, the lamellar phase is stable in experiments. In [67] we wrote a model and studied numerically the stability problem. The thresholds found numerically are coherent with those observed experimentally. Unfortunately, this system can not be use to study the transitions to spherulites. In his thesis, V. Torri studied this system from the mathematical point of view and proved rigorously the above behavior. He was also able to perform some numerical simulations in order to characterize some bifurcation processes [65].

- Diphasic fluids via diffuse interface models.

It has been done through the thesis of F. Boyer [44] and L. Chupin [48]. In these two thesis, in collaboration with the CRPP, models of Cahn-Hilliard type was written in order to simulate in 2-D diphasic fluids. Both newtonian and non newtonian cases were considered. The results of the simulation where compared with the experiments (like spinodal decomposition under a shear stress for example). The Phd advisor was P. Fabrie.

### 3.3.2. *Bifluid flows in micro-channel*

Flows in microchannels are often at low Reynolds numbers. The hydrodynamical parts is therefore not so difficult. However, the main problem is to produce real 3-D simulation covering a large range of situations. For example one wants to describe diphasic flows with surface tension and sometimes surface viscosity. The size of the channel implies that one can observe some very stable phenomena. For example, using a  $T$  junction, a very stable interface between two fluids can be obtained. In a cross junction, one can also have formation of droplets that travel along the channel.



*Figure 1.*

One of the main point is the wetting phenomena at the boundary and the boundary conditions that are fundamental for the description of the flow since the channels are very shallow. The wetting properties can NOT be neglected at all. Indeed, for the case of a two non-miscible fluids system, if one considers no-slip boundary conditions, then since the interface is driven by the velocity of the fluids, the interface between both fluids shall not move on the boundary. The experiments are showing that this is not the case : the interface is moving and in fact all the dynamics starts from the boundary and then propagates to the whole volume of fluids. Therefore, we can not use standard numerical codes and we have to adapt the usual numerical methods to our case to take into account the specificities of our situations. Moreover, we want to obtain reliable models and simulations that can be as simple as possible and that can be used by our collaborators. As a summary,

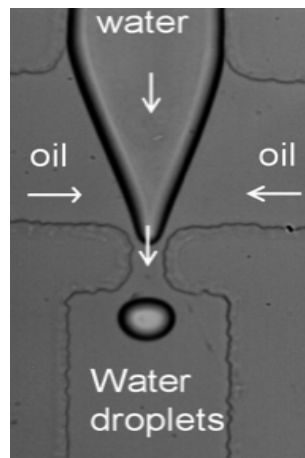


Figure 2.

the main specific points of the physics are : one needs multifluid simulations, the Reynolds number is small (it is therefore easier !), the wetting problems are important, the surface tension problem is also important, flows are really 3-D, the boundary conditions are fundamental due to the size of the channels. Our collaborators in this lab are G. Cristobal, J.-B. Salmon, M. Joanicot, P. Panizza, A. Colin.

A grant (ACI) has been obtained on this subject in 2004 :

<http://www.math.u-bordeaux.fr/~colin/aci/aci.html>.

The projects that we have are the following ones :

- A micro-rheometer.

This is a very simple numerical situation. However, we will give a precise description in order to show what point of view we want to adopt in this project. Using a "T"-junction as in the above picture, one uses two different fluids. Take the picture from above and measure the position of the interface. The fluids are supposed to be Newtonian. Knowing one viscosity and the flow rates, one would like to deduce the viscosity of the other fluid. To do so, we give both viscosity and compute the velocity field and then the flow rates. Then we make a loop in order to deduce the viscosity.

A first article has been written [59] when the velocity field is supposed to be longitudinal, depending only on the transverse variable. We would like to be able to produce a software that could be used with the all process. The situation is in this case obvious from the numerical and mathematical point of view.

- Computations in the "T" junction.

Starting from the above obvious problem, we now consider a much more complex situation. The problem is to compute exactly what occurs at the beginning of the "T" junction or at the cross function producing droplets. We consider two viscous non miscible fluids with viscosities  $\eta_1$  and  $\eta_2$ . Each fluid satisfies the Stokes equation in the domain it occupies. At the interface the velocity is continuous as well as the normal part of the stress tensor. In the case where surface tension has to be taken into account, one imposes that the jump of the stress tensor is equal to the surface tension coefficient multiplied by the curvature times the normal at the interface. These tension surface forces play a major role in micro-channels. The boundary conditions are of Robin type in order to take into account slip at the boundary. Without surface tension, this system has been studied from the mathematical point of view by Demay, Nouri and Poupaud [55]. One can hope an existence result in the case of a diffuse interface, that is when the initial data for  $\eta$  is continuous, even in the case where the surface tension is present. One has to give a weak formulation of this kind of system.



One can also consider complex fluids as well as several types of boundary conditions in order to model the wetting phenomena.

- **Mixing in a microchannel.**

This is a collaboration with F. Boyer (Marseille). In a "Y" junction, one considers the mixing of two fluids in a shallow micro-channel. We make some Hele-Shaw approximations that lead to Darcy's law for the velocities. An order parameter description is used. One therefore obtains Darcy's law for the velocity with coefficients depending on the order parameter. These coefficients take into account for example the slip-length that is used in general boundary conditions describing the wetting. We will justify this system starting from Stokes. Again, the slip condition at the boundary will play a central role. The discretization will be done using finite volumes. This program can be realized in a short term (one year) with comparison with the experiments of J.-B. Salmon in the LOF.

Of course we may have to get rid off the Hele-Shaw hypothesis and we may need to have a real 3-D version for mixing. We will then use the preceding method. These experiments are used in order to measure some reaction rates.

- **Droplets in a micro-channel.** In the case where two non miscible fluids are present in a microchannel, one can obtain one jet of one fluid into a second one. We will study the stability of such a configuration and the formation of droplets. It is a collaboration with P. Panizza that makes such experiments in the LOF. The linear stability property (that may be obtained numerically) is something that can be done in a short term. The study of the formation of the droplets and their propagation in the channel is much more difficult. We will apply a level-set method to this problem.

- We also need to make a systematic study of the wetting properties. This is a quite prospective problem. As said above the interface has to move on the boundary even if the velocity is zero. This is not possible. We use a phenomenological modeling : we impose that the velocity satisfies a Robin type boundary conditions. But a rigorous study of the wetting in a boundary layer has to be made.

- The next step will be to be able to study more complex fluids (viscoelastic, complex fluids like spherulites...). This is for the long term (4-5 years).

- **Crystallization.** A more prospective subject concerns crystallization. Each year, sudden aircraft performance degradation due to ice accretion causes several incidents and accidents. Icing is a serious and not yet totally mastered meteorological hazard due to super-cooled water droplets (liquid water droplets at a temperature below the dew point) that impact on aerodynamic surfaces. Icing results in performance degradations including substantial reduction of engine performance and stability, reduction in maximum lift and stall angle and an increase of drag.

A new approach to in-flight icing analysis based on partial differential equations has been proposed. This new model contains all the physics that cause ice formation from rime to glaze, including conditions categorized as mixed. Glaze and mixed ice form when there is no sufficient heat transfer to immediately freeze all impinging water droplets, with some water running back along the dry surface or the existing ice layer and freezing further downstream. The convective heat transfer controls ice accretion in this case and the thermodynamic model used satisfies the first law of thermodynamics in terms of conservation of mass and energy in a control volume. A continuous form of these equations was derived for the liquid water film. The obtained partial differential equations are shown to be close to the well-known shallow water equations, with the addition of source terms corresponding to the other water phases : solid and vapor. Their discretization was done using a finite volume approach. Four compatibility relations, based on physical considerations, have been defined to insure the well-posedness of the system to solve.

In a mean time project we intend to study if the modeling techniques developed for in-flight icing can be used to model the crystallization process in other industrial applications. A partnership with Rodhia will be created to study and model crystallization occurring in microfluidics.

### 3.3.3. *The application for biological modeling.*

The propagation of a tumor is also a low Reynolds number flow. We propose to adapt the methods developed in microfluidics and complex flows in this framework. We propose to use a model for fluid dynamics to describe the tissue behavior. The macroscopic continuous model is based on Darcy's law which seems to be a good approximation to describe the flow of the tumor cells in the extra-cellular matrix [42], [57], [58]. We develop a two-dimensional model for the evolution of the cell densities. We formulate mathematically the cell densities in the tissue as advection equations for a set of unknown representing the density of cells with position  $(x, y)$  at time  $t$  in a given cycle phase. Assuming that all cells move with the same velocity given by Darcy's law and applying the principle of mass balance, one obtains the advection equations with a source term given by a cellular automata. We assume diffusion for the oxygen  $C$ , the diffusion constants of course depends on the density of cells. The source of oxygen corresponds to the spatial location of blood vessels. The available quantities of oxygen interact with the proliferation rate given by the cellular automata [64]. We plan to add the interaction with a membrane, as well has a mathematical model of angiogenesis. Of course realistic simulation will be 3-D.

## 4. Application Domains

### 4.1. Rhodia for Microfluidics

For microfluidics, the main partner is Rhodia. A new lab has been built recently in Bordeaux. It is a common lab between the CNRS and Rhodia which is the biggest french company of Chemistry. It is called the LOF (Laboratory Of the Future). One of its goals is to develop experimental tools in order to use microfluidics in Chemistry. Microfluidics is the study of fluids in very small quantities, in micro-channels. A typical size of the channel is  $50\mu m \times 50\mu m \times 1cm$ . They are many advantages of using such channels. First, one needs only a few quantity of liquid to analyze. Second, one has very stable flows and quite unusual regimes that allow to make more precise measurements. Some of our collaborators (A. Colin, P. Panizza) moved to this lab.

The idea is to couple numerical simulations with experiments to be able to understand the phenomena, to predict the flows and compute some quantities like viscosity coefficients.

### 4.2. CEA for Laser-plasma interaction

The only industrial contact is the CEA (french nuclear agency). In this center, the construction of a powerful laser (called laser Mega-Joule) began a few years ago. The goal is to simulate in laboratory nuclear fusion. Highly nonlinear regimes of propagation are reached and the propagation of a laser beam in nonlinear media (including plasma) has to be computed.

### 4.3. IFP, Renault and CIRA for flow simulation, analysis and control

To simulate flow over various bodies as a riser pipe or a ground vehicule, our industrial contacts are Institut Francais du Petrole (IFP), Renault and Centro italiano ricerche aerospaziali (CIRA). Shape optimization, passive and active control are also the main objectives in order to reduce drag and to suppress instabilities.

## 5. Software

### 5.1. Microfluidics

Simulation of 2-D diphasic, non newtonian fluids by a Cahn-Hilliard type approach. Finite differences. Cartesian mesh. (collaboration with L. Chupin).

A two-dimensional microfluidic flow code, developed using the simplified models studied in Bordeaux (collaboration with L. Chupin).

Other codes, 2-D and 3-D codes for bifluids flows in micro-channels are being developed.

## 5.2. Laser-plasma interaction

- Two-level Schrödinger-Bloch (3-D). Fourier method in transverse direction. Finite differences in the longitudinal variable. Moving window. (collaboration with B. Nkonga).
- One dimensional Maxwell-Lorentz for non-centrosymmetric crystals ( $\chi^2$  nonlinearity). (collaboration with B. Nkonga).
- One dimensional Landau-Damping : coupling of a Zakharov system in physical space with a diffusion equation in spectral space.
- Two dimensional Raman amplification. Coupling between 4 nonlinear Schrödinger equations and a wave equation.

## 5.3. Flow simulation, analysis and control

### 5.3.1. Flow simulation and analysis.

Two main technologies related to the simulation of incompressible flows exist in the team : DNS methods and Vortex methods.

In DNS, two multigrid codes are available : - NSMulti2D : 2D computational code for solving incompressible Navier-Stokes equations on cartesian meshes in various domains with penalization of obstacles. Boundary conditions such as no-slip, periodical, Dirichlet, artificial on open frontiers on any parts of the boundary are available. - NSMulti3D : Same as previous one in 3D with more restrictive boundary conditions.

In vortex methods : An entirely grid-free and lagrangian 2D code is using vortex blobs to discretize the vorticity field. An axisymmetric version of this program is also available. Another 2D vortex code is using a PIC (Particle-In-Cell) method that permits to decrease the grid diffusion tracking particles with the local velocity field (collaboration with A. Giovannini). Also, a collaboration with G.-H. Cottet related to his 3D vortex code is undertaken.

Finally, some flow analysis and diagnostics tools are developed. - NSAnal : Analysis of 1D signals and 2D pictures of turbulent flows by means of Scilab core. Another software devoted to diagnostics of coherent structures is also available.

### 5.3.2. Low order models, shape optimization and control.

Complex demonstrators for most of the studies undertaken have been developed. Several adjoint codes are available ranging from potential to 2D Euler and 2D compressible Navier-Stokes equations. In collaboration with Dr. D. Quagliarella of CIRA a multiblock adjoint code for RANS models was developed as well as a symbolic manipulation environment to automatically derive the symbolic continuous adjoint of given governing equations (see VKI Ed.Ser. 2003). Such codes are in use at CIRA. For turbomachinery adjoint inverse design, a meridional plane through flow is currently developed for aeroacoustic noise reduction, in collaboration with Michel Roger of the Ecole Centrale de Lyon.

## 6. New Results

### 6.1. Low-Reynolds flows

In the context of low-Reynolds flows, our contribution is in two direction. In [27], [26] we describe an experimental method that allows us to measure the viscosity of a fluid using micro-channel. Basically, we use a coflow. One of the fluid is well-known and we want to obtain the viscosity at different shears of the other. We measure the position of the interface and then solve numerically an inverse problem in order to obtain the viscosity. This is done with A. Colin, M. Joannicot, P. Panizza from the laboratory Rhodia-LOF.

The second point concerning low-Reynolds flow is the introduction of a semi-discrete model for tumor growth. We have coupled continuous equations with a cellular automata that give proliferation rates for the cell. Our simulations are then apply to compute the efficiency of a radio-therapy protocol. It is done in collaboration with B. Ribba from the university of medicine in Lyon.

## 6.2. laser-plasma interaction

**Participants:** Mathieu Colin, Thierry Colin, Cédric Galusinski.

Publications [22], [23], [38] are concerned with the propagation of laser beam in nonlinear media. Usually, such a propagation is described using the NonLinear Schrödinger (NLS) equation. In these three papers we investigate cases where this approach is not valid anymore. The lack of validity of the NLS equation can come from the fact that the pulse is short or with a large spectrum. We propose two alternate methods. In [22], [38] we use directly the full Maxwell system. The main drawback of this system is the computational cost : on has to discretize at the wave-length length which is not possible in realistic physical application. We propose to use a singular equation involving a variable that describes the phase. This variable describes all the oscillations of the solution and allows us to use space-step of the order of the wave-length. This approach is validated on a quadratic nonlinearity in [22] using finite differences scheme and on a cubic one using a spectral approach in [38]. The second method uses a spectral decomposition of the system. It is a systematic approach to introduce an intermediate system that is exact in linear case. It is maybe the more promising approach. We have tested it on model system in [23]. We are now performing the application on Maxwell-type system.

The same kind of approach has been developed in the context of water-waves in [12]. We have introduced intermediate model between the Euler equation with free surface and the KdV equation in order to describe counter-propagating waves.

Concerning plasma physics, we have introduced several systems involving coupling of waves [21], [36]. For each system, we introduce a adapted numerical scheme in order to describe the main feature of the physical context. In [21], we have implemented a scheme that is able to take into account the Raman amplification in a plasma. In [36], the Landau damping phenomena has been described. The next step is to couple these two models.

All of these works are done in collaboration with G. Gallice of CEA CESTA.

The paper [24] is concerned with the so-called Derivative Non Linear Schrödinger equation. This equation is known to have a two-parameter family of solitary waves solutions. We prove orbital stability of these particular solutions for the whole range of parameters values by using variational methods.

## 6.3. Numerical modeling, flow analysis, optimization and control

**Participants:** Héloïse Beaugendre, Charles-Henri Bruneau, Mathieu Colin, Angelo Iollo, Iraj Mortazavi, Mazen Saad.

Numerical simulation of 2D incompressible flows ([19], [13]) :

Validation of a 2D code for solving Navier-Stokes equations on the classical lid-driven cavity test with comparison of the results of the literature and on a new dipole-wall collision benchmark.

Flow analysis ([14], [15], [31]) :

Comparison of various analysis methods adapted to temporal signals and snapshots of turbulent flows.

2D turbulence ([16], [29], [30]) :

Direct numerical simulation of 2D turbulence with coexistence of both direct enstrophy cascade and inverse energy cascade.

Flows in porous media ([17])

Numerical simulation of the miscible displacement of radionuclides in a heterogeneous porous medium with solutions of the COUPLEX benchmark of GdR CNRS MoMAS. The modern techniques of experiment and direct numerical simulation give great quantities of data in the form of vector fields in 3d. The objective analysis and interpretation of such fields constitute a complicated problem. The velocity vector fields can be seldom interpreted directly. In the majority of the cases, the data are first treated in some way and then visualized. This process is prone to biased numerical handling which can affect the conclusions that one obtains from the analysis. In the paper [20] we clarify one of these cases.

In the papers [39], [37], some applications of a precise method to model the transient dynamics of large scale structures in the laminar flow past a bluff body are presented. The flow is described using empirical

eigenfunctions obtained by "proper orthogonal decomposition" and the models are constructed projecting the Navier-Stokes equations onto such eigenfunctions. The linear terms in the expansion coefficients as well as in the control inputs are adjusted to exactly mimic some reference solutions. Applications shown concern the development of flow instabilities leading to vortex shedding and the dynamics of the vortex wake under external actuation.

Two-dimensional and quasi-3D in-flight ice accretion simulation codes have been widely used by the aerospace industry for the last two-decades as an aid to the certification process. The paper [11] proposes an efficient numerical method for calculating ice shapes on simple or complex 3D geometries. The resulting ice simulation system, FENSAP-ICE, is built in a modular fashion to successively solve each of flow, impingement and accretion via field models based on partial differential equations (PDEs). The FENSAP-ICE system results are compared to other numerical and experimental results on 2D and slightly complex 3D geometries. It is concluded that FENSAP-ICE gives results in agreement with other codes calculation results, for the geometries available in the open literature.

The paper [35] presents a model to simulate overland flow genesis induced by shallow water table movements in hillslopes. Variably saturated subsurface flows are governed by the Richards equation discretized by continuous finite elements on unstructured meshes. An obstacle-type formulation is used to determine where saturation conditions are met at the ground surface. The impact of hillslope geometry, boundary conditions, and soil hydraulic parameters on model predictions is investigated on two-dimensional test cases at the metric and hectometric scales. The obstacle-type formulation is also compared with a more detailed model coupling subsurface and overland flow, the latter being described by the Saint-Venant equations in the diffusive wave regime.

The papers [34], [40] give a model of water-gas flows in a porous media and establish the existence of weak solution. We have to deal with a highly nonlinear degenerate parabolic system.

## 7. Contracts and Grants with Industry

### 7.1. Microfluidics

Rhodia already funded a final project of a student of Matmeca. In 2004 a multidisciplinary financial support of CNRS in microfluidics was offered to T. Colin. Moreover, we were awarded for a three year period (2004-2006) by the French ministry of research to develop the microfluidic research (« ACI microfluidique »).

### 7.2. laser-plasma interaction

8 grants have been obtained from the CEA, several in collaboration with B. Nkonga.  
3 PhD thesis were funded by the CEA and the Aquitaine district.  
6 Master thesis (6 month each) were funded by the CEA.

### 7.3. Flow simulation, analysis and control

One contract with CIRA (Italy) has been obtained for years 2003-2005.  
A. Iollo participates to an ANR project for years 2005-2007.  
We had one contract with IFP (Institut Francais du Petrole) during the years 2001-2003 and 3 Matmeca Masters funded by them. Also, a joint patent with IFP on the passive control around oil risers (2004).  
One grant with Renault during years 2003-2004 and 1 Master thesis.  
Members of the team have promoted a EU STREP (trapped vortices) in the 6th framework.  
Iraj Mortazavi was awarded in 2002 by the AMIF program of the ESF for a one month stay in the department of aeronautics of the Imperial College in London.  
STREP Vortex Cell 2050 2005-2007.

## 7.4. CEMAGREF (French agency for water resources)

1 contract, 1 PhD and 4 Master thesis are funded by the CEMAGREF.

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