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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team MERE

Modelling and Water Resources

Sophia Antipolis

THEME BIO

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1. Team

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2. Overall Objectives

2.1. Overall Objectives

Biological WasteWater Treatment Plants (WWTP) are used to transform organic compounds present in wastewater in soluble form (also called substrates) into solids (micro-organisms or biomass also called sludge). In more general terms, such a system where a micro-organism is used to transform substrates into others is called a bioreactor. In the context of wastewater treatment, substrates are consumed by the biomasses under adequate environmental conditions. Once the substrate concentrations have reached normative constraints, the solids (the biomass) and the clean water are separated: the liquid is rejected to the natural environment while the sludge is either incinerated, used in agriculture or, until recently, stored in wetlands. The treatment industry can be considered as the first industry in terms of matter to be processed. Therefore, the design, the control and in more general terms, the optimization of treatment processes are real challenges. Our objective is to better understand these processes in order to optimize their functioning in the presence of uncertainty and of unknown and unmeasured external disturbances. To do so,

1. we approach the problems at two levels: the microscopical scale (the micro-organism) and the macroscopical one (the plant),
2. we use macroscopical modeling and control system science tools to develop new design rules, estimation techniques and control systems that we calibrate on real biological pilot plants.

Our methodology consists in the development of mathematical models of the biological reactions and transports in the reactor. At this stage, we have very strong interactions with micro-biologists. After that we analyze the model with the available mathematical tools or/and through computer simulations. Our main emphasis is put on the effects of the spatial distribution of the biomass. This questioning can be understood at various scales.

- At the macroscopic level we compare the performances of various designs, from infinitely stirred reactors to purely non-mixed reactors through cascade of reactors.

- At the microscopic level we are interested in the growth process of the biomass, limitations caused by the diffusion of the substrate, the role of the biofilms.

We are interested in fundamental questions of microbial ecology, like biodiversity of biomasses, competition and predation since they are at the roots of the understanding of biological wastewater treatment and, at the same time we address very practical questions like the minimization of the size of the bioreactors.

3. Scientific Foundations

3.1. Scientific Foundations

Keywords: *control systems*, *ecology*, *environment*, *mathematical modeling*, *observers*, *process engineering*.

The chemostat is a laboratory device which goes back to the second world war, with the work of Monod and Szilard. It is used to study the growth of micro-organisms. The principle is simple: a continuous flow rate through a constant volume reactor provides nutrients to a population or a community of micro-organisms. At equilibrium the growth-rate must equal the artificial mortality induced by the outflow of the reactor. A simple model, for the case where the reactor is perfectly stirred, is given by a set of two differential equations, one for the variations of the nutrient concentration, the other one for the biomass concentration. This model is based on the classical law of mass action used in the modeling of chemical kinetics: the rate of a reaction is proportional to the product of the concentrations of the two reactants. In the case of population growth this means that the growth-rate of a population depends on the nutrient concentration. This system of two equations has been perfectly well-understood for more than half a century.

The chemostat model is a good first approximation of the running of a wastewater treatment plant. From this simple model one can develop models which incorporate more realistic assumptions like:

- Existence of a complicated trophic chain in the digestion process,
- Consideration of non-perfect mixing inducing diffusion processes,
- Consideration of mass transport in plug-flow reactors,
- Parallel or cascade connections of reactors,
- Re-circulation of the biomass,

which lead to complicated systems of coupled partial differential equations of transport-diffusion type. Due to the presence of non-monotonic kinetics the theory of equations of this type is not yet perfectly understood. Determination of stable stationary solutions is often a question of current research and numerical simulations are used. Moreover the control of industrial plants addresses new questions in the domain of robust control and observers.

Since a Waste Water plant is a microbial ecosystem, microbial ecology is fundamental for the understanding of our processes. An ecosystem is a system in which various populations of different species are interacting between them and reacting to the environmental abiotic parameters. Concepts of competition, predation, symbiosis are used to describe these interactions and try to understand important questions like the biodiversity and the productivity of the ecosystem. The biodiversity is the number of species which is supported by the ecosystem and the productivity measures the rate at which abiotic resources are transformed into biomass. An old prediction of theoretical population models says that, in a constant environment, an ecosystem with n different kinds of resources can support at most n different species (different means that the ways two species use resources are different). This prediction is not realized in wastewater treatment plants where it was demonstrated, using tools of molecular biology (SSCP), that a small number of resources (maintained at a constant level) are able to maintain a huge number of species. This shows that the classical model of the perfectly stirred reactor is no longer valid if one wants to model the biodiversity in the reactor. We explore alternative models based on the consideration of growth-rates which are not solely nutrient-dependent, but are also density-dependent, which means that the growth-rate decreases with the density of the biomass. A special case of density-dependence is the ratio dependence which was much discussed recently.

Since a density-dependent model is a macroscopic model, it is important to understand how the density-dependence is a consequence of the microscopic behaviors of individuals. Since direct observation of the behavior of bacteria is difficult, mathematical modeling is of great help. The hypotheses, at the microscopic level, are expressed in terms of partial differential equations or in terms of individually based models so that macroscopic consequences are derived, either by using mathematical reasonings or computer simulations. Mathematical modeling also helps designing experiments which could validate hypotheses.

4. Application Domains

4.1. Design of Wastewater treatment plants

The question of the optimal design of chemical or biochemical systems has been addressed by several authors during the last thirty years. An important effort has been made by the chemical engineering community to synthesize plants with the smallest possible volume in order to minimize the investment cost. This task turns out to be much more complex in the case of biological systems. One reason for that is the difficulty of finding simple and yet accurate models to represent all the important dynamics of living organisms interacting in a bio-system.

A plant that is made of a cascade of homogeneous Continuous Stirred Tank Reactors (CSTR or chemostats) has a particular practical interest: in most cases, it allows to approximate the behavior of diffusive systems (also called Plug Flow Reactors or PFR) which usually exhibit better performances than a single CSTR. In other terms, a given conversion rate can be obtained with a PFR of smaller volume than the one of a CSTR. However, a PFR is very difficult to operate in practice while CSTR operability and reliability are better.

Biological processes can usually be classified into two classes of systems: micro-biological and enzymatic reactions. In simple terms, micro-biological-based reactions define (bio)reactions where a substrate degradation is associated with the growth of certain organisms while the second, the enzymatic reaction, may be viewed as a chemical reaction with specific kinetic functions.

Given a model of a series of CSTRs, representing either enzyme or micro-biological reactions, and a flow rate to be treated, the problem of determining optimal conditions for a steady-state operation has been studied. In particular, conditions have been proposed to minimize the Total Retention Time (TRT) required to attain a given conversion rate $1 - S_N/S_0$ (here S_0 and S_N denote respectively the input and output substrate concentrations), or equivalently to minimize the total volume of the plant given that the flow rate to be treated is constant.

4.2. Observation and control of wastewater treatment plants

Control problems frequently arise in the context of the study of biological systems such as wastewater treatment plants. In general, in order to cope with disturbances, modeling errors or uncertainty of parameters, one has to take advantage of robust nonlinear control design results. These results are based on central theories of modern non-linear control analysis, like for instance those based on the input-to-state stable (ISS) notion and the back-stepping and the forwarding techniques. Observe that most of these results are based on the construction of families of Control Lyapunov Functions.

Waste water treatments plants are often unstable as soon as bacteria growths exhibit some inhibition. Typically, under a constant feed rate, the wash-out of the reactor (i.e. when biomass is no longer present) becomes an attracting but undesirable equilibrium point. Choosing the dilution rate as the manipulated input is usually a mean for the stabilization about a desired a set point, but the most efficient control laws often require a perfect knowledge of the state variables of the system, namely the online measurement of all the concentrations, which are generally not accessible (for technical or economical reasons). Most often, only a few sensors are available.

A popular way to achieve stabilization of a control dynamical system under partial knowledge of the state is to first design an “observer” or “software sensor” for the reconstruction of the unobserved variables, and then to couple this estimate with a stabilizing feedback control law, if some “separation principle” is satisfied. Unfortunately, in industrial operating conditions, one cannot thoroughly trust the models that were developed and identified in well-controlled environments such as in laboratory experiments. Engineers have to deal with several uncertainties on parts of the model, as well as on the output delivered by the sensors. During the initialization stage or hitches on the process, the system can be far away from the nominal state, where few empirical data are available. Generally, probabilistic hypotheses cannot be justified regarding the nature of the uncertainty for stochastic models to be considered. On the opposite, reasonable bounds on the unknown parts of the models are available, so that uncertainties can be considered as unknown deterministic inputs.

Consequently, robust observers and control laws need to be developed to cope with the particularities of the uncertainty on the models.

4.3. Interpretation of SSCP profiles

The SSCP (Single Strand Conformation Polymorphism) is a molecular analysis technique which allows us to estimate the relative abundance of a given species in a complex ecosystem at a given time t. The result of this analysis is delivered under the form of a graphic in which the x-axis is related to the species while the y-axis gives the relative abundance of the corresponding species. Ideally, under the assumption that two different species do not respond on the same abscissa, a SSCP spectrum would be a succession of rays, the heights of which would correspond to the abundances of these species. From a technical point of view, the method is based on the fact that regions of the DNA remain unchanged during the cellular division, or at least that the variation rate of these regions is very low. Thus, these regions (called 16S) are assumed to be constant and have been designated as being a real signature for a given species. Being able to detect, to mark and to specifically amplify these regions using the PCR (Polymerase Chain Reaction) technique, it has become possible - for about ten years now - to specify "who is there?" in a complex microbial ecosystem. After the PCR, in adequate conditions, the RNA strands are separated on a gel by electrophoresis. The size of the strands and their spatial conformations allow them to be discriminated with respect to the time it takes for each of them to reach a laser detector, the intensity of which depends on the actual quantity of strands (it gives the relative abundance of the corresponding species). Because the detector is not perfect and because the strands of a given species do not move exactly together, the result is not a succession of rays but rather the sum of individual Gaussian-like curves. Furthermore, a real microbial ecosystem may comprise hundreds of species and it is expected that a number of DNA16S strands almost responds at the same time (and thus they are very close in the x-axis of the SSCP spectrum). These facts introduce an important uncertainty about the analysis of such signals.

Until now, the micro-biologists analyze their SSCP profile in the following way. Usually, they empirically identify the visible peaks as being the responses of species which are major quantities in the ecosystem. The height of the peak is used as a measure of its relative abundance into the ecosystem. However, with respect to the uncertainty sources described above, it is straightforward to see that the picture cannot be considered as so simple. For instance, imagine that two species respond on the same x-axis: only one peak (the sum of all DNA16S detected at a given instant by the detector) will be visible on the spectrum and it will be concluded that one species is present at an abundance that is directly related to the magnitude of the peak while, in reality, this peak is due to two (or why not even to a greater number) species. And indeed, our preliminary works show that the results given by these molecular techniques under the form of spectra must be carefully analyzed in order not to draw wrong conclusions. Our objective is to better understand what exactly happens during the analysis in order to identify, characterize and extract the correct information given by this technique.

4.4. Experimentation in ecology

Mathematics and simulations show that substrate dependent models of competition and density-dependent models have radically different predictions in terms of extinction of species. A substrate dependent model is likely to be a reasonably good model for the case of low densities of biomass, the density-dependent model being a good one for high densities. The mathematical treatment on realistic parameters predicts outcomes which are to be tested. In connection with biologists (J.-J. Godon from LBE and R. Arditi from INAPG) we are currently working on this subject.

5. Software

5.1. Software

We produce our own software for the simulations we are currently doing. These softwares are, most of the time, uniquely designed for research purposes. However it might sometimes happen that some software has some wider interest. In that case we try to design it in such a way that it is useful for a larger community.

6. New Results

6.1. Theoretical results

6.1.1. Steady states in a certain class of reaction diffusion-transport equation

Participants: Abdou Khadry Dramé, Claude Lobry, Frédéric Mazenc, Alain Rapaport.

In a previous work (see activity report of 2004) we showed that the asymptotic behavior of the system of two partial differential equations:

$$\begin{array}{l}
\dfrac{\partial S}{\partial t} = -\dfrac{q}{\sigma(z)}\dfrac{\partial S}{\partial z} + d\dfrac{\partial^2 S}{\partial z^2} - \mu(S)X \\
\dfrac{\partial X}{\partial t} = -\dfrac{q}{\sigma(z)}\dfrac{\partial X}{\partial z} + d\dfrac{\partial^2 X}{\partial z^2} + \mu(S)X \\
d\dfrac{\partial S}{\partial z}(t,0) = \dfrac{q}{\sigma(0)}(S(t,0) - S_{in}), \quad d\dfrac{\partial S}{\partial z}(t,l) = 0 \\
d\dfrac{\partial X}{\partial z}(t,0) = \dfrac{q}{\sigma(0)}(X(t,0) - X_{in}), \quad d\dfrac{\partial X}{\partial z}(t,l) = 0
\end{array}$$

(which models a single biomass growing in a bioreactor which is not well-stirred) is the same as the asymptotic behavior of the single equation:

$$\begin{array}{l}
\dfrac{\partial S}{\partial t} = -\dfrac{q}{\sigma(z)}\dfrac{\partial S}{\partial z} + d\dfrac{\partial^2 S}{\partial z^2} - \mu(S)(M - S) \\
\dfrac{q}{\sigma(0)}(S(t,0) - S_{in}), \quad d\dfrac{\partial S}{\partial z}(t,l) = 0
\end{array}$$

The growth rate $\mu(s)$ is supposed to be positive and non-monotonic (increasing, having a maximum, decreasing). Our purpose is to first determine equilibria and next to analyze their stability properties. The two point boundary value problem is solved using asymptotic methods for small and large values of d, which correspond respectively to the plug flow reactor and the perfectly stirred reactor. We prove that, under a mild condition on the growth rate, there exist at most two stable and one unstable solutions. More precisely we have the following : It is known that, under mild conditions, the perfectly stirred reactor has two stable solutions and one unstable. We show that it is the same if d is large (which is not a surprise !) and that, on the opposite side, when d is small (near the perfect plug flow reactor) the stationary solution is unique. For intermediate values of d the analysis is done through computer simulations and the bifurcation diagram is experimentally determined. This work is still in progress.

6.1.2. *Multiple steady state solutions for a cascade of well-stirred reactors with non-monotonic growth-rate*

Participants: Abdou Khadry Dramé, Claude Lobry, Frédéric Mazenc, Alain Rapaport.

We prove that a cascade of well-stirred reactors with a non-monotonic growth-rate may present a complex set of stable equilibria which join together in a unique equilibrium when the number of reactors tends to infinity. The proof is very simple but uses a graphical representation which is not usual. The result is qualitative in the sense that it does not only apply in the case when the growth-rate admits a specific form but also for general non-monotonic growth-rates.

6.1.3. *Interconnection of bioreactors*

Participants: Abdou Khadry Dramé, Claude Lobry, Jérôme Harmand, Alain Rapaport.

Whatever the reason for which it is desirable to optimize a biological process, very few studies related to the design and the control of interconnected biosystems can be found in the literature. Yet, it is particularly important to deal with this problem in the field of WWTP since any treatment process is precisely composed of a number of different tanks connected together by the way of pipes, valves and pumps.

As already mentioned, a cascade of chemostats presents a number of practical interests:

- Non-homogeneous systems (also called tubular or plug flow reactors) may exhibit better performances than homogeneous ones. However, operating a real plug flow system is almost impossible in practice. In such a case, the use of a cascade of homogeneous reactors allows us, in adequate conditions, to approximate the behavior of the plug flow system.
- The presence of a gaseous phase usually increases the homogeneity of the mixed liquid. Again, dividing the reaction scheme into several reactors can be useful to overcome this problem.
- When operating a tubular reactor, a continuous biomass sow is necessary in order to initiate the biological reaction at the entrance of the process. In other terms, it is necessary to have some biomass in the input flow rate. The use of a recirculation can be appropriate. However, again, very few works have studied the influence of a recirculation loop on a biological system.

As a consequence, the study of interconnected biological systems is of particular theoretical as well as of practical importance.

In the project, we have more particularly studied mono-fed and multi-fed cascades of homogeneous bioreactors for which we have analyzed the role of the recirculation on the performances of the system. These studies have led to the proposition of an equivalence principle: conditions have been derived under which there exists an infinity of equivalent systems (of identical volumes) and that only differ by the way they are fed and by the way they are connected together via a recirculation loop. In particular, a graph-based procedure has been proposed to optimize the design of N bioreactors in series [15], [16].

Finally, let us mention that the stability of interconnected nonlinear systems can be guaranteed with the help of small-gain theorems, for which an extension has been proposed in 2004 by members of the team.

6.1.4. Robust observers and robust control

Participants: Jérôme Harmand, Frédéric Mazenc, Alain Rapaport.

Even in the case of perfectly known observable models, the theory of observation of nonlinear systems and synthesis of nonlinear observers presents some technicalities for its application to biological systems (see for instance the contributions of the team in 2004 and the reference [10]).

Nevertheless, our main interest is the study of un-observable models in presence of unknown inputs, that is to say when the collected information facing the uncertainty is not rich enough for a univalued state reconstruction. Since 2003, we have developed a methodolgy for set membership reconstructions [8], based on "intervals observers" (see [13] for a new application in 2005). The spirit of our approach can be summarized in the following way : "guaranteed intervals for the unknown inputs and measures" ⇒ "guaranteed intervals for the state variables". More precisely, for models of the form

$$\dot{x} = f(t, x, u, w(t)), \qquad x(0) = x_0 \tag{1}$$

(where u is the input and $w(t)$ the unknown input) along with observation vectors $y^-(\cdot)$, $y^+(\cdot)$ such that

$$h(t, x(t), u(t), w(t)) \in [y^-(t), y^+(t)], \qquad \forall t \geq t_0,$$

for a given observation model $h(\cdot)$, we design two coupled estimators (instead of one for the usual observable cases) of the form

$$\left\{ \begin{array}{l} \dot{x}^- = f^-(t, x^-, x^+, y^-(t), y^+(t)) \\ \dot{x}^+ = f^+(t, x^-, x^+, y^-(t), y^+(t)) \end{array} \right. \tag{2}$$

such that for any given bounds on unknown $w(\cdot)$ and initial condition x_0, it is possible to initialize system (2) so that the following property is fulfilled

$$x^-(t) \leq x(t) \leq x^+(t), \qquad \forall t \geq t_0, \tag{3}$$

without having to require the convergence of the vectors $x^-(\cdot)$, $x^+(\cdot)$.

We have also investigated the tuning possibilities in the design of interval observers in presence of measurement noise [27]. While estimation (3) remains guaranteed, the performances of the observer, in terms of convergence of $x^+(t) - x_-(t)$ can be adjusted with the help of gains. We face the traditional speed/precision dilemma in this new context of interval estimation.

The control of most of the biological systems poses challenging robust control problems. Indeed, the parameters of the biological models are generally not well-known and delays or unknown dynamics are frequently present. Lyapunov functions are of paramount importance in nonlinear control theory and especially in robust nonlinear control theory: it is very beneficial to have a continuously differentiable Lyapunov function whose derivative along the trajectories of the system can be made negative definite by an appropriate choice of feedback because for example, many proofs of nonlinear disturbance-to-state L^p stability properties rely on Lyapunov functions, control-Lyapunov function (CLF) based control designs guarantee robustness to different types of deterministic and stochastic disturbances, and to unmodeled dynamics. Moreover recent advances in the stabilization of nonlinear delay systems are based on knowledge of continuously differentiable Lyapunov functions. This motivates the problem of constructing strict Lyapunov functions for families of nonlinear systems. We addressed it in several works. In [26] we constructed CLF for systems satisfying conditions of a Jurdjevic-Quinn type. Some of our papers, written before 2005 and [25] are devoted to systems with delay. In particular, we used a Lyapunov function approach to determine bounded control laws which globally asymptotically stabilize linear systems with bounded and delayed inputs and, in [25], the recursive Lyapunov technique called back-stepping (one of the most popular nonlinear techniques of construction of control laws) is adapted to the case where there is an arbitrarily large delay in the input. In [24], robust stability properties are established for a broad family of time-varying systems via the construction of a strict Lyapunov function.

In [35], we explicitly construct strict Lyapunov functions for systems satisfying the stability conditions of the Matrosov theorem. Observe that Lotka-Volterra systems with a globally asymptotically stable equilibrium point usually satisfy these sufficient conditions.

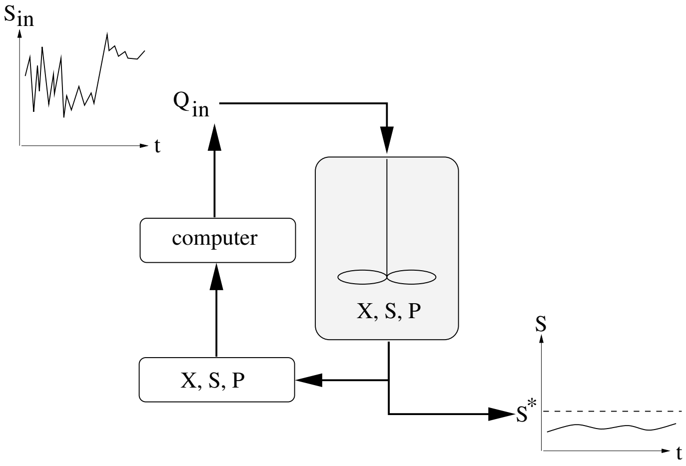

Figure 1. Traditional scheme for piloting bioreactors.

For biological wastewater treatment plants, a standard objective is the regulation of a particular output about a nominal value or a reference trajectory. To achieve this objective, the manipulated variable is traditionally the input dilution rate (see Figure 1). The team has produced several contributions for such control problems in robust frameworks, designing more specifically control laws with the help of intervals observers (see [9] and [14] for a new application in 2005).

In practice, this way of stabilizing bioprocesses implies the necessity of having an upstream storage tank.

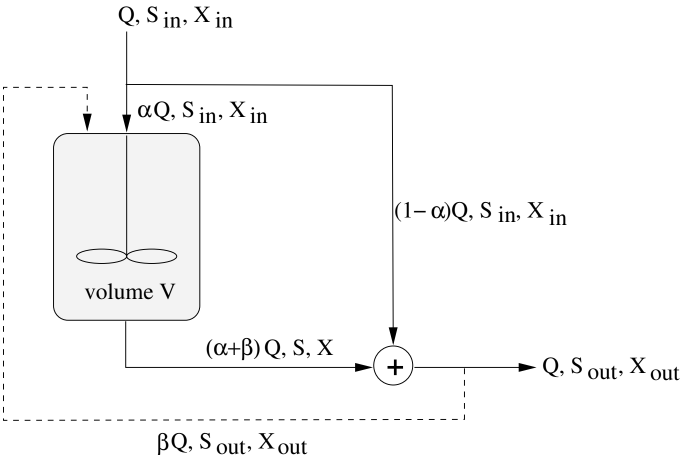

Figure 2. $\alpha \in [0,1]$ *and* $\beta \geq 0$ *are the considered control variables*

We have proposed a new approach, controlling the process by by-pass and recirculation loops instead of the input dilution rate [30], [1]. An upstream storage tank is no longer required and the uncontrolled input rate can be fixed or time-varying (see Figure 2). The control law we have proposed is also robust with respect to unknown disturbances on the input rate as well as uncertainties on the monotonic growth functions. An extension when on-line measurement of the biomass is not available is a work in progress.

6.1.5. Time optimal control of a sequencing batch reactor

Participants: Jérôme Harmand, Claude Lobry, Djalel Mazouni, Alain Rapaport.

From an engineering point of view, biological reactors are classified according to the way they are fed. When treating industrial as well as urban waste-waters, batch processes present a number of advantages with respect to continuous ones. In particular, the reaction rates are usually faster and the separation step, during which the biomass is removed from the effluent to be finally rejected into the environment, is much easier to control than during continuous operation. A batch process operates in a sequential mode (this is why they are called Sequencing Batch Reactors or SBR): the water to be treated is first introduced into a closed tank. Then, the reaction takes place (the biomass degrades the substrates), the biomass settles and the supernatant (clean water) is finally discharged from the process before another cycle begins.

In order to improve the functioning of these processes, we have studied a minimal time optimal problem for a SBR treating both the organic carbon and nitrogen. To do so, two different operating conditions are needed: one aerated period (also called the "aerobic phase") and one without aeration (also called the "anoxic phase"). Depending on the initial concentrations of the different components (biomasses and substrates), the objective is to find the switching instants (from the aerobic phase to the anoxic one or conversely from the anoxic phase to the aerobic one) such that the total reaction time is minimized. Because several components and biological reactions are simultaneously present in the different reaction phases, the problem is far from being solved and a rigorous study has been realized within the framework of the European project EOLI.

In a first step, a reachability study has been realized in 2004: the set of all initial conditions for which there exists a control sequence allowing for the state to reach a final target has been determined. In particular, based on a simple mass balance model of the process under interest [37], it has been established that any initial state in this reachability set can reach the target with a control sequence involving at most two switches (aerobic/anoxic/aerobic) [36]. For practical reasons (minimize the number of switches), a suboptimal time control problem has been defined (determine the switching instants t_1 and t_2 such that any initial state in the reachability set attains the target set in a minimal time using a control sequence (aerobic/anoxic/aerobic)) and solved using the Pontryagin Maximum Principle. In order to complete the picture (apply this control on-line) several observers have been designed and tested with real experimental data during the year 2004. This new control algorithm is now to be implemented on a real 200 liters pilot plant available at the INRA-LBE, Narbonne [30].

6.1.6. Analysis of an SSCP profile

Participants: Jérôme Harmand, Patrice Loisel.

Because of the complexity and the reduction of information due to the analysis, we can at most hope to identify the majority species (species representing a significant percentage). By construction, a majority species generates a peak, but to any peak we cannot systematically associate a majority species. Indeed, a preliminary simulated study showed that for a mixture made up of a very high number of minority species (as it is the case in nature, typically 5000 species with 0.01%) and without any majority species, the SSCP analysis provides a curve with peaks of significant amplitude.

The required goal being to determine the majority species, it is necessary to find a criterion to trust the information given by a peak. A first criterion is the width of the peak (standard deviation for Gaussians). This idea comes from the following observations:

- the variance of the peak of an isolated species does not depend on the species but on specific materials used to make analysis SSCP.

- if, in a mixture of n minority species (with n large), one adds some majority species, one notes that the variance of the peak corresponding to the majority species is of the same order as the variance of an isolated species. On the other hand, for the peaks resulting from contributions of the minority ones, the variance is doubled (or more) with respect to the variance of an isolated species.

The details of this work are reported in [23].

6.1.7. *Models of competition for one resource*

Participants: Claude Lobry, Frédéric Mazenc, Alain Rapaport.

We consider the system:

$$\begin{aligned} \dot{S} &= f(S) - \sum_{i=1}^{n} h_i(S,X_1,\cdots,X_n)X_i \\ \dot{X}_i &= (h_i(S,X_1,\cdots,X_n) - D_i)\,X_i \qquad (i=1,\cdots,n) \end{aligned}$$

under the general following assumptions :

- $f(S)$ is continuous positive between zero and a certain value K and is negative after.
- $\frac{\partial h_i}{\partial s} \geq 0$
- $\frac{\partial h_i}{\partial x_j} \leq 0$,

This systems represents a general competition model for one resource including density dependence of the growth rates h_i. In this completely general form the system is too difficult to be understood, at least for the time being. Thus we have investigated simpler particular cases which have some ecological interest and obtained new results, which is a bit surprising for such an old subject.

- Assume that, for j different from i, we have

  $$\frac{\partial h_i}{\partial x_j} = 0$$

  (which means that each h_i depends only on S and x_i) and that each h_i tends to zero when x_i tends to infinity. Then every equilibrium such that $f'(S_e)$ is strictly negative is locally stable. Under not very restrictive conditions, many species, or even all the species, are present in this stable equilibrium [19].
- Assume moreover that f is decreasing. The equilibrium is unique. Then, under technical assumptions which still have an ecological meaning, this equilibrium is globally asymptotically stable. If f is not decreasing, we can exhibit systems with a locally stable limit cycle (which was already known for competition of two species). This is a work still in progress [33].
- Assume that $f(S) = D(S_{in} - S)$ and $d_i = D$, which is the case of the chemostat, then the unique equilibrium is globally asymptotically stable [18], [29].
- To some extend, the preceding results are valid for the general case when $\frac{\partial h_i}{\partial x_i}$ is large compared to $\frac{\partial h_i}{\partial x_j}$, which means that the intra-species competition is large with respect to the inter-species competition.

Observe that, when each h_i depends only on S, it is well-known that the “competitive exclusion principle” holds. Thus our results can be considered as a contribution to the understanding of the role of the intra-species competition in the persistence of species.

6.2. Applications

6.2.1. Monitoring and estimation techniques for the SBRs

Participants: Jérôme Harmand, Djalel Mazouni, Alain Rapaport.

One of the most important problems when dealing with biological processes is the quasi-systematic lack of sensors able - at a reasonable cost - to deliver on-line information about the composition of the matter to be processed. This is why the development of "software sensors" (observation techniques) are of particular interest in this field of research. When dealing with the optimal time control of SBRs, it was assumed that the entire state of the system was measured on-line. In reality, only parts of the state can be measured and an important experimental work has been realized

1. to generate data that are rich enough to be used for the modeling of a 200 liters SBR pilot plant,
2. to design estimators that use the available on-line information to reconstruct unmeasured state variables,
3. to validate these sensors that will be used for the time-optimal control of the SBR.

6.2.2. Physical bases of density-dependence in the chemostat

Participants: Jérôme Harmand, Claude Lobry.

In cooperation with three biologists (R. Arditi (INAPG), J.-J. Godon (LBE), A. Sciandra (Observatoire Océanographique Villefranche sur Mer)), we established that the density-dependence used in 6.1.7 can be explained by purely abiotic causes [17]. Namely we are able to show that the diffusion of the substrate, associated to the fact that populations are increasing through the division of cells, is a possible explanation of a more intense intra-specific competition than the inter-specific one.

This addresses the question whether, in the real world, chemical communications between individuals (quorum sensing) occur in this context or not. We are trying to start experimental works on this subject.

6.2.3. Software design

Participants: Jérôme Harmand, Patrice Loisel.

A software for the analysis of SSCP profiles is currently designed (see above).

7. Other Grants and Activities

7.1. International

The MERE project-team is very actively involved in cooperation with Africa in two different but related ways.

- C. Lobry, as a former director of CIMPA, has been involved for a long time in cooperation with African mathematical teams. He visits Africa very often and delivers lectures in summer schools or universities.
- The team has a close relationship with the LANI (Laboratoire d'Analyse Numérique et Informatique de l'Université Gaston Berger de Saint-Louis du Sénégal). The LANI is a team of about ten permanent researchers working on partial differential equations applied to problems of development in Sahel and mainly to questions related to water. The LANI is involved in a regional network devoted to the same problems and located in Ouagadougou.
- The team has a close relationship with the Department of Mathematics and the Department of Mechanical Engineering of the Louisiana State University. F. Mazenc is member of the Project “Research in Nonlinear Control Systems Theory: Lyapunov Functions, Stabilization, and Engineering Applications” which has obtained the NSF Standard Grant 0424011.

The MERE project-team interacts also with researchers of CMM (Centro de Modelamiento Matematico, Santiago de Chile) on natural resources modeling problems. The copper industry in Chile produces contaminated water, that can be depolluted by well-selected bacteria.

7.2. European initiatives

7.2.1. *The EOLI project, European contract number ICA4-CT-2002-10012*

EOLI (an acronym for "Efficient Operation of [Low Investment] cost wastewater treatments plants") is an INCO (for International Cooperation) European project (2003-2005) involving eight European/Latin-American partners, the LBE being the INRA partner. It is dedicated to the optimization of a class of WasteWater Treatment Processes (WWTP) through the use of control engineering science tools. The objective of the EOLI project is to design a low-cost, modular and reliable monitoring and control system for wastewater treatment processes dedicated to the treatment of wastewater from urban settlements, especially from those urbanized areas where industries heavily contribute to water pollution. The aerobic treatment of domestic and industrial wastewater by activated sludge is a common process, but the characteristics of many industrial discharges often cause operational problems in continuous flow systems. Therefore, discontinuous processes, as sequencing batch reactors (SBRs), are considered in this project because, in terms of investment and operation costs, process stability, and operation reliability, they are better than the conventional continuous activated sludge process. The monitoring and control system, relying upon information technologies and automatic control, is designed to provide a tool for reliable and efficient operation of SBRs. It is expected to provide a framework for the ISO14001 certification and allows small and medium units (mainly SMEs, not only in Latin America but also in Europe, that cannot afford high investment processes for wastewater treatment) to meet the pollution constraints (like the European Directives 91/271/EEC and 98/15/EEC). The EOLI project is aiming at developing a general, yet adaptable, supervision and monitoring system for the types of wastewater described above. The framework of the EOLI project will integrate the data collected by the sensors, detect fault or abnormal working conditions, and activate model based controllers to optimize the technology and operation of SBRs. The efficient, low-cost and safe operation of the SBRs is expected to be achieved by the integration of high performance controllers, affordable hardware and software sensors and a supervision system to guarantee the reliability of the process operation. In practice, Djalel Mazouni (a PhD student of the LBE granted by EOLI) has taken benefit from the MERE expertise for the development of a time optimal control law of a 200 liters SBR pilot plant available at the LBE.

For further information, see http://www.auto.ucl.ac.be/EOLI/index.html.

8. Dissemination

8.1. Teaching

J. Harmand, C. Lobry and A. Rapaport have organized and participated to two "schools for researchers" (from May the 31st to June the 2nd for the first session and from September the 20st to the 23rd) at the INRA Center of Gruissan about the use of modeling for the understanding of microbial ecology. These two schools are the starting point of the creation of a formal network about microbial ecology at INRA.
Web site of the school: http://www.montpellier.inra.fr/DRN/ECOLE/

C. Lobry delivered courses in Operational Research (16h), mathematical models of bioreactors (20h) and dynamical population models (20h) at Nice University, Institut National d'Agronomie (Paris) and University of Ouagadougou (Burkina Fasso).

A. Rapaport regularly gives lectures on calculus of variation for Master-1 degree, and on control theory for Master-2 “research” degree at Université Montpellier II.

A. Rapaport is in charge of a two-weeks lectures “Mathématiques pour la gestion de ressources renouvelables” at Ecole Nationale Supérieure d'Agronomie de Montpellier. J. Harmand and A. Rapaport deliver each year several lectures and training periods.
Web site: http://www.montpellier.inra.fr/DRN/COURS/

A. Rapaport has organized with the CMM (Centro de Modelamiento Matematico, Santiago de Chile) a one week lecture series at the University of Conception (Chile) on the modeling of natural resources.
Web site: http://www.ing-mat.udec.cl/renewres05

8.2. PhD defended in 2005

Dramé's PhD was supported by a grant from AUF. Until September 2005, he was part time in the MERE project at Montpellier, part time at the University of Saint Louis du Sénégal. The thesis is devoted to the study of the question of existence of multi equilibria in a non-perfectly stirred reactor when the kinetic is non monotonous. He has published three papers in journals (in 2003, 2004 and 2005 [12]) and passed his PhD in September 2005 at Université Montpellier II [11] (committee : H. Attouch, J.-M. Coron (referee), D. Dochain (referee), C. Lobry (co-director), M.-T. Niane (co-director), L. Thibault (president)).

8.3. Ongoing thesis

- Djalel Mazouni is preparing his PhD at the LBE. He should defend his PhD at the beginning of 2006. He is granted by the LBE within the framework of the European EOLI project dedicated to the modeling and the optimization, through the development of control and supervision algorithms, of aerobic SBRs able to treat a large class of wastewater. During these last three years, his work was more particularly devoted to the generation of experimental data (he has started up and has been operating a 200 liters SBR pilot plant since the beginning of the project in 2002), the design of estimation techniques and the development and the validation of a number of innovative time-optimal control laws able to minimize the reaction period lengths. He is the author of four international conference papers (two in 2004 and two in 2005 [36], [37]) and he has submitted one paper in an international journal.
- Maxime Dumont began his PhD Thesis on November the first of 2005. His thesis aims at confronting theoretical ecological concepts to experimental data. His work will consist in proposing a number of experiments with model ecosystems commonly used in biological wastewater plants, such as the nitrification process, in order to validate, or to invalidate the fact that mutualism among individuals and species promotes diversity. The first experiments should be run before the end of the year 2005 at the LBE-INRA in Narbonne.

8.4. Participation to thesis committees

C. Lobry (referee): T. Zerizer "Perturbations de systèmes discrets et applications", Université de Besançon.
A. Rapaport (referee): V. Andrieu "Bouclage de sortie et observateur", Ecole des Mines de Paris (director: L. Praly).

8.5. Conferences, Invited conferences

J. Harmand and C. Lobry have been invited to deliver a conference entitled "Elaboration et justification d'un modèle de compétition de plusieurs espèces pour une ressource" in a workshop organized by the French Academy of Agriculture, Paris, May 24, 2005.

8.5.1. Santiago de Chile

In connection with the lectures given in Chile (see above), A. Rapaport has been invited by the CMM (Centro de Modelamiento Matematico, Santiago de Chile) to deliver a seminar on intra- and inter-specific competition models in the chemostat.

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