INRIA

Project-Team OMEGA

Méthodes numériques probabilistes pour les équations aux dérivées partielles et les mathématiques financières

Sophia Antipolis - Lorraine

THEME NUM

Activity Report

2005

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1. Team

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2. Overall Objectives

2.1. Introduction

The Inria Research team OMEGA is located both at Inria Sophia-Antipolis and Inria Lorraine. The

team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, neurobiology, chemical kinetics.

Our competences cover the mathematics behind stochastic modelling and stochastic numerical methods. We also benefit from a wide experimental experience on calibration and simulation techniques for stochastic models, and on the numerical resolution of deterministic equations by probabilistic methods. We pay a special attention to collaborations with engineers, practitioners, physicists, biologists and numerical analysts.

2.2. Probabilistic numerical methods

Concerning the probabilistic resolution of linear and nonlinear partial differential equations, the OMEGA team studies Monte Carlo methods, stochastic particle methods and ergodic methods. For example, we are interested in fluid mechanics equations (Burgers, Navier-Stokes, etc.), in equations of chemical kinetics and in homogenization problems for PDEs with random coefficients.

We develop simulation methods which take into account the boundary conditions. We provide non asymptotic error estimates in order to describe the global numerical error corresponding to each choice of numerical parameters: number of particles, discretization step, integration time, number of simulations, etc. The key argument consists in interpreting the algorithm as a discretized probabilistic representation of the solution of the PDE under consideration. Therefore part of our research consists in constructing probabilistic representations which allow us to derive efficient numerical methods. In addition, we validate our theoretical results by numerical experiments.

2.3. Stochastic models: calibration and simulation

In financial mathematics and in actuarial science, OMEGA is concerned by market modelling and specific Monte Carlo methods. In particular we study calibration questions, financial risks connected with modelling errors, and the dynamical control of such risks. We also develop numerical methods of simulation to compute prices and sensitivities of various financial contracts.

In neurobiology we are concerned by stochastic models which describe the neuronal activity. We also develop a stochastic numerical method which will hopefully be useful to the Odyssée project to make more efficient a part of the inverse problem resolution whose aim is to identify magnetic permittivities around brains owing to electro-encephalographic measurements.

3. Scientific Foundations

3.1. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and, instead, try to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modelling issues: one has to choose appropriate stochastic models, which requires a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The OMEGA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The OMEGA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The OMEGA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

4.1. Application Domains

OMEGA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Fluid Mechanics, Chemical Kinetics, Finance and Biology.

4.1.1. *Fluid Mechanics*

In Fluid Mechanics OMEGA develops probabilistic methods to solve vanishing vorticity problems and to study complex flows at the boundary, in particular their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our expertise concerns

- The convergence analysis of the stochastic particle methods on theoretical test cases. In particular, we explore speed up methods such as variance reduction techniques and time extrapolation schemes.

- The design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluid (the FENE model). A second one concerns the Lagrangian modelling of turbulent flows and its application in combustion for two–phase flows models (joint collaboration with Électricité de France).

- The Monte Carlo methods for the simulation of fluid particles in a fissured (and thus discontinuous) porous media.

4.1.2. Chemical kinetics

An important part of the work of the OMEGA team concerns the coagulation and fragmentation models.

The areas in which coagulation and fragmentation models appear are numerous : polymerization, aerosols, cement and binding agents industry, copper industry (formation of copper particles), behavior of fuel mixtures in engines, formation of stars and planets, population dynamics, etc.

For all these applications we are led to consider kinetic equations using coagulation and fragmentation kernels (a typical example being the kinetics of polymerization reactions). The OMEGA team aims to analyze and to solve numerically these kinetic equations. By using a probabilistic approach we describe the behavior of the clusters in the model and we develop original numerical methods. Our approach allows to intuitively understand the time evolution of the system and to answer to some open questions raised by physicists and chemists. More precisely, we can compute or estimate characteristic reaction times such as the gelification time (at which there exists an infinite sized cluster) the time after which the degree of advancement of a reaction is reached, etc.

4.1.3. Finance

For a long time now OMEGA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modelling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.4. Biology

For a couple of years OMEGA has studied stochastic models in biology, and developed stochastic methods to analyze stochastic resonance effects and to solve inverse problems. For example, we are concerned by the identification of an elliptic operator involved in the calibration of the magnetic permittivity owing to electro-encephalographic measurements. This elliptic operator has a divergence form and a discontinuous coefficient. The discontinuities make difficult the construction of a probabilistic interpretation allowing us to develop an efficient Monte Carlo method for the numerical resolution of the elliptic problem.

5. New Results

5.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Mireille Bossy, Mamadou Cissé, Madalina Deaconu, Pierre Étoré, Jean-François Jabir, Antoine Lejay, Sylvain Maire, Mohamed El Otmani, Bernard Roynette, Denis Talay, Étienne Tanré, Pierre Vallois.

5.1.1. Euler scheme for SDEs with non-Lipschitz diffusion coefficient

In collaboration with K. Berkaoui (University of Warwick) and A. Diop (BBSP Inc) and motivated by the simulation of financial models, M. Bossy has studied the L^p-convergence rate of a symmetrized Euler scheme for stochastic differential equations of the type

$$X_t = x_0 + \int_0^t b(X_s)ds + \sigma \int_0^t X_s^\alpha dW_s.$$

The analysis has used original arguments, such as a stochastic time change, to face the fact that the diffusion coefficient is not globally Lipschitz. When $1/2 < \alpha < 1$ and under appropriate hypotheses on the drift coefficient b, we obtain the classical rate of convergence $\mathcal{O}(\sqrt{\Delta t})$. We prove that the same rate holds also true when $\alpha = 1/2$ and the hypotheses on b are reinforced.

5.1.2. *Monte Carlo methods for simulating diffusion processes and solving PDE related problems*

Keywords: *Monte Carlo methods*, *backward stochastic differential equations*, *discontinuous media*, *divergence form operator*, *random walk on spheres/rectangles*, *skew Brownian motion*.

M. Deaconu and A. Lejay have worked on a variation of the method of the classical random walk on spheres which is a probabilistic numerical method designed to solve elliptic equations. The new method is based upon an efficient way to simulate the first exit time and position from a rectangle of a Brownian motion [16]. It allows one to overcome some of the limitations of the random walk on spheres: in particular, it concerns more general elliptic operators and boundary conditions. It also allows one to develop a variance reduction technique.

P. Étoré, A. Lejay and M. Martinez have continued to develop several approximation methods for stochastic processes related to elliptic or parabolic operators with discontinuous coefficients in dimension one [19], [49]. These methods rely on a precise analysis of the behavior of these processes when they reach discontinuity points, and on properties of the Skew Brownian motion.

A. Lejay, E. Mordecki (Universidad de la Repúblíca Oriental del Uruguay) and S. Torrès (Universidad de Valparaíso) have also developed an algorithm to solve Backward Stochastic Differential Equations with jumps. This class of equations provides probabilistic interpretations to semilinear integro-partial differential equations.

S. Maire and D. Talay[44] have developed a Monte Carlo method to compute the principal eigenvalue of a neutron transport operator. It is based on the combination of the eigenfunction expansion of the Cauchy problem related to the transport operator and its stochastic representation using the Feynman-Kac formula. Both numerical and theoretical aspects are studied. The extension of this method to the Laplace operator in a domain with Dirichlet boundary conditions has been done by A. Lejay and S. Maire in [43].

In collaboration with E. Gobet (ENSIMAG), S. Maire have computed a spectral approximation of the solutions of general linear partial differential equations using an adaptive Monte Carlo method. The variance and the bias due to the simulations of stochastic processes both decrease geometrically with the number of steps, up to the truncation error. A global solution is obtained, which has the same accuracy as deterministic methods [17]. A new algorithm based on the domain decomposition method has been described in [35], which extends our previous algorithm to piecewise approximations.

All these methods are motivated by problems in physics and finance: estimation of the electrical condictivity in the brain (MEG problem), simulation of diffusion in heterogeneous media, computation of effective coeffients for PDEs at small scales, computation of option prices,...

5.1.3. *Approximation and integration*

In collaboration with C. Deluigi (Université de Toulon et du Var), S. Maire has studied quasi-Monte Carlo methods to compute approximations and integrals of multivariate real-valued functions. They have developed a least square method which produces very accurate quadratures and approximations for multidimensional smooth functions up to dimension 10 [20].

5.1.4. *Statistical estimators of diffusion processes*

In collaboration with J. Jacod (Université Paris 6), A. Lejay and D. Talay have continued to construct and study numerical estimators of the Brownian dimension of a stochastic differential system. This statistical problem is non classical at all. However its resolution should in practice prelude non-parametric estimations of the diffusion coefficients of stochastic differential equations.

P. Étoré and A. Lejay have worked with V. Cvetkovic and A. Frampton (Division of Wfater Resources, KTH, Sweden) to estimate the diffusive behavior of a fluid in a fracture networks.

5.1.5. *Interacting particles system in Lagrangian modelling of turbulent flows*

In the statistical approach of turbulent flows, the Eulerian properties of the fluid are all random fields. This means that the velocity $\mathcal{U}(t, x)$, the pressure and others fundamental quantities depend on possible

realizations $\omega \in \Omega$. In the case of the incompressible Navier Stokes equation in $\mathbb{R}^3$, to compute the averaged velocity, one needs to model the equation of the Reynolds stress and a direct modelling is the so-called k-epsilon turbulence model. An alternative approach [54] consists in describing through a stochastic model the Lagrangian properties of the flow whose laws are linked in a certain way to those of the Eulerian fields. Those models are referred to as Langevin models. For example, the Simplified Langevin model [55], which characterizes particle positions $\mathcal{X}_t$ and particle velocities $\mathcal{U}_t$), is defined as

$$
\begin{cases}
d\mathcal{X}_t = & \mathcal{U}_t dt, \\
d\mathcal{U}_t^i = & \left[-\nabla \langle \mathcal{P} \rangle (t, \mathcal{X}_t) - \left(\frac{1}{2} + \frac{3}{4} C_0 \right) \frac{\langle \varepsilon \rangle (t, \mathcal{X}_t)}{k(t, \mathcal{X}_t)} \left(\mathcal{U}_t - \mathbb{E} \left(\mathcal{U}_t \mid \mathcal{X}_t \right) \right) \right] dt \\
& + \sqrt{C_0 \bar{\varepsilon} (t, \mathcal{X}_t)} dW_t,
\end{cases}
\tag{1}
$$

where C_0, $\langle \varepsilon \rangle (t, x)$ and $k(t, x)$ are supposed to be known. The pressure $\langle \mathcal{P} \rangle (t, x)$ needs to be recovered by the Poisson equation

$$
\nabla^2 \langle \mathcal{P} \rangle = - \sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} \mathbb{E} \left(\mathcal{U}^{(i)} \mathcal{U}^{(j)} \mid \mathcal{X}_t = x \right).
$$

This year, M. Bossy and J-F. Jabir have considered this model as a very particular case of McKean-Vlasov equations with singular kernels, and they have constructed and studied simplified models which keep the main features and difficulties of the above dynamics: in these simplified models, $\langle \varepsilon \rangle$ and k are supposed constant and the pressure gradient is removed, but the kernels are singular and, in the case of flows in a bounded domain, the Lagrangian particle position is restricted to live inside the domain by means of a boundary condition on the spatial evolution of the averaged Eulerian fields. M. Bossy and J-F. Jabir aim to study related interacting particles systems and to prove that their limit distribution is the probability law of the process $(\mathcal{X}_t, \mathcal{U}_t)$ by establishing the "conditional propagation of chaos property" for the particle system (see, e.g., [53]), and proving (weak) existence and uniqueness results for a nonlinear Fokker-Planck equation with non classical boundary conditions.

5.1.6. *Stochastic resonance and neuronal systems*

M. Bossy, D. Talay and É. Tanré have continued their work on the modelling of stochastic resonance effects in the neuronal activity. We have to face important technical difficulties due to the huge complexity of the analytical formulae describing the probability densities of particular stopping times: these random times are the bounds of the spike intervals of the electrical potential along the neurons. We are trying to develop accurate approximate formulae which would allow us to quantify the level of random noise which should be added to the internal noise in order to improve the efficiency of the neuronal activity, in the sense that the period of a periodic electrical input signal is better recognized by the neuronal system.

5.1.7. *Coagulation-fragmentation equations*

Keywords: *Chemical Kinetics*, *Smoluchowski equation*.

In 1916 Smoluchowski introduced a mathematical model to describe coagulation phenomena. Since that, the model evolves and variants of it applies in many fields as polymerization, formation of stars and planets, behavior of fuel mixtures in engines, etc.

The Smoluchowski equation describes the evolution of an infinite system of particles which performs coagulation by pairs. Each particle is characterized by its mass and the *rate of coagulation* of two particles depends on their masses. Denote by $n(k, t)$ the density of particles of mass k at time t in a unit volume. The time evolution of $n(k, t)$ writes:

$$
\begin{cases}
\dfrac{d}{dt} n(k, t) = \dfrac{1}{2} \sum_{j=1}^{k-1} K(j, k-j) n(j, t) n(k-j, t) - n(k, t) \sum_{j=1}^{\infty} K(j, k) n(j, t) \\
n(k, 0) = n_0(k), \ k \geq 1,
\end{cases}
\tag{SD}
$$

where K is the coagulation kernel, assumed to be K symmetric and positive.

This problem is not a classical initial value problem for a system of nonlinear ordinary differential equations.

In [52], we constructed a nonlinear process which *represents* the solution of (SD) and also the continuous version of the Smoluchowski coagulation equation.

This methodology was successfully applied to a problem coming from the modelling of industrial crushers. More precisely, a fundamental problem related to the optimization of the crushing process is to estimate the minimum amount of time and energy required to achieve a prescribed degree of crushing. In collaboration with R. Rebolledo (Pontificia Universidad Católica de Chile) within the framework of the INRIA-CONICYT collaboration programme, E. Tanré develops a model which takes into account the geometry of the crusher, motion of steel's balls, etc. By modelling crushing as a pure fragmentation process, we designed an algorithm to compute residence times in crushers. A paper is in preparation.

In the previous work, the model depends on a kernel which describes the rate of fragmentation of a particle and the distribution of the fragments after breakage. M. Deaconu and E. Tanré have recently started a work that aims to model this kernel. The main idea is to consider also the damage factor for a particle. This parameter increases at each impact even though the particle does not break yet.

5.1.8. *Euler scheme for multivalued stochastic differential equation*

In his thesis and in [50] F. Bernardin has studied the strong convergence of a Euler numerical scheme for multivalued stochastic differential equations (MSDE). The weak convergence rate of this scheme is an open question. F. Bernardin, M. Bossy and D. Talay aim to solve it in order to provide precise convergence rate estimates for Monte Carlo methods for multivalued stochastic models used, e.g., in Random Mechanics. To this end, they are establishing the probabilistic representation of partial differential inclusions in terms of MSDEs.

5.1.9. *Long time behavior of McKean-Vlasov equations.*

Keywords: *McKean Vlasov equation*, *nonlinear stochastic differential equation*, *stochastic particle system.*

In her Ph.D. thesis, Angela Ganz studies an algorithm to compute the limit behavior of McKean-Vlasov equations when time goes to infinity. The objective is to study the efficiency of stochastic particle methods to approximate the stationary solutions of nonlinear Fokker-Planck equations. Our motivation comes from Fluid Mechanics, since several models in Fluid Mechanics can be seen as particular cases of McKean-Vlasov equations.

Preliminary results have been obtained for simplified models such as

$$dX_t = b(X_t)dt + h(t,X_t)dt + dW_t,$$

where h is a perturbation term of the drift b and depends of the probability law μ_t of the random variable X_t. The first objective is to show that, under suitable conditions on the functions b and h, the law μ_t tends to a limit μ_∞ when time goes to infinity, and that μ_∞ is the solution of a stationary nonlinear Fokker-Planck equation. The rate of convergence of μ_t to μ_∞ should also be precised. The second objective is to describe the global error of a stochastic particle method for the stationary nonlinear Fokker-Planck equation under consideration. To this end, inspired by [51] and [57], we consider time discretizations of interacting particle systems related to the McKean-Vlasov equation, and aim to get convergence rate estimates with respect to the discretization step and the number of particles with error constants which are uniform in time.

5.2. Financial Mathematics

Participants: Mireille Bossy, Mamadou Cissé, Pierre Patie, Philip Protter, Benoîte de Saporta, Denis Talay, Etienne Tanré.

5.2.1. *Modelling of financial techniques*

In collaboration with Rajna Gibson (Zürich University), Christophette Blanchet and Sylvain Rubenthaler (Université de Nice Sophia-Antipolis), P. Patie, B. de Saporta, D. Talay and É. Tanré elaborate an appropriate

mathematical framework to develop the analysis of the financial performances of financial techniques which are often used by the traders. This research is funded by NCCR FINRISK (Switzerland) and is a part of its project "Conceptual Issues in Financial Risk Management".

In the financial industry, there are three main approaches to investment: the fundamental approach, where strategies are based on fundamental economic principles, the technical analysis approach, where strategies are based on past prices behavior, and the mathematical approach where strategies are based on mathematical models and studies. The main advantage of technical analysis is that it avoids model specification, and thus calibration problems, misspecification risks, etc. On the other hand, technical analysis techniques have limited theoretical justifications, and therefore none can assert that they are risk-less, or even efficient.

Consider an unstable financial economy. It is impossible to specify and calibrate models which can capture all the sources of instability during a long time interval. Thus it is natural to compare the performances obtained by using erroneously calibrated mathematical models and the performances obtained by technical analysis techniques. To our knowledge, this question has not been investigated in the literature.

We deal with the following model for a financial market, in which two assets are traded continuously. The first one is an asset without systematic risk, called *bond* (or bank account) whose price at time t evolves according to equation

$$dS_t^0 = S_t^0 r dt; \quad S_0^0 = 1.$$

The remaining asset is subject to systematic risk; we shall refer to it as a *stock* and model the evolution of its price at time t with the linear stochastic differential equation

$$dS_t = \sigma \, S_t \, dB_t \, + \, \mu(t) \, S_t \, dt; \quad S_0 = s_0. \tag{2}$$

$(B_t)_{t \geq 0}$ is a standard one-dimensional Brownian motion on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The random process $(\mu(t))_{t \geq 0}$ takes values in $\{\mu_1, \mu_2\}$. The time lengthes between two changes are independent exponential random variables. The difficulty arises here because the trader does not exactly observe the times of change.

This year we extended our earlier results ([11], [38]) in several directions.

In [45] we have characterized optimal allocation strategies in terms of the conditional probability of the drift to be μ_1 knowing the past history of the prices of the assets. Here we have supposed that there is no transaction cost. We prove that this conditional probability is the solution of a stochastic differential equation. This equation can be approximated by a classical Euler scheme. We propose another scheme, inspired by the filtering theory, which seems more efficient. Numerical experiments allow us to compare the approximate allocation strategy based upon the filtering procedure with technical analysis methods, in particular when the parameters of our model are misspecified.

We have also followed two other directions.

On one hand, we try to mathematically estimate the financial performances of various technical analysis techniques. We focus on strategies which are intensively used by the technical traders such as the moving average indicator and the so-called Parabolic SAR indicator. It actually is very difficult to get precise theoretical estimates on the wealth of traders using such indicators. To give a flavour of the technical difficulties of the question, let us denote by S_n the daily stock price at date n and by $M_n = \max_{0 \leq i \leq n} S_i$ the maximum of the stock price up to time n. The parabolic SAR indicator, denoted by P_n, is formally defined by

$$
\begin{aligned}
A_n &= \max(A_n - 1 + 0.02 \cdot 1_{\{S_n > S_{n-1}\}}, 0.2), \; A_0 = 0.02, \\
P_n &= P_{n-1} + A_n(S_n - P_{n-1}).
\end{aligned}
$$

The trader's wealth is a complex functional of the process (P_n). We are trying to get precise informations on the probability law of this functional.

On the other hand, we consider the case where transaction costs cannot be neglected. At any time, the trader is allowed to invest all their wealth in the bond or in the stock. The trader's wealth evolves according to equation

$$\frac{dW_t^\pi}{W_t^\pi} = \pi_t \frac{dS_t}{S_t} + (1-\pi_t)\frac{dS_t^0}{S_t^0} - g_{01}\delta(\Delta\pi_t = 1) - g_{10}\delta(\Delta\pi_t = -1),$$

where $\pi_t \in \{0; 1\}$ is the strategy and g_{01} and g_{10} are the buying and selling costs. This leads to a nonclassical stochastic control problem. We have proved that the value function of this control problem is continuous, satisfies a dynamic programming principle, and is the unique viscosity solution of a Hamilton Jacobi Bellman equation. We have started the numerical analysis of this equation.

5.2.2. *Artificial boundary conditions for nonlinear PDEs in Finance*

In many applications, e.g., in Finance, one needs to solve partial differential equations (PDEs) in unbounded domains. It is the case for option pricing problems or optimal portfolio allocation problems. However, numerical resolutions require that the equations are posed in bounded domains. One thus needs to design boundaries and artifical boundary conditions in such a way that the solution to the localized problem is so close as possible to the solution to the original problem in the whole space.

The probabilistic interpretation of variational inequalities in terms of backward stochastic differential equations is a useful tool to construct proper localized problems. M. Cissé continues his Ph.D. thesis in this direction. A part of his research is done jointly with M. El Otmani, a Ph.D. student from the University Cadi Ayyad of Marrakech. The motivation and the applications concern American option pricing. The localization error of the variational inequalities with artificial Dirichlet boundary conditions is controlled in terms of the misspefication of the solution at the boundary. This result is established owing to a probabilistic interpretation: we have studied a reflected backward stochastic differential equation (RBSDE) coupled with reflected forward backward stochastic differential equation, and we have shown that the solution of this RBSDE provides the viscosity solution of the problem with Dirichlet boundary conditions.

We also have considered variational inequalities related to American–Asian options, and worked on the approximation of the boundary of the exercise domain.

J. Huang has just started his Ph.D. thesis by working on similar questions for the Hamilton–Jacobi–Bellman equation in stochastic control.

5.2.3. *Liquidity Risk*

P. Protter (Cornell University) and D. Talay are addressing the following question. We have the possibility of trading in a risky asset (which we refer to as a stock) with both liquidity and transaction costs. We further assume that the stock price follows a diffusion, and that the stock is highly liquid. We limit our trading strategies to those which change our holdings only by jumps (ie, discrete trading strategies), and we begin with \$0 and 0 shares, and we end with a liquidated portfolio (that is, we no longer hold any shares of the stock), on or before a predetermined ending time T. The question then is, given the structure of the liquidity and transaction costs, what is the optimal trading strategy which will maximize our gains? This amounts to maximizing the value of our risk free savings account. This problem can be solved in this context if it is formulated as a non classical problem in stochastic optimal control. We prove existence and uniqueness results for the related Hamiulton–Jacobi–Bellman equation.

5.3. Stochastic analysis and applications

Participants: Mireille Bossy, Antoine Lejay, Sylvain Maire, Ivan Nourdin, Bernard Roynette, Denis Talay, Étienne Tanré, Pierre Vallois.

In this section we present our results on issues which are more abstract than the preceding ones and, at first glance, might appear decorrelated from our applied studies. However most of them are originally motivated by modelling problems, or technical difficulties to overcome in order to analyze in full generality stochastic numerical methods or properties of stochastic models.

5.3.1. *Stochastic partial differential equations*

Keywords: *Stochastic partial differential equations*, *analytical operator*, *fractional noise*, *mild solutions*, *semi-groups*.

In collaboration with P-L. Lions (Collège de France), M. Bossy, S. Maire, D. Talay and É. Tanré study the long time behavior of viscosity solutions of fully nonlinear stochastic partial differential equations. For particular equations and particular initial conditions, the asymptotic law can be fully explicited. Another direction of research concerns numerical methods to approximate these solutions.

5.3.2. *On first range times of linear diffusions*

Keywords: *Bessel bridges*, *Bessel functions*, *Brownian motion*, *Ray-Knight theorem*, *convexity*, *h-transforms*, *infimum*, *supremum*.

In [29] P. Vallois and P. Salminen (Åbo Akademi University, Finland) consider first range times (with randomised range level) of a linear diffusion on $\mathbb{R}$. Inspired by the observation that the exponentially randomised range time has the same law as a similarly randomised first exit time from an interval, they study a large family of non-negative 2-dimensional random variables (X, X') with this property. The defining feature of the family is $F^c(x, y) = F^c(x + y, 0)$, $\forall x \geq 0, y \geq 0$, where $F^c(x, y) := \mathbb{P}[X > x, X' > y]$. They also explain the Markovian structure of the Brownian local time process when stopped at an exponentially randomised first range time. It is shown that squared Bessel processes with drift are serving hereby as a Markovian element.

Following the visit of P. Vallois at Turku last summer, a continuation of this work with P. Salminen has been investigated in studying the joint distribution of the max-increase and the max-increase of a Brownian motion. One motivation to study the max-increase and the max-increase comes from mathematical finance where the maximum-decrease, also called maximum drawdown (MDD), is used to quantify the riskiness of a stock or any other asset. Related measures used hereby are e.g. the recovery time from MDD and the duration of MDD. We are able to answer to the question arised by Gabor Szekely giving the covariance between the max-increase and the max-increase at a fixed time. This follows from the fact that we are able to determine the joint distribution of the max-increase and the max-increase stopped at an independent and exponential time.

5.3.3. *Limiting laws associated with Brownian motion perturbated by normalized weights*

Keywords: *Bessel processes*, *Ray-Knight's theorems*, *Wiener measure*, *down-crossings*, *enlargement of filtration*, *local time*.

B. Roynette and P. Vallois have continued their studies on weighted Wiener measures.

Let $(\mathbb{P}_x)_{x \in \mathbb{R}}$ be the family of Wiener measures defined on the canonical space $(X_t)_{t \geq 0}$, $(\mathcal{F}_t)_{t \geq 0}$. To an $(\mathcal{F}_t)$-adapted, non negative process $(F_t)_{t \geq 0}$ such that $0 < E_x(F_t) < \infty$, for any $t \geq 0$, $x \in \mathbb{R}$, we associate the probability measure $Q_{x,t}^F$ defined on $(\Omega, \mathcal{F}_t)$ as

$$Q_{x,t}^F(\Gamma_t) = \frac{1}{\mathbb{E}_x[F_t]} \, \mathbb{E}_x[1_{\Gamma_t} F_t], \quad \Gamma_t \in \mathcal{F}_t.$$

In a previous study [56] (see also [24]), we have considered $F_t = \exp\{-\frac{1}{2} \int_0^t V(X_s) ds\}$, where $V : \mathbb{R} \longrightarrow \mathbb{R}_+$ is a Borel function.

In [27], [26] we have considered the more general case $F_t = f(X_t, A_t)$, where $f : \mathbb{R} \times \mathbb{R}^d \longrightarrow]0, +\infty[$ is a Borel function, $(A_t \, ; \, t \geq 0)$ is $(\mathcal{F}_t)$-adapted and $\mathbb{R}^d$-valued. We suppose moreover:

$$\left(Y_t \overset{(\text{def})}{=} (X_t, A_t) \, ; \, t \geq 0 \right) \text{ is a } \quad ((P_x)_{x \in \mathbb{R}} \, ; \, (\mathcal{F}_t)_{t \geq 0}) \text{-Markov process.}$$

We have obtained a "meta-theorem", that is, so a general statement that we have then applied it to various situations. Briefly, the main part of the result is as follows. Let $y_0 = (x_0, a_0)$. Suppose that

$$M_s(y_0, f; y) := \lim_{t \to \infty} \frac{\Lambda_{t-s}(f)(y)}{\Lambda_t(f)(y_0)}$$

exists for all $s \geq 0$ and $y = (x, a) \in \mathbb{R} \times \mathbb{R}^d$, and satisfies some growth condition. Then the measure $Q_{x_0,t}^F$ weakly converges to $Q_{x_0}^M$ when t tends to infinity, that is, $\lim_{t \to \infty} Q_{x_0,t}^F(\Gamma_s) = Q_{x_0}^M(\Gamma_s)$ for all $\Gamma_s \in \mathcal{F}_s$ and $s > 0$, where

$$Q_{x_0}^M(\Gamma_s) = \mathbb{E}_{x_0}[1_{\Gamma_s} M_s], \, \Gamma_s \in \mathcal{F}_s \, ; \, s \geq 0.$$

We then have considered the cases where (A_t) is the Brownian running maximum (S_t) or minimum (I_t), or the Brownian local time at 0 (L_t^0), or the pair (S_t, t), or the triple (S_t, I_t, L_t^0), or the down–crossing process (D_t).

In [25] we have studied weighted measures involving long Brownian bridges. We prove in particular that

$$\lim_{t \to \infty} \mathbb{P}_0(\Gamma_s | S_t = y, X_t = a) = Q_0^{y,a}(\Gamma_s)$$

exists for all $\Gamma_s \in \mathcal{F}_s$ and $s \geq 0$.

Finally, we have proven an extension of the well-known theorem of Pitman: under Q_0^f, (X_t, S_t) is a Markov process such that $(2S_t - X_t, t \geq 0)$ is a 3-dimensional Bessel process started at 0. See [26] for a survey of these results.

5.3.4. *Some connections between (sub)critical branching mechanisms and Bernstein functions*

Jointly with J. Bertoin and M. Yor (Université Paris 6), B. Roynette has obtained the description of some connections between two functional spaces: the space of (sub)critical branching mechanisms and the space of Bernstein functions.

6. Contracts and Grants with Industry

6.1. Collaboration with ADEME-Sophia : local modelling for the wind velocity

Keywords: *Navier-Stokes equations*, *downscaling methods*, *particle in mesh models*.

Participants: Frédéric Bernardin, Mireille Bossy, Jean-François Jabir, Antoine Rousseau.

In February, we started a joint collaboration with the Laboratoire de Météorologie Dynamique (Université Paris 6, École Polytechnique, École Normale Supérieure). This collaboration is funded by the French Environment and Energy Management Agency (ADEME) and concerns the modelling and the simulation of local wind energy resources. We collaborate with Éric Peirano (ADEME), Philippe Drobinski and Tamara Salameh (LMD). Antoine Rousseau (who arrived on October 1st) and Frédéric Bernardin (who arrived on September 1st) concentrate their postdoc research on this subject.

This year we worked on the numerical simulations of turbulent flows using Lagrangian stochastic methods in the context of weather prediction. Starting from some existing numerical results provided by the so-called "MM5" numerical code for weather prediction (LMD, Ecole Polytechnique, Palaiseau), we intend to capture the small scale behavior of the fluid in one single cell of the initial mesh. To this aim, we base our study on a model proposed by S.B. Pope for two-phase turbulent flows (for more details, see 5.1.5). The method consists in injecting a large number of particles in one particular cell of MM5 and, by using some Lagrangian stochastic methods, in computing the velocity and pressure of the flow at small scales, up to turbulence.

Although numerical simulations have not been performed yet, the algorithm is now designed, and we are working on precisely defining each step of the global algorithm which combines numerical methods for computational fluid dynamics as well as methods for stochastic PDEs.

6.2. Meteo–Stoch project

Participants: Mireille Bossy, Jean-François Jabir, Denis Talay.

We also collaborate with the Laboratoire de Météorologie Dynamique within the "Meteo-Stoch" project funded by the French Ministry of Research througgh the ACI "Nouvelles Interfaces des Mathématiques" programme. J-F. Jabir's Ph.D. thesis is funded by a fellowship attached to this project. Our aims are:

To model and simulate: we aim to construct seasonal and local models by using stochastic processes, to qualitatively analyze these models (do they fit typical behaviors of atmospheric variables?), to develop calibration and simulation methods, to analyze the convergence rates of these methods, etc.

To price financial products: we aim to study the effects of our stochastic models on various pricing of financial assets and various risk measures for financial institutions submitted to climatic risks.

We have developed a stochastic weather system coupling the local daily temperature and pluviometry. The temperature is modelled as a Markov-switching autoregressive process whose regime shifts depends on amounts of precipitation. The pluviometry is modelled as a finite-valued Markov chain whose states reflects rainfall degrees. Owing to recent works about ergodic properties of Markov-switching processes [58] and more classical results on Markov chains, we constructed, under weak hypotheses, strongly consistent estimators for relevant parameters (such as the value of regime shifts, the transition of pluviometry conditioned by the noise,...), and we have established their rate of convergence. Our numerical results are reported in subsection 6.3 below.

6.3. Collaboration with EDF-Clamart : Stochastic modelling of local meteorologic variables

Participants: Mireille Bossy, Jean-François Jabir, Denis Talay.

At EDF's request, we have done numerical experiments on a particular case of the stochastic weather model described in subsection 6.2, where the Markov switching process models seasonal fluctuations of local temperature. The mean temperature is removed from data statistical average during a specified season. To calibrate the switching model, we used the estimators mentioned above and 11 years of daily observation coming from METEOFRANCE database. The states of pluviometry are defined by the observation of the amounts of precipitation during the season. After this calibration part, we simulated a realisation of a local weather system. Then this data-gathering permitted us to refine our parameters and to study the behavior of long time series. Our first results seems to prove the validity of our theoretical assumptions and seems to capture some specific climate evolution during the season.

7. Other Grants and Activities

7.1. Other Grants and Activities

A. Lejay is the responsible for the sub-project "Monte Carlo Methods for Fissured Media" within the project "Dispersion in Fissured Media" of the "Groupe de Recherche MOMAS" funded by ANDRA, C.E.A, E.D.F. and C.N.R.S.

A. Lejay received an INRIA "Mini–Appels" Grant to collaborate with V. Cvetkovic (KTH, Sweden).

The team Omega participates to the "Groupe de Recherche GRIP" on stochastic interacting particles and to the European Network AMAMEF on Advanced Mathematical Methods for Finance. D. Talay serves as a member of the scientific committees of these two research networks.

8. Dissemination

8.1. Animation of the scientific community

D. Talay serves as an Associate Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

M. Deaconu and D. Talay are permanent reviewers for the *Mathematical Reviews*.

M. Bossy (up to June) and D. Talay (from June) served as members of the board of the French Society of Applied Mathematics (SMAI).

D. Talay chaired the selection committee for junior permanent research positions at Inria Sophia-Antipolis. He also served as a member of a similar committee at Université Bordeaux 1.

D. Talay has participated to the INRIA delegation which visited the Croucher Foundation and the City University in Hong Kong last Spring.

M. Deaconu serves as a member of the scientific committee of the MAS group (Probability and Statistics) within SMAI.

M. Deaconu serves as a member of the COST International Relations Work Group at INRIA.

M. Deaconu serves as a member of the "Comité des Projets" of LORIA and INRIA Lorraine and of the "COMIPERS — Commission Ingénieurs" of INRIA Lorraine.

M. Deaconu serves as a member of the "Conseil du laboratoire" of IECN and of the "Commission des spécialistes" of the Département de Math'ematiques of Université Nancy 1.

A. Lejay serves as a member of the "Commission de Spécialistes" of Université Louis Pasteur in Strasbourg.

A. Lejay serves as a member of the "Commission des moyens informatiques" of INRIA Lorraine.

P. Vallois is the head of the Probability and Statistics group of Institut Élie Cartan.

P. Vallois serves as a member of the council of the UFR STMIA, the "Conseil du laboratoire" and the "commission de spécialistes" of the Mathematics Department of Université Nancy 1.

B. Roynette and P. Vallois have organized the *Journés de Probabiltités* in Nancy from 5 to 9 September 2005.

P. Vallois has reported to the Ministère de l'Éducation Nationale.

8.2. Animation of workshops

B. Bihain — the head of the Laboratoire de Médecine et Thérapie Moléculaire (INSERM, Nancy) — asked to the Probability and Statistics group at Université Nancy 1 to investigate the proteomic profiling of clinical samples. More precisely, the question involves the differential analysis of the expression levels of a large subset of the proteome of a particular type of clinical specimens to identify those proteins whose change in expression levels might be associated with a given disease process. The statistical tools are Data Analysis, support vectors machine. Jointly with S. Tindel (Université Nancy 1) and J.-M. Monnez (Université Nancy 2), OMEGA Nancy organized a workshop on this topics in May. M. Deaconu, S. Mézière (Université Nancy 2) and A. Koudou (Université Nancy 2) participate to the bio-statistics group. O. Collignon has started a Cifre thesis.

D. Talay and E. Pardoux (Université de Provence) have organized two one–day seminars "Probabilités en Provence Alpes Côte d'Azur" which have gathered probabilists and statisticians working in Marseille, Toulon, Sophia Antipolis and Nice.

8.3. Teaching

D. Talay has a part time position of Professor at École Polytechnique. He also teaches probabilistic numerical methods at Université Paris 6 (DEA de Probabilités) and within the FAME Ph.D. program (Switzerland).

M. Bossy gives a 30h course on "Stochastic calculus and financial mathematics" in the Master IMAFA ("Informatique et Mathématiques Appliquées à la Finance et à l'Assurance", Université de Nice Sophia Antipolis), and a 15h course on "Risk management on energetic financial markets" in the Master "Ingénierie et Gestion de l'Energie" (École des Mines de Paris) at Sophia-Antipolis.

E. Tanré gives a 6h course in the Master IMAFA.

P. Vallois gives courses in Mathematical Finance in a Master at Université Nancy 1.

P. Vallois is the responsible for the DEA courses in Mathematics at Université Nancy 1.

M. Deaconu has given a 30h course in Mathematical Finance in "IUP Sciences Financières", Université Nancy 2.

A. Lejay has given a DEA lecture on Stochastic Differential Equations in Applied Mathematics at Université Nancy 1.

8.4. Ph.D. theses

Olivier Bardou defended his Ph.D. thesis entitled *Contrôle dynamique des erreurs de simulation et d'estimation de processus de diffusion* at Université de Nice in March 22, 2005.

8.5. Participation to congress, conferences, invitations,...

F. Bernardin gave a seminar lecture at the Laboratoire de Mécanique et d'Acoustique de Marseille, CNRS, in October 2005.

M. Bossy gave a series of lectures at the GRIP workhop "Numerical simulations of transport phenomena, particles methods" which was held in March in Sophia Antipolis.

P. Étoré and A. Lejay spent 10 days in Stockholm with a grant from INRIA and KTH to work with V. Cvetkovic and A. Frampton.

P. Étoré gave a talk at the conference *SIAM geoscience 2005* in Avignon in June.

A. Lejay spent two weeks and a half in Chile with a grant INRIA-CONICYT to work with R. Rebolledo and S. Torrès.

A. Lejay gave a talk in the conference *Stochastic homogenization* in Marseille in June and gave four seminars in Toulon, Rennes (IRISA), Le Mans and Rennes (Univ. Rennes 1). He also presented a poster at the annual MOMAS conference in Marseille.

S. Maire has given a talk to the *7th IMACS Congress* in July in Paris and a seminar in Nancy.

A. Rousseau gave seminars at Université de Lyon 1 in November 2005, and at Ecole Normale Supérieure de Cachan (CMLA) in December 2005.

D. Talay co-organized the Conference "Autour des PBs de PB" in Pierre Bernhard's honor which was held in March in Sophia Antipolis.

D. Talay has given a Colloquium Seminar at the Illinois Institute of Technology. He also gave a plenary talk at the Conference on Stochastic Control and Numerics which was held in September at the University of Wisconsin–Milwaukee.

D. Talay gave lectures during the Seminar celebrating Paul Malliavin's 80th birthday which was held in September in Nancy, and during the Seminar on the Discretization of Stochastic Differential Equations which was organized in December in Marseille by F. Delarue (Université Paris 7) and E. Pardoux (Université de Provence).

E. Tanré has spent three weeks in Chile within the INRIA-CONICYT collaboration. He has given seminars at the Pontificia Universidad Catòlica de Chile and at the University of Chile.

E. Tanré has spent one week in Berlin and has given a seminar at Humboldt University.

P. Vallois has given lectures at the meetings : *The second Bachelier Colloqium* in Métabief in January 2005, *Fifth Seminar on stochastic analysis. Random fields and applications* in Ascona in May, *Martingale, Potential and Stochastic Analysis* at Gainesville in November, and the third *Stochastic analysis and probability* in Marrakech in December.

8.5.1. Invitations

The seminar *Théorie et applications numériques des processus stochastiques* organized at Sophia-Antipolis by M. Bossy has received the following speakers: Pierre-Louis Lions (Collège de France), Huyên Pham (Université Paris VII), Antoine Rousseau, (Université Paris-Sud), Ellen Saada (Université de Rouen), Sylvie Méléard (Université Paris X, Modal'X), Julien Guyon (CERMICS - ENPC).

The seminar *Mathématiques financières* organized at Sophia-Antipolis by M. Bossy has received the following speakers: Ragnar Norberg, (London School of economics), Aurélien Alfonsi (CERMICS - ENPC).

M. Bossy and C. Blanchet (Université de Nice Sophia Antipolis) jointly organize a monthly one–day seminar on *Financial Mathematics*. This year, the speakers from oustide the area were N. Oudjane (EDF), J. Aquilina (Cambridge), É. Chevalier (Université d'Évry).

The seminar *Probabilités* organized at Nancy by S. Tindel has received the following speakers: J.-B. Gouere (Lyon 1), V. Schmidt (Ulm), B. de Tilière (Paris 5), M. Sanz-Solé (Barcelona), M. Rousset (Toulouse 3), R. Messikh (Eurandom), O. Couronné (Paris 10), J.-F. Marckert (Versailles), G. Trutnau (Bielefeld), A. Asselah (Marseille 1), C. Stricker (Besançon), J. Garnier (Paris 7), J.-R. Pycke (Paris 6), N. Touzi (ENSAE), Z. Brzezniak (Hull), B. Boufoussi (Marrakech), A. Kebaier (Marne-la-Vallée), V. Bally (Marne-la-Vallée), S. Maire (Toulon), T. Huillet (Cergy), F. Bolley (Toulouse 3), M. Weill (ENS Paris), P. Salminen (Åbo Akademi University, Finland), E. Tanré (INRIA Sophia-Antipolis), F. Delarue (Paris 7), R. Marty (Toulouse), B. de Saporta (INRIA Sophia-Antipolis).

Professor P. Salminen visited P. Vallois in Nancy during one week in November.

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