INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# *Project-Team VEGAS*

# *Effective Geometric Algorithms for Visibility and Surfaces*

*Lorraine*

THEME SYM

# Table of contents

# 1. Team

*VEGAS is a research project of LORIA (Lorraine Research Laboratory in Computer Science and Applications), a laboratory shared by INRIA (National Institute for Research in Computer Science and Control), CNRS (National Center for Scientific Research), Université Henri Poincaré Nancy 1, Université Nancy 2, and INPL (National Engineering Institute of Lorraine).*

**Head of project team**
Sylvain Lazard [Research associate (CR) INRIA]

**Vice-head of project team**
Sylvain Petitjean [Research associate (CR) CNRS]

**Administrative assistant**
Isabelle Herlich [Research technician (TR) INRIA]

**Staff member (INRIA)**
Xavier Goaoc [Research associate (CR)]

**Staff members (University)**
Laurent Dupont [Assistant professor, Université Nancy 2]
Hazel Everett [Professor, Université Nancy 2]

**Ph. D. students**
Julien Demouth [MENESR, since Oct. 2005]
Marc Glisse [ENS fellow, allocation couplée since Sept. 2004]
Maria Pentcheva [MENESR, since Oct. 2004]
Linqiao Zhang [INRIA/McGill University, since Jan. 2004]

**Master student**
Kassiana Mesquita Da Costa [March–Aug. 2005]

# 2. Overall Objectives

## 2.1. Overall Objectives

The main scientific objective of the VEGAS research team is to *contribute to the development of an effective geometric computing* dedicated to *non-trivial geometric objects*. Included among its main tasks are the study and development of new algorithms for the manipulation of geometric objects, the experimentation of algorithms, the production of high-quality software, and the application of such algorithms and implementations to research domains that deal with a large amount of geometric data, notably solid modeling and computer graphics.

Computational geometry has traditionally treated linear objects like line segments and polygons in the plane, and point sets and polytopes in three-dimensional space, occasionally (and more recently) venturing into the world of non-linear curves such as circles and ellipses. The methodological experience and the know-how accumulated over the last thirty years have been enormous.

For many applications, particularly in the fields of computer graphics and solid modeling, it is necessary to manipulate more general objects such as curves and surfaces given in either implicit or parametric form. Typically such objects are handled by approximating them by simple objects such as triangles. This approach is extremely important and it has been used in almost all of the usable software existing in industry today. It does, however, have some disadvantages. Using a tessellated form in place of its exact geometry may introduce spurious numerical errors (the famous gap between the wing and the body of the aircraft), not to mention that thousands if not hundreds of thousands of triangles could be needed to adequately represent the object. Moreover, the curved objects that we consider are not necessarily everyday three-dimensional objects, but also abstract mathematical objects that are not linear, that may live in high-dimensional space, and whose geometry we do not control. For example, the set of lines in 3D (at the core of visibility issues) that are tangent to three polyhedra span a piecewise ruled quadratic surface and the lines tangent to a sphere correspond, in projective five-dimensional space, to the intersection of two quadratic hypersurfaces.

*Effectiveness* is a key word of our research project. By requiring our algorithms to be effective, we imply that the algorithms should be *robust*, *efficient*, and *versatile*. By robust we mean algorithms that do not crash on degenerate inputs and always output topologically consistent data. By efficient we mean algorithms that run reasonably quickly on realistic data where performance is ascertained both experimentally and theoretically. Finally, by versatile we mean algorithms that work for classes of objects that are general enough to cover realistic situations and that account for the *exact geometry* of the objects, in particular when they are curved.

# 3. Scientific Foundations

## 3.1. Theory and applications of three-dimensional visibility

**Keywords:** *3D visibility*, *effective geometry*, *robustness*.

The notion of 3D visibility plays a fundamental role in computer graphics. In this field, the determination of objects visible from a given point, the extraction of shadows or of penumbra boundaries are examples of visibility computations. In global illumination methods (e.g. radiosity algorithms), it is necessary to determine, in a very repetitive manner, if two points of a scene are mutually visible. The computations can be excessively expensive. In radiosity, it is not unusual that 50 to 70% of the simulation time is spent answering visibility queries.

Objects that are far apart may have very complicated and unintuitive visual interactions, and because of this, visibility queries are intrinsically global. This partially explains that, until now, researchers have primarily used ad hoc structures, of limited scope, to answer specific queries on-the-fly. Unfortunately, experience has shown that these structures do not scale up. The lack of a well-defined mathematical foundation and the non-exploitation of the intrinsic properties of 3D visibility result in structures that are not usable on models consisting of many hundreds of thousands of primitives, both from the viewpoint of complexity and robustness (geometric degeneracies, aligned surfaces, etc.).

We have chosen a different approach which consists of computing ahead of time (that is, off-line) a 3D global visibility structure for which queries can be answered very efficiently on-the-fly (on line). The 3D visibility complex – essentially a partition of ray space according to visibility – is such a structure, recently introduced in computational geometry and graphics [32], [34]. We approach 3D global visibility problems from two directions: we study, on the one hand, the theoretical foundations and, on the other hand, we work on the practical aspects related to the development of efficient and robust visibility algorithms.

From a theoretical point of view, we study, for example, the problem of computing lines tangent to four among $k$ polytopes. We have shown much better bounds on the number of these tangents than were previously known [2]. These results give a measure of the complexity of the vertices (cells of dimension 0) of the visibility complex of faceted objects, in particular, for triangulated scenes.

From a practical point of view, we have, for example, studied the problem of the complexity for these 3D global visibility structures, considered by many to be prohibitive. The size of these structures in the worst case is $O(n^4)$, where $n$ is the number of objects in the scene. But we have, in fact, shown that when the objects are uniformly distributed, the complexity is linear in the size of the input [4]. This probabilistic result does not prejudice the complexity observed in real scenes where the objects are not uniformly distributed. However, initial empirical studies show that, even for real scenes, the observed complexity is largely inferior to the theoretical worst-case complexity, as our probabilistic result appears to indicate.

We are currently working on translating these positive signs into efficient algorithms. We are studying new algorithms for the construction of the visibility complex, putting the accent on the complexity and the robustness.

## 3.2. Reliable geometric computations on surfaces

**Keywords:** *Effective geometry*, *model conversion*, *modeling*, *robustness*.

Simple algebraic surfaces cover a variety of forms sufficient for representing the majority of objects encountered in the fields of design, architecture and industrial manufacturing. For instance, it has been estimated that 95% of all mechanical pieces can be well modeled by quadric patches (degree 2 surfaces, including planes, spheres, cylinders and cones) and torii [35]. It is important, then, to be able to process these surfaces in a robust and efficient manner, notably in view of their use in realistic rendering.

In comparison with polygonal representations, modeling and visualization of scenes of quadrics pose new problems. We study, in particular, problems related to the visualization and realistic rendering of such models. We work alongside the members of the ALICE team on the development of the method called virtual meshing which allows us to go beyond the real geometry of objects by creating a geometric abstraction better adapted to light calculations [1].

Early in the rendering process, but along with the development of a tool for illuminating curved surfaces, it is important to have a reliable conversion process from volumetric to surface models. Many conventional modelers are based on the assembly – union, intersection, difference – of simple volumes (a paradigm called Constructive Solid Geometry or CSG), typically quadric volumes. On the other hand, illumination by the radiosity method can only be done on surface representations of objects (called BRep for Boundary Representation). It is necessary, therefore, to be able to pass, in a robust manner, from one representation to the other, an operation known as CSG-BRep conversion, in order to profit from the power of the virtual mesh [1]. The idea is to make the geometric information coherent with the topological information that expresses the relations of proximity and inclusion of different elements.

A fundamental step of this conversion is the computation of the intersection of two primitive volumes. We have recently developed and implemented a robust and near-optimal algorithm for the computation of an exact parametric form of the intersection of two quadrics [14], [24], [25], [26], [27]. Our method is based on the projective formalism, techniques of linear algebra and number theory, and new theorems characterizing the rationality of the intersection. This is the first general approach to the intersection of two quadrics that is usable in practice (as opposed to the approach used until now, that of J. Levin [33]).

Lately we have worked on the use of this general algorithm in an application context. We continue to work on the development of a loop for exact CSG-BRep conversion for models in which the basic primitives are quadric volumes. This work calls for the resolution of algebraic systems for which we collaborate with the members of the SPACES and SALSA projects.

# 4. Application Domains

## 4.1. Computer graphics

Our main application domain is photorealistic rendering in computer graphics. We are especially interested in the application of our work to virtual prototyping, which refers to the many steps required for the creation of a realistic virtual representation from a CAD/CAM model.

When designing an automobile, detailed physical mockups of the interior are built to study the design and evaluate human factors and ergonomic issues. These hand-made prototypes are costly, time consuming, and difficult to modify. To shorten the design cycle and improve interactivity and reliability, realistic rendering and immersive virtual reality provide an effective alternative. A virtual prototype can replace a physical mockup for the analysis of such design aspects as visibility of instruments and mirrors, reachability and accessibility, and aesthetics and appeal.

Virtual prototyping encompasses most of our work on effective geometric computing. In particular, our work on 3D visibility should have fruitful applications in this domain. As already explained, meshing objects of the scene along the main discontinuities of the visibility function can have a dramatic impact on the realism of the simulations.

## 4.2. Solid modeling

Solid modeling, i.e., the computer representation and manipulation of 3D shapes, has historically developed somewhat in parallel to computational geometry. Both communities are concerned with geometric algorithms and deal with many of the same issues. But while the computational geometry community has been mathematically inclined and essentially concerned with linear objects, solid modeling has traditionally had closer ties to industry and has been more concerned with curved surfaces.

Clearly, there is considerable potential for interaction between the two fields. Standing somewhere in the middle, our project has a lot to offer. Among the geometric questions related to solid modeling that are of interest to us, let us mention: the description of geometric shapes, the representation of solids, the conversion between different representations, data structures for graphical rendering of models and robustness of geometric computations.

## 4.3. Fast prototyping

We work in collaboration with CIRTES on rapid prototyping. CIRTES, a company based in Saint-Dié-des-Vosges, has designed a technique called Stratoconception® where a prototype of a 3D computer model is constructed by first decomposing the model into layers and then manufacturing separately each layer, typically out of wood of standard thickness (e.g. 1 cm), with a three-axis CNC (Computer Numerical Controls) milling machine. The layers are then assembled together to form the object. The Stratoconception® technique is cheap and allows fast prototyping of large models.

When the model is complex, for example an art sculpture, some parts of the models may be inaccessible to the milling machine. These inaccessible regions are sanded out by hand in a post-processing phase. This phase is very consuming in time and resources. We work on minimizing the amount of work to be done in this last phase by improving the algorithmic techniques for decomposing the model into layers, that is, finding a direction of slicing and a position of the first layer [8].

# 5. Software

## 5.1. QI

**Participants:** Laurent Dupont, Sylvain Lazard, Sylvain Petitjean.

QI stands for “Quadrics Intersection”. QI is the first exact, robust, efficient and usable implementation of an algorithm for parameterizing the intersection of two arbitrary quadrics, given in implicit form, with integer coefficients. This implementation is based on the parameterization method described in [24], [25], [26], [27] and represents the first complete and robust solution to what is perhaps the most basic problem of solid modeling by implicit curved surfaces.

QI computes an exact parameterization of the intersection of two quadrics with integer coefficients of arbitrary size. It correctly identifies, separates and parameterizes all the connected components of the intersection and gives all the relevant topological information. The parameterizations computed are optimal in terms of their defining functions and near-optimal in terms of the size of the extension on which their coefficients are defined. QI can routinely compute parameterizations of quadrics having coefficients with up to 50 digits in less than 100 milliseconds on an average PC.

QI is written in C++ and builds upon the LiDIA computational number theory library [31] bundled with the GMP multi-precision integer arithmetic [30]. Our implementation consists of roughly 18,000 lines of source code. QI has being registered at the Agence pour la Protection des Programmes (APP). It is distributed under the free for non-commercial use INRIA license and will be distributed under the QPL license in the next release. The implementation can also be queried via a web interface [28].

Since its official release in June 2004, QI has been downloaded six times a month on average and it has been included in the geometric library EXACUS developed at the Max-Planck-Institut für Informatik (Saarbrücken, Germany). QI is also used in a broad range of applications; for instance, it is used in photochemistry for studying the interactions between potential energy surfaces, in computer vision for computing the image of conics seen by a catadioptric camera with a paraboloidal mirror, and in mathematics for computing flows of hypersurfaces of revolution based on constant-volume average curvature.

# 6. New Results

## 6.1. 3D visibility

**Participants:** Julien Demouth, Hazel Everett, Marc Glisse, Xavier Goaoc, Sylvain Lazard, Kassiana Mesquita Da Costa, Sylvain Petitjean, Linqiao Zhang.

On the theme of 3D visibility, we follow two main directions of research, one in which the objects of the scenes are polyhedra, the other in which we consider curved objects.

Our recent results on 3D visibility with polyhedral objects have been the following. We completed and submitted an important result on the number of lines tangent to four among $k$ possibly intersecting arbitrary convex polytopes [19], significantly improving the previous known results on the subject. We also published upper and lower bounds on the number of lines (or, in degenerate cases, of connected components of lines) that are tangent to four triangles in 3D [10] or transversal to line segments in 3D [11]. Related to these problems, G. Lauvaux defended his Ph.D. thesis [8] on the minimization and visualization of the shadow volume (the volume of a model that is inaccessible to a three-axis milling machine) in fast prototyping processes.

Concerning curved objects, we worked on the problem of computing effectively visibility graphs of spheres [29].

Finally we also solved two specific problems concerning two-dimensional visibility problems. We solved the dynamic problem of visibility from a moving viewpoint among points [16]. We also proved theoretical bounds and made an experimental assessment of the size of visibility complex of randomly distributed objects in the plane [18].

## 6.2. Robust computations on surfaces

**Participants:** Laurent Dupont, Sylvain Lazard, Maria Pentcheva, Sylvain Petitjean.

On the theme of robust computation on curved objects, we have presented in a conference in 2003 a new algorithm [5] for computing efficiently and exactly a near-optimal parametric form of the intersection of two quadric surfaces. These theoretical results are major in the sense that the output solution is a rational parameterization whenever one exists and the coefficients are algebraic numbers with at most one extra square root. Furthermore, for each geometric type of intersection, the number of square roots in the coefficients is always minimal in the worst case. Since then we have improved our algorithm in various ways. We have also implemented this algorithm in C++ (see Section 5.1) and we showed that our implementation is extremely efficient in practice, on generic, degenerate as well as “real-life” data. These results were published in conference version last year [6].

We completed this year the theoretical work. This major work resulted in the publication of a long (137 pages) four-part INRIA Research Report [24], [25], [26], [27]. We submitted the first three parts, containing our theoretical results, to the *ACM Transactions on Graphics*. The last part, containing experimental results, appeared in journal form [14]. We also maintain a quadric intersection server available at our Quadrics web page [28].

## 6.3. Combinatorial line geometry

**Participants:** Xavier Goaoc, Sylvain Petitjean.

We obtained two new results on the theme of combinatorial line geometry. We improved the existing bound on the number of geometric permutations of disjoint balls with equal radii in any dimension [13] – a geometric permutation is an ordering that can be realized by a line intersecting all the balls. We also improved the known bounds on Helly numbers for the existence of a line intersecting a collection of disjoint balls of equal radii in 3 dimensions [15]. We then generalized this bound to arbitrary dimension, a result we submitted to *Discrete and Computational Geometry*.

# 7. Other Grants and Activities

## 7.1. National initiatives

### 7.1.1. ACI JemSTIC "Effective geometry for realistic visualization of complex scenes"

**Participants:** Laurent Dupont, Hazel Everett, Xavier Goaoc, Sylvain Lazard, Sylvain Petitjean.

This ACI grant is a ministry grant for the period 2003–2006. The main motivation of this grant is to rework the theoretical bases of fundamental techniques of computer graphics and rendering to speed up computations and move towards a better visual and physical realism. The principal objectives are the effectivity of the methods and the robustness of the calculations. Two problems of particular interest are: theoretical issues in 3D visibility [10], [11], [16], [18], [19], [29] and robust geometric computations on low-degree surfaces [6], [14], [24], [25], [26], [27], [28].

### 7.1.2. Cooperation with other INRIA projects

**Participants:** Laurent Dupont, Hazel Everett, Marc Glisse, Xavier Goaoc, Sylvain Lazard, Sylvain Petitjean.

We work in collaboration with several members of the SALSA group (INRIA Rocquencourt – LIP6) on various problems ranging from 3D visibility to robust computation on curved surfaces. Robust geometric computation on curved objects involves dealing with algebraic polynomials and systems, a specialty of the SALSA group. Our algorithm for computing near-optimal parameterizations of the intersection of quadric surfaces has been developed with D. Lazard [24], [25], [26].

We also work in collaboration with the GEOMETRICA and GALAAD groups (INRIA Sophia-Antipolis). We work mainly with O. Devillers on 3D visibility problems [10], [16], [17], [20], [19], [23], and with M. Teillaud and B. Mourrain on the problem of computing arrangments of 3D curved surfaces and, in particular, on arrangments of quadrics (see Section 7.2.2).

Finally, S. Hornus from ARTIS (INRIA Rhône-Alpes) has visited us and participated in our workshops on 3D visibility [16].

## 7.2. International initiatives

### 7.2.1. McGill-VEGAS associated team

**Participants:** Hazel Everett, Marc Glisse, Xavier Goaoc, Sylvain Lazard, Sylvain Petitjean, Linqiao Zhang.

The McGill-VEGAS associated team (INRIA program) is a joint project between our group and the computational geometry laboratory of McGill University (Montréal). This associated team was started in 2002 under the name McGill-ISA. The research theme is 3D visibility and, more generally, computational geometry.

In this context, we organized the 4th Workshop on Geometry Problems in Computer Graphics. This one-week workshop regrouped 16 people from various places including 4 people from VEGAS and 2 from McGill.

S. Whitesides also visited our group for one week and L. Zhang, who started a Ph.D. in January 2004 co-supervised by S. Whitesides and S. Lazard, visited McGill for two weeks last summer.

### 7.2.2. ARC ARCADIA

**Participants:** Laurent Dupont, Sylvain Lazard, Maria Pentcheva, Sylvain Petitjean.

The ARC ARCADIA (INRIA new investigation grant) is a research project over 2005–2006 regrouping researchers from our group, from the GEOMETRICA project at INRIA Sophia-Antipolis, and from the laboratory of geometric and algebraic algorithms at the National University of Athens. The main objective of this cooperation is to contribute to the unfolding of geometric computing dedicated to quadrics having solid mathematical foundations and to validate theoretical advances by robust and efficient implementations.

In the context of this project we have organized a one-week workshop in Nancy regrouping researchers from ARCADIA (INRIA Lorraine and Sophia-Antipolis and Univ. of Athens) and from MPII (Saarbrücken) and Mainz University. We also organized in Nancy a one-week meeting regrouping researchers from ARCADIA and from the University of Illinois at Urbana-Champaign (UIUC). In addition, M. Teillaud visited LORIA for a week and L. Dupont and S. Lazard participated to the Quadric Day organized at Sophia-Antipolis. Other exchanges took place between INRIA Sophia-Antipolis and the University of Athens.

### *7.2.3. CNRS-INRIA-UIUC cooperation*

**Participant:** Sylvain Lazard.

This collaboration was initiated after the three-month visit of J. Erickson, from the University of Illinois at Urbana-Champaign (UIUC), to our project in the fall of 2004. It is a joint project over 2005–2006 regrouping researchers from UIUC, INRIA Sophia-Antipolis and Lorraine, CNRS LIS, and from the ENS Ulm. The focus of this collaboration is optimization problems in computational topology.

### *7.2.4. PAI STAR*

**Participant:** Xavier Goaoc.

STAR is the French-Korean “programme d’actions intégrées”. This two-year program (2005–2006) supports a collaboration between our group and the group of O. Cheong, from the Korea Advanced Institute for Science and Technology (KAIST), in Daejeon in South Korea. The focus of this cooperation is the study of combinatorial properties of lines in 3 dimensions.

### *7.2.5. Cooperation with MPII Saarbrücken and Mainz University*

**Participants:** Laurent Dupont, Sylvain Lazard, Sylvain Petitjean.

We maintain a collaboration with the group of K. Mehlhorn at MPII, Saarbrücken, and the group of E. Schömer at Mainz University on the topic of reliable geometric computation on surfaces and, in particular, on quadrics. M. Hemmer (Mainz Univ.) and L. Dupont (LORIA) visited each other regularly: 12 short visits, 6 in each direction. In addition, we invited M. Hemmer and E. Berberich (MPII) for a one-week workshop we organized in Nancy on arrangements of quadric surfaces.

## 7.3. Visiting scientists

International visitors:

- G. Vegter (Univ. of Groningen, The Netherlands), invited professor, 1 month;
- D. Bremner (Univ. of New Brunswick, Canada), visiting professor, 2 weeks;
- S.-W. Cheng (HKUST, Hong-Kong), visiting professor, 2 weeks;
- S. Whitesides (McGill Univ., Canada), visiting professor, 1 week;
- B. Speckmann (Eindhoven, The Netherlands), visiting professor, 1 week;
- M. Hemmer (Mainz Univ., Germany), visiting student, 1 week and 6 one or two-days visits;
- E. Berberich (MPII Saarbrücken, Germany), visiting student, 1 week;
- E. Tsigaridas (National University of Athens, Greece), visiting student, 1 week;
- G. Tzoumas (National University of Athens, Greece), visiting student, 1 week.

Visitors from France:

- O. Devillers (INRIA Sophia-Antipolis), 2 weeks;
- S. Hornus (INRIA Rhône-Alpes), 1 week;
- M. Teillaud (INRIA Sophia-Antipolis), 1 week;
- J. Ponce (ENS Ulm – UIUC), 1 week;
- T. Papadopoulo (INRIA Sophia-Antipolis), 1 week.

International visits:

- M. Glisse, Freie Univ., Germany, 3 months;
- X. Goaoc, KAIST, South Korea, 2 weeks;
- X. Goaoc, Rider Univ. and Polytechnic Univ., USA, 2 weeks;
- L. Zhang, McGill Univ., Canada, 2 weeks;
- J. Demouth, KAIST, South Korea, 2 weeks.

# 8. Dissemination

## 8.1. Teaching and training of highly qualified personnel

Several members of the group, in particular the professors, assistant professors and Ph.D. students, actively teach at Université Nancy 2 and INPL. Other members of the group also teach in the Master of Computer Science of Nancy; namely H. Everett and S. Lazard offer the module "Computational geometry and graphics".

In addition to the four Ph.D. students currently in the group, we supervised one Master's student (K. Mesquita da Costa, March–August), one first-year ENS-Cachan internship (G. Batog, mid-June–July), and one MIAGE internship (H. Parmentier, July).

## 8.2. Workshop organisation

We organized two one-week international workshops; typically, these workshops bring together 10 to 20 international researchers and students to work together on research problems.

- H. Everett and S. Lazard co-organized with S. Whitesides (McGill University) the 4th Workshop on Geometry Problems in Computer Graphics (Bellairs Research Institute of McGill University, February 5–11).

- S. Petitjean organized the first Arcadia workshop (Nancy, June 27–July 1).

## 8.3. Invited talks

- H. Everett, P. G. Sorenson Distinguished Graduate Lecture, University of Saskatchewan, Sept 8, 2005.

## 8.4. Participation at conferences and workshops

Members of the group participated in the following events:

- H. Everett, M. Glisse, S. Lazard, M. Pentcheva and L. Zhang participated to the "Journées françaises de géométrie algorithmique", Saint-Pierre de Chartreuse, France, January 24–28, 2005.

- H. Everett, M. Glisse, X. Goaoc and S. Lazard participated to the 21st Symposium on Computational Geometry, Pisa, Italy, June 6–8, 2005.

- X. Goaoc participated to the 8th Korean Workshop on computational geometry and geometric networks, JAIST, Kanazawa, Japan, July 31 – August 5, 2005.

- M. Pentcheva participated to the 13th International Symposium on Graph Drawing, Limerick, Ireland, September 12–14, 2005.

- X. Goaoc participated to the 1009th AMS Sectional Meeting, Bard's College, Annandale-on-Hudson, NY, USA, October 8–9, 2005.

- K. Mesquita da Costa, S. Lazard, and M. Pentcheva participated to the "Journées Informatique et Géométrie", Institut Henri Poincaré, Paris, France, October 13–14, 2005.

- X. Goaoc participated to the workshop "Real Algebra, Quadratic Forms and Model Theory ; Algorithms and Applications", Trimester on real geometry at Institut Henri Poincaré, Paris, November 2–9, 2005.

# 9. Bibliography

## Major publications by the team in recent years

[1] L. ALONSO, F. CUNY, S. PETITJEAN, J.-C. PAUL, S. LAZARD, E. WIES. *The Virtual Mesh: A Geometric Abstraction for Efficiently Computing Radiosity*, in "ACM Transactions on Graphics", vol. 20, n$^o$ 3, 2001, p. 169-201.

[2] H. BRÖNNIMANN, O. DEVILLERS, V. DUJMOVIĆ, H. EVERETT, M. GLISSE, X. GOAOC, S. LAZARD, H.-S. NA, S. WHITESIDES. *On the Number of Lines Tangent to Arbitrary Polytopes in* $\mathbb{R}^3$, in "Proc. of 20th Annual Symposium on Computational Geometry (SoCG'04), New-York", ACM, 2004, p. 46–55.

[3] S.-W. CHENG, O. CHEONG, H. EVERETT, R. VAN OOSTRUM. *Hierarchical decompositions and circular ray shooting in simple polygons*, in "Discrete and Computational Geometry", vol. 32, n$^o$ 3, 2004, p. 401–415.

[4] O. DEVILLERS, V. DUJMOVIĆ, H. EVERETT, X. GOAOC, S. LAZARD, H.-S. NA, S. PETITJEAN. *The expected number of 3D visibility events is linear*, in "SIAM Journal on Computing", vol. 32, n$^o$ 6, Jun 2003, p. 1586-1620.

[5] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics*, in "Proc. of the 19th Annual Symposium on Computational Geometry (SoCG'03), San Diego", ACM, 2003, p. 246–255.

[6] S. LAZARD, L. M. PEÑARANDA, S. PETITJEAN. *Intersecting Quadrics: An Efficient and Exact Implementation*, in "Proc. of 20th Annual Symposium on Computational Geometry (SoCG'04), New-York", ACM, 2004, p. 419–428.

[7] S. PETITJEAN. *A Survey of Methods for Recovering Quadrics in Triangle Meshes*, in "ACM Computing Surveys", vol. 34, n$^o$ 2, 2002, p. 211–262.

## Doctoral dissertations and Habilitation theses

[8] G. LAUVAUX. *La réalisation d'œuvres d'art par prototypage rapide avec le procédé de stratoconception*, Ph. D. Thesis, Université de Reims Champagne-Ardennes, France, LORIA, June 2005.

## Articles in refereed journals and book chapters

[9] C. BORCEA, X. GOAOC, S. LAZARD, S. PETITJEAN. *Common Tangents to Spheres in* $\mathbb{R}^3$, in "Discrete and Computational Geometry", 2005.

[10] H. BRÖNNIMANN, O. DEVILLERS, S. LAZARD, F. SOTTILE. *Lines tangent to four triangles in three-dimensional space*, in "Discrete and Computational Geometry", 2005, http://hal.inria.fr/inria-00000598/en/.

[11] H. BRÖNNIMANN, H. EVERETT, S. LAZARD, F. SOTTILE, S. WHITESIDES. *Transversals to line segments in three-dimensional space*, in "Discrete and Computational Geometry", vol. 34, n$^o$ 3, 2005, p. 381 - 390, http://hal.inria.fr/inria-00000384/en/.

[12] H. Brönnimann, M. Glisse. *Octrees with near optimal cost for ray-shooting*, in "Computational Geometry Theory and Applications", 2005.

[13] O. Cheong, X. Goaoc, H.-S. Na. *Geometric permutations of disjoint unit spheres*, in "Computational Geometry: Theory and Applications", vol. 30, 2005, p. 253-270.

[14] S. Lazard, L. Mariano Peñaranda, S. Petitjean. *Intersecting Quadrics: An Efficient and Exact Implementation*, in "Computational Geometry, Theory and Applications", 2005, http://hal.inria.fr/inria-00000380/en/.

## Publications in Conferences and Workshops

[15] O. Cheong, X. Goaoc, A. Holmsen. *Hadwiger and Helly-type theorems for disjoint unit spheres in R3*, in "Proc. of the 21th Annual Symposium on Computational Geometry, Pisa, Italy", ACM Press, 2005, p. 10-15.

[16] O. Devillers, V. Dujmović, H. Everett, S. Hornus, S. Whitesides, S. Wismath. *Maintaining Visibility Information of Planar Point Sets with a Moving Viewpoint*, in "17th Canadian Conference on Computational Geometry- CCCG'2005, Windsor, Canada", 4 pages, see also [22], 2005, http://www.cccg.ca/proceedings/2005/65.pdf.

[17] O. Devillers, H. Everett, S. Lazard, M. Pentcheva, S. Wismath. *Drawing $K_n$ in Three Dimensions with One Bend per Edge*, in "13th International Symposium on Graph Drawing - GD'2005, University of Limerick, Ireland", 6 pages, see also [23], 2005, http://hal.inria.fr/inria-00000374/en/.

[18] H. Everett, S. Lazard, S. Petitjean, L. Zhang. *An Experimental Assessment of the 2D Visibility Complex*, in "17th Canadian Conference on Computational Geometry - CCCG'2005, Windsor, Canada", 4 pages, 2005, http://www.cccg.ca/proceedings/2005/55.pdf.

## Internal Reports

[19] H. Brönnimann, O. Devillers, V. Dujmović, H. Everett, M. Glisse, X. Goaoc, S. Lazard, H.-S. Na, S. Whitesides. *On the Number of Maximal Free Line Segments Tangent to Arbitrary Three-dimensional Convex Polyhedra*, in Proc. of the 20th "Annual Symposium on Computational Geometry" (SoCG'04), 2004, pp. 46–55, submitted to "SIAM Journal on Computing" in 2005 (28 pages), Research Report, n$^o$ 5671, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5671.html.

[20] H. Brönnimann, O. Devillers, S. Lazard, F. Sottile. *Lines tangent to four triangles in three-dimensional space*, in Proc. of the 16th Canadian Conference on Computational Geometry, 2004, pp. 184–187, to appear in "Discrete and Computational Geometry" in 2005 (12 pages), see also [10], Research Report, n$^o$ 5693, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5693.html.

[21] V. Cortier, X. Goaoc, M. Lee, H.-S. Na. *A note on maximally repeated sub-patterns of a point set*, submitted to "Discrete Mathematics" in 2005 (5 pages), Research Report, n$^o$ 5773, INRIA, December 2005, http://www.inria.fr/rrrt/rr-5773.html.

[22] O. Devillers, V. Dujmović, H. Everett, S. Hornus, S. Whitesides, S. Wismath. *Maintaining Visibility Information of Planar Point Sets with a Moving Viewpoint*, submitted to "International Journal of

Computational Geometry and Applications (IJCGA)" in 2005 (11 pages), see also [16], Research Report, n° 5742, INRIA, November 2005, http://www.inria.fr/rrrt/rr-5742.html.

[23] O. DEVILLERS, H. EVERETT, S. LAZARD, M. PENTCHEVA, S. WISMATH. *Drawing $K_n$ in Three Dimensions with One Bend per Edge*, submitted to "Journal of Graph Algorithms and Applications" in 2005 (10 pages), see also [17], Research Report, n° 5708, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5708.html.

[24] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: I. The Generic Algorithm*, in Proc. of the 19th "Annual Symposium on Computational Geometry" (SoCG'03), 2003, pp. 246–255, submitted to "ACM Transactions on Graphics" in 2005 (28 pages), Research Report, n° 5667, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5667.html.

[25] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: II. A Classification of Pencils*, submitted to "ACM Transactions on Graphics" in 2005 (29 pages), Research Report, n° 5668, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5668.html.

[26] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: III. Parameterizing Singular Intersections*, submitted to "ACM Transactions on Graphics" in 2005 (27 pages), Research Report, n° 5669, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5669.html.

[27] S. LAZARD, L. M. PEÑARANDA, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: IV. An Efficient and Exact Implementation*, in Proc. of the 20th "Annual Symposium on Computational Geometry" (SoCG'04), 2004, pp. 419–428, to appear in "Computational Geometry, Theory and Applications", 2005 (26 pages), Research Report, n° 5670, INRIA, September 2005, http://www.inria.fr/rrrt/rr-5670.html.

## Miscellaneous

[28] VEGAS PROJECT. *QI: a C++ package for parameterizing intersections of quadrics*, 2005, http://www.loria.fr/equipes/vegas/quadrics.html, LORIA, INRIA Lorraine, VEGAS project.

[29] K. MESQUITA DA COSTA. *Visibilité 3D : calculs des segments libres tangents à des sphères unitaires*, Master's Thesis, Université Henri Poincaré - Nancy 1, Technical report, LORIA, 2005.

## Bibliography in notes

[30] *GMP: the GNU MP Bignum Library*, http://www.swox.com/gmp, The Free Software Foundation.

[31] *LiDIA: a C++ Library for Computational Number Theory*, http://www.informatik.tu-darmstadt.de/TI/LiDIA, Darmstadt University of Technology.

[32] F. DURAND. *Visibilité tridimensionnelle : étude analytique et applications*, Ph. D. Thesis, Université Joseph Fourier - Grenoble I, 1999.

[33] J. LEVIN. *A parametric algorithm for drawing pictures of solid objects composed of quadric surfaces*, in "Communications of the ACM", vol. 19, n° 10, 1976, p. 555-563.

[34] M. POCCHIOLA, G. VEGTER. *The visibility complex*, in "International Journal of Computational Geometry and Applications", vol. 6, nº 3, 1996, p. 1-30.

[35] A. REQUICHA, H. VOELCKER. *Solid modeling: a historical summary and contemporary assessment*, in "IEEE Computer Graphics and Applications", vol. 2, nº 1, 1982, p. 9-24.