INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Apics

Analysis and Problems of Inverse type in Control and Signal processing

Sophia Antipolis

THEME NUM

Activity Report

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2. Overall Objectives

2.1. Overall Objectives

The Apics Team is a Project Team since January 2005.

The Team develops constructive methods for modeling, identification and control of dynamical systems.

2.1.1. Research Themes

- Function theory and approximation theory in the complex domain, with applications to frequency identification and design of transfer functions, as well as 2-D inverse boundary problems for the Laplace and Beltrami operators. Development of software for filter identification and the synthesis of microwave devices.
- Inverse potential problems in 3-D and analysis of harmonic fields with applications to source detection and electro-encephalography.
- Control and structure analysis of non-linear systems: continuous stabilization, linearization, and near optimal control with applications to orbit transfer of satellites.

2.1.2. International and industrial partners

- Industrial collaborations with Alcatel Alenia Space (Toulouse and Cannes), Temex (Sophia-Antipolis), Thales AS (Paris), CNES (Toulouse), XLim (Limoges).
- Exchanges with UST (Villeneuve d'Asq), University Bordeaux-I (Talence), University Marseille-I (CMI), CWI (the Netherlands), CNR (Italy), SISSA (Italy), the Universities of Illinois (Urbana-Champaign USA), California at San Diego and Santa Barbara (USA), Michigan at East-Lansing (USA), Vanderbilt University (Nashville USA), Texas A&M (College Station USA), ISIB (Padova, Italy), Beer Sheva (Israel), Leeds (UK), Maastricht and Amsterdam (The Netherlands), TU-Wien (Austria), TFH-Berlin (Germany), Kingston (Canada), Szegëd (Hungary), CINVESTAV (Mexico), ENIT (Tunis), VUB (Belgium), KTH (Stockholm).
- The project is involved in a EMS21-RTG NSF program (with Vanderbilt University), in the ACI "Obs-Cerv" (with the Teams Caiman and Odyssée from Inria-Sophia Antipolis, among others), in the ARC "Sila" (with XLim and the SALSA project at INRIA Rocquencourt), in a STIC Convention between INRIA and Tunisian Universities, in an EPSRC Grant with Leeds University (UK), in the ERCIM "Working Group Control and Systems Theory", in the ERNSI and TMR-NCN European networks, and in a Marie-Curie EIF European program.

3. Scientific Foundations

3.1. Identification and deconvolution

Let us first introduce the subject of Identification in some generality.

Modeling is the process of abstracting the behavior of a phenomenon in terms of mathematical equations. It typically serves two purposes: the first one is to describe the phenomenon with minimal complexity for some specific purpose, the second one is to *predict* its outcome. It is used in most applied sciences, be it for design, control or prediction. However, it is seldom considered as an issue *per se* and today it is usually embedded in some global "optimization" loop.

As a general rule, the user devises a model to fit a parameterized form that reflects his own prejudice, his knowledge of the underlying physical system, and the algorithmic effort he is willing to pay. Such a trade-off usually leads to approximate the experimental data by the prediction of the model when subject to the external excitations assumed to cause the phenomenon under study. The ability to solve this approximation problem, which is often non-trivial and ill-posed, impinges on the practical use of a given method.

It is when assessing the predictive power of a model that one is led to *postulate* the existence of a *true* functional correspondence between data and observations, thereby entering the field of *identification* proper. The predictive power of a model can be expressed in various manners all of which attempt at measuring the difference between the “true system” and the observations. The necessity of taking into account the discrepancy between the observed behavior and the computed behavior naturally induces the notion of *noise* as a corrupting agent of the identification process. This way the noise incorporates to the model, and can subsequently be handled either in a deterministic or stochastic fashion. In deterministic mode, the quality of an identification algorithm rests with its robustness to small errors. This leads to the notion of well-posedness in numerical analysis and of stability of motion in mechanics. However, the noise is most often considered to be random, and then the “true” model is estimated by averaging the data. This notion allows one for a simplified description of complex systems whose underlying mechanisms are not precisely known but plausibly antagonistic. Note that, in any case, *some assumptions* on the noise are required in order to justify the approach (it has to be small in the deterministic case, and must satisfy some independence and ergodicity properties in the stochastic case). These assumptions can hardly be checked in practice, so that the satisfaction of the end-user is the final criterion.

Hypothesizing an exact model also results in the possibility of choosing the data in a manner suited for identifying a specific phenomenon. This often interacts in a complex manner with the *local* character of the model with respect to the data (for instance a linear model is only valid in a neighborhood of a point).

Although identification, from a theoretical perspective, has been mostly the realm of the stochastic paradigm for more than twenty-five years, the Apics team rather develops a deterministic approach to 1-dimensional deconvolution (*i.e.* the identification of linear dynamical systems) which is based on approximating the Fourier-Laplace transform in the complex domain. Of course, the deep links stressed by the spectral theorem between time and frequency domains allow one to partly recast such a framework in a stochastic context. However, the present approach translates the problem of identification into an inverse boundary-value problem for the $\overline{\partial}$ equation, namely the reconstruction from (usually partial) boundary data of an analytic function in a prescribed domain of the complex plane. One feature of this point of view is that it extends to other elliptic partial differential equations, and most naturally to the Laplace and complex Beltrami equations. Beyond these primary examples, some known properties of analytic functions used in the approach still need to be suitably generalized, and a fair portion of the team’s research in inverse problems is currently devoted to such issues for the real Beltrami equation in dimension 2 and the Laplace equation in dimension 3 (see section 6.2).

A prototypical example that illustrates the approach is the harmonic identification of dynamical systems which is widely used in the engineering practice. Here, the data are the response of the system to periodic excitations in its band-width. We look for a stable linear model that accounts for these data in the band-width, while no data are available at high frequencies (which can seldom be measured). In most cases, we want the model to be rational of suitable degree, either because this is imposed by the significance of the parameters or because complexity must remain reasonably low. Other structural constraints, arising from the physics of the phenomenon under study, often superimpose on the model. Note that, in this approach, no statistics are used for the errors, which can be due to corrupted measurements and to the limited validity of the linearity assumption.

We distinguish between an identification step (called non-parametric in a certain terminology) associated with an infinite-dimensional model, and an approximation step in which the order is reduced according to specific constraints on the considered system. The first step typically consists, mathematically speaking, in reconstructing a function, analytic in the right half-plane, knowing its pointwise values on a portion of the imaginary axis. In other terms, the problem is to make the principle of analytic continuation effective on the boundary of the analyticity domain. This is a classical ill-posed issue (the inverse Cauchy problem for the Laplace equation) that we embed into a family of well-posed extremal problems, that may be viewed as a Tikhonov-like regularization scheme related to the spectral theory of analytic operators.

The second step is typically a rational or meromorphic approximation procedure in certain classes of analytic functions on a simply connected domain, say the right half-plane in the case of harmonic identification. To make best possible use of the alloted parameters, it is generally important in this second step to compute optimal or nearly optimal approximants. Rational approximation in the complex plane is a classical and difficult problem, for which only few effective methods exist. In relation to system theory, mainly two difficulties arise: the necessity of controlling the poles of the approximants (to ensure the stability of the model), and the need to handle matrix-valued functions when the system has several inputs and outputs. Moreover, for some inverse problems, the behavior of the poles of best approximants to certain functions constructed from the observations becomes an estimator of the singularities to be detected. This point receives much attention within the team's research.

Concerning this second step, it is worth pointing out that the analogs to rational functions in higher dimensions are the gradients of Newtonian potentials of discrete measures. Very little is known at present on the approximation-theoretic properties of such objects, and a recent endeavor of the project is to study them in the prototypical—though somewhat particular—case of a spherical geometry.

We deal with the above steps in more details through the sub-paragraphs to come. For convenience, we explain them on the circle rather than the line, which is the framework for discrete-time rather than continuous-time systems.

3.1.1. Analytic approximation of incomplete boundary data

Keywords: *extremal problems*, *frequency-domain identification*, *meromorphic approximation*.

Participants: Laurent Baratchart, José Grimm, Juliette Leblond, Jean-Paul Marmorat [CMA, École des Mines de Paris], Jonathan Partington [Univ. Leeds], Fabien Seyfert.

The title refers to the construction of a convolution model of infinite-dimension from frequency data in some bandwidth Ω and some reference gauge outside Ω. The class of models consists of stable transfer functions (*i.e.*, analytic in the domain of stability, be it the half-plane, the disk, etc), and also of transfer functions with finitely many poles in the domain of stability *i.e.*, convolution operators corresponding to linear differential or difference equations with finitely many unstable modes. This issue arises in particular for the design and identification of linear dynamical systems, and in some inverse problems for the Laplacian in dimension two.

Since the question under study may occur on the boundary of planar domains with various shapes when it comes to inverse problems, it is common practice to normalize this boundary once and for all, and apply in each particular case a conformal transformation to recover the normalized situation. The normalized contour chosen here is the unit circle. We denote by D the unit disk, by H^p the Hardy space of exponent p (*i.e.* the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions if $p = \infty$), by R_N the set of all rational functions having at most N poles in D, and by $C(X)$ the set of continuous functions on a space X. We are looking for a function in $H^p + R_N$, taking on an arc K of the unit circle values that are close to some experimental data, and satisfying on $T \setminus K$ some gauge constraints, so that a prototypical Problem is:

(P) Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p + R_N$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

In order to impose pointwise constraints in the frequency domain (for instance if the considered models are transfer functions of lossless systems, see section 4.3.2), one may wish to express the gauge constraint on $T \setminus K$ in a more subtle manner, depending on the frequency:

(P') Let $p \geq 1$, $N \geq 0$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $g \in H^p + R_N$ such that $|g - \psi| \leq M$ a.e. on $T \setminus K$ and such that $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Problem (P) is an extension to the meromorphic case, and to incomplete data, of classical analytic extremal problems (obtained by setting $K = T$ and $N = 0$), that generically go under the name *bounded extremal problems*. These have been introduced and intensively studied by the Team, distinguishing the case $p = \infty$ [50] from the cases $1 \leq p < \infty$, among which the case $p = 2$ presents an unexpected link with the Carleman reconstruction formulas [4].

Deeply linked with Problem (P), and meaningful for assessing the validity of the linear approximation in the considered pass-band, is the following completion Problem:

(P'') Let $p \geq 1$, $N \geq 0$, K an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $h \in L^p(T \setminus K)$ such that $\|h - \psi\|_{L^p(T \setminus K)} \leq M$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

A version of this problem where the constraint depends on the frequency is:

(P''') Let $p \geq 1$, $N \geq 0$, K an arc the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $h \in L^p(T \setminus K)$ such that $|h - \psi| \leq M$ a.e. on $T \setminus K$, and such that the distance to $H^p + R_N$ of the concatenated function $f \vee h$ is minimal in $L^p(T)$ under this constraint.

Let us mention that Problem (P'') reduces to Problem (P) that in turn reduces, although implicitly, to an extremal Problem without constraint, (*i.e.*, a Problem of type (P) where $K = T$) that is denoted conventionally by (P_0). In the case where $p = \infty$, Problems (P') and (P''') can viewed as special cases of (P) and (P'') respectively, but if $p < \infty$ the situation is different. One can also choose different exponents p on K and $T \setminus K$ (the Problem is then said to be of mixed type). This comes up naturally when identifying lossless systems for which the constraint $|h| \leq 1$ must hold at each point while the data, whose signal-to-noise ratio is small at the endpoints of the bandwidth, are better approximated in the L^2 sense. It is perhaps non-intuitive that all these problems have in general no solution when no constraint is provided on $T \setminus K$ (that is, if $M = +\infty$). For instance, considering Problem (P''), a function given by its trace on a subset K of positive measure on the unit circle can always be extended in such a manner that it is arbitrarily close, on K, to a function analytic in the disk; however, it goes to infinity in norm on $T \setminus K$ when the approximation error goes to zero, unless we are in the ideal case where the initial data are *exactly* the trace on K of an analytical function. The phenomenon illustrates the ill-posedness of analytic continuation from the boundary of the analyticity domain, which is germane to the well-known instability of the Cauchy problem for the Laplace equation [74].

The solution to (P_0) is classical if $p = \infty$: it is given by the Adamjan-Arov-Krein (in short: AAK) theory. If $p = 2$ and $N = 0$, then (P_0) reduces to an orthogonal projection. AAK theory plays an important role in showing the existence and uniqueness of the solution to (P'') when $p = \infty$, under the assumption that the concatenated function $f \vee \psi$ belongs to $H^\infty + C(T)$, and for the computation of this solution by solving iteratively a spectral problem relative to a family of Hankel operators whose symbols depend implicitly on the data. The robust convergence of this algorithm in separable Hölder-Zygmund classes has been established [49]. In the Hilbertian case $p = 2$, again for $N = 0$, the solution of (P) is obtained by solving a spectral equation, this time for a Toeplitz operator, depending linearly on a parameter λ that plays the role of a Lagrange multiplier and makes the dependence of the solution implicit in M. The ill-posed character of analytic continuation is to the effect that, if the data are not exactly analytic, the approximation error on K tends to 0 if, and only if, the constraint M on $T \setminus K$ goes to infinity [4]. This phenomenon can be quantified in Sobolev or meromorphic classes of functions f, and asymptotic estimates of the behavior of M and of the error respectively can be obtained, based on a constructive diagonalization scheme for Toeplitz operators due to Rosenblum and Rovnyak, that makes the spectral theorem effective [3]. These results indicate that the error decreases much faster, as M increases, if the data have a holomorphic extension to a neighborhood of the unit disk, this being conceptually interesting for discriminating between nearly analytic data and those that are not close to a linear stable model. From the constructive viewpoint, we face the problem of representing functions through expansions that are specifically adapted to the underlying geometry, for instance, rational bases whose poles cluster at the endpoints of K. Research in this direction is in its infancy.

Problem (P') has been recently solved in the case where $p = 2$ (with $\psi = 0$) which encompasses all mixed problems where the exponent on $T \setminus K$ is greater than 2 [54][19]. It turns out that the solution uniquely exists and that the constraint is saturated pointwise, that is $|g| = M$ a.e. on $T \setminus K$, unless f is the trace on K of an H^2-function satisfying the constraint; the latter fact is perhaps counter-intuitive. Although non-smooth, this infinite-dimensional convex problem has a critical point equation and solves a $min - max$ equation where the multiplier is a function on $T \setminus K$. The solution can be expressed in terms of the multiplier through a normalized Cauchy transform of Carleman type. The case when $\psi \neq 0$ is more delicate in that conditions have to be put jointly on f and ψ for a solution to exist. More details on an algorithmic approach and pending questions can be found in section 6.7.

Smoothness issues in Problems (P) and (P') are both delicate and important in practice. In fact, the solution to such problems is bound to be rather irregular at the endpoints of K unless M is adjusted to f; sufficient conditions for smoothness form a topic of current research.

Let us also emphasize that (P) has many analogs, equally interesting, that occur in different contexts connected to conjugate functions. For instance one may consider the following extremal problem, where the constraint on the approximant is expressed in terms of the real and imaginary parts while the criterion takes only its real part into account:

Let $p \geq 1$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$, and $\alpha, \beta, M > 0$; find a function $g \in H^p$ such that $\alpha \|Re(g-\psi)\|_{L^p(T\setminus K)} + \beta \|Im(g-\psi)\|_{L^p(T\setminus K)} \leq M$ and such that $Re(g-f)$ is of minimal norm in $L^p(K)$ under this constraint.

This yields a natural formulation of issues concerning the Dirichlet-Neumann problem for the Laplace operator (see sections 4.2 and 6.2) where the data and the physical prior information bear on the real and imaginary parts of the analytic function to be recovered.

For $p = 2$, existence and uniqueness of a solution have been established in [76], as well as a constructive procedure which, in addition to the Toeplitz operator that characterizes the solution of (P) in the case $p = 2$ and $N = 0$, also involves a Hankel operator (this extends the results of [70]).

In the non-Hilbertian case, where $p \neq 2, \infty$, but still $N = 0$, the solution of (P) can be deduced from that of (P_0) in a manner analogous to the case $p = 2$, though the situation is more involved as regards duality, because one remains in a convex setup (infinite-dimensional of course), for which local optimization methods can be applied.

Up to now, if $p < \infty$ and $N > 0$, no demonstrably convergent solution to Problem (P_0) is available. This is due to the fact that the problem may display several local *minima*. However, a coherent picture has emerged and rather efficient numerical schemes have been devised, although their convergence has only been established for prototypical classes of functions. The essential features of the approach are summarized below.

First of all, the case $p = 2$ and $N > 0$ of Problem (P_0), which is of particular importance, reduces to rational approximation as described in more details in section 3.1.2. Here, the link with classical interpolation theory, orthogonal polynomials, and logarithmic potentials is strong and fruitful. Second, a general AAK theory in L^p has been proposed which is relatively complete for $p \geq 2$ [8]. Although it does not have, for $p \neq \infty$, the computational power of the classical theory, it has better continuity properties and stresses a continuous link between rational approximation in H^2 (see section 3.1.2) and meromorphic approximation in the uniform norm, allowing one to use, in either context, the techniques available from the other. Hence, similar to the case $p = \infty$, the best meromorphic approximation with at most n poles in the disk of a function $f \in L^p(T)$ is obtained from the singular vectors of the Hankel operator of symbol f between the spaces H^s and H^2 with $1/s + 1/p = 1/2$, the error being here again equal to the $(n+1)$-st singular number of the operator. This generalization has a strong topological flavor and relies on the critical points theory of Ljusternik-Schnirelman as well as on the particular geometry of the Blaschke products of given degree. A matrix-valued version is currently being studied along the same lines. A noticeable common feature to all these problems is the following: the critical point equations express non-Hermitian orthogonality of the denominator (*i.e.*, the polynomial whose zeroes are the poles of the approximant) against polynomials of lower degree, for a complex measure that depends however on this denominator (because the problem is non-linear). This allows one to

1. extend the index theorem to the case $2 \leq p \leq \infty$ [44] in order to approach the problem of uniqueness of a local minimum,
2. characterize the asymptotic behavior of the poles of the approximants for functions with connected singularities that are of particular interest for inverse problems (cf. section 3.1.3),
3. study asymptotic errors with classical techniques of potential theory, which yield estimates to be used in item 1.

In connection with the second and third items above, there are two types of asymptotics, namely weak and strong ones. Weak asymptotics begin to be reasonably understood for functions with branched singularities. Strong asymptotics for non Hermitian orthogonality relations have only been obtained recently in some particular cases, see section 6.6.

In light of these results, and despite the fact that many questions remain open, algorithmic progress is expected concerning (P_0) for $N > 0$ and $p \geq 2$ in the forthcoming years. Subsequently, it is conceivable that the transition from (P_0) to (P) would follow the same lines as in the analytic case [86].

The case where $1 \leq p < 2$ remains largely open, especially from the constructive point of view, because if the approximation error can still be interpreted in terms of singular values, the Hankel operator takes an abstract form which does not lead to a functional identification of its singular vectors. This is unfortunate as this range of values for p is quite interesting: for instance the L^1 criterion induces the operator norm $L^\infty \to L^\infty$ in the frequency domain, which is interesting for damping perturbations. It is plausible that appropriate dualities relate the range $p < 2$ to the range $2 < p$, although this has not yet been established.

A valuable endeavor is to extend to higher dimensions (in particular in 3-D) parts of the the above analysis, where harmonic fields replace analytic functions. On the ball or the half-space, it seems that many of the necessary ingredients are available after the development of real Hardy space theory from harmonic analysis [87], with the notable exception of multiplicative techniques which are unfortunately essential to define Hankel operators. Any progress on these multiplicative aspects would yield corresponding progress in harmonic identification and its use in elliptic inverse problems. Some recent research developments within the team aim in this direction (see section 6.2.3). Similarly, generalizing what precedes to the real Beltrami operator in 2-D is a natural issue with potentially important applications (see section 6.2.2). There, the basic characterization and density properties of traces of solutions on the boundary have only recently been established [43].

3.1.2. Scalar rational approximation

Keywords: *critical point*, *orthogonal polynomials*, *rational approximation*.

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Rational approximation is the second step mentioned in section 3.1 and we first consider it in the scalar case, that is, for complex-valued functions (as opposed to matrix-valued ones). The Problem can be stated as:

Let $1 \leq p \leq \infty$, $f \in H^p$ and n an integer; find a rational function without poles in the unit disk, and of degree at most n that is nearest possible to f in H^p.

The most important values of p, as indicated in the introduction, are $p = \infty$ and $p = 2$. In the latter case, the orthogonality between Hardy spaces of the disk and of the complement of the disk (the last one being restricted to functions that vanish at infinity to exclude the constants) makes rational approximation equivalent to meromorphic approximation, *i.e.*, we are back to Problem (P) of section 3.1.1 with $p = 2$ and $K = T$. Although no demonstrably convergent algorithm is known for a single value of p, the former Miaou project (the predecessor of Apics) has designed a steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed in theory, and it is the first procedure satisfying this property. Roughly speaking, it is a gradient algorithm, proceeding recursively with respect to the order n of the approximant, that uses the particular geometry of the problem in order to restrict the search to a compact region of the parameter space [45]. This algorithm can generate local *minima* if several exist, thus allowing one to discriminate between them. If there is no local *maximum*, a property which is satisfied when the degree is large enough, every local *minimum* can be obtained from an initial condition of lower order. It is not proved, however, that the absolute *minimum* can always be obtained using the strategy of the hyperion or RARL2 software (see section 5.2) that consists in choosing the collection of initial points corresponding to critical points of lower degree; note that we do not know of a counter-example either, still assuming that there is no *maximum*, so there is room for a conjecture at this point.

It is only fair to say that the design of a numerically efficient algorithm whose convergence to the best approximant would be proved is the most important problem from a practical perspective. Meantime, the algorithms developed by the team seem rather effective and although their global convergence has not been established. *A contrario*, it is possible to consider an elimination algorithm when the function to approximate is rational, in order to find all critical points, since the problem is algebraic in this case. This method is surely convergent, since it is exhaustive, but one has to compute the roots of an algebraic system with n variables of degree N, where N is the degree of the function to approximate and there can be as many as N^n solutions among which it is necessary to distinguish those that are coefficients of polynomials having all their roots in the unit disk; the latter indeed are the only ones that generate critical points. Despite the increase of computational power, such a procedure is still unfeasible granted that realistic values for n and N are like a ten and a couple of hundreds respectively (see section 4.3.2).

To prove or disprove the convergence of the above-described algorithms, and to check them against practical situations, the team has undergone a long-haul study of the number and nature of critical points, depending on the class of functions to be approximated, in which tools from differential topology and operator theory team up with classical approximation theory. The study of transfer functions of relaxation systems (*i.e.*, Markov functions) was initiated in [56] and more or less completed in [55], as well as the case of e^z (the prototype of an entire function with convex Taylor coefficients) and the case of meromorphic functions (*à la* Montessus de Ballore) [7]. After these studies, a general principle has emerged that links the nature of the critical points in rational approximation to the regularity of the decrease of the interpolation errors with the degree, and a methodology to analyze the uniqueness issue in the case where the function to be approximated is a Cauchy integral on an open arc (roughly speaking these functions cover the case of singularities of dimension one that are sufficiently regular (see section 3.1.3) has been developed. This methodology relies on the localization of the singularities via the analysis of families of non-Hermitian orthogonal polynomials, to obtain strong estimates of the error that allow one to evaluate its relative decay. Note in this context an analog of the Gonchar conjecture, that uniqueness ought to hold at least for infinitely many values of the degree, corresponding to a subsequence generating the *liminf* of the errors. This conjecture actually suggests that uniqueness should be linked to the ratio of the to-be-approximated function and its derivative on the circle. When this ratio is pointwise greater than 1 (*i.e.*, the logarithmic variation is small), it has been recently proved using Morse theory and the Schwartz lemma that uniqueness holds in degree 1 [12]. The generalization to higher dimensions is an exciting open question.

Another uniqueness criterion has been obtained [8] for rational functions, inspired by the spectral techniques of AAK theory. This result is interesting in that it is not asymptotic and does not require pointwise estimates of the error; however, it assumes a rapid decrease of the errors and the current formulation calls for further investigation.

The introduction of a weight in the optimization criterion is another interesting issue induced by the necessity to balance the information one has at various frequencies with the noise. For instance in the stochastic theory, minimum variance identification leads to assign weights to the errors like the inverse of the spectral density of the noise. It should be noted that most approaches to frequency identification in the engineering practice consists in solving a weighted least-square minimization problem where the design of the weight has to be made so as to obtain satisfactory results using a generic "optimization" toolbox. This leads to consider minimizing a criterion of the form:

$$\left\| f - \frac{p_m}{q_n} \right\|_{L^2(d\mu)} \tag{1}$$

where, by definition,

$$\|g\|^2_{L^2(d\mu)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |g(e^{i\theta})|^2 d\mu(\theta),$$

and μ is a positive finite measure on T, p_m is a polynomial of degree less or equal to m and q_n a monic polynomial of degree less or equal to n. Such a problem is well-posed when μ is absolutely continuous with respect to the Lebesgue measure and has invertible derivative in L^∞. For instance when μ is the squared modulus of an invertible analytic function, introducing μ-orthogonal polynomials instead of the Fourier basis makes the situation similar to the non-weighted case, at least if $m \geq n-1$ [77]. The corresponding algorithm was implemented in the hyperion software (see section 5.5). The analysis of the critical points equations in the weighted case gives various counter-examples to unimodality in maximum likelihood identification [78].

It is worth pointing out that *meromorphic* approximation is better behaved (*i.e.*, essentially invariant) with respect to the introduction of a weight (see section 6.5).

Another kind of rational approximation problems arise in *design* problems, that became over years an increasingly significant part of the team's activity (see sections 4.3, 6.4, and 6.9). These are problems where constraints on the *modulus* of a rational function are sought, and they occur mainly in filter design where the response is a rational function of fixed degree (the complexity of the filter), analytic and bounded by 1 in modulus on the right-half-plane (passivity), whose modulus must be as close as possible to 1 on some subset of the imaginary axis (the pass-band) ans as close as possible to 0 on the complementary subset (the stop-band).

When translated over to the circle, a prototypical formulation consists in approximating the modulus of a given function by the modulus of a rational function of degree n, that is, to solve for

$$\min \left\| |f| - \left|\frac{p_n}{q_n}\right| \right\|_{L^p(T)}.$$

When $p = 2$ this problem can be reduced to a series of standard rational approximation problems, but usually one needs to solve it for $p = \infty$. For this, we observe upon squaring the moduli that the feasibility of

$$\left\| |f|^2 - \left|\frac{p_n}{q_n}\right|^2 \right\|_{L^\infty(T)} < \varepsilon,$$

can be analysed using the Féjèr-Riesz characterization of positive trigonometric polynomials on the unit circle as squared moduli of algebraic polynomials. This reduces the issue to a convex problem in infinite-dimension (because the criterion has to be evaluated at infinitely many points on the unit circle) that constitutes a fundamental tool to deal with rational approximation in modulus. Note that the case where $|f|$ is a piecewise constant functions with values 0 and 1 can also be approached *via* classical Zolotarev problems [85], that can be solved more or less explicitly when the pass-band consists of a single arc. A constructive solution in the case of several arcs (multiband filters) is one recent achievement of the team (see section 6.9). Of course, though the modulus of the response is the first concern in filter design, the variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. As a matter of fact, trading-off abrupt changes in modulus for a moderate derivative of the phase, which are antagonistic effects [68], is an exciting but fairly open issue that needs to be investigated more deeply for the design of high order filters.

From the point of view of design, rational approximants are indeed useful only if they can be translated into physical parameter values for the device to be built. While such problems do not pertain to rational approximation proper, they are of utmost importance in practice. Actually, the fact that a device's response is shaped in the frequency domain whereas the device itself must be specified in the time domain is a major difficulty in the area that reflects the fundamental problem of harmonic analysis. This is where System-Theory enters the scene, as the passage from the frequency response (*i.e.* the transfer-function) to the linear differential or difference equations that generate this response (*i.e.* the state-space representation) is the object of the so-called *realization* process. Algebraically speaking, a realization of a rational matrix H of the variable z is a 4-tuple (A, B, C, D) of real or complex matrices of appropriate sizes such that

$$H(z) = C(zI - A)^{-1}B + D.$$

Since filters have to be considered as multipoles, the issue must indeed be tackled in a matrix-valued context that adds to the complexity. A fair share of the team's research in this direction is concerned with finding realizations meeting certain constraints (imposed by the technology in use) for a transfer-function that was obtained with the above-described techniques. The current approach is to solve algebraic equations in many variables using homotopy methods, which seems to be a path-breaking methodology in the area of filter design (see section 6.8).

3.1.3. Behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Keywords: *discretization of potentials*, *free boundary inverse problems*, *meromorphic approximation*, *orthogonal polynomials*, *rational approximation*, *singularity detection*.

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

We refer here to the behavior of the poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle (which is no restriction in principle thanks to conformal mapping but raises of course difficult questions from the constructive point of view), we find ourselves again in the framework of sections 3.1.1 and 3.1.2, and the invariance of the problem under such transformation was established in [15]. The research so far has focused on functions that are analytic on and outside the contour, and have singularities on an open arc inside the contour.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and also to tackle more constructive issues like uniqueness. However, the original motivation of Apics is to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. This approach to free boundary problems, that are classical in every respect but still quite open, illustrates the point of view of the team and gives rise to an active direction of research at the crossroads of function theory, potential theory and orthogonal polynomials.

As a general rule, critical point equations for these problems express that the polynomial whose roots are the poles of the approximant is a non-Hermitian orthogonal polynomial with respect to some complex measure (that depends on the polynomial itself and therefore varies with the degree) on the singular set of the function to be approximated. New results were obtained in recent years concerning the location of such zeroes. The approach to inverse problem for the Laplacian that we outline in this section appears to be attractive when the singularities are one-dimensional, for instance in the case of a cracked domain (see section 4.2). It can be used as computationally cheap preliminary step to obtain the initial guess of a heavier but more precise numerical optimization. It is rather complementary of the recently popularized MUSIC-type algorithms [73] as it can in principle be used on a single stationary pair of Dirichlet-Neumann data.

When the crack is sufficiently smooth, the approach in question is in fact equivalent to the meromorphic approximation of a function with two branch points, and we were able to prove [15] [48] that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the endpoints of the crack [5]. Moreover the asymptotic density of the poles turns out to be the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are *de facto* well localized if one is able to compute sufficiently many zeros (this is where the method could fail). It is interesting to note that these results apply also, and even more easily, to the detection of monopolar and dipolar sources, a case where poles as well as logarithmic singularities exist. The case of more general cracks (for instance formed by a finite union of analytic arcs) requires the analysis of the situation where the number of branch points is finite but arbitrary. We proved very recently that the poles tend to the contour $\mathcal{C}$ outside of which the function is analytic and single-valued that minimizes the capacity of the condenser $(T, \mathcal{C})$, where T is the exterior boundary of the domain (paper in preparation, see section 6.6). For the definition of a condenser and other basic facts from potential theory, see [85].

It would of course be very interesting to know what happens when the crack is "absolutely non analytic", a limiting case that can be interpreted as that of an infinite number of branch points, and on which very little is known, although there are grounds to conjecture that the endpoints at least are still accumulation points of the poles. This is an outstanding open question for applications to inverse problems (see section refres-fissures). Concerning the problem of a general singularity, that may be two dimensional, one can formulate the following conjecture: if f is analytic outside and on the exterior boundary of a domain D and if K is the minimal compact set included in D that minimizes the capacity of the condenser (T, K) under the constraint that f is analytic and single-valued outside K (it exists, it is unique, and we assume it is of positive capacity in order to avoid degenerated cases), then every limit point (in the weak star sense) of the sequence ν_n of probability measures having equal mass at each pole of an optimal meromorphic approximant (with at most n poles) of f in $L^p(T)$ *has its support in* K *and sweeps out on the boundary of* K *to the equilibrium distribution of the condenser* (T, K). This conjecture, which generalizes the above-mentioned results on 1-D singular sets, is far from being solved in general.

Results of this type open new perspectives in non-destructive control (see section 4.2), in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which a dual approach is thereby proposed: to approximate the boundary conditions and not the equation (as is classically done). Note that the problem of finding a crack suggests non-classical variants of rational and meromorphic approximation where the residues of the approximants must satisfy some constraints in order to take into account the boundary conditions, normal or tangential, along the singularity. In fact, the afore-mentioned results dealing with (unconstrained) meromorphic approximation lead to identify a deformation of the crack (the arc of minimal capacity that links its end points) rather than the crack itself, which is valuable to initialize a heavier direct method but which is not conclusive in itself. In order to limit the deformation which is due to the fact that we did not keep track of all the boundary conditions (especially the fact that the jump across the crack is real), one may consider approximating the complexified solution F of a Neumann problem in a cracked domain D by a meromorphic function of the type $\sum_{j=1}^{n} a_j/(z-z_j) + g(z)$, where g is analytic in D, under the constraint that $\sum_{k\neq j} a_k/(z_j - z_k) + g(z_j)$ is real for each j; in effect, if the poles z_j are distributed along an arc, the above sum is a discrete estimation of the Hilbert transform of the measure defining the function, and enforcing that it is zero should help satisfying the Neumann condition along the arc. Such modifications of the initial problem are only beginning to be considered within the team.

We conclude by mentioning that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is then no longer available, and this makes for a long-term research project with a wide range of applications. It is interesting to mention that the case of sources in dimension three in a spherical geometry, can be attacked with the above 2-D techniques as applied to planar sections (see section 6.2).

3.1.4. Matrix-valued rational approximation

Keywords: *inner matrix*, *rational approximation*, *reproducing kernel space realization theory*.

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, José Grimm.

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (*i.e.*, the degree of a minimal realization in the System-Theoretic sense) generalizes the degree. Hence the problem reads: *Let* $1 \leq p \leq \infty$, $\mathcal{F} \in (H^p)^{m\times l}$ *and* n *an integer; find a rational matrix of size* $m \times l$ *without poles in the unit disk and of McMillan degree at most* n *which is nearest possible to* $\mathcal{F}$ *in* $(H^p)^{m\times l}$. To fix ideas, we may define the L^p norm of a matrix as the p-th root of the sum of the p powers of the norms of its entries.

The main interest of the Apics Team so far lies with the case $p = 2$. Then, the approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [9]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.*, matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices using the so-called Douglas-Shapiro-Shields factorization. The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parameterizations valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. The tangential Schur algorithm [38] provides us with such a parameterization and allowed the team to develop two rational approximation codes (see sections 5.2 and 5.5). The first one is integrated in the endymion software dealing with transfer matrices while the other, which is developed under the Matlab interpreter, goes by the name of RARL2 and works with realizations. Both have been experimented on measurements by the CNES (branch of Toulouse), XLim, and Alcatel Alenia Space (Toulouse), on which they gave high quality results [2] in all cases encountered so far. These codes are now of daily use by Alcatel Alenia Space and XLim, coupled with simulation software like EMXD to design physical coupling parameters for the synthesis of microwave filters made of resonant cavities (see section 7.1).

In the above application, obtaining physical couplings requires the computation of realizations, also called internal representation in System Theory. Among the parameterizations obtained via the Schur algorithm, some have a particular interest from this viewpoint [84], [17]. They lead to a simple and robust computation of balanced realizations and form the basis of the RARL2 algorithm (see section 5.2).

Problems relative to multiple local minima naturally arise in the matrix-valued case as well, but deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The already investigated case of rational functions of the seeked degree (the consistency problem) was solved using rather heavy machinery [6], and that of matrix-valued Markov functions, that are the first example beyond rational function has made progress only recently [20].

In practice, a method similar to the one used in the scalar case has been developed to generate local minima of a given order from those at lower orders. In short, one sets out a matrix of degree n by perturbation of a matrix of degree $n - 1$ where the drop in degree is due to a pole-zero cancellation. There is an important difference between polynomial representations of transfer matrices and their realizations: the former lead to an embedding in a ambient space of rational matrices that allows a differentiable extension of the criterion on a neighborhood of the initial manifold, but not the latter (the boundary is strongly singular). Generating initial conditions in a recursive manner is more delicate in terms of realizations, and some basic questions on the boundary behavior of the gradient vector field are still open.

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.2. Structure and control of non-linear systems

In order to control a system, one generally relies on a model, obtained from *a priori* knowledge, like physical laws, or from experimental observations. In many applications, it is enough to deal with a linear approximation around a nominal point or trajectory. However, there are important instances where linear control does not apply, either because the magnitude of the control is limited or because the linear approximation is not controllable. Moreover, certain control problems, such as path planning, are not local in nature and cannot be solved *via* linear approximations.

Section 3.2.1 describes a problem of this nature, where the controllability of the linear approximation is of little help. Besides, the structural study described in section 3.2.2 aims at exhibiting invariants that can be used, either to bring the study back to that of simpler systems or to lay grounds for a non-linear identification theory. The latter would give information on model classes to be used in case there is no *a priori* reliable information and still the black-box linear identification is not satisfactory.

3.2.1. Feedback control and optimal control

Keywords: *control*, *non holonomic mechanical system*, *non-linear control*, *stabilization of non-linear systems*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet.

Stabilization by continuous state feedback (or output feedback which is a partial information case) consists in designing a control law which is a smooth (at least continuous) function of the state making a given point (or trajectory) asymptotically stable for the closed–loop system. One can consider this as a weak version of the optimal control problem which is to find a control that minimizes a given criterion (for instance the time to reach a prescribed state). Optimal control generally leads to a rather irregular dependence on the initial state; in contrast, stabilization is a *qualitative* objective (*i.e.*, to reach a given state asymptotically) which is more flexible and allows one to impose a lot more regularity.

Lyapunov functions are a well-known tool to study the stability of non-controlled dynamic systems. For a control system, a *Control Lyapunov Function* is a Lyapunov function for the closed-loop system where the feedback is chosen appropriately. It can be expressed by a differential inequality called the "Artstein (in)equation [42]", that looks like the Hamilton-Jacobi-Bellmann equation but is largely under-determined. One can easily deduce from the knowledge of a control Lyapunov function a continuous stabilizing feedback.

The team is engaged in obtaining control Lyapunov functions for certain classes of systems. This can be the first step in synthesizing a stabilizing control but, even when such a control is known beforehand, obtaining a control Lyapunov function can still be very useful to study the robustness of the stabilization, or to modify the initial control law into a more robust one. Moreover, if one has to deal with a problem where it is important to optimize a criterion, and if the optimal solution is hard to compute, one can look for a control Lyapunov function which comes "close" (in the sense of the criterion) to the solution of the optimization problem but leads to a control which is easier to work with.

These constructions are exploited in a joint collaborative research conducted with Alcatel Alenia Space (Cannes), where minimizing a certain cost is very important (fuel consumption / transfer time) while at the same time a feedback law is preferred because of robustness and ease of implementation (see section 7.3).

3.2.2. Transformations and equivalences of non-linear systems and models

Keywords: *classification*, *non-linear control*, *non-linear feedback*, *non-linear identification*.

Participants: Laurent Baratchart, Monique Chyba [Univ. Hawaii (USA)], Jean-Baptiste Pomet.

Here we study certain transformations of models of control systems, or more accurately of equivalence classes modulo such transformations. The interest is two-fold:

- From the point of view of control, a command satisfying specific objectives on the transformed system can be used to control the original system including the transformation in the controller. Of course the favorable case is when the transformed system has a structure that can easily be exploited, for instance when it is a linear controllable system.
- From the point of view of identification and modeling, the interest is either to derive qualitative invariants to support the choice of a non-linear model given the observations, or to contribute to a classification of non-linear systems which is missing sorely today. Indeed, the success of the linear model in control and identification is due to the deep understanding one has of it. In the same manner, a more complete knowledge of invariants of non-linear systems under basic transformations is a prerequisite for a more general theory of non-linear identification.

Concerning the classes of transformations, a *static feedback* transformation of a dynamical control system is a (non-singular) reparametrization of the control depending on the state, together with a change of coordinates in the state space. A *dynamic feedback* transformation of a control system consists of a dynamic extension (adding new states, and assigning them a new dynamics) followed by a state feedback on the augmented system. Let us now stress two specific problems that we are tackling.

3.2.2.1. Dynamic linearization.

The problem of dynamic linearization, still unsolved, is that of finding explicit conditions on a system for the existence of a dynamic feedback that would make it linear.

Over the last years [66], the following property of control systems has been emphasized: for some systems (in particular linear ones), there exists a finite number of functions of the state and of the derivatives of the control up to a certain order, that are differentially independent (*i.e.*, coupled by no differential equation) and do “parameterize all the trajectories”. This property, and its importance in control, has been brought to light in [66], where it is called *differential flatness*, the above mentioned functions being called *flat* or *linearizing functions*, and it was shown, roughly speaking, that a system is differentially flat if, and only if, it can be converted to a linear system by dynamic feedback. On the one hand, this interesting property of the set of trajectories is at least as important in control as the equivalence to a linear system, and on the other hand it gives a handle for tackling the problem of dynamic linearization, namely to find linearizing functions.

An important question remains open: how can one algorithmically decide whether a given system has this property or not, *i.e.*, is dynamically linearizable or not? This problem is both difficult and important for non-linear control. For systems with four states and two controls, whose dynamics is affine in the control (these are the lowest dimensions for which the problem is really non-trivial), necessary and sufficient conditions [10] for the existence of linearizing functions depending on the state and the control (but not on the derivatives of the control) can be given explicitly, but they do point at the complexity of the issue.

From the algebraic-differential point of view, the module of differentials of a controllable system is free and finitely generated over the ring of differential polynomials in d/dt with coefficients in the space of functions of the system, and for which a basis can be explicitly constructed [41]. The question is to find out if it has a basis made of closed forms, that is, locally exact forms. Expressed in this way, it is an extension of the classical integrability theorem of Frobenius to the case where coefficients are differential operators. Together with stability by exterior differentiation (the classical condition), further conditions are required here to ascertain the degree of the solutions is finite, a mid-term goal being to obtain a formal and implementable algorithm to decide whether or not a given system is flat around a regular point. One can further consider sub-problems having their own interest, like deciding flatness with a given pre-compensator, or characterizing “formal” flatness that would correspond to a weak interpretation of the differential equation. Such questions can also be raised locally, in the neighborhood of an equilibrium point.

3.2.2.2. Topological Equivalence

In what precedes, we have not taken into account the degree of *smoothness* of the transformations under consideration.

In the case of dynamical systems without control, it is well known that, away from degenerate (non hyperbolic) points, if one requires the transformations to be merely continuous, every system is *locally* equivalent to a linear system in a neighborhood of an equilibrium (the Hartman-Grobman theorem). It is thus tempting when classifying *control* systems, to look for such equivalence modulo non-differentiable transformations and to hope bring about some robust “qualitative” invariants and perhaps stable normal forms. A Hartman-Grobman theorem for control systems would say for instance, that outside a “meager” class of models (for instance, those whose linear approximation is non-controllable), and locally around nominal values of the state and the control, no qualitative phenomenon can distinguish a non-linear system from a linear one, all non-linear phenomena being thus either of global nature or singularities. Such a statement is wrong: if a system is locally equivalent to a controllable linear system via a bi-continuous transformation—a local homeomorphism in the state-control space—it is *also* equivalent to this same controllable linear system via a transformation that is as smooth as the system itself, at least in the neighborhood of a regular point (in the sense that the rank of the control system is locally constant), see [32] for details; *a contrario*, under weak regularity conditions, linearization can be done by non-causal transformations (see [13]) whose structure remains unclear, but acquires a concrete meaning when the entries are themselves generated by a finite-dimensional dynamics.

The above considerations call for the following question, which is important for modeling control systems: are there local “qualitative” differences between the behavior of a non-linear system and that of its linear approximation when the latter is controllable?

4. Application Domains

4.1. Introduction

The bottom line of the team’s activity is two-fold, namely function theory and optimization in the frequency domain on the one hand, and the control of systems governed by differential equations on the other hand. Therefore one can distinguish between two main families of applications: one dealing with the design and identification of diffusive and resonant systems (these are inverse problems), and one dealing with the control of certain mechanical systems. For applications of the first type, approximation techniques as described in section 3.1.1 allow one to deconvolve linear equations, analyticity being the result of either the use of Fourier transforms or the harmonic character of the equation itself. Applications of the second type mostly concern the control of systems that are “poorly” controllable, for instance low thrust satellites or optical regenerators. We describe all these below in more detail.

4.2. Geometric inverse problems for the Laplace and the Beltrami equation

Keywords: *Beltrami equation*, *Laplace equation*, *inverse boundary problems*, *non destructive control*, *tomography*.

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Localizing cracks, pointwise sources or occlusions in a two-dimensional material, using thermal, electrical, or magnetic measurements on its boundary is a classical inverse problem. It arises when studying fatigue of structures, behavior of conductors, or else electro and magneto-encephalography as well as the detection of buried objects (mines, etc). However, no completely satisfactory algorithm has emerged so far if no initial information on the location or on the geometry is known, because numerical integration of the inverse problem is very unstable. A technique that evolved from the singualr-value decomposition of a parametrix like correlation matrix [82] has become recently popular in the field under the name of MUSIC-type algorithms [73]. The methods we describe are of a different nature, and they are especially valuable when no mutually independent time-varying measurements are available, either because the measurements are stationary or because only few measurements can be made, or else because the superposition of phenomena to be analyzed (*e.g.* the superposition of several sources) are mutually correlated both in time and space. These methods can also be used to approach inverse free boundary problems of Bernouilli type (see section 6.2.2).

The presence of cracks in a plane conductor, for instance, or of sources in a cortex (modulo a reduction from 3-D to 2-D, see below) can be expressed as a lack of analyticity of the (complexified) solution of the associated Dirichlet-Neumann problem that may in principle be approached using techniques of best rational or meromorphic approximation on the boundary of the object (see sections 3.1.1, 3.1.3 and 6.2). In this connection, the realistic case where data are available on part of the boundary only offers a typical opportunity to apply the analytic and meromorphic extension techniques developed earlier.

The 2-D approach proposed here consists in constructing, from measured data on a subset K of the boundary Γ of a plane domain D, the trace on Γ of a function F which is analytic in D except for a possible singularity across some subset $\gamma \subset D$ (typically: a crack). One can then use the approximation techniques described above in order to:

- extend F to all of Γ if the data are incomplete (it may happen that $K \neq \Gamma$ when the boundary is not fully accessible to measurements), for instance to identify an unknown Robin coefficient (see [64] where stability properties of the procedure are established);
- detect the presence of a defect γ in a computationally efficient manner [58];
- obtain information on the location of γ (see [48] [1]).

Thus, inverse problems of geometric type that consist in finding an unknown boundary from incomplete data can be approached in this way [5], often in combination with other techniques [58]. Preliminary numerical experiments have yielded excellent results and it is now important to process real experimental data, that the team is currently busy analyzing. In particular, contacts with the Odyssée Team of Inria Sophia Antipolis (within the ACI “Obs-Cerv”) has provided us with 3-D magneto-encephalographic data from which 2-D information was extracted (see section 6.2). The team also made contact with other laboratories (*e.g.*, Vanderbilt University Physics Dept.) in order to work out 2-D or 3-D data from physical experiments.

We began last year to apply such methods to problems with variable conductivity governed by a 2-D Beltrami equation. The application we have in mind is to plasma confinement for thermonuclear fusion in a tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernouilli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [60]. A joint collaboration on this topic recently started with the Laboratoire J. Dieudonné at the University of Nice, and the CMI-LATP at the University of Marseille I. It has been the object of the post-doctoral stay of E. Sincich and is one of the collaborative research topic with S. Rigat (on leave of absence from the University of Provence) as described in section 6.2.

The goal is first to determine the shape of the surface of the plasma in the chamber from the outer boundary measurements, and in a second step to shape this boundary by choosing some appropriate magnetic flux on this outer boundary (see section 6.2.2).

4.3. Identification and design of resonant systems

Keywords: *filtering device*, *microwave*, *multiplexing*, *surface waves*, *telecommunications*.

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. Resonant systems, either acoustic or electromagnetic based, are prototypical devices of common use in telecommunications. We shall be more specific on two examples below.

4.3.1. Design of surface acoustic wave filters

Participants: Laurent Baratchart, Andrea Gombani, Fabien Seyfert, Martine Olivi.

Surface acoustic waves filters are largely used in modern telecommunications especially for cellular phones. This is mainly due to their small size and low cost. Unidirectional filters, formed of *Single Phase UniDirectional Transducers* (in short: SPUDT) that contain inner reflectors (cf. Figure 1), are increasingly used in this technological area. The design of such filters is more complex than traditional ones.

We are interested here in a filter formed of two SPUDT transducers (Figure 2). Each transducer is composed of cells of the same length τ each of which contains a reflector and all but the last one contain a source (Figure 1). These sources are all connected to an electrical circuit, and cause electro-acoustic interactions inside the piezo-electric medium. In the transducer SPUDT2 represented on Figure 2, the reflectors are positioned with respect to the sources in such a way that near the central frequency, almost no wave can emanate from the transducer to the left ($S_g \approx 0$), this being called unidirectionality. In the right transducer SPUDT1, reflectors are positioned in a symmetric fashion so as to obtain unidirectionality to the left.

Specifications are given in the frequency domain on the amplitude and phase of the electrical transfer function. This function expresses the power transfer and can be written as

$$E(r,g) = 2\,\frac{V_2}{I_0} = \frac{2\sqrt{G_1G_2}\;Y_{12}}{Y_{12}Y_{21}-(Y_{11}+G_1)(Y_{22}+G_2)},$$

where Y is the admittance of the coupling:

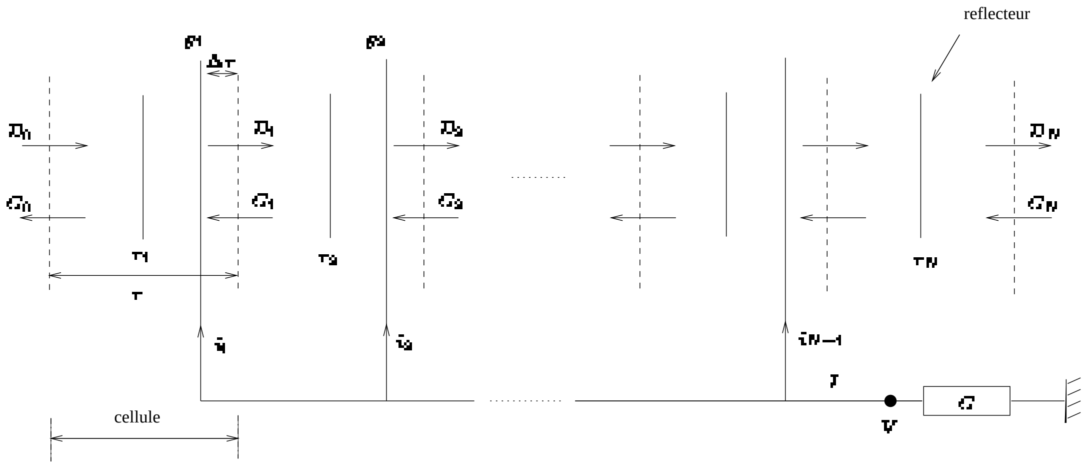

Figure 1. Transducer model.

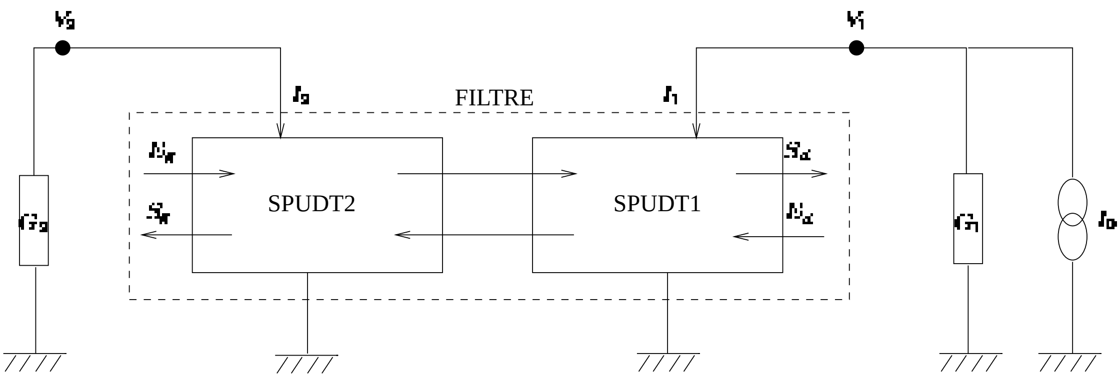

Figure 2. Configuration of the filter

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}.$$

The design problem consists in finding the reflection coefficients r and the source efficiency in both transducers so as to meet the specifications.

The transducers are described by analytic transfer functions called mixed matrices, that link input waves and currents to output waves and potentials. Physical properties of reciprocity and energy conservation endow these matrices with a rich mathematical structure that allows one to use approximation techniques in the complex domain according to the following steps:

- describe the set $\mathcal{E}$ of electrical transfer functions obtainable from the model,
- set out the design problem as a *rational approximation problem* in a normed space of analytic functions:

$$\min_{E \in \mathcal{E}} \|D - E\|,$$

  where D is the desired electrical transfer,
- use a rational approximation software (see section 5.2) to identify the design parameters.

The first item, is the subject of ongoing research. It connects the geometry of the zeroes of a rational matrix to the existence of an inner symmetric extension without increase of the degree (the reciprocal Darlington synthesis, see section 6.4). A collaboration with TEMEX (Sophia-Antipolis) is currently being conducted on the subject.

4.3.2. Hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila, José Grimm, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Each cavity (see Figure 3) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

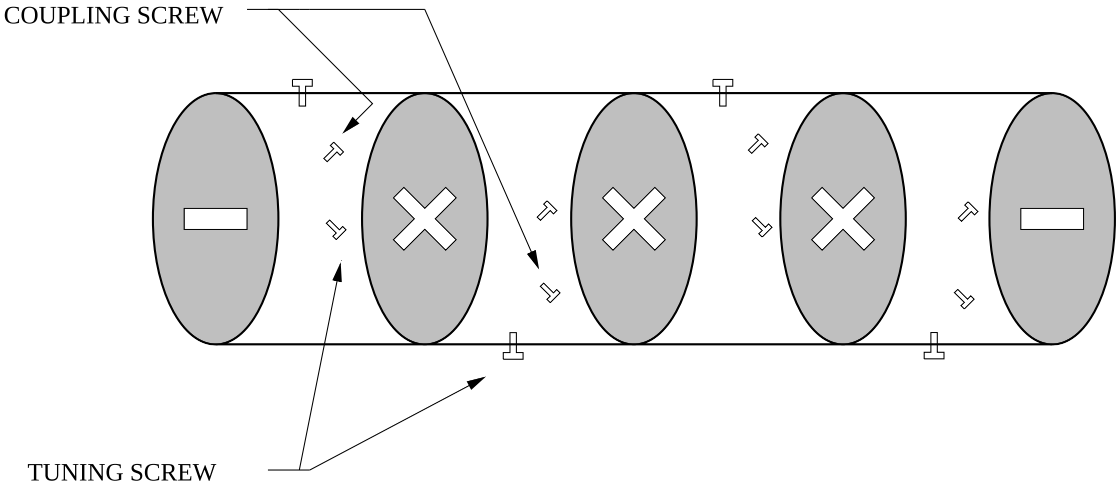

Figure 3. Schematic 4-cavities dual mode filter. Each cavity has 3 screws to couple the modes within the cavity, so that there are 12 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 8 slits are fixed in the design phase.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plug on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the length of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low pass equivalent obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.*, the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, *cf.* section 3.1.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the endymion or RARL2 software (*cf.* section 3.1.4).

- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

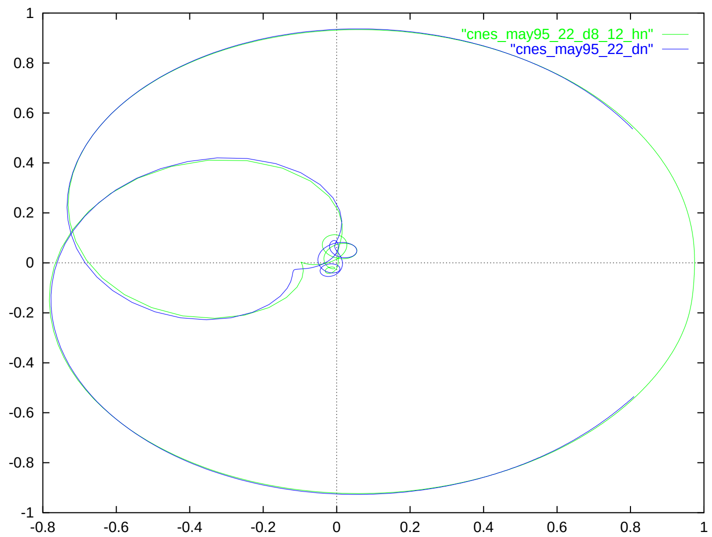

Figure 4. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 4). Non-physical coupling are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

The team investigates today the design of output multiplexors (OMUX) where several filters of the previous type are coupled on a common guide. In fact, it has undergone a rather general analysis of the question “How does an OMUX work?” With the help of numerical simulations and Schur analysis, general principles are being worked out to take into account:

- the coupling between each channel and the “Tee” that connects it to the manifold,
- the coupling between two consecutive channels.

The model is obtained upon chaining the corresponding scattering matrices, and mixes up rational elements and complex exponentials (because of the delays) hence constitutes an extension of the previous framework. Its study is being conducted under contract with Alcatel Alenia Space (Toulouse) (see sections 7.1 and 7.2).

4.4. Spatial mechanics

Keywords: *orbital control*, *satellite*, *spatial mechanics*, *telecommunications*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet.

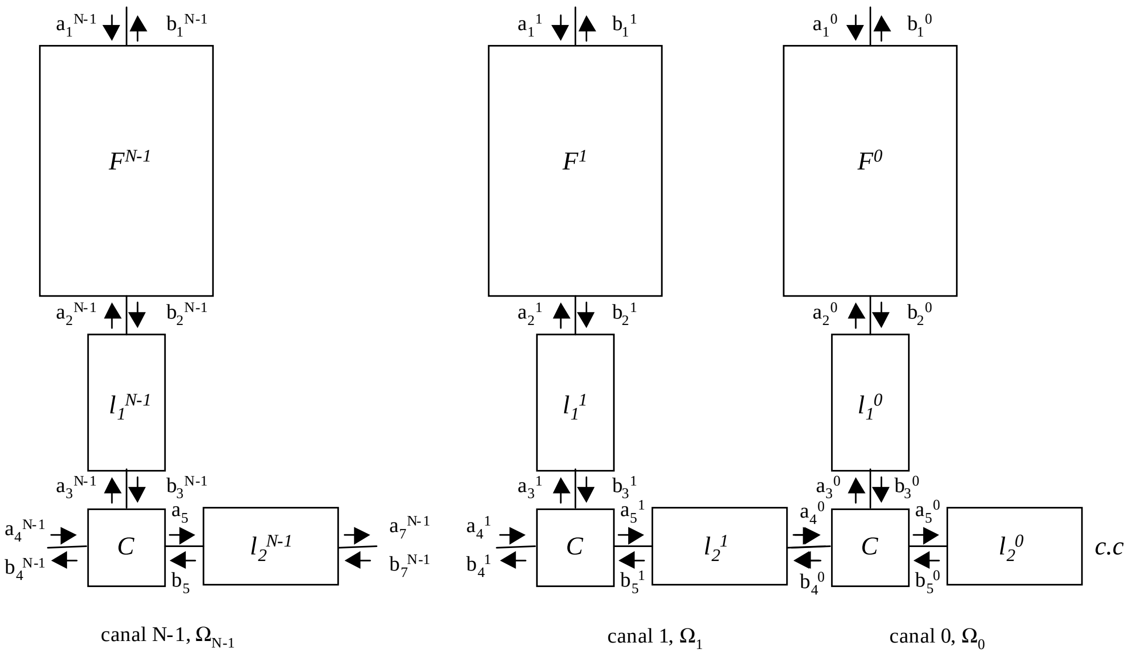

Figure 5. N filters on a manifold. Quantities a and b are incoming and outcoming waves, boxes with an l stand for an adjustable line-length, the 'C' in the square box represents the heart of the Tee (3 by 3 transfer matrix); the big rectangles with an F represent the filters, they are connected to the amplifiers, the bottom row (with the ls and Cs) is the manifold, connected to the antenna on the left, terminated by a cc (short-circuit) on the right.

The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing; see for instance section 4.3 for an illustration.

Of course, this requires that satellites be adequately located and positioned (correct orientation). This problem and similar ones continue to motivate research in control within the team. Generally speaking, aerospace engineering requires sophisticated control techniques for which optimization is often crucial, due to the extreme functioning conditions.

The team has been working for two years on control problems in orbital transfer with low-thrust engines, under contract with Alcatel Space Cannes (see section 7.3). Technically, the reason for using these (ionic) low thrust engines, rather than chemical engines that deliver a much higher thrust, is that they require much less "fuel"; this is decisive because the total mass is limited by the capacity of the launchers: less fuel means more payload, while fuel represents today an impressive part of the total mass.

From the control point of view, the low thrust makes the transfer problem delicate. In principle of course, the control law leading to the right orbit in minimum time exists, but it is quite heavy to obtain numerically and the computation is non-robust against many unmodelled phenomena. Considerable progress on the approximation of such a law by a feedback has been made, and numerical experiments have been conducted (see section 6.13).

4.5. Control of Quantum systems

Participants: Hamza Jirari, Jean-Baptiste Pomet, Pierre Rouchon.

For more than 10 years, physicists have been working on the realization of elementary quantum gates with the goal to build in the future a quantum computer (*cf.* the cavity quantum electrodynamics experiments with circular Rydberg atoms at the Ecole Normale Supérieure in Paris as well as the handling of trapped ions with lasers at Innsbruck University). The main difficulty to overcome for the effective construction of a quantum computer is the decoherence that results form the coupling of Q-bits with their environment: entangled states are difficult to achieve and to maintain over a significant period of time. The goal is to adapt existing control techniques and if necessary to propose new ones for modeling and controlling *open quantum systems*. In particular, for Q-bits coupled with the environment, controllability and disturbance rejection issues arise when trying to design a control that drives the system from one pure quantum state to another (quantum gate) while compensating for the decoherence induced by the environment.

In order to take decoherence into account, one has to use the Heisenberg point of view where the density matrix is used instead of the probability amplitude (Schrödinger point of view); this framework takes into account the coupling with a large environment (reservoir) and its irreversible effects. Under weak coupling and short environment auto-correlation time, the evolution can be described by a differential equation, called the *master equation* which has a well-defined structure under the so-called Lindblad operators [79]; it yields a finite-dimensional bilinear control system, that has not been thoroughly studied up to now. This is the subject of ongoing research. A more sophisticated model is the Bloch-Redfield formalism [61]; it does not have a finite-dimensionnal state (in the control-theoretic sense of this word), but it seems more realistic when the control undergoes fast variations. There is numerical evidence (see [71]) that, in this model, the control can effectively act against dissipation.

This is a very new research topic for the team. We report in section 6.15 on some investigation that started within the post-doctoral stay of Hamza Jirari.

4.6. Non-linear optics

Keywords: *3R regeneration*, *Optics*, *networks*, *optical fibers*, *telecommunications*.

The increased capacity of numerical channels in information technology is a major industrial challenge. The most performing means nowadays for transporting signals from a server to the user and backwards is via optical fibers. The use of this medium at the limit of its capacity of response causes new control problems in order to maintain a safe signal, both in the fibers and in the routing and regeneration devices.

In recent past, the team has worked in collaboration with Alcatel R&I (Marcoussis) on the control of “all-optic” regenerators. Although no collaboration is presently active, we consider this a potentially rich domain of applications

4.7. Transformations and equivalence of non-linear systems

Keywords: *identification*, *mobile cybernetics*, *path planning*.

Participants: Laurent Baratchart, Jean-Baptiste Pomet, David Avanessoff.

The work presented in section 3.2.2 lies upstream with respect to applications. However, beyond the fact that deciding whether a given system is linear modulo an adequate compensator is clearly conceptually important, it is fair to say that “flat outputs” are of considerable interest for path planning [81]. Moreover, as indicated in section 3.2, a better understanding of the invariants of non-linear systems under feedback would result in significant progress in identification.

5. Software

5.1. The Tralics software

Participant: José Grimm [manager].

The development of a LaTeX to XML translator, named Tralics was continued (see section 6.1). TRALICS was sent to the APP in December 2002. Its IDDN number is InterDepositDigitalNumber = IDDN.FR.001.510030.000.S.P.2002.000.31235. Binary versions are available for Linux, Windows and Mac-OS X. Its web page is http://www-sop.inria.fr/apics/tralics. It is now licensed under the CeCILL license version two, see http://www.cecill.info. The latest version is 2.9.

5.2. The RARL2 software

Participants: Jean-Paul Marmorat, Martine Olivi [manager].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see section 3.1.4).

This software takes as input a stable transfer function of a discrete time system represented by

- either its internal realization,
- or its first N Fourier coefficients,
- or discretized values on the circle.

It computes a local best approximant which is *stable, of prescribed McMillan degree*, in the L^2 norm.

It is germane to the arl2 function of endymion from which it differs mainly in the way systems are represented: a polynomial representation is used in endymion, while RARL2 uses realizations, this being very interesting in certain cases. It is implemented in Matlab. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parameterization that has the following advantages

- it incorporates the stability requirement in a built-in manner,
- it allows the use of differential tools,
- it is well-conditioned, and computationally cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum (see section 6.3) by generating, in a structured manner, several initial conditions. Contrary to the polynomial case, we are in a singular geometry on the boundary of the manifold on which minimization takes place, which forbids the extension of the criterion to the ambient space. We have thus to take into account a singularity on the boundary of the approximation domain, and it is not possible to compute a descent direction as being the gradient of a function defined on a larger domain, although the initial conditions obtained from minima of lower order are on this boundary. Thus, determining a descent direction is nowadays, to a large extent, a heuristic step. While this step performs satisfactorily in cases handled so far, it is still unknown how to make it truly algorithmic.

5.3. The RGC software

Participants: Fabien Seyfert, Jean-Paul Marmorat.

The identification of filters modeled by an electrical circuit that was developed by the team (see section 4.3.2) has led to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called “the arrow form” [59] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and also because the assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see 5.6), which is the successor of RGC, solves in a guaranteed manner this constraint realization problem.

5.4. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/personnel/Fabien.Seyfert/Presto_web_page/presto_pres.html. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment);
- automatic determination of an analytic completion, bounded in module for each channel, (see section 6.7);
- rational approximation of fixed McMillan degree;
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies either on hyperion (Unix or Linux only) or RARL2 (platform independent), both rational approximation engines were developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Alcatel Space in Toulouse and a license agreement has been recently negotiated with Thales airborne systems. XLim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingstone, Canada).

5.5. The Endymion software

Participant: José Grimm [manager].

We started the development of Endymion, http://www-sop.inria.fr/apics/endymion/index.html, a software licensed under the CeCILL license version two, see http://www.cecill.info. This software will offer most of the functionalities of hyperion (whose development has been abandoned in 2001), like the *arl2* and *peb2* procedures. It will be much more portable, since it is no more dependent on an external garbage collector or a plotter like *agat*. The symbolic evaluation, based on the Lisp reader, has been tested, debugged and documented.

5.6. Dedale-HF

Participant: Fabien Seyfert.

Dedale-HF is a software meant to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of a given filter characteristics (see section 6.8). Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure6).

Dedale-Hf consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to a particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, *i.e.* the one corresponding to a user-specified filter characteristics. The reference files are computed off line using Groebner basis techniques or numerical techniques based on the exploration a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction of the computational time, say, by a factor of 20.

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale. The software is free of charge for academical research purposes: a registration is however needed in order to have access to the tool's full functionality. Up to now 50 users have registered among the world (mainly: Europe, U.S.A, Canada and China) and 1500 reference files have been downloaded.

6. New Results

6.1. Tralics: a LaTeX to XML Translator

Keywords: *HTML*, *Scanner*, *XML*, *parsing*, *validation*.

Participant: José Grimm.

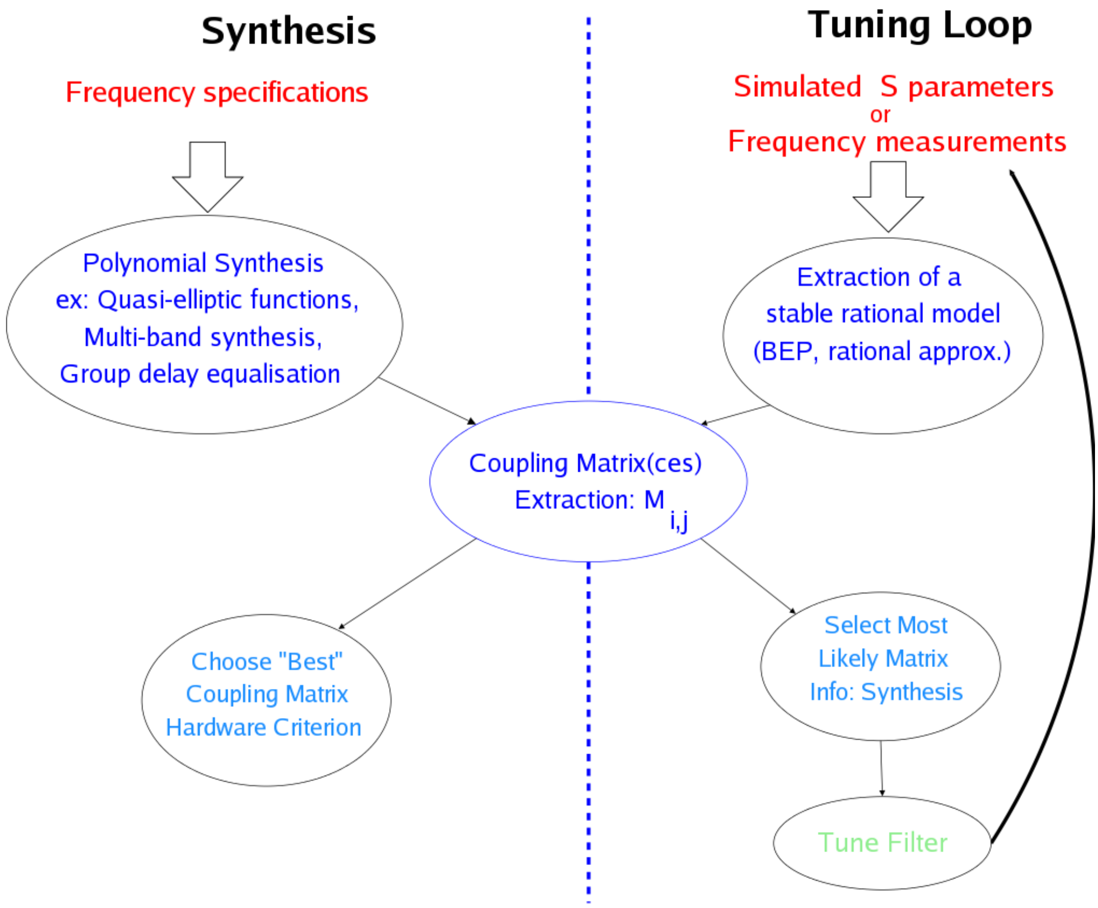

Figure 6. Overall view of the design and tuning process of a microwave filters

The great novelty in the RAWEB2002 (Scientific Annex to the Annual Activity Report of Inria) was the use of XML as intermediate language, and the possibility of bypassing LaTeX. A working group, formed of M.P. Durollet, J. Grimm, L. Pierron, and I. Vatton (not forgetting A. Benveniste, J.-P. Verjus and J.-C. Le Moal) is now in charge of the definition of the tools; in 2003, B. Marmol joined the group, and he is in charge of the dissemination of the package. Christian Rossi joined the group in 2006.

The construction of the raweb is explained schematically on figure 7. The input is either a LaTeX file, or an XML file. Since 2002, the LaTeX to XML translator is the *Tralics* software. An XSLT processor (for instance `xsltproc`, from the Gnome tools) is used to convert the XML either into HTML, or into an XSL-FO document, by adding some formatting instructions (in this phase, we explain for instance that the text font should be Times). This file is formatted by TeX or pdfTeX, thanks to the `xmltex` package that teaches TeX the subtleties of XML and utf-8 encoding, and two packages for the XSL-FO and MathML commands.

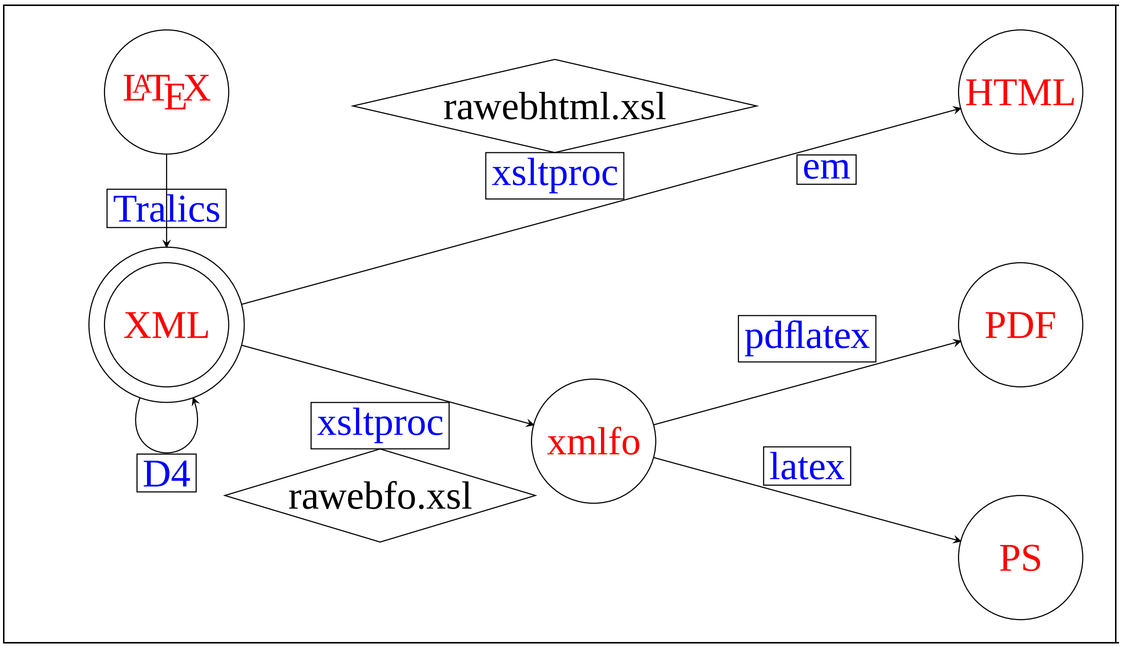

Figure 7. A diagram that explains how the raweb operates. Rectangular boxes contain tools, diamond-shape boxes are style sheets, and circles contain language names. The name 'XML' is in a double circle, it is the central object; the arrow labelled 'D4' that connects it to itself indicates conversion from one DTD to the other, used in 2004. The box containing 'em' represents the Perl script `extract-math.pl` that handles the math formulas; it uses tools borrowed from `latex2html`. This figure was made using the 'pgf' package, a new portable graphic format, not yet understood by Tralics.

A second application of *Tralics* is the following: when researchers wish to publish an Inria Research Report, they send their PostScript or Pdf document, together with the start of the LaTeX source. This piece of document is converted by *Tralics*, using a special configuration file, that extracts only the title page information (author names, abstract, etc). A perl script removes useless pieces, and produces an HTML notice (see for instance http://hal.inria.fr/inria-00070684). As can be easily seen, math formulas like `$(2^n+1)$`, `$n\times n$` are more or less *verbatim* outputs, up to changing the `\catcode` of some characters and redefining all Greek letters and symbols like `\times`.

The main philosophy of *Tralics* is to have the same parser as TeX, but the same semantics as LaTeX. This means that commands like `\chardef`, `\catcode`, `\ifx`, `\expandafter`, `\csname`, etc., that are not described in the LaTeX book and not implemented in translators like latex2html, tth, hévéa, etc., are recognized by Tralics. The program is configurable: the translation depends on certain options and on the `\documentclass`. All element names (except p) can be changed by the user.

Three major versions were released this year, namely 2.7, 2.8 and 2.9. The activity report uses version 2.8, the two research reports, [35] and [36], were updated for version 2.9. The Tralics web page contains the documentation and a link to the sources (the software is licensed under CeCILL, it is an open source free software).

We changed the implementation of the `\pers` command, according to the semantics of the RA2006. Depending on the context, two or three additional arguments are required, their value must belong to a list defined by the raweb team, and given in the *ra.tcf* file. The command `\refercite` was added as a consequence of another change in the semantics of the Raweb.

The concept of *tcf* files was added in version 2.7. For a target type like *ra* (activity report) or *rr* (research report) this file describes how certain commands should be translated. For instance, in the case of *ra*, it will contain the list of research themes, in the case of *rr*, it contains the list of meta-data (author names, abstract, etc.) mentioned above. Before version 2.7, a single file contained everything. In the current distribution there are 14 tcf files, plus a model of *ra.tcf*, the actual file being distributed in Raweb package.

The software can be further parameterised: if you translate a file A, that loads a class B and a package C, then *Tralics* reads files *A.ult*, *B.clt* and *C.plt* instead of the LaTeX equivalents *B.cls* and *C.sty*. A *clt* file has been designed for standard classes (book, report, article) and a *plt* file for all packages in the base directory. All commands dealing with classes and packages (described in Chapter A4 of the LaTeX companion) are now implemented.

The bibtex support was enhanced. It is now possible to define in a configuration file additional entry types, as well as additional fields. The field list of a non-builtin entry type is formed of all standard fields (in some order) plus user defined ones.

We mentioned above an application where mathematical formulas were evaluated by redefining commands and category codes. There is an application of the same type (the goal being to put paper abstracts on the Web) where the same formula has to be typeset twice, once producing a MathML formula, and once a TeX-like formula (for browsers that do not render MathML correctly or at all). A new feature was added in *Tralics*, because the formula has to be tokenized twice. We added an option that allows *Tralics* to issue a math formula (after full expansion) as a TeX formula directly, without having to redefine anything. Moreover, it is possible to tell *Tralics* to produce a non-math formula in simple cases like `$\alpha$` (case of a single character) or `$x^y$` (case of superscript with text only).

We implemented all extensions defined by e-TeX (these are enabled by default in every modern implementation of TeX, as recommended by the LaTeX team). There are two exceptions: the `\middle` command, that does not match MathML specifications, and mixed left-to-right right-to-left writings.

We changed the internal encoding: it is now UTF-8. As a consequence every 16-bit character is a valid character, can have a category code, a lc code, *etc.* Both input and output can be UTF-8 or latin1. Entity names like ‘` `’ are no longer created (character entities are used instead), except for math symbols, like ‘`α`’.

The research report describing *Tralics* has been converted to XML, then to HTML, and put on the web. The second part of this report describes how we did this (the DTD, the style sheets, the *ult* files etc.), as well as another example (a thesis) that was fully converted to XML then HTML. In this case, some parts of the XML file has to be converted to images via LaTeX, using mechanisms similar to those of the Raweb, where only math formulas are translated into images.

6.2. Inverse Problems for 2-D and 3-D elliptic operators

Keywords: *Beltrami equation*, *Laplacian*, *inverse problems*, *non destructive control*, *plasma confinement*, *tomography*.

Participants: Laurent Baratchart, Fehmi Ben Hassen [LAMSIN-ENIT], Maureen Clerc [Odyssée], Imen Fellah [LAMSIN-ENIT], José Grimm, Mohamed Jaoua [UNSA], Juliette Leblond, Moncef Mahjoub [LAMSIN-ENIT], Jean-Paul Marmorat [CMA-EMP], Théodore Papadopoulo [Odyssée], Jonathan R. Partington [Univ. Leeds], Stéphane Rigat, Edward Saff [Univ. Vanderbilt], Maxim Yattselev, Meriem Zghal.

6.2.1. Cauchy problems

Solving overdetermined Cauchy problems for the Laplace equation on an annulus (in 2-D) or a spherical layer (in 3-D) in order to treat incomplete experimental data is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to E.E.G. since the latter involves propagating the initial conditions from the boundary to the center of the domain where the singularities (*i.e.* the sources) are sought. Here, the domain is typically made of several homogeneous layers of different conductivities.

6.2.1.1. 2-D domains

Solving Cauchy problems on a 2-D annulus is the main topic of M. Mahjoub's PhD thesis. This issue arises when identifying a crack in a tube or a Robin coefficient on the inner skull thereof. It can be formulated as a best approximation problem on part of the boundary of a doubly connected domain, and both numerical algorithms and stability results were obtained in this framework [18], [75]. They generalize those previously obtained in simply connected situations [64], [65].

Still in the 2-D case with incomplete data, the geometric problem of finding, in a stable and constructive manner, some unknown (insulating) part of the boundary of a domain was considered in the Ph.D. thesis of I. Fellah. Approximation and analytic extension techniques described in section 3.1.1, together with numerical conformal transformations of the disk, here also provide us with interesting algorithms for the inverse problem under consideration. A related result was recently obtained, namely the L^p existence and uniqueness of the solution to the Neumann problem on a piecewise $C^{1,\alpha}$ domain with inward pointing cusps (note that the endpoints of a crack are such cusps) when $1 < p < 2$. Although it is reminiscent of classical L^p theorems on Lipschitz domains [72], it seems to be a new result and the first one dealing with a cusp while still controlling the conjugate function; the proof uses weighted norm inequalities [15]. Moreover, a Cauchy-type representation for the solution was obtained using properties of Smirnov classes, and the technique generalizes to mixed boundary conditions that occur when the crack is no longer assumed to be a perfect insulator. Describing higher dimensional geometries with cusps to which the result can be extended is an interesting issue.

6.2.1.2. 3-D spherical layers

Cauchy problems on 3-D spherical layers offer an opportunity to state and solve extremal problems for harmonic fields for which an analog of the Toeplitz operator approach to bounded extremal problems [4] has been obtained. More specifically, the density of traces of harmonic gradients in L^2 of an open subset of the 3-D sphere was established, and a Toeplitz operator with symbol the characteristic function of such a subset was defined. Then, a best approximation on the subset of a general vector field by a harmonic gradient under a L^2 norm constraint on the complementary subset can be computed by an inverse spectral equation for the above-mentioned Toeplitz operator. Constructive and numerical aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) are under study and encouraging results have been obtained on numerically simulated data [28], [43].

6.2.1.3. The conductivity operator

With the post-doctoral stay of E. Sincich, a collaboration with the CMI-LATP (University Marseille I) began on elliptic equations corresponding to diffusion processes with variable conductivity. In particular, the 2-D div-grad equation which leads to the so-called *real Beltrami equation* was investigated. In the case of a smooth simply connected domain, we started analyzing this year the existence of solutions in Sobolev $W^{1,p}$ classes for $p > 2$ and the characterization of their traces on the boundary using generalized Cauchy-Riemann equations. We also introduced less regular solutions of Hardy-type (*i.e.* having bounded integral L^p means on

1-D contours tending to the boundary). This should allow for us to state Cauchy problems as bounded extremal issues in classes of generalized analytic functions, whose properties are currently under study [52] together with the behavior of the associated Cauchy and Beurling operators as well as $W^{1,p}$ estimates for generalized Riesz transforms.

The application that initially motivated this study is described in the next section.

6.2.2. Application to modelling of tokamak plasmas

Let us briefly describe a potential application of inverse boundary problems for the Beltrami equation, on a 2-D doubly connected domain, to plasma confinement for thermonuclear fusion in a Tokamak; this collaborative work was started in collaboration with the Laboratoire J. Dieudonné (University of Nice). In the particular case at hand, it seems possible to explicitly compute a basis of solutions (Bessel functions) that should greatly help the computations (see [51] [37]) but the techniques should be valuable more generally.

In the most recent tokamaks, like Jet or ITER, an interesting feature of the level curves of the poloidal flux is the occurrence of a cusp (a saddle point of the poloidal flux, called an X point), and it is desirable to shape the plasma according to a level line passing through this X point for physical reasons relating to the efficiency of the energy transfer.

The problem we have in mind here is of dual Bernoulli type. Classically, the interior Bernoulli problem on a domain Ω (see [67]) is to find a closed subset $A \subset \Omega$ and a harmonic function u in $\Omega \setminus A$ such that $u = 0$ on $\partial\Omega$, $u = 1$ on ∂A, and $\partial u/\partial\nu = Q$ on ∂A, where Q is a given positive constant and ν indicates the outer normal. A natural generalization is obtained on letting u satisfy a more general diffusion equation

$$\text{div}\,(\sigma\nabla u) = 0 \tag{2}$$

in $\Omega \setminus A$, for some non-constant conductivity $\sigma > 0$.

The dual problem arises when both u and $\partial u/\partial n$ are given on the known boundary $\partial\Omega$ while $u = Q$ is constant on ∂A. Note that this issue is *overdetermined*, that is, the boundary data on $\partial\Omega$ have to satisfy some compatibility conditions (of generalized Cauchy-Riemann type). One motivation for the dual problem is the observation that, in the transversal section of a Tokamak (which is a disk if the vessel is idealized into a torus), the so-called poloidal flux is subject to (2) outside the plasma volume for some simple explicit real analytic function σ, while the boundary of the plasma is a level line of this flux [60]. Actually, when looking for a X point, the main interest is attached to the smallest connected so-called "elliptic" solution, which makes for a definite object of study among all other solutions.

When σ is constant and $\partial u/\partial n$ has zero mean on $\partial\Omega$, it is well-known that u has a conjugate function v such that $u + iv$ is holomorphic in $\Omega \setminus A$. More generally, as soon as σ is bounded away from zero and $\sigma\partial u/\partial n$ has zero mean on $\partial\Omega$, a generalized conjugate exists such that $f = u + iv$ satisfies the so-called *real Beltrami equation*:

$$\partial f/\partial\bar{z} = \nu\overline{\partial f/\partial z} \tag{3}$$

where $\nu = (1 - \sigma)/(1 + \sigma)$. Moreover, the Dirichlet-Neumann data for u determine the boundary values of f on $\partial\Omega$ (up to an additive imaginary constant). For fixed A and Q, we intend to study the extremal problem of best approximating these values by (the trace on $\partial\Omega$ of) a solution to (3) under the constraint that it has non-negative real part at most Q on ∂A. This is an infinite-dimensional convex problem whose Lagrange parameter will indicate both how to deform and how to modify A locally in order to improve the criterion.

6.2.3. *Sources recovery in 2-D and 3-D*

The fact that 2-D harmonic functions are real parts of analytic functions allows one to tackle issues in singularity detection and geometric reconstruction from boundary data of solutions to the Laplace equations using the meromorphic and rational approximation tools developed by the team. Some electrical conductivity defaults can be modeled by pointwise sources inside the considered domain. In dimension 2, the question made significant progress in recent years: the singularities of the function (of the complex variable) which is to be reconstructed from boundary measures are poles (case of dipolar sources) or logarithmic singularities (case of monopolar sources). Hence, the behavior of the poles of the rational or meromorphic approximants, described in section 3.1.3, allows one to efficiently locate their position. This is the topic of the article [1], where the related situation of small inhomogeneities connected to mine detection is also considered.

The problem of sources recovery can be handled in 3-D balls by using best rational approximation on 2-D cross sections (disks) from traces of the boundary data on the corresponding circles. It turns out that each of these traces coincides with a 2-D analytic functions in the slicing plane, that has branched singularities inside the disk [14]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework where approximately recovering these singularities can be performed using best rational approximation.

We also started to consider more realistic geometries for the 3-D domain under consideration. A possibility is to parametrize it in such a way that its planar cross-sections are quadrature domains or R-domains. In this framework, best rational approximation can still be performed in order to recover the singularities of solutions to Laplace equations, but complexity issues have to be examined carefully. The preliminary case of an ellipsoid is the topic of the work in progress [39]. Note that it requires the computation of an explicit basis of ellipsoidal harmonics.

Finally, we begin to consider actual 3-D approximation for such inverse problems. Quaternionic analysis seems to be a relevant tool, but the multiplicative side of the theory remains to be developed.

6.2.4. *Application to EEG inverse problems*

In 3-D, epileptic regions in the cortex are often represented by pointwise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electoencephalography). Note that the patient's head is here modeled as a nested sequence of spherical layers. This inverse EEG problem is the object of a collaboration between the Apics and Odyssée Teams through the ACI "Obs-Cerv". A breakthrough was made last year which makes it possible now to proceed via best rational approximation on a sequence of 2-D disks along the inner sphere [14], [27], [53]. The point here is that, up to an additive function harmonic in the 3-D ball, the trace of the potential on each boundary circle coincides with a function having branched singularities in the corresponding disk. The behavior along the family of disks of the poles of their best rational approximants on each circle is strongly linked to the location of the sources, using properties discussed in sections 3.1.3 and 6.6. In the particular case of a unique source, we end up with a rational function which makes for an easy detection; when there are several sources, their localisation requires a slightly more sophisticated machinery to make the convergence of poles of meromorphic approximants effective (see section 6.6). This and other related issues including some preprocessing of the function are still under study.

6.3. Parametrizations of matrix-valued lossless functions

Participants: Bernard Hanzon [Univ. Cork], Jean-Paul Marmorat, Martine Olivi, Ralf Peeters [Univ. Maastricht].

The goal of this work is to implement a stock of parameterizations that could be used for approximation purposes (see section 3.1.4) while taking into account some specific properties induced by the physics such as symmetry, passivity, or some other constraint on the realization matrix like the structure of the coupling in a microwave filter (see section 4.3.2).

Tangential Schur algorithms provide interesting tools to parameterize conservative functions by means of interpolation data. An atlas of charts has been derived from Nevanlinna-Pick interpolation values, in which a function can be represented by a balanced realization computed as a product of unitary matrices from the Schur parameters [17]. Such an atlas presents a number of advantages in view of the approximation problems we have in mind: it ensures identifiability, takes into account the stability constraint, and presents a nice numerical behavior. It has been used in the software RARL2 (se section 5.2). More general interpolation values can be used, associated with a Nudelman (contour integral) interpolation problem. New atlases can also be built, which allow for us to deal with systems having real coefficients. We paid special attention to an atlas which uses a nice mutual encoding property of lossless functions and has been implemented in a new version of the software RARL2. A paper reporting on these has been accepted for publication [80]. All these atlases present a lot of flexibility to design an adapt charts when necessary, for example if one wants to change chart running an optimization process. For example, using a realization in Schur form, a chart can always be found in which all the Schur vectors are zero. Now, the balanced realizations obtained in a given chart possess no particular structure. However, upon choosing the interpolation points at zero and the directions in a particular manner among standard basis vectors, a sub-atlas can be specified in which the balanced canonical forms have a staircase structure with the property that the corresponding controllability matrix is positive upper-triangular. Such canonical forms are of interest because of their nice behaviour under truncation. The corresponding atlas is minimal in that no chart can be left out without loosing the covering property on the manifold. These results will be published in the LAA special issue in honor of Paul Fuhrmann [83]. Up to now, they only concern discrete-time transfer functions. Continuous-time, which is relevant in many applications, is under study and preliminary results have been obtained in the SISO case that relate the well-known Schwartz form to a boundary interpolation problem on the imaginary axis.

6.4. Mathematical modelling of Surface Acoustic Wave filters

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, Fabien Seyfert.

Surface Acoustic Waves (in short: SAW) filters consist in a series of transducers which transmit electrical power by means of surface acoustic waves propagating on a piezoelectric medium. They are usually described by a mixed scattering matrix which relates acoustic waves, currents and voltages. By reciprocity and energy conservation, these transfers must be either lossless, contractive or positive real, and symmetric. In the design of SAW filters, the desired electrical power transmission is specified. An important issue is to characterize analytically the functions that can actually be realized for a given type of filter.

In any case these functions lie in the Schur class, and if they have degree n they can be imbedded into a conservative matrix of McMillan degree at most $n + 2$. This conservative matrix describes the global behavior of the filter. Such a completion problem is known as the Darlington synthesis, and it has always a solution without increase of the McMillan degree in the rational case. However in our case, additional constraints arise from the geometry of the filter, like the symmetry and certain interpolation condition, and these are responsible for the increase of the degree by 2. In [47], a complete mathematical description of such devices is given, including realizations for the relevant transfer functions, as well as a necessary and sufficient condition for symmetric Darlington synthesis preserving the McMillan degree. More generally, in collaboration with P. Enqvist from KTH (Stockholm, Sweden), we characterized the existence of a symmetric Darlington synthesis with specified increase of the McMillan degree: a symmetric extension of a symmetric contractive matrix S of degree n exists in degree $n + k$ if, and only if, $I - SS^*$ has at most k zeros with odd multiplicity [46], [47]. In the language of circuit theory, this result tells us about the minimal number of gyrators to be used in circuit synthesis. These results have been extended to the case of real-valued functions using a frequency domain approach [21]. In these studies, only extensions of twice the initial size have been considered. In view of multi-port synthesis applications it is highly desirable to generalize these results to other types of extensions. Of particular interest is the extension of a scalar Schur function, from a first row extension of any size, to a square symmetric conservative matrix. The techniques developed so far for the symmetric Darlington synthesis should enable us to carry out such a generalization.

Regular contact has been made with TEMEX (Sophia-Antipolis), leading to a new approach in designing an "ideal SAW filter" . This filter has a "symmetric" geometry, the left transducer being reflected from the right transducer, and its scattering matrix S is not only in the Schur class but in fact conservative. The electrical power transmission, on which the specifications are given, is the transmission part of S. This scattering matrix is related through a Cayley transform to the admittance matrix, and in the case of an ideal filter, it is completely determined by the transmission part of the admittance. We then use Zolotarev optimization methods (see section 6.9) to design the admittance. This stage is still not final in that only the poles of the transmission function are optimized whereas the specific links between its denominator and numerator have not yet taken into account. The behavior of the electrical power transmission is very similar to that of the admittance and can be easily tuned using the latter (see Figure 8).

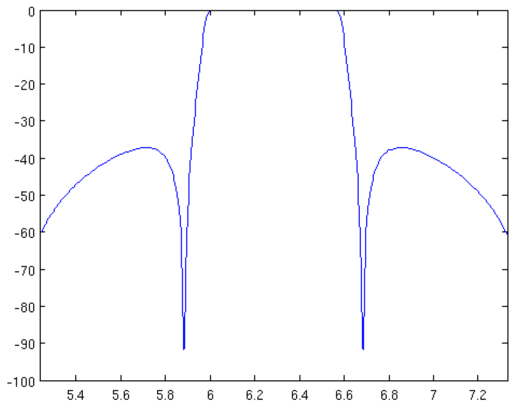

Figure 8. Bode diagram of an ideal filter

6.5. Rational and meromorphic approximation

Participants: Laurent Baratchart, Edward Saff, Maxim Yattselev.

The results of [48] and [15] were exploited last year to prove the convergence in capacity of L^p-best meromorphic approximants on the circle ($p \geq 2$) to Cauchy transforms of complex measure supported on a hyperbolic geodesic plus a rational function. Some mild conditions (bounded variation of the argument and power-thickness of the total variation) were required on the measure. Recall that a sequence of functions is said to converge in capacity if the (logarithmic) capacity of the set where the distance to the limit is greater than ε goes to 0 along the sequence for each fixed $\varepsilon > 0$. An article has been submitted reporting on these results [57]. This year, we proved strong asymptotics for the afore-mentioned functions when the density of the measure does not vanish and its growth at the endpoints is like a fractional power, yielding a strong geometric convergence in this case with a reproduction of the polar singularities of the function with their multiplicities. This result is important for inverse problem of mixed type, like those mentioned in section 6.2.3, where monopolar and dipolar sources are handled simultaneously. Convergence was even obtained on the support of the measure, if the latter has analytic density. An article is currently being written on this topic.

6.6. Behavior of poles

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

It is known after [8] that the denominators of best rational of meromorphic approximants in the L^p norm on a closed curve (say the unit circle T to fix ideas) satisfy for $p \geq 2$ a non-Hermitian orthogonality relation for functions described as Cauchy transforms of complex measures on a curve γ (locus of singularities) contained in the unit disk D. This has been used to assess the asymptotic behavior of the poles of such an approximant when γ is a hyperbolic geodesic arc. More precisely, under weak regularity conditions on the measure, the counting measure of these poles converges weak-star to the equilibrium distribution of the condenser (T, γ) where T is the unit circle. Non asymptotic bounds were also obtained for the sum of the complement to π of the hyperbolic angles under which the poles "see" γ: the sum of these complements over all the poles (they are n in total if the approximant has degree n) is bounded by the aperture of γ plus twice the variation of the argument of the measure (which is independent of n). This produces "hard" testable inequalities for the location of the poles, that should prove particularly valuable in inverse source problems (because they are not asymptotic in nature), see [48] and [15]. We proved this year strong asymptotics that do not deal with the counting measure of the poles (this entails only results in proportion) but with the behavior of *all* of them. They were obtained for Cauchy transforms of smooth nonvanishing complex measures on a hyperbolic geodesic arc in the disk, provided the density increases at least like a fractional power at the endpoints of the arc. This new and interesting result generalizes most of the previous works on a segment [40], [69], and paves the way for further study on uniqueness of local best approximants and inverse source problems. The technical problem facing us is to get rid of the growth assumptions at the endpoints which are induced by the technique (going over to the circle in order to use Fourier analysis and compactness of Hankel operators with continuous symbol). The addition of a rational term which is not singular on the arc to the approximated function has also been handled, generalizing results of Gonchar and Suetin [69]. A numerical illustration is shown in Figures 9-10 for various approximants to the functions F and G given below.

$$\begin{aligned} F(z) &= 7\int_{[-6/7,-1/8]}\frac{e^{it}dt}{z-t} - (3+i)\int_{[2/5,1/2]}\frac{1}{t-2i}\frac{dt}{z-t} + (2-4i)\int_{[2/3,7/8]}\frac{\ln(t)dt}{z-t} \\ &+ \frac{2}{(z+3/7-4i/7)^2} + \frac{6}{(z-5/9-3i/4)^3} + \frac{24}{(z+1/5+6i/7)^4}. \end{aligned}$$

$$G(z) = \int_{[-0.7,0]}\frac{e^{it}}{z-t}\frac{dt}{\sqrt{(t+0.7)(0.4-t)}} + \int_{[0,0.4]}\frac{it+1}{z-t}\frac{dt}{\sqrt{(t+0.7)(0.4-t)}} + \frac{1}{5!(z-0.7-0.2i)^6}$$

The more general situation where γ is a so-called "minimal contour" for the Green potential (of which a geodesic arc is the simplest example) has been settled this year with the same conclusion concerning the weak-* convergence of the counting measure of the poles. The technique uses a potential theoretic analysis of the n-th root of the error and a refinement of Parfenov's estimates on the asymptotics of singular values of Hankel operators. The writing up of this result is underway. It is of particular significance with respect to the determination of several 2-D sources or piecewise analytic cracks from overdetermined boundary data (see sections 3.1.3 and 6.2).

6.7. Analytic extension under pointwise constraints

Participants: Laurent Baratchart, Juliette Leblond, Fabien Seyfert.

To carry out the identification and design of filters under passivity constraints (such constraints are common since passive devices are ubiquitous, including in particular microwave filters), it is natural to consider the mixed bounded extremal problem (P') stated in section 3.1.1. An algorithm to asymptotically solve this problem when $p = 2$ in nested spaces of polynomials was developed last year, and this year a dual approach along the lines of convex optimization theory (although in an infinite-dimensional context) has been investigated. Specifically, the gradient of the dual functional was computed when f lies in the $L \log L$ Zygmund class, and this paves the way for an algorithm with stronger convergence properties than the polynomial one. A connection with normalized Cauchy transforms has also been been carried out, providing a handle to analyze the regularity properties of the solution. More precisely, a condition on the constraint

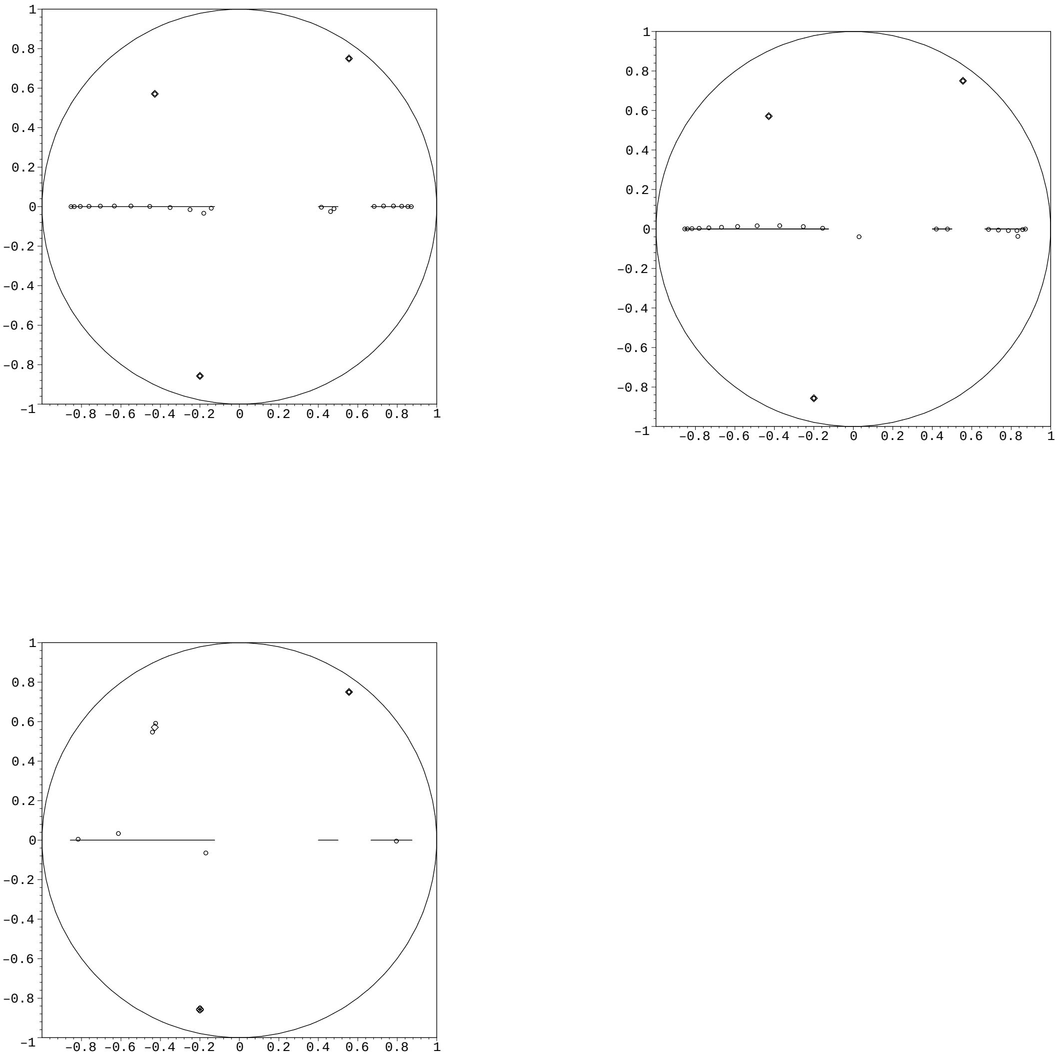

Figure 9. Approximations to the function F; first line: Padé and AAK at degree 30, second line: arl2 at order 13.

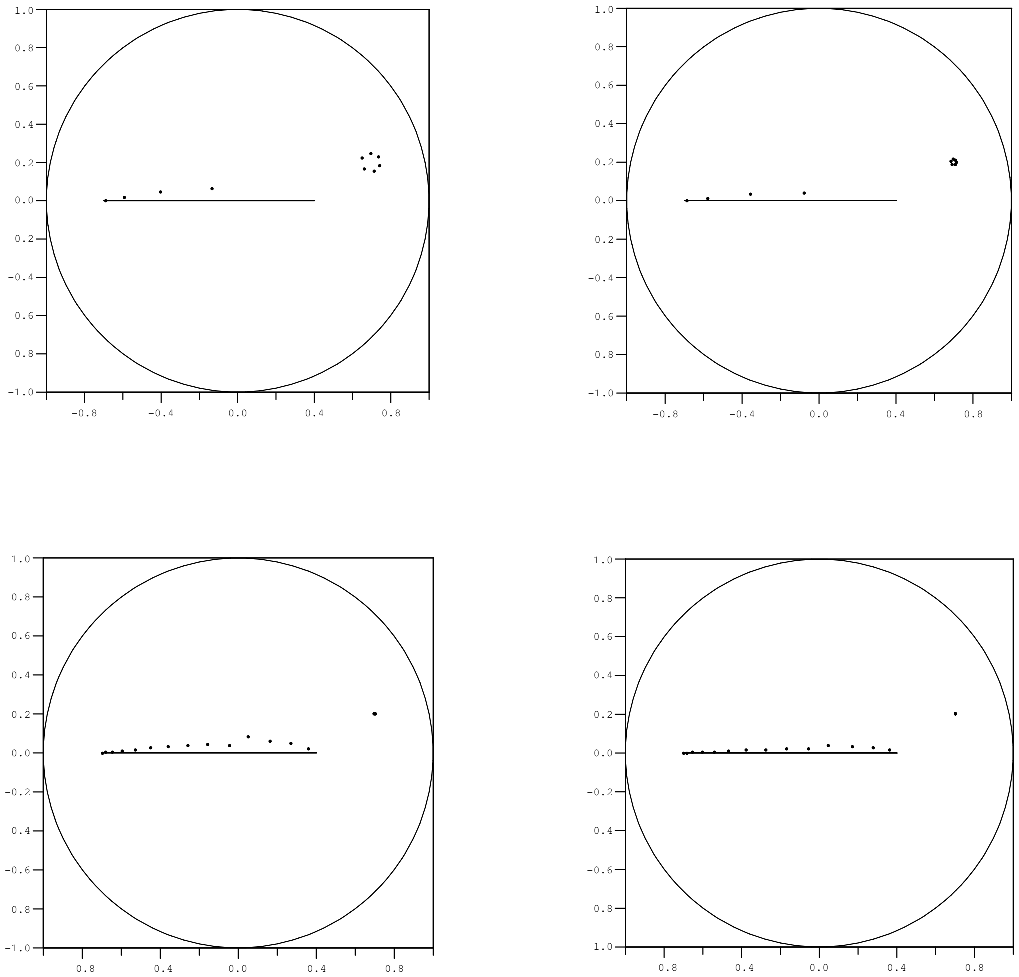

Figure 10. Approximations to the function G; first line: Padé and AAK at order 10, second line, Padé and AAK at order 20.

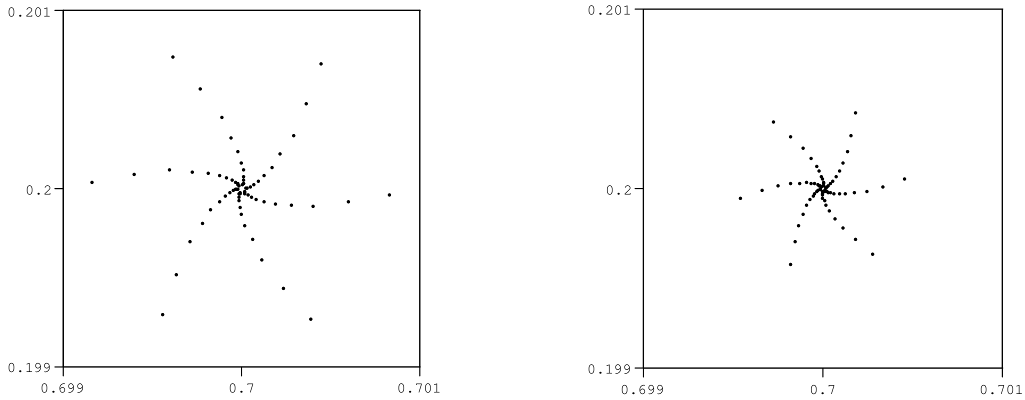

Figure 11. Poles of the n-th approximations to the function G, near the singularity, which is of degree six; left Padé, right AAK; the integer n varies between 21 and 32.

M at the endpoints of the bandwidth K has been given that ensures the continuity of g for smooth f. Such regularity conditions should greatly impinge on the numerical practice of the problem, and should be valuable to estimate delays in waveguides, thereby complementing the existing procedures dealing with this issue in Presto-HF. An article reporting on these results is currently being written.

6.8. Exhaustive determination of constrained realizations corresponding to a transfer function

Participants: Jean Charles Faugère [project SALSA, Rocqencourt], Philippe Lenoir, Stéphane Bila [XLim, Limoges], Fabien Seyfert.

We studied in some generality the case of parameterized linear systems characterized by the following classical state space equation,

$$\sigma(p) \triangleq \left\{ \begin{array}{ll} \dot{x}(t) & = A(p)x(t) + B(p)u(t) \\ y(t) & = C(p)x(t) \end{array} \right. \tag{4}$$

where $p = \{p_1, \cdots, p_r\}$ is a finite set of r parameters and $(A(p), B(p), C(p))$ are matrices whose entries are polynomials (over the field $\mathbb{C}$) of the variables $p_1, ..., p_r$. For a parameterized system σ and $p \in \mathbb{C}^r$ we call $\pi_\sigma(p)$ the transfer function (or transfer matrix) of the system $\sigma(p)$. Some important questions in filter synthesis concern the determination of the following parameterized sets

$$\begin{array}{rl} p \in \mathbb{C}^{r_1},\ E_{\sigma_1}(p) & = \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_1}(p)\} \\ p \in \mathbb{C}^{r_2},\ E_{\sigma_1,\sigma_2}(p) & = \{q \in \mathbb{C}^{r_1}, \pi_{\sigma_1}(q) = \pi_{\sigma_2}(p)\} \end{array} \tag{5}$$

General results were obtained about these sets, in particular a necessary and sufficient condition ensuring that their cardinality is finite. In the special case of coupled-resonators, an efficient algebraic formulation has been derived which allowed us to compute $E_{\sigma(p)}$ for nearly all filter geometries of common use by means of the Gröbner engine FGb developed by the SALSA project at INRIA-Rocqencourt. However for a new class of high order filters first presented in [63] the procedure breaks down because of the complexity of the Gröbner basis computation. This led us to consider instead homotopy methods based on continuation techniques, in order to solve the algebraic system defining $E_{\sigma(p)}$. The usual complexity od such methods, based on the Bezout bound or on mixed volume computations, appeared to be extremely pessimistic in our case because of the degeneracy of our system: for a 10^{th} order filter the Bezout bound is about 10^{44} whereas the number of actual solutions over the ground field $\mathbb{C}$ is only 384. To overcome this difficulty we are currently developing a continuation method which consists in exploring the monodromy group of an algebraic variety by following a family of paths that separate the branch points. More precisely let $h(x,a)$ be a polynomial irreducible system depending on a scalar complex parameter a (x might be multivariate) and generically of dimension zero for a given value of a. Suppose now that we are given a particular solution $h(x_0,a_0)=0$ and we want to compute the complete solution set for the value a_1. It can be shown that lifting a family of paths (in the complex plane) from a_0 to a_1 that separate the branch points (*i.e.* those values of a for which the root-functions $a \to x$ cannot be locally defined) will yield a complete solution set to $h(x,a_1)=0$ (see Figure 12). At the moment, the family of paths is constructed in a brute force manner and leads to heuristics with no formal guarantee on the exhaustivity of the solution set thus obtained but an asymptotic one: provided some real spacing parameter between paths is small enough the algorithm yields a complete solution set. Improvements of these methods (including a systematic way of "chasing" branch points) are now under study.

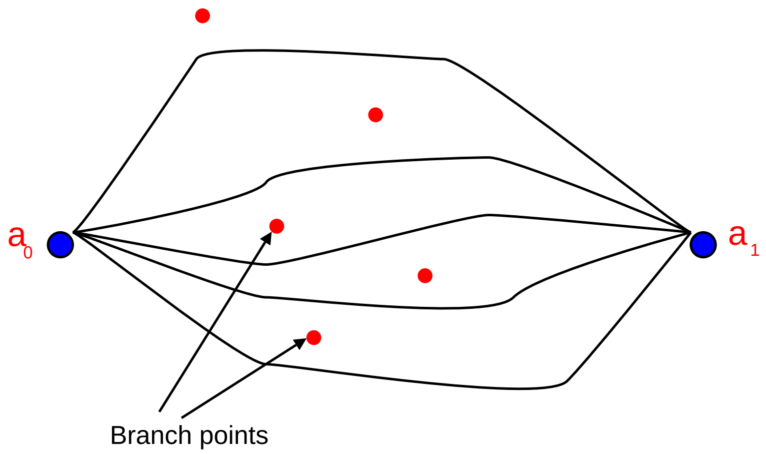

Figure 12. Paths and branch points in the complex plane for the parameter a

For applications of these techniques to microwave filter synthesis, improving the computational time was necessary which led to the design of the software and filter topology database Dedale-HF (see section 5.6). It is based on a continuation method where some particular realizations fibers are computed off-line using a Gröbner basis or else the preceding homotopy method that allows us to speed up considerably on-line computations.

Using our software, it remained to show that filters with multiple solutions topologies are actually realizable in practice. In particular some ambiguities occurring during the tuning step needed to be removed. This was done by introducing the notion of "discriminant experiment" which amounts to trace the fiber of possible realizations on varying a single physical parameter (tuning a screw for example). Techniques to choose this parameter so as to identify among all possible coupling matrices the one implemented by the device have been developed. They allowed us to perform the practical realization of two filters based on the topologies of Figures 13 and 14 that admit respectively fibers of cardinality 15 and 48. This work was carried out in collaboration with XLim. It was supported by the collaborative action ARC Sila funded by INRIA. Application of this work for synthesis of high order multi-band filters was published in [16] and presented in [31].

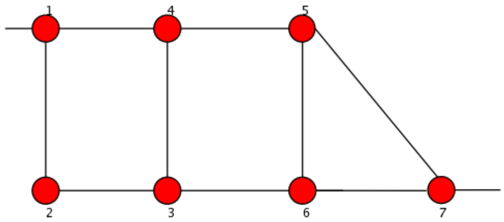

Figure 13. 7th order geometry with "realization fiber" of cardinality 15

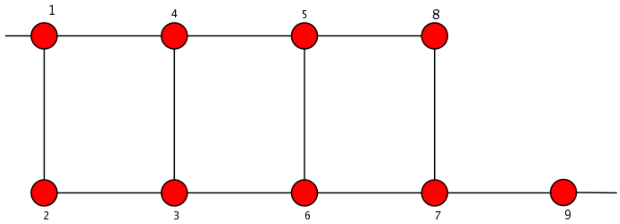

Figure 14. 9th order geometry with "realization fiber" of cardinality 48

6.9. The Zolotarev problem and multi-band filter design

Participants: Vincent Lunot, Philippe Lenoir, Fabien Seyfert.

On introducing the ratio of the numerators of the transmission and reflexion of the scattering matrix, the design of multi-band responses for high-frequency filters (see section 4.3.2) reduces to the following normalized optimization problem of Zolotarev type [85]:

$$\text{letting: } E_{n,m}(K,K') = \{p \in P_m(K), q \in P_n(K') \text{ such that } \forall x \in I\,, \left|\frac{p(x)}{q(x)}\right| \leq 1\},$$

$$\text{solve: } \max_{(p,q)\in E_{m,n}(K,K')} \min_{x\in J} \left|\frac{p}{q}\right| \tag{6}$$

where $I = \bigcup I_i$ (resp. $J = \bigcup J_i$) is a finite union of compact intervals I_i of the real line corresponding to the pass-bands (resp. stop-bands), and $P_m(K)$ stands for the set of polynomials of degree less than m with coefficients in the field K. Depending on physical symmetries of the filter, it is of interest to solve problem (6) for $K = K' = \mathbf{R}$ ("real" problem), $K = \mathbf{C}, K' = \mathbf{R}$ ("mixed" problem), or $K = K' = \mathbf{C}$ ("complex" problem). We have shown that the "real" Zolotarev problem can be decomposed into a sequence of concave maximization problems, the best solution of which yields the optimal solution to the original problem. A characterization in terms of an alternation property has also be given for the solution to each of these sub-problems. Based one this alternation, a Remez type algorithm has been derived. It computes the solutions to these problems in the polynomial case (*i.e.* when the denominator q is fixed), and allows for the computation of a dual-band response (see Figure 16) according to frequency specifications (see Figure 15 for an example from the spacecraft SPOT5 (CNES)). Further, we designed an algorithm in the rational case which, unlike methods based on linear programming, avoids the sampling over all frequencies and is currently under study. This raises the question of the “generic normality” of the approximant with respect to the location of the intervals. This question has not received a definite answer yet. Finally the design of efficient numerical procedures to tackle the “mixed” and the “complex” cases remains a challenging task. These matters will be pursued in V. Lunot’s doctoral work. The Remez algorithm and its application to filter synhtesis are described in [22], [29].

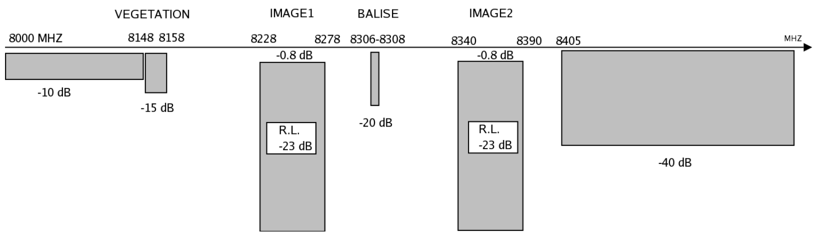

Figure 15. SPOT5 specifications

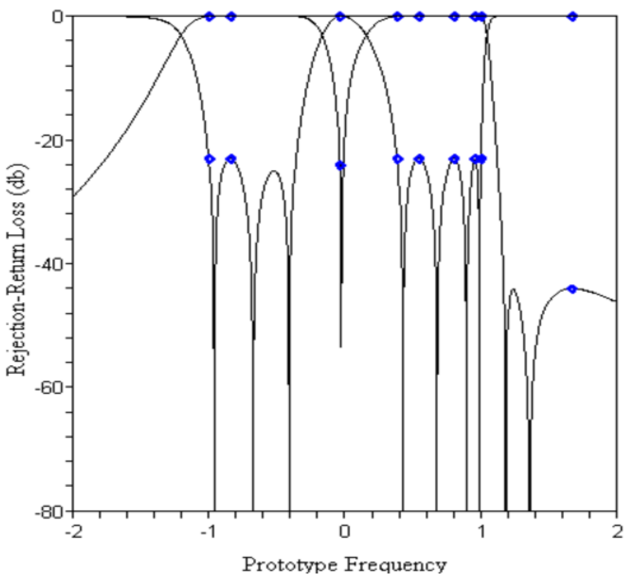

Figure 16. 7^{th} order dual-band response and its critical points

6.10. Frequency approximation and OMUX design

Participants: Laurent Baratchart, Vincent Lunot, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

An OMUX (Output MUltipleXor) can be modeled in the frequency domain through scattering matrices of filters, like those described in section 4.3.2, connected in parallel onto a common guide (see Figure 5). The problem of designing an OMUX with specified performance in a given frequency range naturally translates into a set of constraints on the values of the scattering matrices and of the phase shift introduced by the guide in the considered bandwidth.

An OMUX simulator on a Matlab platform was designed in recent years and used to test some assumptions on the way the OMUX operates. Among them is that each right-section of the OMUX, when the guide gets oriented from the left to the right with its common access lying to the extreme left, acts as a short-circuit in the bandwidth of those channels lying "upstream" (*i.e.* those channels lying to the left of the considered section, so that they are reached first by a wave emanating from the access of the guide). Another assumption is that each channel must reject a little bit in its bandwidth in order to trap energy otherwise reflected by the above-mentioned short-circuit. Under the terms of a recently signed contract with Alcatel Alenia Space (see section 7.2), these assumptions will be used to design a dedicated software to optimize OMUXes, by first trying to optimize a channel when the others are fixed and then looking for a fixed point over all channels.

The direct approach, currently used by the manufacturer, consists in coupling a simulator with a general purpose "optimizer", in order to reduce transmission and reflection wherever they are too large. This yields unsatisfactory results in cases of high degree and narrow bandwidth, in particular because the convergence often fails and multiple initial points must be used resulting in a very lengthy and sometimes unsuccessful search. Besides, manifold-peaks arising from the dilation of the cavities caused by increased temperature (when the satellite gets exposed to the sun), can ruin the design in operational conditions. We expect to be able to produce a multi-phased tuning procedure, first relaxed, channel after channel, then global, using a quasi-Newton method.

6.11. Realization for hybrid nonlinear systems

Participants: Mihaly Petreczky, Jean-Baptiste Pomet.

The paper [30] investigates the realization problem for a class of analytic nonlinear hybrid systems without autonomous switching. Similar to the classical nonlinear realization theory, the realization problem for hybrid systems is translated into a formal realization problem of a class of abstract systems defined on rings of formal power series. Necessary conditions on a given input-output map are presented for the existence of a realization by a hybrid system. A notion analogous to the Lie-rank of nonlinear input-output maps is defined and the above-mentioned necessary condition involves that this Lie-rank should be finite. We also introduced the notion of strong Lie-rank and showed that its finiteness implies the existence of a realization which lies very close to the required hybrid system realization. Thus, the finiteness of the strong Lie-rank can be seen as an "almost" sufficient condition. In the special case of nonlinear analytic systems both the finite Lie-rank and the finite strong Lie-rank conditions presented in the paper reduce to the well-known finite Lie-rank condition. We use the theory of Sweedler-type coalgebras to study the formal realization problem.

This is part of the doctoral work of M. Petreczky on hybrid systems; he visited the team in 2005-2006 under the CTS program (see section 8.3).

6.12. Average control systems

Participants: Alex Bombrun, Jean-Baptiste Pomet.

Motivated by orbit transfer with low-thrust engines (see section 6.13), we developed, for conservative systems with small control, a notion of average control system [26]. Using averaging techniques in this context is rather natural, since the free system produces a fast periodic motion and the *small* control a slow one. In this vein, some recent literature proceeds as follows: the control is assigned, for instance optimal control via Pontryagin's Maximum Principle or some feedback designed beforehand, and then averaging is performed on the resulting ordinary differential equation to analyze its behavior, or rather its limit behavior when the control magnitude tends to zero.

The novelty of the work in [26] is to introduce averaging before assigning the control, hence getting a *control system* that better describes the limit behavior. This concept of average control system is convenient when comparing different control strategies. For instance, it allowed us in [23] to give a partial answer to an open question stated in [62] on estimating the minimum transfer-time between two elliptic orbit when the thrust magnitude tends to zero.

6.13. Feedback for low thrust orbital transfer

Participants: Alex Bombrun, Jonathan Chetboun, Jean-Baptiste Pomet.

The study concerns the control (orbit transfer) of satellites equipped with low thrust engines like plasmic ones, which are efficient with respect to fuel consumption, but deliver a thrust which is much smaller than conventional "chemical" engines (the ratio between the delivered acceleration and the gravity being of the order of 10^{-3}, sometimes less). This problem was raised by Alcatel Alenia Space, and Alex Bombrun's PhD thesis is supported by this company under contract (see section 7.3).

The first results (and maybe the most interesting ones in practice) dealt with the computation of feedback controls, using *ad hoc* Lyapunov functions, that approximated very well time-optimal trajectories [24]. The easy implementation of such control laws makes them attractive as compared to genuine optimal control.

Results obtained this year concern the asymptotic study of this problem when the thrust tends to zero, that captures the "low thrust" effect. We used the average control system described in section 6.12. The result in [25] on discontinuous feedback was motivated by the use of feedback control on the average system.

Another piece of work was concerned with the use of feedback for the controlled three-bodies problem. This was the topic of Jonathan Chetboun's internship. This activity is also supported under contract by Alcatel Alenia Space (Cannes), where the above-mentioned internship partly took place. Within Apics, this research allowed us to numerically simulate a mission like SMART-1 (earth-moon) with feedback controls [33]. A more conceptual understanding of the method still needs to be developed.

6.14. Collision avoidance

Participants: Alex Bombrun, Youssef El Fassy Fihry, Jean-Baptiste Pomet.

Collision avoidance is important, for instance during the formation configuration of a cluster of space vehicles. Motivated by the practical relevance of this problem, we conducted a preliminary study aiming at building "artificial potentials" that depend on the position and velocity. The report [34] mainly contains computations of the boundary set on which this potential should be large to avoid collision. Inside the boundary is the set in which coliision avoidance can be guaranteed.

6.15. Control design for logic quantum gates

Participants: Hamza Jirari, Jean-Baptiste Pomet.

This reasearch has been initiated with the post-doctoral stay of H. Jirari, and its goal was described in section 4.5. The systems under study here are quantum systems of low dimension (1 or 2 Qbits) whose autonomous dynamics is given by the Schrödinger equation and whose interaction with the environement is described by some dissipative term. According to the latter, one distinguishes between the Bloch-Redfeld model and the (more heavily simplified) Lindblad model. In [71], numerical evidence was obtained that in the Bloch-Redfeld model, at least in small dimensional "simple" cases, there exists a control that drives the system from some pure initial state to a target one, while compensating the dissipative effect of the environment. We have pursued this numerical effort on more strongly coupled environments, computing via optimal control techniques. At the same time, we begun the investigation of the control-theoretic structure of the above-mentioned models. In contrast with the Lindblad model, where decoherence is unavoidable in certain cases, it seems that the Bloch-Redfeld model exhibits a more robust behaviour which calls for further understanding.

7. Contracts and Grants with Industry

7.1. Contracts CNES-IRCOM-INRIA

Contract n^{o}04/CNES/1728/00-DCT094 involving CNES, XLIM and INRIA, whose objective is to work out a software package for identification and design of microwave devices. The work at INRIA includes:

- the modeling of delays (see section 4.3.2),
- the exhaustive determination of the coupling coefficients on some case studies (see section 6.8),
- the OMUX stimulator with exact computation of derivatives.

This contract has been renewed for 16 months starting November 2004, in order to develop a generic code for coupling determination and to carry out the optimization of OMUX.

7.2. Alcatel Alenia Space (Toulouse)

A contract reference B00375 has been signed between INRIA and Alcatel Alenia Space (branch of Toulouse), in which INRIA will design and provide a software for OMUX simulation with efficient initial condition for an optimisation algorithm based on recursive tuning of the channels.

7.3. Alcatel Alenia Space (Cannes)

A contract is in the final stage of approval between the two partners. It bears on a Lyapunov-function-based design methodology to set up a feedback law achieving prescribed orbital transfer for a satellite with low-thrust. A numerical Matlab code to demonstrate the validity of the method will be included.

8. Other Grants and Activities

8.1. Scientific Committees

L. Baratchart is a member of the editorial board of *Computational Methods and Function Theory* and *Complex Analysis and Operator Theory*.

8.2. National Actions

Together with project-teams Caiman and Odyssée (INRIA-Sophia Antipolis, ENPC), the University of Nice (J.-A. Dieudonné lab.), CEA, CNRS-LENA (Paris), and some French hospitals (Pitié-Salpêtrière in Paris, Timone in Marseille), we participate in the national action **ACI Masse de données OBS-CERV**, 2003-2006 (inverse problems, EEG).

The post-doctoral training of E. Sincich was funded by INRIA.

8.3. Actions Funded by the EC

The Team was a member of the **Marie Curie multi-partner training site** *Control Training Site*, number HPMT-CT-2001-00278, 2001-2006. See http://www.supelec.fr/lss/CTS/. This network ended April, 2006.

The project is a member of the Working Group Control and System Theory of the **ERCIM** consortium, see http://www.ladseb.pd.cnr.it/control/ercim/control.html.

8.4. Extra-european International Actions

EPSRC grant (EP/C004418) “Constrained approximation in function spaces, with applications”, with Leeds University (UK) and the University Lyon I, 2005-2006.

STIC-INRIA and **AireDéveloppement** grants with LAMSIN-ENIT (Tunis), « Problèmes inverses du Laplacien et approximation constructive des fonctions » (from which M. Zghal and M. El Guenechi received financial support for their trainees).

NSF EMS21 RTG students exchange program (with Vanderbilt University).

8.5. The Apics Seminar

The following scientists gave a talk at the seminar:

- Christophe Prieur, LAAS CNRS Toulouse, *Robust stabilization of nonlinear control systems by means of hybrid feedbacks.*
- Ed B. Saff, Vanderbilt University, *Asymptotics of polynomial zeros: beware of plots!*
- H. Jirari, Institut für Physik, Universität Graz, Autriche, *Contrôle optimal d'un qubit*.
- Luca Rondi, Département de Mathématiques et Informatique, Univ. Trieste, Italie, *A variational approach to the reconstruction of cracks*.
- Mazyar Mirrahimi, INRIA, SOSSO2,. *Identification de paramètres pour un système quantique*
- Sacha Borichev, CMI-LATP, University Marseille I, *Un théorème d'unicité pour l'espace de Korenblum*.
- Maxym Yattselev, *On Baxter's Theorem with Meromorphic Approximation in Mind*.

8.6. Exterior research visitors

- Jonathan Partington, School of Mathematics, Leeds University, U.K.
- Karim Kellay, Stanislas Kupin, Stéphane Rigat, Hassan Youssfi, *et l'équipe d'Analyse et Géométrie*. LATP-CMI, Université Marseille I,
- Fehmi Ben Hassen and Moncef Mahjoub, Lamsin-ENIT, Tunisie.
- Pierre Rouchon, Centre Automatique et Systèmes, Ecole des Mines de Paris.
- Edward B. Saff, Dept. of Mathematics, Vanderbilt University, USA.
- Ugo Boscain, SISSA, Italy.
- Grégoire Charlot, University of Montpellier II.
- Vladimir Peller, University of Michigan at East Lansing, USA.
- Alexei Poltoratski, Texas A&M University, College Station, USA.
- Maxim Yattselev, Vanderbilt University, Nashville, USA.
- Mihaly Petreczky, CWI, Amsterdam, The Netherlands.

9. Dissemination

9.1. Teaching

Courses

- L. Baratchart, DEA Géométrie et Analyse, LATP-CMI, University Marseille I.
- M. Olivi, Mathématiques pour l'ingénieur (Fourier analysis and integration), section Mathématiques Appliquées et Modélisation, 1ère année, Ecole Polytechnique de l'Université de Nice.

Trainees

- Moufida El Guenichi , « Problème inverse d'identification d'un coefficient de Robin non-linéaire : stabilité », co-tutelle with Lamsin-ENIT (Tunis).
- Meriem Zghal, « Problème inverse d'identification d'un coefficient de Robin non-linéaire : algorithmes de résolution », co-tutelle with Lamsin-ENIT (Tunis).

– Jonathan Chetboun (ENPC), « Feedback et poussée faible pour le problème à plus d'un corps central »
– Youssef El Fassy Fihry (École des Mines), « Étude d'ensembles accessibles et dispositifs anti-collision ».

Ph.D. Students

– Alex Bombrun, « Commande optimale, feedback, et tranfert orbital de satellites » (optimal control, feedback, and orbital transfert for low thrust satellite orbit transfer).
– Imen Fellah, "Data completion in Hardy classes and applications to inverse problems", co-tutelle with Lamsin-ENIT (Tunis).
– Vincent Lunot, « Problèmes fréquentiels extrémaux, approximation rationnelle sous contrainte Schur et application à la synthèse de filtres ».
– Moncef Mahjoub, « Complétion de données et ses application à la détermination de défauts géométriques ». co-tutelle with Lamsin-ENIT (Tunis).
– Meriem Zghal, "Meromorphic approximation and inverse problems related to EEG-MEG", co-tutelle with Lamsin-ENIT (Tunis).

Committees

– L. Baratchart was on the PhD reading committee of Mihaly Petreczky (CWI Amsterdam).
– J.-B. Pomet was on the PhD defense committee of Mihaly Petreczky (CWI Amsterdam).
– J. Leblond was on the master committee of Hichem Bouraoui, Lamsin-Enit, Tunis.

9.2. Community service

L. Baratchart is a member of the « commission de spécialistes » (section 25) of the Université de Provence. He was a member of the scientific committee of the conference PICOF 2006

J. Grimm is in charge of organizing the seminar on control and identification.

J. Grimm is a representative at the « comité de centre ». He is a member of the organising committee of PICOF 2006 ("Inverse Problems, Control, and Shape Optimization").

J. Leblond is a substitute member of the « Commission d'évaluation » of INRIA, since September ; she took part to several evaluation seminars and was sitting on hiring committees for CR and DR recruitment and promotion. She was a member of the scientific committee of the conference PICOF 2006. She participates to the working group « Doc » and is in charge with the Séminaires Croisés of the Research Unit.

J. Leblond and J. Grimm were co-editors of the proceedings of the CNRS-INRIA summer school "Harmonic analysis and rational approximation: their role in signals, control and dynamical systems theory" (Porquerolles, 2003) http://www-sop.inria.fr/apics/anap03/index.en.html [11].

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Unit of Sophia Antipolis.

F. Seyfert is a member of the CDL (Comité de Développement Logiciel) of the Research Unit of Sophia Antipolis.

J.-B. Pomet is a representative at the « comité technique paritaire » (CTP).

9.3. Conferences and workshops

A. Bombrun presented communications at the "6th AIMS Conference on Dynamical Systems, Diff. Equations and Applications" in june (Poitiers), and at the "Joint CTS-HYCON Workshop on nonlinear and hybrid systems" in july (Paris).

L. Baratchart gave a conference at IWOTA 2006 (International workshop on operator theory and its applications), Seoul, Korea, in July.

L. Baratchart and M. Yattselev gave a talk at the HSFO-conference (Holomorphic Spaces of Functions and their Operators) at the CIRM, Luminy, France, in July.

L. Baratchart, A. Bombrun and J. Leblond gave communications at MTNS'06 (Mathematical Theory of Networks and Systems), Kyoto, Japan, in July.

J.-B. Pomet was an invited speaker at "Workshop on Geometry of vector distributions, differential equations, and variational problems" (Trieste, Italy, December).

V. Lunot and P. Lenoir gave a talk at the International Microwave Symposium (San Francisco).

F. Seyfert was an invited speaker at "Workshop on Efficient Computation of Gröbner Bases" (Linz, Austria, February)

F. Seyfert gave a talk at the "International Workshop on Microwave Filters", (Toulouse, France, October)

Concerning their joint work with Apics, our collaborators took the following actions.

M. Clerc presented a poster at HBM (Human Brain Mapping) Firenze, Italy, June;

F. Ben Hassen gave a communication at Famelap (Mathematics of Finite Elements), London, UK, in June;

M. Jaoua gave a communication at CARI'06, Bénin, and an invited plenary talk at the Conference « Equations différentielles et Applications » at Annabe, Algeria, both in November.

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