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Team ASPI

Applications statistiques des systèmes de particules en interaction

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2. Overall Objectives

2.1. Overall Objectives

Keywords: *hidden Markov model (HMM)*, *localisation*, *navigation and tracking*, *particle filtering*, *rare event simulation*, *risk evaluation*.

The scientific objectives of ASPI are the design, analysis and implementation of interacting Monte Carlo methods, also known as particle methods, with focus on

- statistical inference in hidden Markov models, e.g. state or parameter estimation, including particle filtering,
- risk evaluation, including simulation of rare events.

The whole problematic is multidisciplinary, not only because of the many scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have already contributed to establish the foundations of the field : target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods, etc. Intuitively speaking, interacting Monte Carlo methods are sequential simulation methods, in which particles

- *explore* the state space by mimicking the evolution of an underlying random process,
- *learn* the environment by evaluating a fitness function,
- and *interact* so that only the most successful particles (in view of the value of the fitness function) are allowed to survive and to get offsprings at the next generation.

The effect of this mutation / selection mechanism is to automatically concentrate particles (i.e. the available computing power) in regions of interest of the state space. In the special case of particle filtering, which has numerous applications under the generic heading of positioning, navigation and tracking, in target tracking, computer vision, mobile robotics, ubiquitous computing and ambient intelligence, sensor networks, etc. each particle represents a possible hidden state, and is multiplied or terminated at the next generation on the basis of its consistency with the current observation, as quantified by the likelihood function. These genetic–type algorithms are particularly adapted to situations which combine a prior model of the mobile displacement, sensor-based measurements, and a base of reference measurements, for example in the form of a digital map (digital elevation map, attenuation map, etc.). In the most general case, particle methods provide approximations of probability distributions associated with a Feynman–Kac flow, by means of the weighted empirical probability distribution associated with an interacting particle system, with applications that go far beyond filtering, in simulation of rare events, simulation of conditioned or constrained random variables, molecular simulation, etc.

ASPI essentially carries methodological research activities, rather than activities oriented towards a single application area, with the objective of obtaining generic results with high potential for applications, and of bringing these results (and other results found in the literature) until implementation on a few appropriate examples, through collaboration with industrial partners.

The main applications currently considered are geolocalisation and tracking of mobile terminals, calibration of models for electricity price, and risk assessment for complex hybrid systems such as those used in air traffic management.

3. Scientific Foundations

3.1. Monte Carlo methods

Monte Carlo methods are numerical methods that are widely used in situations where (i) a stochastic (usually Markovian) model is given for some underlying process, and (ii) some quantity of interest should be evaluated, that can be expressed in terms of the expected value of a functional of the process trajectory, which includes as an important special case the probability that a given event has occurred. Numerous examples can be found, e.g. in financial engineering (pricing of options and derivative securities) [41], in performance evaluation of communication networks (probability of buffer overflow), in statistics of hidden Markov models (state estimation, evaluation of contrast and score functions), etc. Very often in practice, no analytical expression is available for the quantity of interest, but it is possible to simulate trajectories of the underlying process. The idea behind Monte Carlo methods is to generate independent trajectories of this process or of an alternate instrumental process, and to build an approximation (estimator) of the quantity of interest in terms of the weighted empirical probability distribution associated with the resulting independent sample. By the law of large numbers, the above estimator converges as the size N of the sample goes to infinity, with rate $1/\sqrt{N}$ and the asymptotic variance can be estimated using an appropriate central limit theorem. To reduce the asymptotic variance of the estimator, many variance reduction techniques have been proposed. However, running independent Monte Carlo simulations can lead to very poor results, because trajectories are generated *blindly*, and only afterwards are the corresponding weights evaluated. Some of the weights can happen to be negligible, in which case the corresponding trajectories are not going to contribute to the estimator, i.e. computing power has been wasted.

A recent and major breakthrough, a brief mathematical presentation of which is given in 3.2, has been the introduction of interacting Monte Carlo methods, also known as sequential Monte Carlo (SMC) methods, in which a whole (possibly weighted) sample, called *system of particles*, is propagated in time, where the particles

- *explore* the state space under the effect of a *mutation* mechanism which mimics the evolution of the underlying process,

- and are *replicated* or *terminated*, under the effect of a *selection* mechanism which automatically concentrates the particles, i.e. the available computing power, into regions of interest of the state space.

In full generality, the underlying process is a discrete–time Markov chain, whose state space can be finite, continuous (Euclidean), hybrid (continuous / discrete), graphical, constrained, time varying, pathwise, etc., the only condition being that it can easily be *simulated*. The very important case of a sampled continuous–time Markov process, e.g. the solution of a stochastic differential equation driven by a Wiener process or a more general Lévy process, is also covered.

In the special case of particle filtering, originally developed within the tracking community, the algorithms yield a numerical approximation of the optimal filter, i.e. of the conditional probability distribution of the hidden state given the past observations, as a (possibly weighted) empirical probability distribution of the system of particles. In its simplest version, introduced in several different scientific communities under the name of *interacting particle filter* [34], *bootstrap filter* [43], *Monte Carlo filter* [51] or *condensation* (conditional density propagation) algorithm [48], and which historically has been the first algorithm to include a redistribution step, the selection mechanism is governed by the likelihood function : at each time step, a particle is more likely to survive and to replicate at the next generation if it is consistent with the current observation. The algorithms also provide as a by–product a numerical approximation of the likelihood function, and of many other contrast functions for parameter estimation in hidden Markov models, such as the prediction error or the conditional least–squares criterion.

Particle methods are currently being used in many scientific and engineering areas : positioning, navigation, and tracking [44], visual tracking [48], mobile robotics [40], ubiquitous computing and ambient intelligence [45], sensor networks [46], risk evaluation and simulation of rare events [42], genetics, molecular dynamics, etc. Other examples of the many applications of particle filtering can be found in the contributed volume [27] and in the special issue of *IEEE Transactions on Signal Processing* devoted to *Monte Carlo Methods for Statistical Signal Processing* in February 2002, which contains in particular the tutorial paper [29], and in the textbook [59] devoted to applications in target tracking. Applications of sequential Monte Carlo methods to other areas, beyond signal and image processing, e.g. to genetics, and molecular dynamics, can be found in [58].

Particle methods are very easy to implement, since it is sufficient in principle to simulate independent trajectories of the underlying process. The whole problematic is multidisciplinary, not only because of the already mentioned diversity of the scientific and engineering areas in which particle methods are used, but also because of the diversity of the scientific communities which have contributed to establish the foundations of the field : target tracking, interacting particle systems, empirical processes, genetic algorithms (GA), hidden Markov models and nonlinear filtering, Bayesian statistics, Markov chain Monte Carlo (MCMC) methods.

3.2. General framework : Particle approximations of Feynman–Kac flows

The following abstract point of view, developed and extensively studied by Pierre Del Moral [33], [5], has proved to be extremely fruitful in providing a very general framework to the design and analysis of numerical approximation schemes, based on systems of branching and / or interacting particles, for nonlinear dynamical systems with values in the space of probability distributions, associated with Feynman–Kac flows of the form

$$\langle \mu_n, f \rangle = \frac{\langle \gamma_n, f \rangle}{\langle \gamma_n, 1 \rangle} \qquad \text{and} \qquad \langle \gamma_n, f \rangle = \mathbb{E}[f(X_n) \prod_{k=0}^{n} g_k(X_k)] \ ,$$

where X_n denotes a Markov chain with (possibly) time dependent state spaces E_n and with transition kernels Q_n, and where the nonnegative potential functions g_n play the role of selection functions. Feynman–Kac flows naturally arise whenever importance sampling is used : this applies for instance to simulation of rare events, to filtering, i.e. to state estimation in hidden Markov models (HMM), etc. Clearly, the unnormalized linear flow satisfies the dynamical system

$$\langle \gamma_n, f \rangle = \langle \gamma_{n-1}, Q_n(g_n\, f) \rangle = \langle \gamma_{n-1}, R_n\, f \rangle \quad ,$$

with the nonnegative kernel $R_n(x, dx') = Q_n(x, dx')\, g_n(x')$, and the associated normalized nonlinear flow of probability distributions satisfies the dynamical system

$$\langle \mu_n, f \rangle = \frac{\langle \mu_{n-1}, Q_n(g_n\, f) \rangle}{\langle \mu_{n-1}, Q_n\, g_n \rangle} = \langle \overline{R}_n(\mu_{n-1}), f \rangle \qquad \text{where} \qquad \overline{R}_n(\mu) = \frac{\mu\, R_n}{\langle \mu\, R_n, 1 \rangle} \quad ,$$

which can be decomposed in the following two steps

$$\mu_{n-1} \mapsto \eta_n = \mu_{n-1}\, Q_n \mapsto \mu_n = g_n \cdot \eta_n \quad .$$

Conversely, the normalizing constant $\langle \gamma_n, 1 \rangle$, hence the unnormalized (linear) flow as well, can be expressed in terms of the normalized (nonlinear) flow : indeed $\langle \gamma_n, 1 \rangle = \langle \eta_0, g_0 \rangle \cdots \langle \eta_n, g_n \rangle$. To solve these equations *numerically*, and in view of the basic assumption that it is easy to *simulate* r.v.'s according to the probability distributions $Q_n(x, dx')$, i.e. to mimic the evolution of the Markov chain, the original idea behind particle methods consists of looking for an approximation of the probability distribution μ_n in the form of a (possibly weighted) empirical probability distribution associated with a system of particles :

$$\mu_n \approx \mu_n^N = \sum_{i=1}^N w_n^i\, \delta_{\xi_n^i} \qquad \text{with} \qquad \sum_{i=1}^N w_n^i = 1 \quad .$$

The approximation is completely characterized by the set $\Sigma_n = (\xi_n^i, w_n^i\, , \, i = 1, \cdots, N)$ of particle positions and weights, and the algorithm is completely described by the mechanism which builds Σ_k from Σ_{k-1}. In practice, in the simplest version of the algorithm, known as the *bootstrap* algorithm, particles

- are selected according to their respective weights (selection step),
- move according to the Markov kernel Q_k (mutation step),
- are weighted by evaluating the fitness function g_k (weighting step).

The algorithm yields a numerical approximation of the probability distribution μ_n as the weighted empirical probability distribution μ_n^N associated with a system of particles, and many asymptotic results have been proved as the number N of particles (sample size) goes to infinity, using techniques coming from applied probability (interacting particle systems, empirical processes [62]), see e.g. the survey article [33] or the recent textbook [5], and references therein : convergence in $\mathbb{L}^p$, convergence as empirical processes indexed by classes of functions, uniform convergence in time (see also [9], [55]), central limit theorem (see also [53]), propagation of chaos, large deviations principle, moderate deviations principle (see [35]), etc. Beyond the simplest *bootstrap* version of the algorithm, many algorithmic variations have been proposed [37], and are commonly used in practice :

- in the redistribution step, sampling with replacement could be replaced with other redistribution schemes so as to reduce the variance (this issue has also been addressed in genetic algorithms),
- to reduce the variance and to save computational effort, it is often a good idea not to redistribute the particles at each time step, but only when the weights $(w_k^i\, , \, i = 1, \cdots, N)$ are too much uneven.

Most of the results proved in the literature assume that particles are redistributed (i) at each time step, and (ii) using sampling with replacement. Studying systematically the impact of these algorithmic variations on the convergence results is still to be done. Even with interacting Monte Carlo methods, it could happen that some particle ξ_k^i generated in one time step has a negligible weight $g_k(\xi_k^i)$: if this happens for too many particles in the sample $(\xi_k^i\, , \, i = 1, \cdots, N)$, then computer power has been wasted, and it has been suggested to use importance sampling again in the mutation step, i.e. to let particles explore the state space under the action of an alternate wrong mutation kernel, and to weight the particles according to their likelihood for the true model, so as to compensate for the wrong modeling. More specifically, using an arbitrary importance decomposition

$$R_k(x, dx') = Q_k(x, dx')\, g_k(x') = W_k(x, x')\, P_k(x, dx') \ ,$$

results in the following general algorithm, known as the *sampling with importance resampling* (SIR) algorithm, in which particles

- are selected according to their respective weights (selection step),
- move according to the importance Markov kernel P_k (mutation step),
- are weighted by evaluating the importance weight function W_k on the resulting transition (weighting step).

3.3. Statistics of HMM

Keywords: *asymptotic statistics*, *exponential forgetting*, *exponential stability*, *hidden Markov model (HMM)*, *local asymptotic normality (LAN)*.

Hidden Markov models (HMM) form a special case of partially observed stochastic dynamical systems, in which the state of a Markov process (in discrete or continuous time, with finite or continuous state space) should be estimated from noisy observations. The conditional probability distribution of the hidden state given past observations is a well–known example of a normalized (nonlinear) Feynman–Kac flow, see 3.2. These models are very flexible, because of the introduction of latent variables (non observed) which allows to model complex time dependent structures, to take constraints into account, etc. In addition, the underlying Markovian structure makes it possible to use numerical algorithms (particle filtering, Markov chain Monte Carlo methods (MCMC), etc.) which are computationally intensive but whose complexity is rather small. Hidden Markov models are widely used in various applied areas, such as speech recognition, alignment of biological sequences, tracking in complex environment, modeling and control of networks, digital communications, etc.

Beyond the recursive estimation of a hidden state from noisy observations, the problem arises of statistical inference of HMM with general state space, including estimation of model parameters, early monitoring and diagnosis of small changes in model parameters, etc.

Large time asymptotics A fruitful approach is the asymptotic study, when the observation time increases to infinity, of an extended Markov chain, whose state includes (i) the hidden state, (ii) the observation, (iii) the prediction filter (i.e. the conditional probability distribution of the hidden state given observations at all previous time instants), and possibly (iv) the derivative of the prediction filter with respect to the parameter. Indeed, it is easy to express the log–likelihood function, the conditional least–squares criterion, and many other clasical contrast processes, as well as their derivatives with respect to the parameter, as additive functionals of the extended Markov chain.

The following general approach has been proposed :

- first, prove an exponential stability property (i.e. an exponential forgetting property of the initial condition) of the prediction filter and its derivative, for a misspecified model,
- from this, deduce a geometric ergodicity property and the existence of a unique invariant probability distribution for the extended Markov chain, hence a law of large numbers and a central limit theorem for a large class of contrast processes and their derivatives, and a local asymptotic normality property,
- finally, obtain the consistency (i.e. the convergence to the set of minima of the associated contrast function), and the asymptotic normality of a large class of minimum contrast estimators.

This programme has been completed in the case of a finite state space [7], and has been generalized in [36] under an uniform minoration assumption for the Markov transition kernel, which typically does only hold when the state space is compact. Clearly, the whole approach relies on the existence of an exponential stability property of the prediction filter, and the main challenge currently is to get rid of this uniform minoration assumption for the Markov transition kernel [32], [9], so as to be able to consider more interesting situations, where the state space is noncompact.

Small noise asymptotics Another asymptotic approach can also be used, where it is rather easy to obtain interesting explicit results, in terms close to the language of nonlinear deterministic control theory [52]. Taking the simple example where the hidden state is the solution to an ordinary differential equation, or a nonlinear state model, and where the observations are subject to additive Gaussian white noise, this approach consists in assuming that covariances matrices of the state noise and of the observation noise go simultaneously to zero. If it is reasonable in many applications to consider that noise covariances are small, this asymptotic approach is less natural than the large time asymptotics, where it is enough (provided a suitable ergodicity assumption holds) to accumulate observations and to see the expected limit laws (law of large numbers, central limit theorem, etc.). In opposition, the expressions obtained in the limit (Kullback–Leibler divergence, Fisher information matrix, asymptotic covariance matrix, etc.) take here a much more explicit form than in the large time asymptotics.

The following results have been obtained using this approach :

- the consistency of the maximum likelihood estimator (i.e. the convergence to the set M of global minima of the Kullback–Leibler divergence), has been obtained using large deviations techniques, with an analytical approach [49],
- if the abovementioned set M does not reduce to the true parameter value, i.e. if the model is not identifiable, it is still possible to describe precisely the asymptotic behavior of the estimators [50] : in the simple case where the state equation is a noise–free ordinary differential equation and using a Bayesian framework, it has been shown that (i) if the rank r of the Fisher information matrix $\mathcal{I}$ is constant in a neighborhood of the set M, then this set is a differentiable submanifold of codimension r, (ii) the posterior probability distribution of the parameter converges to a random probability distribution in the limit, supported by the manifold M, absolutely continuous w.r.t. the Lebesgue measure on M, with an explicit expression for the density, and (iii) the posterior probability distribution of the suitably normalized difference between the parameter and its projection on the manifold M, converges to a mixture of Gaussian probability distributions on the normal spaces to the manifold M, which generalized the usual asymptotic normality property,
- it has been shown in [56] that (i) the parameter dependent probability distributions of the observations are locally asymptotically normal (LAN) [54], from which the asymptotic normality of the maximum likelihood estimator follows, with an explicit expression for the asymptotic covariance matrix, i.e. for the Fisher information matrix $\mathcal{I}$, in terms of the Kalman filter associated with the linear tangent linear Gaussian model, and (ii) the score function (i.e. the derivative of the log–likelihood function w.r.t. the parameter), evaluated at the true value of the parameter and suitably normalized, converges to a Gaussian r.v. with zero mean and covariance matrix $\mathcal{I}$.

4. Application Domains

4.1. Localisation, navigation and tracking

Keywords: *localisation*, *navigation*, *tracking*.

See 5.1.

Among the many application domains of particle methods, or interacting Monte Carlo methods, ASPI has decided to focus on applications in localisation (or positioning), navigation and tracking [44], which already covers a very broad spectrum of application domains. The objective here is to estimate the position (and also velocity, attitude, etc.) of a mobile object, from the combination of different sources of information, including

- a prior dynamical model of typical evolutions of the mobile,
- measurements provided by sensors,
- and possibly a digital map providing some useful feature (altitude, gravity, power attenuation, etc.) at each possible position,

This Bayesian dynamical estimation problem is also called filtering, and its numerical implementation using particle methods, known as particle filtering, has found applications in target tracking, integrated navigation, points and / or objects tracking in video sequences, mobile robotics, wireless communications, ubiquitous computing and ambient intelligence, sensor networks, etc. Particle filtering was definitely invented by the target tracking community [43], [59], which has already contributed to many of the most interesting algorithmic improvements and is still very active. Beyond target tracking, ASPI is also considering various possible applications of particle filtering in positioning, navigation and tracking, see 7.1.

4.2. Natural resources management and environmental sciences

Keywords: *Bayesian estimation*, *Markov chain Monte Carlo (MCMC)*, *particle filtering*, *renewable resource management*.

Participant: Fabien Campillo.

See 6.4.

We focus on renewable biomass resources (fishery, forest, plant) and agricultural dynamics. Environmental applications give raise to problems with short observations series with a few tens of measurements obtained every day, month or year. Moreover these measurements are of poor quality. The Bayesian inference is adapted to this framework : for a set of observations, we propose a hidden Markov model with given initial law and some structural parameters whose prior laws are given. Monte Carlo methods like Markov chain Monte Carlo (MCMC) and sequential Monte Carlo (particle filtering) should be customized to the present set up [21]. This activity relies on different collaborations with Agrocampus de Rennes, University of Fianarantsao in Madagascar, INRA and CIRAD in Montpellier. It is also supported by the SARIMA program 8.3.

5. Software

5.1. Demos

Participant: François Le Gland [corresponding person].

See 4.1.

To illustrate that particle filtering algorithms are efficient, easy to implement, and extremely visual and intuitive by nature, several demos are available on the site http://www.irisa.fr/aspi/demos/, for localisation, navigation and tracking problems in complex environments, with many geometrical constraints, that would be very difficult to solve with usual Kalman filters. This material has proved very useful in training sessions and seminars that have been organized in response to a demand from an industrial partner (SAGEM, CNES and EDF), and also in teaching. At the moment, the following three demos are available :

Terrain–aided navigation of an aircraft Inertial position and velocity estimates are known to drift away from their true values, and need to be combined with some external source of information. In this demo, noisy measurements of the terrain height below an aircraft are obtained as the difference between (i) the aircraft altitude above the sea level (provided by a pression sensor) and (ii) the aircraft altitude above the terrain (provided by an altimetric radar), and are compared to the terrain height in any possible point (read on the elevation map). A cloud (swarm) of particles explores various possible trajectories generated from inertial navigation estimates and from a model of inertial navigation errors, and are replicated or discarded depending on whether the terrain height below the particle (i.e. at the same horizontal position) matches or not the available noisy measurement of the terrain height below the aircraft.

Positioning and tracking in the presence of obstacles In this demo, several stations cooperate to locate and track a mobile from noisy angle measurements, in the presence of obstacles (walls, tunnels, etc), which make the mobile temporarily invisible from one or several stations.

Indoor navigation of a mobile robot In this demo, a mobile robot is finding its way inside a building, a digital map of which (including walls, doorways, etc.) is provided. The initial position, velocity and orientation of the robot are unknown, and noisy measurements of its rotation and linear displacement are given by an odometer. In addition, a ring of laser sensors detects with some error the distance from the robot to obstacles in sixteen different directions. A cloud (swarm) of particles explores various possible trajectories generated from odometer navigation estimates and from a model of odometer navigation errors, and are replicated or discarded depending on whether the distance from the particle to obstacles matches or not the available noisy measurement of the distance from the robot to the obstacles, in all sixteen directions, and depending also on whether the generated trajectories are compatible with the presence of obstacles.

6. New Results

6.1. Rare event simulation by MCMC in trajectory space

Keywords: *Metropolis–Hastings dynamics*, *population Monte–Carlo*, *rare event*.

Participants: Frédéric Cérou, Arnaud Guyader, Valentine Méar, Julia Charrier.

The estimation of rare event probabilities is a crucial issue in areas such as reliability, telecommunication networks, air traffic management, etc. In complex systems, analytical methods cannot be used, and naive Monte Carlo methods are clearly unefficient to estimate accurately probabilities of order less than 10^{-9}, say. Beside importance sampling, a widespread technique is multilevel splitting, which requires at least some knowledge of the system, to decide where to place the intermediate level sets. In both cases, we need to have some knowledge of the process to find a good set of parameters to run the algotirhm. Moreover, in the case of a rare event to happen before some deterministic final time, we also need the transient behavior of the process, even if it is ergidic. But usually this transient behavior is very difficult to characterize.

To circumvent these difficulties, we have adopted a completly different approach. We assume that the process is in discrete time, such that a trajectory up to final time T is a vector of some finite dimensional space, and we propose to use MCMC approaches to simulate trajectories, given the rare event. Basically, when a proposed transition in the trajectory space is out of a given set, then it is rejected with probability 1. In this very preliminary work, the plain Metropolis–Hastings algorithm showed inefficient because of the large dimension of the state space, but some population Monte Carlo variants showed large improvements. The main difficulty here is to built an efficient Metropolis–like algorithm in very large dimension.

6.2. Particle approximations of Feynman–Kac distributions depending on a parameter

Keywords: *Monte Carlo maximum likelihood (MCML)*, *hidden Markov model (HMM)*.

Participants: François Le Gland.

This is a collaboration with Nadia Oudjane, from the OSIRIS (Optimisation, simulation, risque et statistiques) department of Électricité de France R&D.

In full generality, given nonnegative kernels $R_1, \cdots, R_n$ and a nonnegative measure γ_0, we consider the unnormalized (linear) Feynman–Kac distribution

$$\langle \gamma_n, f \rangle = \int_{E_n} \cdots \int_{E_0} f(x_n) \; \prod_{k=1}^{n} R_k(x_{k-1}, dx_k) \; \gamma_0(dx_0) \quad .$$

A well–known example is provided by the unnormalized conditional probability distribution of the hidden state given past observations, when the hidden state and the observation form jointly a Markov chain : this includes HMM and switching AR models as special cases, with the decomposition $R_k(x, dx') = Q_k(x, dx')\, g_k(x')$ where Q_k is the Markov transition kernel and where the selection function g_k is the likelihood function.

If the nonnegative kernels depend on a parameter, in such a way that the Feynman–Kac distribution is continuous or differentiable w.r.t. the parameter, we would like to design a particle approximation that would have the same regularity properties. The need for such a regularity property arises for instance

- in sensitivity analysis, e.g. in the computation of Greeks, in option pricing,
- in statistics of HMM, see 3.3, e.g. in the evaluation of the derivative w.r.t. the parameter of any contrast function that can be expressed in terms of the conditional probability distribution of the hidden state given past observations.

The smooth particle approximation introduced earlier has been further studied, where a unique interacting particle system is propagated for a given reference value of the parameter, and where importance weights are computed separately for each value of the parameter in a neighborhood of the reference value. Differentiating these importance weights w.r.t. the parameter yields a particle approximation of the linear tangent Feynman–Kac distribution, which coincides with an earlier approach followed in the team, where a particle approximation is derived directly from the linear tangent Feynman–Kac flow. The new results obtained this year for the joint particle approximation of the Feynman–Kac distribution and the linear tangent Feynman–Kac distribution are

- a central limit theorem,
- uniform $\mathbb{L}^p$ error estimates over a neighborhood of the reference value of the parameter,

using an original and promising technique where the importance weights are incorporated into the state variable. A Rao–Blackwellized version of the particle approximation has also been studied, when the underlying state–space model is conditionnally linear Gaussian.

6.3. Sequential data assimilation : ensemble Kalman filter vs. particle filter

Keywords: *data assimilation*, *ensemble Kalman filter (EnKF)*.

Participants: François Le Gland, Vu Duc Tran.

Our first step has been to better understand on simple examples [20] such as the three–dimensionnal Lorenz model, the qualitative behaviour and the performance of the ensemble Kalman filter [38], [39] and other sequential data assimilation methods, and to compare these with several different particle filters. If the Lorenz model is observed at low rate, then the bootstrap particle filters, with or without redistribution, in which particles are propagated with the prediction model only, perform rather poorly, and more advanced particle filters should be used, in which particles are propagated with an observation–driven model. In terms of performance, these more advanced particle filters, with or without redistribution, seem very close to the ensemble Kalman filter.

This preliminary work has motivated our current interest to study the asymptotic behaviour of the ensemble Kalman filter, as the number of ensemble elements increases to infinity. Indeed, very little is known about this question, whereas on the other hand, the asymptotic behaviour of the many different brands of particle filters, as the number of particles goes to infinity, is well understood.

6.4. Monte Carlo methods for Bayesian inference

Keywords: *Bayesian estimation*, *Markov chain Monte Carlo (MCMC)*, *Metropolis-Hastings*, *interacting Monte Carlo methods*, *particle methods*.

Participants: Fabien Campillo, Vivien Rossi.

Markov chain Monte Carlo (MCMC) algorithms [61], [28], [60] allow us to draw samples from a probability distribution π known up to a multiplicative constant. They consist in sequentially simulating a single Markov chain whose limit distribution is π. There exist many techniques to speed up the convergence towards the target distribution by improving the mixing properties of the chain.

An alternative is to run many Markov chains in parallel. The simplest multiple chains algorithm is to make use of parallel independent chains. The recommendations concerning this idea seem contradictory in the literature, as shown by the *many short runs vs. one long run* debate. We can note that independent parallel chains could be a poor idea: among these chains some may not converge, so one long chain could be preferable to many short ones. Moreover, many parallel independent chains can artificially exhibit a more robust behavior which does not correspond to a real convergence of the algorithm.

In practice one however makes use of several chains in parallel. It is then tempting to exchange information between these chains to improve mixing properties of the MCMC samplers. A general framework of population Monte Carlo has been proposed in this context [47], [30].

In the present work [23] we propose an interacting method between parallel chains which provides an independent sample from the target distribution. Contrary to papers previously cited, the proposal law in our work is given and does not adapt itself to the previous simulations. Hence, the problem of the choice of this law still remains.

6.5. Bayesian inference for renewable resources models

Keywords: *Bayesian estimation*, *Markov chain Monte Carlo (MCMC)*, *Metropolis within Hastings*, *Ricker model*, *fishery models*, *interacting Monte Carlo methods*.

Participants: Philippe Cantet, Fabien Campillo, Vivien Rossi, Rivo Rakotozafy.

The approach proposed in 6.4 was applied to the Ricker fishery recruitment model [16], [21]. Markov chain Monte Carlo (MCMC) methods are now widely used to study the evolution of natural resources, such as biomass in fisheries or the dynamics of forests. Although flexible, these methods have however low convergence speeds. By contrast, particle filtering methods are fast but are not well adapted to these models, where measurements are sampled daily, monthly or yearly. Therefore there is no need to treat these measurements in a recursive way, and it seems more appropriate to call upon Monte Carlo methods in interaction, such as population Monte Carlo [5]. In the first case study particle filtering has been applied to calibrate tree-growth models, which are complex and highly nonlinear. MCMC (Metropolis–Hastings) techniques have been compared with particle filtering methods for fitting and identifying such model. The first results showed that MCMC methods are more consuming in terms of computational time. By comparison, particle filtering is faster and less sensitive to prior knowledge, but it also presents a greater variance than the MCMC methods.

6.6. Convolution filter based methods for parameter estimation in general state-space models

Keywords: *Bayesian estimation*, *convolution particle methods*.

Participants: Fabien Campillo, Vivien Rossi.

Consider a hidden Markov model depending on an unknown parameter. The goal is to estimate simultaneously the parameter and the state process based on the observations. In the Bayesian approach, the augmented state variable includes the parameter and is processed by a filtering procedure. These methods suppose that a prior law is given for the parameter and are performed on–line. It is well known that the standard bootstrap particle filter exhibits degenerate behavior in this situation. In [22], [17] we proposed a convolution particle filter that avoid this difficulty.

7. Contracts and Grants with Industry

7.1. Localization and tracking of mobile terminals — contract with FTRD

Participants: Fabien Campillo, François Le Gland, Julien Guillet.

See 4.1.

Contract ALLOC 851 — May 2005/August 2006

The objective was to implement and assess the performance of particle filtering in localisation and tracking of mobile terminals in a wireless network, using network measurements (received signal power strength and possibly TOA (time of arrival)) and a database of reference measurements of the signal power strength, available in a few points or in the form of a digital map (power attenuation map). Generic algorithms have been specialized to the indoor context (wireless local area network, e.g. WiFi) and to the outdoor context (cellular network, e.g. GSM). In particular, constraints and obstacles such as building walls in an indoor environment, street, road or railway networks in an outdoor environment, have been represented in a simplified manner, using a prior model on a graph, e.g. a Voronoï graph as in similar experiments in mobile robotics [57]. To assess the performance of the proposed localisation and tracking algorithms, posterior Cramèr–Rao bounds for a Markov process on a graph have been derived.

The findings of this work is that localisation in outdoor applications using measurements of the signal power strength alone is not yet operational, whereas the situation is much more favorable in indoor applications. This is because the digital maps available for GSM are obtained by running numerical propagation models that do not capture small scale variations, and the solution would be to use additional measurements, such as TOA (time of arrival). A follow–up objective objective would be to update and enrich the initial database of reference measurements, using network measurements collected on–the–fly.

8. Other Grants and Activities

8.1. Data assimilation for air quality (ADOQA) — ARC INRIA

Participants: Fabien Campillo, François Le Gland, Vu Duc Tran.

January 2005/December 2006.

This ARC is coordinated by the project–team CLIME from INRIA Rocquencourt and CEREA / ENPC. Its partners are the project–team IDOPT from INRIA Rhône–Alpes, and INERIS.

One objective of ADOQA is to investigate advanced sequential methods (as opposed to variational methods) for data assimilation of intrinsically nonlinear models, i.e. coupling of numerical models and measured data. In principle, a data assimilation algorithm should propagate uncertainties through the probability distribution of the state variables, whereas current sequential algorithms, such as the Kalman filter and its simplest extensions, only propagate the first two moments. For large–scale systems (physical state of the atmosphere, of the ocean, chemical composition of the atmosphere, etc.), the direct implementation of sequential Monte Carlo methods seems impractical, and simplified, reduced–order models should be used.

Our contribution has been to better understand and compare on simple examples [20] such as the three–dimensionnal Lorenz model, the qualitative behaviour and the performance of the ensemble Kalman filter [38], [39] and other sequential data assimilation methods, with the qualitative behaviour and the performance of particle filters.

8.2. Monte Carlo methods for rare event simulation (RARE) — ARC INRIA

Participants: Frédéric Cérou, Pierre Del Moral, François Le Gland, Arnaud Guyader.

January 2006/December 200R.

This ARC is coordinated by the project–team ARMOR from IRISA / INRIA Rennes. The academic partners are the project–team MATHFI of INRIA Rocquencourt and CERMICS / ENPC, the project–team OMEGA of INRIA Sophia Antipolis and INRIA Lorraine, the project–team MESCAL of INRIA Rhône–Alpes, and the industrial partners are Électricité de France RD, and CENA / DGAC, and its international partners are CWI (Netherlands) and University of Bamberg (Germany).

The objective of RARE is to design and evaluate various Monte Carlo techniques (importance sampling, importance splitting, cross–entropy, etc.), for the simulation of rare but critical events, in several important domain of applications (communication networks, financial risk management, air traffic management, etc.).

Our contribution has been to better understand the asymptotic behaviour of importance splitting methods [18], [14], where intermediate less critical events are introduced, and where trajectories that manage to reach an upper level are replicated into a number of offsprings. Splitting can be achieved in many different ways : in *fixed splitting* for instance, each successful trajectory is given a prescribed deterministic number (possibly depending on the generation number) of offsprings, whereas in *fixed effort* splitting, there is a prescribed deterministic number of trajectories alive at each generation, which amounts to sample with replacement from the successful trajectories at the current stage of the algorithm, and in *fixed performance* splitting a random number of trajectories is simulated, until a prescribed deterministic number of successful trajectories is obtained [15].

It appears that importance splitting can be interpreted in terms of Feynman–Kac distributions, which makes it possible not only to approximate the probability of the rare but critical event, but also to learn which critical trajectories are responsible for the critical event to occur [12]. Challenging issues that are investigated here include the automatic selection of the intermediate sets and their number : asymptotic results have been obtained in the one–dimensional case [31], while in the multi–dimensional case a preliminary objective would be the efficient choice of the importance function, used to define the intermediate sets as level sets.

Another idea would be to use here the concept of adaptive redistribution, routinely used in particle filtering, without any clear mathematical justification : independent trajectories would be simulated, as long as an appropriate criterion (interpreted as the normalisation constant in the Feynman–Kac formulation) remains below a given threshold, and would be replicated / terminated when the monitored criterion would exceed this threshold.

8.3. SARIMA and SARIMA–Madagascar

Participant: Fabien Campillo.

See 4.2 and 6.5.

Within the SARIMA program, Fabien Campillo develops collaborations with the universities of Antananarivo and Fianarantsoa in the field of the probabilistic modeling and the numerical statistical inference for environmental sciences and development. Within this program, Rivo Rakotozafy spend three months per year (one month and a half in 2006) within ASPI in order to prepare an HDR (habilitation à diriger les recherches) under the supervision of Fabien Campillo. The HDR defense is planed for 2008. The results of this collaboration have been presented to the CARI conference in Cotonou [16], see also [21].

9. Dissemination

9.1. Scientific animation

F. Campillo is a member of the committee for the PhD thesis of Ghislain Verdier (École Nationale Supérieure Agronomique de Montpellier and INRA, advisor : Jean–Pierre Vila). He is a member of the «conseil de laboratoire» of IRISA (UMR 6074) and of the «conseil de l'école doctorale de physique, modélisation et sciences pour l'ingénieur de Marseille». He is the INRIA representative for scientific relations with Madagascar, within the SARIMA (support to research activities in computer science and mathematics in Africa) project supported by INRIA and MAE (ministère des affaires étrangères), see 8.3. In relation with this activity, he has spent three weeks in Antananarivo and Fianarantsoa in November and December 2006.

P. Del Moral has organized an invited session on particle methods applied to engineering and physics at the «Journées du groupe MAS» held in Lille in September 2006.

A. Guyader is the coordinator of a reading group at université de Rennes 2, on functional data analysis.

F. Le Gland has reported on the PhD theses of Jaroslav Krystul (Twente University, advisor : Arun Bagchi) and Agnès Lagnoux (université Paul Sabatier, Toulouse, advisors : Dominique Bakry and Pascal Lezaud). He is a member of the committee for the PhD theses of Olivier Rabaste (université de Rennes 1 and ENST Bretagne, advisor : Thierry Chonavel) and Anne Cuzol (université de Rennes 1 and IRISA, advisor : Étienne Mémin), and a member of the committe for the HDR (habilitation à diriger les recherches) of Bruno Tuffin (université de Rennes 1). He has organized an invited session on particle filtering at the XXVI European Meeting of Statisticians held in Toruń in July 2006.

A. Guyader and F. Le Gland are members of the «commission de spécialistes» in applied mathematics (section 26) of université de Rennes 2.

9.2. Teaching

F. Campillo gives a course on Markov models, hidden Markov models, filtering and particle filtering at université de Sud Toulon–Var, within the Master «Mathématiques (Filtrage et traitement des données)» and the Master «Sciences et Technologies (Sciences de la mer, environnement, systèmes)».

F. Le Gland gives a course on Kalman filtering, particle filtering and hidden Markov models, at université de Rennes 1, within the Master STI (école doctorale MATISSE), a 3rd year course on Bayesian filtering and particle approximation, at ENSTA, Paris, within the quantitative finance track, and a 3rd year course on hidden Markov models, at ENST Bretagne, Brest.

9.3. Participation in workshops, seminars, lectures, etc.

Several members of ASPI have given talks in the IRMAR working group on Feynman–Kac formulæ : F. Campillo about particle filtering in practice, F. Le Gland about examples of applications of Feynman–Kac formulæ, and A. Guyader about Feynman–Kac Metropolis algorithms.

In addition to presentations with a publication in the proceedings, and which are listed at the end of the document, members of ASPI have also given the following presentations.

F. Campillo has given talks on Bayesian inference for renewable resource models in the «Statistiques» seminar at ENSAI in March 2006, and in the «Modèles Statistiques à Structure(s) Cachée(s)» seminar at Institut de Modélisation et Mathématique in Montpellier, in October 2006. He has presented the joint work on convolution particle filtering for parameter estimation in general state–space models at the 45th IEEE Conference on Decision and Control (CDC), held in San Diego in December 2006. He has given seminars about Bayesian inference in environemental sciences in the University of Antananarivo and the University of Fianarantsao, Madagascar, in November 2006.

F. Cérou has given a talk on adaptive multilevel splitting for rare event analysis at the ARC RARE (Monte Carlo methods for rare event analysis) kick–off meeting at IRISA in April 2006, see 8.2, and a talk on recent improvements to importance splitting at the 6th International Workshop on Rare Event Simulation (RESIM) held in Bamberg in October 2006.

A. Guyader has given talks on adaptive multilevel splitting for rare event analysis at the «Probabilités et Statistiques» seminar of université de Nice–Sophia Antipolis in April 2006, at the workshop on sequential Monte Carlo methods organized in Oxford in July 2006, at the «Journées du groupe MAS de la SMAI» held in Lille in September 2006, and at the «Probabilités et Statistiques» seminar of université de Lille 1 in December 2006. He has given a talk on nearest neighbor classification in infinite dimension at the «4èmes Journées de Statistique Fonctionnelle et Opératorielle» held in Grenoble in June 2006.

F. Le Gland has given talks on the multilevel splitting approach to rare event simulation at the ARC RARE (Monte Carlo methods for rare event analysis) kick–off meeting at IRISA in April 2006, see 8.2, and at the mini symposium on Stochastic System Theory held at Twente University in September 2006. He has given a talk on particle approximations of Feynman–Kac distributions depending on a parameter at the workshop on sequential Monte Carlo methods organized in Oxford in July 2006. He has given a talk on using noisy georeferenced information sources for navigation and tracking at the «Journées du groupe MAS de la SMAI» held in Lille in September 2006, and at the IEEE workshop on Nonlinear Statistical Signal Processing held in Cambridge in September 2006. He has given a talk on ensemble Kalman filter vs. particle filters at the ARC ADOQA (Data assimilation for air quality) final meeting at INRIA Rhône Alpes in October 2006, see 8.1. He has been one of the main lecturers in the CEA–EDF–INRIA summer school on «Assimilation de données dans la simulation numérique» organized in Saint Lambert in June / July 2006, where he has given a course on particle filtering. J. Guillet has organized the practical sessions related with these lectures.

9.4. Visits and invitations

F. Campillo has been invited in July and September 2006 in the UMR «Analyse des Systèmes et Biométrie» (ASB) at INRA Montpellier. He has spent three weeks in Madagascar in November and December 2006 for the SARIMA project, see 8.3.

F. Le Gland has been invited in November 2006 by Anastasia Papavasiliou in the Department of Statistics of the University of Warwick, and has given there a talk in the CRiSM (Centre for Research in Statistical Methodology) seminar on adapting the number of particles in Monte Carlo methods with interaction.

Sacha Veretennikov, professor in the Department of Statistics of the University of Leeds has visited ASPI for two days in March 2006, and has given a talk on discrete time ergodic filters with wrong initial data.

Josselin Garnier, professor at université de Paris 7 has been invited by ASPI in June 2006 to give a talk on rare event simulation.

Pavel Chigansky, post–doc in the Department of Mathematics of Université du Maine has visited ASPI for two days in November 2006, and has given a talk on the stability of the Bayesian optimal filter with respect to its initial condition.

Rivo Rakotozafy, assistant professor at the University of Fianaranstoa, has been awarded by the French embassy in Antananarivo, Madagascar a grant to support visits to prepare an HDR (habilitation à diriger les recherches) in Madagascar, under the supervision of Fabien Campillo. A related objective is to set up a collaboration between the University of Fianaranstoa and INRIA.

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