

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# *Team Complex*

# Evolution Artificielle et Fractales

Rocquencourt



# **Table of contents**

	Team											
2.	Overall Objectives	1										
	2.1. Overall Objectives	1										
3.	Scientific Foundations	2										
	3.1. Pointwise regularity	2										
	3.2. Multifractal analysis	3 4										
	B. Fractal Processes											
	4. Evolutionary Algorithms, Genetic Algorithms											
	3.5. New evolutionary models	5										
4.	Application Domains											
	4.1. TCP Traffic	5										
	4.2. Signal and Image analysis	6										
_	4.3. Complex interactions	7										
5.	Software											
	5.1. FRACLAB: a Fractal Matlab/Scilab toolbox	7										
	5.2. EASEA : an evolutionary algorithms specification language	8										
	5.3. XCLE - eXtensible Concatenative Language Engine	9										
6.	New Results											
	6.1. Optimization of the bias-variance trade-off for the estimation of the Hölder function	9										
	6.2. Dimension Spectrum for Self-conformal Measures	10										
	<ul><li>6.3. Multifractal analysis of Lévy processes</li><li>6.4. Definition and analysis of processes with prescribed local form</li></ul>	10 11										
	6.5. Multifractal Strings	11										
	6.6. Pointwise Regularity of Fitness Landscapes and adjustment of a Simple ES	11										
	6.7. Cooperative Royal Road Functions	12										
	6.8. Analysis of crossover behaviour for Genetic Programming	13										
	6.9. Overcompression of JPEG images	13										
	6.10. Evolutionary Multifractal Signal Enhancement	13										
	6.11. 2-microlocal signal denoising	15										
	6.12. Texture image segmentation with an evolutionary approach.	15										
	6.13. Stereovision and robot control by the Fly Algorithm	16										
	6.14. Evolutionary mobile robot navigation based on visual landmark detection	16										
	6.15. Modelling Termite Nest formation	17										
	6.16. Interactive Optimisation of Cochlear Implants	17										
	6.17. Optimisation of an E-Learning System	18										
	6.18. Ants paintings on Iterated Function Systems	19										
	6.19. CONSENSUS : Interactive optimisation of a resource allocation problem	19										
	6.20. ArtiE-Fract Improvement	20										
7.	Contracts and Grants with Industry											
	7.1. Contracts and Grants with Industry	22										
8.	Other Grants and Activities											
	8.1. National initiatives	22										
	8.2. European initiatives	22										
	8.3. International initiatives	22										
9.	Dissemination											
	9.1. Organization committees	23										
	9.2. Editorial Boards	23										
	9.3. Other Teaching	23										
	9.4. Invited talks and Scientific popularisation	23										

	9.5.	Ph.D.	These	es																	24	4
10.	Bib	liograp	ohy		 	24	4															

# 1. Team

## Head of project team

Evelyne Lutton [ Research scientist, INRIA, HdR ]

#### Vice head of project team

Jacques Lévy Véhel [ Research scientist, INRIA, HdR ]

#### Administrative Assistant

Nathalie Gaudechoux [ shared with Eiffel2 ]

#### **Associated Engineer**

Loc Fosse [ 18/09/2005 to 17/09/2007 ]

#### **Expert Engineers**

André Balsa [ 17/05/2006 to 16/11/2006 ] Jonathan Chapuis [ 15/02/2005 to 14/02/2006 ] Pierre Grenier [ 15/02/2005 to 14/02/2006 ]

#### Research scientists (external)

Emmanuel Cayla [ Cétoine, L'atelier des Fractales ] Jean Louchet [ Ing. en chef Armement (CR) ] Jean-Marie Rocchisani [ Université Paris XIII ]

#### **Invited researcher**

Michal Rams [ Impan, Warsaw, Poland, 04/01/06 to 28/02/06 ] Franklin Mendivil [ Acadia University, Canada, 04/10/06 to 29/11/06 ] Gabriela Ochoa [ University Simon Bolivar - Caracas, Venezuela, 01/04/06 to 30/06/06 ] Gustavo Olague [ CICESE, Ensenada, Mexico, 01/11/06 to 31/08/07 ] Cynthia Perez-Castro [ CICESE, Ensenada, Mexico, 01/11/06 to 31/08/07 ] Leonardo Trujillo [ CICESE, Ensenada, Mexico, 01/11/06 to 31/08/07 ]

#### **Post-doctoral**

Yann Landrin-Schweitzer [ 01/02/06 to 31/07/06 ] Pierrick Legrand [ 01/03/2006 to 31/08/2006 ]

#### Ph. D. Students

Malek Aichour [ Université de Annaba, Algrie ] Olivier Pauplin [ INRIA, Université Paris 5, avec le projet IMARA ] Grégory Valigiani [ CIFRE-Paraschool, INRIA ]

#### Students intern

Aurélie Bousquet [ 01/03/06 to 30/06/06 ] Sandrine Martin [ 10/07/06 to 31/12/06 ] Anne-Julia Stebel [ 26/06/06 to 08/09/06 ] Michel Tesmer-Tur [ 19/10/05 to 18/04/06 ]

# 2. Overall Objectives

# 2.1. Overall Objectives

**Keywords:** 2-microlocal analysis, Hölder functions, IFS, TCP traffic, evolutionary algorithm, fractal, fractal compression, fractional Brownian motion, genetic algorithm, image analysis, image compression, inverse problem, iterated functions system, large deviations, multifractal analysis, optimisation, signal analysis, stable laws, texture analysis, time-frequency analysis, wavelets.

The tools developed in the *COMPLEX* team deal with the mathematical, algorithmic and computational aspects of the modelling and analysis of complex signals. Examples include radar images, internet or road traffic data, financial series, speech or musical signals, biomedical signals and robotic data.

Research is centred on two synergetic topics:

- Fractal Analysis and Modelling: multifractal analysis, 2-microlocal analysis, fractal stochastic processes.
- Evolutionary Algorithms.

Evolutionary stochastic optimisation methods have proved efficient in the framework of fractal signals and allowed to address formerly unresolved applications. Conversely, analysing the fractal irregularity of signals brings up new elements for the theoretical understanding of evolutionary techniques. Interaction between Evolutionary Algorithms and Fractals is central to the team's research topics.

Applications developed in the team deal with:

- Image and Signals: denoising, segmentation, stereovision, audio2midi,
- Telecom: analysis and modelling of TCP traffic,
- Interactive systems: art and design, data-retrieval, e-learning and resource allocation.

The COMPLEX team also develops several freewares, most notably FRACLAB (a matlab/scilab toolbox for 1D and 2D signal processing) and EASEA (a specification language for evolutionary algorithms).

The COMPLEX team has strong collaborations with IrCcyn in Nantes, with French universities: Orsay (LRI), Calais (LIL), Clermont-Ferrand, and with several foreign universities and research centers: University of St-Andrews (Scottland), CRM Montréal (Quebec), Impan (Poland). The team is involved in the European organisation (former Network of Excellence) EVONET.

The COMPLEX team has industrial contracts with Dassault Aviation, Novartis Pharma (Switzerland), and Paraschool.

# 3. Scientific Foundations

## 3.1. Pointwise regularity

Keywords: 2-microlocal analysis, Hölder exponent, pointwise regularity.

Participants: Jacques Lévy Véhel, Pierrick Legrand.

#### In collaboration with Antoine Echelard (Irccyn).

**Abstract.** In many applications, the local regularity of a function contains information which is essential for further processing. Local regularity may be studied in several ways. We focus on Hölder exponents and 2-microlocal analysis, an extension of Hölder regularity.

Fractal properties of a signal may be analyzed a number of ways. Our team deals with two of these: Local regularity and multifractal analysis.

In the first case, to a signal f(t), one associates a function  $\alpha(t)$ , the *Hölder function* of f, which measures the regularity of f at each point t. This quantity may be evaluated with various tools. For instance, the pointwise Hölder exponent  $\alpha$  of f at  $x_0$  is defined as:

$$\alpha(x_0) = \limsup_{\alpha \to 0} \{ \alpha : \exists c > 0, |f(x) - f(x_0)| \le c |x - x_0|^{\alpha}, |x - x_0| < \rho \}$$

(this definition requires that  $\alpha$  is not an integer and that f is non differentiable).

One may also define a local exponent  $\alpha_l(x_0)$  as:

$$\alpha_l(x_0) = \lim_{\rho \to 0} \sup \left\{ \alpha : \exists c > 0, |f(x) - f(y)| \le c|x - y|^{\alpha}, |x - x_0| < \rho, |y - x_0| < \rho \right\}$$

 $\alpha$  and  $\alpha_l$  are different in general (e.g. for  $f(x) = |x|^{\alpha} \sin \frac{1}{|x|^{\beta}}$ ,  $\alpha(0) = \alpha$ , while  $\alpha_l(0) = \frac{\alpha}{1+\beta}$ ) and have very different properties. For instance,  $\alpha_l$  is stable through differentiation ( $\alpha_l(f', x_0) = \alpha_l(f, x_0) - 1$ ), as  $\alpha$  is not.

As a rule, the smaller  $\alpha(t)$ , the more irregular the function f at t. A discontinuous bounded function has exponent 0, while  $\alpha(t) > 1$  entail that f is differentiable at least once at t. Characterizing signals through their Hölderian regularity has been considered by many authors, both from a theoretical point of view (for instance in relation with wavelet decompositions) and in applications: e.g. turbulence analysis, image segmentation. Such an approach is fruitful when relevant information is contained in the irregularities of the signal, rather than, for instance, in its amplitude or Fourier contents. This occurs in particular when ones tries to detect edges in images, or to analyse non-voiced parts of speech signals. We have partially solved natural questions in this frame, including the characterisation of the Hölder functions, the comparison of the different ways to measure the local regularity, and the problem of their estimation on real signals.

A generalisation of Hölder regularity is provided by 2-microlocal analysis. This analysis allows to describe in great detail the local regularity behaviour. Our work deals with various extensions of 2-microlocal analysis, providing time domain characterisation of 2-microlocal spaces, and the estimation of 2-microlocal quantities from sampled data.

#### **3.2.** Multifractal analysis

Keywords: Hausdorff spectrum, large deviations spectrum, multifractal analysis.

Participants: Jacques Lévy Véhel, Evelyne Lutton.

# In collaboration with Yann Demichel, Claude Tricot (Université de Clermont-Ferrand), Antoine Echelard (Irccyn), Michal Rams (Impan).

**Abstract.** Multifractal analysis provides both a local and a global description of the singularities of a signal: The local description is obtained via the Hölder exponent; The global one is contained in the various multifractal spectra. These multifractal spectra describe geometrically or statistically the distribution of singularities on the support of the signal.

In some situations, the Hölder function of a signal is simple while the signal is irregular. This occurs for instance in the case of the Weierstrass function or the fractional Brownian motion, which are nowhere smooth, but whose Hölder function is constant. There are also irregular signals for which the Hölder function is even more irregular. For instance, f might be continuous but  $\alpha_f$  discontinuous everywhere. A typical example of this situation is the graph of a Fractal Interpolation Function. In such a situation, it is more rewarding to use multifractal analysis than the raw Hölder function: Basically, instead of recording, for each t, the value of the exponent, one groups all the points with same  $\alpha$  into a subset  $E_{\alpha}$ . The irregularity is then characterised globally by computing, for each  $\alpha$ , the Hausdorff dimension  $f_h(\alpha)$  of the set  $E_{\alpha}$ . Thus one evaluates geometrically the "size" of the subsets of the domain of f where a given singularity occurs. Another possibility is to use a statistical description of the distribution of the singularities: More precisely, the *large deviation multifractal spectrum*  $f_g(\alpha)$  estimates the exponential decay speed of the probability to encounter a singularity equal to  $\alpha$  at resolution n, when n tends to infinity.

This kind of analysis, which was first introduced in the study of turbulence, has undergone wide development both in theory (analysis of self-similar measures/functions, in a deterministic or stochastic frame, analysis of capacities, higher-order spectra) and in applications (study of DLA, geophysics, signal/image processing, TCP traffic analysis).

Our work in multifractal analysis deals with theoretical computation of spectra, their comparison (multifractal formalism), and the design of robust estimators in deterministic and stochastic frames.

## **3.3. Fractal Processes**

**Keywords:** *(multi-)fractional Brownian motion, Lévy processes.* **Participant:** Jacques Lévy Véhel.

In collaboration with Olivier Barrière (IrCcyn), Erick Herbin (Dassault), Kenneth Falconer (St Andrews). Abstract. Long-memory processes (i.e. those with slowly decaying autocorrelation) and processes with infinite marginal variance display interesting and sometimes counter-intuitive properties. We study certain of these processes such as (multi-)fractional Brownian motion and Lévy processes. These processes exhibit fractal properties such as self-affinity.

We study processes such as the fractional Brownian motion (fBm) or  $\alpha$ -stables processes, which exhibit fractal properties such as self-affinity  $(x(at) \stackrel{d}{=} a^H x(t))$ , where  $\stackrel{d}{=}$  means equality in distribution), local irregularity, or long range memory (i.e.  $E(x(t)x(t+\tau)) \sim |\tau|^{\beta}$  when  $\tau \to \infty$ ,  $-1 < \beta < 0$ ). These processes have two main features that make them different from << classical>> models:

- α-stables processes have, for α < 2, an infinite variance. This induces discontinuities in the sample paths.</li>
- Long-memory processes exhibit a divergence of the spectral density at 0, which translates into << pseudo-cycles >> of all sizes on the paths.

In both cases, most classical tools (central limit theorem, usual estimators) have to be adapted. Our works deal with the description of certain fractals and multifractal properties of these processes. We also develop extensions that make them more fitted to certain applications. For instance, the local regularity of fBm is almost surely the same at each point. This prevents from using fBm as a model in certain situations (e.g. TCP traffic modelling). We have defined a generalisation of fBm, called multifractional Brownian motion (mBm), which allows to control independently the Hölder exponent at each point.

## 3.4. Evolutionary Algorithms, Genetic Algorithms

**Keywords:** Evolutionary algorithms, deceptivity analysis, genetic algorithms, inverse problems, schema theory, stochastic optimisation.

Participants: Pierrick Legrand, Jacques Lévy Véhel, Yann Landrin-Schweitzer, Evelyne Lutton.

**Abstract.** When using fractal tools for the analysis of complex signals, one often have to deal with large and extremely irregular optimisation problems. Evolutionary algorithms (including Genetic Algorithms) have proven to be powerful tools in this framework, and were able to provide robust solutions, impossible to obtain with other techniques. Conversely, works performed in the team proved also that "fractal" tools were efficient to refine and complement theoretical analysis of simple evolutionary algorithms.

Genetic Algorithms (GA) and more generally evolutionary algorithms (EA) are currently known as efficient stochastic optimisation tools, and are widely used in various application domains. These techniques are based on the evolution of a population of solutions to the problem, the evolution being driven by a "fitness" function that is maximized during the process. Successive populations of solutions are built, with increasingly better fitness (the values of the fitness function increase). Their evolution is based on stochastic operators: selection (the fitness function is used as a sort of "bias" for a random selection in the population), and the "genetic" operators, mainly crossover (combination of two solutions) and mutation (perturbation of a solution). This technique is based on the assumption that well-fitted solutions (also called individuals) can provide better solutions with the help of genetic operators. This assumption can be proven to be connected to some notions of "GA-difficulty" of the function to be optimised.

Theoretical investigations on GAs and EAs generally deal with convergence analysis (and convergence speed analysis on a locally convex optimum for Evolution Strategies), influence of the parameters, GA-easy or GA-difficulty analysis. For a simple GA, analysis is based on several approaches: proof of convergence based on Markov chain modelling [41], [39], deceptive functions analysis, based on Schema analysis and Holland's original theory [43], and finally modelling as dynamical systems, where fractal-like behaviour has been exhibited [44].

From a theoretical viewpoint, some tools, developed in the framework of fractal theory, can be used in order to perform a more accurate analysis of Genetic Algorithms behaviour (mainly based on the schema theory). Actually, an analysis of how GA optimises some "fractal" functions (Hölder functions) makes it possible to model the influence of some parameters of the GA. Such an analysis can then be generalised and gives clues about how to tune some of the GA parameters in order to improve its efficiency. Finally, a further analysis on the same theoretical basis allows the influence of coding in a GA to be analyzed [10].

### 3.5. New evolutionary models

Keywords: Parisian approach, co-evolution and social insects colonies, interactive evolution.

Participants: Yann Landrin-Schweitzer, Evelyne Lutton.

In collaboration with Pierre Collet (Université du Littoral, Calais) and Marc Schoenauer (INRIA-Futurs, TAO).

**Abstract.** The versatility of evolutionary algorithms permits to address optimisation problems that involve non-standard search spaces (lists, graphs, ...). These are very difficult, irregular, impossible to address with other techniques. It is however possible to do "more than optimisation" thanks to artificial Darwinism and population-based methods. This is a major point of our research. We are in particular interested in various evolutionary techniques based on a modified formulation of the problem to be solved: Interactive evolutionary algorithms, co-evolution and "Parisian" evolution, multi-objective optimisation.

Simulated Darwinist evolution can be exploited in various ways, and recent research tends to prove the interest of new evolutionary models. Our works cover several aspects:

- *Parisian approach*: This technique proposed by the team is related to co-evolution techniques. It consists in formulating a problem no longer as the search for an optimum with a population of points in a search space, but as the search for an equilibrium state for a population of "parts" of solutions, that collectively build the searched solution. Such a formulation is not always possible for optimisation problems (the problem has to be split into interdependent subproblems). However, when applicable, this approach is beneficial in terms of efficiency and computation time. It has been applied to inverse problem for IFS, stereovision (the quasi-real-time "flies" algorithm) for obstacle detection, fractal compression and text-retrieval.
- Interactive evolutionary algorithms: When an evolutionary process involves an interaction with a human user (usually fitness evaluation is partly set by the user), one has to reconsider several important points of the evolutionary loop. This research topic is very active. For example, interaction with humans raises several problems, mainly linked to the "user bottleneck," human fatigue. Solutions have to be found in order to avoid systematic and boring interactions. Our work deals with the analysis and development of various user-interaction modes, including Parisian approaches. Current applications include text-retrieval (ELISE), e-learning, over-constrained problems resolution (CONSENSUS), and artistic design (ArtiE-Fract).

# 4. Application Domains

# 4.1. TCP Traffic

Keywords: TCP, multifractal analysis, multifractional Brownian motion.

Participant: Jacques Lévy Véhel.

**Abstract.** Compared to conventional traffic, internet traffic possesses radically different characteristics, whose study requires new tools. In particular, the strong sporadicity has important consequences on the queuing behaviour.

Conventional traffic models generally assume that the arrival processes have short-term memory. It appears that Internet traffic usually does not satisfy such an assumption. In particular, many types of traffic on the Internet are strongly sporadic on several times scales. Recent models based on fBm take into account such features. The success of fBm as a traffic model relies partly on the fact that the long term memory is controlled by a single parameter. As long range dependence is an order 2 statistics, it is natural to enquire whether fBm is also a good model for higher-order statistics of real traffic.

Multifractal analysis allows to answer this question. The multifractal spectrum of fBm is trivial, since fBm is monofractal. We have shown through intensive numerical studies that LAN traffic recorded at Berkeley and CNET exhibits on the contrary a strong multifractal behaviour over 3 to 4 time scales.

The observed spectra evidence differences between incoming and outgoing traffics. Furthermore, the shape of the spectrum of the Berkeley traffic provides information on the stationarity of the process. More generally, the multifractal characteristics of traffic traces have consequences on the queuing behaviour.

Our recent work has dealt with the possible sources of multifractality. We have in particular shown that the very mechanism of TCP is a cause of multifractality.

#### **4.2. Signal and Image analysis**

**Keywords:** 2-microlocal analysis, Hölder exponents, Internet traffic, biomedical signals, change detection, denoising, financial records, interpolation, medical images, multifractal analysis, radar images, segmentation.

Participants: Jacques Lévy Véhel, Pierrick Legrand, Evelyne Lutton.

#### In collaboration with Antoine Echelard (IrCcyn).

**Abstract.** Multifractal processing of signals and images is based on a fine analysis of the local regularity of various measures defined from the data. The corresponding Multifractal spectra are then computed. Contrarily to more classical approaches, there is no filtering. Segmentation, denoising, interpolation or change detection are performed on signal/image points using local as well as global information provided by the spectra.

Signal processing is a task required in many applications, such ECG/EEG and other biological and medical signal analysis, Internet traffic monitoring, financial records analysis, ....

Image analysis is a fundamental component of computer vision problems, with applications in robotics, medical or satellite imaging, ....

Signals and images have often to be "denoised" prior to processing: This is in particular the case for radar images and most medical signals/images. Segmentation is also an important step that provides a description of an image in terms of regions and contours, and that splits signals (in particular biomedical and financial ones) into homogeneous zones. In many applications, one is interested in detecting change points, or variations in sequences of images. Finally, it is sometimes useful to resample the data, e.g. in order to improve resolution

Classical approaches in these domains are based on the general assumption that the available data represent the sampling of an underlying process which is *globally* piecewise regular (e.g. belongs to some Hölder space  $C^{\alpha}$  or some Besov space  $B_{p,q}^{s}$ ). One may then apply for instance certain filters that will yield "gradients" where extrema roughly correspond to contours. Multi-resolution techniques may be used to refine the results. One drawback of this approach is due to preliminary smoothing, resulting in loss of precision. In addition, the hypothesis of a piecewise regular underlying process is not always realistic: Textures for example will in general puzzle these processors.

An alternative approach is to consider that the signal/image represents a function or a measure known at fixed resolution. The irregularities of this measure can then be studied with the help of multifractal analysis. The general principle is the following: First, various measures and capacities are defined from the image grey-levels or signal amplitudes. The corresponding Hölder exponents and multifractal spectra are then computed, providing both local (via  $\alpha$ ) and global (via  $f(\alpha)$ ) information. No hypothesis is made on signal regularity, and there is no prior filtering. We have in particular developed methods that allow to perform segmentation, denoising, interpolation or change detection by using both the local and global regularity

information encompassed in multifractal analysis. Contours, for instance, correspond to points where the multifractal spectrum assume a specific value. Denoising may be achieved by increasing, in a controlled way, the Hölder exponent at each point.

### 4.3. Complex interactions

Keywords: fractal inverse problems, interactive evolutionary algorithms, interactive genetic programming.

Participants: Pierrick Legrand, Jacques Lévy Véhel, Evelyne Lutton.

**Abstract.** We study the use of evolutionary optimisation tools for modelling, controlling or optimising complex interactive systems. In particular, some of them involve fractal inverse problems and multifractal analysis: For interactive design (Artie-Fract software), interactive multifractal denoising (in Fraclab), cochlear implants optimisation (HEVEA project), and termites nest modelling (TERMCAO project).

A standard inverse problem can be formulated the following way: For a given system it is possible to compute an output from input data but reversely it is extremely difficult to estimate the input data that have produced a given output, due to highly non-linear (complex) interactions between input components. In such cases, a "black-box" approach is the only solution: Optimise the input data so that their computed output resembles the given output.

In the domain of fractal analysis, several inverse problems have been successfully addressed using evolutionary optimisation, including the famous inverse problem for IFS, [48], [46], [45]. Our contribution to this domain deals with the use of complex IFS models (mixed, polar) with genetic programming and Parisian approach. The efficient resolution of such fractal inverse problems is crucial to several applications like image compression [47], [42], and fractal antennas optimisation [40].

Additionnally, human interactions in such computer systems tends to add irregularity and unpredictability, but are often necessary to provide useful and efficient algorithms. The example of multifractal image denoising is characteristic : the notion of a "good" denoising strongly depends on the user (a medical practicionner, a photograph, an art expert, etc ...) and on the applicative framework. An additionnal judgement given by the end-user is necessary to identify a satisfying result.

Applications currently considered in the team are artistic interactive design of fractals (ArtiE-Fract, with the Cetoine company), text-retrieval (ELISE, with Novartis-Pharma), resolution of over-constrainted problems for resource allocation (CONSENSUS, in collaboration with the CONSTRAINTS team), termite nest formation (TERMCAO project, with biologists) and cochlear implants optimisation (HEVEA project, with the Avicenne Hospital).

# 5. Software

#### 5.1. FRACLAB: a Fractal Matlab/Scilab toolbox

Participants: Pierrick Legrand, Jacques Lévy Véhel, Michel Tesmer-Tur.

In collaboration with Olivier Barrière, Antoine Echelard (IRCCyN).

FracLab is a general purpose signal and image processing toolbox based on fractal and multifractal methods. FracLab can be approached from two different perspectives:

- Fractal analysis: A large number of procedures allow to compute various fractal quantities associated with 1D or 2D signals, such as dimensions, Hölder exponents or multifractal spectra.
- Signal processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modelling, segmentation, classification, and synthesis.

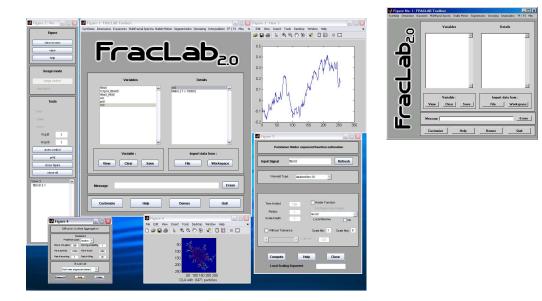


Figure 1. FracLab 2.0 graphical interface

FracLab is not intended to process "fractal" signals (whatever meaning is given to this word), but rather to apply fractal tools to the study of irregular but otherwise arbitrary signals. A graphical interface makes FracLab easy to use and intuitive. In addition, various wavelet-related tools are available in FracLab.

FracLab is a free software. It mainly consists of routines developed in Matlab or C-code interfaced with Matlab and Scilab (a free scientific software package for numerical computations from INRIA). It runs under Linux and Windows environments

The development of FracLab has been continued in 2006: A stand-alone version has been put on line (*i.e.* a version that does not require MatLab to run). We have improved the computation of various multifractal spectra, and added two modules contributed by O. Jones (Univ. of Melbourne, Australia), related to the synthesis and estimation of embedded branching processes.

Fraclab has been downloaded roughly 2500 times between December 2005 and November 2006, by users all around the world. A few dozens laboratories seem to use it. Its use has been acknowledged in several applied research papers.

### 5.2. EASEA : an evolutionary algorithms specification language

Keywords: evolutionary algorithm, stochastic optimisation.

Participants: Jean Louchet, Evelyne Lutton.

#### In collaboration with Pierre Collet (Université du Littoral, Calais).

EASEA (EAsy Specification of Evolutionary Algorithms) was initiated inside the EVO-Lab collaborative action (1999-2000). Its aim was to broaden the access to evolutionary computing by simplify the programmation of EAs, especially for non-computer scientists. A simple specification of an evolutionary algorithm written into an << .ez >> file is used by EASEA. It then produces a C++ source file using the primitives of an underlying evolutionary library. The complex programming tasks are hidden to the user. The description of an evolutionary algorithm then becomes short and simple, and thanks to the EASEA compiler this specification file can be compiled at any place. The current versions (UNIX and Windows) can produce a C++ source file for the GALib or the EO library, or JAVA source files for the DREAM library.

EASEA is now largely used:

- as a teaching support (ENSTA, Ecole Polytechnique, Université du Littoral, Université de Dijon, Ecole Centrale, Ecole des Ponts, CESTI Toulon, University of Massachusetts Dartmouth),
- as a research and industrial development tool (projet SINUS, ENSTA, Laboratoire d'Informatique du Littoral, General Electric (France), Université d'Alger, University of Exeter (UK), Napier University (Ecosse), South-Bank University (Londres), Vrije University of on Amsterdam, University of Dortmund, Universidad de Granada).

A graphical interface, GUIDE is also available. It provides an unified representation of the evolutionary engines (AG, ES, EP, ...), and gives access to unexplored schemes with a versatile presentation.

EASEAv0.7 is available on http://fractales.inria.fr/evo-lab/EVO-easea-engl.html.

### 5.3. XCLE - eXtensible Concatenative Language Engine

Keywords: Compiler, Concatenative Language, Genetic Programming, Open Source Software.

Participant: Yann Landrin-Schweitzer.

An evolution of the OKit project, XCLE has been developed to standardise code management tools for Genetic Programming developers.

XCLE addresses the need to automatically generate and manipulate program code, while retaining performance at the program execution level.

XCLE provides an implementation of basic data types: integers, floats, strings, recursive lists and executable primitives, encapsulated into a generic object type. The API provides the means to integrate program building capabilities into software, handling both the data and code aspects of program generation and execution. The library as a whole provides the necessary framework for manipulating concatenative code.

It constitutes a ready-made basis for a generic genetic operators library, and a tool for code portability and reusability in the GP community. A standardised primitives library and a graphical IDE complement the set of tools offered to developers.

XCLE is currently available at http://varkhan.free.fr/software/xcl/XCLE/.

# 6. New Results

# 6.1. Optimization of the bias-variance trade-off for the estimation of the Hölder function

Participant: Jacques Lévy Véhel.

In collaboration with Olivier Barrière (Irccyn, Nantes), Erick Herbin (Dassault Aviation).

The multifractional Brownian motion (mBm) is a generalization of the celebrated fractional Brownian motion (fBm) where the constant exponent H is replaced with a Hölder continuous function ranging in (0, 1) ([21]). Mbm was invented with the following practical application in mind: Mountains and other earth terrains modeled by fBm are not realistic because fractional Brownian fields have everywhere the same Hölder exponent, as real mountains have a space-varying regularity, due, for instance, to erosion and other phenomena. In the frame on our contract with Dassault aviation, we have put the mbm to use in the modeling of real terrains. Last year, we have solved satisfactorily the problems of synthesizing mBm and estimating its parameters. However, the theoretical properties of our estimator had not been investigated.

The estimator is a wavelet-based one, and depends on two tuning parameters: The width  $\delta$  of the averaging window around the point where one wishes to estimate the exponent, and the number m of levels in the least square regression, needed to eliminate bias. We have computed the risk of the estimator as a function of  $\delta$  and m, and we have shown that there exist optimal values for these parameters, that allow to minimize this risk.

#### 6.2. Dimension Spectrum for Self-conformal Measures

Participant: Jacques Lévy Véhel.

In collaboration with Michal Rams (Institute of Mathematics, Polish Academy of Sciences, Warsow).

We have considered the natural measures associated with a family of conformal iterated function systems satisfying the transversality condition but no separation condition. In this frame, we have been able to compute the exact value of their generalized Renyi dimensions  $D_q$  for q in a certain range.

More precisely, let V be an open and bounded subset of  $\mathbb{R}^d$ . For each parameter  $t \in V$  we consider a conformal iterated function system (IFS)  $(f_i(\cdot, t))_{i=1}^k$  in  $\mathbb{R}^d$  depending on t. We assume this dependence to be smooth (at least  $C^{1+\beta}$ ). We denote by  $\Lambda_t$  the limit set of the IFS, by  $\nu_t$  its natural measure and by s(t) the similarity dimension. It is well known that the Haussdorf dimension of  $\Lambda_t$  verifies  $\dim_H \Lambda_t \leq s(t)$ . We have worked under the assumption that the so-called "transversality condition" introduced by Policott and Simon's holds. We also assume that s(t) < d for all t.

A fine analysis of the properties of the natural measure  $\nu_t$  of the IFS is provided by the computation of the so-called *generalised dimensions* or  $L^q$  spectrum. These are computed as follows. Let, for  $q \ge 0$  and  $\varepsilon > 0$ ,

$$C_q(\nu_t,\varepsilon) = \int (\nu_t(B(x,\varepsilon))^{q-1} d\nu_t(x))$$

 $(B(x,\varepsilon)$  denotes the closed ball of radius  $\varepsilon$  centred at x). For  $q \neq 1$ , one defines the lower and upper q-dimensions as:

$$D_q^-(\nu_t) = \liminf_{\varepsilon \to 0} \frac{\log(C_q(\nu_t, \varepsilon))}{(q-1)\log(\varepsilon)}, \qquad D_q^+(\nu_t) = \limsup_{\varepsilon \to 0} \frac{\log(C_q(\nu_t, \varepsilon))}{(q-1)\log(\varepsilon)}.$$

In case the limit exists, it is called the q-dimension of  $\nu_t$ , denoted  $D_q(\nu_t)$ .

Generalized dimensions are extensively used for the study of chaotic dynamical systems. The existence of the  $L^q$  dimension spectrum is known for the natural measure of the IFS. A result of Hunt and Kaloshin implies that this dimension equals s(t) for all  $q \leq 2$  and for almost all  $t \in V$ . We have proved the following result ([38]):

**Theorem 1** If s(t) < d/2 for all  $t \in V$  then for almost all  $t \in V$  the IFS satisfies the strong open set condition, hence

$$D_q(\nu_t) = s(t)$$

for all q.

If s(t) > d/2 for all  $t \in V$  then for almost all  $t \in V$ 

$$D_q(\nu_t) = s(t)$$

for all  $q \le s(t)/(s(t) - d/2)$ .

#### **6.3.** Multifractal analysis of Lévy processes

Participant: Jacques Lévy Véhel.

In collaboration with Rene Schilling (Univ. Marburg, Germany).

The large deviation spectrum has been computed in the frame of stochastic processes for a large class of Gaussian processes. On the other hand, the Haussdorf multifractal spectrum of Lévy processes is known.

We have shown that the large deviation spectrum of Lévy processes which are not subordinators, is, under certain conditions, equal to:

- 1. for  $\alpha \in [0, 1/\delta], f_q(\alpha) = \alpha \delta$ ,
- 2. for  $\alpha \in (1/\delta, 1/\delta + 1], f_q(\alpha) = 1/\delta \alpha + 1$ ,
- 3.  $f_q(\alpha) = -\infty$  otherwise.

where  $\delta$  is the so-called "Blumenthal-Getoor" index of the process. For subordinators, one has:

- 1. for  $\alpha \in [0, 1/\delta], f_g(\alpha) = \alpha \delta$ ,
- 2.  $f_g(\alpha) = -\infty$  otherwise.

The conditions under which the above holds are technical and pertain to  $\delta$ . We hope to obtain general results shortly.

### 6.4. Definition and analysis of processes with prescribed local form

Participant: Jacques Lévy Véhel.

In collaboration with Kenneth Falconer (Univ. St Andrews, Scotland).

Multifractional Brownian motion (mBm) was introduced in our team as a generalization of fractional Brownian motion (fBm) that allowed to control the local regularity at each point. Subsequently, other classical processes have been extended in the same manner, in particular certain stable processes.

We have tried to develop a general theory that would allow to prescribe the "local form" of stochastic process. In that view, we consider stochastic fields X(t, u) defined on  $\mathbb{R}^2$ . We define Y(t) to be the process on  $\mathbb{R}$  given by Y(t) = X(t, t). We want Y(t) to 'look like' X(t, u) when t is close to u, and we express this in terms of localisability: We say that a process Z(t) on  $\mathbb{R}$  is *localisable* at u with exponent h and local form  $Z'_u(t)$  if

$$\frac{Z(u+rt) - Z(u)}{r^{h}} \xrightarrow{\text{fdd}} Z'_{u}(t) \tag{1}$$

as  $r \searrow 0$ , where  $Z'_u(t)$  is a process on  $\mathbb{R}$  and with convergence in finite dimension distribution. If convergence in (1) is in distribution, we say that Z(t) is *strongly localisable* at u. We are generally interested in the 'fractal' case, when 0 < h < 1.

We have obtained general conditions ensuring that a continuous or cadlag process is (strongly) localisable. We have applied these results to obtain new processes with prescribed local form: These processes, termed *multistable processes*, generalize stable processes by allowing the stability index vary along the path. A realization of a path of such a process is shown on figure 2. Notice how both the regularity and the "intensity of jumps" evolve in time. Such processes could be good models for real-world phenomena ranging from financial logs to the earth surface ([34]).

#### 6.5. Multifractal Strings

Participant: Jacques Lévy Véhel.

In collaboration with Michel Lapidus, John Rock (Univ. Riverside, California) and Franklin Mendivil (Univ. Acadia, Canada).

A fractal string  $\mathcal{L} = \{\ell_j\}_{j=1}^{\infty}$  is simply a countable, non-increasing sequence of lengths whose sum is finite. To  $\mathcal{L}$ , one can associate a bounded open subset of  $\mathbb{R}$ ,  $\Omega$ , which is the union of open intervals with lengths the  $\{\ell_j\}$ .

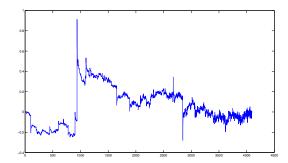


Figure 2. Path of a multistable process.

The one-sided volume of the tubular neighborhood of radius  $\varepsilon$  of  $\partial \Omega$  is

$$V(\varepsilon) = vol_1\{x \in \Omega : d(x, \partial \Omega) < \varepsilon\}.$$

The *Minkowski dimension* of  $\mathcal{L}$  (or  $\Omega$ ) is

$$D = D_{\mathcal{L}} := \inf \{ \alpha \ge 0 : V(\varepsilon) = O(\varepsilon^{1-\alpha}) \text{ as } \varepsilon \to 0^+ \}.$$

The geometric zeta function of  $\mathcal{L}$  is

$$\zeta_{\mathcal{L}}(s) = \sum_{j=1}^{\infty} \ell_j^s$$

One can prove that the abscissa of convergence of the geometric zeta function is precisely the Minkowski dimension. One can consider the meromorphic extension of  $\zeta$  and define the set of complex dimensions of the fractal string  $\mathcal{L}$  contained in some region  $R \subseteq \mathbb{C}$ :

$$\mathcal{D}_{\mathcal{L}}(R) = \{ \omega \in R : \zeta_{\mathcal{L}} \text{ has a pole in } R \}.$$

A number of properties of  $\Omega$  can be deduced from the set of complex dimension. For instance, under mild conditions,  $\Omega$  is Minkowski measurable if and only if there is only one complex dimension with real part equal to  $D_{\mathcal{L}}$ 

We have tried to generalize the theory of complex dimensions in the multifractal frame. One difficulty is that "interesting" sets in multifractal analysis are neither closed nor open, so the theory has to be modified. Likewise, the Minkowski dimension is generally useless in this frame, as most sets are dense.

We have defined two extensions: One is based on the use of the *continuous large deviation spectrum* defined by J. Lévy Véhel and C. Tricot ([35]). The other one uses the Legendre approach to multifractal analysis, and generalizes the structure function ([37]). In both cases, complex dimensions allow to describe more precisely the multifractal behavior of measures.

# 6.6. Pointwise Regularity of Fitness Landscapes and adjustment of a Simple ES

Participants: Jacques Lévy Véhel, Evelyne Lutton.

We have conducted a theoretical and experimental analysis of the influence of the pointwise irregularity of the fitness function on the behavior of an (1+1)ES. Our previous work on this subject suggests that the performance of an EA strongly depends on the irregularity of the fitness function. Several irregularity measures have been derived for discrete search spaces, in order to numerically characterize this type of difficulty for EA. These characterizations are mainly based on regularity exponents. These studies used however a global characterization of fitness regularity, the global Hölder exponent, with experimental validations being conducted on test functions with uniform regularity.

We have is extended these results in two ways: We now deal with continuous search spaces, and pointwise instead of global irregularity is considered. In addition, we have proposed a way to modify the genetic topology so as to accommodate for variable regularity: The mutation radius, that controls the size of the neighborhood of a point, is allowed to vary according to the pointwise irregularity of the fitness function. These results are obtained through a simple theoretical analysis that gives a relation between the pointwise Hölder exponent and the optimal mutation radius. We have verified on numerical examples the validity of this approach ([31]).

### 6.7. Cooperative Royal Road Functions

Keywords: Cooperative coevolution, Parisian approach, test-functions.

Participants: Evelyne Lutton, Gabriela Ochoa.

The parisian approach belongs to the general class of cooperative co-evolutionary algorithms (CCEAs), that represent a natural extension of standard EAs for tackling complex problems. Co-evolutionary algorithms can be generally defined as a class of EAs in which the fitness of an individual depends on its relationship to other members of the population. Several co-evolutionary approaches have been proposed in the literature; they vary widely, but the most fundamental classification relies on the distinction between cooperation and competition. Most work on this domain has been done on competitive models, however there is a increased interest in cooperative models to tackle difficult optimisation problems by means of problems decomposition.

In this work, a set of tuneable test-functions based on Royal-Road functions has been proposed and tested. Experiments prove the computational efficiency of CCEAs on these class of test functions. The problem of finding an "optimal" decomposition is important, and we currently test the automated "emergence" of co-adapted components in multi-populations and mono-populations (i.e Parisian approach) CCEAs.

#### 6.8. Analysis of crossover behaviour for Genetic Programming

Keywords: Genetic Programming.

Participants: Malek Aichour, Evelyne Lutton.

In collaboration with Pierre Collet (Université du Littoral, Calais). PhD partnership with Guelma university, Algeria.

In genetic algorithms, mutation and crossover points are randomly selected. Also, randomness is present in the selection of the parents. A precise analysis of the fitness of the offspring generated by mutation and crossover often provide some important viewpoint on the complexity of the problem to be solved. For instance, it is extremely important to estimate if an operator is destructive or not.

The objective is here to design a new crossover operator for Genetic Programming. Given a selected couple of parents, it is analysed if a locally optimal choice of the crossover node (i.e. a subtrees to be exchanged between parents) improves the search capabilities of a GP, and/or reduces the bloat effect. Classical GP benchmarks as well as a problem of symbolic regression on fractal functions, are used for testing.

#### 6.9. Overcompression of JPEG images

Participants: Jacques Lévy Véhel, Evelyne Lutton.

In collaboration with Franklin Mendivil (Univ. Acadia, Canada).

Many people have realized the following fact: The set of all "reasonable" images is extremely small as compared to the one of all "possible" images. Although the term "reasonable" is vague, the meaning is clear: If one chooses at random the gray level values of all pixels in a  $N \times N$  image, where, say, N = 512 and the gray levels are coded on 8 bits, then the probability that the result looks like a meaningful image is ridiculously small. One may wonder if it is possible to improve the efficiency of the various compression methods by using this remark.

We term attempts of this kind *overcompression*: Overcompression is the process of post-processing compressed images to gain either further size reduction or improved quality by taking advantage of the fact that the set of all "reasonable" images has a sparse structure.

Although overcompression is by no means an easy task, it may be approached by a variety of methods. We have proposed an overcompression scheme for the case of JPEG compressed images.

The JPEG compression format is the most popular image compression method to date. It has served as a standard until recently. Although JPEG has now been surpassed by a new standard, called JPEG 2000, it is still widely used for several reasons.

Our overcompression method improves on the quality of JPEG images by reducing the blocking artifacts commonly encountered with this compression method. These artifacts are reduced by allowing the low frequency coefficients of the DCT to vary slightly. Evolutionary strategies are used in order to guide the modification of the coefficients toward a smoother image ([36]).

Figures 3 and 4 show an example on the well-known Lena image.



Figure 3. Lena with compression factor 5.

#### 6.10. Evolutionary Multifractal Signal Enhancement

**Keywords:** *Evolutionary algorithm, denoising Interpolation, multifractal analysis, signal enhancement.* **Participants:** Pierrick Legrand, Jacques Lévy Véhel, Evelyne Lutton.

In collaboration with Gustavo Olague (EvoVision group, CICESE research center, Mexico). This work has been done under a LAFMI grant.

Signal enhancement, or denoising may be achieved by increasing, in a controlled way, the Hölder exponent at each point. This problem can be formulated as an optimisation problem, i.e. find the regularized signal that is the nearest to an orginal noisy signal. Following a previous work dealing with an estimation of the Hölderian regularity using a wavelet decomposition, we have experimented an estimation of Hölder exponents using the oscillations. The associated optimisation problem becomes more complex, but has been successfully solved using an evolutionary algorithm ([30]).



Figure 4. Compressed Lena optimized with a genetic algorithm.

Using the same optimisation approach, a problem of **interpolation** under constraints can be solved, to build 1D signals or images with a **prescribed regularity**.

#### 6.11. 2-microlocal signal denoising

Participant: Jacques Lévy Véhel.

In collaboration with Antoine Echelard (Irccyn, Nantes).

The celebrated wavelet-thresholding method for signal denoising is known to oversmooth signals: The visual appearance of the denoised signal is often more regular than the one of the original signal, although oscillations (known as "ringing effects") may also appear. We have developed a theoretical approach to measure this phenomenon. It is based on the definition of a "Hölder exponent between two scales", that allows to distinguish various levels of textures in a signal. Depending on the scales, we show that wavelet-thresholding may lead to an infinitely smooth signal. We have then proposed a modified denoising procedure, that allows to recover, under certain conditions, the local regularity of the original signal. We have also obtained pointwise results: At any given point, it is possible to recover the 2-microlocal regularity of the original signal.

These results are made possible thanks to a fine analysis of the effect of a discrete white Gaussian noise on regularity, and to a new estimator of the Hölder exponent of a noisy signal. This estimator is computed from the following quantity:

$$\mathcal{L}_n(p) = \frac{1}{(n-p+1)^2} \sum_{i=p}^n y_i(t)^2,$$

where  $y_i(t)$  is the wavelet coefficient at scale i "above" t, by looking for an integer  $p^*(n)$  such that:

$$\mathcal{L}_n(p^*) = \min_{b \log(n) \le p \le n-b \log(n)} \mathcal{L}_n(p),$$

where b > 1 is a fixed number.

#### 6.12. Texture image segmentation with an evolutionary approach.

**Keywords:** *evolutionary algorithm, stereovision, vision systems for robotics.* **Participants:** Evelyne Lutton, Gustavo Olague, Cynthia Pérez Castro.

#### Collaboration with CICESE, under a LAFMI grant.

In the frame of the general problem of investigating the use of evolutionary computation in computer vision, this work has focused on the problem of texture image segmentation. Segmenting textured images is not a trivial task, and has been studied for decades. Difficulties are for instance due to irregular textures boundaries, that often occur in highly textured scenes. The EvoSeg algorithm that is currently developed, uses knowledge derived from texture analysis to solve the segmentation problem without any a priori information. EvoSeg uses texture features derived from the Gray Level Coocurrence Matrix and optimizes a fitness measure, based on the minimum variance criteria.

## 6.13. Stereovision and robot control by the Fly Algorithm

**Keywords:** *evolutionary algorithm, obstacle avoidance, obstacle detection, parisian approach, stereovision, vision systems for robotics.* 

Participants: Olivier Pauplin, Jean Louchet, Evelyne Lutton.

In collaboration with Arnaud De La Fortelle and Michel Parent (IMARA team).

The Fly algorithm is a stereovision evolutionary algorithm, based on the "parisian approach". It aims to be used in particular in the field of real time obstacle detection and control for mobile robotics and utomated vehicles. It produces a set of 3-D points which gather on the surfaces of obstacles. Those points are evolved following the classical steps of evolutionary algorithms.

The algorithm has been integrated in a vehicule (Cycab) in IMARA project, linked with a stereo vision system, and with Cycab controls.

Improvements of the algorithm itself concern the fitness function and auto-adaptativity. Modules have been added to interpret and exploit the output of the algorithm in order to:

- make the Cycab stop when an obstacle comes in front of it;
- make the Cycab turn to the left (resp. to the right) when an obstacle enters the field of view on the right (resp. on the left);
- draw a map of the encountered obstacles;
- give an estimation of the distance covered (visual odometry).

# 6.14. Evolutionary mobile robot navigation based on visual landmark detection

Keywords: evolutionary algorithm, stereovision, vision systems for robotics.

Participants: Evelyne Lutton, Gustavo Olague, Leonardo Trujillo.

Collaboration with CICESE, under a LAFMI grant.

The current work is related with two aspects on building a robot navigation system that uses visual cues for localization:

- evolving neurocontrollers that perform reactive behaviors,
- synthesizing new local image descriptors useful in object detection/recognition or content based image retrieval (i.e. landmark identification).

The problem of automatically discovering reactive behaviors with Evolutionary Computation is being addressed with two main goals in mind. First, to use a cooperative evolutionary framework based on the Parisian Evolution Concept, where the computational cost of evolving complex neurocontrollers will ideally be diminished by evolving a modular NN controller that is made up of simpler individuals. To this end, the Evolving Neural Networks Through Augmenting Topologies (NEAT) framework will be used due to its concurrent evolution of network weights and topologies, along with the fact that it takes into account speciation as an important part of the framework, which is similar in spirit to the Parisian approach. However, new sharing criteria are investigated, due to the fact that a similarity measure based solely on network topology, as it is presented in NEAT, is believed to be to strong of an assumption. Programming will be carried out in two phases. The initial phase is a simple test of NEAT evolution on a 2D Khepera Simulator; afterwords simulation will be carried out on the 3D simulator provided by the Gazebo/Player architecture that can simulate a Pioneer 2AT robot (this also allows for the simulation of Vision applications).

The second problem, that of evolving local image descriptors is a continuation of the work done in *L. Trujillo* and *G. Olague, "Synthesis of interest point detectors through genetic programming, in Proceedings from GECCO 2006, M. Keijzer et al., eds., Vol.1, (ACM Press 2006), pp. 887–894.*. The new focus is now related to evolving a descriptive measure around extracted local features. To this end, a Genetic Programming approach is proposed, where evolved programs will look to construct discriminative measures that will allow for effective object or scene identification for landmark-based localization of an autonomous mobile robot.

## 6.15. Modelling Termite Nest formation

**Keywords:** *Social insects, fractal growing model, termites.* **Participants:** Aurlie Bousquet, Evelyne Lutton.

In collaboration with Emmanuel Cayla (ESTP), Pascal Jouquet, Michel Lepage (Laboratoire Fonctionnement et Evolution des Systmes Ecologiques, UMR 7525- Ecole Normale Suprieure), Yves Le Goff (Ecole Nationale Suprieure des Arts et Mtiers, Laboratoire Mcanique des Fluides), and Natalie Fortier (INSA Rouen).

The aim of this collaboration with biologists is to understand the mechanisms of nest construction for a particular species of termites (*macrotermes bellicosus*). These termites are living in Africa (in dry as well as forest areas). They build specific structures, rather irregular, but with some characteristic tower-like components. Their nest is composed of several internal and external structures, with food storage area, mushroom plantation (they actually grow and eat a particular species of mushroom), queen chamber and nursery. The nest is a structure that evolves gradually, with respect to the size and age of the colony, as well with respect to the environment and climatic conditions. The challenge is to understand the connections between nest architecture and climatic conditions (and eventually elaborate behavioural models of it).

We currently work on a simplified model of external nest structures, which are usually built by the colony within a single night. The proposed model is based on a population behaviour with elementary social interactions (an ACO model), that has been derived from biologic observations. 2D and 3D simulation proves the capability of such a model to produce fractal structures, similar to the natural ones. Efforts has been derived to the development of several models of elementar termite behaviour. A special attention has been devoted to the chronology of the building (see figure 5). This point particularly, serves as a basis for new experimentation of biologists concerning pheromone distribution and origin of the building material.

### 6.16. Interactive Optimisation of Cochlear Implants

Keywords: Cochlear implants, interactive evolution, medical application.

Participants: Pierrick Legrand, Jacques Lévy Véhel, Evelyne Lutton.

In collaboration with Pierre Collet (LIL, Calais), Claire Bourgeois-République (univ. Bourgogne), Vincent Péan (Innotech), Bruno Frachet (hôpital Avicenne), HEVEA project (French acronym for "Handicap: Etude et Valorisation de l'Ecologie Auditive").

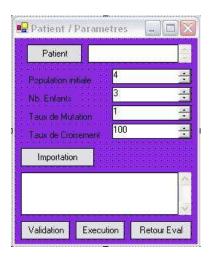




Figure 5. Lateral and transversal cut of an artificial termite nest, colours correspond to chronology (dark = old material).

Cochlear implants are surgically-implanted electronic devices that partially restore hearing of deaf people by electric stimulation of the auditive nerve. The HEVEA project aims at producing improved tuning protocols and devices by: (1) sampling the background noise, (2) characterising the background noise, (3) tune the device with respect to the background noise and (4) automatically select the appropriate parameter setting in real conditions.

This project implements an approach based on both interactive evolution and multi-scale analysis. Items (2) and (4) are classification tasks on usual environmental signals of the patient, that are addressed using a fractal/wavelet approach. Interactive evolution is used in item (3), to produce a device tuning adapted to the patient in a given environment. The interactive evolutionary tuning procedure is now functionnal and is currently tested on a set of patients, using a PDA with a graphical interface shown of figure 6. Evaluation is based on audio tests. Preliminary tests have shown an improvement of patient audition and comfort with interactive evolution.



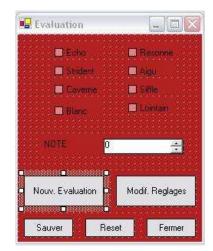


Figure 6. Graphical Interface for pocket PC

## 6.17. Optimisation of an E-Learning System

**Keywords:** Ant Colony Optimisation (ACO), Avatar technology, E-Learning, Elo-rating, Interactive Artificial Evolution.

Participants: Evelyne Lutton, Gregory Valigiani.

In collaboration with Pierre Collet (LIL, Calais), Raphaël Biojout, Yannick Jamont (Paraschool Compagny).

System (ITS) has been implemented within the existing e-learning software of the PARASCHOOL company, in order to help students to find their way among thousands of different items. The system is now operationnal, and the different versions have been tested for real on more than 250,000 users that use the site over the Internet.

The *man-hill optimization* technique stems from a first attempt to use an Ant Colony Optimisation (ACO) algorithm, which revealed unsuited for the task. To the opposite of artificial ants, human students are not controllable: it is not possible to count on innate altruism, their activity is variable (holidays), each student needs a specific treatment, ... All in all, the modifications that needed to be applied to the ACO paradigm were so numerous that it became obvious that the collective use of human students for optimization was indeed a different paradigm that we called "man-hill optimization."

Beyond being a powerful tool for suggesting good exercises, the system showed that it is also very powerful to make sure the e-learning software works well, as it is capable of finding exercices that contain not only syntaxic, but also semantic errors. The system can also point out exercises that are not well placed in the pedagogic progression.

This work also contains a contribution to the automatic rating of students and items (exercises) based on the Elo chess rating system, to an automatic graph construction based on similarity, to a primary s tudy of interactive avatars based on GEStyle to enhance site interactivity...

This e-learning application of the *man-hill optimization* paradigm is but a particular case: all web sites browsed by many users can benefit from this technique to optimize their contents, their structure and make sure that all is going well.

## 6.18. Ants paintings on Iterated Function Systems

Keywords: Ant colonies, Ant paintings, Art and Design, Caml language, Iterated Function Systems.

Participants: Evelyne Lutton, Anne-Julia Stebel.

In collaboration with Nicolas Monmarché (Polytech'Tours), Pierre Weis and Francois Clement (INRIA).

Artificial ant colonies can be used in artistic applications, by simulating a artificial ants that are moving on an image: they follow pheromone paths and depose colors on pixels. The behaviour of artificial ants are controlled by a large set of parameters that created various abstract dynamic paintings, figuring competition or cooperation behaviours.

This internship was aimed at exploring the use artificial ants to color iterated function systems attractors (examples are displayed on figure 7). A prototype has been programmed in Caml language.

#### 6.19. CONSENSUS : Interactive optimisation of a resource allocation problem

Keywords: adaptive search, constrained problems, evolutionary algorithms, resource allocation.

Participants: Loic Fosse, Evelyne Lutton.

In collaboration with Francois Fages (CONSTRAINT Team).

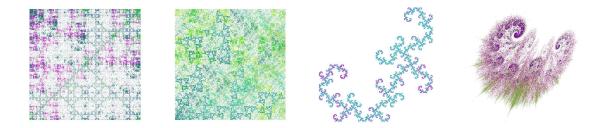


Figure 7. Examples of Ant-Paintings on IFS attractors.

The problem of office affectation on the INRIA Rocquencourt campus can be considered as a complex constraint satisfaction problem: the demand of research teams exceeds the actual resource, and in the same time the constraints and preferences of each team are difficult to represent and tune up within standard constraint satisfaction software. Evolutionary techniques have been used as a complement to constraint satisfaction. Actually many constraints are difficult to express and the relative importance of each constraint is an important factor to efficiently use constraint satisfaction software.

We experimented in 2003 the scheme of a multi-user interactive evolutionary approach for the management of user preferences relative weights, based on a Parisian and multi-population paradigm (on a small size problem). This work has been continued in 2004 (internship of Martin Pernollet), in order to build a prototype for real size testing, based on real data of the Rocquencourt Campus. In 2005 (internship of Sylvain Secherre in the CONTRAINTES team) the constraints expressions and their general balance has been precisely studied on the real size problem.

The Engineering position of Loic Fosse aims at developing a stable version of a real-sized prototype. Consensus can now compute several solutions based on constraints given by all teams, independently from the number of constraints. A grade can be given to this solutions by the users. Using constraints and users grades, the system is able to propose new solutions, i.e new offices distribution maps.

The core of Consensus has been entirely rewritten in C++, to improve performances and maintenance. It now contains:

- A learning algorithm based on the free *Lp Solve* library, to learn implicit preferences from grades attributed to each solutions.
- A search algorithm based on the *adaptive search* algorithm, used to find new solutions after the learning process.
- A statistics tool to get relevant data about solutions.

The interface was also entirely rewritten, using the more recent web technologies (*Php* and *Javascript*). Data are stored in a *Mysql* database, in order to search and visualise solutions, give a grade to each and set teams preferences, with a secured connexion, easily manage databases of users and rooms, and finally control the search process.

#### 6.20. ArtiE-Fract Improvement

Keywords: Art and Design, Fractals, Interactive Evolution, Iterated Function Systems Attractors.

Participants: Jonathan Chapuis, Pierre Grenier, Evelyne Lutton.

In collaboration with Emmanuel Cayla (Cetoine Compagny). This work is a technology-transfer action founded by ANVAR.

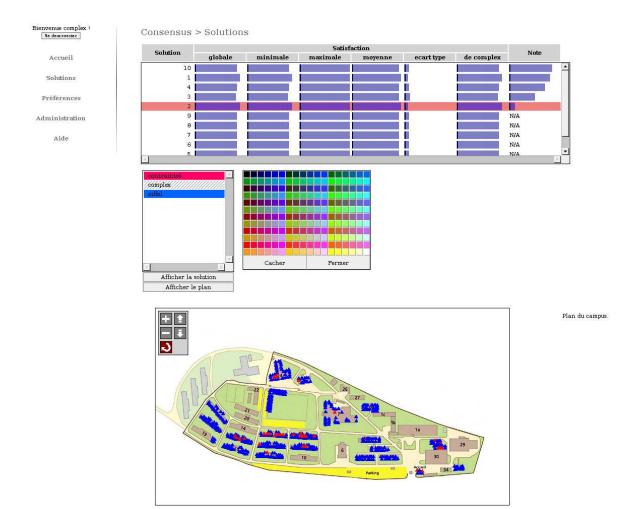


Figure 8. Consensus interface

This action aims at improving several features of the ArtiE-Fract software, in order to build an industrially efficient tool, ArtiE-Fract-V2, was adapted to the end-user technical contraints (textile design and HD video design). ArtiE-Fract-V2 is now running on GTK-2, providing a better interface control and increased reliability. The ANVAR development project successfully ends in April, and ArtiE-Fract-V2 licence has been granted to the Cetoine Society.

Additionnally, a research convention has been signed in 2006 between INRIA, Cetoine, the Angers University, the Lycee de la mode of Cholet and the e-mode technology plattform, in order to experiment new textile applications of the ArtiE-Fract software. Until now, a design internships with the lycee de la mode have been hosted by the Complex team (Sandrine Martin).

# 7. Contracts and Grants with Industry

# 7.1. Contracts and Grants with Industry

The team has contracts with:

- NOVARTIS PHARMA about text-retrieval with evolutionary algorithms (PhD and Post-doctoral position of Yann Landrin-Schweitzer).
- DASSAULT AVIATION on terrain modelling based on mBm.
- PARASCHOOL on evolutionary optimisation of pedagogical path (e-learning, PhD of Gregory Valigiani).
- Innotech, HEVEA project on Cochlar implants optimisation.

# 8. Other Grants and Activities

## 8.1. National initiatives

Our project has collaborations with:

- IrCcyn, Institut de Recherche en Cybernétique et communications de Nantes, since 1996. Areas of collaborations include the study of mBm, 2-microlocal analysis, image analysis and denoising. In addition, the software FracLab has mainly been developed at IRCCYN in the last four years.
- Littoral university (Calais), on e-learning (P. Collet and C. Fonlupt),
- Clermont Ferrand University (C. Tricot) on multifractal analysis.
- Ecole Polytechnique-CRM, Montreal (F. Nekka) on signal/image analysis.

An agreement has been signed in 2006 between INRIA, Cetoine, the Angers University, the Lycee de la mode of Cholet and the e-mode technology plattform, in order to experiment new textile applications of the ArtiE-Fract software. An R&D project is currently built among these partners to be presented to the "Pôle Enfant", a Competitive research pole of the "Pays de la Loire" region.

## 8.2. European initiatives

The team belongs to EvoNet, the European Excellence Network on artificial evolution, and is involved in the european IMPAN SPADE2 project.

# 8.3. International initiatives

The COMPLEX team collaborates with a Mexican research institute (CICESE, Fsica Aplicada, Pr Gustavo Olague) under a LAFMI grant.

# 9. Dissemination

#### 9.1. Organization committees

Complex has organized an international workshop, "Fractal Day", that took place at Inria-Rocquencourt on January 17. Complex was a co-organizer of the "Journées Fractales" in Clermont-Ferrand, November 23-24.

Evelyne Lutton was co-chair (with Hideyuki Takagi) of the "EvoInteraction" Workshop in conjunction with the EuroGP2006 conference, 10-12 April, 2006 Budapest, Hungary. Evelyne Lutton and Hideyuki Takagi will be again co-chair for the second EvoInteraction (Interactive Evolution and Humanized Computational Intelligence) Workshop, to be held in Valencia in April 2007.

Evelyne Lutton, Jacques Lévy Véhel and Fahima Nekka, are organisers of the next "Fractals in Engineering" conference, to be held in Montreal in summer 2008.

Pierre Collet, Evelyne Lutton, and Marc Schoenauer are involved in the organisation of the << Evolution Artificielle '2007 >> conference (Tours, October 2007), and are members of the steering committee of the french association for artificial evolution.

Jean Louchet et Evelyne Lutton have been invited to give a tutorial on "Evolutionary image processing" to the SITIS'06 conference, December 17 - 21, 2006, Hammamet, Tunisia.

Jacques Lévy Véhel presented a tutorial on "'Wavelet-based multifractal analysis of images" at the ICPR conference in Hong-Kong, August 2006.

## 9.2. Editorial Boards

Jacques Lévy Véhel is associate Editor of the journal << FRACTALS >>.

Evelyne Lutton has been co-editor of the Special Issue on Evolutionary Computer Vision and Image Understanding of the Pattern Recognition Letters, and of a book on Genetic and Evolutionary Image Analysis and Signal Processing, with Stefano Cagnoni and Gustavo Olague.

Evelyne Lutton, Stefano Cagnoni and Gustavo Olague are co-editors of a special issue on Evolutionary Computer Vision of the Evolutionary Computation journal.

Evelyne Lutton and Pierre Collet edited a special issue on artificial evolution of the french journal TSI.

J. Lévy Véhel has acted as an expert for the Canadian CRSNG. He is a member of the expert group "Signaux et Traitements Multidimensionnels et Multimodaux".

J. Lévy Véhel has been a referee for IEEE Trans. Image Proc., Fractals, IEE Proc. Vision, SPA.

Evelyne Lutton has been referee for IEEE Transactions on Evolutionary Computation, IEEE Signal Processing Letters, JESA, SMC-PartB.

## 9.3. Other Teaching

- "Fractals and Wavelets" ENSTA (Evelyne Lutton, Jacques Lévy Véhel, 21 h)
- "Fractals and Time-frequency analysis" Centrale de Nantes (Jacques Lévy Véhel, 7 h).
- "Fractals" ESIEA (Jacques Lévy Véhel, 15 h).
- "Fractal Analysis" INT (Jacques Lévy Véhel, 6 h).
- "Artificial Evolution" ENSTA (Evelyne Lutton, Pierre Collet, Cyril Fonlupt, 21 h).

#### 9.4. Invited talks and Scientific popularisation

Evelyne Lutton has been invited to a "Bar des sciences" (April 5 2006, "Bancs de poissons et marchés financiers : peut-on modéliser les comportements collectifs du vivant ?")

Evelyne Lutton, Emmanuel Cayla and Jonathan Chapuis participated to the "Fete de la science" and lead a demo of the ArtiE-Fract software, in Paris, Jardin du Luxembourg, 13-15 october 2006.

Jacques Lévy Véhel has given an invited lecture at the workshop "Fractal Geometry and Dynamics" organized at the Stefan Banach International Mathematical Center, Warsaw, April 2006. He was invited to the workshop "Stochastic Analysis and Related Topics" at University of Marburg, July 2006.

## 9.5. Ph.D. Theses

Gregory Valigiani defended his PhD at the Calais University, on November 10, 2006: "Développement d'un paradigme d'Optimisation par Hommilire et application l'Enseignement Assist par Ordinateur sur Internet".

Jacques Lévy Véhel was a codirector in the thesis of Yann Demichel, whose defence took place on November 24, titled "Analyse fractale et multifractale de processus aléatoires, l'exemple des fonctions de bosses.".

# **10. Bibliography**

## Major publications by the team in recent years

- M. DEKKING, J. LÉVY VÉHEL, E. LUTTON, C. TRICOT (editors). Fractals: Theory and Applications in Engineering, ISBN 1-85233-163-1, Springer Verlag, 1999.
- [2] A. AYACHE, A. BENASSI, S. COHEN, J. LÉVY VÉHEL. Regularity and identification of Generalized Multifractional Gaussian Processes, in "Séminaire de Probabilités XXXVIII - Lecture Notes in Mathematics", Springer-Verlag Heidelberg, vol. 1857, 2004, p. 290-312.
- [3] A. AYACHE, J. LÉVY VÉHEL. The Generalized multifractional Brownian motion, in "Statistical Inference for Stochastic Processes", vol. 3, 2000, p. 7–18.
- [4] A. AYACHE, J. LÉVY VÉHEL. On the Identification of the Pointwise Hölder Exponent of the Generalized Multifractional Brownian Motion, in "Stoch. Proc. Appl.", vol. 111, 2004, p. 119–156.
- [5] J. BARRAL, J. LÉVY VÉHEL. Multifractal Analysis of a Class of Additive Processes with Correlated Non-Stationary Increments, in "Electronic Journal of Probability", vol. 9, 2004, p. 508–543.
- [6] K. DAOUDI, J. LÉVY VÉHEL, Y. MEYER. Construction of continuous functions with prescribed local regularity, in "Journal of Constructive Approximation", vol. 014, n<sup>0</sup> 03, 1998, p. 349–385.
- [7] P. LEGRAND, J. LÉVY VÉHEL, M.-T. DO. Fractal Properties and Characterization of Road Profiles, in "FRACTALS", Vancouver, 2004.
- [8] P. LEGRAND, J. LÉVY VÉHEL. Signal and Image Processing with FracLab, in "FRACTALS", Vancouver, 2004.
- [9] E. LUTTON, J. LÉVY VÉHEL. Hölder functions and Deception of Genetic Algorithms, in "IEEE Transactions on Evolutionary computing", vol. 2, n<sup>o</sup> 2, July 1998.
- [10] E. LUTTON. Genetic Algorithms and Fractals Algorithmes Génétiques et Fractales, Spécialité Informatique, Habilitation à diriger des recherches, Université Paris XI Orsay, 11 Février 1999.

- [11] J. LÉVY VÉHEL. Fractal Approaches in Signal Processing, in "Fractal Geometry and Analysis", C. EVERTSZ, H. PEITGEN, R. VOSS (editors)., World Scientific, 1996.
- [12] J. LÉVY VÉHEL. Introduction to the multifractal analysis of images, in "Fractal Image Encoding and Analysis", Y. FISHER (editor)., Springer Verlag, 1997.
- [13] J. LÉVY VÉHEL, E. LUTTON. Evolutionary signal enhancement based on Hölder regularity analysis, in "EVOIASP 2001, LNCS 37, Lake Como, Italy", E. BOERS, ET AL (editors)., Springer Verlag, 2001.
- [14] J. LÉVY VÉHEL, R. VOJAK. Multifractal Analysis of Choquet Capacities: Preliminary Results, in "Advances in Applied Mathematics", vol. 20, January 1998, p. 1–43.
- [15] J. LÉVY VÉHEL, E. LUTTON, C. TRICOT. Fractals in Engineering: From Theory to Industrial Applications, J. Lévy Véhel, E. Lutton and C. Tricot (Eds), ISBN 3-540-76182-9, Springer Verlag, 1997.
- [16] R. PELTIER, J. LÉVY VÉHEL. Multifractional Brownian Motion, Technical report, n<sup>o</sup> 2645, INRIA, 1995, http://hal.inria.fr/inria-00074045.
- [17] C. TRICOT, J. LÉVY VÉHEL. On various multifractal spectra, in "Fractal Geometry and Stochastics III, Progress in Probability", C. Bandt, U. Mosco and M. Zähle (Eds), Birkhäuser Verlag, vol. 57, 2004, p. 23-42.

#### Year Publications

#### **Books and Monographs**

[18] F. ROTHLAUF, G. SQUILLERO, G. SMITH, J. ROMERO, J. MOORE, P. MACHADO, E. LUTTON, R. DRECHSLER, C. COTTA, E. COSTA, S. CAGNONI, J. BRANKE, H. TAKAGI. Applications of Evolutionary Computing, EvoWorkshops 2006: EvoBIO, EvoCOMNET, EvoHOT, EvoIASP, EvoINTERACTION, Evo-MUSART, and EvoSTOC, Lecture Notes in Computer Science, Softcover, ISBN: 3-540-33237-5. 813 p. LNCS 3907 is now available online., vol. 3907, XXIV, Springer Verlag, Budapest, Hungary, April 10-12 2006, http://www.springeronline.com/3-540-33237-5.

#### **Doctoral dissertations and Habilitation theses**

[19] G. VALIGIANI. Développement d'un paradigme d'Optimisation par Hommilière et application à l'Enseignement Assisté par Ordinateur sur Internet, Ph. D. Thesis, University of Calais, France, November 10 2006.

#### Articles in refereed journals and book chapters

- [20] P. COLLET, P. LEGRAND, C. BOURGEOIS-REPUBLIQUE, V. PEAN, B. FRACHET. Aide au paramétrage d'implants cochléaires par algorithme volutionnaire interactif. Optimisation en traitement du signal et de l'image, in "Trait IC2", P. Siarry Ed., Hermès-Lavoisier., 2006.
- [21] P. COLLET, E. LUTTON, G. VALIGIANI. *Etude Comportementale des Hommilières pour l'Optimisation*, in "EpiNet, EPI Electronic Magazine", n<sup>o</sup> 83, march 2006, http://www.epi.asso.fr/epinet/epinet83.htm.
- [22] P. LEGRAND, C. BOURGEOIS-REPUBLIQUE, V. PEAN, E. HARBOUN-COHEN, J. LÉVY VÉHEL, B. FRACHET, E. LUTTON, P. COLLET. *Interactive evolution for cochlear implants fitting*, in "GPEM", submitted, 2006.

- [23] E. LUTTON, G. OLAGUE, S. CAGNONI. Introduction to the Special Issue on Evolutionary Computer Vision and Image Understanding, in "Pattern Recognition Letters", to appear, 2006.
- [24] E. LUTTON, G. OLAGUE. *Parisian camera placement for vision metrology*, in "Pattern Recognition Letters", vol. 27, n<sup>o</sup> 11, August 2006, p. 1209–1219.
- [25] J. LÉVY VÉHEL, M.-F. DEVAUX, I. TARALOVA, E. BONNIN, J.-F. THIBAULT, F. GUILLON. Contribution of image analysis to the description of enzymatic degradation kinetics for particulate food material, in "Journal of Food Engineering", vol. 77, n<sup>O</sup> Issue 4, December 2006, p. 1096-1107.
- [26] G. OLAGUE, C. PEREZ, C. PUENTE, P. LEGRAND. An artificial life approach to dense stereo disparity, in "Artificial Life and Robotics", submitted, 2006.
- [27] L. TRUJILLO, G. OLAGUE, P. LEGRAND, E. LUTTON. *Regularity-based descriptor computed from local image oscillations*, in "on-line journal of the Optics Society of America, OSX", accepted, 2006.
- [28] G. VALIGIANI, E. LUTTON, Y. JAMONT, R. BIOJOUT, P. COLLET. Automatic Rating Process to Audit a Man-Hill, in "WSEAS Transactions on Advances in Engineering Education", ISSN 1790-1979, vol. 3, n<sup>o</sup> Issue 1, January 2006, p. 1–7.

#### **Publications in Conferences and Workshops**

- [29] P. LEGRAND, J. LÉVY VÉHEL. *Hölderian regularity-based image interpolation*, in "ICASSP06, IEEE International Conference on Acoustics, Speech, and Signal Processing, Toulouse, France", May 14-19 2006.
- [30] P. LEGRAND, E. LUTTON, G. OLAGUE. *Evolutionary denoising based on an estimation of Hölder exponents with oscillations*, in "EVOIASP 2006, 8th European Workshop on Evolutionary Computation in Image Analysis and Signal Processing, Budapest, Hungary", April 10-12 2006.
- [31] E. LUTTON, J. LÉVY VÉHEL. Pointwise Regularity of Fitness Landscapes and the Performance of a Simple ES, in "CEC'06, Vancouver", July 2006.
- [32] G. VALIGIANI, E. LUTTON, P. COLLET. *Adapting the Elo rating*, in "Proceedings of CE'06, Antibes, France", September 18-22 2006.

#### **Internal Reports**

[33] E. LUTTON, Y. LANDRIN-SCHWEITZER, J. LÉVY VÉHEL. Experiments on controlled regularity fitness landscapes, Technical report, n<sup>o</sup> RR-5823, INRIA Rocquencourt, February 2006, https://hal.inria.fr/inria-00070202.

#### **Miscellaneous**

- [34] K. FALCONER, J. LÉVY VÉHEL. Random processes with prescribed local form, submitted.
- [35] M. LAPIDUS, J. LÉVY VÉHEL, J. ROCK. Fractal Strings and Multifractal Zeta Functions, submitted.

[36] J. LÉVY VÉHEL, F. MENDIVIL, E. LUTTON. Overcompressing JPEG images, submitted.

- [37] J. LÉVY VÉHEL, F. MENDIVIL. Local Fractal Strings and Multifractal Strings, submitted.
- [38] M. RAMS, J. LÉVY VÉHEL. Results on the Dimension Spectrum for Self-conformal Measures, submitted.

#### **References in notes**

- [39] R. CERF. Artificial Evolution, European Conference, AE 95, Brest, France, September 1995, Selected papers, vol. Lecture Notes in Computer Science 1063, chap. Asymptotic convergence of genetic algorithms, Springer Verlag, 1995, p. 37–54.
- [40] N. COHEN. Antennas in Chaos : Fractal-Element Antennas, in "Fractals in Engineering 97", Hot Topic Session, Arcachon, France, June 25-27, INRIA, 1997.
- [41] T. E. DAVIS, J. C. PRINCIPE. A Simulated Annealing Like Convergence Theory for the Simple Genetic Algorithm, in "Proceedings of the Fourth International Conference on Genetic Algorithm", 13-16 July, 1991, p. 174–182.
- [42] B. GOERTZEL. *Fractal image compression with the genetic algorithm*, in "Complexity International", vol. 1, 1994.
- [43] D. E. GOLDBERG. Genetic Algorithms and Walsh functions: I. A gentle introduction, II. Deception and its analysis, in "Complex Systems", vol. 3, n<sup>o</sup> 2, April 1989, p. 129–171.
- [44] J. JULIANY, M. D. VOSE. *The Genetic Algorithm Fractal*, in "Evolutionary Computation", vol. 2, n<sup>o</sup> 2, 1994, p. 165–180.
- [45] G. MANTICA, A. SLOAN. Chaotic optimization and the construction of fractals : solution of an inverse problem, in "Complex Systems", vol. 3, 1989, p. 37–62.
- [46] D. J. NETTLETON, R. GARIGLIANO. *Evolutionary algorithms and a fractal inverse problem*, in "Biosystems", Technical note, vol. 33, 1994, p. 221–231.
- [47] L. VENCES, I. RUDOMIN. Fractal compression of single images and image sequences using genetic algorithms, The Eurographics Association, 1994.
- [48] E. R. VRSCAY. Fractal Geometry and Analysis, Kluwer Academic Publishers, chap. Iterated function Systems: theory, applications and the inverse problem, J. Bélair and S. Dubuc, 1991, p. 405–468.