INRIA

Project-Team Coprin

Constraints solving, OPtimization, Robust INterval analysis

Sophia Antipolis

Activity Report

2006

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1. Team

COPRIN is a joint project between Certis (École des Ponts et Chaussées) and INRIA.

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Denny Oetomo [funded by the EU project ARES, arrived in October]

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Xavier Bonnaire [professor, University Technique Federico Santa Maria, Valparaiso, Chile]
Marco Ceccarelli [professor, University of Cassino, Italy, 1 week in July]
Ioannis Emiris [associate professor, Department of Informatics & Telecommunications, National University of Athens, Greece, 1 week in February]
Alexandre Goldsztejn [associate researcher, Computer Science Department of the University of California, Irvine, 2 weeks]
Yahia Lebbah [assistant professor in Computer Science, University of Oran, Algeria, 2 weeks in November]
Maria Cristina Riff [professor, University Technique Federico Santa Maria, Valparaiso, Chile]
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2. Overall Objectives

2.1. Overall Objectives

Keywords: *constraints programming, interval analysis, mechanism theory, multi-precision, numerical robustness, optimal design, robotics, symbolic-numerical calculation, systems solving, uncertainty.*

COPRIN is a joint project between Certis (École des Ponts et Chaussées) and INRIA. Until January 1st, 2006 COPRIN was also a joint project with I3S (UNSA) but one of the project members, M. Rueher, decided to leave COPRIN and to create a new team devoted to constraint programming at I3S. Although the scientific objectives and the application domains of the new team are rather different from those of COPRIN, I3S decided to withdraw from COPRIN (but apart of M. Rueher all the other UNSA staff members of COPRIN decided to remain in the project).

Its scientific objective is to develop and implement systems solving algorithms based on constraints propagation methods, interval analysis and symbolic computation, with interval arithmetic as the primary tool.

We are interested in real-valued system solving ($f(X) = 0$, $f(X) \leq 0$), in optimization problems, and in the proof of the existence of properties (for example it exists X such that $f(X) = 0$ or it exists two values X_1, X_2 such that $f(X_1) > 0$ and $f(X_2) < 0$).

Solutions will be searched within a finite domain (called a *box*) which may be either continuous or mixed (i.e. for which some variables must belong to a continuous range while other variables may only have values within a discrete set). An important point is that we aim to find all the solutions within the domain as soon as the computer arithmetic will allow it: in other words we are looking for *certified* solutions. For example, for 0-dimensional system solving, we will provide a domain that includes one, and only one, solution together with a numerical approximation of this solution, that may further be refined at will using multi-precision.

Our research aims to develop algorithms that can be used for any problem or are specific to a given class of problem, especially problems that are issued from application domains for which we have an internal expertise (such as mechanism theory and software engineering).

Implementation of the algorithms will be performed within the framework of the generic software tool `IcosAlias`, currently under development, whose purpose is to allow one to design and test solving algorithms obtained as the combination of various software modules. `IcosAlias` will be based on the already existing libraries `ICOS` and `ALIAS`, that are still being developed.

As a theoretical complexity analysis of the solving algorithms is usually extremely difficult, the efficiency of the algorithm will be experimentally evaluated through `IcosAlias` or `ALIAS` on various realistic test examples.

Dissemination is also an essential component of our activity as interval analysis based methods are not sufficiently known in the engineering and academic communities.

3. Scientific Foundations

3.1. Scientific Foundations

The scientific objective of the COPRIN project is to develop and implement systems solving and optimization algorithms based on constraints propagation methods, interval analysis and symbolic computation, with interval arithmetic as the primary tool (hence solutions will be searched within a finite domain (called a *box*)) .

The results obtained with these algorithms are certified in the sense that no solution can be missed and that, for 0-dimensional system solving, solutions can be calculated with an arbitrary accuracy. Furthermore some of our algorithms will allow us to deal with systems involving uncertain coefficients.

A system will be constituted by a set of relations that may use all the usual mathematical operators and functions (hence we may deal, for example, with the relation $\sin(x + y) + \log(\cos(e^x) + y^2) \leq 0$) and we are interested in *certified solutions* of real-valued system solving problems ($f(X) = 0$, $f(X) \leq 0$), of optimization problems and in the proof of the existence of properties (for example it exists X such that $f(X) = 0$ or it exists two values X_1, X_2 such that $f(X_1) > 0$ and $f(X_2) < 0$).

Our approach is to develop various operators that will be applied in sequence on a box:

1. *exclusion operators*: these operators determine that there is no solution to the problem within a given box

2. *contractors*: these operators may reduce the size of the box i.e. decrease the width of the allowed ranges for the variables

3. *existence operators*: they allow one to determine that there is a unique solution within a given box and are usually associated to a numerical scheme that enable to compute this solution in a safe way

If a given box is not rejected by the exclusion operators and is not modified by the other operators, then we will bisect one of the variables in order to create two new boxes that will be processed later on. Methods for choosing the bisected variable are also clearly within the scope of the project.

Our research aims to develop operators that can be used for any problem or are specific to a given class of problem, especially problems that are issued from application domains for which we have we have internally an expertise (such as mechanism theory and software engineering). Furthermore we will study symbolic computation based methods:

- to develop an user-friendly interface that will automatically generate an executable program, run it, and return the result to the interface

- to analyze the semantic and syntax of the relations involved in a problem, in order to generate automatically specific operators, or to obtain a better interval evaluation of the expressions (as the evaluation of an expression using interval arithmetic is very sensitive to the expression syntax)

- to allow for the calculation of the solutions with an arbitrary accuracy. For this purpose two approaches are currently being used:

  - certified interval solutions are obtained through a compiled solving program and then a symbolic computation procedure will be used to calculate the solutions up to the desired accuracy

  - a multi-precision interval arithmetic package is used within a symbolic-numeric solving program and allows one to obtain the solutions with an arbitrary accuracy

  The first approach is usually much faster than the second one (by a factor of approximately 1000 to 10000) but may fail to determine solutions if the system is numerically badly conditioned, while the second approach will always find the solutions.

4. Application Domains

4.1. Application Domains

Keywords: *geometric constraints*, *mechanism theory*, *optimal design*, *robotics*.

While the methods developed in the project may be used for a very broad set of application domains (for example we have an activity in control theory and in quantum mechanics), it is clear that the size of the project does not allow all of them to be addressed. Hence we have decided to focus our applicative activities on *mechanism theory* (including *robotics*). In this domain our research focuses on *optimal design* and geometrical modeling of mechanisms, especially for the machine-tool industry, automotive suspensions, virtual reality and medical robotics, which all involve the management of *geometric constraints*. As other domains may exhibit equivalent problems as mechanism theory (e.g. molecular chemistry), they may also be addressed, without constituting a major research axis of the project.

Note that until 2006 we have also addressed problems related to uncertainty management in software engineering. Since the departure of M. Rueher this is no more a research activity for COPRIN.

5. Software

5.1. Introduction

Software is an essential part of the research within COPRIN as our researches may be only experimentally validated. Software developments are addressed along various axes:

1. interval arithmetic: although our purpose is not work in this very specialized area (we generally rely on existing packages) interval arithmetic is an important part of our interval analysis algorithms and we may have to extend the existing packages especially for dealing with multi-precision and arithmetic extensions

2. interval analysis libraries: we daily use two libraries that have been designed in the project and are still under development. A long term work is to develop a generic programming framework that allows for modularity and flexibility, with the objective of being able to test easily new functionalities and to build specific solvers by a simple juxtaposition of existing modules

3. interface to interval analysis: in our opinion interval analysis software must be available within general purpose scientific software (such as `Maple`, `Mathematica`, `Scilab`) and not only as a stand-alone tool. Indeed most end-users will be reluctant to learn a new programming language just to deal with solving problems that are only small elements of a more general problem context. Furthermore interval analysis efficiency may benefit from the functionalities available in the general purpose scientific software.

5.2. Interval arithmetic: BIASK, an implementation of Kaucher arithmetic

Keywords: *interval analysis*, *robots kinematic*, *software*.

Participants: Alexandre Goldsztejn [correspondant], David Daney, Carlos Grandón, Gilles Chabert.

We propose an efficient C package to extend the interval arithmetic to generalized intervals. Together with the concept of modal intervals, this arithmetic allows one to use universally quantified variables where the classical interval arithmetic deals with only existential quantification. The structure of the code is similar and compatible with the well known `BIAS/Profil` library. This software has been used to solve a robot kinematic problem [24] and appears to be efficient for determining an inner approximation of interval systems [19], [20], [25], see section 6.2.4.

5.3. Interval analysis libraries

5.3.1. ALIAS

Keywords: *constraints solving*, *optimization*, *software*, *symbolic computation*.

Participants: Jean-Pierre Merlet [correspondant], Gilles Chabert, David Daney, Yves Papegay, Odile Pourtallier.

The `ALIAS` library (*Algorithms Library of Interval Analysis for Systems*) is a collection of procedures based on interval analysis for systems solving and optimization. Its development has started in 1998.

`ALIAS` is constituted of two parts:

- `ALIAS-C++`: the C++ library (96 000 code lines) which is the core of the algorithms

- `ALIAS-Maple`: the `Maple` interface for `ALIAS-C++` (48 000 code lines). This interface allows one to specify a solving problem within `Maple` and to get the results within the same `Maple` session. The role of this interface is not only to generate automatically the C++ code, but also to perform an analysis of the problem in order to improve the efficiency of the solver. Furthermore, a distributed implementation of the algorithms is available directly within the interface.

Our effort this year has focused on linear algebra aspects:

- improved calculation of the determinant of interval matrices (i.e. matrices whose elements have interval values)
- calculation of the spectral radius of an interval matrix and of its characteristic polynomial
- regularity test of interval matrices and of parametric interval matrices (see section 6.2.1).

The current version of the `ALIAS` library is available through the Web page http://www-sop.inria.fr/coprin/logiciels/ALIAS.

5.3.2. IcosAlias

Participants: Gilles Chabert [correspondant], David Daney, Carlos Grandón, Jean-Pierre Merlet, Bertrand Neveu, Gilles Trombettoni.

The `IcosAlias` software is an interval-based constraint solver, that is being tested and used by different members of COPRIN. The purpose of this software is to offer a highly modular constraint solver platform allowing the rapid prototyping of new ideas, filters and solving strategies.

5.4. Interface

As mentioned in the introduction we believe that one of the major restrictions to the use of interval analysis in engineering is the lack of interface with classical scientific software. `ALIAS` has already an interface for `Maple` and our aim is to provide a similar interface for `Mathematica` and `Scilab`.

5.4.1. Mathematica interface to ALIAS

Participants: Yves Papegay [correspondant], Jean-Pierre Merlet.

We are implementing a Mathematica interface to the `ALIAS` library, as well as a set of interval analysis algorithms.

We aim to provide to the community of Mathematica users a transparent access to the functionalities of `ALIAS` for extending the dissemination of our library. But our main goal is to get an high-level modular interface to `ALIAS` for prototyping, and testing easily and quickly new combinations of interval analysis algorithms, with the benefit of using a computational environment including an arbitrary precision interval arithmetic, and symbolic computation skills as well.

As it is already the case with the Maple interface for `ALIAS-C++`, this interface will also perform an analysis of the problem in order to improve the efficiency of the solver.

5.4.2. Scilab interface to interval analysis

Participants: David Daney [correspondant], Raphaël Pereira.

We have decided to provide an interval analysis interface to `Scilab` with the support of INRIA, that has funded the Associated Engineer Raphaël Pereira for that purpose.

The purpose of this interface is first to to offer a simple syntax for using intervals as any other data types, and to allow to easily prototype interval-based algorithms. Then a `Scilab` toolbox will be proposed that will include initially basic modules (interval arithmetics, linear algebra, polynomials) and ultimately most of COPRIN algorithms. The role of R. Pereira will be to develop a stable toolbox with the help of the project members, to provide a documentation and a support to the end-users.

6. New Results

6.1. Robotics and mechanism theory

Keywords: *accuracy*, *calibration*, *mechanism theory*, *parallel robots*, *robotics*, *singularity*, *wire-driven parallel robots*, *workspace*.

The core of our activity in robotics and mechanism theory is the optimal design of mechanisms and the analysis of parallel robots [14], [29]. We have addressed this year:

- the certified solving of the direct kinematics of parallel robot with modeling uncertainties
- the singularity analysis of parallel robots with modeling uncertainties
- the worst-case accuracy analysis of parallel robots over a given workspace
- the workspace analysis of wire-driven parallel robots
- self-collision test for parallel robots.

This year we have also started the hardware development of our wire-driven parallel robot prototype that will constitute our experimental platform for the next 4 to 8 years.

6.1.1. *Certification of the direct kinematics of a 3-2-1 parallel robot*

Keywords: *certification*, *distance equations*, *uncertainty*, *wire-driven parallel robot*.
Participants: Carlos Grandón, David Daney, Yves Papegay.

We have designed a method that allows one to take into account the uncertainties due to measurement and manufacturing when solving the direct kinematics of a specific type of parallel robot, called the 3-2-1 [23]. This method mixes an algebraic approach and constraint programming techniques designed specifically for distance equations (see section 6.3.2). It has allowed us to certify the result of the measurement device *CATRASYS* (Cassino Tracking System) [36] in the framework of an informal collaboration with Cassino University (Italy).

6.1.2. *Singularity analysis*

Keywords: *interval analysis*, *singularity analysis*, *uncertainty*.
Participants: David Daney, Peter Donelan, Jean-Pierre Merlet.

Singularity analysis is a recurrent research domain for the project. The purpose is to design algorithms allowing to determine if a given workspace for a parallel robot is singularity-free (we remind that in a singular pose the robot cannot be controlled and that the force/torque in its legs may go to infinity, leading to a breakdown of the robot). Checking for singularity amounts to determine if a matrix, the inverse Jacobian of the robot, may become singular at a pose within the workspace. In the past years we have already presented an interval analysis based algorithm that was able to perform this check as soon as an analytical form of the matrix determinant was available. But if uncertainties on the robot modeling are taken into account and are kept as unknowns in the matrix, then the determinant size increases drastically and symbolic tools are no more able to calculate it. Using our work on the regularity of parametric interval matrices (see section 6.2.1) we have designed a very efficient singularity check algorithm that allows to certify that a given workspace is singularity-free even if there are large uncertainties in the robot modeling [28].

6.1.3. *Worst case accuracy for parallel robots*

Keywords: *accuracy*, *interval analysis*.
Participant: Jean-Pierre Merlet.

Errors $\Delta\Theta$ in the measurements of the sensors that are used to control a parallel robot result in errors $\Delta\mathbf{X}$ on the platform positioning. Both types of error are linearly related through the pose dependent Jacobian matrix $\mathbf{J}(\mathbf{X})$ of the robot as $\Delta\mathbf{X} = \mathbf{J}(\mathbf{X})\Delta\Theta$. Unfortunately for closed-loop chains usually only the inverse Jacobian matrix is known in closed-form. Accuracy analysis consists in assuming that the sensor errors are bounded (without lack of generality we may assume that all sensor errors satisfy $|\Delta\Theta_i| \leq 1$) and verifying that the platform positioning errors over a given workspace are lower than a given threshold. In other words we have to check that all solutions in $\Delta\mathbf{X}$ of the set of linear systems $\mathbf{J}^{-1}(\mathbf{X})\Delta\mathbf{X} = \Delta\Theta$ satisfy the threshold constraint. This is a classical problem in interval analysis [38] but the usual methods (such as the Gaussian elimination scheme) does not take into account that the elements of $\mathbf{J}^{-1}$ are not independent (they are functions of the same unknowns) and consequently they provide a large overestimation of the enclosure of the solutions for given ranges for $\mathbf{X}$. We have shown that computing the derivatives of the elements of $\mathbf{J}^{-1}$ with respect to $\mathbf{X}$ and propagating these derivatives in the Gaussian elimination scheme to improve the interval evaluation of the elements appearing in the scheme was sufficient to largely reduce the overestimation of the enclosure of the solution set, leading to a very efficient accuracy algorithm [30].

6.1.4. *Workspace of wire-driven robots*

Keywords: *force analysis*, *interval analysis*, *workspace*.

Participants: Marc Gouttefarde, David Daney, Jean-Pierre Merlet.

The mobile platform of a parallel wire-driven robot is connected to a base by extensible wires. Since wires can only pull and not push on the mobile platform, the relationship between the pose and the feasible wrenches at the mobile platform is an important issue for parallel wire-driven robots which can be addressed in terms of workspace.

Recently, in the case of planar parallel robots with three degrees of freedom, we have proposed an efficient geometric method [11] that allows to determine the workspace, called the wrench-closure workspace (WCW), in which the platform can be fully constrained by the cables (i.e. all wrenches may be imposed on the platform if no limits are assumed on the wire tensions). Since many applications require robots having more than three degrees of freedom (d.o.f.), we have extended the method to the determination of three dimensional cross sections of the WCW of six-d.o.f. wire-driven robots [21], by using the ALIAS library. However the WCW does not take into account the physical constraints $t_{min} \leq t \leq t_{max}$ on the tensions t in the cables ($t_{min} > 0$) and geometric methods are limited to three dimensional workspaces. We have developed a method based on interval analysis that overcomes these limitations. It allows the determination of the wrench-feasible workspace of any n-d.o.f. parallel robot driven by $m \geq n$ cables (generally n=3 or n=6). This method constitutes a necessary basis of the optimal design methodology proposed by the COPRIN project.

6.1.5. *Collision check for parallel robots*

Keywords: *collision*, *interval analysis*.

Participants: David Daney, Jean-Pierre Merlet.

In the past we have investigated the influence of leg interference on the workspace of parallel robots by using a geometrical approach. We have extended the problem in the case where elements linked either to the base or to the platform may interfere with each other or with the legs. Assuming that each element that may potential interfere are finite cylinders we have designed an interval analysis based algorithm that allows one to check if a given 6D workspace is collision-free [27]. This study was motivated by the development of our wire-driven robot (for which it will be necessary to check such interference) and by an industrial contract (see section 7.2).

6.1.6. *Prototype of wire-driven parallel robot*

Keywords: *hardware*, *robot prototype*.

Participants: Nicolas Chleq, David Daney, Marc Gouttefarde, Jean-Pierre Merlet.

The development of our new wire-driven robot has been delayed due to unexpected events related to the funding. With the help of a grant of the Conseil Général des Alpes-Maritimes and our own financial resources we have started the development of this wire robot but with a hardware that will allow only planar motions and with limited possibilities regarding force-feedback. A DREAM engineer has started the integration of the components in October 2006 and we expect to be able to realize the first experiments at the end of the first semester of 2007. We remind that this robot will feature a high mechanical modularity (the location of the wire system may be changed at will and various maximal length changes will be possible) and a very high speed. Using devoted algorithms it will be possible to determine what is the best geometry for a given task and we intend to use this robot in very diverse applications ranging from service robotics (windows washing robot) to medical robotics (e.g. rehabilitation within the 3+3 Med action, see section 8.1.1).

6.2. Linear algebra

6.2.1. *Regularity of parametric interval matrices*

Keywords: *regularity*.

Participants: Jean-Pierre Merlet, David Daney.

Checking the regularity of interval matrices is a well known problem in interval analysis for which there are well known algorithms (Rohn test [37], numerical pre-conditioning, ...). But in many applications (see for example section 6.1.2) interval matrices appear only because we have to consider a matrix whose elements are functions of unknowns which have interval values and we call this type of matrix *parametric interval matrix*. Let $\mathbf{X}$ be a set of unknowns with known interval values and $\mathbf{J(X)}$ be a parametric matrix. Instantiating each variable with all its possible values in its range will lead to a set $\mathbf{J_p}$ of scalar matrices. By substituting in $\mathbf{J}$ the variables by their ranges and interval evaluating each element of the matrix we get an interval matrix $\mathbf{J_i}$, i.e. a set of scalar matrices. It is clear that $\mathbf{J_p} \subset \mathbf{J_i}$ and that $\mathbf{J_p}$ may be much more smaller than $\mathbf{J_i}$ if they are strong dependencies between the elements of $\mathbf{J}$ with respect to $\mathbf{X}$. Hence checking the regularity of $\mathbf{J_i}$ in place of $\mathbf{J_p}$ as done by the algorithm of interval analysis is overkill and we have therefore investigated regularity test methods that allow to better take into account the dependency problem. Our approach is twofold:

1. either we determine an interval matrix $\mathbf{J_m}$ that verify $\mathbf{J_p} \subset \mathbf{J_m} \subset \mathbf{J_i}$ and apply classical interval analysis regularity test,

2. or we specialize regularity test theorems for the case of specific parametric matrices.

For the first approach we have shown that the efficiency of conditioning may be drastically improved by using a symbolic pre-processing. Both the conditioning matrix $\mathbf{K}$ and the parametric matrix $\mathbf{J}$ are kept symbolic and their products $\mathbf{KJ}, \mathbf{JK}$ are computed symbolically. Then a symbolic procedure tries to decrease the number of multiple occurrences of the elements of $\mathbf{X}$ in $\mathbf{KJ}, \mathbf{JK}$. The results are two conditioned matrices that will be used during the regularity test to produce two interval matrices by substituting the interval values for $\mathbf{X}$ and the scalar values for the elements of $\mathbf{K}$. Classical interval analysis regularity tests are then applied on these interval matrices. Such method has proven to be very efficient in the case of the application described in section 6.1.2, leading to a decrease in computation time of more than 10000 compared to classical numerical conditioning [28].

As for the second approach, we have specialized Rohn method for parametric matrices whose rows (or columns) include elements which are linearly dependent with respect to a subset $\mathbf{Y}$ of $\mathbf{X}$. Rohn proves that for an interval matrix if a given set of extremal matrices (i.e. scalar matrices whose elements are the lower or upper bound of the ranges of the interval matrix) have a determinant of same sign, then the interval matrix is regular. We have established that a similar result holds if instead of considering the bound of the intervals we fix the element of $\mathbf{Y}$ to their minimum or maximum value [28]. For example if a parametric matrix has a row written as (x, x) with x in the range [-1,1], then the extremal matrices considered in Rohn test will have rows $(1, 1), (-1, 1), (1, -1), (-1, -1)$ while our test will involve only matrices with rows $(1, 1), (-1, -1)$.

6.2.2. *Enclosure for interval matrix eigenvalues*

Keywords: *characteristic polynomial*, *eigenvalues*, *spectral radius*.

Participants: David Daney, Odile Pourtallier, Alexandre Goldsztejn, Jean-Pierre Merlet.

For many problems it is interesting to prove properties related to the eigenvalues of real matrices, such as minimal and maximal values. For example in control theory the stability of a closed-loop system may be proven if the real part of all eigenvalues is negative and in robotics the eigenvalues of the product of the inverse Jacobian by its transpose is used to measure the amplification factor between the sensor errors and the end-effector positioning errors (see section 6.1.3). In many cases we have to consider not only one matrix but a set of matrices: this may occur if they are uncertainties in the system parameters and/or if we have to prove the property over some state space (for example in robotics over a given workspace for the end-effector). For such cases we have designed an algorithm based on generalized interval arithmetics and on regularity test (see previous section) that allows one to determine bounds for the real eigenvalues. We are currently investigating its extension to complex eigenvalues. We have also designed an algorithm that allows to calculate the interval coefficients of the characteristic polynomial of an interval matrix, that is used in a procedure that finds a bound for the spectral radius of this matrix.

6.2.3. *Improving the efficiency and reliability of linear programming*

Keywords: *linear programming.*

Participants: David Daney, Jean-Pierre Merlet, Pavlos Souldatos.

Interval analysis algorithms are often partially based on linear programming. For example global filters for equations system solving may be based on a linearization of the equations associated to the simplex method for reducing the search space. To preserve the certification properties associated with interval analysis, rounding errors have to be considered when applying linear programming methods.

We have started a step by step study of linear programming methods used in interval analysis in order to propose a certified version (i.e. taking into account round-off errors) of these methods.

6.2.4. *Quantified interval linear systems*

Keywords: *generalized intervals*, *interval linear systems.*

Participants: Gilles Chabert, Alexandre Goldsztejn.

One of the fundamental problem of interval analysis is the determination of the solution set of a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A}$ and $\mathbf{b}$ may be any instance in an interval matrix $[\mathbf{A}]$ and an interval vector $[\mathbf{b}]$. Usually a solution $\mathbf{x}$ is said to satisfy $[\mathbf{A}]\mathbf{x} = [\mathbf{b}]$ if

$$(\exists \mathbf{A} \in [\mathbf{A}]) \ \ (\exists \mathbf{b} \in [\mathbf{b}]) \ \ \mathbf{A}\mathbf{x} = \mathbf{b}.$$

. But we may be interested in other problems e.g. looking for all $\mathbf{x}$ such that

$$(\forall \mathbf{A} \in [\mathbf{A}]) \ \ (\exists \mathbf{b} \in [\mathbf{b}]) \ \ \mathbf{A}\mathbf{x} = \mathbf{b}, \quad \text{or} \quad (\forall \mathbf{b} \in [\mathbf{b}]) \ \ (\exists \mathbf{A} \in [\mathbf{A}]) \ \ \mathbf{A}\mathbf{x} = \mathbf{b}.$$

The set of points satisfying such relations are called *AE-solution sets*. Since the shape of such sets can be very complicated, our objective is to approximate them by interval vectors either outwardly (outer approximation) or inwardly (inner approximation).

We have investigated different approaches to compute outer and inner approximation, inspired from classical interval analysis. Our contributions have been:

1. an extension of the Hansen-Bliek method to right-quantified systems ($[\mathbf{A}]$ remains existentially quantified) for computing an optimal outer approximation of preconditioned systems [9],

2. a generalized interval LU decomposition for computing inner and outer approximations [19],

3. an improvement of the generalized Gauss-Seidel scheme for calculating outer approximation, that includes the management of the division of generalized intervals by a domain that includes 0 [20],

4. a new programming language for modeling AE-solution sets using generalized intervals. We propose a slight change from the state of the art which allows to unify frameworks of AE-solution sets and classical solution sets [20],

5. a test platform with a *branch & prune* solver of parameterized non-linear problems. Inside a domain splitting loop, the system is linearized and tackled with one of the above methods.

6.3. Algebraic systems

6.3.1. *Zero set of uncertain polynomials*

Keywords: *Gershgorin circles*, *extremal polynomials*, *pseudozero set*, *uncertain polynomials.*

Participants: David Daney, Sylvie Icart [UNSA, I3S], Odile Pourtallier.

Determining the zero set of an interval polynomial is necessary for many applications (see for example section 6.2.2). We have designed a localization method based on Gershgorin circles which improves the classical Cauchy or Knuth bounds for "non-uncertain" polynomials (i.e. polynomials with real or complex coefficients) at the price of solving optimization problems with $d - 1$ real variables (d being the degree of the polynomial). We also shown that for determining bounds for the zero set of interval polynomials it is sufficient to determine the roots of some extremal polynomials (i.e. polynomials with real coefficients that are upper or lower bounds of the interval coefficients). It can indeed be shown that the set of real solutions of an interval polynomial is the union of intervals whose boundaries are solutions of some extremal polynomials. Although similar results exist for complex solutions, our experiments have shown that they were not efficient for the practical determination of the complex zeros. We intend to investigate more deeply this direction.

If interval coefficients are obtained for a polynomial because these coefficients are analytical functions of parameters with interval values, an algorithmic approach has allowed us to further reduce the size of the set of real solutions. This is an important result as such structured interval polynomials occur frequently in applications.

6.3.2. *Uncertainty management in distance constraints*

Keywords: *distance constraint*, *inner approximation*, *uncertainty*.

Participants: Carlos Grandón, Bertrand Neveu, Alexandre Goldsztejn.

A distance equations system describes that the Euclidean distance between points in a n-dimensional space is supposed to be known and solving a distance equations system consists in determining the coordinates of the points that are not known. This is a major research axis of the project as this problem appears frequently in various application domains such as robotics (e.g. for determining the pose of a parallel robot), molecular biology (for determining the conformations of a molecular structure) or aeronautics (for object localization), among others.

One of the aim of our research is to solve systems of distance equations where the distances are uncertain (e.g. because they are obtained as measurements). The solutions of this problem are no more isolated points but a continuum of solutions. Because this continuum can be very difficult to express, it is natural to try to approximate it. An approximation of particular interest is one that include all solutions, i.e., an enclosure of the set of solutions of the problem.

Our objective is to approximate this set (that may be constituted of distinct regions called *solution subset*) as precisely as possible, keeping useful information about the different solution subsets and inner and outer zones.

Classical strategies for solving this problem consist in algebraic or numerical techniques based in interval analysis for computing outer approximations. We have developed two strategies to calculate an inner approximation (i.e. a zone containing only solutions):

- a specific quantified elimination algorithm [35]
- an algorithm based on generalized interval evaluation.

The two algorithms have been compared [24], [25] and we have shown that generalized interval evaluation is more efficient.

As a very accurate paving of the solution subsets is not always necessary (in particular when only the number of disconnected subsets is needed), we have studied another approach for computing a rough approximation of each solution subset [26]. It combines a filtering phase with a conditional bisection phase in order to split boxes only when they generate independent solution subsets. Moreover, if inner approximations are required for each solution subset, we have developed a technique, called generalized interval projection, for extending consistent domains [22].

An application of this work has been the calculation of the direct kinematic of a parallel robot (see section 6.1.1) in which the distances are measured by a wire system [23].

6.4. Geometrical constraints

6.4.1. Decomposition of constraint systems

Keywords: *constraint system decomposition*, *geometric constraints*.

Participants: Bertrand Neveu, Gilles Trombettoni, Gilles Chabert.

When a constraint system is decomposable (this feature often appears in CAD where rigid subparts can be computed and assembled), it is interesting to use this decomposition when solving the system, as it greatly simplifies the solving.

Inter-block backtracking (IBB) computes all the solutions of sparse decomposable systems of non-linear equations over the reals. This algorithm, introduced in [1], handles a system of equations previously decomposed into a set of (small) $k \times k$ sub-systems, called blocks. Partial solutions are computed in the different blocks and combined together to obtain the set of global solutions.

When solutions inside blocks are computed with interval-based techniques, IBB can be viewed as a new interval-based algorithm for solving decomposed equation systems. Our previous implementations were based on `Ilog Solver` and its `IlcNum` library but the lack of knowledge on the behavior of this software induces several strong limitations. We have implemented a new version of IBB based on the interval-based library `IcosAlias` developed by G. Chabert. This new version allows IBB to become reliable (no solution is lost) while allowing gaining several orders of magnitude in term of computation time compared to the solving of the whole system [31]. We compared several variants of IBB on a sample of benchmarks, in order to better understand the behavior of IBB. The main conclusion is that the use of an interval Newton operator inside blocks has the most positive impact on the robustness and performance of IBB. Inter block filtering strategies can also sometimes become useful for replacing intelligent backtracking schemes.

6.5. Miscellaneous results

6.5.1. Constructive Interval Disjunction

Keywords: *constructive disjunction*, *filtering*, *intervals*.

Participants: Gilles Trombettoni, Gilles Chabert.

Shaving and constructive disjunction are two main refutation principles used in constraint programming. The shaving principle allows us to compute the singleton arc-consistency (SAC) of finite-domain constraints solving problems (CSP) and the 3B-consistency of numerical CSPs. Considering the domains as unary disjunctive constraints, one can adapt the constructive disjunction, proposed by Van Hentenryck [39] in the nineties, to provide another general-purpose refutation operator. One advantage over the shaving is that the partial consistency performed to refute values in the domains is not entirely lost.

We propose a new filtering operator for numerical CSPs, called CID (Constructive Interval Disjunction), based on constructive disjunction [34], and a hybrid algorithm, called 3BCD, mixing shaving and constructive disjunction. Experiments have been performed on 20 benchmarks. Adding CID to bisection, hull or box consistency, and interval Newton, produces a gain in performance of 1, 2 or 3 orders of magnitude on various benchmarks. 3BCD and adaptive CID filtering algorithms with no additional parameters compare advantageously to the 3B-consistency operator. Finally, the CID principle has led us to design a new splitting strategy.

6.5.2. 2D strip packing problem

Keywords: *bin-packing*, *incomplete optimization methods*, *local search*.

Participants: Gilles Trombettoni, Bertrand Neveu, Maria-Cristina Riff, Xavier Bonnaire.

The 2D strip packing problem is a variant of the well-known combinatorial bin packing problem. It consists in placing predefined rectangles in a 2D strip such that no two rectangles overlap, while minimizing the height of the strip.

We have selected this challenging problem to work in collaboration with Maria-Cristina Riff and Xavier Bonnaire from the University of Santa Maria in Valparaiso (Chile). This collaboration has obtained a financial support from INRIA and CONYCIT (Chile).

The strip packing problem is challenging both for complete and incomplete methods. All the best algorithms use a heuristic method (e.g., Bottom-left fit, Best-fit) to add the rectangles on the strip one by one while maintaining a so-called bottom-left (BL) property. The main drawback is that these heuristics are not really incremental, so that it is difficult to maintain the BL property when one rectangle is removed. This explains, at least partially, the poor results obtained by complete methods (that must remove a rectangle at a given position to put it elsewhere) and by meta-heuristics (that must move a rectangle from a position to another).

We have designed a new property, called "maximal-hole" property, that maintains the full set of rectangle holes of maximal size (a rectangle must be placed in such a hole). As opposed to the BL property, the maximal-hole property can be maintained at a low cost when a rectangle is added (by a heuristic or a complete method), but also when a rectangle is removed.

We have applied this approach for designing metaheuristics with more incremental moves. The first implementation of these metaheuristics inside our INCOP library for incomplete optimization is promising.

6.6. Symbolic tools for modeling and simulation

Keywords: *accuracy*, *code generation*, *modeling*, *reliability*, *simulation*, *symbolic computation*.

Participant: Yves Papegay.

This activity is the main part of a long-term ongoing collaboration with Airbus whose goal is to directly translate the work of aeronautics engineers into digital simulators to accelerate aircraft design. This project already has applications in the aircraft maker development departments.

Modeling and simulation processes usually begin with using scientific theories for describing physical features with formulae and computation algorithms. Based on these models, numerical codes are then implemented for simulation and visualization of these features. In an industrial context, the large number of parameters and equations involved in models make the whole process very long, complex and expensive, especially as reliable and safe codes are required.

Our previous work led to a model edition environment, based on symbolic computation tools, that makes it possible to enter the formulae and the algorithms of the models and to validate them numerically on a reduced set of data. We then worked on how to use these models to generate fully automatically the numerical real-time simulation engines to be plugged in the flight simulator, as well as the technical documentation associated with such simulations, which is indispensable for corporate memory.

In 2004 and 2005 we developed a prototype of C code generation for real time simulation, and we extended its functionalities to automatic evaluation scheduling, generic models, and optimization of the generated code. In 2006, we improved this prototype into a really powerful, robust and complete tool, accepting a wide class of models as inputs, and a lot of constraints on the generated C code. This tool has been tested with the complete model of the landing gear of the next generation of Airbus aircraft.

Theoretical work is in progress on automatizing the initialization of the values of the status parameters of a simulation. Our long term objective is to be able to introduce uncertainties in the parameters for obtaining reliable results that may be used for the optimal design of the system.

7. Contracts and Grants with Industry

7.1. Airbus France

Participant: Yves Papegay.

To improve the production of numerical (flight) simulators from models of aerodynamics, Airbus France is interested in methods and tools like those described in 6.6.

In 2003, a two-years long contract had been set up for prototyping some code generation features. In 2005, this contract has been extended for another two years period. For confidentiality reasons, no further details can be given here.

7.2. Paulstra

Participant: Jean-Pierre Merlet.

Paulstra designs exhaust lines for automotive and is currently proposing a flexible connection between the motor and underbelly exhaust line. For designing this part is is necessary to measure the motor vibration in various operating conditions in order to be able to then replay such vibrations on a vibration line for extensive test of the part. The automotive manufacturer provides a new car and a test circuit to Paulstra which has at most one day to perform the measurements. For these measurements Paulstra was looking for a 6 d.o.f. measurement device that should be easy to insert within the reduced space available in the underbelly of a car. We have proposed them the use of a passive parallel robot using extensible wires as legs. Being given the maximal motion range and the desired measurement accuracy we have started the dimensional synthesis of the device. But it has been necessary to develop a specific self-collision algorithm as the exhaust pipe is located inside the interior of the robot. We have used interval analysis to design a general purpose self-collision test that allows one to test intersection between cylinders which may be either fixed on the base, the platform or associated to the legs [27]. The algorithm is able to detect that a collision occurs within a given workspace or on a given trajectory.

8. Other Grants and Activities

8.1. International and National initiatives

8.1.1. 3+3 Med

Participants: Jean-Pierre Merlet, David Daney.

We have obtained an INRIA grant for a collaborative work with Cassino University, Monastir engineer school and University of Oran for a preliminary work on the development of a wire-driven parallel robot for rehabilitation. Such robot will allow for an objective monitoring of the patient progress (i.e. measuring the range of motion of the injured member during the therapy) and to be able to exert safely a force-controlled rehabilitation protocol with the advantages of being a low-cost platform that is furthermore minimally intrusive for the patient. We have already established what should be the motion measurement ranges for various types of injuries (lower and upper members) together with the amplitude of the desired forces that should be exerted.

8.2. Participation to National and International Conferences

8.2.1. International Conferences

- D. Daney participated to EuComes (European Conf. on Mechanism), Obergurgl, February 21-26 and to IntCP'06, Nantes, France, September 25th, 2006.

- G. Chabert participated to NMA'06 - 6th Int. Conf. on Numerical Methods and Applications, Borovets, Bulgaria (August 2006) and SCAN'06 - 12th Int. Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics, Duisburg, Germany (September 2006).

- M. Gouttefarde presented a paper at the Advances in Robot Kinematics (ARK) Conf. held in Ljubljana, Slovenia, June 2006.

- J-P. Merlet participated to EuComes (European Conf. on Mechanism), Obergurgl, February 21-26, Advances in Robot Kinematics (ARK) Conf., Ljubljana, Slovenia, June 2006, IEEE ICRA in Orlando (May 2006), Chemnitz Parallel Kinematic Seminar (April 2006) and gave an invited talk to the 12th Int. Conf. on Principles and Practice of Constraint Programming (CP 2006), Nantes (September 2006). He was invited to the Gigolag semester on global optimization at the Schrödinger Institute, Vienna (November and December 2006).

- B. Neveu and G. Trombettoni visited the research team of Maria-Cristina Riff Rojas at Federico Santa Maria University in Valparaiso, Chile, during 3 weeks, in May 2006, within the INRIA-CONYCIT cooperation project. Maria-Cristina Riff and Xavier Bonnaire visited in return the COPRIN team in July 2006, during 3 weeks. B. Neveu has presented a paper at the 12th Int. Conf. on Principles and Practice of Constraint Programming (CP 2006), Nantes, France (25-29 September).

- Y. Papegay attended the 8th Int. Mathematica Symposium (IMS'2006), Avignon, June, where his ongoing work on generation of simulators has been presented by his Airbus partner. He also attended the Wolfram 2006 Technology Conf. in October at Urbana Champaign, (Illinois, USA) and gave a talk on C code generation.

- G. Trombettoni participated at the 12th Int. Conf. on Principles and Practice of Constraint Programming (CP 2006), Nantes, France (25-29 September).

8.3. Other Activities

8.3.1. *National Activities*

The "Conseil Général" has allocated a grant of 10 000 euros for the development of the wire robot that we are currently developing for service robotics (see section 6.1.6).

8.3.2. *INRIA activities*

- J-P. Merlet hold the position of CISC (Advisor for Science Information and Communication) of INRIA Sophia and is an elected member of the INRIA Evaluation Board. He was a member of the jury of the competition for hiring junior researcher at INRIA Grenoble

- D. Daney is member of the CUMIR (Comité des Utilisateurs des Moyens Informatiques, Recherche).

8.3.3. *European Activities*

Our proposal ARES for a STREP "Adventure" has been accepted by the EU. The partners of this project are: Scuola Superiore Sant'Anna, Pisa (coordinator), ETH Zürich, Universitat de Barcelona, Centre de Recerca en Bioelectrònica i Nanobiociència.

The purpose of this project is to conceptually investigate and develop a prototype of a system for endoluminal surgery, based on a fleet of modular micro-robots that may be connected to form a reconfigurable mechanism. The project will primarily focus on critical theoretical and technological issues for implementing the concept, including the kinematic analysis of the reconfigurable internal mechanisms, locking/unlocking systems, control and communication, design, fabrication and integration of prototype sensing and actuation modules. ARES has started at the beginning of 2006 and COPRIN is in charge of the reconfigurable strategies of the robots so that it will satisfy given performance criteria in spite of the unavoidable mechanical uncertainties.

9. Dissemination

9.1. Leadership within scientific community

- D. Daney was member of the program committee of IntCP'06.

- J-P. Merlet is suppleant member of the "commission de spécialistes" (61th section) of Nice University and is Chairman of the French section of IFToMM. He is associate editor of the Mechanism and Machine Theory journal, member of the Eucomes (European Conf. on Mechanisms) scientific Committee, of the Scientific Committee of the robotics GDR proposed to CNRS. He is a member of the `Ensemble` working group that promotes the use of interval analysis in the field of Control Theory.

  He has been reviewer for ASME J. of Mechanical Design, Int. J. of Robotics Research, IEEE Trans. on Systems, Man and Cybernetics, IEEE Trans. on Robotics, IEEE/ASME Trans. on Mechatronics.

  J-P. Merlet was a member of LIRMM evaluation board and IEEE ICRA Scientific Committee.

- B. Neveu was member of the Program Committee of "Journées Francophones de programmation par contraintes" (JFPC 2006) and reviewer for conferences CPAIOR 2006, CP 2006, AAAI 2006, ECAI 2006, CARI 2006, IJCAI 2007

- Y. Papegay has been Chair of the Program Committee of the 8th Int. Mathematica Symposium (IMS'2006). He is member of the "commission de spécialistes" (4th section) of the University of French Polynesia.

- Odile Pourtallier has been member of the Organizing Committee and of the Program Committee of the 12th Int. Symposium on Dynamic Games and Application, held in Sophia Antipolis, July 3,6th. She is secretary of the Int. Society of Dynamic Games.

- Gilles Trombettoni has been member of the Program Committee of the CPAIOR 2006 conference (constraint programming, artificial intelligence, operational research). He has reviewed papers for the SAC conference, for the journals AIEDAM and IJCGA.

9.1.1. Activities of general interest

We have produced this year a dissemination movie for undergraduate students that illustrates graphically the basic principle of interval analysis for solving a system of two non-linear equations in two unknowns. It is available on the project Web page and on a CD-ROM with a French and English version. It has been especially well received in the community and is used as an introduction for teaching in various Universities. In the future we intend to realize a version for graduate students that illustrate more advanced concepts and a version for engineers where other possible applications of interval analysis will be presented.

9.2. Teaching

- D. Daney gave course in Medical robotics, DESS Génie Bio-médical, Univ. Nice Sophia Antipolis (15h). .

- G. Chabert gave TD's at EPU Polytech'Nice engineering school (1st and 2nd year), in C and Java programming.

- J-P. Merlet taught 6 hours of robotics at ISIA

- B. Neveu and G. Trombettoni gave lectures on constraint programming in the Computer Science Master at UNSA (20 h) and in EPU at UNSA (20h).

- O. Pourtallier taught 6 hours on game theory to master OSE, at École des Mines de Paris, Sophia Antipolis and 6 hours on optimization, to DESS IMAFA at UNSA.

- G. Trombettoni is an assistant professor in computer science at IUT R&T (networks and telecoms) of Sophia Antipolis.

9.2.1. PhD thesis

- J-P. Merlet was a jury member of 1 PhD and 1 HDR defenses
- B. Neveu was reviewer and jury member of 2 PhD defenses.

9.3. PhD thesis

Current PhD thesis:

1. G. Chabert, *Langage de pilotage et de paramétrage d'algorithmes de résolution de contraintes par intervalles*, University of Nice-Sophia Antipolis (to be defended in December).

2. C. Grandón, *Résolution de systèmes d'équations avec incertitudes*, University of Nice-Sophia Antipolis.

10. Bibliography

Major publications by the team in recent years

[1] C. BLIEK, B. NEVEU, G. TROMBETTONI. *Using Graph Decomposition for Solving Continuous CSPs*, in "Principles and Practice of Constraint Programming, CP'98", LNCS, vol. 1520, Springer, 1998, p. 102-116.

[2] D. DANEY, Y. PAPEGAY, B. MADELINE. *Choosing measurement poses for robot calibration with the local convergence method and Tabu search*, in "Int. J. of Robotics Research", vol. 24, n° 6, June 2005, p. 501-518.

[3] C. JERMANN, G. TROMBETTONI, B. NEVEU, M. RUEHER. *A Constraint Programming Approach for Solving Rigid Geometric Systems*, in "Proceedings of CP'2000,Sixth International Conference on Principles and Practice of Constraint Programming", vol. 1894, LIM, 2000, p. 233-248.

[4] J.-P. MERLET. *Solving the Forward Kinematics of a Gough-Type Parallel Manipulator with Interval Analysis*, in "Int. J. of Robotics Research", vol. 23, n° 3, 2004, p. 221-236.

[5] J.-P. MERLET. *Parallel robots, 2nd Edition*, Springer, 2005.

[6] Y. PAPEGAY. *From Modeling to Simulation with Symbolic Computation: An Application to Design and Performance Analysis of Complex Optical Devices*, in "Proceedings of the Second Workshop on Computer Algebra in Scientific Computing, Munich", Springer Verlag, June 1999.

[7] G. TROMBETTONI. *A Polynomial Time Local Propagation Algorithm for General Dataflow Constraint Problems*, in "Proc. Constraint Programming CP'98, LNCS 1520 (Springer Verlag)", 1998, p. 432–446.

Year Publications

Books and Monographs

[8] Y. PAPEGAY (editor). *Applied Mathematica, Proc. of the 8th Int. Mathematica Symposium*, INRIA, Avignon, Juin 2006.

Articles in refereed journals and book chapters

[9] G. CHABERT, A. GOLDSZTEJN. *Extension of the Hansen-Bliek Method to Right-Quantified Linear Systems*, in "Reliable Computing", to appear.

[10] D. DANEY, N. ANDREFF, G. CHABERT, Y. PAPEGAY. *Interval method for calibration of parallel robots: a vision-based experimentation*, in "Mechanism and Machine Theory", vol. 41, n° 8, August 2006, p. 929-944.

[11] M. GOUTTEFARDE, C. M. GOSSELIN. *Analysis of the Wrench-Closure Workspace of Planar Parallel Cable-Driven Mechanisms*, in "IEEE Trans. on Robotics", vol. 22, n° 3, June 2006, p. 434-445.

[12] C. JERMANN, G. TROMBETTONI, B. NEVEU, P. MATHIS. *Decomposition of Geometric Constraint Systems: a Survey*, in "Int. Journal of Computational Geometry and Applications (IJCGA)", to appear, vol. 16, 2006.

[13] J.-P. MERLET. *Jacobian, manipulability,condition number, and accuracy of parallel robots*, in "ASME J. of Mechanical Design", vol. 128, n° 1, January 2006, p. 199-206.

[14] J.-P. MERLET. *Robust Design of a Deployable Space Telescope*, in "ECMI Newsletter", vol. 39, March 2006, p. 9-11.

[15] Y. PAPEGAY. *Exploring Board Game Strategies*, in "The Mathematica Journal", vol. 10, n° 2, 2006, p. 363-395.

[16] G. TROMBETTONI, M. WILCZKOWIAK. *GPDOF: A Fast Algorithm to Decompose Under-constrained Geometric Constraints: Application to 3D Modeling*, in "Int. Journal of Computational Geometry and Applications (IJCGA)", to appear, vol. 16, 2006.

Publications in Conferences and Workshops

[17] D. DANEY, ETAL . *Calibration of parallel robots: on the elimination of pose-dependent parameters*, in "1st European Conf. on Mechanism Science (Eucomes), Obergurgl", February 21-26 2006.

[18] L. FARENC, Y. PAPEGAY. *Modeling flight dynamics for real-time simulator applications*, in "International Mathematica Symp.", Juin 2006.

[19] A. GOLDSZTEJN, G. CHABERT. *A Generalized Interval LU Decomposition for the Solution of Interval Linear Systems*, in "NMA - 6th International Conference on Numerical Methods and Applications", Springer, 2006.

[20] A. GOLDSZTEJN, G. CHABERT. *On the Approximation of Linear AE-Solution Sets*, in "SCAN - 12th International Symposion on Scientific Computing, Computer Arithmetic and Validated Numerics", 2006.

[21] M. GOUTTEFARDE, J.-P. MERLET, D. DANEY. *Determination of the wrench-closure workspace of 6-dof parallel cable-driven mechanisms*, in "ARK, Ljubljana", June 26-29 2006, p. 315-322.

[22] C. GRANDÓN, G. CHABERT, B. NEVEU. *Generalized Interval Projection: A New Technique fo r Consistent Domain Extension*, in "IJCAI", 2007.

[23] C. GRANDÓN, D. DANEY, Y. PAPEGAY. *Combining CP and Interval Methods for solving the Direct Kinematic of a Parallel Robot under Uncertainties.*, in "Workshop of the Int. Conf. on Principles and Practice of Constraint Programming (IntCP 06)", September 2006.

[24] C. GRANDÓN, A. GOLDSZTEJN. *Inner Aproximation of Distance Constraints with Existentially Quantified Parameters*, in "SAC '06: Proceedings of the 2006 ACM symposium on Applied computing, New York, NY, USA", ACM Press, 2006, p. 1660–1661, http://www-sop.inria.fr/coprin/cgrandon/Publica/index.html.

[25] C. GRANDÓN, A. GOLDSZTEJN. *Quantifier Elimination versus Generalized Interval Evaluation: A comparison on a Special Class of Quantified Constraints*, in "Proc. of the 11th Information Processing and Management of Uncertainty International Conference, IPMU 2006, Paris, France", Editions EDK, 2006, p. 786–793, http://www-sop.inria.fr/coprin/cgrandon/Publica/index.html.

[26] C. GRANDÓN, N. MORALES, G. CHABERT. *A Strategy for solving Systems of Equations with Uncertainties*, in "12th GAMM - IMACS International Symposion on Scientific Computing, Computer Arithmetic and Validated Numerics, SCAN 2006, Duisburg, Germany", 2006.

[27] J.-P. MERLET, D. DANEY. *Legs Interference Checking of Parallel Robots over a Given Workspace or Trajectory*, in "IEEE Int. Conf. on Robotics and Automation, Orlando", May 16-18 2006.

[28] J.-P. MERLET, P. DONELAN. *On the regularity of the inverse jacobian of parallel robot*, in "ARK, Ljubljana", June 26-29 2006, p. 41-48.

[29] J.-P. MERLET. *Interval analysis and robotics*, in "Twelfth International Conference on Principles and Practice of Constraint Programming CP-2006, Nantes", Invited talk, 2006.

[30] J.-P. MERLET. *Computing the worst case accuracy of a PKM over a workspace or a trajectory*, in "5th Chemnitzer Parallelkinematik Seminar, Chemnitz", April 25-26 2006, p. 83-96.

[31] B. NEVEU, G. CHABERT, G. TROMBETTONI. *When Interval Analysis helps Interblock Backtracking*, in "Twelth International Conference on Principles and Practice of Constraint Programming (CP'2006), LNCS 4204", 2006, p. 390–405.

[32] Y. PAPEGAY. *Industrial C code generation, SymbolicC: a C wrapper*, in "Wolfram Technology Conference, Urbana-Champaign", 2006.

[33] M. RIFF, T. ALFARO, X. BONNAIRE, C. GRANDÓN. *EA-MP: An evolutionary algorithm to solve a Mine Planning Problem.*, in "Sixth International Conference on Numerical Methods and Applications - NMA'06, Borovets, Bulgaria", 2006.

[34] G. TROMBETTONI, G. CHABERT. *Constructive Interval Disjunction*, in "IntCP (interval analysis, constraint propagation, applications) Int. workshop at CP'2006 conference", 2006.

Internal Reports

[35] C. GRANDÓN, B. NEVEU. *A Specific Quantifier Elimination for Inner Box Test in Distance Constraints with Uncertainties*, Research Report, n° 5883, INRIA, April 2006, http://hal.inria.fr/inria-00071385.

References in notes

[36] E. OTTAVIANO, M. CECCARELLI, F. THOMAS. *Singularity configurations of a 6-wire parallel architecture*, in "RAAD, Cassino", May 2003.

[37] G. REX, J. ROHN. *Sufficient Conditions for Regularity and Singularity of Interval Matrices*, in "SIAM Journal on Matrix Analysis and Applications", vol. 20, n° 2, 1998, p. 437-445.

[38] S. RUMP. *Verification methods for dense and sparse systems of equations*, in "Topics in Validated Computations-Studies in Computational Mathematics", Elsevier, 1994, p. 63-136.

[39] P. VAN HENTENRYCK, V. SARASWAT, Y. DEVILLE. *Design, Implementation, and Evaluation of the Constraint Language cc(FD)*, in "Constraint Programming: Basic and Trends. Selected Papers of the 22nd Spring School in Theoretical Computer Sciences, Châtillon-sur-Seine, France", Springer-Verlag, 1994.