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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable resolution of geometric and algebraic problems, which are encountered in fields such as CAGD (Computer Aided Geometric Design), robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry, computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical calculations, usually opposed to symbolic calculations, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: geometry, resolution of algebraic systems of equations, and symbolic-numeric links.

3.2. Geometry

We are interested in geometric modeling problems, based on non-discrete models, mainly of semi-algebraic type. Our activities focus in particular on the following points:

3.2.1. Geometry of algebraic varieties

In order to solve effectively an algebraic problem, a preprocessing analysing step is often mandatory. From such study, we will be able to deduce the method of resolution that is best suited to and thus produce an efficient solver, dedicated to a certain class of systems. The effective algebraic geometry provides us tools for analysis and makes it possible to exploit the geometric properties of these algebraic varieties. For this purpose, we focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials.

3.2.2. Geometric algorithms for curved arcs and surface patches

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately the algebraic varieties. Traditional algorithmic geometry deals with problems whose data are linear objects (points, segments, lines) but in very great numbers. Combining these two points of view, we concentrate on problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well known, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

3.2.3. Geometry of singularities and topology

The analysis of singularities for a (semi)-algebraic set provides a better understanding of their structure. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

3.2.4. Geometry, groups, and invariants

The objects in geometrical problems are points, lines, planes, spheres, quadrics, ... Their properties are, by nature, independent from the reference one chooses for performing analytic computations. Which leads us to methods from invariant theory. In addition to the development of symbolic geometric computations that exploit these invariants, we are also interested in developing more synthetic representations for handling those expressions.

3.3. Resolution of algebraic systems

The underlying representation behind a geometric model is often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems. Here are the particular approaches that we develop to handle such questions.

3.3.1. Algebraic methods and quotient structure

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and calculating normal forms in such structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

3.3.2. Duality, residues, interpolation

We are interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

3.3.3. Structured linear algebra and polynomials

The preceding work lead naturally to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and the implications in solving polynomial equations [34].

3.3.4. Decomposition and factorisation

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.3.5. Deformation and homotopy

The behavior of a problem in the vicinity of a data can be interpreted in terms of deformations. Accordingly, the methods of homotopy consist in introducing a new parameter and in following the evolution of the solutions between a known position and the configuration one seeks to solve. This parameter can also be introduced in a symbolic manner, as in the techniques of perturbation of non-generic situations. We are interested in these methods, in order to use them in the resolution of polynomial equations as well as for new algorithms of approximate factorization.

3.4. Symbolic-numeric computation

Either in geometric or algebraic problems, symbolic and numeric computation are closely intertwined. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods, as explained now.

3.4.1. Certification

The numerical problems are often approached locally. However in many situations, it is significant to give global answers, making it possible to certify calculations. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures [30].

3.4.2. Approximation

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities. This approach combines geometrical, algebraic and algorithmic aspects.

3.4.3. Degeneracies and arithmetic

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics. For example, we work on the computation of the sign of expressions, and on approaches combining modular and approximate computations, which speed up the exact answer [25].

4. Application Domains

4.1. CAGD

Keywords: *engineering computer-assisted*, *geometric modeling*.

3D modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). The involved mathematical objects have often an algebraic nature, which are then discretized for easy handling. The treatment of such objects can sometimes be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts of CAGD we have.

4.2. Computer vision and robotics

Keywords: *calibration*, *engineering*, *reconstruction*.

Robotics and computer vision come with specific applications of the methods for solving polynomial equations. That is the case for instance, for the calibration of cameras, robots, computations of configurations and workspace. The resolution of algebraic problems with approximate coefficients is omnipresent.

4.3. Molecular biology and geometrical structures

Keywords: *biology*, *health*.

The chemical properties of molecules intervening in certain drugs are related to the configurations (or conformations) which they can take. These molecules are seen as mechanisms of bars and spheres, authorizing rotations around certain connections, similar to robots series. *Distance geometry* thus plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. The methods we develop are well suited for solving such a problem.

5. Software

5.1. Synaps, a module for symbolic and numeric computations

Keywords: *C++*, *algebraic number*, *bezoutian*, *effective algebraic geometry*, *eigenvalues*, *genericity*, *geometry*, *iterative methods*, *linear algebra*, *links symbolic-numeric*, *polynomials*, *resultant*, *solving*, *sparse matrices*, *stability*, *structured matrices*.

Participants: Ioannis Emiris, Bernard Mourrain [contact person], Jean-Pascal Pavone, Olivier Ruatta, Jean-Pierre Técourt, Philippe Trébuchet, Elias Tsigaridas, Julien Wintz.

See SYNAPS *web site: http://www-sop.inria.fr/galaad/logiciels/synaps/.*

Until recently, symbolic and numeric computations were separated domains: software for manipulating formulas is often not effective for numerical linear algebra; while the numerically stable and efficient tools in linear algebra are usually not adapted to the computations with polynomials.

We design the software SYNAPS (SYmbolic Numeric APplicationS) for symbolic and numerical computations with polynomials. It contains tools to compute with algebraic data structures such as polynomials in one or more variables, ideals, ring quotients, ..., as well as numerical computations on vectors, matrices, iterative processes, ...Specialized tools such as LAPACK, GMP, SUPERLU, RS, GB, ... are also connected and can be imported in a transparent way. A set of solvers for polynomial equations have been developped in this environment and are exploited in geometric computation on algebraic curves and surfaces (topology, intersection, singularity detection and analysis, ...).

These developments are based on C++, and attention is paid to the generic structures so that effectiveness would be maintained. Thanks to the parameterization of the code (*template*) and to the control of their instantiations (*traits, template expression*), they offer generic programming without losing effectiveness. This powerful kernel contains univariate and multivariate algebraic solvers as well as subdivision solvers and several resultant-based methods for projection operations.

Many functionalities of the library are now also available through the computer algebra system MATHEMAGIX, or the geometric modeler AXEL, as dynamic binary libraries.

5.2. Axel, a geometric modeler for algebraic objects

Keywords: *computational algebraic geometry*, *curve*, *implicit equation*, *intersection*, *parameterisation*, *resolution*, *singularity*, *surface*, *topology*.

Participants: Bernard Mourrain, Jean-Pascal Pavone, Julien Wintz [contact person].

See AXEL *web site: http://www-sop.inria.fr/galaad/logiciels/axel/.*

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The application contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This package is now distributed as binary packages as well for linux as for MacOSX, see http://www-sop.inria.fr/galaad/axel/.

5.3. Multires, a maple package for multivariate resolution problems

Keywords: *eigenvalues*, *interpolation*, *linear algebra*, *polynomial algorithmic*, *residue*, *resultant*.

Participants: Laurent Busé [contact person], Ioannis Emiris, Bernard Mourrain, Olivier Ruatta, Philippe Trébuchet.

See MULTIRES *web site: http://www-sop.inria.fr/galaad/logiciels/multires/.*

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, and not their effectiveness, which is achieved in a more adapted environment as SYNAPS. It provides methods to build matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay [33] for projective resultant, Jouanolou [32] combining matrices of Macaulay type, and Bezout and (sparse) resultant on a toric variety [29], [28]. Also being added are a new construction proposed for the residual resultant of a complete intersection [26], functions for computing the degree of residual resultant illustrated in [27], and the geometric algorithm for decomposing an algebraic variety [31]. The Weierstrass method generalized for several variables (presented in [35]) and a method of resolution by homotopy derived from such generalization are implemented as well. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

6. New Results

6.1. Algebra

6.1.1. Solving algebraic equations

Participant: Bernard Mourrain.

Algebraic methods to solve polynomial equations have been described in the tutorial presentation [14]. In the first one, we give an overview of fundamental algebraic properties, used to recover the roots of a polynomial system, when we know the multiplicative structure of its quotient algebra. This involves ingredients such as tables of multiplication, duality, eigenvector computations, Chow form. In a seccond tutorial to be published we describe normal form computation and more precisely border basis methods, that we illustrate several simple examples. In particular, we show its usefulness in the context of solving polynomial systems with approximate coefficients. The main results are recalled and we prove a new result on the syzygies, naturally associated with commutation properties. Finally, we describe an algorithm and its implementation for computing such border bases.

6.1.2. Univariate polynomial solvers

Participants: Ioannis Emiris [Univ. Athens], Bernard Mourrain, Elias Tsigaridas [Univ. Athens].

In [24], we present algorithmic, complexity and implementation results concerning real root isolation of a polynomial of degree d, with integer coefficients of bit size $\leq \tau$, using Sturm (-Habicht) sequences and the Bernstein subdivision solver. In particular, we unify and simplify the analysis of both methods and we give an asymptotic complexity bound of $\tilde{\mathcal{O}}(d^4\tau^2)$. This matches the best known bounds for binary subdivision solvers. Moreover, we generalize this to cover the non square-free polynomials and show that within the same complexity we can also compute the multiplicities of the roots. We also consider algorithms for sign evaluation, comparison of real algebraic numbers and simultaneous inequalities, and we improve the known bounds at least by a factor of d. Finally, we present our `C++` implementation in SYNAPS and some experiments on various data sets. This work has been accepted for publication.

6.1.3. Implicitization of rational hypersurfaces

Participants: Laurent Busé, Marc Chardin [Univ. Paris VI], Jean-Pierre Jouanolou [Univ. Strasbourg].

Recently, a method to compute the implicit equation of a parameterized hypersurface has been developed by the authors. We address here some questions related to this method: optimality of a degree estimate and determination of an extraneous factor that appears when the base points are not locally complete intersections. We then make a link with a resultant computation for the case of rational plane curves and for particular cases of space surfaces. As a consequence, we prove a conjecture of Busé, Cox and D'Andrea. This work has been submitted for publication.

6.1.4. Factors of iterated resultants

Participants: Laurent Busé, Bernard Mourrain.

In this work we were interested in the understanding of the factorization of iterated resultants and discriminants that we encounter for instance in the study of the topology of an algebraic surface. Our contribution was to give all the irreducible factors of two times iterated univariate resultants and discriminants over the integer universal ring of coefficients of the entry polynomials. Moreover, we showed that each factor can be separately and explicitely computed in terms of a certain multivariate resultant. A paper [23] has been submitted for publication.

6.1.5. Discriminants of homogeneous polynomials

Participants: Laurent Busé, Jean-Pierre Jouanolou [Univ. Strasbourg].

This is the continuation of a work, still in progress, started last year which aims to provide a computation study of discriminants of homogeneous polynomials. These objects give a necessary and sufficient condition so that a given variety in a projective space has singularities; they are thus of interest in CAGD since it is useful to detect singularities of a curve or algebraic surfaces which are supposed to represent real objects. This work is done in collboration with Jean-Pierre Jouanolou from the university Louis Pasteur of Strasbourg.

6.1.6. Residue Calculus and Applications

Participants: Mohamed Elkadi, Alain Yger [Univ. Bordeaux].

We present a new algorithm to compute effectively the residue of a polynomial maps inspired by the so-called Arnold's perturbation method. The strategy is to reduce the computations to the case of a map without common zeroes at infinity, and to deduce algebraic relations between the components of this map and the coordinate functions. Then we use a generalized version of the transformation to obtain the residue. Instead of computing symbolic determinants, we rely on constant coefficients of some Laurent series. The algorithm is then applied to solve some questions in Computer Aided Geometric Design.

6.2. Geometry

6.2.1. Geometric modeling with quadric patches

Participants: Bernard Mourrain, Changhe Tu [Shandong Univ.], Jiaye Wang [Shandong Univ.], Wenping Wang [Hong Kong Univ.].

Representing shapes by non-linear model is a challenge in geometric modeling, which aims at developping geometric computation on compact and rich models. Our work on quadric patches is related to this objective. We are interested in developping basic algebraic-geometric operations which are required for exact and efficient manipulations of quadrics in this context. One of the key operation is the intersection of quadrics. We present an efficient method for classifying the morphology of the intersection curve of two quadrics (QSIC) in $\mathbb{PR}^3$, 3D real projective space; including an analysis of singularity, reducibility, the number of connected components, and the degree of each irreducible component, etc. There are in total 35 different QSIC morphologies with non-degenerate quadric pencils, which we characterize by by algebraic condition expressed in terms of the Segre characteristics and the signature sequence of a quadric pencil. We show how to compute a signature sequence with rational arithmetic so as to determine the morphology of the intersection curve of any two given quadrics. Two immediate applications of our results are the robust topological classification of QSIC in computing B-rep surface representation in solid modeling and the derivation of algebraic conditions for collision detection of quadric primitives. This work is submitted for a journal publication.

6.2.2. A computational study of ruled surfaces

Participants: Laurent Busé, Marc Dohm, André Galligo.

Implicitization is a fundamental problem in Computer Aided Geometric Design and there are numerous applications related to it, e.g. the computation of the intersection of two ruled surfaces. The method of μ-bases (also known as "moving lines" or "moving surfaces") constitutes an efficient solution to the implicitization problem. Introduced in 1998 by Cox, Sederberg, and Chen for rational curves, it was generalized later to ruled surfaces. Whereas the curve case is very well understood and we know that the resultant of a μ-basis is the implicit equation to the power d, where d is the degree of the parametrization, this result is still to be shown in its full generality (i.e. for arbitrary d) for ruled surfaces. This work filled this gap by giving a proof, which relies on a geometric idea that reduces the ruled surface case to the curve case. This work has been presented at the EACA conference and is submitted for publication in a journal.

6.2.3. General classification of (1,2) parametric surfaces in $\mathbb{P}^3$

Participants: Thi Ha Lé, André Galligo.

Patches of parametric real surfaces of low degrees are commonly used in Computer Aided Geometric Design and Geometric Modeling. However the precise description of the geometry of the whole real surface is generally difficult to master, and few complete classifications exist. We study surfaces of bidegree (1,2). We present a classification and a geometric study of parametric surfaces of bidegree (1,2) over the complex field and over the real field by considering a dual scroll. We detect and describe (if it is not void) the trace of self-intersection and singular locus in the system of coordinates attached to the control polygon of a patch (1,2) in the box $[0;1] \times [0;1]$. This work has been accepted for publication.

6.2.4. Using Bézier patches of bidegree (1,2) for corn leaf modeling

Participants: Franck Aries [INRA of Avignon], André Galligo, Thi Ha Lé.

Surface modeling has a wide range of applications in botany including canopy models for teledetection, computation of radiative balance and optical properties. The detailed description of the architecture of vegetation canopies is critical for the modeling of many processes. The area, size, shape, orientation and position of the leaves drive the efficiency with which exchanges will occur with the atmosphere, including gas such as water vapor or carbon dioxide, liquids such as water, and radiation. Up to now, models describing canopy functioning are based on very simple approximations of the canopy architecture. A more detailed description is required when investigations focus on the role of particular organs in the canopy, the description of rain interception, the propagation of diseases from organs to others, or the radiative transfer to describe the reflected or emitted fluxes in a range of wavelengths. A corn leaf, as well as many leaves, is formed by a nervure separating 2 ruled surface patches. We investigate such representations with patches of bidegree (1,2). Our approach consists in representing the nervure as a conic or a cubic spline curve, relying on the fact that a line in the parameter space is mapped on a twisted cubic by a parameterization of bidegree (1,2). Then we choose (1,2) Bézier patches delimited by this segment of curve to model the leaf. We present a tool box of algorithms adapted to our purpose, for the patches of bidegree (1,2). This includes fast detection algorithm for selfintersection and intersection, area estimation and computation of normals. In a following paper with a more botany flavor, we will present a realistic model for modeling a corn parcel.

6.2.5. Biquadratic sampling method for intersection of surfaces

Participants: Stéphane Chau, André Galligo, Jean-Pascal Pavone.

In Computer Aided Geometric Design (CAGD), the parameterized surfaces are used delimiting volumes. The computation of the intersection curve between such two surfaces is thus crucial for the description of the CAGD objects. An often used method to address this problem consist in using a mesh for each surface, and then to proceed to their intersection (intersection of triangles). Other methods for the intersection problem deal with global representations of the surfaces such as B-splines, however the usual CAGD procedures (offsetting, drafting, ...) do not conserve this model and procedural surfaces that are based on approximations are required. So, even if the intersection methods are exact, they only provide an approximation of the "real" intersection curve. This can be of bad quality, because the approximations of the surfaces are separated from the intersection process and are somehow "global". In the general case, the only informations that we can access for a parameterized surface are "local" (its evaluation). Thus, a general algorithm is a sampling algorithm. Moreover, in order to design a more robust algorithm which does not avoid possible intersection between two parameterized surfaces S_1 and S_2, we consider the local extreme values of the distance function between S_1 and S_2. We are interested in an intersection method based on approximation by biquadratic patches. It can be seen as a good compromise between the approximation by B-splines and meshes because it is a local approximation (like the meshes case) and the distance function can have extreme values (like the B-spline case). It can be used for the detection of touching points, and bifurcation points which are undetectable by a mesh approximation. This method is almost as efficient as a classical sampling method, but it is more robust. Moreover, this approach requires less evaluations than a classical sampling method, so it could be more interesting for complicated procedural surfaces. Our approach is based on two steps. At first, we build quadtrees hierarchy covering the grids of biquadratic patches and then we have a family of intersection problem between two biquadratic patches. This last problem was treated in work that has been accepted for publication but we also introduce a subdivision method to find the topology of the intersection curves.

6.2.6. Subdivision methods for 2d and 3d implicit curves

Participants: Chen Liang [Univ. of Hong Kong], Bernard Mourrain, Jean-Pascal Pavone.

We describe a subdivision method for handling algebraic implicit curves in 2d and 3d. We use the representation of polynomials in the Bernstein basis associated with a given box, to check if the topology of the curve is determined inside this box, from its points on the border of the box. Subdivision solvers are involved for computing these points on the faces of the box, and segments joining these points are deduced to get a graph isotopic to the curve. Using envelop of polynomials, we show how this method allow to handle efficiently and accurately implicit curves with large coefficients. We report on implementation aspects and experimentations on 2d curves such as ridge curves or self intersection curves of parameterized surfaces, and on silhouette curves of implicit surfaces, showing the interesting practical behavior of this approach.

6.2.7. Intersection and self-intersection of surfaces by means of Bezoutian matrices

Participants: Laurent Busé, Mohamed Elkadi, André Galligo.

The computation of intersection and self-intersection loci of parameterized surfaces is an important task in Computer Aided Geometric Design and computer algebra tools need to be developed further for computing their implicit equations. In this work, we addressed these problems via four resultants with separated variables: two of them are specializations of more general ones and the others are determinantal. We gave a rigorous study in these cases and provide new and useful formulas via adapted computations of Bezoutians. A paper has been submitted for publication.

6.2.8. Topology of curves and surfaces

Participants: Lionel Alberti, George Comte [UNSA], Bernard Mourrain, Jean-Pierre Técourt.

In order to produce a triangulation of an implicit surface, we look at subdivision methods. Our approach to the problem is through singularity theory. By Thom-Mather transversality theorem (which we can guarantee in a computable way) we can prove that in given domains, the surface is topologically trivial (that is, a product of its link with $\mathbf{R}^n$). We can then recursively apply the method to the link of the surface. The classical method uses a CAD decomposition that even in its lighter forms has a complexity depending on the complexity of the projection making the complexity somewhat unrelated to the actual geometry of the surface. With this approach we do not have to rely on a projection and it is hoped that it will actually lead to a practically more efficient algorithm. This work has been submitted for publication. The approach behind this work is described in [12]. Some algebraic issues related to these geometric problems are discussed in [16].

6.2.9. Intersection of algebraic surfaces

Participants: Daouda N'Diatta, Bernard Mourrain, Olivier Ruatta [Univ. Limoges].

Numerical modeling plays an increasingly role in fields at the border between data processing and mathematics. This is the case for example in CAD (computer-aided design), where the objects of a scene or a piece to be built are represented by parameterized curves or surfaces such as NURBS, robotics (problem of the parallel robot, or vision), or molecular biology (rebuilding of a molecule starting from the matrix of the distances between its atoms obtained by NMR). A fundamental operator in this context is the intersection of geometric models, which leads to algebraic questions.

We focus on the problem of computing the topology of a plane algebraic curves and three-dimensional algebraic curves defined as the intersection of two algebraic surfaces. In the case of two parameterized surfaces, in order to reduce to such a situation, we may compute the implicit equation of one of the rational surfaces. This reduce the problem of intersection to the case of a curve defined by an implicit equation in the plane of parameters. Our main concern will be the case of implicit curves, either in the plane or in a three-dimensional space.

The problem of computing the topology of an algebraic curve received a lot of attention in the past literature. Different methods have been experimented, but they suffer from the problem of certifying the topology of the result. At first, we propose an algorithm based on sub-resultant to compute with certainty a simplicial complex isotopic to a plane algebraic curve defined by his implicit equation. Secondly, by using the 2D approach we present a new method to compute with certainty the topology of an algebraic curve in 3D defined by two polynomial equations even when the ideal generate by the two polynomial is not reduced.

These algorithms are beeing implemented in the SYNAPS library.

6.2.10. Resolution of singularities

Participants: Lionel Alberti, Edward Bierstone [Univ. Toronto].

This was done in collaboration with Edward Bierstone, Professor at the Fields Institute, Toronto, Canada.
The local properties of singularities are best understood looking at their resolution and blow-up maps. The structure of resolution maps is not well understood in arbitrary dimension, but we can get a rather good handle on what's happening in dimension 3 for isolated singularities. Resolution is to be understood as a mathematical tool enabling a better handling and understanding of singularities rather than as a goal in itself that will yield efficient algorithms.

6.2.11. Approximate implicitization

Participants: Stéphane Chau, André Galligo.

In order to approximate a general parameterized surface (given by evaluations) by biquadratic patches, we were interested in the approximate implicitization of these polynomial surfaces. The implicitization problem consists in finding the exact algebraic representation of a given rational parametric curve or surface. The classical methods to deal with this problem use the resultants and Groebner bases. But these algebraic approaches are very time-consuming (because of too high degrees) and have the drawback of introducing too many components in excess (so-called "phantom components"). So a very simple patch of surface can have a complicated exact implicit equation because the parameterization is local whereas the algebraic representation is global. Thus, instead of finding the exact representation of a given biquadratic patch, we can wonder how well a given algebraic surface approximates the corresponding parametric surface of bi-degree $(2, 2)$ in the region of interest. This is the approximate implicitization problem and an algebraic approach based on factorization was developed by SINTEF. Our approach consists in the construction of some specific geometric constraints to give a linear system of equations and we use a singular value decomposition to solve it.

6.2.12. Dynamic and generic method for computing an arrangement of implicit curves

Participants: Julien Wintz, Bernard Mourrain.

Arrangements are very important issues in many application fields and have been studied for several years with different points of view. Arrangements are mainly computed using sweep line methods, such as the well known Bentley-Ottman algorithm for computing an arrangement of line segments. These methods are strongly related to the nature of objects and are not well suited for non-trivial objects such as closed curves or surfaces. We propose an arrangement algorithm which is generic in the sense that it is not related to the nature of the objects to be arranged. The algorithm is dynamic, which means, it can be maintained under insertion and deletions of the objects. Objects are always considered with a bounding domain so that we can also consider non convex objects. This domain is subdivided further if the cell is not deemed relevant, what is ensured by a regularity test which depends on the nature of input objects. A cell is relevant if regions can be deduced from the topology of the object within the cell. This general framework has been first tested for computing an arrangement of implicit curves. Type related elements of the algorithm are then specialized for implicit curves. These elements are regularity test and regions computation. A segment of implicit curve is said to be regular in a cell if it is either x-monotonic, y-monotonic or if it contains one and only one singular points and if the number of branches stemming out from this singularity is exactly the number of intersections of the curves with the cell. To compute the number of branches stemming out from a singularity, we use degree theory. To compute the intersection of the curve with the cell border (as well as the topological degree), we use the Bernstein conversion of the polynomials representing the curve. The arrangement is represented by an augmented influence graph on regions obtained by subdivision and merging steps. This algorithm is implemented within the algebraic modeler AXEL and will quite easily be extended to the case of BSpline curves [19].

6.3. Symbolic numeric computation

6.3.1. Solving Toeplitz-block linear systems

Participants: Houssam Khalil, Bernard Mourrain, Michelle Schatzmann [Univ. Lyon I].

Structured matrices appear in various domain, such as scientific computing, signal processing,...Among well-known structured matrices. Toeplitz and Hankel structures have been intensively studied. So far, few investigations have been pursued for the treatment of multi-level structured matrices. We found a relation between Toeplitz matrices and syzygies, and algorithms to compute these syzygies. In following the same way in the univariate case, we gived a relation between bloc-Toeplitz-bloc (TBT) matrices and bivariate syzygies. We are looking for a fast algorithm to compute these syzygies. By computing the SVD of the displacement of a banded TBT matrix we remark that almost of them are very small, which means that we can aproximate this matrix by a small rank matrix. We are trying also to trasform a TBT linear system to another multi-level structured system which was called Cauchy-bloc-Cauchy sytem type, which has a relation with the bivariate interpolation problem.

6.3.2. Algorithms for bivariate polynomial absolute factorization

Participants: Guillaume Chèze, André Galligo, Grégoire Lecerf [CNRS, Univ. Versailles].

Absolute factorization stands for factorization over the algebraic closure of the ground field. This is interesting for the applications; e.g. for a multivariate polynomial with rational coefficients it allows us to decompose further than the rational factorization, for instance into polynomials with real coefficients. This is a long standing problem in Computer algebra.

G. Chèze, André Galligo and G. Lecerf are working on an improvement of an approach initiated by Gao using exact modular and p-adic computations. They already obtained nearly optimal complexity bounds. This work is still in progress.

6.3.3. Geometry and optimisation

Participants: Nikos Pavlidis [University of Patras], Bernard Mourrain, Michael Vrahatis [University of Patras].

In a work developped in the context of our associate team CALAMATA with Greece, we investigated the application of optimisation methods in geometric problems. We have examined the ability of feedforward neural networks to identify the number of real roots of univariate polynomials, and more generally the ability to approximate semi-algebraic sets. The obtained experimental results indicate that neural networks are capable of performing this task with a high accuracy even when the training set is very small compared to the test set (see [15]). We are currently investigating applications of Particule Sworm Optimisation methods to geometric and algebraic problems, such are root isolation, 3D reconstruction from distances in Molecular Biology.

6.3.4. Tensors, symmetric tensors and rank

Participants: Pierre Common [I3S], Gene Golub [Stanford Univ.], Lek-Heng Lim [Stanford Univ.], Bernard Mourrain.

There are several extensions of the Singular Value Decomposition (SVD) to tensors. In this work, we are interested in the Canonical Decomposition, into a minimal sum of outer products of vectors. In particular, this allows us to define a tensor rank that restricts to the matrix rank for order-2 tensors. This decomposition is essential in the process of Blind Identification of Under-Determined Mixtures (UDM), i.e., linear mixtures with more inputs than observable outputs and appear in many application areas, including Speech, Mobile Communications, Machine Learning, Factor Analysis with k-way arrays (MWA), Biomedical Engineering, Psychometrics, and Chemometrics. In this work, we study various properties of symmetric tensors in relation to a decomposition into a sum of outer product of vectors. A rank-1 order-k tensor is the outer product of k vectors. Any symmetric tensor can be decomposed into a linear combination of rank-1 tensors, each of them being symmetric or not. The Rank of a symmetric tensor is the minimal number of rank-1 tsWeensors that are necessary to reconstruct it. The Symmetric Rank is obtained when constituting rank-1 tensors are imposed to be themselves symmetric. It is shown that Rank and Symmetric Rank are generically equal, and that they always exist in an algebraically closed field; in the real field for instance, it is necessary to define several Typical Ranks. The Generic Rank is generally not maximal, contrary to matrices, and is now known for any values of dimension and order. These properties are described in a paper submitted for a journal publication.

7. Other Grants and Activities

7.1. European actions

7.1.1. ACS

Participants: Lionel Alberti, Laurent Busé, Stéphane Chau, André Galligo, Bernard Mourrain [contact person], Jean-Pascal Pavone, Jean-Pierre Técourt, Julien Wintz.

See the ACS *project web site.*
- Acronym: ACS, number FP6-006413
- Title: Algorithms for Complex Shapes.
- Specific Programme: IST
- RTD (FET Open)
- Start date: started 1st May, 2005 - Duration: 3 years
- Participants:
    Univ. Groningen (Netherlands) [coordinating site]
    ETH Zürich (Switzerland),
    Freie Universität Berlin (Germany),
    INRIA Sophia Antipolis (Galaad & Geometrica),
    MPI Saarbrücken (Germany),
    National Kapodistrian University of Athens (Greece),
    Tel Aviv University (Israel)
    GeometryFactory Sarl

- Abstract: computing with complex shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension. Certified topology and numerics, applications to shape approximation, shape learning, robust modeling.

7.1.2. AIM@SHAPE

Participants: Laurent Busé, Emmanuel Briand, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo, Thi Ha Lê, Bernard Mourrain [contact person], Olivier Ruatta, Julien Wintz.

See the AIM@SHAPE *project web site*
- Acronym: aim@shape, number NoE 50766
- Title: AIM@SHAPE, Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing knowledge Embedded in multidimensional digital objects.
- Type of project: network of excellence
- Beginning date: 1st of january 2004 - During: 4 years
- Partners list:
  CNR - Consiglio Nazionale delle Ricerche,
  DISI - Universita di Genova,
  EPFL - Swiss Federal Institute of Technology,
  IGD - Fraunhofer,
  INPG - Institut National Polytechnique de Grenoble,
  INRIA
  CERTH - Center for Research and Technology Hellas,
  UNIGE - Université de Genève,
  MPII - Max-Planck-Institut für Informatik,
  SINTEF,
  Technion CGGC,
  TUD - Darmstadt University of Technology,
  UU - Utrecht University,
  WIS - Weizmann Institute of Science.
- Abstract of the project: it is aimed at coordinating research on representing, modeling and processing knowledge related to digital shapes, where by shape it is meant any individual object having a visual appearance which exists in some (two-, three- or higher- dimensional) space (e.g., pictures, sketches, images, 3D objects, videos, 4D animations, etc.).

7.2. Bilateral actions

7.2.1. Associated team CALAMATA

Participants: Ioannis Emiris, Athanasios Kakargias, Bernard Mourrain [contact person], Nikos Pavlidis, Jean-Pierre Técourt, Elias Tsigaridas, Michael Vrahatis.

The team of Geometric and Algebraic Algorithms at the National University of Athens, Greece, has been associated with GALAAD since 2003. See its web site.

This bilateral collaboration is entitled CALAMATA (CALculs Algebriques, MATriciels et Applications). The Greek team (http://www.di.uoa.gr/~erga/) is headed by Ioannis Emiris. The focus of this project is the solution of polynomial systems by matrix methods. Our approach leads naturally to problems in structured and sparse matrices. Real root isolation, either of one univariate polynomial or of a polynomial system, is of special interest, especially in applications in geometric modeling, CAGD or nonlinear computational geometry. We are interested in computational geometry, actually, in what concerns curves and surfaces.

In 2006, we had the visit of B. Mourrain, J.P. Pavone and J. Wintz to Athens, to work on univariate polynomial subdivision solvers and arrangement problems. M. Vrahatis and K. E. Parsopoulos visited GALAAD, and worked on Particule Sworm Optimisation methods with applications to geometric and algebraic problems.

7.2.2. NSF-INRIA collaboration

Participants: Laurent Busé, André Galligo, Mohamed Elkadi, Bernard Mourrain [contact person], Jean-Pierre Técourt.

The objective of this collaboration between GALAAD and the Geometric Modeling group at Rice University in Houston, Texas (USA) is to investigate techniques from Effective Algebraic Geometry in order to solve some of the key problems in Geometric Modeling and Computational Biology. The two groups have similar interests and complementary strengths. Effective Algebraic Geometry is the branch of Algebraic Geometry that pursues concrete algorithms rather than abstract proofs. It deals mainly with practical methods for representing polynomial curves and surfaces along with robust techniques for solving systems of polynomial equations. Many applications in Geometric Modeling and Computational Biology require fast robust methods for solving systems of polynomial equations. Here we concentrate our collective efforts on solving standard problems such as implicitization, inversion, intersection, and detection of singularities for rational curves and surfaces. To aid in modeling, we shall also investigate some novel approaches to represent shape. In contemporary Computational Biology, many problems can be reduced to solving large systems of low degree polynomial equations. We plan to apply our polynomials solvers together with new tools for analysing complex shapes to help study these currently computationally intractable problems.

7.2.3. PAI Procore collaboration

Participants: Laurent Busé, Stéphane Chau, Yi-King Choi [Hong Kong Univ.], André Galligo, Yang Liu [Hong Kong Univ.], Wenping Wang [Hong Kong Univ.], Julien Wintz.

The objective of this collaboration is to conduct research in algebraic techniques for solving problems in geometric modeling. We will investigate the use of implicit models, for compact and efficient shape representation and processing. The application domains are Computer Aided Geometric Design, Robotics, Shape compression, Computer Biology. We will focus on algebraic objects of small degree such as quadrics, with the aim to extend the approach to higher degree. In particular, we are interested in the following problems:

- Shape segmentation and representation using quadrics.
- Shape processing using quadrics.
- Collision detection for objects defined by quadric surfaces.

Experimentation and validation will lead to joint open source software implementation, dedicated to quadric manipulations. A package collecting these tools will be produced.

7.3. National actions

7.3.1. ANR DECOTES, Tensorial decomposition and applications

Years: 2006-2009.
Partners: I3S, CNRS; LTSI, INSERM; GALAAD, INRIA; SBP, Thales communications.

In signal processing, where high-order statistics are used for blind identification, in data analysis, in sensor array processing, in machine learning, ... tensorial decompositions plays an important role. The aim of this project is to study the key theoretical problems of such decompositions and to devise numerical algorithms dedicated to some selected applications.

7.3.2. ANR GECKO, Geometry and Complexity

Years: 2005-2008
Partners: ALGO, INRIA; GALAAD, UNSA; LIX, Ecole Polytechnique; Univ. Paul Sabatier, Toulouse.

The topics of the project are the study, analysis and implementation of fundamental operations on matrices and polynomials, methods for the resolution of polynomial or analytic equations. It combines numerical analysis, effective algebraic geometry and complexity. See http://gecko.inria.fr/ for more details.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We continued to organize a bi-weekly seminar called "Formes & Formules". The list of talks is available at http://www-sop.inria.fr/galaad/index.php?option=com_content&task=blogcategory&id=23&Itemid=162.

8.1.2. Relations with LSIIT

We are in relation with the Laboratory of Sciences, Computer graphics and Teledetection of Strasbourg (LSIIT, UMR 7005 CNRS-ULP). Arnaud Fabre came in visit one week at GALAAD, visit during which he made demonstration of his achievements in the field of constraint modeling in 3D. He presented a software, CoyoteVR, which allows the modeling of geometric constraints in a virtual reality environment. Scenes built using this tool, are exported in a formalism defined by Julien Wintz, when they worked together, GCML which allow to define with ambiguity a geometric constraint system, using first order logic. Work between Arnaud and Julien lead to a plugin for the definition and the resolution of geometric constraint system in AXEL.

Julien has given, at the days of computer graphics french community 2006, in Bordeaux, a talk on the compilation of knowledge based systems for geometric constraint symbolic resolution. This work initiated before his PhD thesis, under the direction of Pascal Schreck, is continued as a collaboration, which will allow, according to the same principle, automatic compilation of plugins for Axel by using a syntactic base and a semantic base, possibly defined in GCML as well.

Two publications on the application of the compilation process for the generation of symbolic solvers of geometric constraint systems are currently in redaction. After the Afig days, Julien went one week in Strasbourg, which gave the opportunity was the occasion to work at the same time on the constraints plugin for Axel with Arnaud, and on compilation with Pascal. A discussion with Ludovic Sternberger is to define how, porting Axel towards virtual reality can be performed.

8.1.3. Comittee participations

- A. Galligo is a member of the program comittee of the cycle of conference MEGA, the next one, MEGA'07 will be held in Austria.
- B. Mourrain was a member of the programm comittee of the conference SMI'06 (International conf. on Shape Modeling and Applications), Matsushima, Japan, 14-16 June, CASC'06 (International Workshop on Computer Algebra in Scientific Computing), Moldovia, 11-15 September,

8.1.4. Editorial committees

- M. Elkadi, B. Mourrain, R. Piene [Univ. Oslo] are editors of the book "Algebraic Geometry and Geometric Modeling", which appeared as [11].
- L. Busé, M. Elakdi and B. Mourrain are preparing a special issue of Theoretical Computer Science, for the conference *Computational Algebraic Geometry and Applications*, held in Nice on the occasion of the 60th birthday anniversary of André Galligo.

8.1.5. Organisation of conferences and schools

- Bernard Mourrain, Ollivier Ruatta and Jacques-Arthur Weil organised the workshop *Matrices structurées, équations différentielles et calcul formel*, Limoges, 18-19 January.
- Laurent Busé organized with Marc Chardin a week conference at CIRM, Luminy, 15-19 may 2006, *Elimination theory and applications* in honor of Jean-Pierre Jouanolou; 42 international participants; Web page of the conference:
  http://www-sop.inria.fr/galaad/conf/jouanolou/.

- Laurent Busé, Mohamed Elkadi and Bernard Mourrain organized then international conference *Computational Algebraic Geometry and Applications*, at Nice, 2-6 june 2006, on the occasion of the 60th birthday of André Galligo; 40 international participants; Web page: http://www-sop.inria.fr/galaad/conf/06andre/index.html.
- Laurent Busé and Bernard Mourrain organized with C. D'Andrea, R. Goldman, L. González-Vega and M. Sombra, 4-7 september 2006, the conference *Algebraic Geometry and Geometric Modeling 2006*; Web page: http://www.imub.ub.es/aggm06/; 38 international participants.

8.1.6. PHD thesis commitees

- A. Galligo was President of the Jury of the PhD thesis of Mauricio Araya (Inria Sophia, Tropics) at UNSA in November 2006.
- B. Mourrain was referee for the Ph.D of A. Ayad (Univ. Rennes), P. Johansen (Univ. Oslo).

8.1.7. Other comittees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- B. Mourrain is a member of the scientific council of SARIMA.

8.1.8. WWW server

- http://www-sop.inria.fr/galaad/.

8.2. Participation at conferences and invitations

- L. Alberti attended to the Fields Institute summer school "Valuation Theory and Integral Closures in Commutative Algebra", Ottawa, Canada, July 3-22, 2006, to the Fields Institute workshop "Workshop on Computational and Combinatorial Commutative Algebra", Toronto, Canada, July 24-August 4, 2006, to the Fields Institute workshop "Fields-MITACS Industrial Problem-Solving Workshop (FMIPW)", Toronto, Canada, August 14-18, 2006.
- L. Busé was invited to give a talk at the commutative algebra and algebraic geometry seminar of the university of Genova, Italia, february 27-march 3; participated to the conference "Elimination theory and applications", May 15-19, Luminy,France; participated to the conference 'Algebraic Geometry and Geometric Modeling", September 4-7 2006, Barcelona, Spain; was invited to give a talk at the workshop "Algorithms in Algebraic Geometry", september 18-22, Minneapolis, USA; was invited to give a talk at the seminar of the INRIA's team "ALGO", october 23, Rocquencourt, France.
- S. Chau participated to "Computational algebraic geometry and applications", Nice, 2 - 6 june 2006; gave a presentation at "Curves and Surfaces", Avignon, 29 june - 5 july 2006.
- M. Dohm participated to the "CIMPA School on Commutative Algebra" (Hanoi, Vietnam), Dec. 26 - Jan. 6; "Elimination theory and applications" (Luminy, France), May 15-19, Luminy,France; "Computational algebraic geometry and applications" (Nice, France), June 2-6; gave a talk at "Encuentros de lgebra Computacional y Aplicaciones 2006" (Sevilla, Spain), September 7-9
- A. Galligo was invitated to give a lecture on Absolute factorization at the Ecole doctorale de Bordeaux, february 2nd; participated to "Computational algebraic geometry and applications", Nice, 2 - 6 june; to "Curves and Surfaces", Avignon, 29 june - 5 july; gave a presentation at the conference "Algebraic Geometry and Geometric Modeling", September 4-7, Barcelona, Spain; participated to the workshop on Factorization,AIM, Palo Alto (Ca, USA) May 15-19; to ISSAC'06, Genova, Italia, July; to the conference to the memory of M. Bronstein, INRIA Sophia Antipolis, July.
- T. H. Lé participated to *Computational algebraic geometry and applications*, Nice, 2 - 6 june 2006; presentation at *Curves and Surfaces*, Avignon, 29 june - 5 july 2006; to *Forum des jeunes Mathématiciennes*, Paris, 6-7 october 2006.

- B. Mourrain gave a talk at the Dagsthul seminar *Reliable Implementation of Real Number Algorithms: Theory and Practice*, 9-10 january, coorganised the workshop *Matrices structurées, équations différentielles et calcul formel*, Limoges, 18-19 january, participated to the Aim@Shape Management Board and review Meeting, Genova, 7-10 February and 28-31 March, to the ACS workshop, Athens, 8-12 May, the conference *Computational Algebraic Geometry and Application*, Nice, 2-6 june, was invited to give a talk at a minsymposium of the conference *Mathemathical methods for curves and surfaces*, 29 may-2 june, Avignon, gave a talk at a minisymposium of the European Conference on Mathematics for Industry, Madrid, 10-14 july, gave a talk at the *International Congress on Mathematical Software*, Castro Urdiales, Spain, 1-3 september, coorganised the conference *Algebraic Geometry and Geometric Modeling*, Barcelona 4-7 september, gave a talk at the workshop *Algorithms in Algebraic Geometry*, IMA Minneapolis USA, 18-22 september, at the workshop *Software for Algebraic Geometry*, IMA Minneapolis USA, 23-27 october, at the Workshop on *Geometric Modeling and Geometric Computation*, Vorau Austria, 27-29 november, at the worskop on *Global Optimisation, intergating Convexity, Optimization, Logic Programming and Computational Algebraic Geometry*, Vienna Austria, 4-6 december.
- Julien Wintz gave a talk “A framework for geometric constraint satisfaction problem” at the 21st Symposium of Applied Computing, Track 21 : Geometric Computing and Reasoning, Dijon, France, April 24-27, 2006 . He also gave a talk “Subdivision method for computing an arrangement of implicit planar curves” at the conference “Algebraic Geometry and Geometric Modeling”, September 4-7 2006, Barcelona, Spain and another talk “Compilation de systèmes à base de connaissances pour la résolution symbolique de contraintes géométriques” at the “Journées francophones de l’informatique graphique”, Bordeaux, France, 24-27 November 2006 .

8.3. Formation

8.3.1. Teaching at Universities

- Laurent Busé, Master 2nd year of the university of Nice, “Algebraic curves and surfaces for CAGD”, 27 hours.
- Marc Dohm, Licence 2nd year, algebra course, 36h; Licence first year, analysis course, 24h.
- Mohamed Elkadi, Master 1st year: Effective Algebraic Geometry, 60 h, Master 2nd year: Introduction to Elimination Theory, 42 h, Licence 2nd year: Arithmetic, 30 h, Capes: Linear Algebra, 60h.
- André Galligo, Licence (140h), Master (60 h) in mathematics.
- Bernard Mourrain, Master 2nd year of the university of Nice, “Algorithms for curves and surfaces”, 20 hours.

8.3.2. PhD theses in progress

- Lionel Alberti, *Vers une théorie quantitative des singularités*, ED SFA, UNSA.
- Eliman Ba, *Résultants, calculs et applications*,UNSA.
- Stéphane Chau, *Study of singularities used in CAGD*, UNSA.
- Marc Dohm, *Algorithmique des courbes et surfaces algébriques*, UNSA.
- Daouda N’Diatta, *Résultants et sous-résultants et applications*, Univ. Limoges.
- Houssam Khalil, *Matrices structurées en calcul symbolique et numérique*, Univ. Lyon I.
- Thi Ha Lê, *Classification and intersections of some parametrized surfaces and applications to CAGD*, UNSA.
- Julien Wintz, *Méthodes algébriques pour la modélisation géométrique*, INRIA Sophia-Antipolis, ED STIC.

8.3.3. Internships

See the web page of our interships.

- Dominique Dufaye-Santoni, *Isolation de racines réelles de polynômes en une variable* , 1 July 2006 - 30 August 2006.
- Ali Etber, *Mathemagix, vers un système de calcul formel modulaire*, 15 June 2006 - 30 July 2006.
- Chen Liang, *Subdivision methods for the topolopy of implicit 3D curves*, 4 November 2005-30 January 2006.
- Quoc Thinh Nguyen, *Topologie de courbes algébriques implicites*, 1 August 2006 - 30 September 2006.

9. Bibliography

Major publications by the team in recent years

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[6] I.Z. EMIRIS, B. MOURRAIN. *Matrices in Elimination Theory*, in "J. Symbolic Computation, Special Issue on Elimination", vol. 28, 1999, p. 3–44.

[7] A. GALLIGO. *Théorème de division et stabilité en géométrie analytique locale*, in "Ann. Inst. Fourier", vol. 29, 1979, p. 107–184.

[8] A. GALLIGO, S. WATT. *A Numerical Absolute Primality Test for Bivariate Polynomials*, in "Proc. Annual ACM Intern. Symp. on Symbolic and Algebraic Computation", 1997, p. 217–224.

[9] B. MOURRAIN. *Algorithmes et Applications en Géométrie Algébrique*, Ph. D. Thesis, Université de Nice Sophia-Antipolis, September 1997.

[10] B. MOURRAIN, V. Y. PAN. *Multivariate Polynomials, Duality and Structured Matrices*, in "J. of Complexity", vol. 16, n^{o} 1, 2000, p. 110-180.

Year Publications

Books and Monographs

[11] M. ELKADI, B. MOURRAIN, R. PIENE (editors). *Algebraic Geometry and Geometric Modeling*, Mathematics of Visualisation, Springer, 2006.

Articles in refereed journals and book chapters

[12] J. BOISSONNAT, D. COHEN-STEINER, B. MOURRAIN, G. ROTE, G. VEGTER. *Meshing of Surfaces*, Mathematics and Visualisation, Springer-Verlag, 2006, p. 459–478.

[13] G. CHÈZE, A. GALLIGO. *From an approximate to an exact absolute polynomial factorization*, in "J. Symbolic Comput.", vol. 41, n° 6, 2006, p. 682–696.

[14] B. MOURRAIN. *An introduction to algebraic methods for solving polynomial equations*, B. HANZON, M. HAZEWINKEL (editors). , KNAW Press., 2006, p. 49-94.

[15] B. MOURRAIN, N. PAVLIDIS, D. TASOULIS, M. VRAHATIS. *Determining the Number of Real Roots of Polynomials through Neural Networks*, in "Computers and Mathematics with Applications", vol. 39, 2006, p. 527–536.

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Publications in Conferences and Workshops

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