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Project-Team geometrica

Geometric Computing

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THEME SYM

Activity Report

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1. Team

Head of project
Jean-Daniel Boissonnat [DR Inria, HdR]

Vice-head of project
Olivier Devillers [DR Inria, HdR]

Administrative Assistant
Agnès Bessière [TR Inria, assistant of GEOMETRICA and of the projects committee]

Research scientists
Pierre Alliez [CR Inria]
Frédéric Cazals [DR Inria, HdR]
Frédéric Chazal [Maître de Conférence en détachement, HdR]
David Cohen-Steiner [CR Inria]
Sylvain Pion [CR Inria]
Monique Teillaud [CR Inria]
Mariette Yvinec [CR Inria, HdR]

Software development staff
Naceur Meskini [Engineer ODL, until November 2006]
David Rey [Engineer DREAM, since November 2006]
Laurent Saboret [Engineer]
Laurent Rineau [France Telecom since October 2006]

Ph.D. Students
Luca Castelli Aleardi [cosupervised by LIX (Polytechnique)]
Christophe Delage [Allocataire couplé]
Laurent Rineau [ENS until September 2006]
Daniel Russel [Stanford University, Chateaubriand fellowship Mars–September 2006]
Marie Samozino [MENRT monitor fellow]
Abdelkrim Mebarki [grant Algerian governement]
Camille Wormser [Allocataire couplé]
Sébastien Loriot [MENRT monitor fellow]
Trung Nguyen [Alcatel/INRIA]
Pooran Memari [MENRT monitor fellow since October 2006]
Jane Tournois [MENRT monitor fellow since October 2006]
Quentin Mérigot [ENS Lyon since October 2006]

Post-doctoral fellow
Marc Pouget [September–February 2006]
Nico Kruithof [April–September 2006]
Elias Tsigaridas [ARC ARCADIA, October–December 2006]
Christian Rössl [February-December 2006]
Jean-Philippe Pons [January-September 2006]
Benjamin Bouvier [ARC REFLEXP since November 2006]

Student interns
Lakulish Antani [IIT Bombay - India, May-July 2006]
Chinmay Karande [IIT Bombay - India, May-July 2006]
Pedro Machado Manhães de Castro [Internship program, Master1, Universidade Federal de Pernambuco, Recife, Brasil, April–September 2006]
Pooran Memari [Polytechnique, March–September 2006]
Dave Millman [New York University - USA, May-June 2006]
Aditya Parameswaran [IIT Bombay - India, May-July 2006]
Ilya Suslov [Internship program, January–February 2006]

Jane Tournois [EPU Nice Sophia-Antipolis, March-September 2006]
Marc Scherfenberg [Free University of Berlin, May-August 2006]
Chris Wu [New York University - USA, August 2006]
Jihun Yu [New York University - USA, June-August 2006]

2. Overall Objectives

2.1. Overall Objectives

Geometric computing plays a central role in most engineering activities: geometric modelling, computer aided design and manufacturing, computer graphics and virtual reality, scientific visualization, geographic information systems, molecular biology, fluid mechanics, and robotics are just a few well-known examples. The rapid advances in visualization systems, networking facilities and 3D sensing and imaging make geometric computing both dominant and more demanding concerning effective algorithmic solutions.

Computational geometry emerged as a discipline in the seventies and has met with considerable success in resolving the asymptotic complexity of basic geometric problems including data structures, convex hulls, triangulations, Voronoi diagrams, geometric arrangements and geometric optimization. However, in the mid-nineties, it was recognized that the applicability in practice of computational geometry techniques was far from satisfactory and a vigorous effort has been undertaken to make computational geometry more effective. The PRISME project together with several partners in Europe took a prominent role in this research and in the development of a large library of computational geometry algorithms, CGAL.

GEOMETRICA aims at pursuing further the effort in this direction and at building upon the initial success. Its focus is on effective computational geometry with special emphasis on *curves and surfaces*. This is a challenging research area with a huge number of potential applications in almost all application domains involving geometric computing.

The overall objective of the project is to give effective computational geometry for curves and surfaces solid mathematical and algorithmic foundations, to provide solutions to key problems and to validate our theoretical advances through extensive experimental research and the development of software packages that could serve as steps towards a standard for safe and effective geometric computing.

3. Scientific Foundations

3.1. Introduction

The research conducted by GEOMETRICA focuses on three main directions:
- fundamental geometric data structures and algorithms
- robust computation and advanced programming,
- shape approximation and mesh generation.

3.2. Fundamental geometric data structures and algorithms

GEOMETRICA is pursuing long standing research on fundamental geometric data structures and algorithms. GEOMETRICA has a large expertise in Voronoi diagrams and Delaunay triangulations, randomized algorithms, combinatorial geometry and related fields. Recently, we devoted efforts to developing the field of computational geometry beyond linear objects. We are especially interested in developing a theory of curved Voronoi diagrams. Such diagrams allow to model growing processes and have important applications in biology, ecology, chemistry and other fields. They also play a role in some optimization problems and in anisotropic mesh generation. Euclidean Voronoi diagrams of non punctual objects are also non affine diagrams. They are of particular interest to robotics, CAD and molecular biology. Even for the simplest diagrams, e.g. Euclidean Voronoi diagrams of lines, triangles or spheres in 3-space, obtaining tight combinatorial bounds and efficient algorithms are difficult research questions. In addition, effective implementations require to face specific algebraic and arithmetic questions. Working out carefully the robustness issues is a central objective of GEOMETRICA (see below).

In the past years, the main objective of computational geometry has been the design of time efficient algorithms, either from the theoretical point of view of the asymptotic complexity or the more practical aspect of running efficient benchmarks. Surprisingly, less interest has been devoted to improve the space behavior of such algorithms although the problem may become of importance when the main memory is not enough and the system has to swap to find extra memory space; this may happen either for massive data or for small memory devices such as PDA. GEOMETRICA intends to attack this problem from several aspects:
— the compression of geometrical objects for storage or network transmission purposes
— the design of algorithms accessing locally the memory to reduce (but not remove) the swapping in memory
— the design of new data-structures to represent geometrical objects using less memory.
For all these aspects we are interested in both the theoretical asymptotic sizes involved and the practical aspect for reasonable input size. Such a distinction between asymptotic behavior and practical interest may appear strange since we are interested in massive data, but even for a very big mesh of several millions points which can be called "massive data", we are still far from the asymptotic behavior of some theoretical structures.

3.3. Robust computation and advanced programming

An implementation of a geometric algorithm is called *robust* if it produces a valid output for all inputs. Geometric programs are notorious for their non-robustness due to two reasons: (1) geometric algorithms are designed for a model of computation where real numbers are dealt with exactly and, (2) geometric algorithms are frequently only formulated for inputs in general position. As a result, implementations may crash or produce nonsensical outputs. This is observed in all commercial CAD-systems.

The importance of robustness in geometric computations has been recognized for a long time, but significant progress was made only in recent years. GEOMETRICA held a central role in this process, including advances regarding the *exact computation paradigm*. In this paradigm, robustness is achieved by a combination of three methods: *exact arithmetic*, *dedicated arithmetic* and *controlled rounding*.

In addition to pursuing research on robust geometric computation, GEOMETRICA is an active member of a European consortium that develops a large library named CGAL. This library makes extensive use of generic programming techniques and is both a unique tool to perform experimental research in Computational Geometry and a comprehensive library for Geometric Computing. The startup company GEOMETRY FACTORYhas been started in January 2003 to commercialize components from CGAL and to offer services for geometric applications.

3.4. Shape approximation and mesh generation

Complex shapes are ubiquitous in robotics (configuration spaces), computer graphics (animation models) or physical simulations (fluid models, molecular systems). In all these cases, no natural *shape space* is available or when such spaces exist they are not easily dealt with. When it comes to performing calculations, the objects under study must be discretized. On the other hand, several application areas such as Computer Aided Geometric Design or medical imaging require reconstructing $3D$ or $4D$ shapes from samples.

The questions aforementioned fall in the realm of *geometric approximation theory*, a topic GEOMETRICA is actively involved in. More precisely, the generation of samples, the definition of differential quantities (e.g. curvatures) in a discrete setting, the geometric and topological control of approximations, as well as multi-scale representations are investigated. Connected topics of interest are also the progressive transmission of models over networks and their compression.

Surface mesh generation and surface reconstruction have received a great deal of attention by researchers in various areas ranging from computer graphics through numerical analysis to computational geometry. However, work in these areas has been mostly heuristic and the first theoretical foundations have been established only recently. Quality mesh generation amounts to finding a partition of a domain into linear elements (mostly triangles or quadrilaterals) with topological and geometric properties. Typically, one aims at constructing a piecewise linear (PL) approximation with the "same" topology as the original surface (same topology may have several meanings). In some contexts, one wants to simplify the topology in a controlled way. Regarding the geometric distance between the surface and its PL approximation, different measures must be considered: Hausdorff distance, errors on normals, curvatures, areas etc. In addition, the shape, angles or size of the elements must match certain criteria. We call *remeshing* the techniques involved when the input domain to be discretized is itself discrete. The input mesh is often highly irregular and non-uniform, since it typically comes as the output of a surface reconstruction algorithm applied to a point cloud obtained from a scanning device. Many geometry processing algorithms (e.g. smoothing, compression) benefit from remeshing, combined with uniform or curvature-adapted sampling. GEOMETRICA intends to contribute to all aspects of this matter, both in theory and in practice.

Volumetric mesh generation consists in triangulating a given three-dimensional domain with tetrahedra having a prescribed size and shape. For instance, the tetrahedra in a general purpose mesh should be as regular as possible (isotropic case), whereas they should be elongated in certain directions for problem specific meshes. Volumetric mesh generation often makes use of surface mesh generation techniques (*e.g.* to approximate the boundary of the domain or interfaces between two media). Thanks to its strong experience with Delaunay triangulations, GEOMETRICA recently made several contributions to the generation of volumetric meshes, and intends to pursue in this direction.

4. Application Domains

4.1. Geometric modeling and shape reconstruction

Keywords: *Geometric modeling*, *geology*, *medical imaging*, *reverse engineering*, *surface reconstruction*.

Modeling 3D shapes is required for all visualization applications where interactivity is a key feature since the observer can change the viewpoint and get an immediate feedback. This interactivity enhances the descriptive power of the medium significantly. For example, visualization of complex molecules helps drug designers to understand their structure. Multimedia applications also involve interactive visualization and include e-commerce (companies can present their product realistically), 3D games, animation and special effects in motion pictures. The uses of geometric modeling also cover the spectrum of engineering, computer-aided design and manufacture applications (CAD/CAM). More and more stages of the industrial development and production pipeline are now performed by simulation, due to the increased performance of numerical simulation packages. Geometric modeling therefore plays an increasingly important role in this area. Another emerging application of geometric modeling with high impact is medical visualization and simulation.

In a broad sense, shape reconstruction consists of creating digital models of real objects from points. Example application areas where such a process is involved are Computer Aided Geometric Design (making a car model from a clay mockup), medical imaging (reconstructing an organ from medical data), geology (modeling underground strata from seismic data), or cultural heritage projects (making models of ancient and or fragile models or places). The availability of accurate and fast scanning devices has also made the reproduction of real objects more effective such that additional fields of applications are coming into reach. The members of GEOMETRICA have a long experience in shape reconstruction and contributed several original methods based upon the Delaunay and Voronoi diagrams.

4.2. Algorithmic issues in Structural Biology

Keywords: *Molecules*, *docking*.

Two of the most prominent challenges of the post-genomic era are to understand the molecular machinery of the cell and to develop new drug design strategies. These key challenges require the determination, understanding and exploitation of the three-dimensional structure of several classes of molecules (nucleic acids, proteins, drugs), as well as the elucidation of the interaction mechanisms between these molecules.

These challenges clearly involve aspects from biology, chemistry, physics, mathematics and computer science. For this latter discipline, while the historical focus has been on text and pattern matching related algorithms, the amount of structural data now available calls for geometric methods. At a macroscopic scale, the classification of protein shapes, as well as the analysis of molecular complexes requires shape description and matching algorithms. At a finer scale, molecular dynamics and force fields require efficient data-structures to represent solvent models, as well as reliable meshes so as to solve the Poisson-Boltzmann equation.

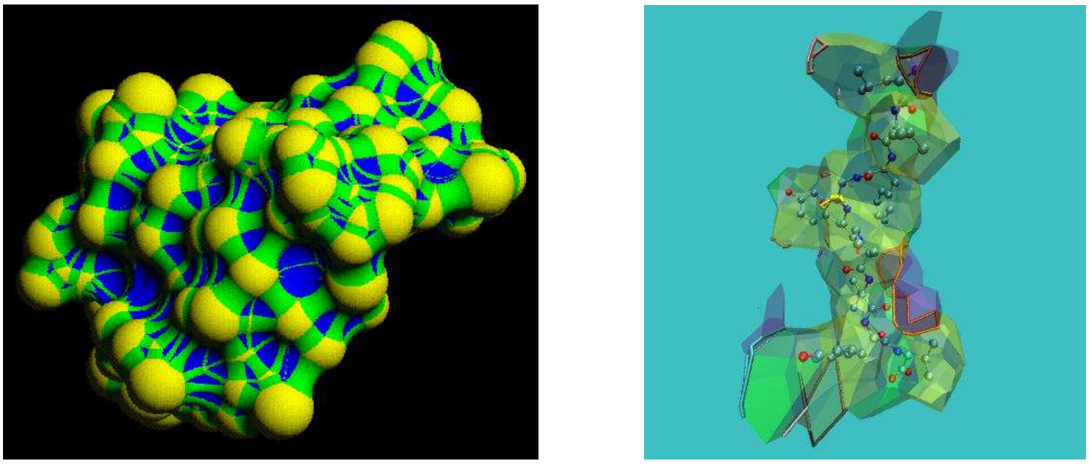

Figure 1. (a) Molecular surface (b) A small peptide surrounded by its interface [56]

4.3. Scientific computing

Keywords: *Unstructured meshes*, *finite element method*, *finite volume method*.

Meshes are the basic tools for scientific computing using finite element methods. Unstructured meshes are used to discretize domains bounded by complex shapes while allowing local refinements. GEOMETRICA contributes to 2D, 3D as well as surface mesh generation. Most of our methods are based upon Delaunay triangulations, Voronoi diagrams and their variants. Affine diagrams are used in the context of volume element methods, while non-affine diagrams are used to generate anisotropic meshes. We investigate in parallel both greedy and variational mesh generation techniques. The greedy algorithms consist of inserting vertices in an initial coarse mesh using the Delaunay refinement paradigm, while the variational algorithms consists of minimizing an energy related to the shape and to the size of the elements. Our goal is to show the complementarity of these two paradigms. Quadrangle surface meshes are also of interest for reverse engineering and geometry processing applications. Our approach consists of sampling a set of curves on the surface so as to control the final edge alignment, the mesh sizing and the regularity of the quadrangulation.

5. Software

5.1. CGAL, the computational geometry algorithms library

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Hervé Brönnimann, Frédéric Cazals, Frank Da, Christophe Delage, Olivier Devillers, Andreas Fabri, Julia Flötotto, Philippe Guigue, Menelaos Karavelas, Abdelkrim Mebarki, Naceur Meskini, Sylvain Pion [contact person], Marc Pouget, François Rebufat, Laurent Saboret, Monique Teillaud, Radu Ursu, Mariette Yvinec.

http://www.cgal.org

CGAL is a C++ library of geometric algorithms initially developed within two European projects (project ESPRIT IV LTR CGAL december 97 - june 98, project ESPRIT IV LTR GALIA november 99 - august 00) by a consortium of eight research teams from the following institutes: Universiteit Utrecht, Max-Planck Institut Saarbrücken, INRIA Sophia Antipolis, ETH Zürich, Tel Aviv University, Freie Universität Berlin, Universität Halle, RISC Linz. The goal of CGAL is to make the solutions offered by the computational geometry community available to the industrial world and applied domains.

The CGAL library consists in a kernel, a basic library and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes : we can choose between Cartesian or homogeneous coordinates, use different types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

The basic library provides a number of geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d-dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface (concept).

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview, SCILAB...).

GEOMETRICA is particularly involved in the arithmetic issues that arise in the treatment of robustness issues, the kernel, triangulation packages and their close applications such as alpha shapes, general maintainance...

CGAL is about 500,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Solaris, Cygwin...), Visual C++ (Windows), Intel C++... Version 3.2.1 has been released in July 2006. The previous main release, CGAL 3.1, has been downloaded 15000 times from our web site, during the 18 months period where it was the main version. Moreover, CGAL is now directly available as packages for the Debian and Fedora Linux distributions.

CGAL is distributed under an Open Source license (LGPL or QPL depending on which packages), and is commercialized by GEOMETRY FACTORY, a startup company started in 2003 by A. Fabri.

5.2. Web services

5.2.1. A web service for surface reconstruction

Participant: David Cohen-Steiner.

In collaboration with Frank Da and Andreas Fabri. http://cgal.inria.fr/Reconstruction/.

The surface reconstruction algorithm developed by D. Cohen-Steiner and F. Da using CGAL is available as a *web service*. Via the web, the user uploads the point cloud data set to the server and obtains a VRML file of the reconstructed surface, which gets visualized in the browser of the user. This allows the user to check if the algorithm fits his/her needs, to learn how to adjust the parameters, before contacting INRIA to obtain an executable. It also provides us with the opportunity to collect real-world data sets used for testing and improving our reconstruction algorithms.

5.2.2. Modeling macro-molecular interfaces

Participant: Frédéric Cazals.

As described in section 6.7.2, we recently proposed an interface model of (macro-)molecular interfaces based upon power diagrams [23]. The corresponding software, *Intervor*, has been made available to the community from the web site http://bombyx.inria.fr/Intervor/intervor.html. Our publication appeared in the September issue of *Protein Science*, yet, the server has been used about 200 times in July - August. To the best of our knowledge, this code is the only publicly available one for analyzing interfaces in complexes.

6. New Results

6.1. Introduction

The presentation of our new results follows the three main directions recalled in Section 3.1
– fundamental geometric data structures and algorithms,
– robust computation and advanced programming,
– shape approximation and mesh generation.

Contributions on the first item are the construction of Voronoi diagrams of spheres, several results related to visibility and graph algorithms for reporting maximal cliques.

Work on the second research direction includes the design and a preliminary implementation of a certified curved kernel for CGAL. In addition, to further extend the robustness and the efficiency of CGAL, geometric constructions are now filtered in a way similar to predicates.

Work on the third axis gained momentum and is subdivided in surface approximation, mesh generation, computation and stability of geometric features, and compact data structures.

Lastly, we report on some applications in molecular biology, visualization and signal processing.

6.2. Combinatorics, data structures and algorithms

Keywords: *Voronoi diagrams*, *compact data structures*.

6.2.1. Curved Voronoi diagrams

Participants: Jean-Daniel Boissonnat, Camille Wormser, Mariette Yvinec.

Voronoi diagrams are fundamental data structures that have been extensively studied in Computational Geometry. A Voronoi diagram can be defined as the minimization diagram of a finite set of continuous functions. Usually, each of those functions is interpreted as the distance function to an object. The associated Voronoi diagram subdivides the embedding space into regions, each region consisting of the points that are closer to a given object than to the others. We may define many variants of Voronoi diagrams depending on the class of objects, the distance functions and the embedding space. Affine diagrams, i.e. diagrams whose cells are convex polytopes, are well understood. Their properties can be deduced from the properties of polytopes and they can be constructed efficiently. The situation is very different for Voronoi diagrams with curved regions. Curved Voronoi diagrams arise in various contexts where the objects are not punctual or the distance is not the Euclidean distance. This work [16] is a survey of the main results on curved Voronoi diagrams. We describe in some detail two general mechanisms to obtain effective algorithms for some classes of curved Voronoi diagrams. The first one consists in linearizing the diagram and applies, in particular, to diagrams whose bisectors are algebraic hypersurfaces. The second one is a randomized incremental paradigm that can construct affine and several planar non-affine diagrams.

6.2.2. Complexity of Delaunay triangulation for points on lower-dimensional polyhedra

Participant: Olivier Devillers.

In collaboration with N. Amenta (University of California at Davis) and D. Attali (LIS-Grenoble)

Following previous 3D results of Attali and Boissonnat, we show that the Delaunay triangulation of a set of points distributed nearly uniformly on a polyhedron (not necessarily convex) of dimension p in d-dimensional space is $O(n^{(d-1)/p})$. For all $2 \leq p \leq d-1$, this improves on the well-known worst-case bound of $O(n^{\lceil d/2 \rceil})$ [50].

6.2.3. Bregman Voronoi diagrams of probabilistic distributions

Participant: Jean-Daniel Boissonnat.

In collaboration with F. Nielsen (Sony Computer Science Laboratories) and R. Nock (University of Antilles-Guyane)

We investigate a framework for defining and building Voronoi diagrams for a broad class of distortion measures called Bregman divergences, that includes not only the traditional (squared) Euclidean distance, but also various divergence measures based on entropic functions. As a by-product, Bregman Voronoi diagrams allow one to define information-theoretic Voronoi diagrams in statistical parametric spaces based on the relative entropy of distributions. We show that for a given Bregman divergence, one can define several types of Voronoi diagrams related to each other by convex duality or embedding. Moreover, we can always compute them indirectly as power diagrams in primal or dual spaces, or directly after linearization in an extra-dimensional space as the projection of a Euclidean polytope. Finally, our paper proposes to generalize Bregman divergences to higher-order terms, called k-jet Bregman divergences, and touch upon their Voronoi diagrams [43].

6.2.4. Compact representation of geometric data structures

Participants: Luca Castelli-Aleardi, Olivier Devillers, Abdelkrim Mebarki.

In collaboration with G. Schaeffer (LIX-Palaiseau)

We address the problem of representing the connectivity information of geometric objects using as little memory as possible. As opposed to raw compression issues, the focus is here on designing data structures that preserve the possibility of answering incidence queries in constant time. We propose in particular the first optimal representations for 3-connected planar graphs and triangulations, which are the most standard classes of graphs underlying meshes with spherical topology. Optimal means that these representations asymptotically match the respective entropy of the two classes, namely 2 bits per edge for 3-c planar graphs, and 1.62 bits per triangle or equivalently 3.24 bits per vertex for triangulations [38]. These results remain rather theoretical because the asymptotic behavior is far from the practical one, mainly due to a sub-linear term in $O\left(\frac{n \log \log n}{\log n}\right)$ which is actually not negligible. We are investigating some simplified framework inspired by the theoretical method but taking care of practical issues [37].

6.2.5. Exact circle arrangements on a sphere, with applications in structural biology

Participants: Frédéric Cazals, Sébastien Loriot.

Given a collection of circles in a sphere, we adapt the Bentley-Ottmann algorithm to the spherical setting to compute the *exact* arrangement of the circles [58]. Assuming the circles are induced by balls, we also extend the algorithm to report the balls covering each face of the arrangement. The algorithm consists of sweeping the sphere by a meridian, which is non trivial because of the degenerate cases and the algebraic specification of event points. The paper focuses on the construction of the arrangement, the predicates and algebraic questions being developed in a companion paper.

This construction is motivated by the calculation of parameters describing multi-body contacts in structural biology, so as to go beyond the classical exposed and buried surface areas. As an illustration, statistics featuring spectacular changes wrt traditional observations on protein - protein complexes are provided.

6.3. Geometric computing

Keywords: *algebraic degree of algorithms*, *computational geometry*, *degeneracies*, *perturbation*, *predicates*, *robustness*.

6.3.1. Perturbations and vertex removal in Delaunay and regular 3D triangulations

Participants: Olivier Devillers, Monique Teillaud.

Though Delaunay and regular triangulations are very well known geometric data structures, the problem of the robust removal of a vertex in a three-dimensional triangulation is actually a problem in practice. We propose a simple method that allows to remove any vertex even when the points are in very degenerate configurations [61]. The solution is available in CGAL (see Section 6.8.11).

6.3.2. Operations on curves

Participants: Pedro Machado Manhães de Castro, Sylvain Pion, Daniel Russel, Ilya Suslov, Monique Teillaud [contact person], Elias Tsigaridas.

Work on the CGAL curved kernel has continued, following the past years work, in two main directions.

The first direction consisted in research on methods to improve the new package for manipulations of circular arcs in the plane that was released in CGAL 3.2 [71] (see Section 6.8.6). In particular, different possible representations of algebraic numbers were studied and compared. Geometric filtering techniques were also studied [74], [70], [69]. These improvements will be included in the next release. An extension of this work to the 3D case, which raises new issues, is in progress. This work is also mentioned in [27].

The second research direction consists in defining concepts for the manipulation of algebraic curves of general degree, with the goal of a CGAL implementation. This is done in collaboration with our ACS and Arcadia partners [52], [51].

6.3.3. Robust construction of the extended three-dimensional flow complex

Participants: Frédéric Cazals, Sylvain Pion.

The Delaunay triangulation and its dual, the Voronoi diagram, are ubiquitous geometric complexes. From a topological standpoint, the connexion has recently been made between these constructions and the Morse theory of distance functions. In particular, algorithms have been designed to compute the flow complex induced by the distance functions to a point set.

This paper [57] develops the first complete and robust construction of the extended flow complex, which in addition of the stable manifolds of the flow complex, also features the unstable manifolds. A first difficulty comes from the interplay between the degenerate cases of Delaunay and those which are flow specific. A second class of problems comes from cascaded constructions and predicates —as opposed to the standard in-circle and orientation predicates for Delaunay. We deal with both aspects and show how to implement a complete and robust flow operator, from which the extended flow complex is easily computed. We also present experimental results.

6.3.4. Lazy exact evaluation of geometric computations

Participant: Sylvain Pion.

*In collaboration with A. Fabri (*GEOMETRY FACTORY*)*

We present [41] a generic C++ design to perform efficient and exact geometric computations using lazy evaluations. Exact geometric computations are critical for the robustness of geometric algorithms. Their efficiency is also critical for most applications, hence the need for delaying the exact computations at run time until they are actually needed. Our approach is generic and extensible in the sense that it is possible to make it a library that users can extend to their own geometric objects or primitives. It involves techniques such as generic functor adaptors, dynamic polymorphism, reference counting for the management of directed acyclic graphs and exception handling for detecting cases where exact computations are needed. It also relies on multiple precision arithmetic as well as interval arithmetic. We apply our approach to the whole geometric kernel of CGAL.

6.3.5. On real root isolation

Participant: Elias Tsigaridas.

In collaboration with Ioannis Emiris (National University of Athens, Greece).

[62] presents the average-case bit complexity of subdivision-based univariate solvers, namely those named after Sturm, Descartes, and Bernstein. By real solving we mean real root isolation. We prove bounds of $\tilde{\mathcal{O}}_B(N^5)$ for all methods, where N bounds the polynomial degree and the coefficient bitsize, whereas their worst-case complexity is in $\tilde{\mathcal{O}}_B(N^6)$. In the case of the Sturm solver, our bound depends on the number of real roots. Our work is a step towards understanding the practical complexity of real root isolation. This enables a better juxtaposition against numerical solvers, the latter having worst-case complexity in $\tilde{\mathcal{O}}_B(N^4)$.

In [81], we present algorithmic, complexity and implementation results concerning real root isolation of integer univariate polynomials using the continued fraction expansion of real algebraic numbers. One motivation is to explain the method's good performance in practice. We improve the previously known bound by a factor of $d\tau$, where d is the polynomial degree and τ bounds the coefficient bit size, thus matching the current record complexity for real root isolation by exact methods (Sturm, Descartes and Bernstein subdivision). Namely our complexity bound is $\tilde{\mathcal{O}}_B(d^4\tau^2)$ using a standard bound on the expected bit size of the integers in the continued fraction expansion. Moreover, using a homothetic transformation we improve the expected complexity bound to $\tilde{\mathcal{O}}_B(d^3\tau)$ under the assumption that $d = \mathcal{O}(\tau)$. We compute the multiplicities within the same complexity and extend the algorithm to non square-free polynomials.

6.4. Surface approximation

Keywords: *Computational topology*, *geometric probing*, *implicit surfaces*, *point set surfaces*, *surface learning*, *surface meshing*.

6.4.1. Reconstruction with Voronoi centered radial basis functions

Participants: Marc Alexa, Pierre Alliez, Marie Samozino, Mariette Yvinec.

We consider [45], [80] the problem of reconstructing a surface from scattered points sampled on a physical shape. The sampled shape is approximated as the zero level set of a function. This function is defined as a linear combination of compactly supported radial basis functions. We depart from previous work by using as centers of basis functions a set of points located on an estimate of the medial axis, instead of the input data points. Those centers are selected among the vertices of the Voronoi diagram of the sample data points. Being a Voronoi vertex, each center is associated with a maximal empty ball. We use the radius of this ball to adapt the support of each radial basis function. Our method can fit a user-defined budget of centers: the selected subset of Voronoi vertices is filtered using the notion of lambda medial axis, then clustered to fit the allocated budget.

6.4.2. Designing quadrangulations with discrete harmonic forms

Participants: Pierre Alliez, David Cohen-Steiner.

In collaboration with Mathieu Desbrun and Yiying Tong from Caltech.

We introduce a framework for quadrangle meshing of discrete manifolds. Based on discrete differential forms, our method hinges on extending the discrete Laplacian operator (used extensively in modeling and animation) to allow for line singularities and singularities with fractional indices. When assembled into a singularity graph, these line singularities are shown to considerably increase the design flexibility of quad meshing. In particular, control over edge alignments and mesh sizing are unique features of our approach. Another appeal of our method is its robustness and scalability from a numerical viewpoint: we solve a sparse linear system to generate a pair of piecewise-smooth scalar fields whose isocontours form a pure quadrangle tiling, with no T-junctions [47].

6.4.3. Periodic global parameterization

Participant: Pierre Alliez.

In collaboration with Nicolas Ray, Bruno Lévy and Wan Chiu Li from Loria, and Alla Sheffer from University of British Columbia.

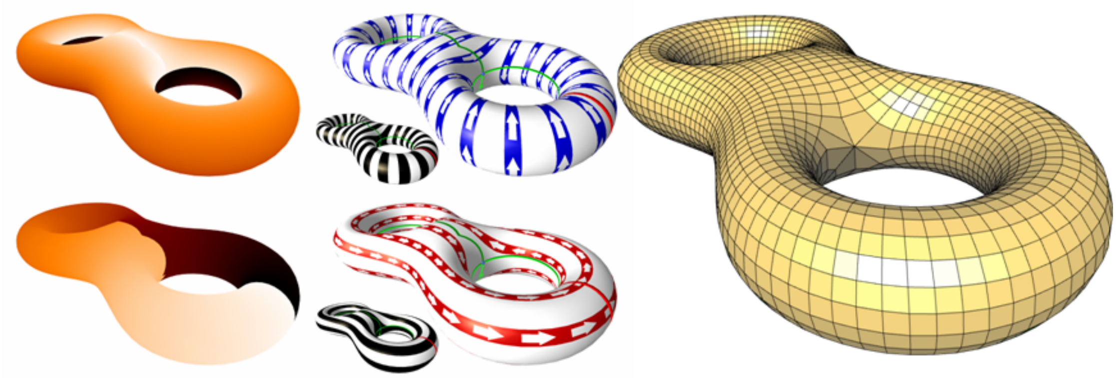

Figure 2. Quadrangle surface tiling.

We present a new globally smooth parameterization method for surfaces of arbitrary topology [31]. Our method does not require any prior partition into charts nor any cutting. The chart layout (i.e., the topology of the base complex) and the parameterization emerge simultaneously from a global numerical optimization process. Given two orthogonal piecewise linear vector fields, our method computes two piecewise linear periodic functions, aligned with the input vector fields, by minimizing an objective function. The bivariate function they define is a smooth parameterization almost everywhere, except in the vicinity of the singular points of the vector field, where both the vector field and the derivatives of the parameterization vanish. Our method can construct quasi-isometric parameterizations at the expense of introducing additional singular points in non-developable regions where the curl of the input vector field is non-zero. We also propose a curvature-adapted parameterization method, that minimizes the curl and removes those additional singular points by adaptively scaling the parameterization. In addition, the same formalism is used to allow smoothing of the control vector fields. We demonstrate the versatility of our method by using it for quad-dominant remeshing and T-spline surface fitting. For both applications, the input vector fields are derived by estimating the principal directions of curvatures.

6.5. Mesh generation

Keywords: *Isotropic meshing*, *anisotropic meshing*, *level sets*, *tetrahedral meshing*, *triangle meshing*.

6.5.1. A generic software design for Delaunay refinement meshing

Participants: Laurent Rineau, Mariette Yvinec.

We present [78] a generic software designed to implement meshing algorithms based on the Delaunay refinement paradigm. Such a meshing algorithm is generally described through a set of rules guiding the refinement of mesh elements. The central item of the software design is a generic class, called a mesher level, that is able to handle one of the rules guiding the refinement process. Several instantiations of the mesher level class can be stacked and tied together to implement the whole refinement process. As shown in this paper, the design is flexible enough to implement all currently known mesh generation algorithms based on Delaunay refinement. In particular it can be used to generate meshes approximating smooth or piecewise smooth surfaces, as well as to mesh three dimensional domains bounded by such surfaces. It also adapts to algorithms handling small input angles and various refinement criteria. This design highly simplifies the task of implementing Delaunay refinement meshing algorithms. It has been used to implement several meshing algorithms in the CGAL library.

6.5.2. A Lagrangian approach to dynamic interfaces through kinetic triangulations

Participants: Jean-Daniel Boissonnat, Jean-Philippe Pons.

We propose [44] a robust and efficient Lagrangian approach for modeling dynamic interfaces between different materials undergoing large deformations and topology changes, in two dimensions. Our work brings an interesting alternative to popular techniques such as the level set method and the partical level set method, for two-dimensional and axisymmetric simulations. The principle of our approach is to maintain a two-dimensional triangulation which embeds the one-dimensional polygonal description of the interfaces. Topology changes can then be detected as inversions of the faces of this triangulation. Each triangular face is labeled with the type of material it contains. The connectivity of the triangulation and the labels of the faces are updated consistently during deformation, within a neat framework developed in computational geometry: kinetic data structures. Thanks to the exact computation paradigm, the reliability of our algorithm, even in difficult situations such as shocks and topology changes, can be certified. We demonstrate the applicability and the efficiency of our approach with a series of numerical experiments in two dimensions. Finally, we discuss the feasibility of an extension to three dimensions.

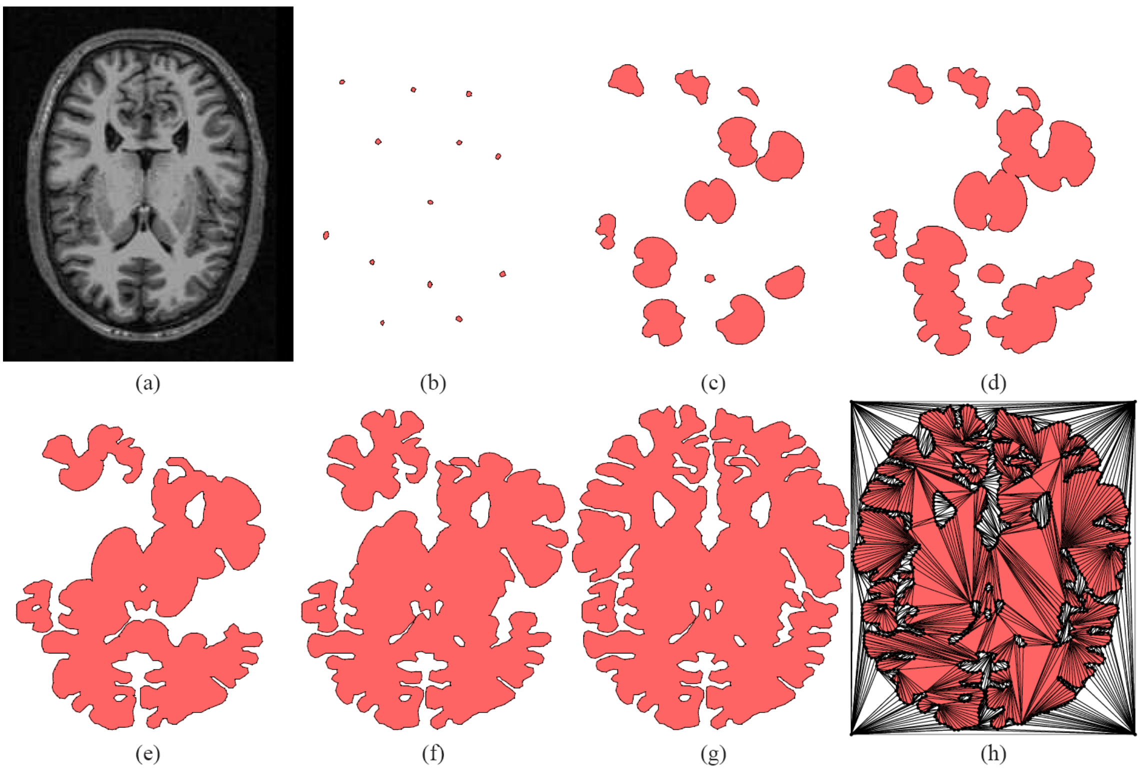

Figure 3. Application to brain segmentation: (a) MR image, (b-g) different stages of the evolution, (h) labeled triangulation of the final shape.

6.6. Computation and stability of geometric features

Keywords: *Geometric inference*, *computational topology*.

6.6.1. Vines and vineyards by updating persistence in linear time

Participant: David Cohen-Steiner.

In collaboration with Herbert Edelsbrunner and Dmitriy Morozov from Duke University.

Persistent homology is the mathematical core of recent work on shape, including reconstruction, recognition, and matching. Its pertinent information is encapsulated by a pairing of the critical values of a function, visualized by points forming a diagram in the plane. The original persistence algorithm computes the pairs from an ordering of the simplices in a triangulation and takes worst-case time cubic in the number of simplices. The main result of this paper [40] is an algorithm that maintains the pairing in worst-case linear time per transposition in the ordering. A side-effect of the algorithm's analysis is an elementary proof of the stability of persistence diagrams in the special case of piecewise-linear functions. We use the algorithm to compute 1-parameter families of diagrams which we apply to folding trajectories of proteins.

6.6.2. A sampling theory for compacts sets in Euclidean space

Participants: Frédéric Chazal, David Cohen-Steiner.

In collaboration with A. Lieutier from Dassault Systèmes and LMC/IMAG.

We introduce [39] a parameterized notion of feature size that interpolates between the minimum of the local feature size, and the recently introduced weak feature size. Based on this notion of feature size, we propose sampling conditions that apply to noisy samplings of general compact sets in Euclidean space. These conditions are sufficient to ensure the topological correctness of a reconstruction given by an offset of the sampling. Our approach also yields new stability results for medial axes, critical points and critical values of distance functions.

6.6.3. Second fundamental measure of geometric sets and local approximation of curvatures

Participant: David Cohen-Steiner.

In collaboration with J.M. Morvan from IGD (Lyon).

Using the theory of normal cycles, we associate with each geometric subset of a Riemannian manifold a —tensor-valued— curvature measure, which we call its second fundamental measure [25]. This measure provides a finer description of the geometry of singular sets than the standard curvature measures. Moreover, we deal with approximation of curvature measures. We get a local quantitative estimate of the difference between curvature measures of two geometric subsets, when one of them is a smooth hypersurface.

6.7. Applications

6.7.1. Design of interferometric telescopes

Participants: Jean-Daniel Boissonnat, Trung Nguyen.

In collaboration with P. Blanc and F. Falzon (Alcatel Alenia Space)

Given a disk O in the plane called the objective, we want to find n small disks $P_1, ..., P_n$ called the pupils such that $\bigcup_{i,j=1}^{n} P_i \ominus P_j \supseteq O$, where $\ominus$ denotes the Minkowski difference operator, while minimizing the number of pupils, the sum of the radii or the total area of the pupils. This problem is motivated by the construction of very large telescopes from several smaller ones by so-called Optical Aperture Synthesis. In this paper [42], we provide exact, approximation and heuristic solutions to several variations of the problem.

6.7.2. Revisiting the Voronoi description of protein-protein interfaces

Participant: Frédéric Cazals.

In collaboration with F. Proust, R. Bahadur(CNRS, Orsay) and J. Janin (CNRS, Orsay).

This paper [23] develops a model of macromolecular interfaces based on the Voronoi diagram and the related alpha-complex, and we test its properties on a set of 96 protein-protein complexes taken from the Protein Data Bank. The Voronoi model provides a natural definition of the interfaces, and it yields values of the number of interface atoms and of the interface area that have excellent correlation coefficients with those of the classical model based on solvent accessibility. Nevertheless, some atoms that do not loose solvent accessibility are part of the interface defined by the Voronoi model. The Voronoi model provides robust definitions of the curvature and of the connectivity of the interfaces, and leads to estimates of these features that generally agree with other approaches. Our implementation of the model allows an analysis of protein-water contacts that highlights the contacts of structural water molecules at protein-protein interfaces.

6.7.3. Geometric, topological and contact analysis of interfaces in macro-molecular complexes

Participant: Frédéric Cazals.

Understanding the *sociology* of interactions between the proteins encoded in a genome is a central question of structural biology, and interface models between molecules forming a complex are instrumental in this perspective. Qualifying interface atoms as atoms loosing solvent accessibility in the complex, or pairs of atoms within a distance threshold, several interface models have been proposed. Yet, until recently, no interface model existed to answer coherently (if at all) the following questions: can one bridge the gap from atoms loosing solvent accessibility to interface pairs? is the interface flat or curvy? is it connected or not (does it have a multi-patch structure)? is a connected component of the interface simply connected or not (does it have a hole)? what is precisely the role played by interface structural water?

Using the α-complex of the Van der Waals balls, a construction derived from the Voronoi diagram, we designed such an interface model, and validated it on the usual database of co-crystallized protein-protein complexes. This paper [56] is a methodological contribution aiming at easing the access of the interface model to structural biologists. As such, the following topics are covered: (i) the geometric principles underlying the interface model (ii) the definitions of the interface and its extension to accommodate structural water (iii) the statistics one can compute from the interface model (iv) the Software *Intervor* and the associated web site. These presentations are accompanied by illustrations and insights on protein - protein complexes.

6.8. Software

Keywords: *C++ standardization*, *CGAL packages*, *CORE*, *PYTHON*, *SCILAB*.

6.8.1. Generic programming and the CGAL library

Participant: Monique Teillaud.

In collaboration with Efi Fogel (University of Tel-Aviv)

This work [28] discusses geometric programming, and shows how the generic programming paradigm is used in CGAL to overcome problems encountered when implementing effective computational geometry algorithms.

6.8.2. CG-LAB :an interface between CGAL and SCILAB

Participants: Naceur Meskini, Sylvain Pion.

We have continued the implementation of a SCILAB toolbox dedicated to geometric computing, based on CGAL, named CGLAB. A third release 1.1 of this toolbox has been made in 2006. It provides the basic Delaunay triangulations in any dimension, various 2D meshing algorithms, 2D convex hull computations, 2D stream lines placements, 2D and 3D function interpolation and 3D isosurface mesh generation. CGLAB is available under the Open Source LGPL license, and is subject to the constraints of the underlying CGAL packages. The web site is http://cglab.gforge.inria.fr/.

6.8.3. CGAL-PYTHON: PYTHON bindings around CGAL

Participants: Naceur Meskini, Sylvain Pion.

We have started the implementation of Python bindings around CGAL. PYTHON is a powerful, general purpose, interpreted language, which is increasingly used in scientific areas such as molecular biology, and various application areas. A first beta release 0.9 of this software has been made available in 2006. It provides bindings around several CGAL packages: 2D and 3D triangulations, Delaunay triangulations, 2D constrained triangulations, 2D mesh generator, 2D and 3D Alpha shapes, 3D Polyhedron, 2D convex hull, several geometric optimization algorithms, and the 2D and 3D kernel. CGAL-PYTHON is available under the Open Source LGPL license, and is subject to the constraints of the underlying CGAL packages. The web site is http://cgal-python.gforge.inria.fr/.

6.8.4. C++ standardization of interval arithmetic

Participant: Sylvain Pion.

In collaboration with H. Brönnimann (Polytechnic University Brooklyn), and G. Melquiond (ARENAIRE).

We have submitted a revision of our proposal of specification of interval arithmetic [35], [36], [17] to the C++ standardization committee. We hope that by having this tool standardized, better ans faster implementations will exist, and we also hope that certified computations will be considered by more programmers. Some industry members of the C++ standardization committee like Sun Microsystems are also very interested in getting interval arithmetic standardized.

Our proposal is a basic template class parameterized by a floating point type, and provide some functions around it. Due to implementation cost constraints, we had to make choices between the simplicity of the implementation, and functionality. This appears for example in the choice we made for the specification of comparison operators between intervals. An accompanying proposal has also been drafted in the process, specifying the interface for doing multi-valued logic [68]. We have presented our proposals in front of the C++ standardization committee (ISO WG21), where they received good support so far.

6.8.5. New CGAL package for skin surfaces

Participant: Nico Kruithoff.

Skin surfaces, introduced by Edelsbrunner have a rich and simple combinatorial and geometric structure that makes them suitable for modeling large molecules in biological computing. Meshing such surfaces is often required for further processing of their geometry, like in numerical simulation and visualization.

A skin surface is defined by a set of weighted points (input balls) and a scalar called the shrink factor. If the shrink factor is equal to one, the surface is just the boundary of the union of the input balls. For a shrink factor smaller than one, the skin surface becomes tangent continuous, due to the appearance of patches of spheres and hyperboloids connecting the balls.

This package constructs a mesh isotopic to the skin surface defined by a set of balls and a shrink factor using an algorithm proposed by Nico Kruithof and Gert Vegter.

An optimized algorithm is implemented for meshing the union of a set of balls.

6.8.6. New CGAL package for 2D circular kernel

Participants: Monique Teillaud, Sylvain Pion.

The work done in previous years led to the release in CGAL 3.2 of a new package providing users with elementary manipulations of circular arcs in the plane [71]. This package extends the standard CGAL kernel by adding new types of points that can have non-rational coordinates, line segments whose endpoints can be of this new type, and circular arcs. Among other functionalities, exact comparisons, as well as constructions of intersections between these new objects can be performed.

This package will be improved in the next release, after the research done this year (see Section 6.3.2).

The 2D Circular kernel is under evaluation by Dassault Systèmes (that bought a research license from GEOMETRY FACTORY) and by GEOMETRY FACTORYfor a VLSI (Very Large Scale Integration) company.

6.8.7. New CGAL package for spatial sorting

Participant: Christophe Delage.

Recent work show the influence of insertion order on the performance of incremental computational geometry algorithms. With a careful, yet fast pre-ordering of the input data, many CGAL algorithms can expect several-fold speed-ups.

The Spatial_sorting package is a first attempt to tackle this issue. It provides functions to sort 2D and 3D points in a way that dramatically improves triangulation algorithms, and others.

Comprehensive benchmarking is to be done on the many incremental algorithms implemented in CGAL. We also plan to extend this package to sort other geometric objects such as circles, line segments, circular arcs, *etc.*

6.8.8. New CGAL package for estimating differential properties

Participants: Frédéric Cazals, Marc Pouget.

Consider a sampled smooth surface, and assume we are given a collection of points P about a given sample p. We aim at estimating the differential properties up to any fixed order of the surface at point p from the point set $P^+ = P \cup \{p\}$. More precisely, first order properties correspond to the normal or the tangent plane; second order properties provide the principal curvatures and directions, third order properties provide the directional derivatives of the principal curvatures along the curvature lines, etc. This new package, *Jet_fitting_3*, implements the Cazals-Pouget method based on polynomial fitting to perform such estimations. Datasets amenable to such a processing are naturally unstructured point clouds, as well as meshes —whose topological information may be discarded.

6.8.9. New CGAL package for reporting ridges and umbilics

Participants: Frédéric Cazals, Marc Pouget.

Given a smooth surface, a ridge is a curve along which one of the principal curvatures has an extremum along its curvature line. An umbilic is a point at which both principal curvatures are equal. Umbilics are special points on the ridge lines. Ridges are curves of *extremal* curvature and therefore encode important informations used in segmentation, registration, matching and surface analysis. The new package *Ridge_3* implements algorithms to report umbilics and ridges on a surface given as a triangular mesh. Differential quantities associated to the mesh vertices are assumed to be given for these algorithms. If not, these quantities may be computed from the *Jet_fitting_3* package just mentioned.

6.8.10. Improved predicates for the CGAL package of Voronoi diagram of circles

Participants: Christophe Delage, Dave Millman.

A new way of expressing the predicates for the 2D additively weighted Voronoi diagram was proposed, which results in simpler computations. These new predicates make the Apollonius_graph_2 CGAL package significantly faster than the current implementation available in CGAL 3.2 and will be integrated into the code base.

6.8.11. Vertex removal in the CGAL 3D regular triangulation

Participants: Christophe Delage, Olivier Devillers, Monique Teillaud.

The work done on symbolic perturbations (Section 6.3.1) allowed to provide the 3D regular triangulation class [73] with vertex removal. This new functionality was integrated in CGAL 3.2. As far as we know, CGAL is the only software offering this functionality.

7. Contracts and Grants with Industry

7.1. GeometryFactory

The initial development phase of the CGAL library has been made by a European consortium. In order to achieve the transfer and diffusion of CGAL in the industry, a company called GEOMETRY FACTORYhas been founded in January 2003 by Andreas Fabri (http://www.GeometryFactory.com).

The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORYis a link between the researchers from the computational geometry community and the industrial users.

It offers licenses to interested companies, and provides support. There are contracts in various domains such as CAD/CAM, medical applications, GIS, computer vision...

GEOMETRY FACTORYis keeping close contacts with the original consortium members, and in particular with GEOMETRICA.

7.2. Alcatel Alenia Space

Participants: Jean-Daniel Boissonnat, Trung Nguyen.

In collaboration with P. Blanc and F. Falzon (Alcatel Alenia Space).

The goal of this study is to optimize pupil configurations for extended source imaging based on optical interferometry.

The motivation for this work comes from the observation of the Earth from a geostationary orbit (i.e. at a distance of $\sim$ 36000 km) with a resolution of 1 m. A simple calculus shows us that we would need a telescope having a diameter of approximately 20 m for an optical wavelength of $\sim$ 500 nm. Needless to say such an instrument dimension is not adapted to the observation from space and the use of interferometric telescopes (Optical Aperture Synthesis, OAS) is to be considered in this case. We pursue a geometric approach [42].

7.3. France-Telecom

Participants: Olivier Devillers, Laurent Rineau, Mariette Yvinec.

In collaboration with Jean-François Morlier (France Telecom).

The goal of this study is to compute an abstract representation of an antenna network for mobile phone using Voronoï diagrams.

The work started October 15th.

8. Other Grants and Activities

8.1. National initiatives

8.1.1. Color

Participants: Mariette Yvinec, Marc Scherfenberg.

The MEDMESH color involves the teams Geometrica, Asclépios, Caiman and Odyssée from INRIA Sophia Antipolis and the Neurophysiology and Neuropsychology Laboratory of the hospital *La Timone* in Marseille. The goal is to test the mesh generator developped in Geometrica on real meshing problems provided by the other teams of the Color. See for examples figure 4.

The web site of the Medmaeh color is at the url: http://www-sop.inria.fr/geometrica/collaborations/Medmesh.

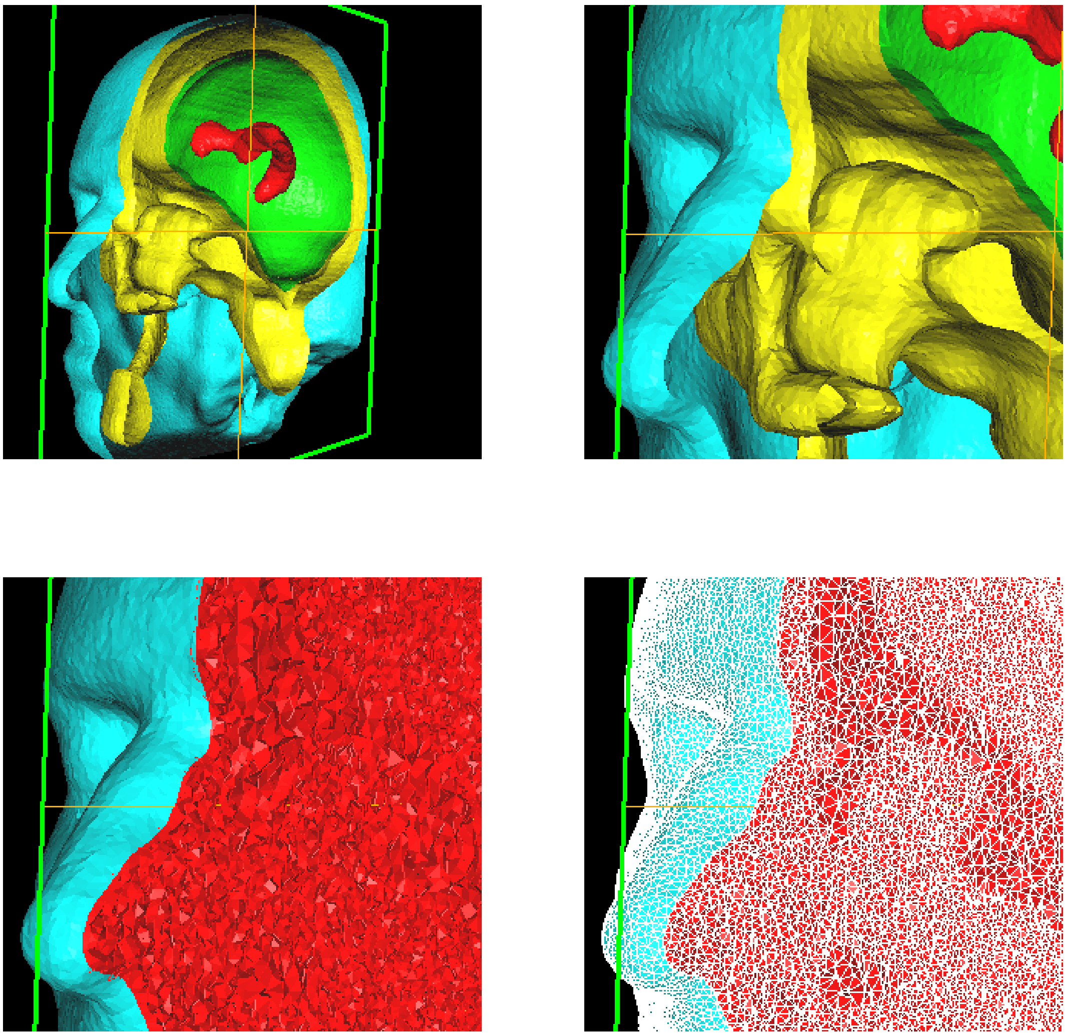

Figure 4. A mesh of a head generated from a few 3D images. The head volume is structured by four surfaces: the skin, the skull, the brain and the ventricule. Each region is meshed with a density related to the wavelength of electromagnetic waves in the correspondant material.

8.1.2. ACI

Participants: Pierre Alliez, Luca Castelli Aleardi, Olivier Devillers, Abdelkrim Mebarki.

We are a member of GEOCOMP an initiative from the national «Action Concertée Incitative Masses de Données». The project involves
— the «Laboratoire d'Informatique de l'Ecole Polytechnique (LIX), CNRS, Ecole Polytechnique»,
— the «Laboratoire Bordelais de Recherche en Informatique (LaBRI), CNRS, Université Bordeaux I»,
— the «Service de Physique Théorique, CEA Saclay» and the GEOMETRICA team.

The project investigates compression schemes and compact data structures devoted specifically to geometrical objects.

Project web page: http://www.lix.polytechnique.fr/Labo/Gilles.Schaeffer/GeoComp/

8.1.3. INRIA Cooperative Research Initiative (ARC) Arcadia

Participants: Sylvain Pion, Daniel Russel, Monique Teillaud, Elias Tsigaridas.

The main objective of this ARC is to contribute to the unfolding of a geometric computing dedicated to quadrics having solid mathematical foundations and to validate theoretical advances by robust and efficient implementations.

Arcadia is a collaboration between GEOMETRICA and VEGAS at INRIA Lorraine. A foreign group is also involved, from the National University of Athens.

Project web page: http://www.loria.fr/~petitjea/Arcadia/

8.2. European initiatives

8.2.1. STREP FET Open ACS

INRIA (teams GALAAD and GEOMETRICA) participates to the IST project ACS.
– Acronym : ACS, numéro IST-006413
– Title : Algorithms for complex shapes with certified topology and numerics.
– Specific program : IST
– STREP (FET Open)
– Starting date : may 1st, 2005 - Duration : 3 years
– Participation mode of INRIA : Participant
– Other participants : Rijksuniversiteit Groningen, Eidgenössische Technische Hochschule Zürich, Freie Universität Berlin, Institut National de Recherche en Informatique et Automatique, Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V., National Kapodistrian University of Athens, Tel Aviv University, GeometryFactory Sarl
The ACS project aims at advancing the state of the art in computing with complex shapes. Current technolgy can cope well with curves in the plane and smooth surfaces in three-dimensional space. We want to address a larger class of shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension.

Increasingly demanding applications require efficient and robust algorithms for complex shapes. Topics that arise and that we address are shape approximation (including meshing and simplification), shape learning (including reconstruction and feature extraction), as well as robust modeling (including boolean operations). Our work on these topics will be closely intertwined with basic research on shape representations.

A unique and ambitious feature of our approach is the guaranteed quality of all data structures and algorithms we plan to develop. Through certified topology and numerics, we will be able to prove that the output is topologically and numerically consistent, according to prespecified criteria. A software prototype, dealing with a restricted class of complex shapes, will demonstrate the feasibility of our techniques in practice.

The web site of the project includes a detailled description of the objectives and all results http://acs.cs.rug.nl.

8.2.2. Network of Excellence aim@shape

INRIA is part of the Network of Excellence:
– Acronym : AIM@SHAPE
– Title: Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing Knowledge Embedded in multidimensional digital objects).
– Reference: 506766
– Start Date: 2004-01-01
– Duration: 48 months
– Contract Type: Network of Excellence
– Action Line: Semantic-based knowledge systems
– Project Funding: 5.74 Million Euros.
– Other participants :
– IMATI, Genova, Italy.
– University of Genova, Italy.
– EPFL, Lausanne, Switzerland.
– Fraunhofer Institute, Germany.
– INPG, France.
– Center for Research and Technology, Greece.
– University of Geneva, Switzerland.
– SINTEF, Norway.
– TECHNION, Israel.
– Weizmann Institute, Israel.
– Utrecht University, Netherlands.
The mission of AIM@SHAPE is to advance research in the direction of semantic-based shape representations and semantic-oriented tools to acquire, build, transmit, and process shapes with their associated knowledge. We foresee a generation of shapes in which knowledge is explicitly represented and, therefore, can be retrieved, processed, shared, and exploited to construct new knowledge. The attainment of a new vision of shape knowledge is achieved by: the formalisation of shape knowledge and the definition of shape ontologies in specific contexts; the definition of shape behaviours which formalise the interoperability between shapes; the delineation of methods for knowledge-based design of shapes and the definition of tools for semantics-dependent mapping of shapes. The web site of the network includes a detailled description of the objectives and some results http://www.aim-at-shape.net.

8.3. International initiatives

8.3.1. Associated team Genepi

Participants: Sylvain Pion, Monique Teillaud.

We are involved in an INRIA associated team with Chee Yap (New York University) and Hervé Brönnimann (Polytechnic University Brooklyn), around the subjects of generic programming and robustness of geometric algorithms. This work includes the specification of algorithms in terms of concepts of geometries. It also includes the interface between algorithms and data structures, as well as collaborations on robustness issues around curved objects.

8.3.2. Scientific and Technological Cooperation between France and Israel

Participants: Jean-Daniel Boissonnat, David Cohen-Steiner, Mariette Yvinec.

In the framework of the Research Networks Program in Medical and Biological Imaging from the High Council for Scientific and Technological Cooperation between France-Israel, we have obtained a financial support for the following project *Geometric reconstruction of organs from freehand ultrasound*. Our israelian partner is the Technion-Israel Institute of Technology, located in Haifa.

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Editorial boards of scientific journals

- J-D. Boissonnat is a member of the editorial board of *Theoretical Computer Science*, *Algorithmica*, *International Journal of Computational Geometry and Applications*, *Computational Geometry : Theory and Applications*, and *The Visual Computer*.
- M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.
- S. Pion (chair), M. Teillaud and M. Yvinec are members of the CGAL editorial board.
- P. Alliez is a member of the editorial board of *The Visual Computer*.

9.1.2. Conference program committees

- Pierre Alliez was member of the paper committee of the Eurographics Symposium on Geometry Processing 2006, Pacific Graphics 2006, ACM Symposium on Solid and Physical Modeling 2006, and IEEE International Conference on Shape Modeling and Applications 2006.
- Jean-Daniel Boissonnat was member of the scientific committee of the International Conference on Curves and Surfaces, 2006, and of the paper committee of the Eurographics Symposium on Geometry Processing 2006.
- Frédéric Cazals was member of the paper committee of the Eurographics Symposium on Geometry Processing 2006, of the Symposium on Point Based Graphics 2006, and of the ACM Symposium on Solid and Physical Modeling 2006.
- Mariette Yvinec was member of the program committee of the European Symposium on Algorithms, ESA 2006.

9.1.3. Ph.D. thesis and HDR committees

- Pierre Alliez was reviewer for the Ph.D. thesis of Martin Marinov (RWTH Aachen).
- Jean-Daniel Boissonnat was a member of the Habilitation dissertations of H. Delingette, G. Malandain, F. Nielsen and X. Pennec (Université de Nice). He was a member of the Ph.D. thesis committee of G. Melquiond (ENS Lyon).
- Frédéric Cazals was reviewer for the Ph.D. thesis of Julie Bernauer (Univ. Orsay).
- Olivier Devillers was reviewer for the Ph.D. thesis committee of Arnaud Gelas (INSA Lyon), and was a member of the thesis committee of Luca Castelli Aleardi (Polytechnique).
- Mariette Yvinec was reviewer for the Ph.D. thesis committee of Rémi Allègre, Université de Lyon, December 2006.
- Sylvain Pion was a member of the Ph.D. thesis committee of Zilin Du, New York University, March 2006.

9.1.4. INRIA committees

- Pierre Alliez is member of the « Comité des cours et colloques » at INRIA Sophia-Antipolis.
- Agnès Bessière is member of the « Comité des utilisateurs des moyens informatiques des services de l'INRIA Sophia-Antipolis » (CUMIS)
- Agnès Bessière, Olivier Devillers and Pierre Alliez are members of the « comité de centre » at INRIA Sophia-Antipolis.
- Jean-Daniel Boissonnat is chairman of the INRIA Evaluation Board.
- Frédéric Cazals is member of the INRIA Scientific Steering Committee of INRIA (COST).
- Sylvain Pion is a member of the « Commission du Développement Logiciel » (CDL) at INRIA Sophia-Antipolis.
- Monique Teillaud is a member of the INRIA Evaluation Board and of the INRIA Sophia Antipolis Commission for hygiene and security.
- Mariette Yvinec is member of the « Comité des utilisateurs des moyens informatiques de recherche de l'INRIA Sophia-Antipolis » (CUMIR)

9.1.5. Other committees

- Jean-Daniel Boissonnat is member of the « Commission de spécialistes » of the Ecole Normale Supérieure de Paris.
- Frédéric Cazals is member of the « Commission de spécialistes » of the Mathematics Department of the University of Bourgogne, Dijon, France.
- Olivier Devillers is member of the «Conseil scientifique de l'école doctorale STIC» at Nice University.
- Sylvain Pion is member of the experts group of AFNOR for the C++ language.

9.1.6. WWW server

http://www-sop.inria.fr/geometrica/

The GEOMETRICA project maintains on its web site a collection of comprehensive sheets about the subjects presented in this report, as well as downloadable softwares. Web services feature a surface reconstruction server (see section 5.2.1), as well as a server dedicated to modeling protein-protein interfaces (see section 5.2.2).

9.2. Teaching

9.2.1. Teaching responsibilities

- In the «Master STIC» of Nice Sophia-Antipolis University Olivier Devillers chairs the second year research speciality : «Image et géométrie pour le multimédia et la modélisation du vivant»
- Olivier Devillers is professor «Chargé de cours» at École Polytechnique.

9.2.2. Teaching at universities

- MPRI (Master Parisien de Recherches Informatiques) (2006-2007), Cours de 2ième annee , Géométrie algorithmique : Diagrammes de Voronoï, Triangulations et Maillages, Jean-Daniel Boissonnat and Mariette Yvinec (24h).
- École Polytechnique (Palaiseau), Computational Geometry (2006-2007), 40h (O. Devillers).
- École Polytechnique (Palaiseau), Java programming (2006-2007), exam (O. Devillers).
- ISIA (Sophia-Antipolis), Computational Geometry (2006-2007), 10h (O. Devillers).
- Maîtrise Informatique (2005-2006) (Nice), Computational Geometry, 24h (O. Devillers 12h, A. Mebarki 12h).
- Master STIC-IGMMV-ISI (Sophia-Antipolis), Algorithmic frameworks for geometry (2006-2007), 15h (O. Devillers).
- Master STIC-IGMMV-ISI (Sophia-Antipolis), Implementing computational geometry (2005-2006), 20h (P. Alliez).
- Master STIC-IGMMV (Sophia-Antipolis), Surfaces and meshes (2005-2006), (F. Cazals - 9h, P. Alliez - 6h).
- Master IVR - Grenoble, Maillages et Surfaces, (F. Cazals - 6h; D. Attali (CNRS) - 6h).
- Master MIGS - Dijon, Maillages et surfaces (P. Alliez - 6h), Algorithmes en biologie structurale (F. Cazals - 6h).

9.2.3. Internships

Internship proposals can be seen on the web at http://www-sop.inria.fr/geometrica/positions/
- Lakulish Antani, *Mesh Sizing using Additively Weighted Voronoi Diagrams*, IIT Bombay.
- Chinmay Karande, *Partial geometric shape matching using product graphs* IIT Bombay.
- Aditya Parameswaran, *Robust flow complex implementation, and rounding for curved objects* IIT Bombay.
- Pedro Machado Manhães de Castro, *Improvements of the* CGAL *2d circular kernel and design of a 3D circular kernel*.
- Pooran Memari, *Reconstruction from cross-sections*, EPU Nice Sophia-Antipolis [84].
- Marc Scherfenberg, *Unstructured mesh generation for modelling human exposure to electromagnetic fields*.
- Ilya Suslov, *Benchmarking of arrangements of circular arcs*.
- Jane Tournois, *Maillages 2D optimisés* [86], EPU Nice Sophia-Antipolis.
- Dave Millman, *Improved predicates for the Apollonius diagram of* CGAL.

- Jihun Yu, *Redesign of the* CORE *library*.
- Chris Wu, *Quadrangle surface tiling*.

9.2.4. Ongoing Ph.D. theses

- Christophe Delage, *Non affine Voronoi diagrams*, ENS-Lyon.
- Sébastien Loriot, *Modélisation mathématique, calcul et classification de poches d'arrimage de médicament sur les protéines*, université de Bourgogne.
- Abdelkrim Mebarki, *Structures de données compactes pour la géométrie*, université de Nice-Sophia Antipolis.
- Pooran Memari, *Reconstruction from cross-sections*, université de Nice-Sophia Antipolis.
- Thanh-Trung Nguyen, *Geometric Optimization for the Conception of Telescopes*.
- Laurent Rineau, *Maillages tétraédriques*, Université de Paris VI.
- Marie Samozino, *Filtrage, simplification et représentation multirésolution d'objets géométriques reconstruits*, Université de Nice-Sophia Antipolis.
- Camille Wormser, *Maillages et diagrammes anisotropes*.

9.2.5. Ph.D. defenses

- Luca Castelli, *Compression et entropie d'objets pour la synthèse d'images* en cotutelle avec l'École Polytechnique. Defense date: 12/12/2006.

9.3. Participation to conferences, seminars, invitations

9.3.1. Invited talks

Members of the project have presented their published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars or summer schools.
- «Constructing affine and curved Voronoi diagrams», The World a Jigsaw: Tessellations in the Sciences, Lorentz Center, Leiden, J-D. Boissonnat.
- «CGAL, the Computational Geometry Algorithms Library», The World a Jigsaw: Tessellations in the Sciences, Lorentz Center, Leiden, P. Alliez.
- «Surface mesh generation by Delaunay refinement», university of Gronningen, J-D. Boissonnat.
- «La géométrie algorithmique : des triangles aux formes», Journées de l'Académie des sciences : la science en mouvement, Sophia-Antipolis, J-D. Boissonnat.
- «Non-linear computational geometry: an introduction», *Excursions in Algorithmics: A late festschrift for Franco P. Preparata*, Providence, USA, J-D. Boissonnat.
- «Bregman Voronoi diagrams», Stanford university, J-D. Boissonnat.
- «Compact representation of geometric data structures», *Mc Gill seminar*, Montreal, Canada, O. Devillers.
- «Succinct representations of triangulations and planar maps», *Excursions in Algorithmics: A late festschrift for Franco P. Preparata*, Providence, USA, O. Devillers.
- «Revisiting the description of Protein-Protein interfaces», INRA Jouy-en-Josas Seminar, MIA Dpt, F. Cazals.
- «Robustesse Numérique en Géométrie Algorithmique, et application pratique dans CGAL», *Journées Informatique et Géométrie*, Lyon, S. Pion.
- «Topological Persistence», Lorentz Center, Leiden University, Netherlands, March 2006. D. Cohen-Steiner.
- «Topological Persistence», Technische Universität Berlin, November 2006. D. Cohen-Steiner.
- «From triangles to curves», Monique Teillaud, Invited talk at the European Workshop on Computational Geometry, Delphi, March 2006. http://www-sop.inria.fr/geometrica/team/Monique.Teillaud/talks/EWCG.pdf
- «Quadrangle Surface Tiling», P. Alliez. Invited talk at the 11th International Workshop on Vision, Modeling and Visualization 2006, RWTH Aachen, Germany.

9.3.2. The Geometrica seminar

http://www-sop.inria.fr/geometrica/seminars/
The GEOMETRICA seminar featured presentations from the following visiting scientists:
Bernhard Kornberger, University of Graz (Austria)

Sylvain Lazard, Vegas project team (Loria)
Elias Tsigaridas, Projet Geometrica (INRIA Sophia-Antipolis).
Chantal Prevost, Université PMC (Paris).
Craig Gotsman, Technion (Haifa, Israel).
Stéphane Redon, Projet I3D (INRIA Rhône-Alpes).
Pascal Reuter, Projet IPARLA, (LaBri-INRIA).
Chinmay Karande, IIT Bombay.
Olivier Lichtarge, Baylor College of Medecine (Houston, Texas).
Mario Botsch, ETH Zurich.
Hervé Brönnimann, Polytechnic University Brooklyn (New York, USA).
Steve Oudot, Stanford University (USA).
Nico Kruithof, RUG (Groningen, Netherlands).
Antoine Vigneron, INRA (Jouy-en-Josas).
Joël Janin, CNRS-Orsay.
Marco Craizer, PUC (Rio de Janeiro, Brésil).
Marc Pouget, Projet Geometrica (INRIA Sophia-Antipolis).
Peter Giblin, Liverpool University (Royaume-Uni).
Stephane Birmanns, Health Science Center, Texas University (Houston, USA).
Jean Ponce, University of Illinois (Urbana-Champaign, USA).

9.3.3. Scientific visits

- S. Pion visited New York University for one week.
- S. Pion visited the Max Planck Institut in Saarbruecken for one week.
- J.-D. Boissonnat visited Stanford University for one week.
- O. Devillers visited Mc Gill University (Montreal) for one week.

GEOMETRICA has hosted the following scientists:

- Zilin Du, NYU, Janvier 2006.
- Marcos Craizer, PUC Rio de Janeiro, February 2006.
- Chandrajit Bajaj, the University of Texas at Austin.
- Craig Gotsman, Professor at Technion, October 2006.
- Sylvain Lazard, Vegas (Loria), one week in December 2006.
- Bernhard Kornberger, Graz, two weeks in December.

9.3.4. Distinctions

- J-D. Boissonnat has been nominated Chevalier de l'Ordre National du Mérite.
- J-D. Boissonnat received the «Grand Prix EADS des sciences de l'information et applications».
- P. Alliez was awarded the EUROGRAPHICS young researcher award.

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