



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*Project-Team Magique-3D*

*Modélisation avancée en Géophysique 3D*

*Futurs*

THEME NUM

*Activity*  
*R* *eport*

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# 1. Team

*Magique3D is a joint project of INRIA Futurs, LMA (Laboratoire de Mathématiques Appliquées– CNRS UMR 5142, Université de Pau et des Pays de l'Adour) and MIGP (Laboratoire de Modélisation et d'Imagerie en Géosciences de Pau– CNRS UMR 5212, Université de Pau et des Pays de l'Adour)*

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# 2. Overall Objectives

## 2.1. Overall Objectives

The MAGIQUE-3D project is presented jointly by two laboratories of University of Pau (Department of Applied Mathematics - LMA associated with CNRS, and Department of Modeling and Imaging in Geosciences - MIGP, also associated with CNRS). Gathering several researchers of different backgrounds in geophysics, physics, mathematics and scientific computing, the main purpose of this project is to establish a link between progress in high-resolution 3D scientific computing and various fields of geophysics. We wish on the one hand to develop sophisticated modeling tools (by integrating physical aspects of the phenomena under study) and to validate them in a rigorous way, and on the other hand to apply them to real cases of geophysical interest.

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A first strength of this project is its intrinsically multi-disciplinary character. Moreover, the topics studied can lead to applications in fields other than seismology or seismic studies in the oil industry, for instance medical tomography, non destructive testing of materials or underwater acoustics.

A second strength of this project is that it is strongly related to the regional and national industrial environment, in particular regarding how to go from theoretical studies of relatively complex media to real applications of the methods developed for real cases encountered in the field. Our main industrial partner is TOTAL, whose main research center is located in Pau. We develop strong collaborations with this petroleum company. The project could also establish links with the industrial valorization unit of University of Pau (ValUPPA), which would ensure close contacts between the researchers of the project and the PME/PMI (small and medium-size companies) of the region of Pau.

Since the MAGIQUE-3D team has been created (January 2005), its overall objectives have been reorganized to answer to numerous questions from its main industrial partner TOTAL. At the end of 2006, the project is composed of two main topics that structure the original parts of the research activities of the group. The first topic, entitled 'Depth Imaging', is related to modeling of seismic wave propagation in complex geological structures, taking into account underlying physical phenomena. The main part of our research program has been defined jointly by working groups composed of members of MAGIQUE-3D and of our main industrial partner TOTAL. The second topic, that could be given the general title 'Advanced modeling in wave propagation', is related to the realistic numerical simulation of complex three-dimensional geophysical phenomena and its comparison with real data recorded in the field. We also participate in different research programs that allow us to work on other aspects of scientific computing in the context of external collaborations.

## 3. Scientific Foundations

### 3.1. Modeling

The main activities of Magique-3D in modeling concerns the derivation and the analysis of models that are based on mathematical physics and are suggested by geophysical problems. In particular, Magique-3D considers equations of interest for the oil industry and focus on the development and the analysis of numerical models which are well-adapted to solve quickly and accurately problems set in very large or unbounded domains as it is generally the case in geophysics.

### 3.2. Depth Imaging

The research program of MAGIQUE-3D for depth imaging is divided into two topics that both deal with the solution of the wave equation in a complex 3D medium but that differ in terms of the methods that are used. The first topic is entitled "One-way and two-way models" and is based on numerical micro-local analysis. This is interesting because there are few researchers working on numerical methods based on micro-local analysis. From a practical point of view, such investigations are supposed to lead to the solution of approximate wave equations but they have the advantage of giving rise to fast solution methods which, if not accurate in general situation, can provide approximate solutions that can subsequently be used as data to initialize implicit schemes for solving full wave equations. The second topic is entitled "High-performance methods for solving wave equations". It is more classical and deals with the development of methods of resolution that are innovative in terms of their numerical implementation but that are often based on classical methods such as finite elements.

### 3.3. Wave propagation in porous media

The propagation of waves in porous media can be of interest in many applications. MAGIQUE-3D develops different axes of research on this subject. By using numerical methods like finite differences, finite elements or meshless techniques like the various boundary integral methods (Boundary Integral Method, Indirect Boundary Element Method, Meshless Galerkin Method...) we aim at solving the equations describing porous

media. In a first attempt we will solve these equations by performing improvements of absorbing boundary conditions by using different alternative formulations like the Perfect Matched Layer techniques and high-order paraxial methods. By using a Convolutional Perfectly Matched layer formulation similar to the one developed by Dimitri Komatitsch and Roland Martin (Geophysics 2006) to propagation of isotropic and anisotropic elastic case, we solve the PML absorbing boundary conditions with more than one auxiliary memory variable (instead of one in the isotropic and anisotropic cases and for each component of the velocity and stresses). The establishment of stability criteria for this specific PML procedure is currently under development with Abdelaâziz Ezziani, Roland Martin and Dimitri Komatitsch.

Abdelaâziz Ezziani did his PhD on numerical modeling of wave propagation in viscolastic and porous media based on a finite element method [26]. Dimitri Komatitsch is an expert on numerical modeling of elastic or viscoelastic seismic wave propagation, for instance using the three-dimensional spectral-element method, which is a finite-element method with an exactly diagonal mass matrix. In the context of a collaboration with Jeroen Tromp (Division of Geological and Planetary Sciences, California Institute of Technology (Caltech), USA), Dimitri Komatitsch and Abdelaâziz Ezziani have started to collaborate on writing a variational formulation of the poroelastic wave equation written in displacement that would remain explicit and therefore be suitable for the spectral-element method. In that context, in 2007 they will welcome Cristina Morency, who is a postdoctoral researcher at Caltech and who will spend several months in Pau to implement that variational formulation and fully test it.

### 3.4. Numerical methods

In the 1990s, numerical modeling techniques that made it possible to study large 3D problems started to emerge. However, many research groups continued to carry out two-dimensional calculations because of the computational cost of these methods in 3D, and of the difficulty of implementing them (one needs to run them on large multiprocessor machines, based on specific programming techniques such as multitasking). In the last thirty years, several methods have been used for the numerical calculation of synthetic seismograms in complex geological models, first in two dimensions, and more recently in three dimensions. The finite-difference technique [44] is the most popular, and was applied successfully to local or regional models [38], [45]. Another largely used technique is the pseudospectral method, which uses global bases of Chebyshev or Legendre polynomials [25]. However, in many cases of practical interest, these traditional methods suffer from limitations such as numerical dispersion or numerical anisotropy. We thus have in recent years developed a new technique, called the spectral element method, which had been introduced initially in fluid dynamics [41], and that we applied for the first time to the propagation of waves in 3D structures (see for example [35], [34]). This work showed the superiority of the spectral element method over more traditional numerical techniques in terms of precision, weak numerical dispersion, and geometrical flexibility making it possible to adapt it to large and complex 3D models [32], [33]. Such precision is a crucial advantage for the resolution of forward or inverse problems in seismology.

The spectral-element technique developed by Dimitri Komatitsch and his coworkers is based on a variational formulation of the wave equation, and combines the flexibility of a finite element method with the precision of a global pseudospectral method. The finite element grid is adapted to all major discontinuities of the geological model. In order to maintain a relatively constant grid resolution in the entire model in terms of the number of grid points per wavelength, and to reduce the computing time, the size of the elements is decreased with depth in a geometrically-conforming fashion, which allows us to preserve an exactly diagonal mass matrix in the method. The effects of attenuation and anisotropy are taken into account in the technique.

We applied this technique to a large number of real geophysical cases of practical interest, for example the study of strong ground motion and of the associated seismic risk in the densely-populated Los Angeles basin region. This area consists of a basin of great dimension (more than 100 km x 100 km) which is one of the deepest sedimentary basins in the world (the sedimentary layer has a maximum thickness of 8.5 km right underneath downtown Los Angeles), and therefore one of the most dangerous because of the resulting amplification of the seismic waves. In the case of a small recent earthquake in Hollywood (of magnitude MW = 4.2 on September 9, 2001), well recorded by more than 140 stations of the TriNet seismic network of Southern

California, we managed for the first time to fit the three components of the vector displacement, while most of the previous studies concentrated on the vertical component only [39], [46], [40], while still obtaining a good fit to the recorded data down to relatively short periods (2 seconds). This study clearly showed how useful sophisticated 3D numerical modeling techniques can be in such a context [31].

Topography also plays a significant role for the characteristics of the "ground roll", i.e., surface waves recorded by the oil industry in field acquisition experiments, and its effects are essentially three-dimensional. We will use the 3D spectral-element code (SPECFEM3D) developed by D. Komatitsch and his coworkers to generate synthetic data for various 2D and 3D configurations. Some recent articles show that the presence of heterogeneities in the subsurface not only contributes to attenuate the recorded signal but also to delay coherent events. It is also difficult, under these conditions, to distinguish between a distribution of heterogeneities and a mere stratification of layers. We will also carry out 3D simulations with complex topographies. Effects of amplification due to conversions of body waves into Rayleigh surface waves in regions of sharp topography have been observed in the field. Amplitude variations of the signal of an order of magnitude can appear along the recording antenna [43], [37]. Because of diffraction phenomena, complex arrivals are also observed. We will try to reproduce these effects with our 3D numerical simulations of seismic wave propagation.

A few years ago, Dimitri Komatitsch started to work on this topic of the numerical modeling of the effect of topography on seismic wave propagation with Jacques Muller (TOTAL) and Patrice Ricarte (IFP) within the framework of the "Foothills" project of TOTAL. At the time, calculations were carried out exclusively in 2D because of the computer resources available. A very significant problem for the oil industry is indeed to study models located in foothill basins, in which the topography and the presence of the weathered surface layer (Wz) plays a crucial role on the quality of the seismic data recorded. It is now necessary to generalize such calculations, and obviously nowadays to carry them out in 3D using the seismic modeling tools that we have developed. Roland Martin is currently in the process of using such techniques to apply it to another real case in South America in which the very thin weathered zone makes the simulation very difficult to perform at high-frequency if classical numerical simulation techniques such as staggered finite-difference methods are used.

## 4. Software

### 4.1. Software

The MAGIQUE-3D project is based (in part) on existing software packages, which are already validated, portable and robust. The SPECFEM3D software package, developed at the California Institute of Technology (USA) by Dimitri Komatitsch and Jeroen Tromp, and which is still actively maintained by Dimitri Komatitsch and his colleagues from University of Pau, allows the precise modeling of seismic wave propagation in complex three-dimensional geological models. Phenomena such as anisotropy, attenuation (i.e., anelasticity), fluid-solid interfaces, rotation, self-gravitation, as well as crustal and mantle models can be taken into account. The software is written in Fortran95 with MPI message-passing on parallel machines. It won the Gordon Bell Prize for best performance of the Supercomputing'2003 conference. In 2006, Dimitri Komatitsch has established a new collaboration with the Barcelona Supercomputing Center (Spain) to work on further optimizing the source code to prepare it for very large runs on future petaflops machines to solve either direct or inverse problems in seismology. Optimizations will focus on improving load balancing, reducing the number of cache misses and switching from blocking to non-blocking MPI communications to improve performance on very large systems. Because of its flexibility and portability, the code has been installed and run successfully on a large number of platforms (for example IBM Power, Linux PC, SGI, HP Compaq DEC, NEC, Earth Simulator, Sun, CRAY), and is used on a daily basis by more than 100 academic institutions throughout the world.



## 5. New Results

### 5.1. Maxwell system in strongly absorbing layers

**Participants:** H el ene Barucq, Mathieu Fontes, Dimitri Komatitsch.

We have develop a model for modeling the propagation of waves in absorbing media. The first study we realize concerns the long-time behavior of the electromagnetic field propagating into a bianisotropic medium. We have proven that there exists an energy which is the combination of the first and second-order electromagnetic energies which is exponentially decreasing if the field propagates into a strictly star-shaped domain bounded by a Silver-M uller boundary. This is a new result because we had to consider the case where both the electric and the magnetic field are not divergence free. The second study consists in extending the first one to the case of PML models. We began by writing a PML version of the model and we have proven that we have to replace the functions tensors by pseudodifferential operators which corresponds to the case of a bianisotropic medium with memory effects. We have then established the problem admits a single solution and we have perform a numerical analysis of the behavior of the electromagnetic energy. Our numerical investigations show that the energy is quickly decreasing.

### 5.2. Mixed Hybrid Methods for the Helmholtz Equation

**Participants:** Mohamed Amara, Angela Bernardini ., Rabia Djellouli.

The standard Finite Element Method (FEM) is based on continuous piecewise polynomials Galerkin approximation. This approach provides a quasi-optimal numerical method for elliptic boundary value problems in the sense that the accuracy of the numerical solution differs only by a constant  $C$  from the best approximation obtained from a finite element method. Then this property guarantees good performances of computations at any mesh resolution for the Laplace operator but it can not be preserved for other cases. For example, for the Helmholtz equation, the constant  $C$  increases with the wave number as a consequence of a lost of ellipticity as the wave number becomes large. This phenomenon is well-known as a pollution effect [22]. It is due to numerical dispersion errors and FEM are able to cope with large wave numbers only if the mesh resolution is also increased suitably. In order to avoid the pollution effect, numerous discretization techniques have been developed. They include the weak element method for the Helmholtz equation, the Galerkin/least-squares method, the quasi-stabilized finite element method, the partition of unity method, the residual-free bubbles for the Helmholtz equation, the ultra-weak variational method, the least squares method, and recently a discontinuous Galerkin method has been introduced by Farhat et al., [27], [28]. In this work, we present a new mixed hybrid method in the spirit of [23] for the solution of the Helmholtz equation in the high-frequency regime. Our approach is based on the local approximation of the solution by oscillating finite element polynomials using quadrilateral shaped elements. The shape functions have been chosen in such a way that they oscillate more and more as the wave number increases. In this way the oscillations of the Helmholtz solution are already included in the local approximation of the solution. But the continuity across each interior element interface is lost and it is weakly enforced by introducing suitable Lagrange multipliers which could penalize the computational burden of the method. In fact the discontinuous nature of the approximation enables us to use a static condensation of primal variables prior to assembly. Consequently, the cost of the new method is determined by the total number of Lagrange multipliers introduced at each edge of the finite element mesh, and by the sparsity pattern of the corresponding system matrix.

### 5.3. Dirichlet-to-Neumann vs Bayliss-Gunzburger-Turkel operators for the modelling of elliptical scatterers at low and mid frequency

**Participants:** H el ene Barucq, Rabia Djellouli, Anne Ga elle Saint-Guirons.

The solution of a scattering problem generally involves the coupling of the physical model with Absorbing Boundary Conditions (ABC). The efficiency of such conditions has a big impact on the accuracy of the numerical solution. Thus their construction is one of the main step of the numerical method handling. If the ABC design is generally based on high frequency assumptions, numerical investigations illustrate the fact that most of the ABC are valid at low frequency also. In [30], the effect of the wave number on the performance of local ABC has been done for the cases of the circle and the sphere. In this work we construct a new family of Dirichlet-to-Neumann (DtN) conditions for elliptical shaped scatterers which is quite general for applications in scalar acoustic problems. The conditions apply to a scalar acoustic problem governed by the Helmholtz equation. The DtN maps are designed to be local because we aim at keeping the sparsity of the discrete finite element matrix. Then we analyze their performance at low and mid frequencies by considering in a first time the radiator problem both for the two and the three-dimensional cases. This analysis allows us to select the best DtN condition and is based on a modal analysis involving Mathieu functions (2D) [21] or spheroidal wave functions (3D) [29]. In a second time, we do the same comparisons by considering the scattering problem which is solved by using the On-Surface-Radiation-Condition method [36]. We can also enhance the effect of the eccentricity on the performance of the conditions. Next, we complete the theoretical study by comparing the efficiency of the DtN operators with Bayliss-Gunzburger-Turkel (BGT) operators which were formerly studied by Reiner et al. [42]. The comparison is done for the radiator and the scattering problems. Our conclusions are the following. For a thin ellipse (the eccentricity is close to 1), the second-order DtN condition outperforms the second-order BGT one while both conditions tend to give rise to similar results for the sphere (the eccentricity is zero). The supremacy of the DtN condition is more and more obvious as the frequency increases to begin really significant at high frequency.

## 5.4. Optimization of the GSP for Depth Imaging

**Participants:** H el ene Barucq, Nicolas Le Goff, Roland Martin, Frank Prat.

We have considered the question of optimizing the GSP code by adopting different approaches, as follows. First of all we have speed up the computation of the wave field by changing its representation as a series in which we have used a suitable arrangement of the terms in such a way that at least the  $2j$ -th and the  $(2j + 1)$ -th terms are computed in the same time. Hence the time computing is reduced considerably since it is at least divided by two. Secondly, the structure of the code has been parallelized. This work was really necessary because the code, which was developed by MAGIQUE-3D has been next installed at TOTAL . For a run at 3D, the initial version required 7 Go of high memory which is prohibitive to go further into the solution of the inverse problem. The parallelization has been optimized by enforcing a better share of the works between each process while the first version consisted in sending the auxiliary computations to the main process which had to work a lot and penalized the computing performances. The new algorithm improves significantly the computing performances of the GSP code. For a velocity model consisting of  $400 \times 400 \times 200$  points, we have used 8 processes and we got the numerical results after 2 hours, by using 200 Mo of high memory. Using the first version of the code, the main process required 2 Go of high memory alone for a smaller velocity model of  $128 \times 128 \times 50$  points and it was not possible to consider a model of  $400 \times 400 \times 200$  points. At present time, some investigations are done to estimate the improvements we realize when comparing the performance of the GSP code with the SPECFEM code.

Very encouraging results are obtained between the two methods. For a 2D three horizontal layered and acoustic medium, the seismograms obtained with both methods show the same travel times and the reflections due to the several interfaces are quite the same. The parallel versions of both codes provide similar computational times for few processes while the GSP code is faster (from a few seconds to a few minutes) than the Specfem code (from few seconds to 1 hour) by one order of magnitude for 20, 50 and 100 processors. At 26Hz, and for a heterogeneous medium model provided by TOTAL, the same kind of mesh (18km by 11km = 15000 by 4000 points) is considered for both methods. The GSP code seems to last few minutes when Specfem2D lasts an hour. The main interfaces can be captured in both cases but Specfem2D is much more accurate than GSP. This seems to be due to the fact that the periodic boundary conditions are not tapered efficiently at the lateral boundaries (except the upper free surface) and that the lateral variations of the material properties are not adequately taken into account by GSP. However, It is extremely important to emphasize that GSP seems

to be a good candidate for seismic reflection, and a much faster procedure than Specfem2D to discriminate the main features of the subsurface geological structures. Both programs have been optimized in synchronous and asynchronous mode for High Performance Computing applications. Improved padding techniques will be added in the near future in order to reduce the spurious modes appearing in the seismograms produced by GSP and coming from the lateral periodic boundary conditions.

## 5.5. Space-time mesh refinement for wave propagation in aeroacoustics

**Participants:** Abdelaâziz Ezziani, Patrick Joly.

We are interested in space-time mesh refinement methods for wave propagation in aeroacoustics. We have developed a method which is applicable to zero order perturbations of symmetric hyperbolic systems in the sense of Friedrichs (Linearized Euler equations are of this type). The method is based on the one hand on the use of a conservative higher order discontinuous Galerkin approximation for space discretization and a finite difference scheme in time, on the other hand on appropriate discrete transmission conditions between the grids. We use a discrete energy technique to drive the construction of the matching procedure between the grids and guarantee the stability condition. Moreover, under suitable geometrical conditions on the grids, this method is fully explicit.

## 5.6. Convolution PML (C-PML)

**Participants:** Dimitri Komatitsch, Roland Martin.

The Perfectly Matched Layer absorbing boundary [24] condition has proven to be very efficient from a numerical point of view for the elastic wave equation to absorb both body waves with non-grazing incidence and surface waves. However, at grazing incidence the classical discrete Perfectly Matched Layer method suffers from large spurious reflections that make it inefficient for instance in the case of very thin mesh slices, in the case of sources located very close to the edge of the mesh, and/or in the case of receivers located at very large offset. In addition, most classical Perfectly Matched Layer models use a split version of the fields, which makes them more difficult to implement and more expensive to use numerically. We have demonstrated how to improve the Perfectly Matched Layer at grazing incidence for the differential anisotropic elastic wave equation based on an unsplit convolution technique. We have illustrated the efficiency of this improved Convolution Perfectly Matched Layer based on two-dimensional numerical benchmarks using a finite-difference method on very thin mesh slices for an isotropic material and show that results are significantly improved compared to the classical Perfectly Matched Layer technique. We have also shown that the technique is efficient in the case of anisotropic materials.

## 5.7. Scientific visualization, segmentation, surface approximation with faults and/or rapidly varying data in the Geosciences

**Participants:** Christian Gout, Dimitri Komatitsch, Anne-Gaëlle Saint-Guirons.

In many problems of geophysical interest, one has to deal with data that exhibit complex fault structures. This occurs for instance when describing the topography of seafloor surfaces, mountain ranges, volcanoes, islands, or the shape of geological entities, that can present large and rapid variations due for instance to the presence of faults in the structure. The usual approximation methods used to lead to instability phenomena or undesirable oscillations. The key point to get a good approximant consists in precisely define the locations of the large variations and the faults. To do that, we propose an automatic method that uses segmentation methods (developed by Gout and Le Guyader) to segment the Lagrange dataset in order to get several patches delimited by large gradient (or faults) of the dataset. Then, having the knowledge about the location of the discontinuities of the surface, we can generate a mesh (which takes into account the set of discontinuities) and a spline approximant is then computed.

The other part of this topic is the Fault modeling in 3D complex geophysical data. This kind of problems is of crucial interest in Geosciences as it constitutes an important exploration gap for reservoir characterization. Moreover, it is well known that interpretation of faults in seismic data is today a time consuming manual task... and reducing time from exploration to production of an oil field has great economical benefits. A first work has been done in 2006 (Gout and Le Guyader [6]) but specific geophysical visualization problems limit the 3D applications of these results. A collaboration (linked to this topic) with M.P. Cani (INRIA Rhône Alpes) has been rescheduled for 2007/2008.

The Ph. D. thesis (CIFRE at the University of Pau and TOTAL) of Guilhem Dupuy "Creation et manipulation de maillages de grandes tailles: applications aux Geosciences" under the direction of Dimitri Komatitsch and Bruno Jobard (Pau, Lab. d'Informatique) is also strongly linked to this theme.

## 6. Contracts and Grants with Industry

### 6.1. Contracts with TOTAL

- Modélisation et simulation numérique pour la migration terrestre par équations d'ondes tridimensionnelles.  
Period: 2006 October - 2009 september, Management: INRIA Futurs, Amount: 108000 euros.
- Résolution de l'équation d'Helmholtz 3D par une méthode de Galerkin discontinue DGM utilisant des bases d'ondes planes.  
Period: 2006 October - 2009 september, Management: INRIA Futurs, Amount: 108000 euros.
- Period: 2007 January - 2007 December, Management: INRIA Futurs, Amount: 43000 euros.
- Contrat annuel du Laboratoire de Modélisation et d'Imagerie en Géosciences (MIGP) CNRS UMR 5212 avec TOTAL.  
Period: January 1, 2006 - December 1, 2006, Management: CNRS, Amount: 170000 euros.
- Couplage de modèles thermomécaniques réservoirs-puits, Bourse CIFRE  
Period: 2005 October - 2008 september, Management: ADERA, Amount: 51000 euros.

### 6.2. Contract with CSUN

In the context of the Associate Team MAGIC.

Period: 2006 January - 2008 December, Total Amount: 46000 USD

## 7. Other Grants and Activities

### 7.1. Competence Network on Wave Equations

**Participants (outside the MAGIQUE-3D group):** Abderrhamane Bendali (INSA Toulouse and CERFACS), Hervé Chauris (École des Mines de Paris), Olivier Lafitte (Université Paris 13), Serge Gratton (CERFACS), Jérôme Le Rousseau (Université Aix-Marseille I), Jean Roman (Scalapplix, INRIA Futurs, Bordeaux)

MAGIQUE-3D maintains active collaborations with TOTAL. In the context of depth imaging and with the collaboration of Henri Calandra from TOTAL, MAGIQUE-3D coordinates research activities dealing with the development of high-performance numerical methods for solving wave equations in complex media. This project involves French academic researchers in mathematics and in geophysics, and is funded by TOTAL. The current partners of the network were selected during a kickoff meeting held in Pau in June 2006. The main areas of interest and research topics of the program are now defined and a draft proposal has been submitted in September 2006. In this context, two contracts have been initiated between TOTAL and INRIA in the context of the Ph.D of C. BALDASSARI and M. GRIGOROSCUTA. In the same time, a Ph.D is funded by TOTAL at Cerfacs with S. Gratton as advisor and the Ph.D student is supposed to work with A. BERNARDINI from MAGIQUE-3D during 2007.

The different partners plan to work jointly on topics dealing with solving wave equations. Funds from TOTAL will be used to organize working groups and support Ph.D. and post-doctoral positions. Moreover, considering the main areas of interest and research topics of the program, links with ANR could be established and could provide an additional source of funding. To our knowledge, our network is the first in the French research community to establish links between industrial and academic researchers in the context of a long-term research program managed by an INRIA team.

## 7.2. ANR Project NUMASIS

MAGIQUE-3D participates in an ANR research program called NUMASIS managed by J.F. Méhaut (INRIA Rhône-Alpes, Grenoble). In this context we naturally collaborate with SCALAPPLIX and RUNTIME from INRIA futurs (Bordeaux). The main idea of the NUMASIS project is that multiprocessor machines of tomorrow will possess a NUMA architecture introducing multiple levels of hierarchy in computers (multi-modules, multi-core chips, multithreading systems, etc). To use them efficiently, parallel applications must have powerful tools making it possible to guide the distribution of execution and data flows without compromising their portability. The NUMASIS project proposes to evaluate the functionalities provided by current systems, to apprehend their limitations, to design and implement new management mechanisms for processes, data and communications within the basic software (operating system, libraries). The application field selected for NUMASIS is seismology, which appears to us to be representative of current needs in scientific computing.

Numerical modeling of seismic wave propagation in complex geological media is one of the significant research topics in seismology. Various approaches will be studied and their adequacy compared to specificities of NUMA machines will be evaluated. The various calculations will be based on modern numerical algorithms such as spectral elements, high-order finite differences or finite elements applied to realistic 3D models. The NUMASIS project will study problems of parallel algorithms (distribution, scheduling) making it possible to optimize the calculations based on these schemes by using as efficiently as possible the execution frameworks developed for these NUMA architectures.

The NUMASIS project was submitted in 2005 and accepted by the Calcul Intensif et Grilles de Calcul program of ANR. The project officially started on January 1, 2006 under reference ANR-05-CIGC-002. It will end on December 31, 2008. In the context of this project, MAGIQUE-3D has obtained 84000 euros for three years, which will be used mostly to fund the salary of a software engineer (David Michéa, who has joined MAGIQUE-3D on November 1, 2006) for two years.

## 7.3. Collaborations with other INRIA projects

MAGIQUE-3D participates in an ANR research program called NUMASIS and managed by J.F. Méhaut (INRIA Rhône-Alpes, Grenoble). In this context we naturally collaborate with SCALAPPLIX and RUNTIME from INRIA futurs (Bordeaux). Moreover we can expect to develop links with POEMS from INRIA Rocquencourt after the arrival of two young researchers (J. DIAZ and A. EZZIANI) who prepared their Ph.D in this project.

## 7.4. International collaborations

### 7.4.1. Visits

- D. Komatitsch has spent 4 weeks last Spring as an Invited Professor at the Barcelona Supercomputing Center (Spain) in the context of the HPC-Europa program to work on high-performance simulations of seismic wave propagation following large earthquakes.
- R. Martin has spent 3 weeks last summer in Mexico (4 to 23 August) as a visitor. He worked with Dr. Carlos Ortiz-Aleman in the context of a collaboration with the Mexican Oil Institute (Instituto Mexicano del Petroleo) on Multiphase flow Tomography by global optimization method. A 3D finite volume code for elliptic problems has been produced and coupled to an inverse problem stochastic solver (accelerated simulated annealing/Gauss-Newton procedure). This allows us to perform a general parallel inverse problem solver that will be coupled to seismic and potential techniques and to attempt 3D subsurface imaging.

- C. Gout has spent one month (july) at the University of Hawai'i (Mano'a campus, Honolulu, USA) to work on surface approximation from rapidly varying Lagrange data and/or bathymetric data.
- H. Barucq has spent 2 weeks at California States University at Northridge (CSUN).
- A. Bernardini has spent 4 weeks at CSUN.
- A. G. Saint-Guirons has spent 4 weeks at CSUN.

#### **7.4.2. Associate team MAGIC**

Since January 2006, the team is associated to a team located at CSUN (California State University at Northridge) which is managed by R. Djellouli. Our common program research takes part of the activities we develop in modelisation essentially. Two PhD. students from MAGIQUE-3D are participating to these works and they have the opportunities to be current visitors at CSUN. We are funding both by the DREI from INRIA and CSUN and our collaborations will continue in this context next year, after a partial evaluation from INRIA. One of the next main events for the next year will be the organization of an international Workshop at Pau on the program research we develop for the Helmholtz equation.

## **8. Dissemination**

### **8.1. Scientific animation**

In the context of the network on the wave equation, MAGIQUE-3D organises a seminar at the University of Pau, in collaboration with TOTAL. Twice a month, we invite somebody to talk on non standard numerical methods for solving wave problems.

In the context of depth imaging activities, MAGIQUE-3D will co-organize a congress on harmonic analysis, signal theory and meshless methods in April 2007 (see <http://www.math.estia.fr/>). The participation of MAGIQUE-3D to the organization has been decided after H. BARUCQ was invited to give a plenary talk on numerical microlocal analysis applied to oil industry, on the occasion of this congress.

### **8.2. Teaching**

#### **8.2.1. Lecture**

- Lecture/course to Master students (39 hours) at University of Pau, France, on "Calcul Parallèle pour les Sciences de la Terre" ("Parallel computing in the Earth Sciences")
- Lecture/course to Master students (39 hours) at University of Pau, France, on "Modélisation en Géophysique" ("Geophysical modeling")
- Lecture/course to Master students (45 hours) at University of Pau, France, on "Problèmes de Scattering" ("Scattering Problems")

#### **8.2.2. Training Courses**

Invited lecture (3 hours) at the UNESCO training course at the ICTP Center in Trieste (Italy), "The spectral-element method and three-dimensional seismology" for the "Eighth Workshop on three-dimensional modeling of seismic wave generation, propagation and their inversion", September 2006.

### 8.3. Participation in conferences and other scientific events

- D. KOMATITSCH, invited talk at the UNESCO ICTP Center in Trieste (Italy), "The spectral-element method and three-dimensional seismology" for the "Eighth Workshop on three-dimensional modeling of seismic wave generation, propagation and their inversion", September 2006.
- D. KOMATITSCH, invited professor in the context of the High-Performance Computing (HPC)-Europa program at the Barcelona Supercomputing Center, Spain: 4 weeks (May 15 - June 9, 2006).
- R. MARTIN and D. KOMATITSCH. "An optimized Convolution-Perfectly Matched Layer (C-PML) absorbing technique for 3D seismic wave simulation based on a finite-difference method". EGU2006, General Assembly 2006 Vienna, Austria, 2-7 April 2006.
- C. GOUT, invited talk at the NSF-CNRS Optimal Synergy of Engineering and Life Sciences, Title of the talk: Segmentation in the Geosciences and Life sciences, Palais des Papes, Avignon, France, June 2006.
- C. GOUT, Fault reconstruction on 3D geophysical datasets, conference Curves and Surfaces V, July 2006.
- C. GOUT, Segmentation under constraints: a survey, conference Curves and Surfaces V, July 2006.
- C. GOUT, invited talk at the University of Troms (Norway) on surface approximation from rapidly varying data, December 2005.

### 8.4. PhD Thesis

#### 8.4.1. Achieved

- FONTES Mathieu, Title: *Propriétés mathématiques de modèles géophysiques pour l'absorption des ondes. Application aux conditions de bords absorbants*, Funding: Ministère de la Recherche, Beginning: 2002, October, End: 2006, July, Director: H. BARUCQ, Co-director: D. KOMATITSCH
- DUBOIS Loïc, Title: *Etude mécanique de la crise sismique sud-islandaise de juin 2000 par modélisation numérique tridimensionnelle : effets rhéologiques et géométriques*, Université Paul Sabatier, Toulouse, 2006, November, Director: D. KOMATITSCH, Co-director: K. FEIGL, Observatoire Midi-Pyrénées de Toulouse.
- KADA KLOUCHA Chakib, Title: *Modélisation numérique d'écoulements diphasiques incompressibles à surface libre*, Funding: Ministère des Affaires Etrangères-BAF - INRIA Rocquencourt, Beginning: 2002, October, End: 2006, November 30, Director: M. Amara, Co-director: F. Dabaghi (50%) CR INRIA

#### 8.4.2. In course

- BALDASSARI Caroline, Title: *Modélisation et simulation numérique pour la migration terrestre par équations d'ondes tridimensionnelles*, Funding: TOTAL group, Management: INRIA Futurs, Beginning: 2006, October, Director: H. BARUCQ.
- GRIGOROSCUA Magdalena, Title: *Simulation numérique 3D des équations d'Helmholtz*, Funding: TOTAL group, Management: INRIA Futurs, Beginning: 2006, October, Director: M. AMARA.
- MADEC Ronan, Funding: Beginning: 2006, October, Director: D. KOMATITSCH.
- SAINT-GUIRONS Anne-Gaëlle Title: *Développement et Analyse de conditions aux limites absorbantes pour des modèles géophysiques*, Beginning: 2005, October, Director: H. BARUCQ, Co-director: R. DJELLOULI.
- LIZAIK Layal, Title: *Couplage de modèles thermomécaniques d'écoulements polyphasiques réservoirs-puits*, Funding: Bourse CIFRE - Groupe TOTAL, Beginning: 2005, October, Director: M. Amara, Co-director: D. Capatina, MC Univ. Pau (50%)

- PETRAU Agnès, Title: *Couplage adaptatif de modèles multidimensionnels d'écoulements estuariens*, Funding: Collectivités Territoriales, Management: Université de Pau, Beginning: 2006, October, Director: M.Amara, Co-director: D. Trujillo, MC Univ. Pau (50%)

### 8.4.3. PhD reviews

- VETILLARD Jocelyne, Title: *Quelques méthodes d'analyse et de contrôle de problèmes aéroacoustiques*, CNAM Paris, Director: P. DESTUYNDER, Reviewer: M.AMARA, Date: 2006, december 8.
- ZERBIB Nicolas, Title: *Sous-structuration et décomposition de domaines pour la résolution des équations de Maxwell*, INSA Toulouse - CERFACS, Director: A. BENDALI, Reviewer: M.AMARA, Date: 2006, march 31.
- ZIMMERMAN Sébastien, Title: *Etude et implémentation de méthodes de volumes finis pour les fluides incompressibles*, Université Clermont Ferrand 2, Director: R. TOUZANI, Reviewer: M.AMARA, Date: 2006, february 13.
- MOUYSSSET Vincent, Title: *Une méthode de sous-domaines pour la résolution des équations de Maxwell instationnaires en présence d'un ensemble non-connexe d'objets diffractants*, Université Paul Sabatier Toulouse III, Director: P. MAZET, Reviewer: H. BARUCQ, Date: 2006, november.
- ISHIZAWA ESCUDERO Oscar Anil, Title: *Diffraction multiple des ondes en milieu aléatoire: application à l'étude de l'effet site-ville*, Ecole Centrale de Paris, Reviewer: D.KOMATITSCH, Date: 2006, may 11.

## 9. Bibliography

### Year Publications

#### Articles in refereed journals and book chapters

- [1] M. AMARA, C. BERNARDI, V. GIRAULT, F. HECHT. *Formulation fonction-courant et tourbillon du problème de Stokes dans un domaine bidimensionnel multiplement connexe*, in "Comptes Rendus de l'Académie des Sciences", vol. 342, n° 8, 2006, p. 617-622.
- [2] M. AMARA, D. CAPATINA, D. TRUJILLO. *ariational approach for the multiscale modeling of a river flow. Part 1: Derivation of hydrodynamical models*, in "SIAM Multiscale Modeling and Simulation", submitted, 2006.
- [3] M. AMARA, D. CAPATINA, D. TRUJILLO. *Stabilized finite element method for the Navier Stokes equations with non standard boundary conditions*, in "Mathematics of Computation", accepted, 2006.
- [4] M. AMARA, R. DJELLOULI, C. FARHAT. *Convergence analysis of a discontinuous Galerkin method with plane waves and Lagrange multipliers for the solution of Helmholtz problems*, in "SIAM Journal on Numerical Analysis", submitted, 2006.
- [5] H. BARUCQ, B. DUQUET, F. PRAT. *True-Amplitude one-way propagation in heterogeneous media*, in "J. of Scientific Computing", submitted, 2006.
- [6] H. BARUCQ, M. FONTES. *Well-posedness and exponential stability of Maxwell-like systems coupled with strongly absorbing layers*, in "J. Math. Pures Appl.", to appear, 2006.



- [7] H. BARUCQ, M. FONTES, D. KOMATITSCH. *Mathematical and numerical analysis of a perfectly matched layer Maxwell system involving pseudodifferential operators*, in "J. of Computational Physics", submitted, 2006.
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- [9] E. CHALJUB, D. KOMATITSCH, J. P. VILOTTE, Y. CAPDEVILLE, B. VALETTE, G. FESTA. *Spectral Element Analysis in Seismology*, in *Advances in Wave Propagation in Heterogeneous Media*, in "Advances in Geophysics", R.-S. WU, V. MAUPIN (editors)., Elsevier, to appear, 2006.
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- [14] D. KOMATITSCH, R. MARTIN. *An unsplit convolution Perfectly Matched Layer improved at grazing incidence for the three-dimensional differential anisotropic elastic wave equation*, in "Geophysics", submitted, 2006.
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