INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team mc2

Modeling, control and computations: applications to fluid mechanics and laser-plasma interaction.

Futurs

THEME NUM

Activity Report

2006

Table of contents

1. Team

Head of project-team
Thierry Colin [PR, Institut Universitaire de France, HdR]

Vice-head of project team
Angelo Iollo [PR, Université Bordeaux 1, HdR]

Permanent researchers
Héloïse Beaugendre [MC, Université Bordeaux 1]
Charles-Henri Bruneau [PR, Université Bordeaux 1, HdR]
Mathieu Colin [MC, Université Bordeaux 1]
Thierry Colin [PR, Institut Universitaire de France, HdR]
Cédric Galusinski [PR, Université Toulon, HdR]
Angelo Iollo [PR, Université Bordeaux 1, HdR]
Iraj Mortazavi [MC, Université Bordeaux 1]
Olivier Saut [Research Associate (CR)]

Associate members
Patrick Fischer [MC, Université Bordeaux 1]
Khodor Khadra [Visiting technical staff]
Guy Métivier [PR, Institut Universitaire de France, HdR]
Mazen Saad [PR, École Centrale Nantes, HdR]

Ph. D. students
Sylvain Bénito [funded by the french ministry of research]
Marcelo Buffoni [funded by a Lagrange fondation (Italy)]
Julien Dambrine [funded by INRIA and the Aquitaine district]
Delphine Deyperas [funded by Renault and the CNRS]
Federico Galizio [funded by the italian ministry of research]
Elodie Jammouillé [funded by the CEMAGREF]
Edoardo Lombardi [funded by the french ministry of research]
Sandra Tancogne [funded by the Aquitaine district]
Haysam Telib [funded by the italian ministry of research]
Paul Vigneaux [funded by the french ministry of research]
Jessie Weller [funded by the french ministry of research]

Post-doctoral fellows
Michel Bergman

Technical Staff
Frederic Chantalat [Engineer funded by the ANR.]

2. Overall Objectives

2.1. Overall Objectives

The aim of this project is to develop numerical codes in order to explain, control, simulate and predict some phenomena coming from physics, chemistry or engineering. These codes will be constructed around modeling tools that are developed in the project.

2.1.1. Modeling tools, control and applications

2.1.1.1. Modeling

In our work, modeling is:

- *Phenomenological*: use of ad-hoc models in order to represent some precise phenomena. In nonlinear optics, one uses non linear modeling in order to describe interaction between laser and matter like for example the quadratic nonlinearity in order to describe frequency doubling. The construction of nonlinear differential laws for the stress within the context of visco-elastic fluids follows the same lines: one first construct the linear response of the medium (in terms of polarization) or of the fluid (in terms of the stress) to an oscillatory source term; then one adds nonlinear terms in order to model the nonlinear response of the medium (frequency doubling, self-focusing...) or of the fluid (normal forces). Input/output models in control theory are useful to describe the result of a process without describing it precisely. The wall conditions in micro-fluidics (fluids in micro-channels) are often taken heuristically in order to model the slip at the boundary. It has a fundamental importance since the use of several kinds of surface can drastically influence the mixing properties, the formations of droplets, etc...This will determine the boundary conditions that have to be used in the numerical codes.
- *Asymptotical*: using asymptotic expansions, we derive simpler models containing all the pertinent phenomena. Examples of such a process are the derivation of the nonlinear Schrödinger equation in nonlinear optics or for water-waves and internal waves; the boundary layer equations in fluid mechanics; the penalization method for the simulation of incompressible flows with obstacles or the analysis of ribblets in micro-fluidics that are used to control the mixing of the fluids. Another example is the use of shallow fluid model in order to have fast prediction (Hele-Shaw approximation in micro-fluidics, Boussinesq type model for water waves or internal waves).
- *Numerical*: direct numerical tools are used to represent the physical phenomena. A typical example is the POD (proper orthogonal decomposition) and its use in control theory and to obtain quick simulations.

2.1.1.2. Analysis, optimization and control

- *Analysis*: Modern experiment techniques and direct numerical simulation give large amounts of data in the form of two and three dimensional vector fields. Analysis and objective interpretation of such fields constitute a complicated problem. The velocity vector fields can rarely be interpreted directly. This procedure is prone to arbitrary numerical manipulations which may affect the conclusions which one resumes from analysis. A typical example is the determination of scaling factors in turbulence: inappropriate windowing effects may give virtual scaling factors. We develop tools based on solid mathematical backgrounds to interpret fluid flows on more rigorous grounds.
- *Shape optimization*: In theory, shape optimization in ground vehicle aerodynamics does not present any difference compared to optimization in aeronautics, a field in which the adjoint method is used as an automatic tool of design in industrial realities like Dassault and Boeing. In practice on the other hand, an important difference remains in the level of approximation used to determine the aerodynamic coefficients. Indeed, although these are strongly non stationary detached flows, in car industry one is often satisfied with a result obtained by a solution of the stationary averages Navier-Stokes equations. Moreover, the industries of the automobile field use mainly commercial codes and they do not have access to the sources. For all these reasons there is obviously an industrial interest to develop powerful numerical methods adapted to the solution of the continuous adjoint equations of the stationary averaged Navier-Stokes equations.
- *Control and real time computations*: The use of feedback to control flows of the industrial type is accompanied by two strong, crippling constraints from a point of view of practical applications. On the one hand, in order for the control to have a sense, it is necessary that the energy balance of the operation being positive. In other words, energy spent to control the flow must be lower

than the discounted profit. In addition, the controller used in the loop of feedback must be able to react in real time with information coming continuously from the sensors to adapt the signal to be sent to the actuators. Various control methods exist to this end. However, the extension of these techniques to more realistic problems proves to be delicate. In particular, when the dimension of the problem increases (to fix the orders of magnitude of a realistic problem, Spalart and Al, 1997 estimate that for an aerofoil of plane in flying conditions, i.e., for $Re = O(10^7)$, it is necessary to employ approximately 10^{11} points and to integrate the equations on approximately 10^6 time steps) these approaches may become inapplicable. Other methods must thus be considered. Within the framework of this research program, we propose to couple small-scale models to reduced order models built starting from existing off-line computations and data-bases.

2.1.1.3. Applications

Our methods are applied to three areas.

1. *Fluid dynamics*: Turbulence, drag reduction, stress reduction, form optimization. The challenges are :
   - Find scale factor for turbulent flows cascades in realistic configurations,
   - The control of the drag of a vehicle in order to decrease the fuel consummation,
   - The control of the stress for a pipe line or an off-shore platform.
2. *Interface problems and complex fluids*: micro-fluidics, bifluid flows, miscible fluids, tumor propagations, complex fluids, environmental problems (coastal flows, porous medium). The challenges are:
   - To obtain reliable and simple models that can be used by our partner Rhodia (for micro-fluidics).
   - The obtaining of tumor growth models including some mechanics.
3. *Nuclear fusion*: laser-matter interaction (with a plasma, a crystal). The challenge is mainly to be able to couple several complex phenomena (Raman, Doppler, Landau, ...).

The final objective of our project is to obtain reliable numerical simulations of various phenomena that can be used by our collaborators from other laboratories or from the industrial community. Our main partners on this project will be Renault, IFP, CIRA (Centro italieno ricerche aerospatiali), CPMOH (Laboratory of Physics, Bordeaux 1 University) for 1., the CEA (french nuclear agency) and the laboratory CELIA (joint laboratory between Bordeaux 1 university, CEA and CNRS) for 3., the LOF (Laboratory Of the Future, joint lab between Rhodia (biggest french company of chemistry) and the CNRS, located near the campus of Bordeaux) and the medical school of Lyon for 2..

2.1.2. Production of numerical codes

This is the main goal of our project. We want to develop numerical codes in order to simulate the models that ere developed with our partners. Two kinds of numerical simulation are concerned.

2.1.2.1. Fluid mechanics

1. *2D and 3D simulations at high Reynolds number.* We develop direct simulation methods: multi-grid techniques, vortex methods, Detached Eddy Simulation (DES). The applications that we have in mind are the turbulence, the flow around a vehicle, the stress on a pipe-line. The penalization method is used in order to take into account the obstacles.
2. *Multi-fluid flows and interface problems:* We perform 2D and 3D simulations of multi-fluid flows using level set methods and mixing models. We describe propagation of tumors and interactions tumor-membranes in the same framework. The applications are micro-fluidics, propagation of tumor, porous media and complex fluids.

The way we will organize our work from the technical point of view is the following one. We will build a platform (called eLYSe) using only cartesian, regular meshes. This choice comes from the fact that we want to address interface problems using level set methods. For these interface problems, we will have to compute the curvature of the interface with high precision (in micro-fluidics, the surface tension is the leading order phenomena and small errors on the computation of the curvature give rise to instabilities). The level set technology is now very accurate on structured mesh, we therefore made this choice. However, we want to address cases with complex geometry and/or obstacles. We will therefore use systematically penalization methods. The idea is to have a uniform format on all the team that consists in several boxes:

1. Definition of the geometry and of the penalization zones.
2. Specification of the model in itself (bifluid or not, Newtonian or not, mixing or not, presence of membranes etc...).
3. the boundary conditions that have to be imposed by a penalization operator.
4. The solvers.
5. Graphic interface.

As said before, the interface problems and the interaction with a membrane will be handled by level set methods as well as the shape optimization problem. So this platform can be used for direct numerical simulation as well as for shape optimization and control.

The main effort concerning modeling will concern point 2) and 3) (model and boundary conditions). We do not plan for the moment to make special research effort on the solver part and we will use the solver available in the literature.

This platform will have two roles: the first one will be to allow a comprehensive treatment for the computation of complex fluids with interface, membranes, adapted to the world of physical-chemistry and micro-fluidics and for solving shape optimization problems. The second role will be to keep a set of numerical modules that will be devoted to more specific applications (for example multi-grid methods or vortex methods for the study of turbulence). We therefore need to have some unified standards for the geometry, for the graphic interface, within the team, but it is of course hopeless to consider 3-D turbulence and low-Reynolds flows in a micro-channel with the same code!

2.1.2.2. Dispersive equations

We want to achieve the same goal in the context of dispersive equations. We will use finite difference methods as well as spectral methods for extensions of Schrödinger, Zakharov, KdV and Boussinesq equations. The heart of the methods are often order 2, centered scheme coupled with relaxation methods in order to take into account the conserved quantities of the models. Again, we want to develop standards within the team in order to apply our computations to both laser-plasma interaction and water-waves phenomena. One of the main problem is to couple models made of equations sometime using the time t as evolution variable and sometime using the direction of propagation z as evolution variable.

3. Software

3.1. Interface problems

- Simulation of 2-D diphasic, non newtonian fluids by a Cahn-Hilliard type approach. Finite differences. Cartesian mesh. (collaboration with L. Chupin).
- A two-dimensional microfluidic flow code, developed using the simplified models studied in Bordeaux (collaboration with L. Chupin).
- Other codes, 2-D and 3-D codes for bifluids flows in micro-channels are being developed.
- MicroixFlow: 2-D microfluidic flow code in a cross-type geometry.

- M3D: 3-D microfluidic flow code in a "T"-type geometry.
- CaT-Cell-2D: 2D code for modeling avascular tumor growth and accounting for the cell cycle and mechanical effects due to the extra-cellular matrix.
- CaT-Cell-3D: 3D code for modeling avascular tumor growth and accounting for the cell cycle and mechanical effects due to the extra-cellular matrix.

3.2. Laser-plasma interaction

- Two-level Schrödinger-Bloch (3-D). Fourier method in transverse direction. Finite differences in the longitudinal variable. Moving window. (collaboration with B. Nkonga).
- One dimensional Maxwell-Lorentz for non-centrosymmetric crystals ($\chi 2$ nonlinearity). (collaboration with B. Nkonga).
- One dimensional Landau-Damping: coupling of a Zakharov system in physical space with a diffusion equation in spectral space.
- Two dimensional Raman amplification. Coupling between 4 nonlinear Schrödinger equations and a wave equation.
- MB-NLc: Two dimensional code for ultrashort pulses propagation in nonlinear crystals based on Maxwell-Bloch equations.

3.3. Flow simulation, analysis and control

3.3.1. Flow simulation and analysis.

Two main technologies related to the simulation of incompressible flows exist in the team: DNS methods and Vortex methods. In DNS, two multigrid codes are available:

- NSMulti2D: 2D computational code for solving incompressible Navier-Stokes equations on cartesian meshes in various domains with penalization of obstacles. Boundary conditions such as no-slip, periodical, Dirichlet, artificial on open frontiers on any parts of the boundary are available.
- NSMulti3D: Same as previous one in 3D with more restrictive boundary conditions.

In vortex methods: An entirely grid-free and lagrangian 2D code is using vortex blobs to discretize the vorticity field. An axisymmetric version of this program is also available. Another 2D vortex code is using a PIC (Particle-In-Cell) method that permits to decrease the grid diffusion tracking particles with the local velocity field (collaboration with A. Giovannini). Also, a collaboration with G.-H. Cottet related to his 3D vortex code is undertaken.

Finally, some flow analysis and diagnostics tools are developed.

- NSAnal: Analysis of 1D signals and 2D pictures of turbulent flows by means of Scilab core.
- Another software devoted to diagnostics of coherent structures is also available.

3.3.2. Porous media

- Richards: 2D FE unstructured computational code for fluid simulations in porous media variably saturated.

3.3.3. Low order models, shape optimization and control

Complex demonstrators for most of the studies undertaken have been developed. Several adjoint codes are available ranging from potential to 2D Euler and 2D compressible Navier-Stokes equations. In collaboration with Dr. D. Quagliarella of CIRA a multiblock adjoint code for RANS models was developed as well as a symbolic manipulation environment to automatically derive the symbolic continuous adjoint of given governing equations (see VKI Ed.Ser. 2003). Such codes are in use at CIRA. For turbo-machinery adjoint inverse design, a meridional plane through flow is currently developed for aeroacustic noise reduction, in collaboration with Michel Roger of the Ecole Centrale de Lyon.

4. New Results

4.1. Interface problems

Participants: Thierry Colin, Cédric Galusinski, Olivier Saut, Sandra Tancogne, Paul Vigneaux.

4.1.1. Microfluidics

We consider models for the simulation of curvature-driven incompressible bifluid flows, where the surface tension term is discretized explicitly. From this formulation a numerical stability condition arises. Because the stability condition induced by surface tension, proposed by Brackbill *et al.* (1992) for Navier-Stokes equation, involves the density but not the viscosity : this condition is only valid in certain flow regimes (high Reynolds numbers). It is therefore necessary to derive another stability condition, for both low and medium Reynolds numbers. In the following, we propose a new theoretical estimation of this stability condition induced by surface tension for Navier-Stokes equation which is also the stability condition for Stokes equation. This stability condition is induced by the explicit discretization of the surface tension term and avoids oscillatory behavior of the interface around an asymptotic shape of interface. These oscillations are related to what is known in the literature as *parasitic currents*. We illustrate our analysis with numerical simulations of microfluidic flows using Level Set method. In addition, we propose a method to reduce computational cost induced by this stability condition for low flow velocities. This is described in [44] and we presented in [36], [37], [35].

4.1.2. Tumor growth modeling

We have introduced a multi-scale model for avascular tumor growth. This age-structured model accounts for the cell cycle (the set of steps a cell has to undergo before its division).

This model also takes into account for the mechanical effects due to an extra-cellular membrane surrounding the tumor.

In a first paper [28], the macroscopic continuous model was based on Darcy's law which seems to be a good approximation to describe the flow of the tumor cells in the extra-cellular matrix [45], [50], [51]. However, with Darcy's law one can not easily add mechanical effects as an elastic force due to a membrane.

For this matter, we have used a level-set formulation to localize and compute the elastic force due to the membrane [16]. The velocity of the movement due to the cellular division is computed through a Stokes equations. Cell densities moves at the velocity of this movement and progress in the cell cycle. At one point of this cell cycle, the environmental conditions (concentration of oxygen...) are checked and the division only occurs if the environment is favorable. Otherwise, cells enter a quiescent state in which they wait for the environment to become favorable again.

As an example, we show in Fig. 1, the final tumor size in an experiment with variable elastic force. The shape of the tumor is noticeably sensible to the elasticity of the surrounding membrane.

This model and its discretization are described in a forthcoming paper [41].

4.2. Laser-plasma and laser-matter interaction

Participants: Mathieu Colin, Thierry Colin, Cédric Galusinski, Olivier Saut.

4.2.1. Ultrashort pulses propagation in nonlinear crystals

We already have studied 2 and 3 level Maxwell-Bloch systems in order to describe propagations in gas [48], [49]. In [47], we have presented a Maxwell-Bloch model adequate to describe laser-matter interaction of ultrashort pulses in nonlinear crystal. Initially this model was discretized using naive extensions of Yee's scheme for the Maxwell equations [54] in one and two dimensions of space [53], [52].

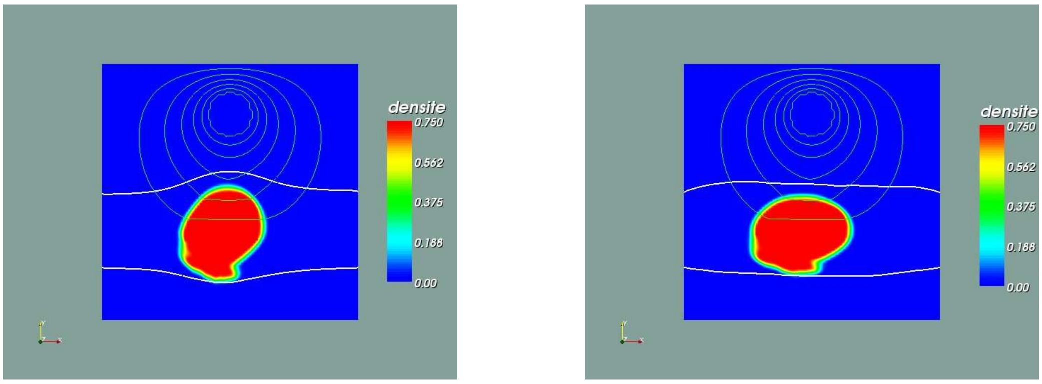

Figure 1. Tumoral growth in a channel modeling a galactophorous duct. The green isolines represent the concentration of oxygen (that the cells need in order to divide). Left: without elastic tension. Right: with elastic tension.

To overcome the computational burden, we have developed two new schemes [14] which are much more efficient to study our model. The first scheme uses a pseudo-spectral method. The time derivatives are obtained by finite differences, but the spatial derivatives are obtained by mean of a Fourier transform. This scheme is much simpler than the previous one as the use of staggered grids in space for the electric and magnetic fields are no longer mandatory to ensure a second-order scheme. Because of the wraparound effect of the discrete Fourier transform, absorbing layers have to be added to the computational domain. The last scheme presented is a FDTD scheme, which should address the main drawback of our first scheme [52]. The three components of the nonlinear polarization are now computed at the same points in space. The nonlinear polarization term is not explicitly involved in the Maxwell equations.

Thanks to these schemes, we are now able to study propagation over larger distance than before and to observe for instance auto-focusing of a laser beam of high intensity with our microscopic models.

4.2.2. Laser-plasma interaction

Concerning plasma physics, we have introduced several systems involving coupling of waves [22], [46]. For each system, we introduce a adapted numerical scheme in order to describe the main feature of the physical context. In [22], we have implemented a scheme that is able to take into account the Raman amplification in a plasma. We have also developed a wave version of the Zakharov system introduced in [30] by replacing the Schrödinger equations by Klein Gordon equations. We have obtained a convergence theorem and we have done some numerical simulations which justify in a certain sense the time envelope approximation. This work will be submitted soon.

In [46], the Landau damping phenomena has been described. The next step is to couple these two models. All of these works are done in collaboration with G. Gallice of CEA CESTA.

4.3. Numerical modeling, flow analysis, optimization and control

Participants: Héloïse Beaugendre, Charles-Henri Bruneau, Mathieu Colin, Angelo Iollo, Iraj Mortazavi.

4.3.1. Low order models

A proper orthogonal decomposition (POD) is used to construct low order models to solve incompressible fluids subject to Navier-Stokes equations [23]. A new strategy is under development to enrich the POD basis in order to solve more general problems in real time with the help of Michel Bergmann (INRIA post-doc).

4.3.2. Optimization

4.3.2.1. Aerodynamic optimization

The aerodynamic optimization problem of turbo-machinery blades is addressed by means of an inverse problem and its adjoint equations. The blade geometry is simplified by considering it a thickless flow surface. The shape of the blade leading edge and the load distribution over the blades are the design variables. The flow is modeled by the compressible Euler equations in an axisymmetric framework. A set of discrete adjoint equations are employed to determine the sensitivities of the blade aerodynamic characteristics with respect to the above mentioned design parameters. Finally, the sensitivities are fed to a gradient based algorithm to optimize performance. The result is shown in Fig. 2.

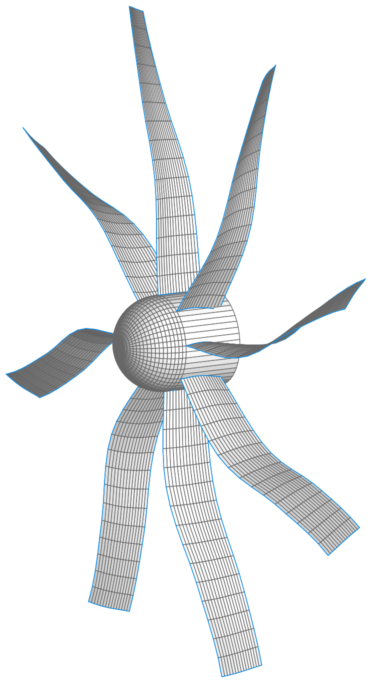

Figure 2. Optimizing a blade geometry.

4.3.2.2. Inverse problems

As a preamble for drag-reducing studies on Stokes' flows, inverse obstacle problems governed by Laplace and Poisson equations have been considered. Dirichlet boundary conditions have been applied on both the top/bottom edges of the computational domain and the geometry interface, whereas Neumann boundary conditions have been used on the remaining edges. Shape optimization is handled by finite differences discretization of the continuous adjoint equations on a cartesian mesh. A level-set function φ, sign-changing across the interface, enables the localization of the body by providing the distance of each node from its border.

At each step, after determination of the shape-derivative, and alteration of the boundary, Fast-Marching methods operate a redistanciation process. Both direct and adjoint equations are solved by the GMRES algorithm thanks to a penalization method. However, an iterative procedure aiming at finding a consistent extension of the solution inside the penalized zone is added so that the gradient evaluation along the normal, often inaccurate due to the use of this type of mesh can be significantly improved. Even though Level-Set

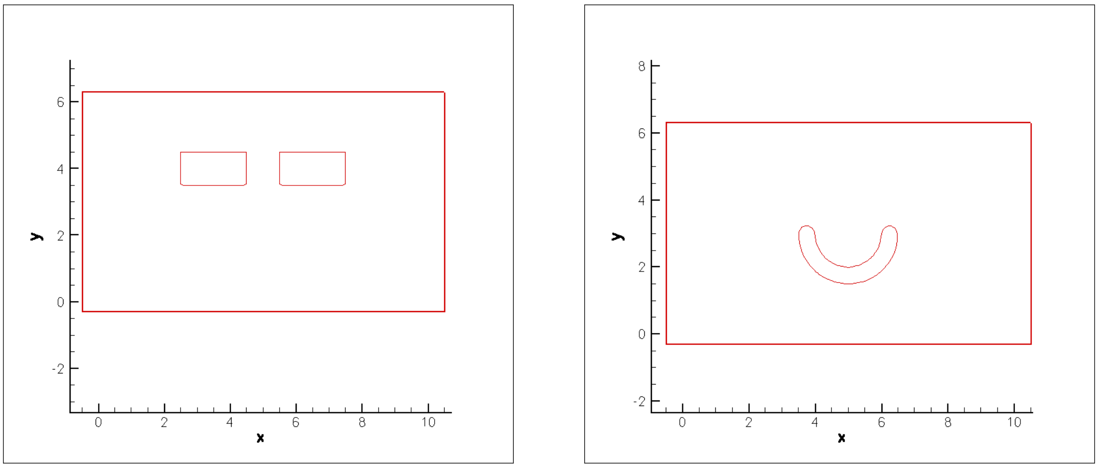

Figure 3. Inverse problems and level-set methods. Known the potential and the electric field, find the object which is embedded in the physical domain. Left initial guess. Right solution.

methods prove to be quite efficient in terms of dealing with topological changes, such inverse problems appear to be severely ill-posed, causing gradient-based methods to fall into local minima regions. Thus, a study of the choice of the objective function has been carried out showing the relevance of a multi-scale approach where large, medium, and small contributions follow one another along iterations in W-shaped cycles. Another way to make the problem more sensitive to shape variations was inspired by scanning techniques used in medical imagery. The angle from which the obstacle is observed is modified by rotating the obstacle during the optimization algorithm thanks to a fifth-order WENO scheme for transport equation. This method also has a drastic influence on speeding up convergence.

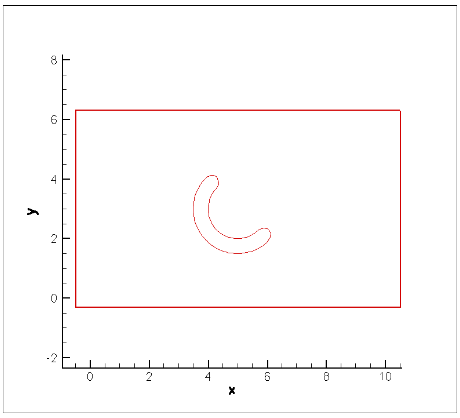

Figure 4. Inverse problems and level-set methods. Known the potential and the electric field, find the object which is embedded in the physical domain. Result after 900 optimization steps.

In July 2006 Guillaume Gancel defended his PhD on the numerical simulation and sensitivity analysis of an inverse problem on water distribution systems (advisers: P. Fabrie and I. Mortazavi) in collaboration with Cemagref-Bordeaux and a new thesis, with the same advisers, is started in november 2006, to advance this work towards new sensitivity and control analysis.

4.3.3. Control

The research on the active and passive flow control strategies was the cornerstone of 2006 work.

4.3.3.1. Passive control of bluff bodies flow

A direct numerical simulation of flows in a lid-driven cavity and in a channel behind arrays of cylinders is performed [18], [31]. A filtering is achieved by means of wavelet packets and possible biases are analyzed. The coexistence of both direct enstrophy cascade and inverse energy cascade are studied computing energy and enstrophy spectra and fluxes. New applications of porous devices to different simplified car geometries were performed. Important progress in the decrease of the aerodynamical drag was achieved (collaboration with Patrick Gillieron (Renault)). A BDI PhD thesis in order to develop 3D control methods for simplified car geometries is started in October 2006 in collaboration with Renault Company (advisers: CH Bruneau et Iraj Mortazavi). We are also members of a GDR (Groupement De Recherche) of CNRS: "Controle de Decollements".

4.3.3.2. Active flow control

In active flow control a Vortex-In-Cell code was modified in order to implement several closed-loop and open-loop control techniques in a backward-facing step channel geometry. Promising results were achieved for transitional flows (collaboration with Emmanuel Creuse (projet SIMPAF INRIA Futurs) and Andre Giovannini (IMF Toulouse)).

4.3.4. Flow analysis

A method is proposed to estimate the velocity field of an unsteady flow using a limited number of flow measurements. The method is based on a non-linear low dimensional model of the flow and on expanding the velocity field in terms of empirical basis functions. Applications may range from feedback low control to monitoring of the fl ow in nonaccessible regions. The method that we devised exploits an idea which is similar to that at the basis of the Kalman filter. A sample result is shown on Fig. 5.

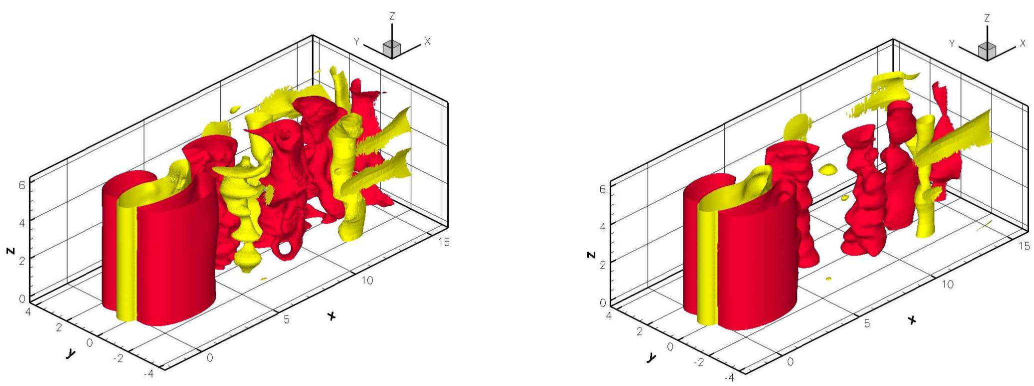

Figure 5. Flow-field estimates from a limited number of measurements on the boundary. Dynamic estimation, i.e., boundary measurements and low-order model. Left: isolines of the U-component field. Right: reconstruction from shear stress sensors on the wall.

4.3.4.1. Porous Media

The GDR MOMAS focuses its work on the numerical modeling of the nuclear waste management problems. In collaboration with Serge Huberson (LEA Poitiers) we propose to solve this problem by coupling a finite-element method to solve the variably saturated subsurface flows, Richards equation, with a particle method for the transport equation. In [11] the Richards equation is discretized by continuous finite elements on unstructured meshes. An obstacle-type formulation is used to determine where saturation conditions, and thus seepage face conditions, are met at the ground surface. To be able to answer the following question: "how much is a conservative flow field needed for low order pollutant transport simulation?" we present in [29] a particular non-standard nonconformal mixed finite-element approximations of Richards equation. The obtained scheme is a cell-centered scheme for the pressure p and the velocity u ensuring a local conservation property at the level of the mesh and is often called finite volume box scheme. Simulations are done using both conformal and nonconformal finite elements methods coupled with the particle method for the transport equation. Also the particle methods are used to analyze the uncertainties comparing several random and probabilistic approaches with a fine parametric study on the constant or variable parameters.

4.3.4.2. Modeling 3D complex flow

Icing results in performance degradations including substantial reduction of engine performance and stability, reduction in maximum lift and stall angle and an increase of drag. [12] proposes an efficient numerical method for calculating ice shapes on simple or complex 3D geometries. The resulting ice simulation system, FENSAP-ICE, is built in a modular fashion to successively solve each flow, impingement and accretion via field models based on partial differential equations (PDEs). One of the most important challenges in understanding the performance degradation is the accurate prediction of complex and massively separated turbulent flows. Spalart et al. proposed DES as a cost-effective and plausibly accurate approach for predicting flows experiencing massive separation. The DES model is being developed in the code FluidBox from the project-team INRIA Futurs ScAlApplix. This model is being developed in a new module called NSdesSA. Two classes of schemes have been implemented: classical finite volume schemes and the more recent residual distribution schemes. Several low Mach preconditioning techniques are also implemented. The code has been parallelized with and without overlap of the domains.

4.3.4.3. Simulation and analysis of 2D and 3D vortical flows

A direct numerical simulation of flows in a lid-driven cavity and in a channel behind arrays of cylinders is performed. A filtering is achieved by means of wavelet packets and possible biases are analysed. The coexistence of both direct enstrophy cascade and inverse energy cascade are studied computing energy and enstrophy spectra and fluxes [19], [17], [30].

Work was also performed on the determination of the unknown viscosity of a fluid with respect to the location of the interface between two fluids in a micro channel [26].

Finally, I. Mortavizi collaborate also with the Aeronautics Department of the Imperial College of London on the geometrical identification of Coherent Structures in three-dimensional flows and the accuracy of such approaches (Collaborators: Denis Doorly and Spencer Sherwin).

5. Contracts and Grants with Industry

5.1. Microfluidics

Rhodia already funded a final project of a student of Matmeca. In 2004 a multidisciplinary financial support of CNRS in microfluidics was offered to T. Colin. Moreover, we were awarded for a three year period (2004-2006) by the French ministry of research to develop the microfluidic research (“ACI microfluidique”).

We are partners of an ANR project SCAN in the Nanotechnology program of ANR. Others partners are Rhodia, Saint-Gobain and the Ecole Supérieure de Physique-Chimie industrielles de Paris. The leader of the project is Rhodia. The goal of the project is the construction of micro-rheometers and the study of mixing processes. We have obtain 1,5 year of post-doctoral position and funding for the team (3 years program).

5.2. Laser-plasma interaction

8 grants have been obtained from the CEA, several in collaboration with B. Nkonga. 3 PhD thesis were funded by the CEA and the Aquitaine district. 6 Master thesis (6 month each) were funded by the CEA.

With the CEA, we start the study of new model for coupling Raman amplification with Landau damping. We also want to implement 3-D models for propagation of short pulses in crystals.

5.3. Flow simulation, analysis and control

One contract with CIRA (Italy) has been obtained for years 2003-2005. A. Iollo is responsible an ANR project for years 2005-2007. We had one contract with IFP (Institut Francais du Petrole) during the years 2001-2003 and 3 Matmeca Masters funded by them. Also, a joint patent with IFP on the passive control around oil risers (2004). One grant with Renault during years 2003-2004 and 1 Master thesis.

Members of the team have promoted a EU STREP Vortex Cell 2050 (trapped vortices) in the 6th framework. Iraj Mortazavi was awarded in 2002 by the AMIF program of the ESF for a one month stay in the department of aeronautics of the Imperial College in London.

Since 2006, we are partner of an ANR project on advanced numerical methods named COMA. One PhD is funded by Renault.

A project funded by the Aquitaine Region concerning real time numerical simulations was awarded to members of the team.

5.4. CEMAGREF (French agency for water resources)

1 contract, 1 PhD and 4 Master thesis are funded by the CEMAGREF.

5.5. Airbus

DESGIVRE is a three years project between Airbus and MC2 INRIA Futurs. Title "DES analysis of aircraft performance degradation due to ice accretion". The overall objectives of this proposal are the following:

1. Analysis of the DES approach;
2. Develop the DES model for the simulation of 3D turbulent flow;
3. Discuss the issues that impact the method, including the underlying RANS turbulence model and the simulation design for DES (grids and choice of time steps);
4. Highlight the capabilities of the method and draw a summary of some of the current research activities in DES;
5. Use Airbus test cases to answer the following question: is it possible and advisable to use DES to quantify the performance degradation due to icing;
6. Analysis of the available post-processing for DES. Draw perspectives on the use of Proper Orthogonal Decomposition (POD) for DES simulations.

6. Dissemination

6.1. Leadership within scientific community

6.1.1. Conferences, meetings and tutorial organization

6.1.1.1. Conference talks

Aside from seminars in french Universities, the members of MC2 gave numerous talks in international conferences.

- Favier J., Cordier L., Kourta A. et Iollo A. (2006) Calibrated POD Reduced-Order Models of massively separated flows in the perspective of their control. Proceedings of FEDSM2006. 2006 ASME Joint US Â European Fluids Engineering Summer Meeting. July 17-20, 2006, Miami, FL.
- Buffoni M., Camarri S., Iollo A., Salvetti M.V. (2006) Low-order modeling of 3D laminar flows past a confined square cylinder. EUROMECH 6th European Fluid Mechanics Conference. Stockholm, Sweden. June 26 Â 30.
- Gallizio F., Iollo A. and Zannetti L. (2006) Prandtl-Batchelor channel flows past plates at normal incidence. EUROMECH 6th European Fluid Mechanics Conference. Stockholm, Sweden. June 26 Â 30.
- Weller J., Buffoni M. and Iollo A. (2006) Modèles d'ordre réduit et décomposition de domaine. Journées activités Universitaires de Mécanique. La Rochelle, 31 août et 1er septembre.
- Telib H., Iollo A. and Zannetti L. (2006) Modeling and optimization of a propeller by means of inverse problems. Third international Conference in Inverse Problems: Modeling and Simulation. Oludeniz (Fethiye, Mugla) May 29 - June 02. Turkey.
- Lombardi E., Buffoni M., Camarri S., Iollo A. and Salvetti M.V. (2006) Modeling and identification of an unsteady 3D flow by an accurate reduced order model. European Drag Reduction and Flow Control Meeting. Ischia, Italy. April 10 Â 13.
- Buffoni M., Telib H. and Iollo A. (2006) Boundary conditions by reduced order modeling. Proceedings of ICCFD4. Fourth International Conference on Computational Fluid Dynamics. Ghent, Belgium. July 10 Â 14.
- Beaugendre, H. and Ern, A., (2006) , Finite volume box scheme for a certain class of nonlinear conservative laws in mixed form, Proceeding of ICCFD4. Fourth International Conference on Computational Fluid Dynamics. Ghent, Belgium. July 10-14.
- Bruneau, C.H., Fischer, P. and Kellay, H. (2006) , Identification and role of coherent structures in two dimensional turbulence, Proceeding of ICCFD4. Fourth International Conference on Computational Fluid Dynamics. Ghent, Belgium. July 10-14.
- Bruneau, C.H., Mortazavi, I. and Gilliéron, P. (2006), Flow regularization and drag reduction around blunt bodies using porous devices, ERCOFTAC, European Drag reductions conference, Ischia, Italy.
- Galusinki, C., Khadra, K. and Vigneaux, P. (2006), Level set method for bifluid flows in microchannels, 7th world Congress on Computational Mechanics.
- Mathieu Colin: June 2006, invited speaker in the special session "Non linear dispersive waves" of the 6th international conference "Dynamics systems and differential equations".
- Thierry Colin: January 2006 Kagurazaka Seminar, Tokyo University of science, invited speaker.
- Thierry Colin: Workshop: "Asymptotic Analysis of PDEs", Sapporo university, February 2006.
- Thierry Colin: Invited speaker in the French national congress of numerical analysis 2006.
- Thierry Colin: July 16-22 2006, invited speaker in the mini-symposium "Numerical and computational aspects of interface problems and applications" during the 7th World Congress on Computational Mechanics, organized by UCLA and Northwestern university in Los Angeles.
- Thierry Colin: Invited speaker in the seminar on nonlinear propagation organized in Saclay by the Nuclear energy direction of the CEA, december, 14th 1006.
- Olivier Saut, June 2006, invited speaker in the special session "Biomathematics" of the 6th international conference "Dynamics systems and differential equations".
- Olivier Saut, July 2006, Summer School MCRTN "Modeling, Mathematical Methods and Computer Simulation of Tumour Growth and Therapy", Kolymbari, Greece.

6.1.1.2. Conference organisation

- Mathieu Colin, Thierry Colin: Co-organization with J. Bona and D. Lannes of a special session "Nonlinear-waves" in the 6th international conference "Dynamics systems and differential equations" in Poitiers, june 2006.
- Thierry Colin: Organization of two days on dispersive problems in Bordeaux, march 06 for the doctor Honoris Causa ceremony for Jerry Bona.
- Thierry Colin: Member of the scientific committee of the summer school "Ondes et matière d'Aquitaine" organized by the CEA CESTA (september 2006).
- Thierry Colin: Member of the scientific committee of the workshop "Challenges actuels en mécanique des fluides: application à la biologie, aux ondes de surface et à la microfluidique" au CIRM (06).
- Thierry Colin: Member of the organization committee of the workshop :"Modélisation et calcul scientifique liés aux problèmes du bâtiment : problèmes, méthodes et enjeux", 8 décembre sur le Campus de Savoie Technolac.

6.1.2. Invitation in foreign Universities

- Thierry Colin: January 2006: University of Osaka, Tokyo University of Science, University of Sapporo, 3 weeks, invited by N. Hayashi, K. Kato and T. Ozawa.
- Thierry Colin: November 2006 University of Columbia (Michael Weinstein) and University of Illinois at Chicago (UIC) (J. Bona).
- A. Iollo was invited for a 2 weeks visit during September 2006 at the Mathematics and Computing Technology Group of The Boeing Company, Seatlle, USA.
- I. Mortazavi was invited one week in Imperial College of London, UK.
- I. Mortazavi was invited one week in Politecnico di Torino, Italy.

6.1.3. Administrative duties

- Charles-Henri Bruneau: Member of the hiring committee for applied mathematics in Orsay, Bordeaux 1 and Bordeaux 2.
- Charles-Henri Bruneau: Member of several research group of CNRS.
- Charles-Henri Bruneau: Member of the administrative committee of the Mathematics and Computer Science division of the University of Bordeaux 1.
- Charles-Henri Bruneau: Member of the executive board of the International Conferences on Computational Fluid Dynamics.
- Charles-Henri Bruneau: In charge of one academic year of the engineering school MATMECA.
- Charles-Henri Bruneau: In charge of the MASTER of Mathematics Engineering of Bordeaux 1.
- Charles-Henri Bruneau: Member of the executive board of the 4th Int. Conf. Computational Fluid Dynamics, Gent July 2006.
- Mathieu Colin: In charge of the exam of entry in the engineering school MATMECA.
- Mathieu Colin: Director of the communication of the engineering school MATMECA.
- Mathieu Colin: Member of the hiring committee for applied mathematics in Bordeaux.
- Thierry Colin: Member of the scientific committee of the Mathematics and Computer Science division of the University of Bordeaux 1.
- Thierry Colin: Member of the hiring committee for applied mathematics in Bordeaux.
- Thierry Colin: Member of the scientific committee of the university of Bordeaux .

- Thierry Colin: Administration board of the engineering school MATMECA.
- Thierry Colin: Scientific expert for the ministry of research.
- Thierry Colin: Head of the applied math. Lab, University of Bordeaux (55 faculties).
- Thierry Colin: Member of the editorial board of the series of books "Mathématiques et Applications", edited by Springer and the SMAI (french Society for Industrial and Applied Math.), 40 books have already been published.
- Thierry Colin: responsible of the applied math. team inside the math. lab., university of Bordeaux.
- Angelo Iollo: Member of the steering committee of grants endowed by the Royal Society (for a collaboration with Prof. Chernyshenko), by INTAS (for a collaboration with Prof. Meleshko) and by EU (STREP - VortexCell2050).
- A. Iollo: Member of the steering committee of EU (STREP - VortexCell2050).
- A. Iollo: Director of the Mathematical Engineering Department.
- Iraj Mortazavi: Head of international exchanges of engineering school Matmeca.
- Iraj Mortazavi: Head of industrial collaborations and internships of the Scientific Computing M.Sc.
- Iraj Mortazavi: Member of the hiring committee for applied mathematics in Bordeaux.

6.2. Ph. D. Theses

6.2.1. Theses defended in 2006

- R. Balaouar (funded by the CEA and the Aquitaine district) Coupling of Raman instability and Landau damping in plasma physics. Contact at the CEA: G. Gallice. December 2006. Advisors: T. Colin and C. Galusinski.
- T. Bouchères (funded by the CEA and the Aquitaine district) on the modeling for Raman scattering in nonlinear optics. Contact at the CEA: A. Bourgeade. February 2006. Advisors: T. Colin and B. Nkonga.
- G. Gancel (funded by the CEMAGREF) on the numerical simulation and sensitivity analysis of water distribution systems. Advisors: P. Fabrie and I. Mortazavi

6.2.2. Theses started in 2006

- J. Dambrine (funded by INRIA and the Aquitaine district). Models for mixing in microfluidics. Collaboration with Rhodia.
- D. Deyperas (BDI with Renault). Advanced active and passive control strategies for 3D flows around simplified car geometries.
- E. Jammouillé (funded by the CEMAGREF). Numerical Modeling of optimization and control of water distributed systems.
- E. Lombardi (funded by the french ministry of research).

7. Bibliography

Major publications by the team in recent years

[1] C.-H. BRUNEAU, H. KELLAY. *Coexistence of two inertial ranges in two-dimensional turbulence*, in "Phys. Rev. E", vol. 71, n^{o} 10, 2005.

[2] C.-H. BRUNEAU, I. MORTAZAVI. *Passive control of bluff body flows using porous media*, in "Int. J. for Num. Meth. in Fluids", vol. 46, 2004.

[3] C.-H. BRUNEAU, M. SAAD. *The 2D lid-driven cavity problem revisited*, in "Computers & Fluids", vol. 35, n° 3, 2006, p. 326–348.

[4] S. CHERNYSHENKO, B. GALLETTI, A. IOLLO, L. ZANNETTI. *Trapped vortices and favorable pressure gradient*, in "Journal of Fluid Mechanics", vol. 482, 2003, p. 235–255.

[5] M. COLIN, T. COLIN. *On a quasilinear Zakharov system describing laser-plasma interactions*, in "Differential Integral Equations", vol. 17, 2004, p. 297–330.

[6] T. COLIN, C. GALUSINSKI, H. KAPER. *Long waves in micromagnetism*, in "Communications in PDE", vol. 27, 2002, p. 1625–1658.

[7] T. COLIN, B. NKONGA. *Numerical model for light interaction with two level atoms medium.*, in "Physica D", vol. 188, 2004, p. 92–118.

[8] B. GALLETTI, C.-H. BRUNEAU, L. ZANNETTI, A. IOLLO. *Low-order modelling of laminar flow regimes past a confined square cylinder*, in "Journal of Fluid Mechanics", vol. 503, 2004, p. 161–170.

[9] H. TELIB, M. MANHART, A. IOLLO. *Analysis and low order modeling of the inhomogeneous transitional flow inside a T-mixer*, in "Physics of Fluids", vol. 16, n° 8, 2004, p. 2717–2731.

[10] A.-S. WUNNENBURGER, A. COLIN, T. COLIN, D. ROUX. *Undulation instability under shear: a model to explain the different orientations of a lamellar phase under shear*, in "Eur. Phys. J. E", vol. 2, n° 3, 2000, p. 277–283.

Year Publications

Articles in refereed journals and book chapters

[11] H. BEAUGENDRE, A. ERN, T. ESCLAFFER, E. GAUME, I. GINZBURG, C. KAO. *A seepage face model for the interaction of shallow water tables with the ground surface: Application of the obstacle-type method*, in "Journal of Hydrology", vol. 329, 2006, p. 258–273.

[12] H. BEAUGENDRE, F. MORENCY, W. HABASHI. *Development of a second generation inflight icing simulation code*, in "Journal of Fluids Engineering", vol. 128, n° 2, March 2006, p. 378–387.

[13] R. BELAOUARD, T. COLIN, G. GALLICE, C. GALUSINSKI. *Theoretical and numerical study of a quasi-linear Zakharov system describing Landau damping*, in "M2AN", to appear, 2006.

[14] A. BOURGEADE, O. SAUT. *Numerical methods for the bidimensional Maxwell-Bloch equations in nonlinear crystals*, in "J. Comput. Phys.", vol. 213, n° 2, 2006, p. 823–843.

[15] A. BOURGEADE, O. SAUT. *Propagation d'impulsions laser ultracourtes dans un cristal non linéaire*, in "ESAIM Proc", to appear, 2006.

[16] D. BRESCH, T. COLIN, E. GRENIER, B. RIBBA, O. SAUT, C. VERDIER. *Modèles avec paramètre d'ordre, level set et interface diffuse : Application à la cancérologie*, in "ESAIM Proc", to appear, 2006.

[17] C.-H. BRUNEAU, P. FISCHER. *Spectra and filtering: a clarification*, in "Int. J. Wavelets, Multiresol. Info. Proc.", to appear, 2006.

[18] C.-H. BRUNEAU, I. MORTAZAVI. *Control of vortex shedding around a pipe section using a porous sheat*, in "Int. J. Offshore and Polar Engg.", vol. 16, n° 2, 2006, p. 90–96.

[19] C.-H. BRUNEAU, M. SAAD. *The behaviour of high Reynolds flows in a driven cavity*, in "Computational Fluid Dynamics J.", to appear, 2006.

[20] M. BUFFONI, S. CAMARRI, A. IOLLO, M. SALVETTI. *Low-dimensional modelling of a confined three-dimensional wake flow*, in "Journal of Fluid Mechanics", vol. 569, 2006, p. 141–150.

[21] S. CHERNYSHENKO, A. IOLLO, G. DI CICCA, Z. HU, N. SANDHAM, A. SMIRNOV. *Placebo verification technique in the analysis and interpretation of turbulent velocity fields*, in "Mekhanika Zhidkosti i Gaza", to appear, 2006.

[22] M. COLIN, T. COLIN. *A numerical model for the Raman Amplification for laser-plasma interaction*, in "Journal of Computational and Applied Math.", vol. 193, n° 2, 2006, p. 535–562.

[23] B. GALLETTI, A. BOTTARO, C.-H. BRUNEAU, A. IOLLO. *Accurate Model Reduction of Transient and forced wakes*, in "European J. Mech. B/Fluids", to appear, 2006.

[24] G. GANCEL, I. MORTAZAVI, O. PILLER. *Coupled numerical simulation and sensitivity assessment for quality modelling for water distribution systems*, in "Applied Math. Letters", vol. 19, 2006, p. 1313–1319.

[25] P. GUILLOT, A. COLIN, S. QUINIOU, G. CRISTOBAL, M. JOANICOT, C-H. BRUNEAU, T. COLIN. *Un rhéomètre sur puce microfluidique*, in "La Houille Blanche", vol. 3, 2006.

[26] P. GUILLOT, P. PANIZZA, J. SALMON, M. JOANICOT, A. COLIN, C.-H. BRUNEAU, T. COLIN. *A viscosimeter on a microfluidic chip*, in "Langmuir", vol. 22, n° 14, 2006, p. 6438–6445.

[27] B. RIBBA, T. COLIN, S. SCHNELL. *A multiscale mathematical model of cancer growth and radiotherapy efficacy: The role of cell cycle regulation in response to irradiation*, in "Theoretical Biology and Medical Modeling", 2006, p. 3–7.

[28] B. RIBBA, O. SAUT, T. COLIN, D. BRESCH, E. GRENIER, J-P. BOISSEL. *A multiscale mathematical model of avascular tumor growth to investigate the therapeutic benefit of anti-invasive agents*, in "Journal of Theoretical Biology", vol. 243, 2006, p. 532–541.

Publications in Conferences and Workshops

[29] H. BEAUGENDRE, A. ERN. *Finite volume box scheme for a certain class of nonlinear conservative laws in mixed form*, in "ICCFD, Springer", The Fourth International Conference on Computational Fluid Dynamics, Ghent, Belgium, July 2006.

[30] C.-H. BRUNEAU, P. FISCHER, H. KELLAY. *Identification and role of coherent structures in two dimensional turbulence*, in "4th Int. Conf. on Comp. Fluid Dyn.", 2006.

[31] C.-H. BRUNEAU, I. MORTAZAVI, P. GILLIÉRON. *Flow regularisation and drag reduction around blunt bodies using porous devices*, in "ERCOFTAC, European Drag Conference, Ischia, Italy", 2006.

[32] C.H. BRUNEAU, I. MORTAZAVI, P. GILLIÉRON. *Flow regularisation and drag reduction around blunt-bodies using porous devices*, in "Proceedings of the 2nd IUTAM Symp. Flow Control and MEMS (London, 19-22 September 2006), Edited by: J. Morrison, Springer", to appear, 2006.

[33] E. CREUSE, A. GIOVANNINI, I. MORTAZAVI. *Active control of a transitional flow over a backward-facing step*, in "Procedings of the 4th Int. Conf. on CFD (ICCFD4: Gent, 10-14 July 2006), Edited by: H. Deconinck and E. Dick, Springer", to appear, 2006.

[34] J. FAVIER, L. CORDIER, A. KOURTA, A. IOLLO. *Calibrated POD Reduced-Order Models of massively separated flows in the perspective of their control*, in "Proceedings of FEDSM2006. 2006 ASME Joint US Â European Fluids Engineering Summer Meeting", July 2006.

[35] C. GALUSINSKI, K. KHADRA, P. VIGNEAUX. *Level Set method for bifluid flows in microchannels*, in "7th World Congress on Computational Mechanics - WCCM VII - Hyatt Regency Century Plaza Hotel, Los Angeles, California, July 16 - 22", July 2006.

[36] P. VIGNEAUX. *Droplets in microchannels with Level Set method*, in "ECCOMAS CFD 2006, Proceedings of the European Conference on Computational Fluid Dynamics 2006, Egmond aan Zee, The Netherlands, September 5-8, 2006", P. WESSELING, E. OATE, J. PÉRIAUX (editors). , 2006.

[37] P. VIGNEAUX. *Level Set method for Curvature-driven Flows in Microfluidics*, in "Computational Fluid Dynamics 2006, Proceedings of the Fourth International Conference on Computational Fluid Dynamics, ICCFD4, Ghent, Belgium, July 10-14, 2006", to appear, Springer, 2006.

[38] P. VIGNEAUX. *Méthode Level Set pour des écoulements en microcanaux*, 2006.

Miscellaneous

[39] H. BEAUGENDRE, A. ERN. *Conservative velocity fields using Crouzeix-Raviart finite elements for ground water flows*, 2006.

[40] H. BEAUGENDRE, A. ERN, S. HUBERSON. *How much is a conservative flow field needed for low order pollutant transport simulation?*, 2006.

[41] D. BRESCH, T. COLIN, E. GRENIER, B. RIBBA, O. SAUT. *Computational modeling of avascular tumor growth*, in preparation, 2006.

[42] C. GALUSINSKI, M. SAAD. *A nonlinear degenerate system modeling water-gas in reservoir flow*, submitted for publication, 2006.

[43] C. GALUSINSKI, M. SAAD. *Two compressible immiscible fluids in porous media*, submitted for publication, 2006.

[44] C. GALUSINSKI, P. VIGNEAUX. *Level-Set method and stability condition for curvature-driven flows*, submitted for publication, 2006.

References in notes

[45] D. AMBROSI, L. PREZIOSI. *On the closure of mass balance models for tumor growth. Math Models Method*, in "Appl Sci", vol. 12, 2002, p. 737–754.

[46] R. BELAOUARD, T. COLIN, G. GALLICE, C. GALUSINSKI. *Theoretical and numerical study of a quasi-linear Zakharov system describing Landau damping*, 2005.

[47] C. BESSE, B. BIDÉGARAY, A. BOURGEADE, P. DEGOND, O. SAUT. *A Maxwell-Bloch model with discrete symmetries for wave propagation in nonlinear crystals: an application to KDP*, in "M2AN Math. Model. Numer. Anal.", vol. 38, n^{o} 2, 2004, p. 321–344.

[48] T. BOUCHÈRES, A. BOURGEADE, T. COLIN, B. NKONGA, B. TEXIER. *Study of a mathematical model for stimulated Raman scattering*, in "Math. Models Methods Appl. Sci.", vol. 14, n^{o} 2, 2004, p. 217–252.

[49] T. COLIN, B. NKONGA. *Numerical model for light interaction with two level atoms medium*, in "Phys. D", vol. 188, n^{o} 1-2, 2004, p. 92–118.

[50] H.P. GREENSPAN. *Models for the Growth of a Solid Tumor by diffusion*, in "Stud Appl Math", vol. 4, 1972, p. 317–340.

[51] H.P. GREENSPAN. *On the growth and stability of cell cultures and solid tumors*, in "J Theor Biol", vol. 56, 1976, p. 229–242.

[52] O. SAUT. *Étude numérique des nonlinéarités d'un cristal par résolution des équations de Maxwell-Bloch*, Ph. D. Thesis, INSA Toulouse, 2003.

[53] O. SAUT. *Computational modeling of ultrashort powerful laser pulses in an anisotropic crystal*, in "J. Comput. Phys.", vol. 197, n^{o} 2, 2004, p. 624–646.

[54] K. YEE. *Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media*, in "IEEE Trans. Antennas Propag.", vol. AP-14, 1966, p. 302–307.