INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team POEMS

Wave propagation: Mathematical Analysis and Simulation

Rocquencourt

THEME NUM

Activity Report

2006

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2. Overall Objectives

2.1. Overall Objectives

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar)and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and INRIA (UMR 2706). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.
Beyond the general objective of contributing to the progress of the scientific knowledge, three goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,
- the treatment of complex problems whose simulation is close enough to real life situations,
- the development of original mathematical and numerical techniques,
- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

3. Scientific Foundations

3.1. Scientific Foundations

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unkonwn solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to $e^{i\omega t}$, where $\omega > 0$ denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation) :

$$-c^2 \Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelization, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengthes (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behaviour of the solution and its knowledge will have a great influence on the choice of a numerical method.
Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, ...)
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models in various situations,
- imaging techniques and inverse problems related to wave propagation.

These areas constitute the main fields of interest for the Project POEMS.

4. Application Domains

4.1. Application Domains

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

4.1.1. Acoustics.

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently.

Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modelling until the numerical approximation of existing models. Our works in this area are partially supported by EADS (and Airbus). The final objective is to reduce the noise radiated by Airbus planes.

Vibroacoustics, which concerns the interaction between sound propagation ans vibrations of thin structures, also raises up a lot of relevant research subjects. Our collaboration with EADS on this subject, with application to the confort of the cockpits of airplanes, allowed us to develop a new research direction about time domain integral equations.

A particularly attractive application concerns the simulation of musical imstruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufactoring of new instruments. The modeling and simulation of the timpani and of the guitar have been carried out in collaboration with A. Chaigne of ENSTA.

4.1.2. Electromagnetism.

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations poses new and challenging mathematical questions.

Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, and design of optoelectronic micro-components.

Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). Finally micro and nano optics have seen recently fantastic technological developments, and there is a real need for tools for the numerical simulation in these areas.

Our team has taken a large part in this research in the past few years. In the beginning, our activity was essentially concerned with radar furtivity (supported by the French Army and Aeronautic Companies). Now, it is evolving in new directions thanks to new external (academic and industrial) contacts:

- We have been developing since 2001 a collaboration with ONERA on EM modeling by higher order methods (theses of S. Pernet and M. Duruflé).
- As partners of ONERA, we have been selected by the CEG (a research organism of the French Army) to contribute to the development of a general computational code in electromagnetism. The emphasis is on the hybridization of methods and the possibility of incorporating specific models for slits, screens, wires,...
- We have been participating since 2002, to the ARC HEADEXP concerning the simulation of electromagnetic waves in the brain.

- Optics is becoming again a major application topic. In the past our contribution to this suject was quite important but remained at a rather academic level. Our recent contacts with the company ATMEL (on the modelling of optical filters) and with the Institut d'Electronique Fondamentale (Orsay) (we have initiated with them a research program about the simulation of micro and nano opto-components) are motivating new research in this field.

4.1.3. Elastodynamics.

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view. Our activity on this topic, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research, has seen a most welcomed rebound through new academic and industrial contacts.

The two major application areas of elastodynamics are geophysics and non destructive testing. A more recent interest has also been brought to fluid-stucture interaction problems.

- In geophysics, one is interested in the propagation of elastic waves under ground. Such waves appear as natural phenomena in seisms but they are also used as a tool for the investigation of the subterrain, mainly by the petroleum industry for oil prospecting (seismic methods). This constitutes an important field of application for numerical methods. Our more recent works in this area have been motivated by various research contracts with IFP (French Institute of Petroleum), IFREMER (French Research Institute for the Sea) or SHELL (which have supported, at least partially, the PhD theses of S. Fauqueux, A. Ezziani and J. Diaz).
- Another important application of elastic waves is non-destructive testing: the principle is typically to use ultra-sounds to detect the presence of a defect (a crack for instance) inside a metallic piece. This topic is the object of an important cooperation with EDF (French Company of Electricity) in view on the application to the control of nuclear reactors. This collaboration has motivated some of the most important and innovative scientific achievements of the project with the theses of C. Tsogka, G. Scarella and J. Rodriguez.

  At a more academic level, we have been interested in other problems in the domain of elastic waves in plates (in view of the application to non-destructive testing) through our participation to the GDR Ultrasons. In this framework, we have developped our resarches on multi-modal methods, exact transparent conditions or shape reconstruction of plates of variable cross section.
- Finally, we have recently been led to the study of fluid-solid interaction problems (coupling of acoustic and elastic waves through interfaces) as they appear in underwater seismics (IFREMER) and stemming from ultra-sound propagation in bones (in contact with the Laboratoire d'Imagerie Paramétrique of Paris VI University).

4.1.4. Gravity waves.

These waves are related to the propagation of the ocean swell. The relevant models are derived from fluid mechanics equations for incompressible and irrotational flows. The applications concern in large part the maritime industry, if particular the questions of the stability of ships, sea keeping problems, wave resistance,... The application we have recently worked on concerns the stabilization of ships and off-shore platforms (contract with DGA).

5. Software

5.1. Advanced software

- **MELINA :** This software has been developed under the leadership of D. Martin for several years in order to offer to the researchers a very efficient tool (in Fortran 77 and object oriented) for easily implementing finite element based original numerical methods for solving partial differential equations. It has specific and original potential in the domain of time harmonic wave problems (integral representations, spectral DtN conditions,...). Nowadays, it is fully functional in various application areas (acoustics and aeroacoustics, elastodynamics, electromagnetism, water waves). It is an open source software with on line documentation available at http://perso.univ-rennes1.fr/daniel.martin/melina/. The software is regularly used in about 10 research laboratories (in France and abroad) and number of research papers have published results obtained with MELINA (see the Web site). Moreover, every 2 years, a meeting is organized which combines a workshop which teaches new users with presentations by existing users.
  During the last four years, apart from various local improvements of the code, new functionalities have been developed:
  - Higher order finite elements (up to 10^{th} order),
  - Higher order quadrature formulae,
  - DtN boundary conditions in 3D.

  A new C++ version of the software is under development. We will take advantage of this evolution for extending the class of finite elements (mixed elements, tensor valued elements, ...).
- **LSM** : This software is a Fortran-90 code coupled with a Matlab interface. It solves the inverse acoustic and electromagnetic scattering problem using the Linear Sampling Method and the Tikhonov regularization. This code has been developed by H. Haddar. A parallel version has been produced by M. Fares from Cerfacs. This code was provided to and used by researchers at the university of Delaware (E. Darrigrand, P. Monk), Cerfacs (M. Fares) and the University of Genova (M. Piana). A 2-D version of this code coupled with the forward solver of the Helmholtz equation (provided by F. Collino) is under construction and should be available on the project web-site before the end of 2004.

5.2. Prototype software

- **ACOUS2D :** This software was written in the frame of S. Fauqueux's thesis. Property of INRIA. It concerns the simulation of transient acoustic waves in an a 2D inhomogeneous medium based on a mixed formulation of spectral elements. Sources are spherical and reflecting boundaries can be of Dirichlet or Neumann types and unbounded domains are taken into account by using PML.
- **ELASTIC2D :** Same characteristics and author as ACOUS2D for transient linear elastodynamic waves. The media can also be anisotropic. Property of IFP, INRIA owns a copy for research purposes.
- **ELASTIC3D :** Same as ELASTIC2D in 3D.
- **RAPH-ELAS :** This code, developed by J. Rodriguez, is devoted to solve the linear elastodynamic equations in 2D. This solver, that is based on ELAST-2D (developed by C. Tsogka), includes the possibility of doing recursive local space-time mesh refinement of arbitrary ratio. RAPH-ELAS will be included in ATHENA-2D, the code of the electricity company EDF.
- **Contact2D :** This code, developed on the basis of ELAST-2D by G. Scarella, solves 2D elastodynamics equations in heterogeneous media in the presence of cracks modeled with pure unilateral contact conditions. It has been implemented as a part of the code ATHENA-2D (EDF).
- **VISCO2D :** This code, written by A. Ezziani in the framework of collaborations with IFREMER and SHELL , is an extension of the mixed finite element code ELAST-2D of C. Tsogka to viscoelastic media (generalized Zener's models). sold to SHELL.

- **APE2D :** This has been written by A. Ezziani in the framework of a contract with SHELL. It concerns the simulation of waves in poro-elastic media (Biot's model) by higher order finite elements with mass lumping and PML's for open boundaries.
- **FLUID-STRUCT2D :** This software was written in the frame of S. Fauqueux's thesis during a 6 weeks stay at Caltech. Property of IFP, INRIA owns a copy for research purposes. It models the propagation of a transient acoustic wave in a solid through a fluid in 2D. Its purpose is the modeling of acoustic waves in the sea for seismic prospection. The numerical models in fluid ans solid are the same as those used in ACOUS2D and ELASTIC2D.
- **Flusol2d :** This, developped by J. Diaz on the basis of ELAST-2D, is aimed at solving fluid-structure interaction problem in the case of a plane interface in two dimensions. It is based on a mixed dual-dual formulation : a variational formulation where the pressure in the fluid and the velocities in the solid are searched in an L^2-like space and the velocities in the fluid and the stresses in the solid are searched in an $H(div)$ like space. This code is used by IFREMER.
- **Flusol3d :** This software solves fluid-structure interaction problems in three dimensions in general geometries . It is based on a primal-primal formulation (a variational formulation where the pressure in the fluid and the velocities in the solid are searched in an $H^1 - like$ space and the stresses in the solid are searched in an $L^2 - like$ space) and spectral finite elements.
- **MAXANIR :** Modeling transient TM Maxwell's equations by a mixed edge element method on hexahedric meshes with mass-lumping. The media can be inhomogeneous and anisotropic. Sources can be spherical or plane waves, reflecting boundaries are metallic and unbounded domains are taken into account by using PML. This software was written by G. Cohen. Property of INRIA.
- **MAXWELL2D :** Modeling transient TE or TM Maxwell's equations rewritten in a wave equation formalism, which enables to use mixed spectral elements instead of edge elements. The media can be inhomogeneous and anisotropic. Sources can be spherical or plane waves, reflecting boundaries are metallic and unbounded domains are taken into account by using PML. This software was written by S. Fauqueux in the frame of a start-up incubation. Property of INRIA.
- **MAX2D :** This code, written by P. Ciarlet and E. Jamelot, solves time dependent 2D Maxwell's equations in singular domains, using Lagrange finite elements and particular treatments of singularities.
- **MAXTETRA3D :** This code has been developed by C. Poirier, H. Haddar and S. Vérité. The object is the resolution 3D time domain Maxwell's equations using tetrahedric second order edge elements with mass lumping. It uses the automatic 3D mesh generator NetGen developed by A. Schröbel. This code has been used for the ARC HEADEXP.
- **GeDeOND :** Modeling transient 3D Maxwell's equations by a discontinuous Galerkin method on hexahedric meshes. The media can be inhomogeneous and anisotropic. Sources can be spherical or plane waves, reflecting boundaries are metallic and unbounded domains are taken into account by using PML. This software was written in the frame of S. Pernet's thesis at ONERA-Toulouse. Property of INRIA and ONERA.
- **MONTJOIE :** This code has been written by M. Duruflé in the framework of a collaboration with ONERA. It concerms the resolution, by volumic methods, of the Helmholtz equation and the time-harmonic Maxwell's equations, both in 2-D and 3-D. This code uses spectral finite element method on quadrilateral/hexahedral meshes for the scalar case. It uses finite edge element for the vectorial case.
- **MODALOPT :** This code, written by E. Lunéville, based on multi-modal decomposition of waveguides of variable cross-section, is able to solve inverse problems (by minimization techniques) such as shape optimization or shape identification.

6. New Results

6.1. Introduction

We have chosen to group our research into 7 distinct parts. Of course this partition is somewhat arbitrary and overlap is possible (a given work could appear in several categories).

6.2. Numerical methods for time domain wave propagation

6.2.1. Higher-Order Methods in space

Participants: Gary Cohen, Nicolas Castel, Edouard Demaldent, Marc Duruflé.

In the frame of his thesis, N. Castel studied the dipersion relations of discontinuous Galerkin methods (DGM) and applied this study to reflection-transmission analysis. This analysis was never done yet. First results show the introduction of a phase-shift when crossing a discontinuity of media.

E. Demaldent started his thesis at ONERA-Palaiseau under the direction of G. Cohen. He implemented different kinds of high-order quadrilateral elements. In particular, he studied different strategies to compute singular integrals which are much more difficult to compute for high-order elements. The first result show a real gain of quadrilateral elements versus triangular ones.

M. Duruflé finished his thesis in February. He was taken in a post-doc period of six months in our project. In the frame of this post-doc, he studied, in collaboration with G. Cohen, the effect of numerical dissipation on spectral DGM. This study lead to a novel spurious-free formulation of spectral edge elements in the time domain. On the other hand, several methods of local time-stepping were studied.

6.2.2. Higher order time discretization of second order hyperbolic problems

Participants: Jean-Charles Gilbert, Patrick Joly.

We are concerned here with a very classical problem, namely the numerical approximation of second order hyperbolic problems, more precisely problems of the form

$$\frac{d^2u}{dt^2} + \mathcal{A}u = 0, \tag{1}$$

where $\mathcal{A}$ is a linear unbounded positive selfadjoint operator in some Hilbert space V. This appears to be the generic abstract form for a large class of partial differential equations used for wave propagation in various domains of application, in particular in acoustics, electromagnetism, and elasticity. A lot of work in the literature has been devoted to the construction of higher order methods for the space discretization of (1) (using, finite differences, finite elements, discontinuous Galerkin methods,...). Much less work has been devoted to the natiral question of higher order time discretization. This work is a contribution to this problem.

We first study the even order $(2m)$ schemes obtained by the modified equation method. We show that the corresponding CFL upper bound for the time step remains bounded when the order of the scheme increases, which can be somewhat penalizing.

This observation leads us to propose variants of these schemes constructed to optimize the CFL condition: this is formulated as an optimization problem in a space of polynomials of given degree. Despite some unpleasant properties (the objective function is nonconvex and even discontinuous at the solution!), this problem can be fully analyzed. In particular, we prove the existence and uniqueness of the solution and give necessary and sufficient conditions of optimality.

These conditions are exploited to design an algorithm for the effective numerical solution of the optimization problem. The obtained results are more than satisfactory with respect to our original objective. They suggest some conjectures. If they were true, this would mean that we would be able to produce schemes of arbitrary high order in time and whose computational cost would be almost independent of the order. Of course, this is a preliminary work and much has still to be done, including the following items:

- The effective efficiency of the new schemes should be tested on realistic wave propagation problems.
- The impact of the modification of the initial schemes (the ones which are based on the modified equation technique) on the effective accuracy (we are only guaranteed that the order of approximation is preserved) should be analyzed thorough numerical dispersion studies.

6.2.3. Fast solvers for evolution equations

Participant: Jing-Rebecca Li.

This work concerns the efficient and accurate solution of the heat equation in unbounded domains based on integral representation.

We solve the heat equation in unbounded domains by efficiently evaluating the integral representation of the solution. Part of the computation is done in the Fourier domain (transform of the spatial variables) so that the algorithm complexity is linear (rather than quadratic, as in a naive approach) in the number of time steps.

Several recently developed numerical tools were necessary for the practical implementation of this new approach, including the spectral approximation of the free space heat kernel, accurate evaluation of layer potentials, the non-uniform FFT, the fast Gauss transform, and they have been incorporated into the code.

There exist currently many numerical algorithms treating complicated phenomena that do not handle diffusion into a large or unbounded domain in a satisfactory fashion. This new heat equation solver is mainly intended to be integrated into these algorithms to improve their performance. In particular, we have coupled the new heat solver to an existing code which models dendritic solidification using a phase field model and obtained good results.

6.2.4. Mixed spectral finite element methods for vibroacoustics

Participants: Gary Cohen, Marc Duruflé, Pascal Grob, Patrick Joly.

This corresponds to the first part of the PhD thesis of P. Grob, which has been defended in May. The subject is the numerical simulation of the acoustic radiation of the vibrations of a thin mechanical structure (typically a plate). This is done in collaboration with EADS.

P. Grob has studied a method based on coupling 2D (for the plate) and 3D (for the fluid) mixed spectral elements of different orders. The most recent part of the work is a partial contribution to the justification (from both theoretical and numerical points of view) of the use of such elements and more particularly the effect of the use of quadrature approximation (an essential step of the method) on the accuracy of the computation. The redaction of the corrresponding article is under way.

6.2.5. Local time stepping and discontinuous Galerkin methods for symmetric first order hyperbolic systems.

Participants: Abdelazziz Ezziani, Patrick Joly.

This new thematic is developed via the post-Doc of A. Ezziani in collaboration with Airbus. The general framework of this collaboration is the hybridization of numerical domains for the time-domain solution of Linearized Euler equations in aeroacoustics.

We have been interested in non conforming space-time mesh refinement methods for wave propagation in aeroacoustics, in the spirit of previous work in electromagnetism and elastodynamics. We have developed a method which is applicable to zero order perturbations of symmetric hyperbolic systems in the sense of Friedrichs (Linearized Euler equations are of this type) The method is based on the one hand on the use of a conservative higher order discontinuous Galerkin approximation for space discretization and a finite difference scheme in time, on the other hand on appropriate discrete transmission conditions between the grids. We use a discrete energy techniques to drive the construction of the matching procedure between the grids and guarantee the stability condition. Moreover, under suitable geometrical conditions on the grids, this method is quasi explicit.

The method that we developed last year for a 1-2 mesh refinement (the time step is twice larger in the coarse grid) has been extended to any rational refinement rate. It has been generalized also to a class of dissipative discontinuous Galerkin methods, who have the reputation to have better stability properties than purely conservative methods : the key point is to remove the dissipation additional terms associated to the interface.

This work has been presented at the CEA-INRIA-EDF school on discontinuous Galerkin methods in November.

6.2.6. *Combining integral equations and finite elements in vibroacoustics*

Participants: Pascal Grob, Patrick Joly.

This is the second aspect of the PhD thesis of P. Grob.

Let us recall that we have developed an original method based on the coupling between spectral finite elements for the mechanical part of the problem and retarded potentials for the acoustic part. The origimality of the coupling procedure is that the stability of the method is theoretically guaranteed through the comnservation of a discrete energy. Moreover, the stability condition is the same as in the absence of he coupling. The corresponding code has been successfully implemented and the work has been accepted for publication in SIAM J. on Scientific computing.

6.2.7. *The Singularity Expansion Method*

Participants: Christophe Hazard, François Loret.

The only new fact about this topic is the submission of an article (by C. Hazard and F. Loret) which presents the application of the method to the two-dimensional problem of an elastic plate floting at the free surface of the sea, as well as numerical results (problem dealt with in F. Loret's thesis).

6.3. Time-harmonic diffraction problems

6.3.1. *Locally perturbed infinite periodic media*

Participants: Sonia Fliss, Patrick Joly, Jing-Rebecca Li.

Periodic media play a major role in applications, in particular in optics for micro and nano-technology. From the point of view of applications, one of the main interesting features is the possibility offered by such media of selecting ranges of frequencies for which waves can or can not propagate. Mathematically, this property is linked to the gap structure of the spectrum of the underlying differential operator appearing in the model.

Of course, there is a need for efficient numerical methods for computing the propagation of waves inside such structures. In real applications, the media are not really periodic but differ from periodic media only in bounded regions (small with respect to the total size of the propagation domain). In such a case, a natural idea is to reduce the pure numerical computations to these regions and to try to take advantage of the periodic structure of the problem outside: this is particularly of interest when the periodic regions contain a large number of periodicity cells.

The first situation we have studied concerns the computation of the propagation of acoustic waves in a **locally perturbed infinite periodic waveguide**, namely an infinite structure which is periodic in one privileged direction (the propagation direction) and bounded in the other transverse variables. We investigate the question of finding artificial (but exact) boundary conditions to reduce the numerical computation to a neighborhood of this perturbation (for more details see the article of the three participants which appears in *Commun. Comput. Phys.*).

A direct extension of the waveguide problem is the transmission of an incident wave from a constant media to a periodic one. Using the Floquet-Bloch Transform, we show that we can reduce this computation to the study of finding exact boundary conditions for a set of periodic half-waveguides with quasi-periodic conditions. Thus the DtN operator of the periodic half space problem is constructed with these waveguide DtN operators. Thanks to this method, we can compute easily the transmission ans reflection coefficients and then the transmitted and diffracted wave. We show in the following figures a simple example of two different medias for which we represent the refraction index (see figure 1). We consider an incident plane wave whose direction is indicated in figure 1 and we represent the total field for different frequencies (see figure 2).

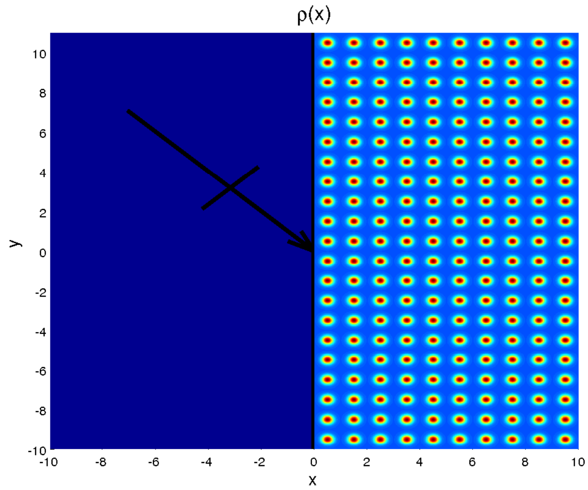

Figure 1. The refraction index in the two half space and the direction of the incident plane wave ($\theta = \pi/4$)

The last situation studied is the **2D-plane** case: we are interested in propagation media which are a local perturbation of an infinite structure which is periodic in two directions. Physically, the defect may be a geometric defect, a perturbation of a periodic refraction index or a local source. For simplicity, we will restrict ourselves to the case where the periodicity cell presents at least two symmetries, which is often the case in the applications. By revisiting the Floquet-Bloch Theory, we propose a method for constructing DtN operators. We show in particular that we can factorize the DtN operator as a product of two operators. The first one is the DtN operator corresponding to a periodic half space problem. The second one is the solution of an affine operator equation. The idea is to solve this equation using not physical variables but Floquet-Bloch variables. That means that instead of considering that the operator is defined on L^2 functions of the real line, we consider that it is defined on $L^2([0, L] \times [0, 2\pi/L])$. Numerically, the interest is then to replace the discretization of a (1D) infinite set by the discretization of a (2D) compact set. This work is currently in progress.

The first perspectives of this work is to generalize this study to a 3D problem and to analyse the extension of this method to elasticity and electromagnetism. This work applied to elasticity takes part to the collaboration with EADS and their interest on the composites materials. Applied to electromagnetism and photonic crystals

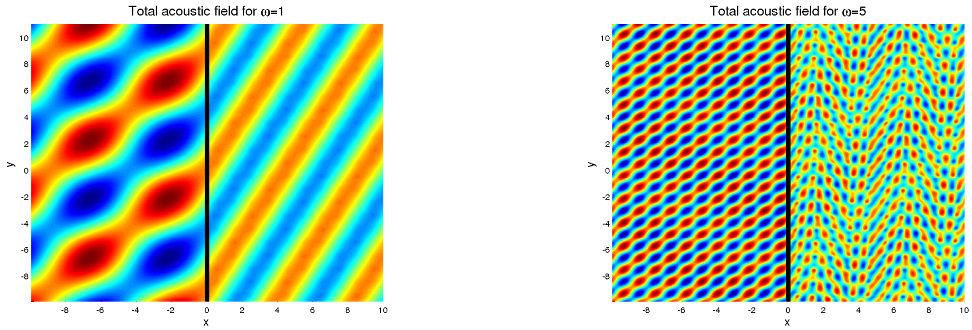

Figure 2. The total field fot two frequencies $\omega = 1$ (to the left) and $\omega = 5$ (to the right)

modelization, it enters in the framework of the ANR collaboration with the laboratory of Fundamental Electronics of Orsay University.

6.3.2. Reduced basis method for harmonic wave propagation problems

Participant: Jeronimo Rodríguez.

This work is developed in collaboration with J. S. Hesthaven (Brown University) and Y. Maday (Paris VI). When solving problems related with active control or shape optimization we are often interested on the systematic evaluation of an output $s(\mu)$ that depends on an input $\mu \in \mathcal{D} \subset \mathbb{R}^p$. Both quantities are usually related by the evaluation of a functional $l(\cdot;\mu)$ over $u(\mu) \in X$ solution of a PDE

$$a(u(\mu), v; \mu) \ = \ f(v; \mu), \quad v \in X.$$

The large number of evaluations needed on this kind of applications enforces the necessity of using numerical methods that are not only accurate but also very fast. A natural approach to follow consists on computing a good approximation of the PDE solution (that we will denote the *true approximation*) using a Galerkin approach and then evaluating the output using this discrete function. Let us denote by $\mathcal{N}$ the dimension of the approximation space. The solution provided by this approach will be accurate enough is we use fine meshes and/or higher order polynomials for the computation. Unfortunately, this is in general too expensive in terms of CPU time, the method being too slow for the purposes of our applications as $\mathcal{N}$ might be too large.
To improve the speed of this first approximation we apply the reduced basis method. This approach is based on the assumption that the solution of the PDE depends smoothly on the input parameters. In this way, we might expect that the approximation space given by the span of some solutions of the PDE, i. e.,

$$X_N \ = \ \text{span}\{u(\mu), \quad \mu \in \mathcal{S}_N\},$$

for a set of "well selected" parameters

$$\mathcal{S}_N \ = \ \{\mu_j \in \mathcal{D}, \ j \in \{1, \cdots, N\}\},$$

will provide a good approximation of the solution for any other value. As soon as these functions are well adapted to the problem we might expect that $N \ll \mathcal{N}$. In most applications, it has been noticed an spectral rate of convergence towards the true approximation.

Under some assumptions on the operators a very effective off-line on-line strategy can be followed. The off-line part is performed once and for all. The number of operations of the on-line part of the algorithm will only depend on N and the parameter complexity of the functional, being thus, very fast. Another important feature of the reduced basis method is that a rigorous a posteriori error estimator that certifies the reduced basis solution can be provided. Currently, the research on this topic is mainly related to the improvement of the estimator efficiency and the possibility of relaxing the assumptions allowing the off-line on-line computational strategy.

6.3.3. Modeling of meta-materials in electromagnetism.

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Carlo Maria Zwölf.

C. M. Zwölf (PhD student) has been working in this field since the end of 2004, under the joint supervision of A. S. Bonnet-Ben Dhia and P. Ciarlet. After the study of a simplified scalar model, we consider Maxwell's equations for the electric field, involving wave transmission between media with opposite sign dielectric and/or magnetic constants. Since the sign of the constants shifts at the interfaces, the main mathematical question is whether there exists a solution to this set of equations. To provide a positive answer, we build a three-field variational formulation equivalent to the original set of equations and we show that, under some suitable conditions, the formulation is well-posed in the sense that it fits into the coercive plus compact framework. Interestingly, this formulation can be discretized, and numerical experiments are under way.

6.3.4. Scattering by a locally non uniform open waveguide

Participants: Christophe Hazard, Anne-Sophie Bonnet-Ben Dhia.

The aim of this common work with Lahcen Chorfi and Ghania Dhakia is to study the scattering by a localized defect in an open acoustic waveguide. Using a spectral approach based on the diagonalization of the transverse operator, we have proved the existence and uniqueness of the solution to the time-harmonic wave equation together with a suitable radiation condition at infinity.

6.3.5. Aeroacoustics

Participants: Jean-François Mercier, Anne-Sophie Bonnet-Ben Dhia, Eve-Marie Duclairoir.

To improve our method to solve scattering problems in shear flows, we have developed a "low Mach number" model, rigorously valid for flows of low velocities, but in practice giving good approximations for non-small Mach numbers (up to at least 0.4). This model consists in deriving, from the non-local link (convolution formula consisting in a highly oscillating integral for low Mach numbers) between the vorticity (rotational of the displacement) and the displacement, a local relation easier to deal with in the regularized Galbrun's equations. In particular, this simplified model allows to adapt easily the numerical method to the 3D case.

6.3.6. Coefficients d'influence pour les méthodes d'équations intégrales

Participant: Marc Lenoir.

Les méthodes d'équations intégrales variationnelles, ou méthodes d'éléments finis de frontière, constituent une méthode de choix pour la résolution numérique des problèmes de diffraction, tout particulièrement dans le domaine de l'électromagnétisme. Depuis quelques années, la mise au point d'une technique d'approximation des coefficients hors diagonaux par développement en harmoniques sphériques, dite méthode des multipôles, a permis d'accéder à des fréquences jusqu'ici inaccessibles.

Une partie des difficultés qui apparaissent lors de la mise en œuvre de cette méthode peut être imputée à un calcul imprécis ou même erroné des coefficients proches de la diagonale, qui découlent du calcul d'une intégrale singulière. La méthode que nous avons mise au point repose sur l'utilisation d'une formule de quadrature relative aux fonctions homogènes, qui dans le cas tridimensionnel s'applique directement à la solution élémentaire du laplacien, soit en fait au premier ordre du développement en fonctions homogènes de la solution élémentaire d'une équation du second ordre à coefficients constants. L'extension au cas bidimensionnel, qui fait intervenir un logarithme est aisée. Un compte rendu à l'Académie a été accepté.

6.4. Absorbing boundary conditions and absorbing layers

6.4.1. Exact bounded PML's with singularly growing absorption

Participants: Eliane Bécache, Andres Prieto.

For the time-harmonic Helmholtz equation, the idea of using a singular layer parameter in order to get exact bounded PMLs has been proposed in a recent paper of Bermudez, Hervella, Prieto and Rodriguez. Last year, we have started to work on the extension of this model to time dependent waves, essentially on a numerial point of view. This year, we focused on the theoretical analysis of this model, i.e. the proof of existence and uniqueness and of the exact character of the bounded layers. The main difficulty is related to the functional framework, which involves the use of weighted spaces.

6.4.2. On the stability of numerical schemes for approximating PMLs in corners

Participants: Eliane Bécache, Andres Prieto.

In several papers, some instabilities of PMLs have been reported, starting at the corners. Trying to understand these phenomena with a code developed in our lab, and based on a discretisation using spectral mixed finite elements in space and second order finite differences in time, we also observed instabilities, for a time step satisfying the CFL condition in the interior domain, which were removed while decreasing the time step. Analyzing the scheme, we have obtained an explicit CFL condition depending on the parameter of the layer, first for the lower order scheme, and more recently for any order. We have then proposed a new scheme which does not deteriorate the stability of the global scheme. These results have been validated experimentally.

6.5. Waveguides and resonances

6.5.1. The multimodal method

Participants: Christophe Hazard, Anne-Sophie Bonnet-Ben Dhia, Eric Lunéville.

The application of the multimodal method was studied in the context of a closed acoustic waveguide with slowly varying cross-section. In this situation, the exponential behaviour of the modal components of the field is included in the choice of the basis functions in the longitudinal direction. This allows to use larger elements in this direction, which was confirmed by numerical experiments.

6.5.2. Leaky waves in open waveguides

Participants: Christophe Hazard, Anne-Sophie Bonnet-Ben Dhia.

The extension of the multimodal approach to open waveguides leads naturally to the concept of leaky modes. This notion is similar to the notion of resonances which are the basic objects for the application of the Singularity Expansion Method in scattering problem. An attempt was made to summarize the use of leaky modes in the existing literature. In the case of a slab waveguide, we have compared some numerical techniques to compute approximations of these modes. This work comes within the framework of the ANR *SimNanoPhot* (common project with Institut d'Electronique Fondamentale, Orsay).

6.5.3. Elastic waveguides

Participants: Vahan Baronian, Anne-Sophie Bonnet-Ben Dhia, Eric Lunéville.

We have developed a method to solve the diffraction problem of Lamb waves in a 2D elastic waveguide. The diffraction is produced by a defect, for instance a crack.
The method is based on a coupling between a modal decomposition of the field in the safe part of the waveguide, and uses a finite element representation of the field in the bounded region containing the defect.
For the modal decomposition, the field must be described by a 4-vector (Ux, Uy, Sxx and Sxy) composed of the displacement and the horizontal stress. By using the symmetry properties of the Lamb waves (right going and left going modes), we can split the initially 4-vector in two 2-vectors (X = [Ux,Sxy] and Y = [Sxx,Uy]) described by only two set of modal components (Xn and Yn).

Moreover the modal 2-vectors (Xn and Yn) have a bi-orthogonality relation (due to Fraser) that allows us to project X (resp. Y), on the modal basis Yn (resp. Xn). Consequently, from the knowledge of the modal decomposition of one 2-vector Y (resp. X), we can build an impedance relation Y = C.X (resp. X = C.Y) on each transverse section of the safe part of the waveguide.
The idea is to use this impedance relation as a boundary condition on the fictitious boundaries of the computational domain. Unfortunately, this is not directly tractable in a classical displacement variational approach, since X (resp. Y) involves a component of the stress field, which is not available. So the originality of our method is to build a mixed variational formulation, introducing the restriction of Sxy (resp. Sxx) on the fictitious boundaries as new unknowns.
The technique has been first validated in the case of a safe bounded wave-guide to check the transparency of the boundary operator, and then applied to the case of a cracked one.

6.5.4. Acoustic waveguides with absorbing walls

Participants: Jean-François Mercier, Anne-Sophie Bonnet-Ben Dhia.

This work is developed in collaboration with S. Poernomo and E. Redon. In order to solve scattering problems in presence of treated walls, we want to couple a finite element approximation around the source to a modal expansion on the duct modes to select the outgoing solution. The modes are not orthogonal in presence of a uniform flow and classical methods based on the construction of a Dirichlet-to-Neumann operator fail. However we have been able to build alternative transparent boundary conditions thanks to the modes, both theoretically and numerically. Moreover modes become "nearly" orthogonal when the mode order increases which ensures the convergence of the method.

6.6. Asymptotic methods and approximate models

6.6.1. Asymptotic models for thin slots

Participants: Christophe Kirsch, Patrick Joly, David Sanchez.

These works are in the continuation of the PhD thesis of S. Tordeux who was devoted to the complete asymptotic analysis .

In the framework of the Post-Doc of D. Sanchez, we have extended tha first part of the thesis of S.Tordeux to the construction and the analysis of a 2D-3D approximate model for the time harmonic (3D) Maxwell equations in media including slots. The analysis is now complete.

With the stage of O. Bessis, we have also looked at the propogation of wave inside a network of thin slots in 2D. The objective is to understand how this can be modeled as a system of 1D PDE's in a networs of segments with a particular attention to the node connection conditions. We are particularly interested in trying to improving the existing models (see for instance the works by Kuchment, Schatzman-Rubinstein)

In the framework of the Post-Doc of C. Kirsch, we have looked at the numerical aspects of the problem of the junction of a slot with an exterior domain. In 2D, for the Helmholtz equation, we have proposed a mixed formulation of the transmission problem slot / exterior domain which is of primal-dual type. An interesting point with this method is that the error analysis shows that this method does not suffer from any locking phenomenon.

6.6.2. Asymptotic models for thin wires

Participants: Xavier Claeys, Patrick Joly, Houssem Haddar, Marc Durufle, Francis Collino.

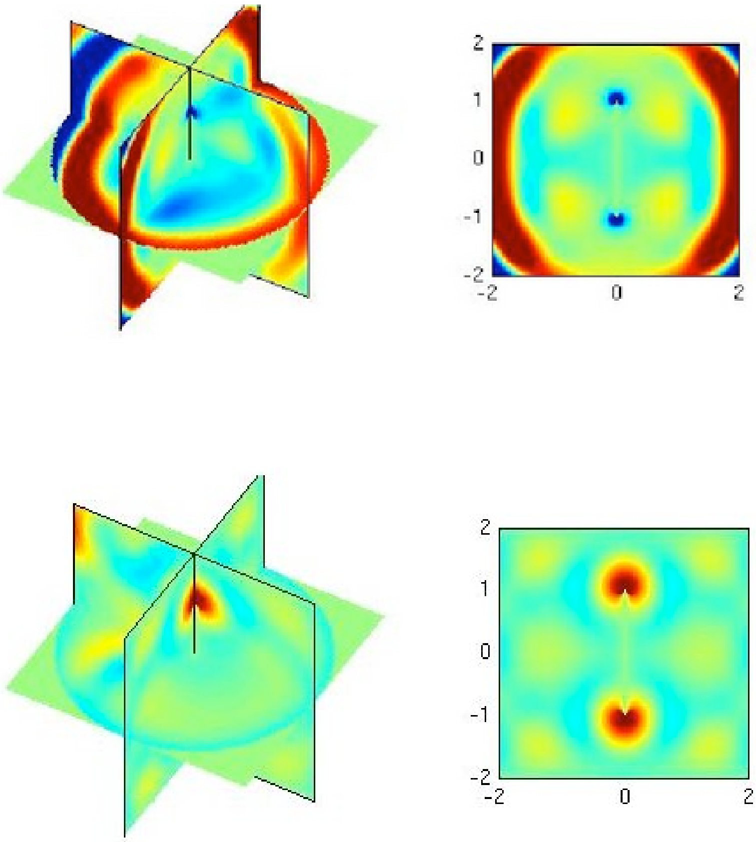

Figure 3. Diffraction of an electromagnetic wave by a finite wire. Snapshots (2D and 3D) of the modulus of the electric field at two successive instants (up and down pictures)

The purpose of this work is the construction of efficient theoretical and numerical models for the diffraction of (acoustic or electromagnetic) waves by perfectly conducting thin wires. At the current time, two models are related to this topic in electrodynamics. The first one is a physical model for antennas called the Pocklington model, and the second one is a numerical model for thin wires in finite difference schemes called the Holland model. To our knowledge, none of these models has been given any theoretical convincing justification up to now. Our works aims at deriving, justifying (and possibly improving) approximate models of the same nature. Our approach is the following:

1. For a given problem, we dervive an asymptotic analysis with respect to a small parameter, the thickness ε of the wire.
2. We use the previous to derive and / or justify simple approximate models suitable for efficient numerical computations. This includes the comparison with other existing models.
3. We propose and analyse numerical schemes for these simplified models.

Concerning asymptotic expansions, our recent contributions in this direction are the following:

- For 2D problems, last year, X.Claeys, H.Haddar and P.Joly achieved a complete asymptotic analysis of this problem using matched asymptotic expansions for the diffraction of acoustic waves by a small obstacle in 2D problems(with both Dirichlet and Neumann conditions).
- For 3D scalar problems (Helmholtz equations) with Dirichlet conditions, We have shown that the technique of matched asymptotic expansions can still be applied, at least for a straight wire, thanks to the use of spheroïdal coordinates. The algorithm for the complete formal asymptotic expansion of the solution has been derived. The firsat term of the asymptotics has been completely justified and the analysis should be extended to higher orders without any problem.
- For 3D vectorial problems (Maxwell equations), spheroïdal coordinates can still be used associated to an adapted representation of vector fields in such a system of coordinates. The algebra of the corresponding computations is quite complex. This work is in progress.

Concerning approximate models, the actual state of the work is the following:

- We propose a model based on the ideas of the fictitious domain method with Lagrange multipliers with an additional assumption : traces of the unknown solutions (or of first derivatives of this solution) are suppsed to be constant along each cross section of the wire. The corresponding model admits a mixed variational formulation having good properties (in particuler energy conservation in the time dependent case.
- The asymptotic analysis has permitted us to completely justify such a model for scalar problems. The error is in $O(\varepsilon)$ far from the wire and $O(\sqrt{\varepsilon})$ close to the wire.
- A by product of our analysis is a mathematical justification of the Pocklington model that appears to be of similar accuracy. However, we believe that our model ias more flexible and adapted to numerical computations, in particular in time domain.

Our most recent progress concerns the numerical treatment of our approximate model in the simple 2D case.

- A brutal Galerkin approximation of finite element type of the mixed formulation suffers from numerical locking phenomenon : typically it provides an error in $O(\varepsilon + h/\varepsilon)$ where h is the space step. In practice, it makes impossible to take h significantly larger then ε ahich is a priori the practical interest of the wire model.
- We propose an enriched Galerkin method by enriching the approximation space for the primal variable with one special basis function (an ε dependent "singular" function.deduced from the exact asymptotic expansion. The theory we have been able to establish for the moment predicts an error in $O(\log|\varepsilon|(\varepsilon + h))$ which proves that the locking phenomenon has been circumvented. Actual numerical computations show that the accuracy is even much better (we conjecture that, far fron the wire the error is $O(\varepsilon{+}h^m)$ with finite elements of order m).

- By decomposing the approximate solution into its “singular” part and its “finite element” part, we obtain an equivalent scheme wich is of the same nature of the well known Holland model used for FDTD computations. This observation has (at least) two interests :
  - It provides a generalization of the Holland model outside the FDTD framework.
  - It provides theoretical justification of the Holland model.

The perspectives are the following:

- Use higher order terms of the asymptotics to get more accurate approximate models.
- Extend the above numerical procedure to the scalar 3D case.
- Generalize the complete approach to 3D Maxwell’s equations.

6.6.3. Quasi-singularities for Maxwell’s equations

Participants: Patrick Ciarlet, Samir Kaddouri.

We model the corona discharge, around a ’rounded’ corner. The goal is to determine the value of the charge density at the tip of this corner. To that aim, we compute the electrostatic potential in an unbounded domain, with a boundary that includes a ’rounded’ corner of small curvature radius ϵ. The 2D cartesian and axisymmetric cases have been solved and in particular, an explicit relationship describing the value of the charge density (with respect to ϵ) has been established. The same analysis is currently carried out in 2D1/2 prismatic and axisymmetric geometries. The resulting method has been implemented numerically, and tested on a number of test-cases. Among others, it has been compared to an integral method, which is more more costly to implement. The results are comparable.

6.6.4. Approximate models for hydrodynamic instabilities

Participants: Patrick Joly, Anne-Sophie Bonnet-Ben Dhia, Kamel Berriri.

This new topic has been developed within the framework of the PhD thesis of K. Berriri.

When one studies the propagation of sound in shear flows, one is faced to the apparition of well known instability phenomena : the Kelvin-Helholtz instability. This question is particularly crucial when the Mach profile of the flow presents strong variations: when one tries to model such flow with discontinuities, the natural transmission conditions lead to strongly ill-posed evolution problem.

This clearly appears when one looks for the fundamental solution of Galbrun’s equation via the Cagniard-De Hoop’s method which, in addition to classical phenomena observed in presence of diopter (transmission relection phenomena head waves), c;early exhibits the contribution the instability waves.

Using asymptotic analysis boundary layer techniques, we have been able to propose approximate transmission conditions that may lead to a well posed problem and permit us to model hydrodynamic instabilities.

The mathematical analysis of the approximate problem is related to the study of a non self-adjoint eigenvalue problem. The analysis of this problem is non trivial and emphasizes the influence of the properties of the Mach profile (smoothness, convexity) on the well-posedness of the related initial value problems.

This analysis points out the difficulties for the construction of a stable discretization scheme but also suggests some ideas to get such a scheme. The design and the implementation of such a method will be the next step of this study.

6.7. Imaging and inverse problems

6.7.1. Near-field sampling methods

Participant: Houssem Haddar.

We pursued the development of so-called qualitative methods in electromagnetic inverse scattering problems by extending the recently introduced sampling algorithm (RG-LSM) based on the use of the reciprocity gap principle to 3-D electromagnetic problems. This work has been carried out in collaboration with F. Cakoni from the university of Delaware and M'B Fares from CERFACS. As compared with the classical linear sampling method, the new algorithm uses Cauchy data on a given surface and avoids the need for computing the Green tensor of the background medium. We gave the theoretical foundation of this algorithm in the case of scattering problems from embedded perfect conductors and also anisotropic inclusions. We also performed some numerical tests using synthetic data that demonstrated the viability of both imaging methods (LSM and RG-LSM) in the context of imaging buried perfect conductors in a two-layered medium. These first results are not representative of all the potentials of RG-LSM but only aimed at validating this method in a simplified configuration. A more detailed numerical work, including for instance anisotropic inclusions and complex backgrounds in under preparation.

Parallel to the development of imaging techniques, we also explored the use of sampling methods to provide qualitative informations on the scatterer physical properties. With that perspective, and in collaboration with F. Cakoni, we proposed an algorithm to estimate the conductivity of a thin coating of an anisotropic inclusion without the need of a priori knowledge of the anisotropic inclusion properties. The algorithm uses the solution of so-called interior transmission problem, which can approximated by the indicator function of the RG-LSM algorithm. The numerical validation of such approach is under investigations.

6.7.2. Inverse crack problem

Participants: Houssem Haddar, Fabrice Delbary.

Within the framework of the phd thesis of F. Delbery, defended in 2006, and in collaboration with A. Ben Abda, we pursued our research on the use of reciprocity gap principle to characterize the shape of planar cracks and their impedance boundary condition from acoustic overdetermined measurements at a given boundary. We demonstrated that satisfactory estimates of the impedance can be obtained from quasi-explicit reconstructions of the solution jumps across the crack. Numerical examples have shown good performances of this approach. The reconstruction of non planar crack, with Dirichlet boundary condition, has also been studied by using an algorithm based of RG-LSM described above. Satisfactory numerical results has been obtained for the 2-D problem from synthetic data.

6.7.3. Conformal mapping and the inverse electrostatic problem

Participant: Houssem Haddar.

In collaboration with R. Kress, we have recently developed a new simple and fast numerical scheme for solving two-dimensional inverse boundary value problems for the Laplace equation that model nondestructive testing and evaluation via electrostatic imaging. The inverse problem consists of reconstructing the interior boundary Γ_0 from the Cauchy data on Γ_1 of a harmonic function satisfying a homogeneous Dirichlet or Neumann boundary condition on the unknown interior boundary curve Γ_0. The reconstruction method consists of two parts: In a first step, by successive approximations a nonlocal and nonlinear ordinary differential equation is solved to determine the boundary values of a holomorphic function Ψ on the outer boundary circle C_1 of an annulus B. Then in a second step via regularizing a Laurent expansion in the sense of Tikhonov an ill-posed Cauchy problem is solved to determine Ψ in the annulus and the unknown Γ_0 as the image $\Psi(C_0)$ of the interior boundary circle C_0 of B.

We extended this approach to the case of a homogeneous impedance boundary condition. The analysis and the numerical implementation of the method differ from the limiting cases of the Dirichlet and Neumann condition since the impedance problem in the annulus B that is associated with the impedance problem in the original domain D depends on the conformal map Ψ. We proved uniform convergence of the algorithm for sufficiently large impedances, i.e. when we are clsoe to the Dirichlet problem, and gave numerical validating examples. We are currently investigating a modification of the scheme in order remove this restrictive assumption on the impedance.

6.7.4. *Quasi-reversibility*

Participants: Laurent Bourgeois, Eric Lunéville.

A very recent contribution concerns the method of quasi-reversibility to solve ill-posed Cauchy problems for an elliptic equation. It consists of transforming the initial second-order and ill-posed problem into a family (depending on a small parameter) of fourth-order and well-posed problems. This method, which was first proposed 40 years ago, still arises questions, from a theoretical and a numerical point of view. Concerning numerical aspects, we developed both a mixed formulation of quasi-reversibility (which enables the use of simple finite elements) and a non-conforming formulation in 2D (with the help of the Morley's finite element). In both cases, we analyzed convergence of the solution of quasi-reversibility to the true solution, when the small parameter of the method as well as the size of the mesh tend to zero. In the second case, we also proposed a strategy based on the Morozov's principle and using duality in the sense of Fenchel-Rockafellar in order to make an efficient choice for these two small parameters. Concerning theoretical aspects, there is some work in progress concerning general stability results for ill-posed elliptic problem (which in particular lead to convergence rates for the method of quasi-reversibility), with the help of Carleman estimates. Some results were recently obtained in smooth domains, and it is natural to wonder if they still hold in non-smooth domains, for example in domains with corners.

6.7.5. *Time reversal*

Participants: Christophe Hazard, Chokri Ben Amar.

The works recently made in this topic are twofold. On one hand, numerical simulations of DORT method (with the code MELINA) were realized for a time-harmonic reversal mirror located in a closed waveguide, and compared to the case of propagation in the free space. On the other hand, we have studied the case of a time-dependent reversal mirror, in particular the effect of an iterative process which optimizes the energy scattered by the obstacles.

7. Contracts and Grants with Industry

7.1. Contract POEMS-DGA

Participants: Patrick Ciarlet, Christophe Hazard, Grace Hechme.

This contract concerns the Singular Expansion method for time dependent problems and the resolution of transient Maxwell's equations in singular domains.

7.2. Contract POEMS-EADS-1

Participants: Gary Cohen, Pascal Grob, Patrick Joly.

This contract is about the numerical simulation of time dependent vibro-acoustics phenomena using coupled methods (3D / 2D finite elements, retarded potentials / 2D finite elements)

7.3. Contract POEMS-EADS-2

Participants: Sonia Fliss, Patrick Joly.

This contract is about the numerical simulation of elastic wave propagation in composite materials (periodic structures with a defect) in the time harmonic regime.

7.4. Contract POEMS-EADS-3

Participants: Anne-Sophie Bonnet-Ben Dhia, Eve-Marie Duclairoir, Jean-François Mercier.

This contract is about the numerical simulation of frequency domain aeroacoustics using Galbrun's equations and regularized finite element techniques.

7.5. Contract POEMS-Airbus

Participants: Abdelaâziz Ezziani, Patrick Joly.

This contract is about the hybridation of time domain numerical techniques in aeroacoustics (Linearized Euler equations).

7.6. Contract POEMS-ONERA Palaiseau

Participants: Gary Cohen, Marc Duruflé.

This contract is about the numerical simulation of time harmonic wave propagation using higher order discretization methods (mixed finite elements, Discontinuous Galerkin)

7.7. Contract POEMS-ONERA-CE Gramat

Participants: Gary Cohen, Patrick Joly.

This contract is about hybrid methods for the time domain solution of Maxwell's equations.

8. Other Grants and Activities

8.1. National Cooperations

- SimNanoPhot : project of the ANR in collaboration with IEF (Institut d'Electronique Fondamentale) of the University of Orsay. It concerns the modelization of micro and nano-structures in optics.
- GDR Ultrasons: this GDR, which regroups more than regroup 15 academoc and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.
- We have been involved in several scientific manifestations of the GDR Ondes.
- A new ANR (RNTL) project *MOHYCAN: MOdélisation HYbride et Couplage semi-ANalytique pour la simulation du CND*.
  Topic: *On the coupling of the finite element code ATHENA with the semi-analitic code CIVA. Non-destructif testing.* Collaborators: CEA-LIST (main contact), EDF and CEDRAT.

8.2. International Cooperations

- The project is involved in the INRIA/NSF collaboration "Collaborative Effort on Approximate Boundary Conditions For Computational Wave Problems" with J. Hesthaven (Brown University) and P. Petropoulos (New Jersey University).
- The Project is involved in a STIC project with the LAMSIN of ENIT (Tunis) with A. Ben Abda and N. Gmati.
- The Project is member of the Associate Team ENEE between INRIA and Maghreb.
- The GDR Ultrasons has been extended to a collaboration with United Kingdom.
- Collaboration with *Universidade de Santiago de Compostela*, Santiago de Compostela, A Coruña, Spain. (Since August 2005). Topic: *Absorción del ruido de banda ancha en campo difuso mediante control activo multicanal de la impedancia de sistemas absorbentes multicapa. (Acoustics)* Collaborators: A. Bermúdez, J-L Ferrín.
- Participation to MATHmONDES 2, 2nd British-French Workshop on Mathematical Techniques for Wave Problems, University of Manchester, June 2006.

8.3. Visiting researchers

Liliana Borcea, Professor at Rice University, specialist on Imaging methods, has been visiting the Project for six months.

9. Dissemination

9.1. Various academic responsibilities

- A. S. Bonnet-Ben Dhia became the Head of the Electromagnetism Group at CERFACS (Toulouse)
- A. S. Bonnet-Ben Dhia is in charge of the relations between l'ENSTA and the Master "Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)".
- H. Haddar was a member of the Comité des bourses of INRIA Rocquencourt.
- H. Haddar is "mediateur" for the PhD students of INRIA Rocquencourt.
- P. Joly is a member of the Commission de Spécialistes of the University Paris VII.
- P. Joly is a member of the Post Docs Commission of INRIA Rocquencourt.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- P. Joly is an expert for the MRIS (Mission pour l'Innovation et la Recherche Scientifique) of DGA (Direction Générale de l'Armement)
- P. Joly has been co-editor (with V. Gobin of ONERA) of special issue of the Comptes Rendus de l'Académie des Sciences on Mathematical and Numerical Modeling in Electromagnetism, following the Journée Scientifique ONERA on the same topic in 2005.
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- Le nouveau site web du Projet Poems à été conçu et mis au point par M. Lenoir (avec le concours de M. Diamantini (ENSTA)). Mise en place effective prévue dédut 2007.
- The Project organizes the monthly Seminar Poems (Coordinators: X. Claeys, J. F. Mercier)
- Organization of a course on Discontinuous Galerkin Methods for Wave Propagation Problems (in the framework of CEA-EDF-INRIA Schools), Rocquencourt, November 2006.
- Organization of the "Journee Techniques Asymptotiques" (Ensta Paris, June 2006) (X. Claeys)
- The Project has participated to the "Journee des nouveaux arrivants" at INRIA Rocquencourt (E. Bécache, X. Claeys, S. Fliss, P. Joly)
- Evaluation Committee of the de "Unite de Mathématiques Appliquées" of ENSTA, (July 2006)

9.2. Teaching

- Laurent Bourgeois
  - *Outils élémentaires d'analyse pour les EDP*, ENSTA, Paris
  - *Contrôle optimal des EDP*, ENSTA, Paris
- Anne-Sophie Bonnet-Ben Dhia
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA, 2006
  - *Propagation d'ondes* , Master de Dynamique des Structures et Couplages, Ecole Centrale de Paris, 2006
  - *Guides acoustiques* , Master de Dynamique des Structures et Couplages, Ecole Centrale de Paris, 2006
  - *Propagation dans les guides d'ondes*, ENSTA, 2006
  - *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques*, ENSTA, 2006

- Xavier Clays
  - *Géométrie différentielle élémentaire*, UVSQ, Versailles
  - *Méthode des éléments finis*, ENSTA, Paris
- Gary Cohen
  - *Cours de Master II: Méthodes numériques pour les équations des ondes*, Université de Paris-Dauphine October-December 2006
  - *École des Ondes: Mixed-spectral discontinuous Galerkin methods for the wave equations*, Inria-Rocquencourt 27th November-1st October 2006
- Sonia Fliss
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Fonctions de la variable complexe*, ENSTA
- Houssem Haddar
  - *Calcul Scientifique* , Ecole des Mines, Paris
  - *Cours Eléments Finis*, ENSTA, Paris
- Christophe Hazard
  - *Théorie Spectrale*, Master Paris 6, Parcours Analyse Numérique et EDP, 2006
  - *Théorie Spectrale et application aux guides optiques*, ENSTA 2ème année, 2006
  - Outils élémentaires d'analyse pour les EDP, ENSTA 1ère année, 2006
- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Outils élémentaires d'analyse pour les EDP*, ENSTA, Paris
  - *Introduction to the mathematical analysis of linear wave propagation*, Master course, University of Santiago de Compostela (Spain), June 2006
  - *Introduction to numerical methods for elastic wave propagation*, Advanced course on waves in pre-stressed materials, CISM, Udine (Italie), September 2006
  - *Raffinement de maillage espace-temps et méthodes de Galerkin discontinues*, Ecole des Ondes, INRIA, Rocquencourt, Novembre 2006
  - *Méthodes volumiques pour la propagation des ondes*, College Polytechnique, Paris, Novembre 2006
- Marc Lenoir
  - *Résolution des problèmes d'acoustique par équations intégrales*, Cours à l'Université Catholique du Chili, Santiago, Janvier 2006
  - *Fonctions d'une ariable complexe*, ENSTA, 2006
  - *Méthodes d'équations intégrales*, ENSTA, 2006
  - *Théorie*, Master M&S de l'INSTN, 2006
- Jing Li
  - *Introduction a la discretisation des EDPs*, ENSTA, Paris
  - *Calcul Scientifique*, Ecole des Mines, Paris
- Jean-François Mercier
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA 2006

  - *Fluides compressibles*, ENSTA 2006
  - *Variable complexe*, ENSTA 2006
  - *Ondes dans les milieux continus*, ENSTA 2006
- Jeronimo Rodríguez
  - *Optimisation quadratique*, ENSTA, Paris
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Simulation numérique*, ENSTA, Paris
- Carlo Maria Zwölf
  - Monitorat, University of Versailles-Saint Quentin.

9.3. Participation in Conferences, Workshops and Seminars

- Anne Sophie Bonnet-Ben Dhia
  - *Acoustic scattering in presence of a mean flow*, workshop on Advances in Computational Scattering, Banff, february 2006
  - *Application of the Cagniard-de Hoop method to aeroacoustics*, MATHmONDES 2 2nd BRITISH-FRENCH WAVES MEETING, Manchester, june 2006
  - *Propagation du son dans un écoulement : simulation numérique du régime périodique établi*, 38ème Congrès National d'Analyse Numérique, Guidel, may 2006
  - *Propagation acoustique dans un écoulement discontinu, ou comment résoudre un problème fortement mal posé*, 2ème Journées sur les Equations Différentielles et leurs Applications. Annaba, november 2006.
  - *De la fibre optique au pot d'échappement, suivez le guide d'ondes*, Conférence à l'Ecole Alsacienne, november 2006.
- Laurent Bourgeois
  - *Locating an obstacle in a 3D finite depth ocean using the convex scattering support*, PICOF'06, Nice (France), April 2006
  - Invited speaker to the "Séminaire EDP et applications" of the "Institut Elie Cartan", Nancy (France), May 2006
- Patrick Ciarlet
  - Séminaire à l'Université de Zürich (Zürich, Suisse), January 2006
- Xavier Clayes
  - *Méthode asymptotique pour la diffraction par des fils minces*, Journee Techniques Asymptotiques, Ensta Paris, June 2006
  - *Asymptotic models for diffraction by thin wires*, AIMS'6th International Conference on Dynamical Systems and Differential Equation, Poitiers, June 2006
  - *Asymptotic models for diffraction by thin wires*, Seminar in Lamsin, Tunis, July 2006
- G. Cohen
  - *Méthodes volumiques d'ordre élevé en électromagnétisme*, Journées electromagnetisme et guerre electronique, Toulouse, March 21-22 2006
  - *Méthodes volumiques d'ordre élevé en électromagnétisme*, CEM06, St Malo, April 4-6 2006

  - *Higher-order numerical methods for Maxwell's equations*, Duke University, Durham, USA, June 7 2006
  - *High-Order Numerical Methods for Maxwell's Equations on Unstructured Meshes*, Recent Mathematical and Computational Developments of Maxwell's Equations: Challenges and Frontiers, Weihai, China, July 24-28 2006

- M. Duruflé
  - *A discontinuous Galerkin method Q_r applied to aeroacoustics*, ECCOMAS CFD 2006, Egmond aan Zee, The Netherlands, September 5-8 2006

- Sonia Fliss
  - *Computation of Harmonic Wave Propagation in Locally Perturbed Infinite Periodic Media*, MATHmONDES-2, Manchester, United Kingdom, June 2006
  - *Propagation des ondes dans des milieux périodiques infinis localement perturbés*, Séminaire du LAMSIN, Tunis, Juillet 2006
  - *Computation of Harmonic Wave Propagation in Locally Perturbed Infinite Periodic Media*, Mathematical Colloquium of the Universität Karlsruhe, Karlsruhe, Germany, November 2006

- H. Haddar
  - *Invitation from March 19 till March 26*, Appl. Math. Departement, UDEL.
  - *A Class of Non-iterative Methods Applied to Microwave Tomography at a Fixed Frequency*, PIERS'06, Cambridge, March 2006
  - *An application of conformal mapping to image non perfectly conducting inclusions*, PICOF'06, Nice, April 2006
  - *Imagerie electrostatique : une approche originale basee sur les applications conformes.*, Neuvieme rencontre tunisienne d'ete en mathematique : E.D.P, Analyse Non Lineaire et Geometrie, ENIT- LAMSIN, July 2006
  - *Scientific sejourn from May to July 2006*, LAMSIN, Tunis.
  - *"Imagerie par micro-ondes basee sur les donnees de Cauchy" (application a la detection d'anisotropies)*, CMAP Seminar, Ecole polytechnique, November 2006

- C. Hazard
  - *Focalisation sélective par retournement temporel d'ondes acoustiques*, Seminar POEMS, ENSTA, april 2006.

- P. Joly
  - *Variational methods for time dependent non destructive testing experiments*, Invited conference, AFPAC 06, Wye (England), January 2006
  - *Mathematical analysis and asymptotic models for wave propagation in media with thin slots*, Seminar at ETH Zurich (Switzerland), April 2006
  - *Analyse asymptotique de modèles de propagation d'ondes dans des milieux comportant des fentes minces*, Seminar at Collège de France, Paris, April 2006
  - *Mathematical analysis and asymptotic models for wave propagation in media with thin slots*, Seminar at the University of Santiago de Compostela (Spain), June 2006
  - *Approximate models for linear aeroacoustics in nearly discontinuous flows.*, Invited conference, Third Workshop on Numerical Methods for Evolution Equations, Heraklion (Crete), September 2006

  - *Le temps et sa discrétisation dans les équations aux dérivées partielles*, “Conference TIPE à l’attention des professeurs de classes préparatoires”, ENSTA, Paris, October 2006
  - *Analyse asymptotique de modèles de propagation d’ondes dans des milieux comportant des fentes minces*, Invited conference, Annaba (Algeria), November 2006

- Samir Kaddouri
  - *Computing electromagnetic charge densities at rounded corners : an improver Peek’s formula*, AIMS’6th International Conference on Dynamical Systems and Differential Equation, Poitiers, June 2006
  - *Développement asymptotique pour le calcul de densité de charge à la surface d’une pointe*, Journée Techniques Asymptotiques, Ensta Paris, June 2006.

- Christophe Kirsch
  - *Non reflecting boundary conditions for time dependent multiple scattering problems*, MATHmONDES2, Second British-French Workshop on Mathematical Techniques for Wave Problems, Manchester (England), June 2006.
  - *Modèles de fentes pour la propagation d’ondes*, Journée Techniques Asymptotiques, Ensta Paris, June 2006

- M. Lenoir
  - Invitation pour un mois à l’Université Catholique du Chili (Prof. Mario Duran)

- Jing-Rebecca Li
  - *Efficient and accurate solution of the heat equation in unbounded domains based on integral representation*, Seminaire POEMS, Roquencourt, France, Novembre 2006
  - *Efficient and accurate solution of the heat equation in unbounded domains based on integral representation*, SFB Seminar, Bonn, Germany, Decembre 2006

- Jean-François Mercier
  - *Simulation numérique de la propagation d’onde en présence d’un écoulement uniforme et d’une paroi traitée*, 8^e^ Congrès de la Société Française d’Acoustique, Tours, april 2006
  - *Simulation numérique par éléments finis d’une multi-diffusion acoustique*, 8^e^ Congrès de la Société Française d’Acoustique, Tours, april 2006
  - *Finite element computation of acoustic multiple scattering*, MATHmONDES 2, 2^nd^ British-French Workshop on Mathematical Techniques for Wave Problems, Manchester, june 2006
  - *Simulation numérique d’une multi-diffusion acoustique : comparaison avec le modèle de Foldy*, Quatrièmes journées du GDR-US : étude de la propagation ultrasonore en milieux non-homogènes en vue du contrôle non destructif, Presqu’île de Giens, may 2006

- David Sanchez
  - *Modèles de fentes pour la propagation d’ondes*, Journée Techniques Asymptotiques, Ensta Paris, June 2006
  - *Ondes longues en ferromagnétisme*, Séminaire Poems, INRIA Rocquencourt
  - *Ondes longues en ferromagnétisme*, Séminaire EDP de Strasbourg (IRMA), Strasbourg
  - *Ondes longues en ferromagnétisme*, Séminaire de Mathématiques Appliquées de Clermont-Ferrand, Strasbourg

- Jeronimo Rodríguez

- *Conservative methods for the discretization of wave propagation problems with local time stepping.* Seminar POEMS, École Nationale Supérieure de Techniques Avancées, Paris, France. May 11th 2006.
- *Space-time mesh refinement methods for elastodynamics.* GDR 2501. Presqu'île de Giens. France. May 14th 2006 – May 19th 2006.
- *Reduced basis method for harmonic wave propagation problems.* Seminar at Laboratoire de Mécanique et Acoustique (LMA), Marseille, France. November 21st 2006.
- *Reduced basis output bounds for harmonic wave propagation problems.* CEA-EDF-INRIA School on Discontinuous Galerkin Methods, INRIA Rocquencourt, France. November 27th 2006 – December 1st 2006.

- C. M. Zwölf
  - *Etude variationnelle pour la transmission d'ondes électro-magnétiques entre milieux de constantes caractéristiques de signes opposés*, Seminar POEMS, ENSTA, june 2006

10. Bibliography

Year Publications

Books and Monographs

[1] G. DERVEAUX, P. JOLY, C. TSOGKA. , V. A. DOUGALIS, J. EKATERINARIS, N. KAMPANIS (editors). *Numerical methods for elastic wave propagation*, Numerical Insight, To appear, CRC Press.

Doctoral dissertations and Habilitation theses

[2] K. BERRIRI. , Ph. D. Thesis, Université Paris 9, december 2006.

[3] F. DELBARY. *Identification de fissures par des ondes acoustiques*, Ph. D. Thesis, Université Paris 6, may 2006.

[4] M. DURUFLÉ. *Intégration numérique et éléments finis d'ordre élevé appliqués aux équations de Maxwell en régime harmonique.*, Ph. D. Thesis, Université Paris 9, February 2006.

[5] P. GROB. *Méthodes numériques de couplage pour la vibroacoustique instationnaire: Eléments finis spectraux d'ordre élevé et potentiels retardés*, Ph. D. Thesis, Université Paris 9, May 2006.

Articles in refereed journals and book chapters

[6] F. ASSOUS, J. P. CIARLET, E. GARCIA, J. SEGRÉ. *Time-dependent Maxwell's equations with charges in singular geometries.*, in "Comput. Methods Appl. Mech. Engrg.", vol. 196, 2006, p. 665–681.

[7] F. ASSOUS, P. CIARLET. *Vlasov-Maxwell simulations in singular geometries*, in "ICCS'06, Reading, UK, Part IV", Lecture Notes in Computer Science, Springer, 2006, p. 623–630.

[8] R. BARTHELMÉ, P. J. CIARLET, E. SONNENDRÜCKER. *Two- and three-field formulations for wave transmission between media with opposite sign dielectric constants*, in "Math. Models Meth. App. Sci.", To appear.

[9] C. BEN AMAR, N. GMATI, C. HAZARD, K. RAMDANI. *Numerical simulation of time-harmonic reversal mirrors for acoustic waves*, in "SIAM J. Appl. Math.", 2006.

[10] A.-S. BONNET-BEN DHIA, K. BERRIRI, P. JOLY. *Régularisation de l'équation de Galbrun pour l'aéroacoustique en régime transitoire*, in "ARIMA", 2006.

[11] A. S. BONNET-BEN DHIA, P. J. CIARLET, C. M. ZWÖLF. *Two- and three-field formulations for wave transmission between media with opposite sign dielectric constants*, in "J. Comput. Appl. Math.", to appear.

[12] A.-S. BONNET-BEN DHIA, E.-M. DUCLAIROIR, G. LEGENDRE, J.-F. MERCIER. *Time-harmonic acoustic propagation in the presence of a shear flow*, in "Journal of Computational and Applied Mathematics", 2006.

[13] L. BOURGEOIS. *Convergence rates for the quasi-reversibility method to solve the Cauchy problem for Laplace's equation*, in "Inverse problems", vol. 22, 2006, p. 413-430.

[14] L. BOURGEOIS, C. CHAMBEYRON, S. KUSIAK. *Locating an obstacle in a 3D finite depth ocean using the convex scattering support*, in "accepted in Journal of Computational and Applied Mathematics", 2006.

[15] E. BÉCACHE, A.-S. BONNET-BEN DHIA, G. LEGENDRE. *Perfectly matched layers for time-harmonic acoustics in the presence of a uniform flow*, in "SIAM Journal on Numerical Analysis", to appear.

[16] A. CHAIGNE, G. DERVEAUX, P. JOLY. *Améliorer la guitare acoustique*, in "Dossiers Pour la Science", vol. 52, May 2006, p. 74–75.

[17] N. CHAIGNE, P. JOLY, L. RHAOUTI. *Le son des timbales*, in "Dossiers Pour la Science", vol. 52, May 2006, p. 66–72.

[18] P. J. CIARLET, S. KADDOURI. *Multiscaled asymptotic expansions for the electric potential: surface charge densities and electric fields at rounded corners*, in "Math. Models Meth. App. Sci.", To appear.

[19] P. J. CIARLET, S. KADDOURI. *Justification de la loi de Peek en électrostatique*, in "C. R. Acad. Sci. Paris, Ser. I", vol. 343, 2006, p. 671–674.

[20] P. J. CIARLET, G. LEGENDRE. *Well-posedness of the Drude-Born-Fedorov model for chiral media.*, in "Math. Models Meth. App. Sci.", To appear.

[21] P. CIARLET, B. JUNG, S. KADDOURI, S. LABRUNIE, J. ZOU. *The Fourier singular complement method for the Poisson problem. II. Axisymmetric domains*, in "Numer. Math.", vol. 102, n^{o} 4, 2006, p. 583–610.

[22] M. CLAUSEL, M. DURUFLÉ, P. JOLY, S. TORDEUX. *A mathematical analysis of the resonance of the finite thin slots*, in "Appl. Numer. Math.", vol. 56, n^{o} 10-11, 2006, p. 1432–1449.

[23] G. COHEN, X. FERRIERES, S. PERNET. *A Spatial High-Order Hexahedral Discontinuous Galerkin Method to Solve Maxwell's Equations in Time Domain*, in "J. of Computational Physics", to appear.

[24] G. COHEN, P. GROB. *Mixed Higher Order Spectral Finite Elements for Reissner Mindlin Equations*, in "SIAM journal SISC", to appear.

[25] F. COLLINO, T. FOUQUET, P. JOLY. *Conservative space-time mesh refinement methods for the FDTD solution of Maxwell's equations*, in "J. Comput. Phys.", vol. 211, n^{o} 1, 2006, p. 9–35.

[26] J. DIAZ, P. JOLY. *A time domain analysis of PML models in acoustics*, in "Comput. Methods Appl. Mech. Engrg.", vol. 195, n^{o} 29-32, 2006, p. 3820–3853.

[27] M. DURUFLÉ, H. HADDAR, P. JOLY. *Higher order generalized impedance boundary conditions in electromagnetic scattering problems*, in "C. R. Phys. Acad. Sci. Paris", vol. 7, n^{o} 5, 2006, p. 533–542.

[28] J. C. GILBERT, P. JOLY. *Higher order time stepping for second order hyperbolic problems and optimal CFL conditions*, in ""Numerical Analysis and Scientific Computing for PDEs and their Challenging Applications"", Springer, To appear.

[29] P. GROB, P. JOLY. *Conservative coupling between finite elements and retarded potentials. Application to vibroacoustics*, in "SIAM J. Sci. Comput.", To appear.

[30] C. HAZARD, F. LORET. *Generalized eigenfunction expansions for scattering problems with an application to water waves*, in "Proceedings of the Royal Society of Edinburg, A", 2006.

[31] S. JOB, E. LUNÉVILLE, J.-F. MERCIER. *Diffraction of an acoustic wave by a plate in a uniform flow: a numerical approach*, in "J. of Comp. Acoustics", to appear.

[32] P. JOLY, S. TORDEUX. *Asymptotic analysis of an approximate model for time harmonic waves in media with thin slots*, in "M2AN Math. Model. Numer. Anal.", vol. 40, n^{o} 1, 2006, p. 63–97.

[33] P. JOLY, S. TORDEUX. *Matching of asymptotic expansions for wave propagation in media with thin slots. I. The asymptotic expansion*, in "Multiscale Model. Simul.", vol. 5, n^{o} 1, 2006, p. 304–336 (electronic).

[34] M. LENOIR. *Influence coefficients for variational integral equations*, in "C. R. Acad. Sci.", vol. 343, 10 2006, p. 561–564.

[35] J. LI. *Low order approximation of the spherical nonreflecting boundary kernel for the wave equation.*, in "Linear Algebra and its Applications", to appear.

Publications in Conferences and Workshops

[36] C. HAZARD. *Computing resonances for large time simulation of scattering phenomena*, MAFELAP 2006, 6 2006.

Internal Reports

[37] A.-S. BONNET-BEN DHIA, M. LENOIR. *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, Technical report, ENSTA, 2006.

[38] S. TORDEUX. *Un problème de Laplace non standard en milieu non borné*, Technical report, n^{o} RR-5799, INRIA, January 2006, http://hal.inria.fr/inria-00070224.