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Project-Team Apics

Analysis and Problems of Inverse type in Control and Signal processing

Sophia Antipolis - Méditerranée

THEME NUM

Activity Report

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1. Team

Team Leader
Laurent Baratchart [DR Inria, HdR]

Deputy Team Leader
Jean-Baptiste Pomet [CR Inria]

Administrative Assistant
France Limouzis [AI Inria, partial time in the team]

Staff Member
José Grimm [CR Inria]
Juliette Leblond [DR Inria, HdR]
Martine Olivi [CR Inria]
Alban Quadrat [CR Inria]
Stéphane Rigat [MC Univ]
Fabien Seyfert [CR Inria]

Ph. D. Students
Alex Bombrun [Fellow, Inria, until February]
Imen Fellah [Co-advised, LAMSIN-ENIT Tunis, until July]
Vincent Lunot [Fellow, Inria]
Moncef Mahjoub [Co-advised, LAMSIN-ENIT Tunis]
Meriem Zghal [Co-advised, LAMSIN-ENIT Tunis]

Engineers
Rania Bassila [IA INRIA]

Scientific Advisors
Andrea Gombani [LADSEB-CNR, Padova, Italy]
Mohamed Jaoua [Univ. Nice-Sophia Antipolis]
Jean-Paul Marmorat [Centre de mathématiques appliquées de l'école des mines de Paris]
Jonathan Partington [Univ. Leeds, UK]
Pierre Rouchon [CAS École des Mines de Paris]
Edward Saff [Univ. Vanderbilt, Nashville, USA]

Post-Doctoral Fellows
Hamza Jirari [Univ. Graz, until february]
Maxim Yattselev [Univ. Vanderbilt, from September]

Visitors and Trainees
Fahmi Ben Hassen [Lamsin-Enit, Tunis, three weeks]

2. Overall Objectives

2.1. Overall Objectives

The Team aims at designing and developing constructive methods in modeling, identification and control of dynamical, resonant and diffusive systems.

2.1.1. Research Themes

- Function theory and approximation theory in the complex domain, with applications to frequency identification of linear systems and 2-D inverse boundary problems for the Laplace and Beltrami operators.
- System and circuit theory with applications to the modeling of analog microwave devices. Development of dedicated software for the synthesis of such devices.
- Inverse potential problems in 3-D and harmonic analysis with applications to non-destructive control and magneto/electro-encephalography.
- Control and structure analysis of non-linear systems with applications to orbit transfer of satellites.

2.1.2. International and industrial partners

- Industrial collaboration under contract with Alcatel Alenia Space (Toulouse and Cannes), Thales AS (Paris), CNES (Toulouse), XLim (Limoges).
- Exchanges with UST (Villeneuve d'Asq), University Bordeaux-I (Talence), University of Orléans (MAPMO), University of Pau (EPI INRIA Magic-3D), University Marseille-I (CMI), CWI (the Netherlands), SISSA (Italy), the Universities of Illinois (Urbana-Champaign USA), California at San Diego and Santa Barbara (USA), Michigan at East-Lansing (USA), Vanderbilt University (Nashville USA), Texas A&M (College Station USA), ISIB (CNR Padova, Italy), Beer Sheva (Israel), RMC (Kingston, Canada), University of Erlangen (Germany), Leeds (UK), Maastricht University (The Netherlands), Cork University (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), Kingston (Canada), Szegëd (Hungary), CINVESTAV (Mexico), ENIT (Tunis), VUB (Belgium), KTH (Stockholm).
- The project is involved in a EMS21-RTG NSF program (with Vanderbilt University), in the INRIA Team Enée associated with LAMSIN-ENIT (including the EPI Anubis and Poems), in an EPSRC Grant with Leeds University (UK), in the ERCIM "Working Group Control and Systems Theory", in the ANR projects AHPI (Math., coordinator) and FILIPIX (Telecom.).

3. Scientific Foundations

3.1. Identification and approximation

Modeling means abstracting the behavior of a phenomenon in terms of mathematical equations, and is generally used to *predict* the outcome of this phenomenon. Modeling involves typically two steps: choosing a model class and estimating "optimal" parameters within this class. The model class mainly reflects the available knowledge of the underlying physical system, and the algorithmic effort the user is willing to pay. Indeed, the second step usually consists in approximating the experimental data by the prediction of the model. The ability to solve this approximation problem, which is often non-trivial and ill-posed, impinges on the effectiveness of a method.

Below, we shall be mainly interested in stable linear time-invariant systems, in particular resonant ones, and also in isotropically diffusive systems.

If only finitely many real parameters need to be adjusted, as in the case of systems described by ordinary differential equations, this process is often called *parametric identification* and it has long been the realm of the stochastic paradigm. When the parameters are sought in some functional space, the term *inverse problem* is most commonly used and the techniques dwell more on functional analysis. This clear-cut distinction of course gets blurred in practice. In all cases numerical optimization has become an all-pervasive tool.

The Apics team's research on this topic is mostly concerned with identifying elliptic differential equations from boundary data. Special emphasis is put on the function-theoretic aspects of the approximation step. The basic features of our approach are perhaps best captured when discussing the classical example of harmonic identification for stable linear dynamical systems (*i.e.* 1-D causal convolution operators), which is a standard issue in engineering practice. Here, one measures the response of the system to periodic excitations in its band-width, thereby estimating the Fourier transform of the kernel in some working range of frequencies. As the impulse response is supported on the positive time-axis by causality, this Fourier transform extends analytically to the right half-plane, thereby defining the so-called *transfer function* of the system. One then seeks a stable transfer-function, that is, a function analytic in the complex right half-plane, whose trace on the imaginary axis approximately meets that Fourier transform. No data are available outside the band-width, as high frequencies can seldom be measured and often show nonlinear behaviour. Thereby the identification problem translates into an inverse boundary-value problem for the $\overline{\partial}$ equation, namely to recover a function, analytic in a plane domain, from (incomplete) boundary data. This point of view naturally extends to other elliptic partial differential equations in two dimensions, in particular to the Laplace and complex Beltrami equations. Motivated by free boundary problems in plasma control, inverse conductivity issues from impedance tomography, and questions of source recovery arising in magneto/electro-encephalography, a good deal of the team's research on inverse problems today aims at generalizing this approach to the real Beltrami equation in dimension 2 and to the Laplace equation in dimension 3.

In many important applications of harmonic identification (in particular for resonant systems, see section 4.3), one wants the model to be rational of suitable degree, either because this brings physical significance to the parameters which is necessary for design, or because the complexity must be kept reasonably low. Other structural constraints, arising from the physics of the phenomenon under study, often superimpose on the model: for instance a passive system must be modelled by a transfer-function of modulus at most 1, a passive network must have a symmetric scattering matrix, etc... This leads us to split this identification problem in two: first we seek a stable but infinite (numerically: high) dimensional model to fit the data, and second we approximate it by a lower order model reflecting further properties of the original system.

In some sense, the first step amounts to set up a reference behaviour for the model at *each* frequency. Mathematically speaking, it consists in reconstructing a function analytic in the right half-plane (the transfer function), from its values on an interval of the imaginary axis (the band-width). In other words, one wants to make the principle of analytic continuation effective from part of the boundary of the analyticity domain. This classical ill-posed issue (*i.e.* the inverse Cauchy problem for the Laplace equation) we will embed into a family of well-posed extremal problems, that may be viewed as a regularization scheme of Tikhonov-type. These problems are infinite-dimensional but convex. Note that, by the Cauchy-Riemann equations, the tangential derivative of the imaginary part of an analytic function is the normal derivative of its real part. The question above can therefore be recast as a Dirichlet-Neumann problem with overdetermined data on part of the boundary. As such, it occurs in many more inverse problems than harmonic identification, and it arises naturally in higher dimensions when analytic functions get replaced by gradients of harmonic functions (see sections 4.2 and 6.2).

The second step aims at reducing the complexity while bringing physical significance to the design parameters. It typically consists of a rational approximation procedure with prescribed number of poles in certain classes of analytic functions. To make the best out of the parameters, it is usually of importance to compute truly optimal or at least near-optimal approximants. Rational approximation in the complex domain is a classical but difficult nonconvex problem, for which few effective methods exist. In relation to system theory, two specific difficulties superimpose on the classical situation, namely one must control the region where the poles of the approximants lie in order to ensure the stability of the model, and one has to handle matrix-valued functions when the system has several inputs and outputs, in which case the number of poles must be replaced by the McMillan degree.

Below, rational or more generally meromorphic approximation with prescribed number of poles will be used to approach other inverse problems beyond harmonic identification. In fact, the way the singularities of the approximant (*i.e.* its poles) relate to the singularities of the approximated function (that may include branchpoints, essential singularities etc...) is an all-pervasive theme in approximation theory that receives much attention within the team. The bottom line is that, for appropriate classes of functions, the location of the poles of the approximant can be used as an estimator of the singularities of the approximated function (see sections 4.2 and 6.2). In this context, the analog of rational functions in higher dimensions consists of gradients of discrete Newtonian potentials.

We provide further details on the two steps mentioned above. in the sub-paragraphs to come, still stressing harmonic identification, in connection with the recovery of analytic functions, as an illustrative example.

3.1.1. Analytic approximation of incomplete boundary data

Keywords: *Beltrami equations*, *Dirichlet-Neumann problems*, *Hardy spaces*, *extremal problems*, *harmonic functions*.

Participants: Laurent Baratchart, José Grimm, Juliette Leblond, Jean-Paul Marmorat [CMA, École des Mines de Paris], Jonathan Partington [Univ. Leeds], Stéphane Rigat, Emmanuel Russ [Univ. Marseille III], Fabien Seyfert.

Given a planar domain D, the problem is to recover an analytic function from its values on a subset of the boundary of D. In harmonic identification, D is the right half-plane and the analytic function is the transfer functions of a stable system. However, since the recovery issue may arise on domains with various shapes when it comes to inverse problems, it is convenient to normalize D and apply in each particular case a conformal transformation to meet the normalization. In the simply connected case, which is that of the half-plane, we fix D to be the unit disk, so that its boundary is the unit circle T. We denote by H^p the Hardy space of exponent p which is the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which make it possible to speak of (traces of) analytic functions on the boundary.

A standard extremal problem on the disk is [68], [71], [89]:

> (P_0) Let $1 \leq p \leq \infty$ and $f \in L^p(T)$; find a function $g \in H^p$ such that $g - f$ is of minimal norm in $L^p(T)$.

When seeking an analytic function in D which approximately matches some measured values f on a subarc K of T, the following generalization of (P_0) naturally arises:

> (P) Let $1 \leq p \leq \infty$, K a subarc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behaviour capsulizing the expected behaviour of the model off K, while M is the admissible error with respect to this expectation. The value of p reflects the type of stability which is sought and how much one wants to smoothen the data.

To fix terminology we generically refer to (P) as a *bounded extremal problem*. The solution to this convex infinite-dimensional optimization problem can be obtained upon iteratively solving spectral equations for appropriate operators, that involve a Lagrange parameter, with right hand-side given by the solution to (P_0) for some weighted concatenation of f and ψ [46], [43]. The underlying operator is of Toeplitz type when $p = 2$, thereby stressing an unexpected link with Carleman's reconstruction formulas [34]. The operator is of Hankel type when $p = \infty$, in which case the solution rests on the Adamjan-Arov-Krein (in short: AAK) theory. The latter connects the singular decomposition of Hankel operators to best meromorphic approximants in $L^\infty(T)$, and would allow for us to treat the extended version of (P) where g, instead of being analytic, is allowed a prescribed number of poles in D [45]. When $1 < p < \infty$ but $p \neq 2$, the computations become more involved, as the duality mapping is non-linear. The situation when $p = 1$ is essentially open from the constructive viewpoint.

In all cases, the ill-posed character of analytic continuation is reflected by the denseness of traces of H^p-functions in $L^p(K)$ (if $p = \infty$ this requires f to be continuous on K). It is to the effect that, if the data are not *exactly* analytic, the constraint $\|g - \psi\|_{L^p(T\setminus K)} \leq M$ becomes an equality and the approximation error $\|g - f\|_{L^p(K)}$ tends to 0 if, and only if, M goes to infinity [43]. When $p = 2$ this phenomenon has been quantified according to the smoothness of f, as estimates of the error as a function of M can be derived from a diagonalization procedure for Toeplitz operators [2]. The error in fact decreases much faster, when M increases, when the data are holomorphic in neighborhood of D; this is of interest to discriminate between nearly analytic data and those lying far from representing a linear stable model.

Various modifications of (P) can be made to meet specific needs. For instance when dealing with lossless transfer functions (see section 4.3.2), one may want to express the constraint on $T \setminus K$ in a pointwise manner and thus turn to

> (P') Let $1 \leq p \leq \infty$, K a subarc arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M \in L^p(T \setminus K)$; find a function $g \in H^p$ such that $|g - \psi| \leq M$ a.e. on $T \setminus K$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Problem (P') was recently solved when $p = 2$ and $\psi = 0$ [49]. It turns out that the solution uniquely exists and that $|g| = M$ on $T \setminus K$, unless f is the trace on K of an H^2-function satisfying the constraint. This fact is perhaps counter-intuitive, and actually makes (P') attractive for design purposes. Although (P') is not smooth, the solution can then be expressed in terms of a Lagrange multiplier, which is this time a *function* on $T \setminus K$, through a normalized Cauchy transform of Carleman type. Another approach, based on polynomial discretization, connects to classical Tchebychev approximation. The case when $\psi \neq 0$ is more delicate in that conditions have to be put jointly on f and ψ for a solution to exist.

Somewhat dual to (P) is the extension problem where one seeks $h \in L^p(T \setminus K)$ such that the concatenated function $f \vee h$ is as close as possible to H^p under the constraint that $\|h - \psi\|_{L^p(T\setminus K)} \leq M$ [45].

Yet another variation of (P), aiming at solving inverse boundary-value problems from mixed Dirichlet-Neumann data, goes as follows (see sections 4.2 and 6.2):

> (P'') Let $1 < p < \infty$, K be an arc of the unit circle T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$, and $M > 0$; find a function $g \in H^p$ such that $\|\mathrm{Im}(g - \psi)\|_{L^p(T\setminus K)} \leq M$ and $\mathrm{Re}(g - f)$ is of minimal norm in $L^p(K)$ under this constraint.

When $p = 2$, the solution to (P'') involves both Toeplitz and Hankel operators [75]). When $p = 1, \infty$, the above formulation is ill-posed. When slightly modified, it gives rise to a new identification technique of Robin coefficients, which is interesting in particular for corrosion control. see [21].

For using (P), (P'), and (P'') to tackle inverse problems, it is natural to ask about the continuity of g with respect to f, ψ and M. As a general rule, L^p continuity holds if $p < \infty$, but only weak-* continuity if $p = \infty$ Smoothness also is a delicate issue, for the solution is rather irregular at the endpoints of K unless M and ψ are adjusted to f. Sufficient conditions for smoothness form a topic of current research, in relation to the behaviour of Cauchy singular integrals on an arc at its endpoints. In the team's target applications to harmonic identification of resonant systems, as implemented in PRESTO-HF, the regular behaviour of such systems at high frequencies is used together with extension formulations to circumvent this problem (see section 5.4).

The above-mentioned problems can be stated on an annular geometry rather than a disk. For $p = 2$ the solution proceeds much along the same lines [80], [79], although the spectral properties of the Toeplitz operators involved are rather different. When K is the outer boundary, (P) regularizes a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from overdetermined Dirichlet-Neumann data on the outer boundary (see section 6.2.1). Interestingly perhaps, (P'') becomes a tool to approach Bernoulli type problems for the Laplacian, where overdetermined observations are made on the outer boundary and we *seek the inner boundary* knowing it is a level curve of the flux. In fact, solving (P'') with small M for some initial guess of the inner boundary, and using the Lagrange parameter to locally deform this guess so as to improve the error, yields a descent algorithm tending either to a solution or a topological singularity (see section 6.2.2).

Considerable effort is currently payed by the team to carry over bounded extremal problems and their solution to more general settings. Such generalizations are twofold: on the one hand APICS considers 2-D diffusion equations with variable conductivity, on the other hand it investigates the ordinary Laplacian in R^n. The applications we seek are to the determination of free boundaries in plasma control for the first case, and to source detection in electro/magneto-encephalography for the second case.

A diffusion equation in dimension 2 can be viewed as the compatibility condition of a so-called real Beltrami equation [74]. This way analytic functions get replaced by quasi-conformal maps in problems (P_0) and (P). Hardy spaces of solutions, which are a little more general than classical Sobolev solutions, have been introduced by the team when $1 < p < \infty$ to imbed such maps in $L^p(T)$, and the ill-posedness of (P) again translates into the denseness of solutions in $L^p(K)$. The underlying Toepliz and Hankel operators, as well as the expansions of solutions needed to constructively handle such problems, are now under study. The goal is to solve the analog of (P'') in this context to approach Bernoulli-type problems (see section 6.2.1).

At present, bounded extremal problems for the n-D Laplacian are considered on half-spaces or balls. Following [90], Hardy spaces are defined as gradients of harmonic functions satisfying L^p growth conditions on inner hyperplanes or spheres. Such a definition is especially suited for the study of Dirichlet-Neumann problems. Toeplitz operators with scalar symbol have been introduced to solve the analog of (P) when $p = 2$. From the constructive viewpoint, spherical harmonics offer a reasonable substitute to Fourier expansions [38]. Only very recently we defined operators of Hankel type that connect to the solution of (P_0) in the BMO norm. The goal is to solve the analog of (P) on spherical shells to approach inverse diffusion problems across a conductor layer.

3.1.2. Meromorphic and rational approximation

Keywords: *critical point theory*, *meromorphic approximation*, *orthogonal polynomials*, *rational approximation*.

Participants: Laurent Baratchart, José Grimm, Vincent Lunot, Jean-Paul Marmorat, Martine Olivi, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

Let as before D designate the unit disk, T the unit circle, and $\overline{H}^p$ be the Hardy space of exponent p of the *complement* of D in the complex plane. By definition, $h(z)$ belongs to $\overline{H}^p$ if, and only if $h(1/z)/z$ belongs to H^p. We further put R_N for the set of rational functions with at most N poles in D, which allows us to define the meromorphic functions in $L^p(T)$ as the traces of functions in $H^p + R_N$.

A natural generalization of problem (P_0) is

> (P_N) Let $1 \le p \le \infty$, $N \ge 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Problem (P_N) aims at recovering a solution of the inhomogeneous Laplace equation $\Delta u = \mu_N$, with μ_N some (unknown) linear combination of N Dirac masses, from overdetermined Dirichlet-Neumann data.

Only for $p = \infty$ and continuous f is it known how to solve (P_N) in closed form. The unique solution is given by the AAK theory, that allows one to express g_N in terms of the singular vectors of the Hankel operator with symbol f. The continuity of g_N as a function of f only holds for stronger norms than uniform, *e.g.* Besov norms [87].

The case $p = 2$ is of special importance. Specifically, if we write $f = f^+ + f^-$ upon splitting the Fourier expansion of f between coefficients of non-negative and strictly negative index respectively, and if r_N is a best approximant to f^- from R_N in $L^2(T)$, then $g_N = f^+ + r_N$. In particular when $f \in \overline{H}^2$, so that $f^+ = 0$, then (P_N) reduces to rational approximation. Moreover, it turns out that r_N has no pole outside D and vanishes at infinity, hence it is a *stable* rational approximant to f. However, in contrast with the situation when $p = \infty$, this approximant may *not* be unique.

To perform the second step of the identification scheme described in section 3.1, namely rational approximation with a prescribed number of poles to a function analytic in the right half-plane, one can map the latter conformally to the complement of D and solve (P_N) with $p = 2$ for the transformed function on T. Although no demonstrably convergent algorithm is known, the former Miaou project (the predecessor of Apics) has designed a steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; it seems the only procedure meeting this property. Roughly speaking, it is a gradient algorithm that proceeds recursively with respect to the order N of the approximant, using the particular geometry of the problem to restrict the search to a compact region of the parameter space [41]. If there is no local *maximum*, a property which is satisfied when N is large enough, then every local *minimum* can be obtained from *some* initial condition of lower degree. Still, it is not known whether the absolute *minimum* can always be obtained using the strategy of the Endymion or RARL2 software consisting in choosing initial conditions corresponding to *critical points* of lower degree (see sections 5.2 and 5.5). The algorithm, however, has proved rather effective in all applications carried out so far (see section 4.3). It is interesting to note that when f is rational (which is no real restriction in practice), finding all the critical points of (P_N) when $p = 2$ is an algebraic problem that might, in principle, be handled *via* elimination theory. Despite the increase of computational power, though, such a procedure is far too complex for realistic values of N and of $\deg f$ (see section 4.3.2).

The current approach is still not satisfactory in that the convergence to global minimum is not established in general. In this regard, APICS has undergone a long-haul study of the number and nature of critical points, in which tools from differential topology and operator theory team up with classical approximation theory. The main discovery is that the nature of the critical points (*e.g. local minima*, saddles...) depends on the decrease of the interpolation error to f as N increases. Besides, an index theorem can be proved [50] to the effect that, in the generic case where all critical points are non-degenerate, they are finite in number and the sum of their indices add up to 1; recall that the index of a critical point is 1 if its Morse index is even and -1 if it is odd. Based on this, sufficient conditions have been developed for a local *minimum* to be unique. This technique requires strong error estimates that are often difficult to obtain, and most of the time only hold for N large. Examples where uniqueness or asymptotic uniqueness has been proved this way include transfer functions of relaxation systems (*i.e.*, Markov functions) [53], [52], the exponential function (the prototype of an entire function with convex Taylor coefficients), and meromorphic functions (*à la* Montessus de Ballore) [7]. The case where f is the Cauchy integral on a hyperbolic geodesic arc of a Dini-continuous function which does not vanish "too much" has been very recently answered in the positive, see section 6.5. We conjecture that uniqueness is linked to the ratio of the to-be-approximated function and its derivative on the circle. When the latter is greater than 1 (*i.e.*, the logarithmic variation is small), it has been proved that uniqueness indeed holds in degree 1 [39]. The generalization to higher degree is an exciting open question.

An analog to AAK theory has been carried out for $2 \leq p < \infty$ [8]. Although not computationally as powerful, it has better continuity properties and stresses a continuous link between rational approximation in H^2 (see section 3.1.2) and meromorphic approximation in the uniform norm, allowing one to use, in either context, techniques available from the other. Hence, similar to the case $p = \infty$, the best meromorphic approximation with at most n poles in the disk of a function $f \in L^p(T)$ is obtained from the generalized singular vectors of a Hankel operator with symbol f. This generalization has a strong topological flavor and relies on the critical points theory of Ljusternik-Schnirelman as well as on the particular geometry of the Blaschke products of given degree.

A common feature to all these problems is that critical point equations express non-Hermitian orthogonality relations for the denominator of the approximant. This is used in an essential manner to assess the behavior of the poles of the approximants to functions with branched singularities, which is of particular interest for inverse problems (*cf.* sections 3.1.3, 6.6).

When $1 \leq p < 2$, problem (P_N) is still fairly open.

In higher dimensions, the analog of problem (P_N) is the approximation of a vector field with gradients of potentials generated by N point masses instead of meromorphic functions. The issue is by no means understood at present, and is a major endeavour of future research problems.

Certain constrained rational approximation problems, of special interest in identification and design of passive system, arises when putting additional conditions on the approximant, for instance that it should be smaller than 1 in modulus. How good the approximation to a given function f under this constraint is unclear, since best approximants tend to violate the constraint by winding around f on T. The team begins to investigate such questions using hyperbolic interpolation. Specifically, in light of the progress made on classical Schur interpolation in the last years [77], the convergence properties of the multipoint Schur interpolation are currently under study, see section 6.5.

The introduction of a weight in rational approximation is another interesting issue, which is useful in order to balance the information with the noise. For instance in the stochastic theory, minimum variance identification leads to assign weights to the errors like the inverse of the spectral density of the noise. Extensions of the above-mentioned algorithms were implemented in Endymion, based on the work in [81]. The analysis of the critical points equations in the weighted case gives various counter-examples to unimodality in maximum likelihood identification [82].

Another kind of rational approximation problem arises in *design* problems, that became over years an increasingly significant part of the team's activity (see sections 4.3, 6.4, and 6.8). These are problems where constraints on the *modulus* of a rational function are sought, and they occur mainly in filter design where the response is a rational function of fixed degree (the complexity of the filter), analytic and bounded by 1 in modulus on the right-half-plane (passivity), whose modulus must be as close as possible to 1 on some subset of the imaginary axis (the pass-band) ans as close as possible to 0 on the complementary subset (the stop-band).

When translated over to the circle, a prototypical formulation consists in approximating the modulus of a given function by the modulus of a rational function of degree n, that is, to solve for

$$\min \left\| |f| - \left|\frac{p_n}{q_n}\right| \right\|_{L^p(T)}.$$

When $p = 2$ this problem can be reduced to a series of standard rational approximation problems, but usually one needs to solve it for $p = \infty$. The case where $|f|$ is a piecewise constant function with values 0 and 1 can also be approached *via* classical Zolotarev problems [88], that can be solved more or less explicitly when the pass-band consists of a single arc. A constructive solution in the case of several arcs (multiband filters) is one recent achievement of the team (see section 6.8). Of course, though the modulus of the response is the first concern in filter design, the variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. As a matter of fact, trading-off abrupt changes in modulus for a smaller derivative of the phase is an important issue in frequency design which currently under investigation within the team under contract with the CNES, see section 6.8.

From the point of view of design, rational approximants are indeed useful only if they can be translated into physical parameter values for the device to be built. While such problems do not pertain to rational approximation proper, they are of utmost importance in practice. Actually, the fact that a device's response is shaped in the frequency domain whereas the device itself must be specified in the time domain is a major difficulty in the area that reflects the fundamental problem of harmonic analysis. This is where System-Theory enters the scene, as the correspondence between the frequency response (*i.e.* the transfer-function) and the linear differential or difference equations that generate this response (*i.e.* the state-space representation) is the object of the so-called *realization* process. Algebraically speaking, a realization of a rational matrix H of the variable z is a 4-tuple (A, B, C, D) of real or complex matrices of appropriate sizes such that

$$H(z) = C(zI - A)^{-1}B + D.$$

Since filters have to be considered as multipoles, the issue must indeed be tackled in a matrix-valued context that adds to the complexity. A fair share of the team's research in this direction is concerned with finding realizations meeting certain constraints (imposed by the technology in use) for a transfer-function that was obtained with the above-described techniques. The current approach is to solve algebraic equations in many variables using homotopy methods, which seems to be a path-breaking methodology in the area of filter design (see section 6.7).

3.1.3. Behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Keywords: *discretization of potentials*, *free boundary inverse problems*, *meromorphic approximation*, *orthogonal polynomials*, *rational approximation*, *singularity detection*.

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

We refer here to the behavior of the poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle, one we oneself again in the framework of sections 3.1.1 and 3.1.2; the invariance of the problem under conformal mapping was established in [5]. The research so far has focused on functions that are analytic on and outside the contour, and have singularities on an open arc inside the contour.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and also to tackle constructive issues like uniqueness. However, the original motivation of Apics is to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities.

As a general rule, critical point equations for these problems express that the polynomial whose roots are the poles of the approximant is a non-Hermitian orthogonal polynomial with respect to some complex measure (that depends on the polynomial itself and therefore varies with the degree) on the singular set of the function to be approximated. New results were obtained in recent years concerning the location of such zeroes. The approach to inverse problem for the Laplacian that we outline in this section appears to be attractive when the singularities are one-dimensional, for instance in the case of a cracked domain (see section 4.2). It can be used as computationally cheap preliminary step to obtain the initial guess of a heavier but more precise numerical optimization. It is rather complementary of the popular MUSIC-type algorithms [78] as it can in principle be used on a single stationary pair of Dirichlet-Neumann data.

When the crack is sufficiently smooth, the approach in question is in fact equivalent to the meromorphic approximation of a function with two branch points, and we were able to prove [5][3] that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the endpoints of the crack [44]. Moreover the asymptotic density of the poles turns out to be the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are *de facto* well localized if one is able to compute sufficiently many zeros (this is where the method could fail). It is interesting to note that these results apply also, and even more easily, to the detection of monopolar and dipolar sources, a case where poles as well as logarithmic singularities exist. The case of more general cracks (for instance formed by a finite union of analytic arcs) requires the analysis of the situation where the number of branch points is finite but arbitrary. We proved very recently that the poles tend to the contour $\mathcal{C}$ outside of which the function is analytic and single-valued that minimizes the capacity of the condenser $(T, \mathcal{C})$, where T is the exterior boundary of the domain (paper in preparation, see section 6.6). For the definition of a condenser and other basic facts from potential theory, see [88].

It would of course be very interesting to know what happens when the crack is "absolutely non analytic", a limiting case that can be interpreted as that of an infinite number of branch points, and on which very little is known, although there are grounds to conjecture that the endpoints at least are still accumulation points of the poles. This is an outstanding open question for applications to inverse problems (see section 6.2). Concerning the problem of a general singularity, that may be two dimensional, one can formulate the following conjecture: if f is analytic outside and on the exterior boundary of a domain D and if K is the minimal compact set included in D that minimizes the capacity of the condenser (T, K) under the constraint that f is analytic and single-valued outside K (it exists, it is unique, and we assume it is of positive capacity in order to avoid degenerated cases), then every limit point (in the weak star sense) of the sequence ν_n of probability measures having equal mass at each pole of an optimal meromorphic approximant (with at most n poles) of f in $L^p(T)$ *has its support in* K *and sweeps out on the boundary of* K *to the equilibrium distribution of the condenser* (T, K). This conjecture, which generalizes the above-mentioned results on 1-D singular sets, is far from being solved in general.

Results of this type open new perspectives in non-destructive control (see section 4.2), in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which a dual approach is thereby proposed: to approximate the boundary conditions and not the equation (as is classically done).

Let us point out that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is then no longer available, and this makes for a long-term research project with a wide range of applications. It is interesting to mention that the case of sources in dimension three in a spherical geometry, can be attacked with the above 2-D techniques as applied to planar sections (see section 6.2).

3.1.4. Matrix-valued rational approximation

Keywords: *inner matrix*, *rational approximation*, *realization theory*, *reproducing kernel space*.

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, José Grimm.

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (*i.e.*, the degree of a minimal realization in the System-Theoretic sense) generalizes the degree.

The problem we want to consider reads: *Let* $\mathcal{F} \in (H^2)^{m \times l}$ *and* n *an integer; find a rational matrix of size* $m \times l$ *without poles in the unit disk and of McMillan degree at most* n *which is nearest possible to* $\mathcal{F}$ *in* $(H^2)^{m \times l}$. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [9]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (*i.e.*, matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices using the so-called Douglas-Shapiro-Shields factorization. The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parameterizations valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. The tangential Schur algorithm [35] provides us with such a parameterization and allowed the team to develop two rational approximation codes (see sections 5.2 and 5.5). The first one is integrated in the Endymion software dealing with transfer matrices while the other, which is developed under the Matlab interpreter, goes by the name of RARL2 and works with realizations. Both have been experimented on measurements by the CNES (branch of Toulouse), XLim, and Alcatel Alenia Space (Toulouse), on which they gave high quality results [42] in all cases encountered so far. These codes are now of daily use by Alcatel Alenia Space and XLim, coupled with simulation software like EMXD to design physical coupling parameters for the synthesis of microwave filters made of resonant cavities (see section 7.1).

In the above application, obtaining physical couplings requires the computation of realizations, also called internal representation in System Theory. Among the parameterizations obtained via the Schur algorithm, some have a particular interest from this viewpoint [10]. They lead to a simple and robust computation of balanced realizations and form the basis of the RARL2 algorithm (see section 5.2).

Problems relative to multiple local minima naturally arise in the matrix-valued case as well, but deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The already investigated case of rational functions of the sought degree (the consistency problem) was solved using rather heavy machinery [6], and that of matrix-valued Markov functions, that are the first example beyond rational function has made progress only recently [40].

In practice, a method similar to the one used in the scalar case has been developed to generate local minima of a given order from those at lower orders. In short, one sets out a matrix of degree n by perturbation of a matrix of degree $n-1$ where the drop in degree is due to a pole-zero cancellation. There is an important difference between polynomial representations of transfer matrices and their realizations: the former lead to an embedding in a ambient space of rational matrices that allows a differentiable extension of the criterion on a neighborhood of the initial manifold, but not the latter (the boundary is strongly singular). Generating initial conditions in a recursive manner is more delicate in terms of realizations, and some basic questions on the boundary behavior of the gradient vector field are still open.

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.2. Structure and control of non-linear systems

In order to control a system, one generally relies on a model, obtained from *a priori* knowledge, like physical laws, or from experimental observations. In many applications, it is enough to deal with a linear approximation around a nominal point or trajectory. However, there are important instances where linear control does not apply, either because the magnitude of the control is limited or because the linear approximation is not controllable. Moreover, certain control problems, such as path planning, are not local in nature and cannot be solved *via* linear approximations.

Section 3.2.1 describes a problem of this nature, where the controllability of the linear approximation is of little help. Besides, the structural study described in section 3.2.2 aims at exhibiting invariants that can be used, either to bring the study back to that of simpler systems or to lay grounds for a non-linear identification theory. The latter would give information on model classes to be used in case there is no *a priori* reliable information and still the black-box linear identification is not satisfactory.

3.2.1. Feedback control and optimal control

Keywords: *control*, *non holonomic mechanical system*, *non-linear control*, *stabilization of non-linear systems*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet.

Stabilization by continuous state feedback (or output feedback which is a partial information case) consists in designing a control law which is a smooth (at least continuous) function of the state making a given point (or trajectory) asymptotically stable for the closed–loop system. One can consider this as a weak version of the optimal control problem which is to find a control that minimizes a given criterion (for instance the time to reach a prescribed state). Optimal control generally leads to a rather irregular dependence on the initial state; in contrast, stabilization is a *qualitative* objective (*i.e.*, to reach a given state asymptotically) which is more flexible and allows one to impose a lot more regularity.

Lyapunov functions are a well-known tool to study the stability of non-controlled dynamic systems. For a control system, a *Control Lyapunov Function* is a Lyapunov function for the closed-loop system where the feedback is chosen appropriately. It can be expressed by a differential inequality called the “Artstein (in)equation [37]”, that looks like the Hamilton-Jacobi-Bellmann equation but is largely under-determined. One can easily deduce from the knowledge of a control Lyapunov function a continuous stabilizing feedback.

The team is engaged in obtaining control Lyapunov functions for certain classes of systems. This can be the first step in synthesizing a stabilizing control but, even when such a control is known beforehand, obtaining a control Lyapunov function can still be very useful to study the robustness of the stabilization, or to modify the initial control law into a more robust one. Moreover, if one has to deal with a problem where it is important to optimize a criterion, and if the optimal solution is hard to compute, one can look for a control Lyapunov function which comes “close” (in the sense of the criterion) to the solution of the optimization problem but leads to a control which is easier to work with.

These constructions are exploited in a joint collaborative research conducted with Alcatel Alenia Space (Cannes), where minimizing a certain cost is very important (fuel consumption / transfer time) while at the same time a feedback law is preferred because of robustness and ease of implementation (see section 7.3).

3.2.2. Transformations and equivalences of non-linear systems and models

Keywords: *classification*, *non-linear control*, *non-linear feedback*, *non-linear identification*.

Participants: Laurent Baratchart, Monique Chyba [Univ. Hawaii (USA)], Jean-Baptiste Pomet.

Here we study certain transformations of models of control systems, or more accurately of equivalence classes modulo such transformations. The interest is two-fold:

- From the point of view of control, a command satisfying specific objectives on the transformed system can be used to control the original system including the transformation in the controller. Of course the favorable case is when the transformed system has a structure that can easily be exploited, for instance when it is a linear controllable system.
- From the point of view of identification and modeling, the interest is either to derive qualitative invariants to support the choice of a non-linear model given the observations, or to contribute to a classification of non-linear systems which is missing sorely today. Indeed, the success of the linear model in control and identification is due to the deep understanding one has of it. In the same manner, a more complete knowledge of invariants of non-linear systems under basic transformations is a prerequisite for a more general theory of non-linear identification.

Concerning the classes of transformations, a *static feedback* transformation of a dynamical control system is a (non-singular) reparametrization of the control depending on the state, together with a change of coordinates in the state space. A *dynamic feedback* transformation of a control system consists of a dynamic extension (adding new states, and assigning them a new dynamics) followed by a state feedback on the augmented system. Let us now stress two specific problems that we are tackling.

Dynamic linearization. The problem of dynamic linearization, still unsolved, is that of finding explicit conditions on a system for the existence of a dynamic feedback that would make it linear. It is equivalent to the problem of finding a “parameterization” of the solutions by a certain number of functions that are not constrained by differential equations, i.e. a formula that gives the general solution as a function of these functions and their derivatives, i.e. a nonlinear differential operator whose order is sometimes called the order of the parameterization. Existence of such a parameterization has been emphasized over the last years [69] as very important and useful in control. A system is *differentially flat* when such a parameterization exists, with the property that each solution in turn uniquely determines the parameterizing function, which is then obtained by applying a differential operator to the solution.

It was shown, roughly speaking, that a system is differentially flat if, and only if, it can be converted to a linear system by dynamic feedback. Thus, flatness turns out to be a property of the set of trajectories which is of importance for path-planning and also gives a handle for tackling the problem of dynamic linearization.

An important question remains open: how can one algorithmically decide whether a given system has this property or not, *i.e.*, is dynamically linearizable or not? This problem is both difficult and important for non-linear control. The mathematical difficulty is that no a priori bound is known on the order of the above mentioned differential operator giving the parameterization. Within the team, results on low dimensional systems have been obtained [11], see also [14]; the above mentioned difficulty is not solved for these systems but results are given with artificially prescribed bounds on this order. These characterizations point at the complexity of the issue.

From the algebraic-differential point of view, the module of differentials of a controllable system is free and finitely generated over the ring of differential polynomials in d/dt with coefficients in the ring of functions on the system's trajectories. A basis for this module can be explicitly constructed [36]. The question is to find out if there exists a basis consisting of closed, that is, locally exact forms. Expressed in this way, it is an extension of the classical integrability theorem of Frobenius to the case where coefficients are differential operators. Together with stability by exterior differentiation (the classical condition), further properties are required to ascertain that the degree of the solutions is finite, a mid-term goal being to obtain a formal and implementable algorithm to decide whether or not a given system is flat around a regular point. One can further consider interesting sub-problems like deciding flatness with a given pre-compensator, or characterizing "formal" flatness that would correspond to a weak interpretation of the differential equation. Such questions can also be raised locally, in the neighborhood of an equilibrium point.

Topological Equivalence. In what precedes, we have not taken into account the degree of *smoothness* of the transformations under consideration.

In the case of dynamical systems without control, it is well known that, away from degenerate (non hyperbolic) points, if one requires the transformations to be merely continuous, every system is *locally* equivalent to a linear system in a neighborhood of an equilibrium (the Hartman-Grobman theorem). It is thus tempting when classifying *control* systems to look for such equivalence modulo non-differentiable transformations, hoping to bring about some robust "qualitative" invariants and perhaps stable normal forms. A Hartman-Grobman theorem for control systems would say for instance, that outside a "meager" class of models (for instance, those whose linear approximation is non-controllable), and locally around nominal values of the state and the control, no qualitative phenomenon can distinguish a non-linear system from a linear one, all non-linear phenomena being thus either of global nature or singularities. Such a statement is wrong: if a system is locally equivalent to a controllable linear system via a bi-continuous transformation—a local homeomorphism in the state-control space—it is *also* equivalent to this same controllable linear system via a transformation that is as smooth as the system itself, at least in the neighborhood of a regular point (in the sense that the rank of the control system is locally constant), see [51] for details; *a contrario*, under weak regularity conditions, linearization can be done by non-causal transformations (see [15]) whose structure remains unclear, but acquires a concrete meaning when the entries are themselves generated by a finite-dimensional dynamics.

The above considerations call for the following question, which is important for modeling control systems: are there local "qualitative" differences between the behavior of a non-linear system and that of its linear approximation when the latter is controllable?

3.3. Algebraic analysis approach to mathematical systems theory

Keywords: *algebraic analysis*, *computer algebra*, *linear systems*, *module theory*.

Participant: Alban Quadrat.

Many systems coming from mathematical physics, applied mathematics and engineering sciences can be described by means of systems of ordinary or partial differential equations, difference equations, differential time-delay equations... In the case of linear systems, these systems can be defined by means of matrices with entries in non-commutative algebras of functional operators such as differential operators, shift operators, time-delay operators, difference operators...

The methods of *algebraic analysis* [1] give a way to intrinsically study a linear functional system by means of its associated finitely presented left module over a non-commutative polynomial ring of functional operators. Thanks to the works of B. Malgrange, V. Palamodov, J. Bernstein, M. Kashiwara and other, algebraic analysis

[1] J.-E. Bjork, Rings of Differential Operators, North-Holland Publishing Company (1979); M. Kashiwara, Algebraic Study of Systems of Partial Differential Equations, Mémoires de la Société Mathématiques de France 63 (1992); V. P. Palamodov, Linear Differential Operators with Constant Coefficients, Grundlehren der mathematischen Wissenschaften 168, Springer (1970).

yields new results and information about the algebraic and analytic properties of linear functional systems, their solutions and associated algebraic and geometric invariants.

The research of APICS on such topics combines Gröbner bases techniques over some non-commutative polynomial rings with the development of new algorithms of algebraic analysis in order to effectively check classical properties of module theory (e.g., existence of a non-trivial torsion submodule or r-pure torsion submodules, torsion-freeness, reflexiveness, projectiveness, stably freeness, freeness, simple or decomposable modules), give their system-theoretical interpretations (existence of autonomous elements or successive parametrizations, existence of minimal/injective parametrizations or Bézout equations) and compute important tools of homological algebra (e.g., (minimal) free resolutions, split long exact sequences, extension and torsion functors, projective and Krull dimensions, Hilbert power series). The developed algorithms are implemented in various symbolic packages that are used to apply our results to systems theory (e.g., parameterizability, flatness, autonomous elements, equivalences of systems, factoring and decomposing linear functional systems) and to mathematical physics (e.g., research of potentials, computations of the field equations and the conservation laws).

4. Application Domains

4.1. Introduction

The bottom line of the team's activity is two-fold, namely function theory and optimization in the frequency domain on the one hand, and the control of systems governed by differential equations on the other hand. Therefore one can distinguish between two main families of applications: one dealing with the design and identification of diffusive and resonant systems (these are inverse problems), and one dealing with the control of certain mechanical systems. For applications of the first type, approximation techniques as described in section 3.1.1 allow one to deconvolve linear equations, analyticity being the result of either the use of Fourier transforms or the harmonic character of the equation itself. Applications of the second type mostly concern the control of systems that are "poorly" controllable, for instance low thrust satellites or optical regenerators. We describe all these below in more detail.

4.2. Geometric inverse problems for the Laplace and the Beltrami equation

Keywords: *Beltrami equation*, *Laplace equation*, *inverse boundary problems*, *non destructive control*, *tomography*.

Participants: Laurent Baratchart, Amel Ben Abda [ENIT, Tunis], Fehmi Ben Hassen [ENIT, Tunis], Imen Fellah [ENIT, Tunis], José Grimm, Mohamed Jaoua [UNSA], Juliette Leblond, Moncef Mahjoub [LAMSIN-ENIT], Jonathan R. Partington [Univ. Leeds], Stéphane Rigat, Emmanuel Russ [Univ. Provence], Edward Saff [Univ. Vanderbilt], Maxim Yattselev.

Localizing cracks, pointwise sources or occlusions in a material, using thermal, electrical, or magnetic measurements on its boundary is a classical inverse problem. It arises when studying fatigue of structures, behavior of conductors, or else electro and magneto-encephalography as well as the detection of buried objects (mines, etc). However, no completely satisfactory algorithm has emerged so far if no initial information on the location or on the geometry is known, because numerical integration of the inverse problem is very unstable. A technique that evolved from the singular-value decomposition of the correlation matrix [85] is popular in the field under the name of MUSIC-type algorithms [78]. The methods we describe are of a different nature, and they are especially valuable when no mutually independent time-varying measurements are available, either because the measurements are stationary or because only few measurements can be made, or else because the superposition of phenomena to be analyzed (*e.g.* the superposition of several sources) are mutually correlated both in time and space. These methods can also be used to approach inverse free boundary problems of Bernoulli type (see section 6.2.2).

The presence of cracks in a plane conductor, for instance, or of sources in a cortex (modulo a reduction from 3-D to 2-D, see below) can be expressed as a lack of analyticity of the (complexified) solution of the associated Dirichlet-Neumann problem that may in principle be approached using techniques of best rational or meromorphic approximation on the boundary of the object (see sections 3.1.1, 3.1.3 and 6.2). In this connection, the realistic case where data are available on part of the boundary only offers a typical opportunity to apply the analytic and meromorphic extension techniques developed earlier.

The 2-D approach proposed here consists in constructing, from measured data on a subset K of the boundary Γ of a plane domain D, the trace on Γ of a function F which is analytic in D except for a possible singularity across some subset $\gamma \subset D$ (typically, a crack or a discrete set of pointwise sources). One can then use the approximation techniques described above in order to:

- extend F to all of Γ if the data are incomplete (it may happen that $K \neq \Gamma$ when the boundary is not fully accessible to measurements), for instance to identify an unknown Robin coefficient (see [67] where stability properties of the procedure are established);
- detect the presence of a defect γ in a computationally efficient manner [59];
- obtain information on the location of γ (see [3], [1]).

Thus, inverse problems of geometric type that consist in finding an unknown boundary from incomplete data can be approached in this way [44], often in combination with other techniques [59]. Preliminary numerical experiments have yielded excellent results and it is now important to process real experimental data, that the team is currently busy analyzing. In particular, contacts with the Odyssée project-team of Inria Sophia Antipolis has provided us with 3-D magneto-encephalographic data from which 2-D information was extracted (see section 6.2). The team also made contact with other laboratories (*e.g.*, Vanderbilt University Physics Dept.) in order to work out 2-D or 3-D data from physical experiments.

The team is beginning to adapt such methods to problems with variable conductivity governed by a 2-D Beltrami equation. The application we have in mind is to plasma confinement for thermonuclear fusion in a Tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [62]. A joint collaboration on this topic recently started with the Laboratoire J. Dieudonné at the University of Nice, and the CMI-LATP at the University of Marseille I. It is one of the collaborative research topics with S. Rigat as described in section 6.2. The goal is first to determine the shape of the surface of the plasma in the chamber from the outer boundary measurements, and in a second step to shape this boundary by choosing some appropriate magnetic flux on this outer boundary (see section 6.2.2).

4.3. Identification and design of resonant systems

Keywords: *filtering device*, *microwave*, *multiplexing*, *surface waves*, *telecommunications*.

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. Resonant systems, either acoustic or electromagnetic based, are prototypical devices of common use in telecommunications. We shall be more specific on two examples below.

4.3.1. Design of surface acoustic wave filters

Participants: Laurent Baratchart, Andrea Gombani, Fabien Seyfert, Martine Olivi.

Surface acoustic waves filters are largely used in modern telecommunications especially for cellular phones. This is mainly due to their small size and low cost. Surface Acoustic Waves (in short: SAW) filters consist in a series of transducers (see figure 1) which transmit electrical power by means of surface acoustic waves propagating on a piezoelectric medium. They are usually described by a mixed scattering matrix which relates acoustic waves, currents and voltages. By reciprocity and energy conservation, these transfers must be either lossless, contractive or positive real, and symmetric. In the design of SAW filters, the desired electrical power transmission is specified. An important issue is to characterize analytically the functions that can actually be realized for a given type of filter.

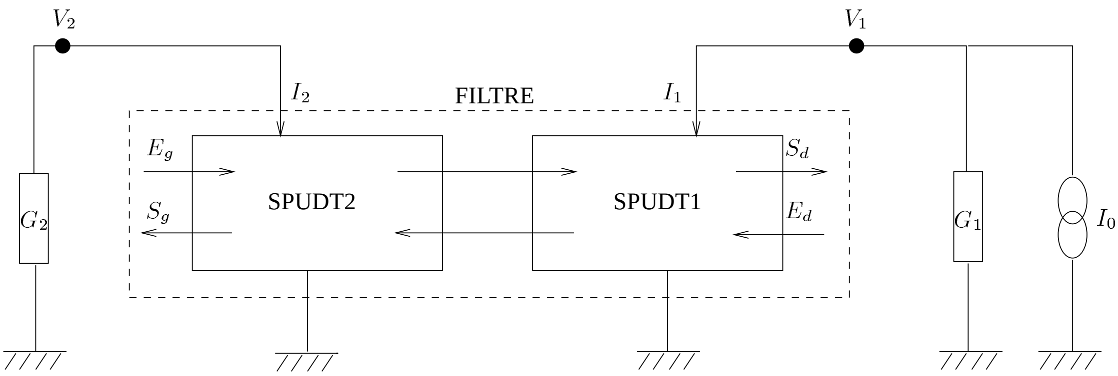

Figure 1. Configuration of the filter

4.3.2. Hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila, José Grimm, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Each cavity (see Figure 2) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

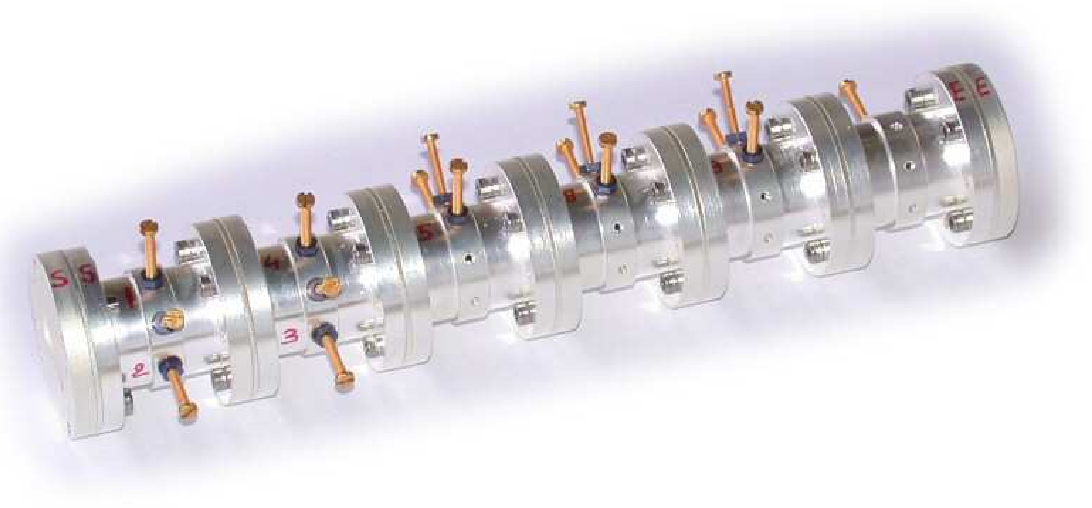

Figure 2. Picture of a 6-cavities dual mode filter. Each cavity (unless the last one) has 3 screws to couple the modes within the cavity, so that there are 16 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 11 slits are fixed in the design phase.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low pass equivalent obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.*, the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, *cf.* section 3.1.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this

example) via the Endymion or RARL2 software (*cf.* section 3.1.4).

- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

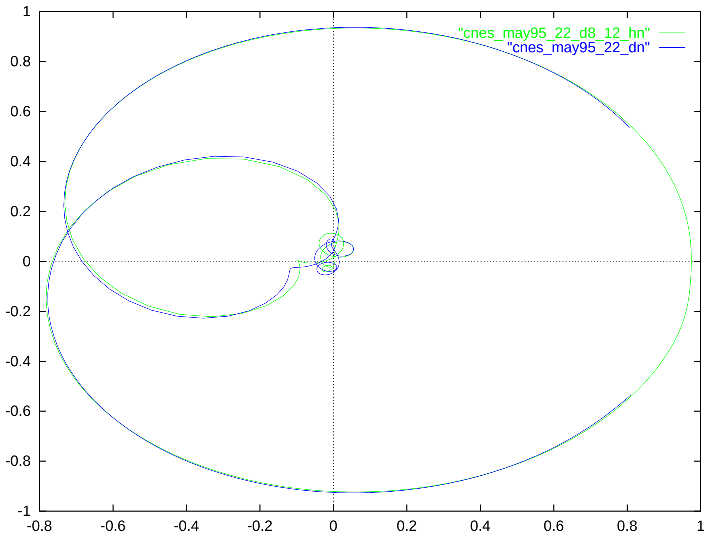

Figure 3. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 3). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

The team investigates today the design of output multiplexors (OMUX) where several filters of the previous type are coupled on a common guide. In fact, it has undergone a rather general analysis of the question "How does an OMUX work?" With the help of numerical simulations and Schur analysis, general principles are being worked out to take into account:

- the coupling between each channel and the "Tee" that connects it to the manifold,
- the coupling between two consecutive channels.

The model is obtained upon chaining the corresponding scattering matrices, and mixes up rational elements and complex exponentials (because of the delays) hence constitutes an extension of the previous framework. Its study is being conducted under contract with Alcatel Alenia Space (Toulouse) (see sections 7.1 and 7.2).

4.4. Spatial mechanics

Keywords: *orbital control*, *satellite*, *spatial mechanics*, *telecommunications*.

Participants: Alex Bombrun, José Grimm, Jean-Baptiste Pomet.

The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing; see for instance section 4.3 for an illustration.

Of course, this requires that satellites be adequately located and positioned (correct orientation). This problem and similar ones continue to motivate research in control within the team. Generally speaking, aerospace engineering requires sophisticated control techniques for which optimization is often crucial, due to the extreme functioning conditions.

The team has been working for two years on control problems in orbital transfer with low-thrust engines, under contract with Alcatel Space Cannes (see section 7.3). Technically, the reason for using these (ionic) low thrust engines, rather than chemical engines that deliver a much higher thrust, is that they require much less "fuel"; this is decisive because the total mass is limited by the capacity of the launchers: less fuel means more payload, while fuel represents today an impressive part of the total mass.

From the control point of view, the low thrust makes the transfer problem delicate. In principle of course, the control law leading to the right orbit in minimum time exists, but it is quite heavy to obtain numerically and the computation is non-robust against many unmodelled phenomena. Considerable progress on the approximation of such a law by a feedback has been made, and numerical experiments have been conducted (see section 6.13).

4.5. Control of Quantum systems

Keywords: *Heisenberg equation*, *Quantum gates*, *optimal control*.

Participants: Hamza Jirari, Jean-Baptiste Pomet, Pierre Rouchon.

For more than 10 years, physicists have been working on the realization of elementary quantum gates with the goal to build in the future a quantum computer (*cf.* the cavity quantum electrodynamics experiments with circular Rydberg atoms at the Ecole Normale Supérieure in Paris as well as the handling of trapped ions with lasers at Innsbruck University). The main difficulty to overcome for the effective construction of a quantum computer is the decoherence that results form the coupling of Q-bits with their environment: entangled states are difficult to achieve and to maintain over a significant period of time. The goal is to adapt existing control techniques and if necessary to propose new ones for modeling and controlling *open quantum systems*. In particular, for Q-bits coupled with the environment, controllability and disturbance rejection issues arise when trying to design a control that drives the system from one pure quantum state to another (quantum gate) while compensating for the decoherence induced by the environment.

In order to take decoherence into account, one has to use the Heisenberg point of view where the density matrix is used instead of the probability amplitude (Schrödinger point of view); this framework takes into account the coupling with a large environment (reservoir) and its irreversible effects. Under weak coupling and short environment auto-correlation time, the evolution can be described by a differential equation, called the *master equation* which has a well-defined structure under the so-called Lindblad operators [83]; it yields a finite-dimensional bilinear control system, that has not been thoroughly studied up to now. This is the subject of ongoing research. A more sophisticated model is the Bloch-Redfield formalism [64]; it does not have a finite-dimensional state (in the control-theoretic sense of this word), but it seems more realistic when the control undergoes fast variations. There is numerical evidence (see [76]) that, in this model, the control can effectively act against dissipation.

This is a very new research topic for the team. We report in section 6.12 on some investigation that started within the post-doctoral stay of Hamza Jirari.

5. Software

5.1. The Tralics software

Participant: José Grimm [manager].

The development of the LaTeX to XML translator, named Tralics, was continued (see section 6.1). A new version was sent to the APP in February 2007, its IDDN number is InterDepositDigitalNumber = IDDN.FR.001.510030.001.S.P.2002.000.31235. Binary versions are available for Linux, Windows and MacOS X. Its web page is http://www-sop.inria.fr/apics/tralics. It is now licensed under the CeCILL license version two, see http://www.cecill.info. Latest release is version 2.11, dated 29-11-2007.

5.2. The RARL2 software

Participants: Jean-Paul Marmorat, Martine Olivi [manager].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see section 3.1.4) http://www-sop.inria.fr/apics/RARL2/rarl2-eng.html.

This software takes as input a stable transfer function of a discrete time system represented by

- either its internal realization,
- or its first N Fourier coefficients,
- or discretized values on the circle.

It computes a local best approximant which is *stable, of prescribed McMillan degree*, in the L^2 norm.

It is akin to the arl2 function of Endymion from which it differs mainly in the way systems are represented: a polynomial representation is used in Endymion, while RARL2 uses realizations, this being very interesting in certain cases. It is implemented in Matlab. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parameterization that has the following advantages

- it incorporates the stability requirement in a built-in manner,
- it allows the use of differential tools,
- it is well-conditioned, and computationally cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum (see section 6.3) by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification 4.3.2 as well as sources or cracks recovery 4.2. It was released to the universities of Delft, Maastricht, Cork and Brussels. The paramatrization embodied in RARL2 was recently used for a multi-objective control synthesis problem provided by ESTEC-ESA (The Netherlands) 6.3.

5.3. The RGC software

Participants: Fabien Seyfert, Jean-Paul Marmorat.

The identification of filters modeled by an electrical circuit that was developed by the team (see section 4.3.2) has led to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called "the arrow form" [60] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and also because the assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see 5.6), which is the successor of RGC, solves in a guaranteed manner this constraint realization problem.

5.4. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/personnel/Fabien.Seyfert/Presto_web_page/presto_pres.html. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment);
- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree;
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies either on hyperion (Unix or Linux only) or RARL2 (platform independent), both rational approximation engines were developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Alcatel Space in Toulouse and a license agreement has been recently negotiated with Thales airborne systems. XLim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingstone, Canada).

5.5. The Endymion software

Participant: José Grimm [manager].

We continued the development of *Endymion*, http://www-sop.inria.fr/apics/endymion/index.html, a software licensed under the CeCILL license version two, see http://www.cecill.info. This software will offer most of the functionalities of hyperion (whose development has been abandoned in 2001). It is much more portable, not depending any more on an external garbage collector. Memory management has been made easier by adding a type PolynomL, that lies between Polynom (basic polynomial type) and Lisp (generic object); such objects are automatically destroyed after use. The rational approximation algorithm *arl2* and the bounded extremal problem *bep2* are implemented but not yet fully debugged.

5.6. Dedale-HF

Participant: Fabien Seyfert.

Dedale-HF is a software meant to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of a given filter characteristics (see section 6.7). Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 4).

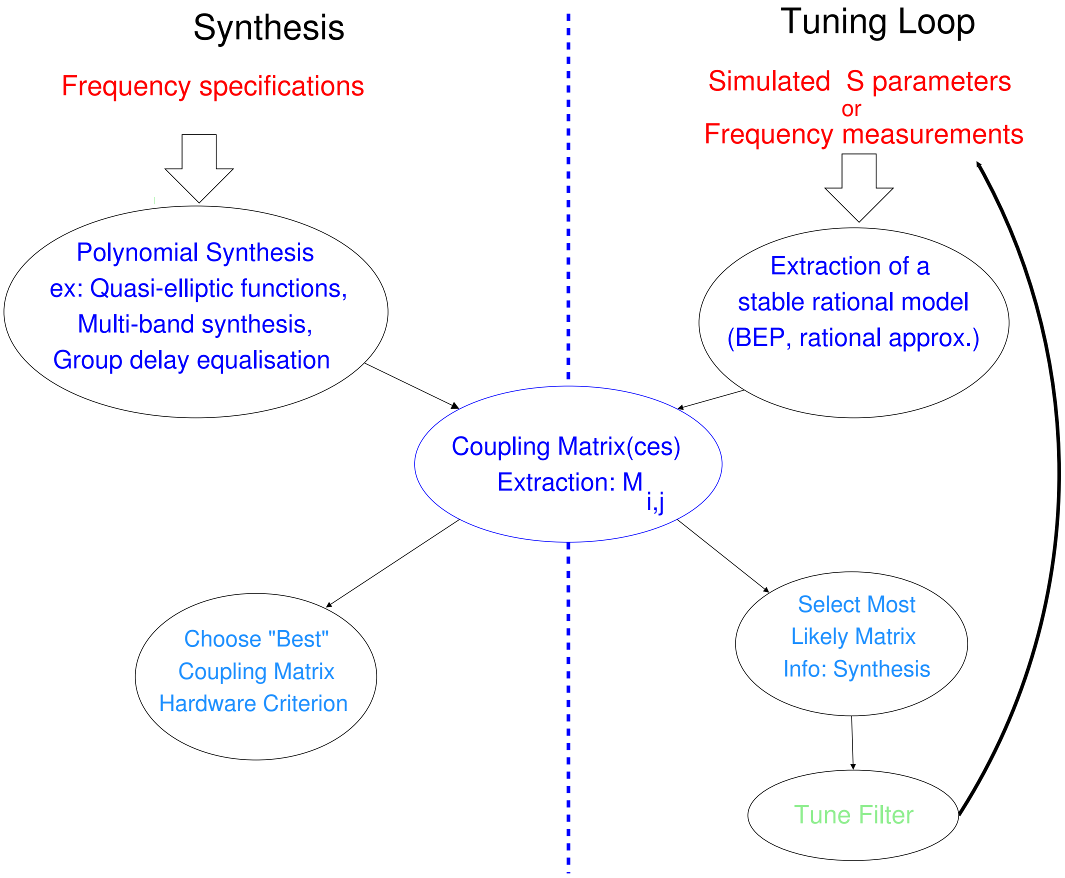

Figure 4. Overall view of the design and tuning process of a microwave filter

Dedale-Hf consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to a particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, *i.e.* the one corresponding to a user-specified filter characteristics. The reference files are computed off line using Groebner basis techniques or numerical techniques based on the exploration a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction of the computational time, say, by a factor of 20.

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale/WebPages. The software is free of charge for academical research purposes: a registration is however needed in order to have access to the tool's full functionality. Up to now 90 users have registered among the world (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

As mentioned in 6.7 an extension of this software that handles symmetrical networks is under construction.

5.7. Library OreModules

Keywords: *Constructive algebraic analysis*, *Gröbner bases*, *constructive homological algebra*, *control theory*, *linear functional systems*, *mathematical physics*, *mathematical systems theory*.

Participants: Thomas Cluzeau [ENSIL, Limoges], Anna Fabiańska [U. of Aachen, Germany], Alban Quadrat [correspondent], Daniel Robertz [U. of Aachen, Germany].

The OREMODULES library of Ore_algebra (*Ore_algebra* is a part of the commercial release of Maple) is dedicated to the study of linear functional systems defined over certain Ore algebras of functional operators and their applications in mathematical systems theory and mathematical physics.

The main novelty of OREMODULES is to combine the recent developments of the Gröbner bases over some non-commutative polynomial rings with new algorithms of algebraic analysis in order to effectively check classical properties of module theory (e.g., existence of a non-trivial torsion submodule, torsion-freeness, reflexiveness, projectiveness, stably freeness, freeness), give their system-theoretical interpretations (existence of autonomous elements or successive parametrizations, existence of minimal/injective parametrizations or Bézout equations) and compute important tools of homological algebra (e.g., (minimal) free resolutions, split exact sequences, extension functors, projective or Krull dimensions, Hilbert power series).

The abstract language of homological algebra used in the algebraic analysis approach carries over to the implementations in OREMODULES: up to the choice of the domain of functional operators which occurs in a given system, all algorithms are stated and implemented in sufficient generality such that linear systems defined over the Ore algebras developed in the Maple package of *Ore_algebra* are covered at the same time. Applications of the OREMODULES package to mathematical systems theory are illustrated in a large library of examples.

The STAFFORD package of OREMODULES contains an implementation of constructive versions of J. T. Stafford's famous but difficult theorem stating that every ideal over the Weyl algebras $A_n(k)$ and $B_n(k)$ (k is a field of characteristic 0) can be generated by two generators. Based on this implementation and on algorithmic results recently obtained by the authors of this package, two algorithms have been implemented which compute bases of free modules over the Weyl algebras $A_n(\mathbb{Q})$ and $B_n(\mathbb{Q})$.

The forthcoming QUILLEN-SUSLIN package of OREMODULES, developed by A. Fabiańska (University of Aachen) with the help of A. Quadrat, contains an implementation of the famous Quillen-Suslin theorem. In particular, this implementation allows us to compute bases of free modules over a commutative polynomial rings with coefficients in the field $\mathbb{Q}$ and in the principal ideal domain $\mathbb{Z}$.

The MORPHISMS package of OREMODULES was developed by T. Cluzeau (ENSIL, Limoges) and A. Quadrat in order to handle some homological tools such as computations of some morphisms between two finitely presented modules over Ore algebras, compute kernel, coimage, image and cokernel of such morphisms and projectors. Using the packages STAFFORD and QUILLEN-SUSLIN, these results allow us to compute factorizations as well as finding some decompositions of linear systems over Ore algebras. In terms of module theory, the MORPHISMS package gives some methods to test if two modules are isomorphic, if a given module contains a submodule (reducible modules) or if it can be written as the direct sum of two submodules. Applications of the MORPHISMS package to mathematical physics and mathematical systems theory are illustrated in a library of examples.

6. New Results

6.1. Tralics: a LaTeX to XML Translator

Keywords: *HTML*, *MathML*, *XML*.

Participant: José Grimm.

The major use of *Tralics* remains the production of the RAWEB (Scientific Annex to the Annual Activity Report of Inria), as explained schematically on figure 5. The input is a LaTeX file, converted by *Tralics* into an XML file; this file is converted to another XML file, conforming to a new DTD, via *xslrproc*; this new file is converted to HTML or XSLT-FO via the use of style sheets; the XSL-FO file formatted into Pdf by pdfTeX, thanks to the *xmltex* package that teaches TeX the subtleties of XML and utf-8 encoding, and two packages for the XSL-FO and MathML commands. This process is completely automatic: it suffices to call *make*; another possibility consists in sending a tar file with the sources to *iRAbot* (the Inria Raweb Robot). Once authors have finished writing their contributions, all files are send a common place, and processed by a tools names RalyX. Since 2002, a lot of people have worked on these tools, for instance M.P. Durollet, J. Grimm, C. Rossi, B. Marmol, A.M. Vercoustre, A. Benveniste, I. Vatton, J.-P. Verjus, J.-C. Le Moal, L. Pierron.

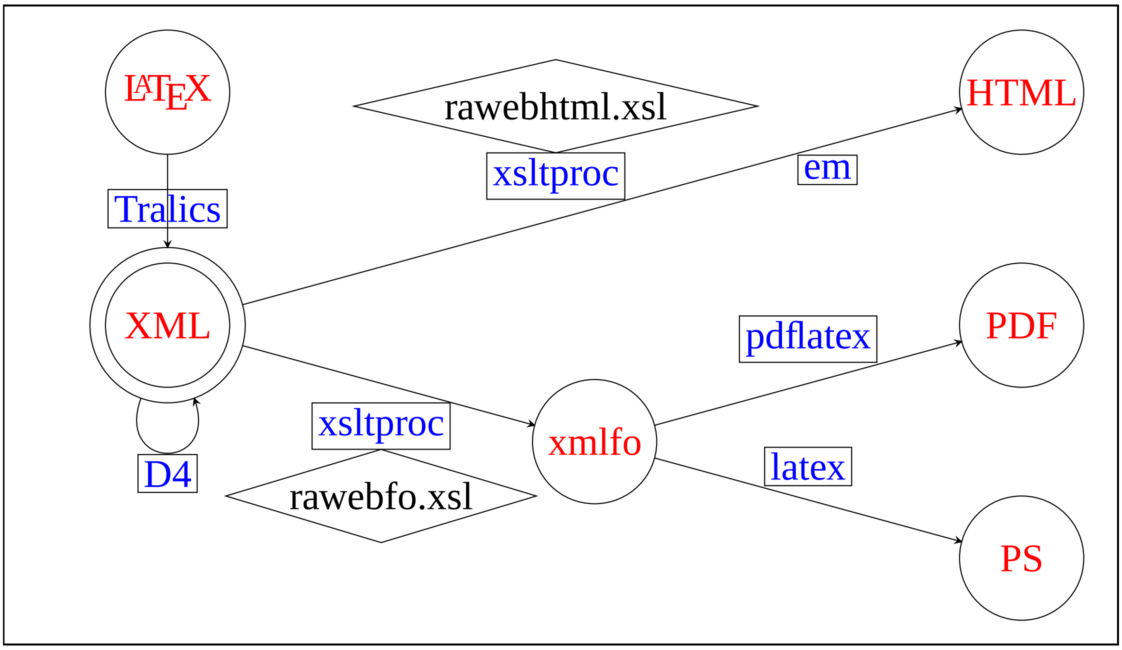

Figure 5. A diagram that explains how the raweb operates. Rectangular boxes contain tools, diamond-shape boxes are style sheets, and circles contain language names. The name 'XML' is in a double circle, it is the central object; the arrow labelled 'D4' that connects it to itself indicates conversion from one DTD to the other, used in 2004. The box containing 'em' represents the Perl script `extract-math.pl` that handles the math formulas; it uses tools borrowed from `latex2html`. This figure was made using the 'pgf' package, a new portable graphic format, not yet understood by Tralics.

A second application of *Tralics* is the production of metadata (author, title, keywords, abstracts) on HTML servers. One could imagine that a server like HAL computes all relevant meta data from the TeX source (as was the case for the Inria server before the use of HAL). A typical example is Cedram (http://www.cedram.org (Centre de diffusion de revues académiques mathématiques), that publishes for instance the Annales de l'Institut Fourier, where you have to pay for the full paper, but the abstract, produced by *Tralics* is free.

The main philosophy of *Tralics* is to have the same parser as TeX, but the same semantics as LaTeX. This means that commands like `\chardef`, `\catcode`, `\ifx`, `\expandafter`, `\csname`, etc., that are not described in the LaTeX book and not implemented in translators like latex2html, tth, hévéa, etc., are recognized by Tralics. The program is configurable: the translation depends on certain options and on the `\documentclass`. All element names (except p) can be changed by the user. A great number of extensions have been added (for instance, the internal encoding is UTF-8, commands like `\catcode` apply to all Unicode characters, all extensions of ϵ-TeXlike `\unless` are available).

Two major versions have been released this year, namely 2.10 in May (redesigned for the 2007 activity report) and 2.11 in November. The two reports [73] and [32], were updated. Almost all math constructions found in the TeXbook or the LaTeX companion are now working. These are described in [31], which is also available as an XHTML+MathML page on the Tralics web page, that contains also the documentation and a link to the sources (the software is licensed under CeCILL, it is an open source free software). We also provide support for Content markup, via commands of the form `\mathcn`.

Among the fine points that were enhanced this year, let us mention the following ones: the implementation of the `\pers` command, which was modified so that if a Team belongs to more than one Research Center, then you have to specify to which one each person belongs (a default value can be indicated). The number of Hdr is counted. A major difference between previous versions is that no intermediate TeX file is generated: in *Simplified RA* mode, the preprocessor has been removed, as well as all its fatal errors, the order of modules is not changed any more.

The input encoding mechanism was also enhanced: it is possible to specify the encoding (via the inputenc package) anywhere in the document. The MathML processor has been enhanced, as described in [30]. New packages, like braket have been implemented.

6.2. Inverse Problems for 2-D and 3-D elliptic operators

Keywords: *Beltrami equation*, *Laplacian*, *inverse problems*, *non destructive control*, *plasma confinement*, *tomography*.

Participants: Laurent Baratchart, Rania Bassila, Fehmi Ben Hassen [LAMSIN-ENIT], Maureen Clerc [Odyssée], Imen Fellah, José Grimm, Mohamed Jaoua [UNSA], Juliette Leblond, Moncef Mahjoub, Jean-Paul Marmorat [CMA-EMP], Théodore Papadopoulo [Odyssée], Jonathan R. Partington [Univ. Leeds], Alban Quadrat, Stéphane Rigat, Edward Saff [Univ. Vanderbilt], Maxim Yattselev, Meriem Zghal.

6.2.1. Cauchy problems

Solving overdetermined Cauchy problems for the Laplace equation on an annulus (in 2-D) or a spherical layer (in 3-D) in order to treat incomplete experimental data is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG since the latter involves propagating the initial conditions from the boundary to the center of the domain where the singularities (*i.e.* the sources) are sought. Here, the domain is typically made of several homogeneous layers of different conductivities.

6.2.1.1. 2-D domains

Solving Cauchy problems on a 2-D annulus is the main topic of M. Mahjoub's PhD thesis. This issue arises when identifying a crack in a tube or a Robin coefficient on the inner skull thereof. It can be formulated as a best approximation problem on part of the boundary of a doubly connected domain, and both numerical algorithms and stability results were obtained in this framework [80], [79]. They generalize those previously obtained in simply connected situations [67] and have been efficiently coupled with the sources recovery issue in a 2D model of the EEG problem [58].

Still in the 2-D case with incomplete data, the geometric problem of finding, in a stable and constructive manner, some unknown (insulating) part of the boundary of a domain was considered in the Ph.D. thesis of I. Fellah [13]. Approximation and analytic extension techniques described in section 3.1.1, together with numerical conformal transformations of the disk, here also provide us with interesting algorithms for the inverse problem under consideration.

6.2.1.2. 3-D spherical layers

Cauchy problems on 3-D spherical layers offer an opportunity to state and solve extremal problems for harmonic fields for which an analog of the Toeplitz operator approach to bounded extremal problems [43] has been obtained. More specifically, the density of traces of harmonic gradients in L^2 of an open subset of the 3-D sphere was established, and a Toeplitz operator with symbol the characteristic function of such a subset was defined. Then, a best approximation on the subset of a general vector field by a harmonic gradient under a L^2 norm constraint on the complementary subset can be computed by an inverse spectral equation for the above-mentioned Toeplitz operator. Constructive and numerical aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) are under study and encouraging results have been obtained on numerically simulated data [38].

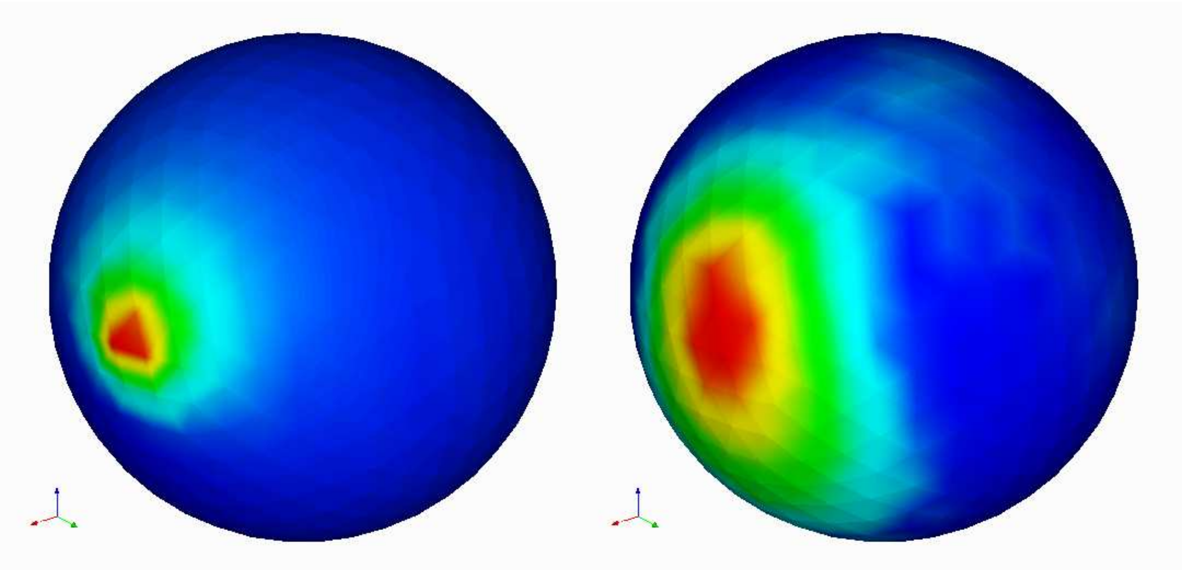

Figure 6. explicit data and (BEP) solution

The corresponding problem in L^∞ is considerably more difficult. Very recently, in a collaborative work with A. Bonami and S. Grellier (université d'Orléans) in the framework of the ANR project AHPI, we obtained that the BMO distance between a bounded vector field on the sphere and a bounded harmonic gradient is within a constant of the norm of a Hankel-like operator, acting on L^2 divergence-free vector fields with values in L^2 gradients. Such Hankel operators were apparently not introduced before. This result, which is based on the atomic decomposition for H^1 and the div-curl lemma, opens new perspectives on the analogy between extremal problems on the disk and the sphere.

6.2.1.3. The conductivity operator

In collaboration with the CMI-LATP (University Marseille I) and in the framework of the ANR AHPI, the team considers 2-D diffusion processes with variable conductivity. In particular the so-called 2-D *real Beltrami equation*, whose compatibility condition is the 2-D div-grad equation, was investigated. In the case of a smooth domain, and for a smooth ($W^{1,\infty}$) conductivity, we analyzed the Dirichlet problem for solutions in Sobolev $W^{1,p}$ classes, $1 < p < \infty$. The traces of such solutions on a strict subsets K of the boundary turn out to be dense in $W^{1-1/p,p}(K)$. The behavior of the associated Cauchy and Beurling operators as well as $W^{1,p}$ estimates for generalized Riesz transforms have already been obtained [48], [27]. We also introduced less regular solutions of Hardy-type (*i.e.* having bounded integral L^p-means on a system of curves tending to the boundary). Their traces merely lie in L^p of the boundary, a space which is better suited to identification from pointwise measurements than the more regular $W^{1-1/p,p}$. Again these traces turn out to be dense on strict subsets of the boundary. This should allow for us to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a manner similar to what we did for analytic functions as discussed in section 3.1.1. The case of a conductivity that is merely in L^∞, which is important for inverse conductivity problems, is now under study. Then, it is still unknown whether solutions exist for all p.

The application that initially motivated this work is described in the next section.

6.2.2. Application to free boundary problems and plasma control

Let us briefly describe a potential application of inverse boundary problems for the Beltrami equation, on a 2-D doubly connected domain, to plasma confinement for thermonuclear fusion in a Tokamak; this collaborative work was started in collaboration with the Laboratoire J. Dieudonné (University of Nice). In the particular case at hand, it seems possible to explicitly compute a basis of solutions (Bessel functions) that should greatly help the computations, see [47], but the techniques should be valuable more generally.

In the most recent Tokamaks, like Jet or ITER, an interesting feature of the level curves of the poloidal flux is the occurrence of a cusp (a saddle point of the poloidal flux, called an X point), and it is desirable to shape the plasma according to a level line passing through this X point for physical reasons relating to the efficiency of the energy transfer.

The problem we have in mind here is of dual Bernoulli type. Classically, the interior Bernoulli problem on a domain Ω (see [70]) is to find a closed subset $A \subset \Omega$ and a harmonic function u in $\Omega \setminus A$ such that $u = 0$ on $\partial\Omega$, $u = 1$ on ∂A, and $\partial u/\partial\nu = Q$ on ∂A, where Q is a given positive constant and ν indicates the outer normal. A natural generalization is obtained on letting u satisfy a more general diffusion equation

$$\mathrm{div}\,(\sigma\nabla u) = 0 \tag{1}$$

in $\Omega \setminus A$, for some non-constant conductivity $\sigma > 0$.

The dual problem arises when both u and $\partial u/\partial n$ are given on the known boundary $\partial\Omega$ while $u = Q$ is constant on ∂A. Note that this issue is *overdetermined*, that is, the boundary data on $\partial\Omega$ have to satisfy some compatibility conditions (of generalized Cauchy-Riemann type). One motivation for the dual problem is the observation that, in the transversal section of a Tokamak (which is a disk if the vessel is idealized into a torus), the so-called poloidal flux is subject to (1) outside the plasma volume for some simple explicit real analytic function σ, while the boundary of the plasma is a level line of this flux [62]. Actually, when looking for a X point, the main interest is attached to the smallest connected so-called “elliptic” solution, which makes for a definite object of study among all other solutions.

When σ is constant and $\partial u/\partial n$ has zero mean on $\partial\Omega$, it is well-known that u has a conjugate function v such that $u + iv$ is holomorphic in $\Omega \setminus A$. More generally, as soon as σ is bounded away from zero and $\sigma\partial u/\partial n$ has zero mean on $\partial\Omega$, a generalized conjugate exists such that $f = u + iv$ satisfies the so-called *real Beltrami equation*:

$$\partial f/\partial\bar{z} = \nu\overline{\partial f/\partial z} \tag{2}$$

where $\nu = (1 - \sigma)/(1 + \sigma)$. Moreover, the Dirichlet-Neumann data for u determine the boundary values of f on $\partial\Omega$ (up to an additive imaginary constant). For fixed A and Q, we intend to study the extremal problem of best approximating these values by (the trace on $\partial\Omega$ of) a solution to (2) under the constraint that it has non-negative real part at most Q on ∂A. This is an infinite-dimensional convex problem whose Lagrange parameter will indicate both how to deform and how to modify A locally in order to improve the criterion.

6.2.3. Sources recovery in 2-D and 3-D

The fact that 2-D harmonic functions are real parts of analytic functions allows one to tackle issues in singularity detection and geometric reconstruction from boundary data of solutions to the Laplace equations using the meromorphic and rational approximation tools developed by the team. Some electrical conductivity defaults can be modeled by pointwise sources inside the considered domain. In dimension 2, the question made significant progress in recent years: the singularities of the function (of the complex variable) which is to be reconstructed from boundary measures are poles (case of dipolar sources) or logarithmic singularities (case of monopolar sources). Hence, the behavior of the poles of the rational or meromorphic approximants, described in section 3.1.3, allows one to efficiently locate their position. This is the topic of the article [1], where the related situation of small inhomogeneities connected to mine detection is also considered, and of [58] for a 2D version of the EEG problem, where a first analytic completion step has been performed on the data.

The problem of sources recovery can be handled in 3-D balls by using best rational approximation on 2-D cross sections (disks) from traces of the boundary data on the corresponding circles. It turns out that each of these traces coincides with a 2-D analytic functions in the slicing plane, that has branched singularities inside the disk [4]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework where approximately recovering these singularities can be performed using best rational approximation.

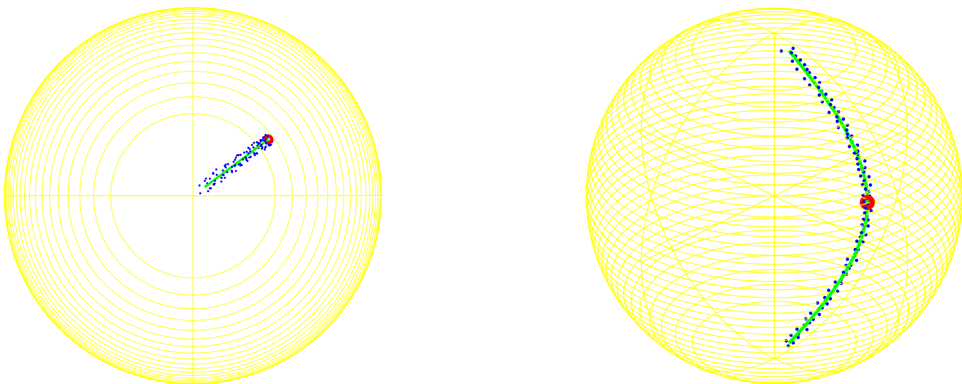

Figure 7. Localization of a single source, above and profile views

We also started to consider more realistic geometries for the 3-D domain under consideration. A possibility is to parametrize it in such a way that its planar cross-sections are quadrature domains or R-domains. In this framework, best rational approximation can still be performed in order to recover the singularities of solutions to Laplace equations, but complexity issues have to be examined carefully. The preliminary case of an ellipsoid requires the preliminary computation of an explicit basis of ellipsoidal harmonics [28], [86] and is the topic of the PhD thesis of M. Zghal.

6.2.4. Application to EEG inverse problems

In 3-D, epileptic regions in the cortex are often represented by pointwise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electoencephalography). Note that the patient's head is here modeled as a nested sequence of spherical layers. A breakthrough was made which makes it possible now to proceed via best rational approximation on a sequence of 2-D disks along the inner sphere [4]. The point here is that, up to an additive function harmonic in the 3-D ball, the trace of the potential on each boundary circle coincides with a function having branched singularities in the corresponding disk. The behavior along the family of disks of the poles of their best rational approximants on each circle is strongly linked to the location of the sources, using properties discussed in sections 3.1.3 and 6.6. In the particular case of a unique source, we end up with a rational function which makes for an easy detection; when there are several sources, their localisation requires a slightly more sophisticated machinery to make the convergence of poles of meromorphic approximants effective (see section 6.6). This and other related issues including some preprocessing of the function are still under study. This inverse EEG problem is the object of a collaboration between the Apics and Odyssée Teams, through the ACI "Obs-Cerv" during last years and now around the associate engineer R. Bassila. Her task consists in achieving a dedicated numerical interface between available softwares from both teams, "FindSources3D".

6.3. Parametrizations of matrix-valued lossless functions

Participants: Bernard Hanzon [Univ. Cork], Jean-Paul Marmorat, Martine Olivi, Ralf Peeters [Univ. Maastricht].

We mentioned in section 3.1.4 the role of parameters defining an atlas of charts in optimization problems on a manifold. Such parametrizations are available for lossless transfer functions and have been implemented in our rational approximation softwares. Charts were obtained from Schur analysis, the parameters being Nevanlinna-Pick interpolation values [10]. These parametrizations present a number of advantages from an algorithmic viewpoint : flexibility, implementation facilities, numerical behavior. Our results concerning more general interpolation parameters, associated with a Nudelman (contour integral) interpolation problem have been published this year [23]. This last parametrization has been used in an application to multi-objective control which is the object of a contract (n^{o} 20084/06/NL/JD) between CMA-EMP and ESTC-ESA (Noordwijk, NL). The problem of multi-objective control synthesis leads to a linear matrix inequality which has to be minimized with respect to a fixed degree stable system called the Youla parameter. The parameterization is used to control the poles part of the Youla parameter. The linear part is obtained by the semidefinite minimization of the LMI.

Now, the balanced realizations obtained in an arbitrary chart possess no particular structure. However, a particular form of the realization matrix can present some interest either because it explicitly contains the physical parameters (coupling parameters between resonators for the coupling matrix of an hyperfrequency filters, see section 4.3.2) or because of its nice behaviour under truncation. We have some results in that connection. Upon choosing the interpolation points at zero and the directions in a particular manner among standard basis vectors, a minimal atlas can be specified in which the balanced realizations have a staircase structure with the property that the corresponding controllability matrix is positive upper-triangular [24].

Up to now, our parametrizations only concern discrete-time transfer functions. In continuous-time, which is relevant in many applications, we have established a connection in the SISO case between the well-known Schwartz form and a boundary interpolation problem on the imaginary axis. These results have been presented at the Benelux meeting on systems and control (Lommel, Belgium, March 2007).

6.4. Lossless completion of a Schur function

Participants: Laurent Baratchart, Andrea Gombani, Martine Olivi, Fabien Seyfert.

Darlington Synthesis is an old problem dating back to pre-transistor times (1939), and it was studied in a system theoretic framework in the seventies. In recent years, with the advent of the information technology society, synthesis problems became important again. Indeed, new devices are being built whose design rests on optimizing a linear finite-dimensional approximation around the nominal frequency. These models are subject to various constraints coming from the underlying physical device, and parametrizing them is a prerequisite to optimization, see section 6.9 for the example of resonant cavities.

Darlington Synthesis turns out to show up naturally when modelling SAW filters, which are used in most cheap telecommunication devices (e.g. mobile phones)4.3.1. In this application the physical law of reciprocity induces a symmetry constraint. In mathematical terms, given a $p \times p$ symmetric Schur (contractive) function S, the problem is to imbed S into symmetric lossless function $\widehat{S}$ of twice the original size. One of course looks for a minimal degree completion, or dually one imposes the degree to be minimal and seeks a minimal size completion. In collaboration with P. Enqvist from KTH (Stockholm, Sweden), we characterized the existence of a $(2p \times 2p)$ symmetric Darlington synthesis with specified increase of the McMillan degree: a symmetric completion of a symmetric contractive matrix S of degree n exists in degree $n + k$ if, and only if, $I - SS^*$ has at most k zeros with odd multiplicity [16]. In the language of circuit theory, this results tells us about the minimal number of gyrators to be used in circuit synthesis. Using a frequency domain approach, these results have been extended to the case of real-valued functions. These results have been presented at LynSys2007 (Canberra, February). In view of multi-port synthesis applications, the completion of a scalar Schur function, or of a unitary row, to a square conservative matrix is currently under study.

6.5. Rational and meromorphic approximation

Participants: Laurent Baratchart, Vincent Lunot, Edward Saff, Maxim Yattselev.

The results of [3] and [5] have been extensively used over the last two years to prove the convergence in capacity of L^p-best meromorphic approximants on the circle (*i.e.* solutions to problem (P_N) of section 3.1.2) when $p \geq 2$, for those functions f that can be written as Cauchy transforms of complex measures supported on a hyperbolic geodesic arc $\mathcal{G}$ [57]. A rational function can also be added to f without modifying the results, which is useful for applications to inverse sources problems. Some mild conditions (bounded variation of the argument and power-thickness of the total variation) were required on the measure. Here, we recall that convergence in capacity means that the (logarithmic) capacity of the set where the error is greater than ε goes to 0 for each fixed $\varepsilon > 0$. Deepening the analysis, we were able this year to quantify this convergence, namely it is geometric with pointwise rate $\exp\{-1/C - G\}$ where C is the capacity of the condensor $(T, \mathcal{G})$ and G the Green potential of the equilibrium measure. The results can be adapted to somewhat general interpolation schemes, and two articles were submitted for publication on the subject [55], [54]. From this work it follows that the counting measures of the poles of the approximants converge, in the weak-* sense, to the Green equilibrium distribution on $\mathcal{H}$. In particular the poles cluster to the endpoints of the arc, which is of fundamental use in the team's approach to source detection (see section 6.2.3).

The technique we just described only yields convergence in capacity and n-th root asymptotics. To obtain strong asymptotics, additional assumptions must usually be made on the approximated function. This year, we proved strong asymptotics for the afore-mentioned Cauchy integrals when the density of the measure is Dini-smooth, and vanishes in at most finitely many points like a small fractional power. Moreover,the polar singularities of the function are asymptotically reproduced by the approximants with their multiplicities. This result is important for inverse problem of mixed type, like those mentioned in section 6.2.3, where monopolar and dipolar sources are handled simultaneously. Convergence even holds on the support of the measure if the latter is analytic. The method is to translate the critical point equation into trigonometric orthogonality on T where harmonic analysis techniques can be used. Analyzing the orthogonality in terms of Hankel and Toeplitz spectral equations, the conclusion ultimately follows from estimates on the essential spectrum of Hankel operators. This type of result is new, for the density may vanish at some points (although in a control manner). When relaxed to multipoint interpolation schemes instead of optimal approximation, it allows one to consider rather general arcs that need not be geodesic, which is also new. An article is being written on this topic [56]. Moreover, these results yield bounds on the multiplicity of the singular values of the underlying Hankel operators [91].

The sharp asymptotics mentioned above have been used to obtain uniqueness of a critical point (hence of a local *minimum*) in rational $L^2(T)$-approximation to the corresponding functions f. The method of proof, based on the index theorem and on some estimates of the Hessian matrix, closely follows [52], but appeals to [3] in order to bound the distance of the poles from $\mathcal{H}$. This is the first uniqueness result for Cauchy integral of *complex* measures.

Through the doctoral work of V. Lunot, a different rational approximation problem, has been taken up. More precisely, we considered the approximation of a function f in the Schur class (*i.e.* those functions analytic in D and bounded by 1 in modulus) by a rational function of given degree which is also in the Schur class. This problem is quite important for the modelling of passive systems. Our technique dwells on the multipoint Schur algorithm [65]. After the seminal work in [77], the links between the classical Schur interpolants and the orthogonal polynomials on the circle associated to the Clark measure representing $(1+zf)/(1-zf)$ are well understood. When the multipoint algorithm is used on a sequence of points that remains in a compact subset of D, corresponding generalizations were carried over with orthogonal polynomials replaced by orthogonal rational functions. However, in order to efficiently approximate functions that do not extend analytically across T, it is necessary to let the interpolation points tend to T. Using weighted polynomials approximation [88], we constructed a distribution of interpolation points in D that produces a hyperbolic analog to the Szegö theorem. Namely, the hyperbolic distance between f and the n-th interpolant goes to zero when integrated against the Poisson kernel at the n-th interpolation point. This result lends perspective to the use of interpolation points near the boundary in order to locally reduce the approximation error. This piece of research will be part of V. Lunot's doctoral dissertation.

6.6. Behavior of poles

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

It is known after [8] that the denominators of best rational of meromorphic approximants to a function f in the L^p norm on a closed curve (say the unit circle T to fix ideas) satisfy for $p \geq 2$ a non-Hermitian orthogonality relation when f is the Cauchy transform of a complex measure on a curve γ (the locus of singularities) contained in the unit disk D. This has been used in the last two years to assess the asymptotic behavior of the poles of the approximants when γ is a hyperbolic geodesic arc. Specifically, under weak regularity conditions on the measure, the counting measure of the poles converges weak-star to the equilibrium distribution of the condenser (T,γ) [55], [54]. Let us mention that the more general situation where γ is a so-called "minimal contour" for the Green potential (of which a geodesic arc is the simplest example) has been settled with the same conclusion. This technical result is still under writing. It is of particular significance to locate several 2-D sources or piecewise analytic cracks from overdetermined boundary data (see sections 3.1.3 and 6.2).

Strong asymptotics were also obtained last year, that deal with the behavior of *all* the poles and not just a full proportion of them. They were established for f the Cauchy transform of a smooth nonvanishing complex measure on a hyperbolic geodesic arc in the disk, provided the density increases at least like a fractional power at the endpoints of the arc. This year, using Power's theorem on the essential spectrum of Hankel operators, we were able to handle cases where the density *does* vanish. On the one hand, this vanishing is severely restricted: it may only occur in finitely many points and like a fractional power with a bound on the exponent. On the other hand, it is known that conditions have to be put on the zeroing for such a result to hold, and it is apparently first of its kind. Although the bound is likely to be far from optimal, this sheds light on the fundamental phenomenon of spurious poles.

The case where a rational function is added to the approximated function has also been handled, generalizing results of Gonchar and Suetin [72]. A numerical illustration is shown in Figures 8-9 for various approximants to the functions F and G given below.

$$\begin{aligned} F(z) &= 7\int_{[-6/7,-1/8]}\frac{e^{it}dt}{z-t} - (3+i)\int_{[2/5,1/2]}\frac{1}{t-2i}\frac{dt}{z-t} + (2-4i)\int_{[2/3,7/8]}\frac{\ln(t)dt}{z-t} \\ &+ \frac{2}{(z+3/7-4i/7)^2} + \frac{6}{(z-5/9-3i/4)^3} + \frac{24}{(z+1/5+6i/7)^4}. \end{aligned}$$

$$G(z) = \int_{[-0.7,0]}\frac{e^{it}}{z-t}\frac{dt}{\sqrt{(t+0.7)(0.4-t)}} + \int_{[0,0.4]}\frac{it+1}{z-t}\frac{dt}{\sqrt{(t+0.7)(0.4-t)}} + \frac{1}{5!(z-0.7-0.2i)^6}$$

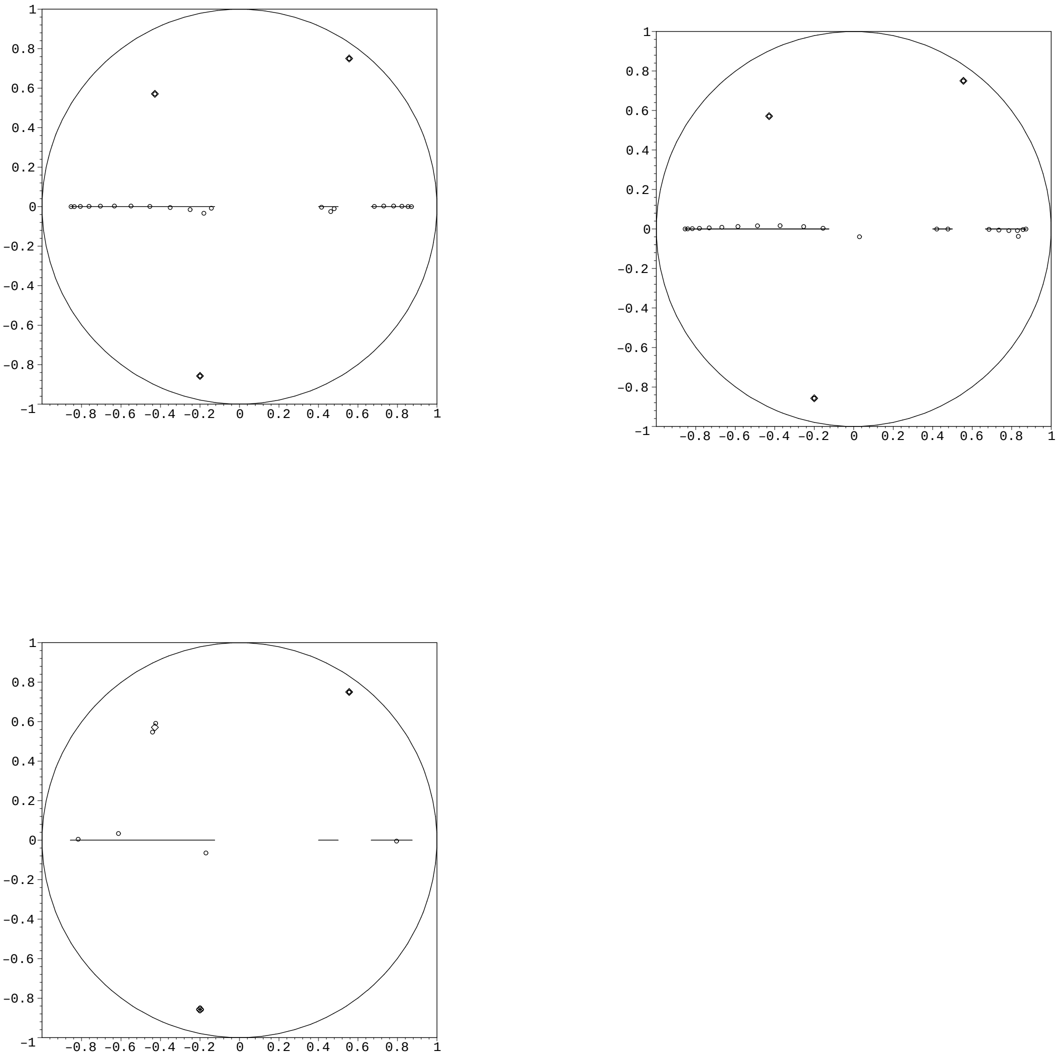

Figure 8. Approximations to the function F; first line: Padé and AAK at degree 30, second line: arl2 at order 13.

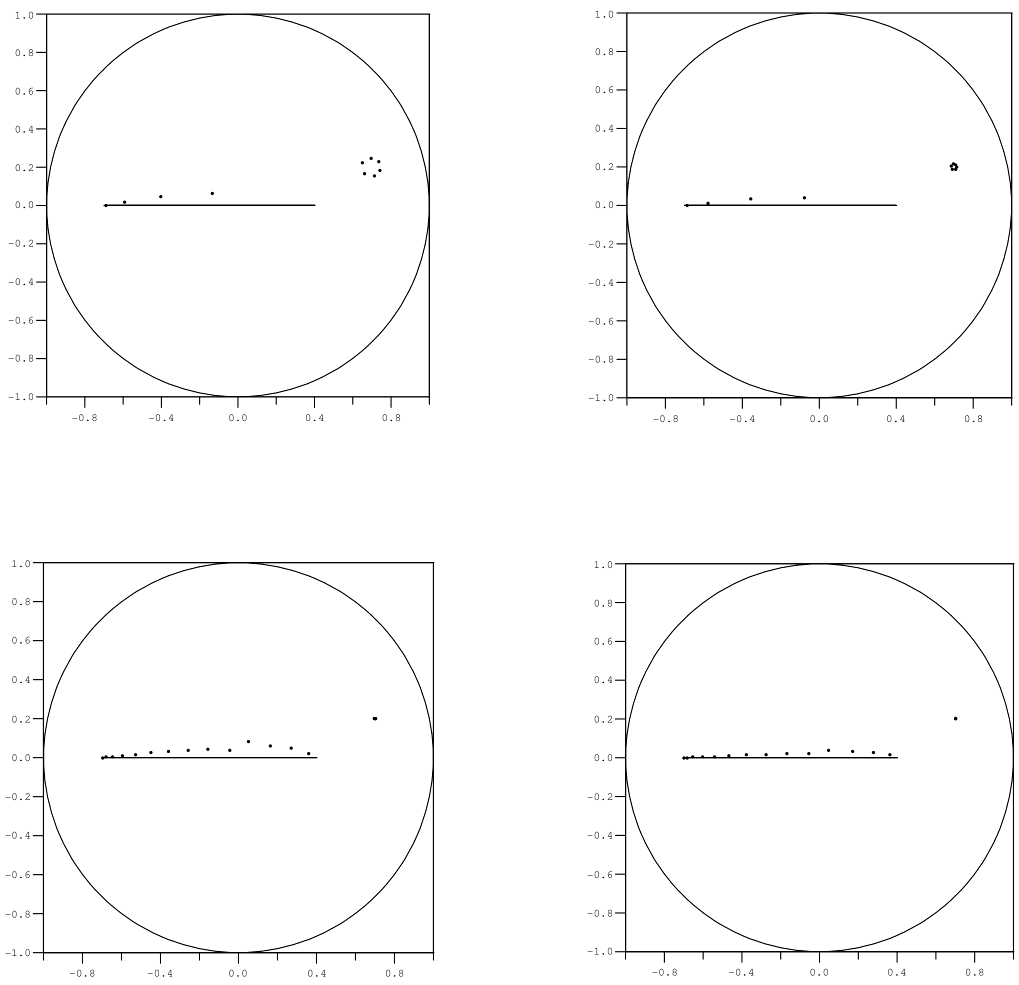

Figure 9. Approximations to the function G; first line: Padé and AAK at order 10, second line, Padé and AAK at order 20.

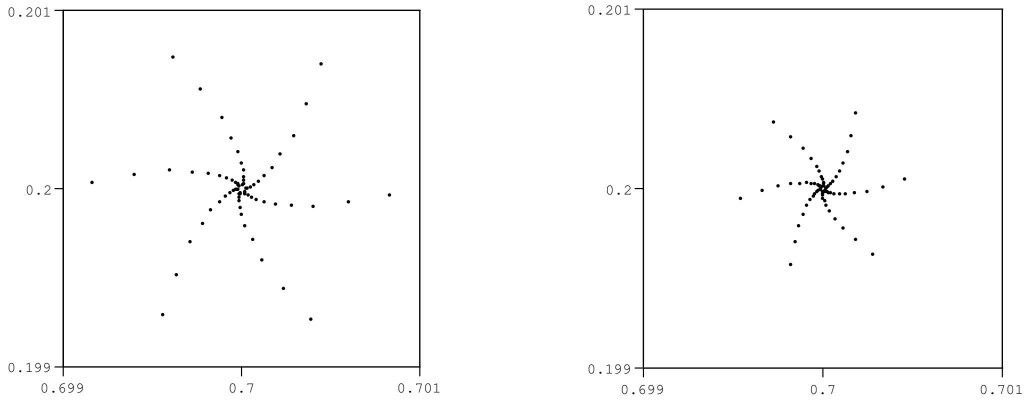

Figure 10. Poles of the n-th approximations to the function G, near the singularity, which is of degree six; left Padé, right AAK; the integer n varies between 21 and 32.

6.7. Exhaustive determination of constrained realizations corresponding to a transfer function

Participants: Jean Charles Faugère [project SALSA, Rocquencourt], Philippe Lenoir, Stéphane Bila [XLim, Limoges], Fabien Seyfert.

Groomed by industrial users like Thales Alenia-Space, we started to analyze the realizations of 2x2 lossless scattering systems whose scattering response $(S_{i,j})$ satisfies the so-called *self-reciprocal* condition $S_{1,1} = S_{2,2}$. In the filter-design community, it was common belief that every self-reciprocal response can be realized in the form of an electrical circuit with plane symmetry. In short, such circuits should be identical when viewed from each access. Whereas it is clear that symmetric circuits have self-reciprocal responses, we showed this year that the converse does not hold. More precisely the algebraic variety of auto-reciprocal responses decomposes into several prime components. Only one of them consists of systems that can be realized by symmetric circuits. In particular we gave examples of some dual band responses that have no symmetric physical realizations whatsoever. In addition, we connected the prime decomposition of the variety with an interlacing property of the transmission and reflexion zeros. This yields an "almost visual" procedure to decide whether a symmetric network can realize a given frequency response.

In future work, we will focus on the practical implementation of this analysis within the software Dedale-Hf 5.6.

A general survey on applications of our work to microwave filters synthesis has been published [26].

6.8. The Zolotarev problem and multi-band filter design

Participants: Vincent Lunot, Philippe Lenoir, Fabien Seyfert.

This piece of research is part of V. Lunot's doctoral work. The theoretical developments took place over the last two years, while deepenings of the numerical aspects were carried out in 2007. The problem goes as follows. On introducing the ratio of the transmission and reflexion entries of a scattering matrix, the design of a multi-band filter response (see section 4.3.2) reduces to the following optimization problem of Zolotarev type [88]:

$$\text{letting: } E_{n,m}(K,K') = \{p \in P_m(K), q \in P_n(K') \text{ such that } \forall x \in I\,, \left|\frac{p(x)}{q(x)}\right| \leq 1\},$$

$$\text{solve: } \max_{(p,q)\in E_{m,n}(K,K')} \min_{x\in J} \left|\frac{p}{q}\right| \tag{3}$$

where $I = \bigcup I_i$ (resp. $J = \bigcup J_i$) is a finite union of compact intervals I_i of the real line corresponding to the pass-bands (resp. stop-bands), and $P_m(K)$ stands for the set of polynomials of degree less than m with coefficients in the field K. Depending on the physical symmetry of the filter, it is interesting to solve problem (3) either for $K = K' = \mathbf{R}$ (“real” problem) or $K = \mathbf{C}, K' = \mathbf{R}$ (“mixed” problem), or else $K = K' = \mathbf{C}$ (“complex” problem). The “real” Zolotarev problem can be decomposed into a sequence of concave maximization problems, whose solution we were able to characterize in terms of an alternation property. Based one this, a Remez-like algorithm has been derived in the polynomial case (*i.e.* when the denominator q of the scattering matrix is fixed), which allows for the computation of a dual-band response (see Figure 12) according to the frequency specifications (see Figure 11 for an example from the spacecraft SPOT5 (CNES)). We have designed an algorithm in the rational case was well which, unlike linear programming, avoids sampling over all frequencies. This raises the issue of the “generic normality” (*i.e.* the maximum degree) of the approximant with respect to the geometry of the intervals. This question remains open. The design of efficient procedures to tackle the “mixed” and “complex” cases remains a challenge. A preliminary version of a software by V. Lunot to treat the complex case has been released this year to our academic partners: Xlim and the Royal Military College of Canada. Applications of the Remez algorithm to filter synthesis are described in [61], [84]. An article on the general approach based on linear programming has been accepted for publication [22].

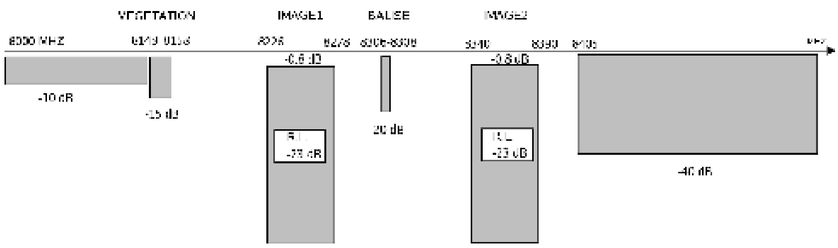

Figure 11. SPOT5 specifications

6.9. Synthesis and Tuning of broad band microwave filters

Participants: Smain Amari [Royal Military College of Canada (RMC), Kingstone], Magued Bekheit [RMC], Fabien Seyfert.

A collaboration has started with the Royal Military College of Canada on the modeling and synthesis of broad band microwave filters. Until now, the team’s work on microwave filters has focused on narrow-band devices where the ratio between the pass-band and the middle frequency is about 1%. This allows the use of low-pass transformations leading to abstract equivalent circuits containing non-physical elements, like impedance inverters (*i.e.* frequency-independent coupling) Recent applications in the GSM sector calls however for the use of filters with broader pass-band. Whereas the low-pass formalism clearly breaks down for relative pass-bands above 5%, the expansion of the filter’s response over finitely many modes of the Helmholtz equation can still be done, justifying the rational character of the response in his case too. We showed,

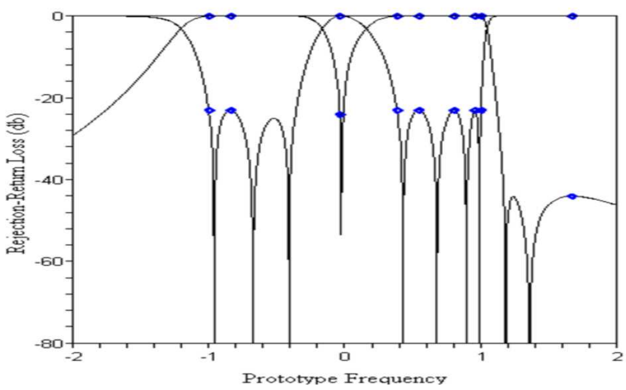

Figure 12. 7^{th} order dual-band response and its critical points

that using a modified version of Zolotarev's problem 6.8 with non-polynomial weight, it is again possible to synthesize optimal responses in the high frequency domain. Moreover an equivalent circuit formalism involving "physical"capacitive couplings was developed in order to help the actual realization of the device. The tuning requires a reliable version of the Presto-Hf software in the high frequency domain. In this new setting where no low-pass transformation can be made, challenging numerical problems arising from the relative smallness of the approximation band in the underlying bounded extremal problems must be overcome. This will be a major research effort of the team in the future. Preliminary results on the realization of suspended strip line filters with relative band up to 30% (in collaboration with the university of Ulm, Germany) are encouraging.

Another pending issue is that of controlling the group delay, that is, the derivative of the phase of p/q. The latter becomes large around abrupt changes of the modulus, and an approach based on perturbing the "optimal" poles to keep the logarithmic derivative more flat is currently being investigated.

6.10. Frequency approximation and OMUX design

Participants: Laurent Baratchart, Vincent Lunot, Jean-Paul Marmorat [CMA-EMP], Fabien Seyfert.

An OMUX (Output MUltipleXor) can be modeled in the frequency domain through scattering matrices of filters, like those described in section 4.3.2, connected in parallel onto a common guide. The problem of designing an OMUX with specified performance in a given frequency range naturally translates into a set of constraints on the values of the scattering matrices and of the phase shift introduced by the guide in the considered bandwidth.

An OMUX simulator on a Matlab platform was designed in recent years and used to check various assumptions about how the OMUX operates; in particular, that each right-section of the OMUX (when the guide gets oriented from the left to the right with common access to the extreme left) acts as a short-circuit to those channels lying "upstream" in their bandwidth. Another assumption was that each channel must reflect a certain amount of energy in its bandwidth in order to trap waves otherwise reflected by the above-mentioned short-circuit. Under the terms of a contract with Alcatel Alenia Space (see section 7.2), these assumptions have been used to design a dedicated software to optimize OMUXes whose first version has just been released to AAS.

The direct approach, used up to now by the manufacturer, consists in coupling a simulator with a general purpose "optimizer", in order to reduce transmission and reflection wherever they are too large. This does not perform well for high degree and narrow bandwidth, where convergence often fails and multiple initial points must be used, resulting in a lengthy and sometimes unsuccessful search. The software proceeds by adding channels recursively, applying to the new channel the above short-circuit and reflection-in-the bandwidth rules. This yields an initial guess for the global “optimizer” which seems to regularly outperform those currently used by AAS. More extensive tests are being conducted. The problem of “manifold-peaks” has not been improved yet, and would require a dedicated study.

6.11. Necessary conditions for dynamic linearization

Participant: Jean-Baptiste Pomet.

No new result was obtained this year on this topic, but the article [14] has appeared. It mostly reports on results from David Avanessoff’s PhD. The subject is the dynamic linearizability, or parametrizability (see section 3.2.2) of control systems with 3 states and 2 controls; this essentially covers the same range of systems as those with 4 states and 3 controls that are affine with respect to the control.

Such systems were previously studied in [11] where it is shown they can be “(x, u)-flat” only if they admit a parametrization of order at most (1,2), *i.e.* one parametrizing function is differentiated at most twice and the other once. We conjecture that other systems cannot be parametrized at all (*i.e.* for no order). What we prove in [14] is that they do not admit parametrizations of order (3,3) nor of order $(2, k)$ for any k.

This result is stronger than those in [11] and also much more readable, as the latter reference used computer algebra in the proofs.

6.12. Control design for logic quantum gates

Participants: Hamza Jirari, Jean-Baptiste Pomet.

This research has been initiated with the post-doctoral stay of H. Jirari, and its goal was described in section 4.5. The systems under study here are quantum systems of low dimension (1 or 2 Qbits) whose autonomous dynamics is given by the Schrödinger equation and whose interaction with the environment is described by some dissipative term. According to the latter, one distinguishes between the Bloch-Redfeld model and the (heavily simplified) Lindblad model. In [76], numerical evidence was obtained that for the Bloch-Redfeld model, at least in simple small dimensional cases, there exists a control that drives the system from some pure initial state to a target one, while compensating the dissipative effect of the environment. We have pursued this numerical effort on more strongly coupled environments, computing *via* optimal control techniques. We also begun the investigation of the control-theoretic structure of such models. In contrast with the Lindblad model, where decoherence is unavoidable in some cases, it seems that the Bloch-Redfeld model exhibits a more robust behavior which calls for further understanding.

6.13. Feedback for low thrust orbital transfer

Participants: Alex Bombrun, Jean-Baptiste Pomet.

Most of this section accounts for results obtained in Alex Bombrun’s PhD work; the thesis was defended on March 12, see [12].

It deals with the orbit transfer of satellites equipped with low thrust engines like plasmic ones, which are energy-efficient but deliver a much smaller thrust than conventional “chemical” engines: the ratio between acceleration and gravity is like 10^{-3}, sometimes less. This problem was raised by Alcatel Alenia Space, who supports our research effort on the subject (see section 7.3).

Low thrust satellites are prototypical of conservative systems with small controls, *i.e.* control systems whose drift is a conservative dynamical system having one or more first integrals, and whose control is constrained to be very small. Here, the drift is the Kepler system, a completely integrable Hamiltonian system with five independent first integral in a six-dimensional phase space. In the terminology of [12], a *Kepler control system* is a system in dimension n whose drift has $n - 1$ first integral and compact trajectories (for the satellite, this is true only for negative energy, in the so-called elliptic domain) and where the control is small in the sense of certain asymptotic properties as the bound on the control tends to zero.

The first results concern the computation of feedback controls, using *ad hoc* Lyapunov functions.These approximate surprisingly well time-optimal trajectories. The easy implementation of such control laws makes them attractive as compared to genuine optimal control. Here the $n-1$ first integrals are an easy means to build control Lyapunov functions since any function of these first integral can be made monotone decreasing by a suitable control. The surprising result is that fitting a Lyapunov function as close as possible to *one particular* time-minimum trajectory yields a feedback control that performs excellently on a wide range of initial conditions. This is under writing for publication. To make the study more systematic, we started an asymptotic analysis when the thrust tends to zero. Control Lyapunov functions can then be tested on the average control system described in section 6.14; this research is still in course.

6.14. Average control systems

Participants: Alex Bombrun, Jean-Baptiste Pomet.

For the class of *Kepler control systems* that we just defined in section 6.13, a notion of *average control system* is introduced in [12]. Using averaging techniques in this context is rather natural, since the free system produces a fast periodic motion and the *small* control a slow one; averaging is a widespread tool in perturbations of integrable Hamiltonian systems, and the small control is in some sense a "perturbation". In some recent literature, one proceeds as follows: the control is pre-assigned, for instance to time optimal control via Pontryagin's Maximum Principle or else to some feedback designed beforehand. Then, averaging is performed on the resulting ordinary differential equation, whose limit behavior is analyzed when the control magnitude tends to zero.

The novelty of [12] (see also [63]) is to average *before* assigning the control, hence getting a *control system* that describes the limit behavior better. For that reason, the average control system is a convenient tool when comparing different control strategies.

It allowed us to answer an open question stated in [66] on the minimum transfer-time between two elliptic orbit when the thrust magnitude tends to zero, see [17].

Under some controllability conditions that are trivially satisfied in the case at hand, we proved that the average system is one where the velocity set has nonempty interior, i.e. all velocity directions are allowed at any point, and the constraint is convex; mathematically this yields a Finsler structure (in the same way as a controllable system without drift with a quadratic constraint on the control yields a sub-Riemanian structure). An article is in progress on these results; the latter can be found in [12].

6.15. Computational methods in mathematical systems theory

Participants: Thomas Cluzeau [ENSIL, Limoges], Anna Fabiańska [U. of Aachen, Germany], Alban Quadrat, Daniel Robertz [U. of Aachen, Germany].

6.15.1. Applications of the Quillen-Suslin theorem and cd implementation

In 1955, Serre conjectured that every row vector with coefficients in a commutative polynomial ring $A = k[x_1, ..., x_n]$ over a field k which admits a right-inverse over A can be completed into a square matrix with a non-zero constant determinant. This conjecture was independently proved by Quillen and Suslin in 1976 and is nowadays called the *Quillen-Suslin theorem*. Within module theory, the Quillen-Suslin theorem means that a projective A-module is free and the computation of the unimodular matrix gives an explicit basis of the free module. Constructive proofs of the Quillen-Suslin theorem have largely been studied in the symbolic computation literature but no implementation was available till now. A Maple implementation of the Quillen-Suslin theorem called QUILLENSUSLIN has recently been achieved by A. Fabiańska (University of Aachen, Germany) in collaboration with A. Quadrat and is demonstrated in [20], [29]. Based on the Quillen-Suslin theorem, we solve in [20], [29] open problems in mathematical systems theory (e.g., Lin-Bose's conjectures, computation of (weakly) left/right coprime factorizations of multivariate rational transfer matrices, computation of flat outputs and injective parametrizations of flat differential time-delay systems, Lie-Bäcklund equivalence of flat differential time-delay linear systems). The corresponding algorithms have been implemented by the authors in the package QUILLENSUSLIN.

6.15.2. Constructive version of Stafford's theorem, implementation and applications

A well-known but difficult result in non-commutative algebra due to J. T. Stafford asserts that every projective left module M over the Weyl algebras $D = A_n(k)$ or $B_n(k)$ of differential operators with polynomial/rational coefficients (k is a field of characteristic 0) with $\mathrm{rank}_D(M) \geq 2$ is free. The purpose of [25] is to present a constructive proof of this result as well as an effective algorithm for the computation of bases of M. This new algorithm has been implemented by the authors in the package STAFFORD of the library OREMODULES [18]. The main motivation for studying this open algorithmic problem is to effectively compute injective parametrizations and flat outputs of flat under-determined linear systems of partial differential equations (Monge problem) [25].

6.15.3. Factorization & decomposition problems

The factorization and decomposition problems of linear systems of ordinary differential or difference equations have long been studied in the mathematical and symbolic computation literature. The main novelty of the approach developed in [19] is to recast these problems within the framework of algebraic analysis and to extend them to general linear systems defined over certain classes of functional operators called Ore algebras (e.g., partial differential equations, differential time-delay equations, recurrence equations). In particular, [19] gives general conditions for a linear functional system to admit non-trivial factorizations and decompositions. Under certain conditions (free modules), we prove that the system is equivalent to a block triangular or a block diagonal system. The different algorithms have been implemented in the package MORPHISMS of OREMODULES [18]. Using the packages STAFFORD and QUILLENSUSLIN, MORPHISMS gives us a constructive way to study the factorization and decomposition problems for general linear functional systems. A library of examples demonstrates the package MORPHISMS on examples coming from mathematical physics and control theory.

6.15.4. Extension problem

Within an algebraic analysis approach, we constructively solve in [33] the following open problem in multidimensional systems theory: given two linear systems S_1 and S_2, parametrize all the linear systems S which contain S_1 as a subsystem and satisfy that the quotient S/S_1 is isomorphic to S_2. These results are applied to parametrize all the equivalence classes of linear systems S which admit a fixed parametrizable subsystem S_1 and satisfy that S/S_1 is isomorphic to a fixed autonomous system S_2. The different algorithms have been implemented in the package MORPHISMS and illustrated on different classical examples of differential time-delay linear systems.

7. Contracts and Grants with Industry

7.1. Contracts CNES-IRCOM-INRIA

Contract (reference INRIA: 2470, CNES: 60465/00) n^{o} 04/CNES/1728/00-DCT094 involving CNES, XLIM and INRIA, whose objective is to work out a software package for identification and design of microwave devices. The work at INRIA concerns the design of multiband filters with constraints on the group delay. The problem is to control the logarithmic derivative of the modulus of a rational function, while meeting specifications on its modulus.

7.2. Alcatel Alenia Space (Toulouse)

A contract (reference INRIA: 1931, AAS: B00375) has been signed between INRIA and Alcatel Alenia Space (branch of Toulouse), in which INRIA will design and provide a software for OMUX simulation with efficient initial condition for an optimisation algorithm based on recursive tuning of the channels.

7.3. Alcatel Alenia Space (Cannes)

The aim of this contract (reference INRIA: 2267, AAS: B02173) was to demonstrate the feasability of the approach taken in A. Bombrun's PhD for low-thrust satellite orbit transfer. A Lyapunov-function-based design method has been successfully implemented in a numerical Matlab code and validated on real transfer situations.

8. Other Grants and Activities

8.1. Scientific Committees

L. Baratchart is a member of the editorial board of *Computational Methods and Function Theory* and *Complex Analysis and Operator Theory*.

J. Leblond is a member of the scientific committee of PICOF'08 (4th International Conference on Inverse Problems, Control and Shape Optimization).

A. Quadrat is an associate editor of the international journal *Multidimensional Systems and Signal Processing* (Springer).

8.2. National Actions

The team was awarded two ANR grants in 2007.

The first one, entitled AHPI (Analyse Harmonique et Problèmes Inverses), is a "Projet blanc" in Mathematics involving INRIA-Sophia (APICS, coordinator), the Université de Provence (LATP), the Université Bordeaux I (LATN), the Université d'Orléans (MAPMO), INRIA-Futurs (Magic 3D), the Université de Pau. The coordinator is L. Baratchart. It aims at developing Harmonic Analysis techniques to approach inverse problems in seismology, Electro-encephalography, tomography and nondestructive control.

The second one, entitled FILIPIX, is a "Projet Thématique en Télécommunications", involving INRIA-Sophia (APICS), XLIM, AAS (Centre de Toulouse, coordinator).

8.3. Actions Funded by the EC

The project is a member of the Working Group Control and System Theory of the **ERCIM** consortium, see http://www.ladseb.pd.cnr.it/control/ercim/control.html.

A collaboration entitled *Computational Methods in Mathematical Systems Theory* between the Apics project and the Lehrstuhl B für Mathematik RWTH - Aachen was supported by the grant PAI Procope. Within this framework, we have received the visits of Daniel Robertz, Anna Fabiańska and Mohamed Barakat.

8.4. Extra-european International Actions

EPSRC research grant EP/F020341/1 (Operator theory in function spaces on finitely-connected domains), with Leeds University (UK) and the University Lyon I, 2007-2009.

INRIA-DGRST-Univ. tunisiennes (STIC) with LAMSIN-ENIT (Tunis), « Problèmes inverses du Laplacien et approximation constructive des fonctions » (from which M. Zghal received financial support for her PhD). Apics est l'une des EPI partenaires (avec Anubis et Poems) du LAMSIN au sein de l'équipe associée Enée.

NSF EMS21 RTG students exchange program (with Vanderbilt University).

8.5. The Apics Seminar

The following scientists gave a talk at the seminar:

- Andrei Agrachev, SISSA Trieste, Acad des Sciences Moscou, *Geometry of Rolling Bodies and Optimal Control*.
- Silvère Bonnabel, CAS, ENSMP, *Observateurs et Symmétries*.
- Mohamed S. Boudellioua, Sultan Qaboos University, Oman, *Equivalence and Factorization of Multivariate Polynomial Matrices*.
- José Grimm, *Putting math on the Web with Tralics*.
- Amadeo Irigoyen, Équipe d'analyse complexe, Univ. Paris VI, *Quelques applications de la théorie d'approximation nonlinéaire en théorie inverse de Sturm-Liouville*.
- Ian Jermyn, Inria Ariana, *Contours actifs d'ordre supérieur et champs de phase pour la compréhension d'image*.
- Hamza Jirari, *Contrôle optimal en mécanique quantique*.
- Vincent Lunot, *Un problème de Zolotarev. Application à la synthèse de filtres multi-bandes*.
- Alban Quadrat, *Factorisation et décomposition des systèmes linéaires fonctionnels*.
- Thomas Ransford, Université de Laval, Canada, *Pseudospectres et croissance de puissances*.
- Eva Sincich, RICAM, Austrian Academy of Sciences, Linz, *Stability for some inverse scattering problems*.

9. Dissemination

9.1. Teaching

Courses

- L. Baratchart, DEA Géométrie et Analyse, LATP-CMI, University Marseille I.
- J. Leblond, Centre Montessori, collège, Mouans-Sartoux.
- M. Olivi, Mathématiques pour l'ingénieur (Fourier analysis and integration), section Mathématiques Appliquées et Modélisation, 3ème année, Ecole Polytechnique Nice-Sophia Antipolis.
- A. Quadrat, ISIA, Master affiliated to the Ecole des Mines de Paris (computer algebra).

Ph.D. Students

- Alex Bombrun, « Les Transferts Orbitaux à Faible Poussée : Optimalité et Feedback » (optimal control, feedback, and orbital transfert for low thrust satellite orbit transfer). Defended March 12, 2007. [12]
- Imen Fellah, « Complétion de données dans les espaces de Hardy et problèmes inverses pour le Laplacien en 2D », co-tutelle with Lamsin-ENIT (Tunis), [13].
- Vincent Lunot, « Problèmes fréquentiels extrémaux, approximation rationnelle sous contrainte Schur et application à la synthèse de filtres ».
- Moncef Mahjoub, « Approximation harmonique dans une couronne et applications à la résolution numérique de quelques problèmes inverses », co-tutelle with Lamsin-ENIT (Tunis).
- Meriem Zghal, « Constructive aspects of some inverse problems (Cauchy, sources) for Laplace equation in ellipsoidal domains », co-tutelle with Lamsin-ENIT (Tunis).

Committees

- L. Baratchart was on the PhD defense committee of Philippe Jaming, Univ. Orléans.
- L. Baratchart was on the “soutenance d’habilitation” committee of Maureen Clerc, Univ. Nice.
- J. Leblond was on the PhD defense committee of Imen Fellah, Lamsin-Enit, Tunis.
- M. Olivi was on the PhD defense committee of Tom D’haene, Vrije Universiteit Brussel.

9.2. Community service

L. Baratchart is INRIA’s representative at the « conseil scientifique » of the Université de Provence.

J. Grimm is a representative at the « comité de centre ».

J. Leblond is a member of the « Commission d’évaluation » of INRIA.She is in charge with the Séminaires Croisés of the Research Centre, participates to the working group « Méditerranée 3+3 » and has participated to the working group « Accueil des stagiaires étrangers ».

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Unit of Sophia Antipolis.

A. Quadrat was a member of the Jury Concours CR2, INRIA Futurs, Lille 2007.

J.-B. Pomet is a representative at the « comité technique paritaire » (CTP).

F. Seyfert is a member of the CDL (Comité de Développement Logiciel) of the Research Unit of Sophia Antipolis.

9.3. Conferences and workshops

L. Baratchart, J. Grimm, J. Leblond and M. Olivi attended the 2007 ERNSI Meeting in San Servolo (Italy), where M. Olivi presented a talk on “Real Symmetric Darlington Synthesis.”

V. Lunot gave a talk at the International Microwave Symposium “Optimal Synthesis for Multi-Band Microwave Filters”, Honolulu, U.S.A, June

J. Leblond and S. Rigat attended the International Conference in Mathematics in honor of G. Henkin, IHP, Paris, June.

J. Leblond attended the forum on information and communication science and technologies, to celebrate 40 years of activity at INRIA, Lille, December.

L. Baratchart was an invited speaker at the colloquium of the Université de Bourgogne, Dijon, May.

L. Baratchart was an invited speaker at the Workshop “Modern Approaches in Orthogonal Polynomials”, Banff, Alberta, CN., November.

L. Baratchart, J. Leblond and S. Rigat gave communications at the workshop “ Analyse microlocale et harmonique pour les problèmes inverses ”, CIRM, Marseille, March.

L. Baratchart was an invited speaker and S. Rigat gave a communication at Mathestia, Bidart, April.

L. Baratchart gave a communication at the conference “Special functions, information theory and Mathematical Physics”, Granada, SP., September.

L. Baratchart and S. Rigat participated while J. Leblond gave a communication at the meeting of the ANR project AHPI, Pau, October.

J. Leblond was an invited speaker at MEMO, a conference on numerical methods and modelling to celebrate the 10th birthday of LAMSIN, Tunis, December. December.

J.-B. Pomet gave a talk at the conference in honor of Claude Lobry, St Louis, (Senegal).

J.-B. Pomet was an invited speaker at the 80^{th} RCP meeting on interactions between math and Physics, Strasbourg (France).

A. Quadrat gave two talks at the Journées Nationales de Calcul Formel, Luminy (France), 29/01-02/02.

A. Quadrat gave a talk at the working group EDP-MOSAR of the GDR MACS, ENST (Paris), 23-23/03.

A. Quadrat was an invited speaker at the Journées Doctorals-Journées Nationales du GDR MACS, Reims (France), 09-11/07.

A. Quadrat was an invited speaker at the Worskop on Control Distributed Parameter Systems (CDPS 2007), Namur (Belgium), 23-27/07, in the session dedicated to the tribute to F. Callier's scientific career.

A. Quadrat gave two talks at the Journée “Intéractions entre théorie algébrique et calcul scientifique: Etat de l'art et applications”, CNAM (Paris), 18/09.

A. Quadrat gave a talk at the Journées de l'ANR GECKO, INRIA Sophia Antipolis, 19-21/11.

Concerning their joint work with Apics, our collaborators took the following actions:

A. Gombani gave a communication at LynSys2007, Canberra, February.

M. Mahjoub gave a communication at the Mediterranean Conference on Biomathematics, Cairo, Egypt, June.

J.R. Partington gave two talks at the regular seminars of analysis, at Glasgow University (Feb.) and at King's College in London (Nov.).

R. Peeters gave a communication at the "Benelux meeting on systems and control" (Lommel, Belgium, March).

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Major publications by the team in recent years

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[12] A. BOMBRUN. *Les Transferts Orbitaux à Faible Poussée : Optimalité et Feedback*, Ph. D. Thesis, École des Mines de Paris, March 2007.

[13] I. FELLAH. *Complétion de données dans les espaces de Hardy et problèmes inverses pour le Laplacien en 2D*, Ph. D. Thesis, ENIT-Lamsin, Univ. Tunis II, July 2007.

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