INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team Commands

Control, Optimization, Models, Methods and Applications for Nonlinear Dynamical Systems

Futurs

THEME NUM

Activity Report

2007

Table of contents

1. Team

Head of project-team
J. Frédéric Bonnans [DR Inria, HdR]

Vice-head of project-team
Hasnaa Zidani [Enseignante chercheur, ENSTA]

Administrative assistant
Fabienne Lancry [Since Sept. 07]

Research scientist
Jérôme Bolte [Maître de Conférences, U. Paris VI, since Sept. 1]
Pascal Jaisson [PhD and Prof. agrégé, started Sept. 1]
Pierre Martinon [CR Inria; Post-Doc until Aug. 31, CR since]

External collaborators
Emmanuel Trélat [Professeur, U. Orléans, HdR]

Ph. D. student
Romain Apparigliato [EDF, Logilab]
Grégory Emiel [EDF, IMPA]
Audrey Hermant [DGA]
Nadia Megdich [Ater at Paris XI until August 07, at Paris IX since]
Francisco Silva [CORDI PhD fellowship, started Sept. 15]
Elisabeth Ottenwaelter [IUT Paris]

Post-Doctorant fellow
Emiliano Cristiani [since Sept. 1st]
Nicolas Forcadel [since Oct. 1st]
Amin Ghorbel [Feb. to July 2007]
Stefania Maroso [Until July 2007, while being Ater at Paris VII]

Visiting scientist
Nikolai Osmolovskii [Systems Research Institute, Warszawa, 6 months]

Student internship
Soledad Aronna [U. Rosario, Argentina, 3 months]
Saad Serghini [EMI, Rabat, 3 months]

2. Overall Objectives

2.1. Scientific directions

Commands is a team with a global view on dynamic optimization in its various aspects: trajectory optimization, geometric control, deterministic and stochastic optimal control, stochastic programming, dynamic programming and Hamilton-Jacobi-Bellman approach.

Our aim is to derive new and powerful algorithms for solving numerically these various problems, with applications in several industrial fields. While the numerical aspects are the core of our approach it happens that the study of convergence of these algorithms and the verification of their well-posedness and accuracy raises interesting and difficult theoretical questions, such as, for trajectory optimization: qualification conditions and second-order optimality condition, well-posedness of the shooting algorithm, estimates for discretization errors; for the Hamilton-Jacobi-Bellman approach: accuracy estimates, strong uniqueness principles when state constraints are present, for stochastic programming problems: sensitivity w.r.t. the probability laws, formulation of risk measures.

2.2. Industrial impact

At the same time we are deeply involved in applications of critical importance, in particular: trajectories of space vehicles (in collaboration with CNES, the French space agency), as well as management, storage and trading of energy resources (in collaboration with EDF, GDF and TOTAL).

2.3. Highlights

2.3.1. Singular arcs for launcher trajectories

We have studied the advantage of having a singular arc for real world launcher trajectories. Although no such phenomenon seems to occur with the present Ariane V launcher, we have shown the possibility of having one when the launcher has bigger aerodynamic reference area and specific impulsion. The principal investigator for this study was P. Martinon. See the report [29].

2.3.2. Fast resolution of HJB equations

We have developed a fast numerical code (for dimensions 2 to 4) on sparse grids for the resolution of Hamilton-Jacobi-Bellman equations whose solutions take only values 0 and 1. This code computes, in a few seconds, capture basins and associated time optimal trajectories. We are currently aiming to use this code for solving a climbing problem (for aircrafts) with maximal final mass, under a structural constraint on dynamic pressure.

3. Scientific Foundations

3.1. Deterministic optimal control

3.1.1. Historical perspective

For deterministic optimal control we will distinguish two approaches, *trajectory optimization*, in which the object under consideration is a single trajectory, and the Hamilton-Jacobi-Bellman approach, based on dynamic principle, in which a family of optimal control problems is solved.

The roots of deterministic optimal control are the “classical” theory of the calculus of variations, illustrated by the work of Newton, Bernoulli, Euler, and Lagrange (whose famous multipliers were introduced in [146]), with improvements due to the “Chicago school”, Bliss [71] during the first part of the 20th century, and by the notion of relaxed problem and generalized solution (Young [181]).

Trajectory optimization really started with the spectacular achievement done by Pontryagin’s group [166] during the fifties, by stating, for general optimal control problems, nonlocal optimality conditions generalizing those of Weierstrass. This motivated the application to many industrial problems (see the classical books by Bryson and Ho [101], Leitmann [149], Lee and Markus [148], Ioffe and Tihomirov [138]). Since then, various theoretical achievements have been obtained by extending the results to nonsmooth problems, see Aubin [51], Clarke [104], Ekeland [122]. Substantial improvements were also obtained by using tools of differential geometry, which concern a precise understanding of optimal syntheses in low dimension for large classes of nonlinear control systems, see Bonnard, Faubourg and Trélat [98].

Overviews of numerical methods for trajectory optimization are provided in Pesch [163], Betts [69]. We follow here the classical presentation that distinguishes between direct and indirect methods.

Dynamic programming was introduced and systematically studied by R. Bellman during the fifties. The HJB equation, whose solution is the value function of the (parameterized) optimal control problem, is a variant of the classical Hamilton-Jacobi equation of mechanics for the case of dynamics parameterized by a control variable. It may be viewed as a differential form of the dynamic programming principle. This nonlinear first-order PDE appears to be well-posed in the framework of *viscosity solutions* introduced by Crandall and Lions [109], [110], [107]. These tools also allow to perform the numerical analysis of discretization schemes. The theoretical contributions in this direction did not cease growing, see the books by Barles [55] and Bardi and Capuzzo-Dolcetta [54].

A interesting by-product of the HJB approach is an expression of the optimal control in feedback form. Also it reaches the global optimum, whereas trajectory optimization algorithms are of local nature. A major difficulty when solving the HJB equation is the high cost for a large dimension n of the state (complexity is exponential with respect to n).

3.1.2. Direct methods for trajectory optimization

The so-called *direct methods* consist in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure. So the two main problems are the choice of the discretization and the nonlinear programming algorithm. A third problem is the possibility of refinement of the discretization once after solving on a coarser grid.

3.1.2.1. Rough control discretization.

Many authors prefer to have a coarse discretization for the control variables (typically constant or piecewise-linear on each time step) and a higher order discretization for the state equation. The idea is both to have an accurate discretization of dynamics (since otherwise the numerical solution may be meaningless) and to obtain a small-scale resulting nonlinear programming problem. See e.g. Kraft [143]. A typical situation is when a few dozen of time-steps are enough and there are no more than five controls, so that the resulting NLP has at most a few hundreds of unknowns and can be solved using full matrices software. On the other hand, the error order (assuming the problem to be unconstrained) is governed by the (poor) control discretization. Note that the integration scheme does not need to be specified (provided it allows to compute functions and gradients with enough precision) and hence general Ordinary Differential Equations integrators may be used.

3.1.2.2. Full discretization.

On the other hand, a full discretization (i.e. in a context of Runge-Kutta methods, with different values of control for each inner substep of the scheme) allows to obtain higher orders that can be effectively computed, see Hager [127], Bonnans [89], being related to the theory of partitioned Runge-Kutta schemes, Hairer et al. [128]. In an interior-point algorithm context, controls can be eliminated and the resulting system of equation is easily solved due to its band structure. Discretization errors due to constraints are discussed in Dontchev et al. [119]. See also Malanowski et al. [153].

3.1.2.3. Direct shooting.

For large horizon problems integrating from the initial time to the final time may be impossible (finding a feasible point can be very hard !). Analogously to the indirect method of multiple shooting algorithm, a possibility is to add (to the control variables), as optimization parameters, the state variables for a given set of times, subject of course to “sticking” constraint. Note that once more the integration scheme does not need to be specified. Integration of the ODE can be performed in parallel. See Bock [72].

3.1.2.4. Parametrization by output variables.

Recent proposals were made of methods based on a reformulation of the problem based on (possibly flat) *output variables*. By the definition, control and state variables are combinations of derivatives of these output variables. When the latter are represented on a basis of smooth functions such as polynomials, their derivatives are given linear combinations of the coefficients, and so the need for integration is avoided. One must of course take care of the possibly complicated expression of constraints that can make numerical difficulties. The numerical analysis of these methods seems largely open. See on this subject Petit, Milam and Murray [164].

3.1.2.5. Collocation and pseudospectral methods.

The collocation approach for solving an ODE consists in a polynomial interpolation of the dynamic variable, the dynamic equation being enforced only at limited number of points (equal to the degree of the polynomial). Collocation can also be performed on each time step of a one-step method; it can be checked than collocation methods are a particular case of Runge-Kutta methods.

It is known that the polynomial interpolation with equidistant points is unstable for more than about 20 points, and that the Tchebycheff points should be preferred, see e.g. [77]. Nevertheless, several papers suggested the use of pseudospectral methods Ross and Fahroo [172] in which a single (over time) high-order polynomial approximation is used for the control and state. Therefore pseudospectral methods should not be used in the case of nonsmooth (e.g. discontinuous) control.

3.1.2.6. Robustness.

In view of model and data uncertainties there is a need for robust solutions. Robust optimization has been a subject of increasing importance in recent years see Ben-Tal and Nemirovski [59]. For dynamic problems taking the worst-case of the perturbation at each time-step may be too conservative. Specific remedies have been proposed in specific contexts, see Ben-Tal et al. [58], Diehl and Björnberg [115].

A relatively simple method taking into account robustness, applicable to optimal control problems, was proposed in Diehl, Bock and Kostina [116].

3.1.2.7. Nonlinear programming algorithms.

The dominant ones (for optimal control problems as well as for other fields) have been successively the augmented Lagrangian approach (1969, due to Hestenes [135] and Powell [167], see also Bertsekas [65]) successive quadratic programming (SQP: late seventies, due to [131], [168], and interior-point algorithms since 1984, Karmarkar [141]. See the general textbooks on nonlinear programming [66], [82], [160].

When ordered by time the optimality system has a "band structure". One can take easily advantage of this with interior-point algorithms whereas it is not so easy for SQP methods; see Berend et al. [103]. There exist some very reliable SQP softwares SNOPT, some of them dedicated to optimal control problems, Betts [68], as well as robust interior-point software, see Morales et al. [156], Wächter and Biegler [180], and for application to optimal control Jockenhövel et al. [139].

3.1.2.8. Our contributions in direct methods

We have developed a general SQP algorithm [70], [87] for sparse nonlinear programming problems, and the associated software for optimal control problems; it has been applied to atmospheric reentry problems, in collaboration with CNES [88].

More recently, in collaboration with CNES and ONERA, we have developed a sparse interior-point algorithm with an embedded refinement procedure. The resulting TOPAZE code has been applied to various space trajectory problems [103], [102], [147]. The method takes advantage of the analysis of discretization errors, is well-understood for unconstrained problems [89].

3.1.3. Indirect approach for trajectory optimization

The indirect approach eliminates control variables using Pontryagin's maximum principle, and solves the two-points boundary value problem (with differential variables state and costate) by a single or multiple shooting method. The questions are here the choice of a discretization scheme for the integration of the boundary value problem, of a (possibly globalized) Newton type algorithm for solving the resulting finite dimensional problem in $I\!R^n$ (n is the number of state variables), and a methodology for finding an initial point.

3.1.3.1. Discretization schemes

The choice of the discretization scheme for the numerical integration of the boundary value problem can have a great impact on the convergence of the method. First, the integration itself can be tricky. If the state equation is stiff (the linearized system has fast modes) then the state-costate has both fast and unstable modes. Also, discontinuities of the control or its derivative, due to commutations or changes in the set of active constraints, lead to the use of sophisticated variable step integrators and/or switching detection mechanisms, see Hairer et al. [129], [130]. Another point is the computation of gradients for the Newton method, for which basic finite differences can give inaccurate results with variable step integrators (see Bock [72]). This difficulty can be treated in several ways, such as the so-called "internal differentiation" or the use of variational equations, see Gergaud and Martinon [125].

3.1.3.2. Specific formulations for constrained problems and singular arcs

Most optimal control problems include control and state constraints. In that case the formulation of the TPBVP must take into account entry and exit times of boundary arcs for these constraints, and (for state constraints of order at least two) times of touch points (isolated contact points). In addition for state constrained problems, the so-called "alternative formulation" (that allows to eliminate the "algebraic variables, i.e. control and state, from the algebraic constraints) has to be used, see Hartl, Sethi and Vickson [134].

Another interesting point is the presence of singular arcs, appearing for instance when the control enters in the system dynamics and cost function in a linear way, which is common in practical applications. As for state contraints, the formulation of the boundary value problem must take into account these singular arcs, over which the expression of the optimal control typically involves higher derivatives of the Hamiltonian, see Goh [126] and Robbins [169].

3.1.3.3. Homotopy approaches

As mentioned before, finding a suitable initial point can be extremely difficult for indirect methods, due to the small convergence radius of the Newton type method used to solve the boundary value problem. Homotopy methods are an effective way to address this issue, starting from the solution of an easier problem to obtain a solution of the target problem (see Allgower and Georg [49]). It is sometimes possible to combine the homotopy approach with the Newton method used for the shooting, see Deuflhard [114].

3.1.3.4. Other related approaches

With a given value of the initial costate are associated (through in integration of the reduced state-costate system) a control and a state, and therefore a cost function. The latter can therefore be minimized by ad-hoc minimization algorithms, see Dixon and Bartholomew-Biggs [117]. The advantage of this point of view is the possibility to use the various descent methods in order to avoid convergence to a local maximum or saddle-point. The extension of this approach to constrained problems (especially in the case of state constraints) is an open and difficult question.

3.1.3.5. Our contributions in indirect methods

We have recently clarified under which properties shooting algorithms are well-posed in the presence of state constraints. The (difficult) analysis was carried out in [86], [84]. A related homotopy algorithm, restricted to the case of a single first-order state constraint, has been proposed in [85].

We also conducted a study of optimal trajectories with singular arcs for space launcher problems. The results obtained for the generalized three-dimensional Goddard problem (see [155]) have been successfully adapted for the numerical solution of a realistic launcher model (Ariane 5 class).

Furthermore, we continue to investigate the effects of the numerical integration of the boundary value problem and the accurate computation of Jacobians on the convergence of the shooting method. As initiated in [125], we focus more specifically on the handling of discontinuities, with ongoing work on the geometric integration aspects (Hamiltonian conservation).

3.1.4. Geometric control

Geometric approaches succeeded in giving a precise description of the structure of optimal trajectories, as well as clarifying related questions. For instance, there have been many works aiming to describe geometrically the set of attainable points, by many authors (Krener, Schättler, Bressan, Sussmann, Bonnard, Kupka, Ekeland, Agrachev, Sigalotti, etc). It has been proved, in particular, by [*Krener-Schättler, SICON, 1989*], that, for generic single-input control-affine systems in $I\!R^3$, $\dot{x} = X(x) + uY(x)$, where the control satisfies the constraint $|u| \leq 1$, the boundary of the accessible set in small time consists of the surfaces generated by the trajectories x_+x_- and x_-x_+, where x_+ (resp. x_-) is an arc corresponding to the control $u = 1$ (resp. $u = -1$); moreover, every point inside the accessible set can be reached with a trajectory of the form $x_-x_+x_-$ or $x_+x_-x_+$. It follows that minimal time trajectories of generic single-input control-affine systems in $I\!R^3$ are locally of the form $x_-x_+x_-$ or $x_+x_-x_+$, i.e., are bang-bang with at most two switchings.

This kind of result has been slightly improved recently by Agrachev-Sigalotti, although they do not take into account possible state constraints.

3.1.4.1. Our contributions.

In [96], we have extended that kind of result to the case of state constraints: we described a complete classification, in terms of the geometry (Lie configuration) of the system, of local minimal time syntheses, in dimension two and three. This theoretical study was motivated by the problem of atmospheric re-entry posed by the CNES, and in [97], we showed how to apply this theory to this concrete problem, thus obtaining the precise structure of the optimal trajectory.

3.1.5. Hamilton-Jacobi-Bellman approach

This approach consists in calculating the value function associated with the optimal control problem, and then synthesizing the feedback control and the optimal trajectory using Pontryagin's principle. The method has the great particular advantage of reaching directly the global optimum, which can be very interesting, when the problem is not convex.

3.1.5.1. Characterization of the value function

From the dynamic programming principle, we derive a characterization of the value function as being a solution (in viscosity sense) of an Hamilton-Jacobi-Bellman equation, wich is a nonlinear PDE of dimension equal to the number n of state variables. Since the pioneer works of Crandall and Lions [109], [110], [107], many theoretical contributions were carried out, allowing an understanding of the properties of the value function as well as of the set of admissible trajectories. However, there remains an important effort to provide for the development of effective and adapted numerical tools, mainly because of numerical complexity (complexity is exponential with respect to n).

3.1.5.2. Numerical approximation for continuous value function

Several numerical schemes have been already studied to treat the case when the solution of the HJB equation (the value function) is continuous. Let us quote for example the Semi-Lagrangian methods [124], [123] studied by the team of M. Falcone (La Sapienza, Rome), the high order schemes WENO, ENO, Discrete galerkin introduced by S. Osher, C.-W. Shu, E. Harten [133], [137], [136], [161], and also the schemes on nonregular grids by R. Abgrall [48], [47]. All these schemes rely on finite differences or/and interpolation techniques which lead to numerical diffusions. Hence, the numerical solution is unsatisfying for long time approximations even in the continuous case.

3.1.5.3. Discontinuous case

In a realistic optimal control problem, there are often constraints on the state (reaching a target, restricting the state of the system in an acceptable domain, ...). When some controlability assumptions are not satisfied, the value function associated to such a problem is discontinuous and the region of discontinuity is of great importance since it separates the zone of admissible trajectories and the nonadmissible zone.

In this case, it is not reasonable to use the usual numerical schemes (based on finite differences) for solving the HJB equation. Indeed, these schemes provide poor approximation quality because of the numerical diffusion.

3.1.5.4. Our contributions on the HJB approach

Discrete approximations of the Hamilton-Jacobi equation for an optimal control problem of a differential-algebraic system were studied in [80].

Numerical methods for the HJB equation in a bilevel optimization scheme where the upper-level variables are design parameters were used in [83]. The algorithm has been applied to the parametric optimization of hybrid car engines.

Within the framework of the thesis of N. Megdich, we have studied new antidiffusive schemes for HJB equations with discontinuous data [76], [74]. One of these schemes is based on the Ultrabee algorithm proposed, in the case of advection equation with constant velocity, by Roe [171] and recently revisited by Després-Lagoutière [113], [112]. The numerical results on several academic problems show the relevance of the antidiffusive schemes. However, the theoretical study of the convergence is a difficult question and is only partially done [75].

3.2. Stochastic optimal control

Optimal stochastic control problems occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky and non-risky assets, super-replication with uncertain volatility, management of power resources (dams, gas). Air traffic control is another example of such problems.

3.2.1. Stochastic programming

By stochastic programming we mean stochastic optimal control in a discrete time (or even static) setting; see the overview by Ruszczynski and Shapiro [46]. The static and single recourse cases are essentially well-understood; by contrast the truly dynamic case (multiple recourse) presents an essential difficulty Shapiro [176], Shapiro and Nemirovski [175]. So we will speak only of the latter, assuming decisions to be measurable w.r.t. a certain filtration (in other words, all information from the past can be used).

3.2.1.1. Dynamic programming.

In the standard case of minimization of an expectation (possibly of a utility function) a *dynamic programming principle* holds. Essentially, this says that the decision is a function of the present state (we can ignore the past) and that a certain reverse-time induction over the associated values holds. Unfortunately a straighforward resolution of the dynamic programming principle based on a discretization of the state space is out of reach (again this is the curse of dimensionality). For convex problems one can build lower convex approximations of the value function: this is the *Stochastic dual dynamic programming* (SDDP) approach, Pereira and Pinto [162]. Another possibility is a *parametric approximation of the value function*; however determining the basis functions is not easy and identifying (or, we could say in this context, learning) the best parameters is a nonconvex problem, see however Bertsekas and J. Tsitsiklis [67], Munos [157].

3.2.1.2. Tree based algorithms.

A popular approach is to sample the uncertainties in a structured way of a *tree* (branching occurs typically at each time). Computational limits allow only a small number of branching, far less than the amount needed for an accurate solution Shapiro and Nemirovski [175]. Such a poor accuracy may nevertheless (in the absence of a more powerful approach) be a good way for obtaining a reasonable policy. Very often the resulting programs are linear, possibly with integer variables (on-off switches of plants, investment decisions), allowing to use (possibly dedicated) mixed integer linear programming codes. The tree structure (coupling variables) can be exploited by the numerical algorithms, see Dantzig and Wolfe [111], Kall and Wallace [140].

3.2.1.3. Monte Carlo approaches

By Monte Carlo we mean here sampling a given number of independent trajectories (of uncertainties). In the special case of optimal stopping (e.g., American options) it happens that the state space and the uncertainty space coincide. Then one can compute the transition probabilities of a Markov chain whose law approaches the original one, and then the problem reduces to the one of a Markov chain, see [99]. Let us mention also the quantization approach, see [52].

In the general case a useful possibility is to compute a tree by agregating the original sample, as done in [120].

3.2.1.4. Controlling risk

Maximizing the expectation of gains can lead to a solution with a too high probability of important losses (bankruptcy). In view of this it is wise to make a compromise between expected gains and risk of high losses. A simple and efficient way to achieve that may be to maximize the expectation of a utility function; this, however, needs an ad-hoc tuning. An alternative is the mean-variance compromise, presented in the case of portfolio optimization in Markowitz [154]. A useful generalization of the variance, uncluding dissymetric functions such as semideviations, is the theory of deviation measures, Rockafellar et al. [170].

3.2.1.5. Risk measures

Another possibility is to put a constraint on the level of gain to be obtained with a high probability value say at least 99%. The corresponding concept of value-at-risk leads to difficult nonconvex optimization problems, although convex relaxations may be derived, see Shapiro and Nemirovski [158].

Yet the most important contribution of the recent years is the axiomatized theory of risk measures Artzner et al. [50], satisfying the properties of monotonicity and possibly convexity.

In a dynamic setting, risk measures (over the total gains) are not coherent (they do not obey a dynamic programming principle). The theory of *coherent risk measures* is an answer in which risk measures over successive time steps are inductively applied; see Ruszczyński and Shapiro [173]. Their drawback is to have no clear economic interpretation at the moment. Also, associated numerical methods still have to be developed.

3.2.1.6. Links with robust optimization

The study of relations between chance constraints (constraints on the probability of some event) and robust optimization is the subject of intense research. The idea is, roughly speaking, to solve a robust optimization (some classes of which are tractable in the sense of algorithmic complexity). See the recent work by Ben-Tal and Teboulle [60].

3.2.2. Continuous-time stochastic optimal control

The case of continuous-time can be handled with the Bellman dynamic programming principle, which leads to obtain a characterization of the value function as solution of a second order Hamilton-Jacobi-Bellman equation [110], [108].

3.2.2.1. Theoretical framework

Sometimes this value function is smooth (e.g. in the case of Merton's portfolio problem, Oksendal [183]) and the associated HJB equation can be solved explicitly. Still, the value function is not smooth enough to satisfy the HJB equation in the classical sense. As for the deterministic case, the notion of viscosity solution provides a convenient framework for dealing with the lack of smoothness, see Pham [165], that happens also to be well adapted to the study of discretization errors for numerical discretization schemes [144], [56].

3.2.2.2. Numerical approximation

The numerical discretization of second order HJB equations was the subject of several contributions. The book of Kushner-Dupuis [145] gives a complete synthesis on the chain Markov schemes (i.e Finite Differences, semi-Lagrangian, Finite Elements, ...). Here a main difficulty of these equations comes from the fact that the second order operator (i.e. the diffusion term) is not uniformly elliptic and can be degenerated. Moreover, the diffusion term (covariance matrix) may change direction at any space point and at any time (this matrix is associated the dynamics volatility).

3.2.2.3. Our contributions on stochastic optimal control

In the framework of the thesis of R. Apparigliato (that will finish at the end of 2007) we have studied the robust optimization approach to stochastic programming problems, in the case of hydroelectric production, for one valley. The main difficulty lies with both the dynamic character of the system and the large number of constraints (capacity of each dam). We have also studied the simplified electricity production models for respecting the “margin” constraint. In the framework of the thesis of G. Emiel and in collaboration with CEPEL, we have studied large-scale bundle algorithms for solving (through a dual “price decomposition” method) stochastic problems for the Brazilian case.

For solving stochastic control problems, we studied the so-called Generalized Finite Differences (GFD), that allow to choose at any node, the stencil approximating the diffusion matrix up to a certain threshold [95]. Determining the stencil and the associated coefficients boils down to a quadratic program to be solved at each point of the grid, and for each control. This is definitely expensive, with the exception of special structures where the coefficients can be computed at low cost. For two dimensional systems, we designed a (very) fast algorithm for computing the coefficients of the GFD scheme, based on the Stern-Brocot tree [92]. The GFD scheme was used as a basis for the approximation of an HJB equation coming from a super-replication problem. The problem was motivated by a study conducted in collaboration with Société Générale, see [73].

Within the framework of the thesis of Stefania Maroso, we also contributed to the study of the error estimate of the approximation of Isaac equation associated to a differential game with one player [90], and also for the approximation of HJB equation associated with the impulse problem [91].

3.3. Optimal control of partial differential equations

The field has been strongly influenced by the work of J.L. Lions, who started its systematic study of optimal control problems for PDEs in [150], in relation with singular perturbation problems [151], and ill-posed problems [152]. A possible direction of research in this field consists in extending results from the finite-dimensional case such as Pontryagin's principle, second-order conditions, structure of bang-bang controls, singular arcs and so on. On the other hand PDEs have specific features such as finiteness of propagation for hyperbolic systems, or the smoothing effect of parabolic sytems, so that they may present qualitative properties that are deeply different from the ones in the finite-dimensional case.

3.3.1. Controllability

The study of controllability properties for infinite dimensional systems is an illustration of this point. In view of the finiteness of propagation for hyperbolic systems, instantaneous controllability is not possible (as it is in the finite-dimensional case) and specific tools have to be developped. The theme has been very active since the eighties, see e.g. [61]. When analyzing whether the solution of the PDE can be driven to a given final target by means of a finite energy control applied say on a part of the boundary of the domain, it follows from functional analysis that the desired surjectivity property is equivalent to "strong" injectivity of the adjoint mapping, and it can be shown easily that the latter is nothing but an observability property (similarly to the finite-dimensional case). The Hilbert Uniqueness Method consists in determining the solution with minimal energy (solution of some linear quadratic optimal control). It is by now well known that, for hyperbolic equations such as wave-like equations, the discretization of the HUM method fails for most numerical schemes, due to to high frequency spurious solutions; specific remedies such as Tychonoff regularization, multigrid methods, mixed finite elements, numerical viscosity and filtering of high frequencies are described in [177], [182]. Observability/controllability properties depend in a very sensitive way on the class of PDE under consideration. The heat and wave equations behave in a significantly different way, because of their different behavior with respect to time reversal.

3.3.2. Optimal control of variational inequalities

Unilateral systems in mechanics, plasticity theory, multiphases heat (Stefan) equations, etc. are described by inequalities; see Duvaut and Lions [121], Kinderlehrer and Stampacchia [142]. For an overview in a finite dimensional setting, see Harker and Pang [132]. Optimizing such sytems often needs dedicated schemes with specific regularization tools, see Barbu [53], Bermúdez and Saguez [64]. Nonconvex variational inequalities can be dealt as well in Controllability of such systems is discussed in Brogliato et al. [100].

3.3.3. Sensitivity and second-order analysis

As for finite-dimensional problems, but with additional difficulties, there is a need for a better understanding of stability and sensitivity issues, in relation with the convergence of numerical algorithms. The second-order analysis for optimal control problems of PDE's in dealt with in e.g. [78], [174]. No much is known in the case of state constraints. At the same time the convergence of numerical algorithms is strongly related to this second-order analysis.

3.3.4. Coupling with finite-dimensional models

Many models in control problems couple standard finite dimensional control dynamics with partial differential equations (PDE's). For instance, a well known but difficult problem is to optimize trajectories for planes land-off, so as to minimize, among others, noise pollution. Noise propagation is modeled using wave like equations, i.e., hyperbolic equations in which the signal propagates at a finite speed. By contrast when controlling furnaces one has to deal with the heat equation, of parabolic type, which has a smooting effect. Optimal control laws have to reflect such strong differences in the model.

3.3.5. Applications

Let us mention some applications where optimal control of PDEs occurs. One can study the atmospheric reentry problem with a model for heat diffusion in the vehicle. Another problem is the one of traffic flow, modeled by hyperbolic equations, with control on e.g. speed limitations. Of course control of beams, thin structures, furnaces, are important.

3.3.6. Our contributions in control and optimal control of PDEs

An overview of sensitivity analysis of optimization problems in a Banach space setting, with some applications to the control of PDEs of elliptic type, is given in the book [93]. See also [78].

We studied various regularization schemes for solving optimal control problems of variational inequalities: see Bonnans and D. Tiba [94], Bonnans and E. Casas [79], Bergounioux and Zidani [63]. The well-posed of a "nonconvex" variational inequality modelling some mechanical equilibrium is considered in Bonnans, Bessi and Smaoui [81].

In Coron and Trélat [105], [106], we prove that it is possible, for both heat like and wave like equations, to move from any steady-state to any other by means of a boundary control, provided that they are in the same connected component of the set of steady-states. Our method is based on an effective feedback procedure which is easily and efficiently implementable. The first work was awarded SIAM Outstanding Paper Prize (2006).

4. Application Domains

4.1. Application Domains

Our techniques are generic, but we have especially in view the fields of Aerospace trajectories (rockets, planes), automotive industry (car design), chemical engineering (optimization of transient phases, batch processes).

There are also important economic applications such as the optimization of storage and management, especially of natural and power resources, portfolio optimization.

5. Software

5.1. Software

1. **COTCOT (Conditions of Order Two and COnjugate Times)**. Freeware developed by B. Bonnard, J.-B. Caillau and E. Trélat in 2004-2005. This package of routines, callable from MATLAB, generates, for a given optimal control problem, the equations of the maximum principle using automatic differentiation (based on ADIFOR). A FORTRAN code is generated and mex files are created for MATLAB. Numerical integration of the underlying differential equations is led, and the solution of the associated shooting problem is then obtained using FORTRAN codes interfaced with MATLAB.

   This software, motivated by our studies for the CNES, was used to solve the atmospheric reentry problem and the orbit transfer problem. A more specific use of that software is currently of interest for EADS. The latter supports a study implementing a specific tool for the orbit transfer.
2. **SIMPLICIAL software for indirect shooting[1].** Developed since 2003 by P. Martinon in collaboration with J. Gergaud (ENSEEIHT Toulouse) and E. Hairer (Univ. Genève). This package of Fortran90 routines implements several variants of indirect shooting methods for optimal control problems, with an embedded simplicial homotopy approach. Special care is given to the numerical integration of the shooting function and the precise computation of its jacobian, especially in the discontinuous case. Originally designed for the resolution of bang-bang low thrust orbital transfers for the CNES, the software has been since then extended to handle problems with singular arcs.

[1] **Available at http://www.cmap.polytechnique.fr/~martinon/simplicial**

3. **LAUNCHER software for space launcher trajectory optimization.** Based on the SIMPLICIAL package, this software has been specifically developed for the CNES by P. Martinon for the study of singular arcs in realistic launcher problems, in the framework of a contract with the CNES within the OPALE group.
4. **TOPAZE code for trajectory optimization.** Developed in the framework of the PhD Thesis of J. Laurent-Varin, supported by CNES and ONERA. Implementation of an interior-point algorithm for multiarc trajectory optimization, with built-in refinement. Applied to several academic, launcher and reentry problems.
5. **SOHJB code for second order HJB equations**. Developped since 2004 in C++. It solves the stochastic HJB equations in dimension 2. The code based on the Generalized Finite Differences, include a step of decomposition of the covariance matrices in elementary diffusions pointing towards grid points. The implementation is very fast and was mainly tested on academic examples.
6. **Sparse HJB-Ultrabee**. Developped by N. Megdich, in collaboration with O. Bokanowski (Paris 6 & 7), in Scilab to deal with optimal control problems with 2 or 3 state variables. A specific software dedicated to space problems is currently developped, with E. Cristiani, in C++, in the framework of a contract with the CNES.

6. New Results

6.1. Trajectory optimization

6.1.1. Second-order Conditions for State Constrained Optimal Control

Participants: F. Bonnans, A. Hermant.

In [40] we considered optimal control problem of an ordinary differential equation with several pure state constraints, of arbitrary orders, as well as mixed control-state constraints. We assume (i) the Hamiltonian to be strongly convex and the mixed constraints to be convex w.r.t. the control variable, and (ii) a linear independence condition of the active constraints at their respective order to hold. We give a complete analysis of the smoothness and junction conditions of the control and of the constraints multipliers. This allows us to obtain, when there are finitely many nontangential junction points, a theory of no-gap second-order optimality conditions and a characterization of the well-posedness of the shooting algorithm. These results generalize those obtained in the case of a scalar-valued state constraint and a scalar-valued control.

6.1.2. Stability Analysis of Optimal Control Problems with a Second-order State Constraint

Participant: A. Hermant.

The reference [43] gives stability results for nonlinear optimal control problems subject to a regular state constraint of second-order. The strengthened Legendre-Clebsch condition is assumed to hold, and no assumption on the structure of the contact set is made. Under a weak second-order sufficient condition (taking into account the active constraints), we show that the solutions are Lipschitz continuous w.r.t. the perturbation parameter in the L^2 norm, and Hölder continuous in the L^∞ norm. We use a generalized implicit function theorem in metric spaces by Dontchev and Hager [118]. The difficulty is that multipliers associated with second-order state constraints have a low regularity (they are only bounded measures). We obtain Lipschitz stability of a "primitive" of the state constraint multiplier.

6.1.3. Second-order analysis of optimal control problems with control and initial-final state constraints

Participants: F. Bonnans, N. Osmolovskii.

We have an ongoing work on the characterization of either "bounded strong" or "Pontryagin mimina" satisfying a quadratic growth condition, for optimal control problems of ordinary differential equations with constraints on initial-final state and control. No Clebsch-Legendre condition is assumed. This extends previous work by A. Milyutin and N. Osmolovskii where the control constraints were assumed to be uniformly linearly independent.

6.1.4. Multidimensional singular arcs

Participants: F. Bonnans, P. Martinon, E. Trélat (Univ. Orléans).

With J. Laurent-Varin (Direction des lanceurs, CNES Evry). We started in Fall 2006 a study of the multi-dimensional singular arc that can occur in the atmospheric flight of a launcher. The physical reason for not having a bang-bang control (despite the fact that the hamiltonian function is affine w.r.t. the control), is that aerodynamic forces may make a high speed ineffective. We investigate variants of Goddard's problems for nonvertical trajectories. The control is the thrust force, and the objective is to maximize a certain final cost, typically, the final mass. In this article, performing an analysis based on the Pontryagin Maximum Principle, we prove that optimal trajectories may involve singular arcs (along which the norm of the thrust is neither zero nor maximal), that are computed and characterized. Numerical simulations are carried out, both with direct and indirect methods, demonstrating the relevance of taking into account singular arcs in the control strategy. The indirect method we use is based on our previous theoretical analysis and consists in combining a shooting method with an homotopic method. The homotopic approach leads to a quadratic regularization of the problem and is a way to tackle with the problem of nonsmoothness of the optimal control.

6.1.5. Study of optimal trajectories with singular arcs for space launcher problems

Participants: F. Bonnans, P. Martinon, E. Trélat (Univ. Orléans).

With J. Laurent-Varin (Direction des lanceurs, CNES Evry). In the frame of a research contract with the CNES (french space agency), we started in Fall 2006 a study of the singular arcs that can occur in the atmospheric climbing phase of a space launcher. These singular arcs correspond to time intervals where the optimal thrust level of the launcher is neither maximal nor zero, contrary to the usual bang-bang command law (which comes from the fact that the Hamiltonian of the system is affine with respect to the control). The physical reason behind this phenomenon is that aerodynamic forces may make high speed ineffective (namely the drag term, proportional to the speed squared).

We first investigate three-dimensional (i.e. nonvertical) variants of the Goddard problem, with the thrust force as the control and typically maximizing the final mass as the cost. Performing an analysis based on the Pontryagin Maximum Principle, we prove that optimal trajectories may involve singular arcs, that are computed and characterized. Numerical experimentations are carried out, both with indirect and direct methods, demonstrating the relevance of taking into account the singular arcs in the control strategy. The indirect method we primarily use is based on our previous theoretical analysis and consists in combining a shooting method with an homotopic method (continuation). The homotopic approach involves a quadratic regularization of the problem and is a way to handle the nonsmoothness of the optimal control, providing both sufficient information on the singular structure of the control and a suitable initial point for the shooting method. These results are published in [29].

Then we tackle a heavy multi-stage launcher problem (an Ariane V flight to the geostationary transfer orbit) with a realistic physical model for the thrust and drag forces. As a preliminary result, we first solve the complete flight with stage separations, at full thrust. Then we focus on the first atmospheric climbing phase, to investigate the possible existence of optimal trajectories with singular arcs. Once again, we primarily use an indirect shooting method (based on Pontryagin's Maximum Principle), coupled to a continuation (homotopy) approach. We introduce a new way to determine the singular control, the usual conditions based on the derivatives of the Hamiltonian with respect to the control being unusable due to the presence of tabulated data in the physical model. The solutions we obtain are confirmed by some additional experiments with a direct method. We study two slightly different launcher models, and observe that modifying parameters such as the aerodynamic reference area and specific impulsion can indeed lead to optimal trajectories with or without singular arcs. Future developments include the study of next generation launchers, reusable and/or aerolifted.

This work led to the development of two numerical codes for solving optimal control problems with singular arcs by a combined shooting-homotopy method. The first one is for the Goddard problem (SIMPLICIAL, freely available), and the second is specifically designed for the CNES heavy launcher problems (LAUNCHER).

6.1.6. Geometric control

6.1.6.1. Singular trajectories of control-affine systems

Participant: E. Trélat.

With Y. Chitour (U. Paris XI) and F. Jean (ENSTA). When applying methods of optimal control to motion planning or stabilization problems, some theoretical or numerical difficulties may arise, due to the presence of specific trajectories, namely, minimizing singular trajectories of the underlying optimal control problem. In this article, we provide characterizations for singular trajectories of control-affine systems. We prove that, under generic assumptions, such trajectories share nice properties, related to computational aspects; more precisely, we show that, for a generic system – with respect to the Whitney topology –, all nontrivial singular trajectories are of minimal order and of corank one. These results, established both for driftless and for control-affine systems, extend our previous results. As a consequence, for generic control-affine systems (with or without drift) defined by more than two vector fields, and for a fixed cost, there do not exist minimizing singular trajectories. Besides, we prove that, given a control-affine system satisfying the Lie algebra rank condition, singular trajectories are strictly abnormal, generically with respect to the cost. We then show how these results can be used to derive regularity results for the value function and in the theory of Hamilton-Jacobi equations, which in turn have applications for stabilization and motion planning, both from the theoretical and implementation issues.

6.1.6.2. On the stabilization problem for nonholonomic distributions

Participant: E. Trélat.

With L. Rifford (U. Nice). Let M be a smooth connected and complete manifold of dimension n, and Δ be a smooth nonholonomic distribution of rank $m \leq n$ on M in this article. We prove that, if there exists a smooth Riemannian metric on Δ for which no nontrivial singular path is minimizing, then there exists a smooth repulsive stabilizing section of Δ on M. Moreover, in dimension three, the assumption of the absence of singular minimizing horizontal paths can be dropped in the Martinet case. The proofs are based on the study, using specific results of nonsmooth analysis, of an optimal control problem of Bolza type, for which we prove that the corresponding value function is semiconcave and is a viscosity solution of a Hamilton-Jacobi equation, and establish fine properties of optimal trajectories.

6.1.7. Nonlinear optimal control via occupation measures and LMI-relaxations

Participant: E. Trélat.

With J.-B. Lasserre, D. Henrion and C. Prieur, LAAS, Toulouse. We consider in [44] the class of nonlinear optimal control problems (OCP) with polynomial data, i.e., the differential equation, state and control constraints and cost are all described by polynomials, and more generally for OCPs with smooth data. In addition, state constraints as well as state and/or action constraints are allowed. We provide a simple hierarchy of LMI (linear matrix inequality)-relaxations whose optimal values form a nondecreasing sequence of lower bounds on the optimal value. Under some convexity assumptions, the sequence converges to the optimal value of the OCP. Preliminary results show that good approximations are obtained with few moments.

6.1.8. Application to hydropower models

Participants: S. Aronna, F. Bonnans.

With P. Lotito, U. Tandil (Argentina). In the framework of the STIC AmSud “Energetic Optimization” project and of the internship of S. Aronna, we have established some qualitative properties of optimal controls for a continuous-time hydrothermal electricity production model. We show that in the case of several “parallel” dams, nonuniqueness of optimal trajectories may occur, and we characterize them in some cases. Then we discuss singular arcs using a reformulation of controls that allows to separate the “linear” and “nonlinear controls.

6.1.9. Hamilton-Jacobi-Bellman approach

6.1.9.1. Antidiffusive schemes for first order HJB equations

Participants: E. Cristiani, N. Forcadel, N. Megdich, H. Zidani.

In collaboration with O. Bokanowski (Lab. JLL, Paris 7), we have continued the study of antidiffusive numerical schemes for HJB equations coming from state-constrained optimal control problems (RDV problems, target problems, capture basin).

In [39], we prove the convergence of a non-monotonous scheme for a one-dimensional first order Hamilton-Jacobi-Bellman equation of the form $v_t + \max_\alpha (f(x,\alpha)v_x) = 0$, $v(0,x) = v_0(x)$. The scheme is related to the HJB-Ultrabee scheme suggested in [15], which has an anti-diffusive behavior, but which convergence was not proved. We show for general discontinuous initial data a first-order convergence of the scheme, in L^1-norm towards the viscosity solution, we also derive an error estimate (in L^1-norm).

Recently, P. Jaisson has started to extend these results to the convergence proof of the N-bee scheme also proposed in [15].

The computation of the capture basins can be reduced to the resolution of an HJB equation whose solution takes only values 0 and 1 (0 in the area of admissible trajectories and 1 in the nonadmissible area). For this situation, the HJB-ultrabee scheme is particularly adapted and allows an accurate localization of the interface (front) between the zone of 0 and that of 1. Taking into account this nice property, we developed numerical codes (for dimensions 2 and 3) on sparse grids allowing to store only the meshs crossed by the front. This implementation leads to a better management of the storage capacity and an important gain of computing time. Several numerical examples were performed validating the method (see the forthcoming thesis of N. Megdich). Moreover, we currently work on an optimized code in C++. Our objective being to carry out a fast code in 4d to deal with the climbing problem of a space shuttle (current contract with CNES).

6.1.9.2. Fast Marching Method for general HJB equations

Participants: E. Cristiani, N. Forcadel, H. Zidani.

In connection with the recruitment of E. Cristiani et N. Forcadel (post-doc positions), we have started to work on the generalization of the Fast-Marching Methods (FMM) for solving general HJB equations.

6.2. Stochastic optimal control

Optimal stochastic control problems occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky assets.

6.2.1. Convergence results on Howard's algorithm

Participants: S. Maroso, H. Zidani.

In [38], we have studied the Howard's algorithm for the resolution of $\min_{a\in\mathcal{A}} (B^a x - b^a) = 0$ where B^a is a matrix, b^a is a vector (possibly of infinite dimension), and $\mathcal{A}$ is a compact set. Under a monotonicity assumption on the matrices B^a, a global super-linear convergence result is obtained.

In the particular case of an obstacle problem of the form $\min(Ax - b,\, x - g) = 0$ where A is an $N \times N$ matrix satisfying a monotonicity assumption, the convergence of Howard's algorithm may be achieved in no more than N iterations, instead of the usual 2^N bound. Still in the case of obstacle problem, we established the equivalence between Howard's algorithm and a primal-dual active set algorithm [62].

We also propose an Howard-type algorithm for a "double-obstacle" problem of the form $\max(\min(Ax - b, x - g), x - h) = 0$.

We finally illustrate the algorithms on the discretization of nonlinear PDE's arising in the context of mathematical finance (American option, and Merton's portfolio problem), and for the double-obstacle problem.

6.2.2. Numerical approximation for a super-replication problem

Participants: S. Maroso, H. Zidani.

In collaboration with O. Bokanowski[2], and B. Bruder[3], we study a super-replication problem in dimension 2. The main difficulty for this problem comes from the non-boundedness of the control set. First we give a characterisation of the value function ϑ as unique viscosity solution of a second order HJB equation. The main difficulty here lies on the fact that the control variable is unbounded.

In [37], we study an approximation scheme for the HJB equation based on the generalized finite differences algorithm introduced in [95], [92]. We prove the existence, uniqueness of a bounded discrete solution. We also verify the monotonicity and stability of the scheme. Moreover, we give a consistance error approximation. Then, by using the same arguments as in [57], we prove the convergence of the discrete solutions towards the value function ϑ, when the discretization step size tends to 0. Moreover, we have perfomed several numerical tests to validate the theoretical convergence result.

6.2.3. Pricing of Amer-Asian options

Participants: S. Serghini, H. Zidani.

Within a framework of Internship (PFE) of Saad Serghini (from EMI School, Rabat), we studied the pricing of Amer-Asian options. This pricing leads to a second order variational inequations whose diffusion term (matrix of covariance) may be degenerated and is dominated by an advection term. In order to limit the numerical diffusion, in particular when covariance is cancelled, we tested and compared several antidiffusive schemes (Van-Leer, N-bee, Discrete Galerkin). See the report [37].

6.3. Short and middle term electricity production planning

Participants: G. Emiel, F. Bonnans.

With C. Sagastizábal, CEPEL, Rio de Janeiro. In the framework of G. Emiel's PhD thesis we study possible evolutions of non-smooth optimization algorithms when dealing with large scale problems. One of the main motivations is the resolution of stochastic optimization problems through Lagrangian decomposition. Those problems arise in particular in mid-term production planning. The past year focused on two approaches : dynamic Lagrangian relaxation, and incremental resolution.

6.3.1. Dynamic Lagrangian Relaxation :

The Lagrangian relaxation framework is commonly used in the resolution process of complex optimization problems. For example, it allows to generate bounds for mixed-integer linear programming problems. However, when the number of dualized constraints is very large (exponential in the dimension of the primal problem), explicit dualization is not possible. To reduce the dual dimension, heuristics were proposed in the literature that involve a separation procedure to dynamically select a restricted set of constraints to be dualized along the iterations. This relax–and–cut type approach has shown numerical efficiency. Belloni and Sagastizábal have obtained recently primal-dual convergence when using an adapted bundle method for the dual step, under minimal assumptions on the separation procedure. In [42], we extend these results to the subgradient scheme, widely used in the mixed-integer literature.

6.3.2. Incremental resolution :

When dealing with the mid-term production planning problem, Lagrangian relaxation of coupling constraints yield a separable dual function. It can be decomposed in several dual sub-functions. The idea of an incremental resolution is to take full advantage of this structure by making a dual iteration after the evaluation of each sub-function. Indeed, computational times resulting from the sub-problems resolutions are preponderant. Hence, we may achieve a significant amelioration by adopting such an algorithmic scheme. It has already been studied by Nedić and Bertsekas when using a subgradient algorithm for the dual phase. We applied this

[2]Lab. Jacque-Louis Lions, Paris 7

[3]Lab. Probabilités et modèles alétoires, Paris 7, and Soc. Générale.

approach in conjunction with Bundle algorithms. The recent theory of approximated bundle methods provides a nice framework for the convergence study. Numerical results still need to be further investigated but seem promising.

6.3.3. Efficient optimization technique for weekly unit commitment

Participants: R. Apparigliato, J.P. Vial, R. Zorgati.

The Unit Commitment Problem (UCP) consists of defining the minimal-cost power generation schedule for a given set of power plants. Due to many complex constraints, the deterministic UCP, even in its deterministic version, is a challenging large-size, non-convex, non-linear optimization problem, but there exist nowadays efficient tools to solve it. For a very short term horizon, the deterministic UCP is satisfactory; it is currently used for the daily scheduling in an industrial way. For the two/four-week time horizon which we are concerned with, uncertainty becomes significant and cannot be ignored anymore, making it necessary to treat the UCP as a stochastic problem. Dealing with uncertainty introduces a level of complexity that is of an order of magnitude higher than in the deterministic case. Thus, there is a need to design new stochastic optimization techniques and models, that are implementable in an industrial context.

6.3.3.1. Hydraulic management with robust optimization

The first part of the PhD of R. Apparigliato focusses on the problem of hydraulic management. One of the principal uncertainty factors, affecting hydro production system, is reservoir inflows. Indeed, these are highly variable. We can forecast these inflows on the two next weeks and estimate the maximal distances on the associated trend. Optimal decisions obtained with a deterministic approach aren't robust for inflow realizations and, so, lead to constraints violations. In this context, we suggest approaching this problem with the help of robust optimization, in order to obtain robust production decisions. This choice is justified first of all by the necessity of implementing a new approach and making a comparison with respect to existing techniques. Furthermore it is justified due to the important economical interests in the definition of a robust hydraulic management on the weekly horizon. Such an approach was tested on a representative valley of French system. The valley is managed on 7 days (84 time steps) and consists on 3 reservoirs and 6 production groups.

Third year: The third year of the PhD served for finalizing this study, for making a summary of the works of the first two years on this subject. The robust approaches were compared with a deterministic one, the so called Deterministic with Periodic Revision (DPR), close to current practices in exploitation. The main results show that robust models allow us to decrease strongly the volume constraints violations (by about 75 - 95% in our example) until making them almost nil. The costs are only increased by 0,5%. The size of the model is almost the same as the original deterministic one. The results obtained by the robust approach are thus satisfactory and allow us to eventually implement it in an operational context, thus taking uncertainty into account. A large number of oral communications have been given on this subject. An internal report which introduces robust optimization was published [179] as well as an application of the hydraulic problem [36]. An article is in preparation.

6.3.3.2. The supply shortage hedging

Abstract: In the deregulation context of the electric markets, the weekly unit commitment could play a leading role, in complement of its current role of making scheduling for the daily horizon: the management of the production margin, defined as the difference between the total offer and the total demand, taking into account stochasticity affecting the electrical system. This problem consists of determining which optimal decisions, according to a certain economical criteria, satisfy:

- hedge against supply shortage risk in order to satisfy demand in 99% of the situations,
- sell on markets the surplus of production with regard to a safety margin, as far as the legal and commercial measures allow it.

At present, there is no operational tool to satisfy needs relative to the role of management of the production margin and the risk. This established fact doesn't allow us to approach the problems of the risk measure, hedging or even that of trading. The second year of the PhD was partially dedicated to the formulation and to the resolution of the problem of the active management of the margin. From an analysis, a first formulation taking into account the interruption contracts and market products was made with the help of chance constraints and implemented for tests. This formulation is for the moment only time dependent (open loop).

Third year: Several research axes were examined in order to improve this open-loop formulation: formulation of the problem allowing the modification maintenance schedules of thermal units, improvement of the computation of the production margin,... However, the effort was mainly focussed on the application of a formulation in closed loop, that is a control dependent on the stochastic process and of time. It could permit us to make decisions function of the uncertainty history. We propose to apply a new approach: stochastic programming with step decision rules (SPSDR). Multi-stage stochastic programming is known to suffer from an exponential increase of the number of variables (decisions and states) when the number of steps increases linearly. The SPSDR approach [178] tries to approximate the solution of such problems by combining several techniques. The first idea is to work with independent experts [159]. Every expert works on the basis of small pool of scenarios, drawn randomly in the original set of scenarios, according to the initial stochastic process. The second idea is that experts work with decision rules, combining step functions. Optimal decision rules are then combined to build the final decision rule. This last one is then applied on a large number of scenarios. First results are relatively encouraging. The determination of the margin process is described in a EDF note [45]. The formulations of the problem in open and closed loop have lead to several oral communications.

6.4. Optimal control of partial differential equations

6.4.1. Control for Fast and Stable Laminar-to-High-Reynolds-Numbers Transfer in a 2D Navier-Stokes Channel Flow

Participant: E. Trélat.

With J.-M. Coron (U. Paris-Sud) and R. Vazquez (U. Sevilla). We consider the problem of generating and tracking a trajectory between two arbitrary parabolic profiles of a periodic 2D channel flow, which is linearly unstable for high Reynolds numbers. Also known as the Poiseuille flow, this problem is frequently cited as a paradigm for transition to turbulence. Our procedure consists in generating an exact trajectory of the nonlinear system that approaches exponentially the objective profile. Using a backstepping method, we then design boundary control laws guaranteeing that the error between the state and the trajectory decays exponentially in L^2, H^1, and H^2 norms. The result is first proved for the linearized Stokes equations, then shown to hold locally for the nonlinear Navier-Stokes system. see the report [41].

6.4.2. A penalization approach for tomographic reconstruction of binary axially symmetric objects

Participant: E. Trélat.

With R. Abraham and M. Bergounioux, U. Orléans. We propose in this work a variational method for tomographic reconstruction of blurred and noised binary images based on a penalization process of a minimization problem settled in the space of bounded variation functions. We prove existence and/or uniqueness results and derive an optimality system, both for the minimization problem and its penalized version. Numerical simulations are provided to demonstrate the relevance of the approach. See [11].

6.4.3. Control of ferromagnetic materials

Participant: E. Trélat.

With G. Carbou (U. Bordeaux) and S Labbé (IMAG, Grenoble). We investigate the problem of controlling the magnetic moment in a ferromagnetic nanowire submitted to an external magnetic field in the direction of the nanowire. The system is modeled with the one dimensional Landau-Lifschitz equation. In the absence of control, there exist particular solutions, which happen to be relevant for practical issues, called travelling walls. In this paper, we prove that it is possible to move from a given travelling wall profile to any other one, by acting on the external magnetic field. The control laws are simple and explicit, and the resulting trajectories are shown to be stable. See [32].

6.5. Optimization tools

6.5.1. Tame Optimization

Participants: J. Bolte, F. Alvarez, H. Attouch, A. Daniilidis, A.S. Lewis, J. Munier, M. Shiota.

An important part of the work described below relies on recent developments on model theoretic structures (o-minimal structures) which generalize axiomatically the qualitative properties of semialgebraic sets (see van den Dries, Shiota). Semialgebraic sets are subsets of IR^n defined by a finite number of polynomial equalities and inequalities. Finite union/intersection and complement of semialgebraic sets are semialgebraic; more importantly linear projections of semialgebraic sets remain semialgebraic (Tarski Seideberg principle). These facts yield remarkable stability properties as well as a kind of "finiteness of pathologies" principle. An illustration of these considerations could be as follows: take a bounded semialgebraic set $A \subset IR^n \to IR$ and a polynomial function $P : IR^n \times IR^m \to IR$, then the *nonsmooth* function $y \to \sup_{x \in A} P(x, y)$ has a semialgebraic graph (stability) and is smooth everywhere save perhaps on an finite union of manifolds of low dimension (finitess/tameness of pathologies). Keeping in mind the semialgebraic model, an o-minimal structure over IR is a boolean collection $\mathcal{O}$ of subsets of $\cup_{n \in IN} IR^n$ enjoying (in particular) two major properties: the family is stable with respect to linear projection and "one-dimensional" sets are exactly finite union of intervals. A function/point-to-set mapping is said to belong to such a structure if its graph belongs to $\mathcal{O}$. This "theoretical" extensions of real algebraic geometry could seem useless if the only example of o-minimal structure was given by the collection of semialgebraic sets. Two major breakthroughs by Gabrielov (globally subanalytic sets) and Wilkie (log-exp structure) have shown that o-minimal structure are numerous. This fact is, to our opinion, of high importance for applied mathematics.

The striking stability results enjoyed by such structures can be indeed used to show that many finite-dimensional optimization problems are (or could be) formulated within this setting. This was the starting point for the use of such a theory in variational analysis and for the study of some related optimization algorithms. This being said, a general idea to understand what could be obtained in this framework, is to think "o-minimal" problems -which are often qualified in a more vivid way as "tame"- as problems which are generically well-posed or well-behaved.

More specifically the works we present here were developed in view of the study of two following general problems

- convergence of gradient methods in a nonsmooth and nonconvex setting,
- convergence of Newton's method either smooth or nonsmooth.

Particular attention was dedicated to the analysis of convergence rate of such methods and to what is usually called *global convergence*. This last term means that the sequence/curve generated by the algorithm/dynamical system converges to a specific equilibrium despite the fact that a continuum of critical points may be involved.

6.5.1.1. Tame mappings are semismooth

Superlinear convergence of the Newton method for nonsmooth equations requires a "semismoothness" assumption. In [16] we have proved that locally Lipschitz functions definable in an o-minimal structure (in particular semialgebraic or globally subanalytic functions) are semismooth. Semialgebraic, or more generally, globally subanalytic mappings present the special interest of being γ-order semismooth, where γ is a *positive* parameter. As an application of this new estimate, we prove that the error at the kth step of the Newton method behaves like $O(2^{-(1+\gamma)^k})$.

6.5.1.2. The Łojasiewicz inequality for nonsmooth subanalytic functions with applications to subgradient dynamical systems

In [17], we show that, given a real-analytic function $f : IR^n \to IR$ and a critical point $a \in IR^n$, the Łojasiewicz inequality asserts that there exists $\theta \in [\frac{1}{2}, 1)$ such that the function $|f - f(a)|^{\theta} \|\nabla f\|^{-1}$ remains bounded around a. We have extended the above result to a wide class of nonsmooth functions (that possibly admit the value $+\infty$), by establishing an analogous inequality in which the derivative $\nabla f(x)$ can be replaced by any element x^* of the subdifferential $\partial f(x)$ of f. Like its smooth version, this result provides new insights into the convergence aspects of subgradient-type dynamical systems. Provided that the function f is sufficiently regular (for instance, convex or lower-C^2), the bounded trajectories of the corresponding subgradient dynamical system can be shown to be of finite length. Explicit estimates of the rate of convergence are also derived.

6.5.1.3. On the convergence of the proximal algorithm for nonsmooth functions involving analytic features

In [14], we study the convergence of the proximal algorithm applied to nonsmooth functions that satisfy the Łojasiewicz inequality around their generalized critical points. Typical examples of functions complying with these conditions are continuous semialgebraic or subanalytic functions. Following Łojasiewicz's original idea, we have proved that any bounded sequence generated by the proximal algorithm converges to some generalized critical point. We also obtain convergence rate results which are related to the flatness of the function by means of Łojasiewicz exponents. Apart from the sharp and elliptic cases which yield finite or geometric convergence, the decay estimates are of order $O(k^{-s})$, where $s \in (0, +\infty)$ depends on the flatness of the function.

6.5.1.4. Clarke subgradients of stratifiable functions

In [18], we have established the following result: if the graph of a lower semicontinuous real-extended-valued function $f : IR^n \to IR \cup \{+\infty\}$ admits a Whitney stratification (so in particular if f is a semialgebraic function), then the norm of the gradient of f at $x \in \operatorname{dom} f$ relative to the stratum containing x bounds from below all norms of Clarke subgradients of f at x. As a consequence, we have obtained a Morse-Sard type theorem as well as a nonsmooth extension of the Kurdyka-Łojasiewicz inequality for functions definable in an arbitrary o-minimal structure. It is worthwhile pointing out that, even in a smooth setting, this last result generalizes Kurdyka inequality by removing the boundedness assumption on the domain of the function.

6.5.1.5. A unifying local convergence result for Newton's method in Riemannian manifolds

In [12], we consider the problem of finding a singularity of a vector field X on a complete Riemannian manifold. Inspired by previous work of X. Wang, and Zabrejko-Nguen on Kantorovich's majorant method, our approach relies on the introduction of an abstract one-dimensional Newton's method obtained using an adequate Lipschitz-type radial function of the covariant derivative of X. Our main theorem asserts that if the one-dimensional method is well-posed and converges, then the Newton's method for X inherits of both its qualitative and *quantitative* specifities. A specialization of this result permits to recover three famous results, namely the Kantorovich, Smale and Nesterov-Nemirovskii theorems. Concerning real-analytic vector fields the convergence criterion does not involve any curvature terms as it was the case in the pioneering work of Dedieu et al. The result is an exact equivalent of Smale α-theorem in a Riemannian setting.

7. Contracts and Grants with Industry

7.1. Contracts

1. INRIA-EDF (PhD of R. Apparigliato), 2005-2007. *Application of recourse optimization for risk management in short term power planning*. Involved researchers: **F. Bonnans**.
2. INRIA-EDF (PhD of G. Emiel), 2005-2008. *Solving large scale problems for middle term power planning*. Involved researchers: **F. Bonnans**.
3. ENSTA-CNES (OPALE pole framework), 2007. *HJB approach for the atmospheric re-entry problem*. Involved researchers: F. Bonnans, **H. Zidani**.

4. ENSTA-DGA, 2007-2008. *Study of HJB equations associated to motion planning*. Involved researchers: N. Forcadel, **H. Zidani**.
5. Univ. Orléans-CNES (OPALE pole framework), 2007. *Singular arcs in the launchers problem*. Involved researchers: F. Bonnans, P. Martinon (INRIA), **E. Trélat**.

8. Other Grants and Activities

8.1. International collaborations

- In the setting of the STIC AmSud project on "Energy Optimization" we have a collaboration with P. Lotito (U. Tandil) on deterministic continuous-time models for the optimization of hydrothermal electricity.
- With Claudia Sagastizábal, CEPEL, Rio de Janeiro : we are currently analysing some approaches for stochasting programming, with application to the production of electricity.
- With Felipe Alvarez (CMM, Universidad de Chile, Santiago) we study the logarithmic penalty approach for optimal control problems.
- Italy: U. Roma (Sapienza). With M. Falcone: numerical methods for the resolution of HJB equations.
- Portugal: U. Aveiro. With D.F.M. Torres. Invariant laws and optimal syntheses.

8.2. Visiting Scientists

Long-term visits: N. Osmolovskii (6 months, SRI, Warsaw, Poland).

Short-term visits: invited professors, Claudia Sagastizábal and Mikhail Solodov (1 week, IMPA, Rio de Janeiro, Brazil), Hector Ramirez-Cabrera (1 week, CMM and U. Chile, Santiago), Pablo Lotito (2 weeks, Argentina).

Student internships: Soledad Aronna, U. Rosario, Argentina, 3 months, and Saad Serghini, EMI, Rabat, 3 months.

9. Dissemination

9.1. Editorial boards and scientific community

- F. Bonnans is one of the three Corresponding Editors of "ESAIM:COCV" (Control, Optimisation and Calculus of Variations), and Associate Editor of "Applied Mathematics and Optimization", and "Optimization, Methods and Software".
- F. Bonnans is member of the Council-at-Large of the Mathematical Programming Society (2006-2009), member of the Optimal Control Technical Committee 2.4 of IFAC (International Federation of Automatic Control), 2005-2008, and member of the board of the SMAI-MODE group (2007-2010). He is one of the organizers of SPO (Séminaire Parisien d'Optimisation, IHP, Paris).
- E. Trélat is Associate Editor of "ESAIM:COCV" (Control, Optimisation and Calculus of Variations) and of "International Journal of Mathematics and Statistics".

9.2. Teaching

- F. Bonnans

1. Associate Professor, Ecole Polytechnique (Courses on Operations Research and Optimal Control, 50 h), and Course on Continuous Optimization, Mastere de Math. et Applications, Filière "OJME", Optimisation, Jeux et Modélisation en Economie, Université Paris VI (18 h).
2. *Introduction to stochastic programming*. I Escuela Franco LatinoAmericana de Optimizacion Energetica. Pergamino (Argentina), 8 h, April 23-28, 2007.

- N. Forcadel
  1. *Numerical methods in finance*, 12h (third year of ENSTA- M2 MMMEF, U. Paris 1).
  2. Minicourse *Fast Marching Method*, Automn School on "Introduction to numerical methods for moving boundaries", Ensta, November 12-14, 2007.
- A. Hermant
  1. ENSTA: *Quadratic Optimization* (first year, 12h), *Dynamical systems and introduction to automatic control* (first year, 12h).
  2. ENSMP (Mines): Mathematics (integration), first year, 26h.
- P. Jaisson
  1. *Course on partial differential equations*, L3 (third year), University of Versailles Saint Quentin (UVSQ), 36h.
- E. Trélat
  1. Professor of Mathematics, University of Orléans where he teaches continuous optimization, numerical analysis, automatic, and optimal control. He is responsible of the Master of Mathematics which involves in particular automatic and control.
  2. Courses on *Optimal Control* in ENSTA (22 h) and in the Master ATSI of University Paris-Sud (12h).
- H. Zidani - Professeur at ENSTA (70h)
  1. Courses at ENSTA: *Quadratic Optimization* (first year, 21h), "Front propagation" course (third year), *Hamilton-Jacobi-Bellman approach to Optimal Control* (third year, 21 h), "Numerical methods for front propagation" (third year, 21 h),
  2. 'Courses in the Master "Ingénierie Mathématiques", U. of Paris-Sud Orsay.

     ATS of University Paris-Sud : 'Optimal control" (30 h).
- N. Megdich
  1. As duty of the half ATER position: *Automatic control* (Lessons, Master 1, 30h, Supervision of personal work, Master 1, and Exercices, Master 2, 30h).

9.3. Organisation of conferences

- Autumn School, Course *Introduction to numerical methods for moving boundaries*, Ensta, 12-14 Novembre, 2007. N. Forcadel and H. Zidani, members of organizing commitee.
- CEA-EDF-INRIA Course *Optimal control: Algorithms and Applications*, Rocquencourt, 30 May-1st June 2007. F. Bonnans, organizer.
- Journées CODE 07 (Conférence de la SMAI sur l'Optimisation et la Décision), Institut Henri Poincaré, 18-20 avril 2007. F. Bonnans, member of organizing commitee.
- CIFA 2008, Bucharest. E. Trélat, in charge of the stream "Optimization and Control of nonlinear systems".

- E. Trélat : ANR SICOMAF Thematic day “Control and Ferromagnetism”, Université Paris-Sud 11 (Orsay), June 21, 2007.

9.4. Invitations as plenary speakers

- E. Trélat: Czech-French-German Conference on Optimization, Heidelberg, Sept. 17-21, 2007, and Conference SMAI 2007, Praz-sur-Arly, June 4-8.
- F. Bonnans: Europt-OMS joint meeting: 2nd Conference on Optimization Methods and Software and 6th EUROPT Workshop "Advances in Continuous Optimization", Prague, July 4-7, 2007.

9.5. Talks presented at conferences

- R. Apparigliato
    1. Andrieu, L. and Apparigliato, R. and Lisser, A. and Tang, A.: *Stochastic Optimization under Risk Constraints: Application to Hedging Problem in Electrical Industry*. Power Systems Management Conference, Athens, 5-8 June 2007.
    2. Andrieu, L. and Apparigliato, R. and Lisser, A. and Tang, A.: *Stochastic Optimization under Risk Constraints: Application to Hedging Problem in Electrical Industry*. 11th International Conference on Stochastic Programming, Vienna, 27-31 August 2007.
    3. Apparigliato, R. and Thénié, J. and Vial, J.-P.: *Step decision rules for multistage stochastic programming: Application on a Hedging Problem in Electrical Industry*. 11th International Conference on Stochastic Programming, Vienna, 27-31 August 2007.
    4. Apparigliato, R. and Vial, J.P. and Zorgati, R.: *Gestion hebdomadaire d’une vallée hydraulique par optimisation robuste*. Conférence conjointe FRANCORO-ROADEF, Grenoble, 20-23 Feb. 2007.
    5. Apparigliato, R. and Vial, J.P. and Zorgati, R.: *Weekly management of a hydraulic valley by robust optimization*. 3ème Cycle Romand de Recherche Opérationnelle, Zinal (Suisse), 4-8 March 2007
- J. Bolte
    1. *Characterizations of Kurdyka-Lojasiewicz inequality*. International Conference on "Nonconvex Programming, Local and Global Approaches. Theory, Algorithms and Applications". INSA, Rouen, Dec. 17-21.
- F. Bonnans
    1. S. Aronna, F. Bonnans, P. Lotito: *Optimal control techniques for hydropower production*. International Conference on "Nonconvex Programming, Local and Global Approaches. Theory, Algorithms and Applications". INSA, Rouen, Dec. 17-21.
- G. Emiel
    1. G. Emiel, C. Sagastizabal: *Dynamic subgradient methods*. European Conference on Operational Research EURO XXII, Prague, July 2007
    2. G. Emiel: *Modeling dependency for Credit Risk*. Mathematics and Finance : Research in Options, Rio de Janeiro, Oct. 2007.
- A. Hermant
    1. *Conditions d’optimalité du second-ordre pour les problèmes de commande optimale avec contraintes sur l’état. Application au tir*. Journée des Doctorants du CMAP, Ecole Polytechnique, March 7, 2007.

2. *Conditions d'optimalité du second-ordre pour les problèmes de commande optimale avec contraintes vectorielles sur l'état ; application au tir*. Conférence sur l'Optimisation et la Décision CODE 2007, Paris, April 18-20, 2007.
3. *Stability and sensitivity analysis for optimal control problems with a first-order state constraint and application to continuation methods*. 23rd IFIP Conference on System Modeling and Optimization Cracow, Poland, July 23-27, 2007.
4. *Stability and sensitivity analysis for optimal control problems with a first-order state constraint and application to continuation methods*. 13th Czech-French-German Conference on Optimization Heidelberg, Sept. 17-21, 2007.

- N. Megdich
  1. *A fast anti-dissipative method for the minimum time problem. Application to atmospheric re-entry*. Conférence sur l'Optimisation et la Décision CODE 2007, Paris, April 18-20, 2007.
- P. Martinon
  1. *Détection de commutations et utilisation des équations variationnelles pour la méthode de tir*. Conférence sur l'Optimisation et la Décision CODE 2007, Paris, April 18-20, 2007.
  2. *SNumerical conservation of first integrals for an optimal control problem with control discontinuities*. 13th Czech-French-German Conference on Optimization Heidelberg, Sept. 17-21, 2007.
- E. Ottenwaelter
  1. *Schémas numériques de résolution de l'équation de Hamilton-Jacobi-Bellman de la commande optimale stochastique*. Journée des Doctorants du CMAP, Ecole Polytechnique, March 7, 2007.
- E. Trélat
  1. R. Abraham, M. Bergounioux, E. Trélat: *A penalization approach for tomographic reconstruction of binary axially symmetric objects*. International Conference on "Nonconvex Programming, Local and Global Approaches. Theory, Algorithms and Applications". INSA, Rouen, Dec. 17-21.

9.6. Invitations to seminars

- R. Apparigliato: *Gestion hebdomadaire d'une vallée hydraulique par optimisation robuste*. Séminaire scientifique de l'Institut Français du Pétrole, Rueil-Malmaison, 26 Oct. 2007.
- N. Forcadel: *Résultats d'homogénéisation pour la dynamique des dislocations et pour certains systèmes de particules*. Séminaire d'analyse appliquée, Université, Nov.26, 2007.
  Résultats d'homogénéisation pour la dynamique des dislocations Groupe de travail Mécanique des fluides, U.P.S. Toulouse, Oct. 25, and Groupe de travail d'homogénéisation, U. Paris, Nov. 19, 2007, and Séminaire d'analyse numérique, Université de Rennes, Dec. 20, 2007.
- E. Trélat: *Regularity of the value function in optimal control; applications to viscosity solutions and to stabilization*. Séminaire d'Analyse Appliquée, Univ. Brest (Jan. 16), Seminar of Mathematics, U. Orléans, (Feb 1st)., "Image day", U. Orléans, (Oct. 25).
- H. Zidani: *Numerical Methods for Hamilton-Jacobi Equations*. Séminaire du Lab. Jacques-Louis Lions, Université Paris-VI, Paris (March 8).

10. Bibliography

Major publications by the team in recent years

[1] M. BERGOUNIOUX, H. ZIDANI. *A fully Discrete Approximation for a Control Problem of Parabolic Variational Inequalities*, in "SIAM J. Numerical Analysis", vol. 39, n° 6, 2002, p. 2014-2033.

[2] J. Bonnans, J. C. Gilbert, C. Lemaréchal, C. Sagastizábal. *Numerical Optimization: theoretical and numerical aspects*, Universitext, second edition, Springer-Verlag, Berlin, 2006.

[3] J. Bonnans, A. Hermant. *Well-Posedness of the Shooting Algorithm for State Constrained Optimal Control Problems with a Single Constraint and Control*, in "SIAM J. Control Optimization", 2007.

[4] J. Bonnans, S. Maroso, H. Zidani. *Error estimates for a stochastic impulse control problem*, in "Appl. Math. and Optim.", vol. 55, n^o 3, 2007, p. 327–357.

[5] J. Bonnans, A. Shapiro. *Perturbation analysis of optimization problems*, Springer-Verlag, New York, 2000.

[6] J. Bonnans, H. Zidani. *Consistency of generalized finite difference schemes for the stochastic HJB equation*, in "SIAM J. Numerical Analysis", vol. 41, 2003, p. 1008-1021.

[7] B. Bonnard, L. Faubourg, E. Trélat. *Mécanique céleste et contrôle des véhicules spatiaux*, Springer-Verlag, 2006.

[8] N. Bérend, J. Bonnans, J. Laurent-Varin, M. Haddou, C. Talbot. *An Interior-Point Approach to Trajectory Optimization*, AIAA J. Guidance, Control and Dynamics, to appear, Rapport de Recherche, n^o 5613, INRIA, 2005, http://hal.inria.fr/inria-00070395.

[9] J.-M. Coron, E. Trélat. *Global steady-state controllability of 1-D semilinear heat equations*, in "SIAM J. Control Optimization", vol. 43, 2004, p. 549–569.

[10] J. Gergaud, P. Martinon. *Using switching detection and variational equations for the shooting method*, in "Optimal Control Applications and Methods", vol. 28, n^o 2, 2007, p. 95–116.

Year Publications

Articles in refereed journals and book chapters

[11] R. Abraham, M. Bergounioux, E. Trélat. *A penalization approach for tomographic reconstruction of binary axially symmetric objects*, in "Applied Mathematics of Optimization", to appear, preliminary version : preprint HAL 2007, 28 pages, 2007.

[12] F. Alvarez, J. Bolte, J. Munier. *A Unifying Local Convergence Result for Newton's method in Riemannian Manifolds*, in "Foundation of Computational Mathematics", To appear, 2007.

[13] J. André, F. Bonnans, L. Cornibert. *Planning reinforcement on gas transportation networks with optimization methods*, in "European Journal of Operational Research", To appear, 2007.

[14] H. Attouch, J. Bolte. *On the convergence of the proximal algorithm for nonsmooth functions involving analytic features*, in "Mathematical Programming B", To appear, 2007.

[15] O. Bokanowski, H. Zidani. *Anti-diffusive schemes for linear advection and application to Hamilton-Jacobi-Bellman equations*, in "J. Sci. Computing", vol. 30, n^o 1, 2007, p. 1 – 33.

[16] J. Bolte, A. Daniilidis, A. Lewis. *Tame mappings are semismooth*, in "Math. Prog. B", 2007.

[17] J. BOLTE, A. DANIILIDIS, A. LEWIS. *The Lojasiewicz inequality for nonsmooth subanalytic functions with applications to subgradient dynamical systems*, in "SIAM Optimization", vol. 17, n^{o} 4, 2007, p. 1205-1223.

[18] J. BOLTE, A. DANIILIDIS, A. LEWIS, M. SHIOTA. *Clarke subgradients of stratifiable functions*, in "SIAM Optimization", vol. 18, n^{o} 2, 2007, p. 556-572.

[19] J. BONNANS, M. ANDRÉ. *Fast computation of the leastcore and prenucleolus of cooperative games*, in "RAIRO-RO", To appear, 2007.

[20] J. BONNANS, A. HERMANT. *No Gap Second order Optimality Conditions for Optimal Control Problems with a single State Constraint and Control*, in "Mathematical Programming, Series B", Online First DOI 10.1007/s10107-007-0167-8, 2007.

[21] J. BONNANS, A. HERMANT. *Well-Posedness of the Shooting Algorithm for State Constrained Optimal Control Problems with a Single Constraint and Control*, in "SIAM J. Control Optimization", vol. 46, n^{o} 4, 2007, p. 1398- -1430.

[22] J. BONNANS, A. HERMANT. *Stability and Sensitivity Analysis for Optimal Control Problems with a First-order State Constraint and application to continuation methods*, in "ESAIM:COCV", To appear, 2008, http://www.cmap.polytechnique.fr/~hermant/first-order.pdf.

[23] J. BONNANS, S. MAROSO, H. ZIDANI. *Error estimates for a stochastic impulse control problem*, in "Applied Mathematics and Optimization", vol. 55, n^{o} 3, 2007, p. 327- -357.

[24] B. BONNARD, J.-B. CAILLAU, E. TRÉLAT. *Second order optimality conditions in the smooth case and applications in optimal control*, in "ESAIM COCV", vol. 13, n^{o} 2, 2007, p. 207–236.

[25] N. BÉREND, J. BONNANS, M. HADDOU, J. LAURENT-VARIN, C. TALBOT. *An Interior-Point Approach to Trajectory Optimization*, in "J. Guidance, Control and Dynamics", vol. 30, n^{o} 5, 2007, p. 1228- -1238.

[26] Y. CHITOUR, F. JEAN, E. TRÉLAT. *Singular trajectories of control-affine systems*, in "SIAM J. Control Optim", To appear, 2007.

[27] J.-M. CORON, E. TRÉLAT. *Tout est sous contrôle*, in "Matapli", n^{o} 83, 2007, p. 59–73.

[28] J. GERGAUD, P. MARTINON. *Using switching detection and variational equations for the shooting method*, in "Optimal Control Applications & Methods", vol. 28), n^{o} 2, 2007, p. 95–116.

[29] P. MARTINON, J. BONNANS, E. TRÉLAT. *Singular arcs in the generalized Goddard's Problem*, in "J. Optimization Theory Applications", to appear, preliminary version : INRIA report RR-6157, 2007, http://hal.inria.fr/inria-00139273.

[30] L. RIFFORD, E. TRÉLAT. *On the stabilization problem for nonholonomic distributions*, in "J. Eur. Math. Soc.", To appear, 2007.

Publications in Conferences and Workshops

[31] B. BONNARD, J.-B. CAILLAU, E. TRÉLAT. *Second order optimality conditions with applications*, in "Proceedings of the Sixth AIMS International Conference, Poitiers (2006)", To appear in Discrete Contin. Dyn. Syst, 10 pages, 2007.

[32] G. CARBOU, S. LABBÉ, E. TRÉLAT. *Control of travelling walls in a ferromagnetic nanowire*, suppl., 2007.

[33] J.-M. CORON, E. TRÉLAT, R. VAZQUEZ. *Fast Tracking of Poiseuille Trajectories in Navier-Stokes 2D Channel Flow*, in "Proceedings of CTS-HYCON Workshop, Paris (2006)", To appear in International Scientific and Technical Encyclopedia, ISTE, 2007.

[34] C. PRIEUR, E. TRÉLAT. *On two hybrid robust optimal stabilization problems*, in "Proceedings of the 13th IFAC Workshop on Control Applications of optimisation (CAO'06)", Internat. J. Tomography Stat., vol. 6, 2007, p. 86–91.

Internal Reports

[35] F. ALVAREZ, J. BONNANS, J. LAURENT-VARIN. *Asymptotic expansion of the optimal control under logarithmic penalty: worked example and open problems*, technical report, n^o 6170, INRIA, avril 2007, http://hal.inria.fr/inria-00143515.

[36] R. APPARIGLIATO, J.-P. VIAL, R. ZORGATI. *Optimisation Robuste Linéaire - Application : gestion court terme d'une vallée hydraulique*, Technical report, EDF R&D, Dept OSIRIS, H-R32-2007-00658-FR, 1 av. Général de Gaulle, F-92141 Clamart, 2007.

[37] O. BOKANOWSKI, S. MAROSO, B. BRUDER, H. ZIDANI. *Numerical approximation for superreplication problem under gamma constraints*, technical report, INRIA, 2007.

[38] O. BOKANOWSKI, S. MAROSO, H. ZIDANI. *Some convergence results for Howard's algorithm*, technical report, INRIA, 2007, http://hal.inria.fr/inria-00179549.

[39] O. BOKANOWSKI, N. MEGDICH, H. ZIDANI. *Convergence of a non-monotone scheme for Hamilton-Jacobi-Bellman equations with discontinuous data*, technical report, INRIA, 2007, http://hal.inria.fr/inria-00193157.

[40] J. BONNANS, A. HERMANT. *Second-order Analysis for Optimal Control Problems with Pure State Constraints and Mixed Control-State Constraints*, Technical report, n^o RR-6199, INRIA, 2007, http://hal.inria.fr/inria-00148946.

[41] J.-M. CORON, E. TRÉLAT, R. VAZQUEZ. *Control for Fast and Stable Laminar-to-High-Reynolds-Numbers Transfer in a 2D Navier-Stokes Channel Flow*, Preprint Hal, 23 pages, Technical report, 2007.

[42] G. EMIEL, C. SAGASTIZÁBAL. *Dynamic subgradient methods*, Second report, EDF-IMPA contract, Technical report, IMPA, July 2007.

[43] A. HERMANT. *Stability Analysis of Optimal Control Problems with a Second-order State Constraint*, Technical report, INRIA, November 2007, http://hal.inria.fr/inria-00186968/fr.

[44] J.-B. LASSERRE, D. HENRION, C. PRIEUR, E. TRÉLAT. *Nonlinear optimal control via occupation measures and LMI-relaxations*, Preprint Hal, 26 pages, Technical report, 2007.

[45] R. ZORGATI, R. APPARIGLIATO, J.-M. LUEL. *Couverture et arbitrage sur les marchés en gestion hebdomadaire de la production électrique. Partie I: Marge de production et Risque physique à 1%*, Technical report, EDF R&D, Dept OSIRIS, H-R37-2007-01772-FR, 1 av. Général de Gaulle, F-92141 Clamart, 2007.

References in notes

[46] A. RUSZCZYŃSKI, A. SHAPIRO (editors). *Stochastic Programming*, Handbook in Operations Research and Management, vol. 10, Elsevier, Amsterdam, 2003.

[47] R. ABGRALL. *Numerical discretization of boundary conditions for first order Hamilton-Jacobi equations*, in "SIAM J. Numerical Analysis", vol. 41, n^o 6, 2003, p. 2233–2261.

[48] R. AGBRALL, S. AUGOULA. *High order numerical discretization for Hamilton-Jacobi equations on triangular meshes*, in "J. Scientific Computing", vol. 15, n^o 2, 2000, p. 197–229.

[49] E. ALLGOWER, K. GEORG. *Numerical Continuation Methods*, Springer-Verlag, Berlin, 1990.

[50] P. ARTZNER, F. DELBAEN, J. EBER, D. HEATH. *Coherent measures of risk*, in "Mathematical Finance", vol. 9, n^o 3, 1999, p. 203–228.

[51] J. AUBIN, H. FRANKOWSKA. *Set-valued analysis*, Birkhäuser, Boston, 1990.

[52] V. BALLY, G. PAGÈS, J. PRINTEMS. *A quantization tree method for pricing and hedging multidimensional American options*, in "Math. Finance", vol. 15, n^o 1, 2005, p. 119–168.

[53] V. BARBU. *Optimal control of variational inequalities*, Research Notes in Mathematics, vol. 100, Pitman, Boston, 1984.

[54] M. BARDI, I. CAPUZZO-DOLCETTA. *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations*, Systems and Control: Foundations and Applications, Birkhäuser, Boston, 1997.

[55] G. BARLES. *Solutions de viscosité des équations de Hamilton-Jacobi*, Mathématiques et Applications, vol. 17, Springer, Paris, 1994.

[56] G. BARLES, E. JAKOBSEN. *Error bounds for monotone approximation schemes for Hamilton-Jacobi-Bellman equations*, in "SIAM J. Numerical Analysis", vol. 43, n^o 2, 2005, p. 540–558 (electronic).

[57] G. BARLES, P. SOUGANIDIS. *Convergence of approximation schemes for fully nonlinear second order equations*, in "Asymptotic Analysis", vol. 4, 1991, p. 271–283.

[58] A. BEN-TAL, B. GOLANY, A. NEMIROVSKI, J.-PH.VIAL. *Supplier-Retailer Flexible Commitments Contracts: A Robust Optimization Approach*, in "Manufacturing and Service Operations Management", vol. 7, n^o 3, 2005, p. 248-273.

[59] A. BEN-TAL, A. NEMIROVSKI. *Robust convex optimization*, in "Mathematics of Operations Research", vol. 23, 1998, p. 769–805.

[60] A. BEN-TAL, M. TEBOULLE. *An old-new concept of convex risk measures: the optimized certainty equivalent*, in "Mathematical Finance", vol. 17, n^{o} 3, 2007, p. 449–476.

[61] A. BENSOUSSAN, G. D. PRATO, M. DELFOUR, S. MITTER. *Representation and control of infinite-dimensional systems*, Second edition, Birkhäuser, Boston, 2007.

[62] M. BERGOUNIOUX, M. HADDOU, M. HINTERMÜLLER, K. KUNISCH. *A comparison of a Moreau-Yosida-based active set strategy and interior point methods for constrained optimal control problems*, in "SIAM Journal on Optimization", vol. 11, 2000, p. 495–521 (electronic).

[63] M. BERGOUNIOUX, H. ZIDANI. *A fully Discrete Approximation for a Control Problem of Parabolic Variational Inequalities*, in "SIAM J. Numerical Analysis", vol. 39, n^{o} 6, 2002, p. 2014-2033.

[64] A. BERMÚDEZ, C. SAGUEZ. *Optimal control of a Signorini problem*, in "SIAM Journal on Control and Optimization", vol. 25, 1987, p. 576–582.

[65] D. BERTSEKAS. *Constrained optimization and Lagrange multipliers methods*, Academic Press, New York, 1982.

[66] D. BERTSEKAS. *Dynamic programming and optimal control (2 volumes)*, Athena Scientific, Belmont, Massachusetts, 1995.

[67] D. BERTSEKAS, J. TSITSIKLIS. *Neuro-dynamic programming (2nd edition)*, Athena Scientific, Belmont, Mass., 1996.

[68] J. BETTS. *Practical methods for optimal control using nonlinear programming*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2001.

[69] J. BETTS. *Survey of Numerical Methods for Trajectory Optimization*, in "AIAA J. of Guidance, Control and Dynamics", vol. 21, 1998, p. 193-207.

[70] G. BLANCHON, J. DODU, J. BONNANS. *Optimisation des réseaux électriques de grande taille*, in "Analysis and optimization of systems", A. BENSOUSSAN, J. LIONS (editors), Lecture Notes in Information and Control Sciences, vol. 144, Springer-Verlag, Berlin, 1990, p. 423-431.

[71] G. BLISS. *Lectures on the Calculus of Variations*, University of Chicago Press, Chicago, Illinois, 1946.

[72] H. BOCK. *Numerical treatment of inverse problems in chemical reaction kinetics*, in "Modelling of Chemical Reaction Systems, Heidelberg", Springer Series in Chemical Physics, vol. 18, Springer, 1981, p. 102–125.

[73] O. BOKANOWSKI, B. BRUDER, S. MAROSO, H. ZIDANI. *Numerical approximation of a superreplication problem under gamma constraints*, RR Inria, to appear, Technical report, 2007.

[74] O. BOKANOWSKI, N. MEGDICH, H. ZIDANI. *An adaptative antidissipative method for optimal control problems*, in "ARIMA", vol. 5, 2006, p. 256–271.

[75] O. BOKANOWSKI, N. MEGDICH, H. ZIDANI. *On the convergence of a non monotone scheme for HJB equations*, in "Preprint, to appear", 2007.

[76] O. BOKANOWSKI, H. ZIDANI. *Anti-diffusive schemes for linear advection and application to Hamilton-Jacobi-Bellman equations*, in "J. Sci. Computing", vol. 30, n° 1, 2007, p. 1–33.

[77] J. BONNANS. *Optimisation Continue*, Dunod, Paris, 2006.

[78] J. BONNANS. *Second order analysis for control constrained optimal control problems of semilinear elliptic systems*, in "Journal of Applied Mathematics & Optimization", vol. 38, 1998, p. 303-325.

[79] J. BONNANS, E. CASAS. *An extension of Pontryagin's principle for state-constrained optimal control of semilinear elliptic equations and variational inequalities*, in "SIAM J. Control Optimization", vol. 33, n° 1, 1995, p. 274-298.

[80] J. BONNANS, P. CHARTIER, H. ZIDANI. *Discrete approximations of the Hamilton-Jacobi equation for an optimal control problem of a differential-algebraic system*, in "Control and Cybernetics", vol. 32, 2003.

[81] J. BONNANS, R. B. FOURATI, H. SMAOUI. *The obstacle problem for water tanks*, in "Journal de Mathématiques Pures et Appliquées", vol. 82, n° 11, 2003, p. 1527–1553.

[82] J. BONNANS, J. C. GILBERT, C. LEMARÉCHAL, C. SAGASTIZÁBAL. *Numerical Optimization: theoretical and numerical aspects*, Universitext, second edition, Springer-Verlag, Berlin, 2006.

[83] J. BONNANS, T. GUILBAUD, A. KETFI-CHERIF, C. SAGASTIZÁBAL, D. VON WISSEL, H. ZIDANI. *Parametric optimization of hybrid car engines*, Technical report, n° RR-4674, INRIA, 2003, http://hal.inria.fr/inria-00071911.

[84] J. BONNANS, A. HERMANT. *Second-order Analysis for Optimal Control Problems with Pure State Constraints and Mixed Control-State Constraints*, in "Annals of I.H.P. - Nonlinear Analysis", Submitted, preliminary version: INRIA report RR-6199, 2006, http://hal.inria.fr/inria-00148946.

[85] J. BONNANS, A. HERMANT. *Stability and Sensitivity Analysis for Optimal Control Problems with a First-order State Constraint*, in "ESAIM:COCV", 2007, http://hal.inria.fr/inria-00087573.

[86] J. BONNANS, A. HERMANT. *Well-Posedness of the Shooting Algorithm for State Constrained Optimal Control Problems with a Single Constraint and Control*, in "SIAM J. Control Optimization", 2007.

[87] J. BONNANS, G. LAUNAY. *Sequential quadratic programming with penalization of the displacement*, in "SIAM J. Optimization", vol. 5, 1995, p. 792-812.

[88] J. BONNANS, G. LAUNAY. *Large Scale Direct Optimal Control Applied to a Re-Entry Problem*, in "AIAA J. of Guidance, Control and Dynamics", vol. 21, 1998, p. 996-1000.

[89] J. BONNANS, J. LAURENT-VARIN. *Computation of order conditions for symplectic partitioned Runge-Kutta schemes with application to optimal control*, in "Numerische Mathematik", vol. 103, n° 1, 2006, p. 1–10.

[90] J. BONNANS, S. MAROSO, H. ZIDANI. *Error estimates for stochastic differential games: the adverse stopping case*, in "IMA, J. Numerical Analysis", vol. 26, 2006, p. 188-212.

[91] J. BONNANS, S. MAROSO, H. ZIDANI. *Error estimates for a stochastic impulse control problem*, in "Appl. Math. and Optim.", vol. 55, n° 3, 2007, p. 327–357.

[92] J. BONNANS, E. OTTENWAELTER, H. ZIDANI. *Numerical schemes for the two dimensional second-order HJB equation*, in "ESAIM: M2AN", vol. 38, 2004, p. 723-735.

[93] J. BONNANS, A. SHAPIRO. *Perturbation analysis of optimization problems*, Springer-Verlag, New York, 2000.

[94] J. BONNANS, D. TIBA. *Pontryagin's principle in the control of semilinear elliptic variational inequalities*, in "Applied Mathematics and Optimization", vol. 23, n° 3, 1991, p. 299-312.

[95] J. BONNANS, H. ZIDANI. *Consistency of generalized finite difference schemes for the stochastic HJB equation*, in "SIAM J. Numerical Analysis", vol. 41, 2003, p. 1008-1021.

[96] B. BONNARD, L. FAUBOURG, G. LAUNAY, E. TRÉLAT. *Optimal control with state constraints and the space shuttle re-entry problem*, in "Journal of Dynamical and Control Systems", vol. 9, n° 2, 2003, p. 155–199.

[97] B. BONNARD, L. FAUBOURG, E. TRÉLAT. *Optimal control of the atmospheric arc of a space shuttle and numerical simulations by multiple-shooting techniques*, in "Math. Models Methods Applied Sciences", vol. 15, n° 1, 2005, p. 109–140.

[98] B. BONNARD, L. FAUBOURG, E. TRÉLAT. *Mécanique céleste et contrôle des véhicules spatiaux*, Mathématiques et Applications, vol. 51, Springer-Verlag, 2006.

[99] M. BROADIE, P. GLASSERMAN, Z. HA. *Pricing American options by simulation using a stochastic mesh with optimized weights*, in "Probabilistic constrained optimization, Dordrecht", Nonconvex Optim. Appl., vol. 49, Kluwer Acad. Publ., 2000, p. 26–44.

[100] B. BROGLIATO, M. MABROUK, A. Z. RIO. *On the controllability of linear juggling mechanical systems*, in "Systems Control Lett.", vol. 55, n° 4, 2006, p. 350–367.

[101] A. E. BRYSON, Y.-C. HO. *Applied optimal control*, Hemisphere Publishing, New-York, 1975.

[102] N. BÉREND, J. BONNANS, J. LAURENT-VARIN, M. HADDOU, C. TALBOT. *An Interior-Point Approach to Trajectory Optimization*, in "AIAA J. Guidance, Control and Dynamics", vol. 30, n° 5, 2005, p. 1228- -1238.

[103] N. BÉREND, J. BONNANS, J. LAURENT-VARIN, M. HADDOU, C. TALBOT. *Fast Linear Algebra for Multiarc Trajectory Optimization*, in "Large scale nonlinear optimization", G. D. PILLO, M. ROMA (editors), Nonconvex Optimization and Its Applications, vol. 83, Springer, 2006, p. 1–14.

[104] F. CLARKE. *A new approach to Lagrange multipliers*, in "Mathematics of Operations Research", vol. 2, 1976, p. 165-174.

[105] J.-M. CORON, E. TRÉLAT. *Global steady-state controllability of 1-D semilinear heat equations*, in "SIAM J. Control Optimization", vol. 43, 2004, p. 549–569.

[106] J.-M. CORON, E. TRÉLAT. *Global steady-state stabilization and controllability of 1-D semilinear wave equations*, in "Commun. Contemp. Mathematics", vol. 8, n^{o} 4, 2006, p. 535–567.

[107] M. CRANDALL, L. EVANS, P.-L. LIONS. *Some properties of viscosity solutions of Hamilton-Jacobi equations*, in "Trans. Amer. Math. Soc", vol. 282, 1984, p. 487-502.

[108] M. CRANDALL, H. ISHII, P.-L. LIONS. *User's guide to viscosity solutions of second order partial differential equations*, in "Bull. American Mathematical Society (New Series)", vol. 27, 1992, p. 1–67.

[109] M. CRANDALL, P.-L. LIONS. *Viscosity solutions of Hamilton Jacobi equations*, in "Bull. American Mathematical Society", vol. 277, 1983, p. 1–42.

[110] M. CRANDALL, P.-L. LIONS. *Two approximations of solutions of Hamilton-Jacobi equations*, in "Mathematics of Computation", vol. 43, 1984, p. 1–19.

[111] G. DANTZIG, P. WOLFE. *Decomposition principle for linear programs*, in "Operations Research", vol. 8, 1959, p. 101-111.

[112] B. DESPRÉS, F. LAGOUTIÈRE. *Contact discontinuity capturing schemes for linear advection and compressible gas dynamics*, in "J. Sci. Comput.", vol. 16, 2001, p. 479-524.

[113] B. DESPRÉS, F. LAGOUTIÈRE. *A non-linear anti-diffusive scheme for the linear advection equation*, in "C. R. Acad. Sci. Paris, Série I, Analyse numérique", vol. 328, 1999, p. 939-944.

[114] P. DEUFLHARD. *Newton Methods for Nonlinear Problems*, Springer-Verlag, Berlin, 2004.

[115] M. DIEHL, J. BJÖRNBERG. *Robust dynamic programming for min-max model predictive control of constrained uncertain systems*, in "IEEE Transactions on Automatic Control", vol. 49, n^{o} 12, 2004, p. 2253–2257.

[116] M. DIEHL, H. BOCK, E. KOSTINA. *An approximation technique for robust nonlinear optimization*, in "Mathematical Programming", vol. 107, 2006, p. 213 - 230.

[117] L. DIXON, M. BARTHOLOMEW-BIGGS. *Adjoint-control transformations for solving pratical optimal control problems*, in "Optimal Control, Applications and Methods", vol. 2, 1981, p. 365-381.

[118] A. L. DONTCHEV, W. W. HAGER. *Lipschitzian stability for state constrained nonlinear optimal control*, in "SIAM Journal on Control and Optimization", vol. 36, n^{o} 2, 1998, p. 698–718 (electronic).

[119] A. DONTCHEV, W. HAGER, V. VELIOV. *Second-order Runge-Kutta approximations in control constrained optimal control*, in "SIAM Journal on Numerical Analysis", vol. 38, 2000, p. 202–226 (electronic).

[120] J. DUPAČOVÁ, N. GRÖWE-KUSKA, W. RÖMISCH. *Scenario reduction in stochastic programming. An approach using probability metrics*, in "Math. Program.", vol. 95, n^{o} 3, Ser. A, 2003, p. 493–511.

[121] G. DUVAUT, J. LIONS. *Les inéquations en mécanique et en physique*, Dunod, Paris, 1972.

[122] I. EKELAND. *Nonconvex minimization problems*, in "Bulletin of the American Mathematical Society", vol. 1(New series), 1979, p. 443-474.

[123] M. FALCONE, R. FERRETTI. *Semi-Lagrangian schemes for Hamilton-Jacobi equations, discrete representation formulae and Godunov methods*, in "Journal of Computational Physics", vol. 175, 2002, p. 559–575.

[124] M. FALCONE, R. FERRETTI. *Convergence analysis for a class of high-order semi-Lagrangian advection schemes*, in "SIAM J. Numer. Anal.", vol. 35, n^{o} 3, 1998, p. 909–940 (electronic).

[125] J. GERGAUD, P. MARTINON. *Using switching detection and variational equations for the shooting method*, in "Optimal Control Applications and Methods", vol. 28, n^{o} 2, 2007, p. 95–116.

[126] B. GOH. *Necessary Conditions for Singular extremals involving multiple control variables*, in "J. SIAM Control", vol. 4, 1966, p. 716–731.

[127] W. HAGER. *Runge-Kutta methods in optimal control and the transformed adjoint system*, in "Numerische Mathematik", vol. 87, n^{o} 2, 2000, p. 247–282.

[128] E. HAIRER, C. LUBICH, G. WANNER. *Geometric numerical integration*, Springer Series in Computational Mathematics, vol. 31, Springer-Verlag, Berlin, 2002.

[129] E. HAIRER, S. P. NØRSETT, G. WANNER. *Solving ordinary differential equations. I*, Springer Series in Computational Mathematics, Second, vol. 8, Springer-Verlag, Berlin, 1993.

[130] E. HAIRER, G. WANNER. *Solving ordinary differential equations. II*, Springer Series in Computational Mathematics, Second, vol. 14, Springer-Verlag, Berlin, 1996.

[131] S. HAN. *Superlinearly convergent variable metric algorithms for general nonlinear programming problems*, in "Math. Programming", vol. 11, 1976, p. 263–282.

[132] P. HARKER, J.-S. PANG. *Finite dimensional variational inequality and nonlinear complementarity problems: A survey of theory, algorithms and applications*, in "Mathematical Programming, series B", vol. 48, 1990, p. 161-220.

[133] E. HARTEN. *ENO schemes with subcell resolution*, in "J. Computational Physics", vol. 83, 1989, p. 148–184.

[134] R. HARTL, S. SETHI, R. VICKSON. *A survey of the maximum principles for optimal control problems with state constraints*, in "SIAM Review", vol. 37, 1995, p. 181–218.

[135] M. HESTENES. *Multiplier and gradient methods*, in "Journal of Optimization Theory and Applications", vol. 4, 1969, p. 303-320.

[136] C. HU, C.-W. SHU. *A discontinuous Galerkin finite element method for Hamilton-Jacobi equations*, in "SIAM J. on Scientific Computing", vol. 21, n^{o} 2, 1999, p. 666–690 (electronic).

[137] C. HU, C.-W. SHU. *A discontinuous Galerkin finite element method for Hamilton-Jacobi equations*, in "SIAM J. on Scientific Computing", vol. 21, n^{o} 2, 1999, p. 666–690 (electronic).

[138] A. IOFFE, V. TIHOMIROV. *Theory of Extremal Problems*, Russian Edition: Nauka, Moscow, 1974, North-Holland Publishing Company, Amsterdam, 1979.

[139] T. JOCKENHÖVEL, L. T. BIEGLER, A. WÄCHTER. *Dynamic Optimization of the Tennessee Eastman Process Using the OptControlCentre*, in "Computers and Chemical Engineering", vol. 27, 2003, p. 1513–1531.

[140] P. KALL, S. WALLACE. *Stochastic Programming*, Wiley, Chichester, 1994.

[141] N. KARMARKAR. *A new polynomial time algorithm for linear programming*, in "Combinatorica", vol. 4, 1984, p. 373-395.

[142] D. KINDERLEHRER, G. STAMPACCHIA. *An introduction to variational inequalities and their applications*, Academic Press, New York, 1980.

[143] D. KRAFT. *Finite-difference gradients versus error-quadrature gradients in the solution of parameterized optimal control problems*, in "Optimal Control, Appl. and Methods", vol. 2, 1981, p. 191-199.

[144] N. KRYLOV. *On the rate of convergence of finite-difference approximations for Bellman's equations with variable coefficients*, in "Probability Theory and Related Fields", vol. 117, 2000, p. 1–16.

[145] H. KUSHNER, P. DUPUIS. *Numerical methods for stochastic control problems in continuous time*, Applications of mathematics, Second edition, vol. 24, Springer, New York, 2001.

[146] J.-L. LAGRANGE. *Mécanique analytique*, reprinted by J. Gabay, 1989, Paris, 1788.

[147] J. LAURENT-VARIN, N. BÉREND, J. BONNANS, C. TALBOT. *An efficient optimization method dealing with global RLV (ascent and descent) trajectories*, in "Proc. 56th International Astronautical Congress, Fukuoka October 17-21", 2005.

[148] E. LEE, L. MARKUS. *Foundations of optimal control theory*, John Wiley, New York, 1967.

[149] G. LEITMANN. *An introduction to optimal control*, Mc Graw Hill, New York, 1966.

[150] J. LIONS. *Contrôle optimal de systèmes gouvernés par des équations aux dérivées partielles*, Dunod, Paris, 1968.

[151] J. LIONS. *Perturbations singulières dans les problèmes aux limites et en contrôle optimal*, Lecture Notes in Mathematics, vol. 323, Springer-Verlag, Berlin, 1973.

[152] J. LIONS. *Contrôle des systèmes distribués singuliers*, Dunod, Paris, 1983.

[153] K. MALANOWSKI, C. BÜSKENS, H. MAURER. *Convergence of approximations to nonlinear optimal control problems*, in "Mathematical programming with data perturbations, New York", Lecture Notes in Pure and Appl. Math., vol. 195, Dekker, 1998, p. 253–284.

[154] H. MARKOWITZ. *The elimination form of the inverse and its application to linear programming*, in "Management Science. Journal of the Institute of Management Science. Application and Theory Series", vol. 3, 1957, p. 255–269.

[155] P. MARTINON, J. BONNANS, E. TRÉLAT. *Singular arcs in the generalized Goddard's Problem*, in "J. Optimization Theory Applications", To appear. Preprint: INRIA Report RR-6157, 2007.

[156] J. MORALES, J. NOCEDAL, R. WALTZ, G. LIU, J. GOUX. *Assessing the Potential of Interior Methods for Nonlinear Optimization*, Unpublished manuscript, 2001.

[157] R. MUNOS. *Policy gradient in continuous time*, in "J. Machine Learning Research", vol. 7, 2006, p. 771–791.

[158] A. NEMIROVSKI, A. SHAPIRO. *Convex approximations of chance constrained programs*, in "SIAM J. Optim.", vol. 17, n^o 4, 2006, p. 969–996 (electronic).

[159] Y. NESTEROV, J.-P. VIAL. *Confidence level solutions for stochastic programming*, Technical report, Department of Management Studies, University of Geneva, 2001.

[160] J. NOCEDAL, S. WRIGHT. *Numerical optimization*, Springer-Verlag, New York, 1999.

[161] S. OSHER, C.-W. SHU. *High essentially nonoscillatory schemes for Hamilton-Jacobi equations*, in "SIAM J. Numer. Anal.", vol. 28, n^o 4, 1991, p. 907-922.

[162] M. PEREIRA, L. M. PINTO. *Multi-stage stochastic optimization applied to energy planning*, in "Mathematical Programming", vol. 52, n^o 2, Ser. B, 1991, p. 359–375.

[163] H. J. PESCH. *A practical guide to the solution of real-life optimal control problems*, in "Control and Cybernetics", vol. 23, 1994, p. 7–60.

[164] N. PETIT, M. MILAM, R. MURRAY. *A new computational method for optimal control of a class of constrained systems governed by partial differential equations*, in "IFAC", 2002.

[165] H. PHAM. *Optimisation et Contrôle Stochastique Appliqués à la Finance*, n^o 61, Springer Verlag, 2007.

[166] L. PONTRYAGIN, V. BOLTYANSKI, R. GAMKRELIDZE, E. MICHTCHENKO. *The Mathematical Theory of Optimal Processes*, Wiley Interscience, New York, 1962.

[167] M. POWELL. *A method for nonlinear constraints in minimization problems*, in "Optimization", R. FLETCHER (editor), Academic Press, New York, 1969, p. 283-298.

[168] M. POWELL. *Algorithms for nonlinear constraints that use Lagrangian functions*, in "Mathematical Programming", vol. 14, 1978, p. 224-248.

[169] H. ROBBINS. *A generalized Legendre-Clebsch condition for the singular case of optimal control*, in "IBM J. of Research and Development", vol. 11, 1967, p. 361-372.

[170] R. ROCKAFELLAR, S. URYASEV, M. ZABARANKIN. *Optimality conditions in portfolio analysis with general deviation measures*, in "Mathematical Programming", vol. 108, n^o 2-3, Ser. B, 2006, p. 515–540.

[171] P. ROE. *Some contributions to the modelling of discontinuous flows*, in "Lectures in Applied Mathematics", vol. 22, 1985, p. 163–193.

[172] I. ROSS, F. FAHROO. *Pseudospectral knotting methods for solving optimal control problems*, in "AIAA Journal of Guidance, Control and Dynamics", vol. 27, 2004, p. 397–508.

[173] A. RUSZCZYŃSKI, A. SHAPIRO. *Conditional risk mapping*, in "Math. Oper. Res.", vol. 31, n° 3, 2006, p. 544–561.

[174] A. RÖSCH, F. TRÖLTZSCH. *Sufficient second-order optimality conditions for an elliptic optimal control problem with pointwise control-state constraints*, in "SIAM J. Optim.", vol. 17, n° 3, 2006, p. 776–794 (electronic).

[175] A. SHAPIRO, A. NEMIROVSKI. *On complexity of stochastic programming problems*, in "Continuous optimization, New York", Appl. Optim., vol. 99, Springer, 2005, p. 111–146.

[176] A. SHAPIRO. *Statistical inference of multistage stochastic programming problems*, Technical report, School of Industrial and Systems Engineering, Georgia Institute of Technology, 2002.

[177] L. TEBOU, E. ZUAZUA. *Uniform boundary stabilization of the finite difference space discretization of the 1-d wave equation*, in "Advances in Computational Mathematics", vol. 26, 2007, p. 337-365.

[178] J. THÉNIÉ, J.-P. VIAL. *Step decision rules for multistage stochastic programming : a heuristic approach*, Technical report, HEC Genève, Logilab, HEC Genève, Logilab, 2006.

[179] J.-P. VIAL, R. APPARIGLIATO. *Optimisation Robuste Linéaire - Une introduction succinte*, Technical report, EDF R&D, Dept OSIRIS, H-R32-2006-04801-FR, 1 av. Général de Gaulle, F-92141 Clamart, 2007.

[180] A. WAECHTER, L. BIEGLER. *On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming*, in "Mathematical Programming Series A", vol. 106, 2006, p. 25-57.

[181] L. YOUNG. *Lectures on the calculus of variations and optimal control theory*, W. B. Saunders Co., Philadelphia, 1969.

[182] E. ZUAZUA. *Optimal and approximate control of finite-difference approximation schemes for the 1D wave equation*, in "Rend. Mat. Appl. (7)", vol. 24, n° 2, 2004, p. 201–237.

[183] B. ØKSENDAL. *Stochastic differential equations*, Universitext, Sixth, Springer-Verlag, Berlin, 2003.