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Project-Team Coprin

Constraints solving, OPtimization, Robust INterval analysis

Sophia Antipolis - Méditerranée

THEME SYM

Activity Report

2007

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1. Team

COPRIN is a joint project between Certis (École des Ponts et Chaussées) and INRIA.

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2. Overall Objectives

2.1. Overall Objectives

Keywords: *constraints programming*, *interval analysis*, *mechanism theory*, *multi-precision*, *numerical robustness*, *optimal design*, *robotics*, *symbolic-numerical calculation*, *systems solving*, *uncertainty*.

COPRIN is a joint project between Certis (École des Ponts et Chaussées) and INRIA.

Its scientific objective is to develop and implement systems solving algorithms based on constraints propagation methods, interval analysis and symbolic computation, with interval arithmetic as the primary tool. The academic goals of these algorithms is to provide *certified solutions* to generic problems (e.g. to calculate all solutions of a system of equations within a search space) or to manage the *uncertainties* of the problems (e.g. to provide an enclosure of all solutions of a system of equations whose coefficients are intervals). These academic goals may also be declined in applicative goals. For example we may determine a domain that describes all possible dimensions of a mechanism that has to satisfy a set of performance requirements. Being given this domain it will be possible to determine nominal dimensions for the mechanism so that even if there are bounded variation between the real dimensions and the nominal one, then the real mechanism will still satisfy the requirements: hence we will be able to manage manufacturing uncertainties for the real process.

Our research aims to develop algorithms that can be used for any problem or are specific to a given class of problem, especially problems that are issued from application domains for which we have an internal expertise (such as mechanism theory and robotics).

A key point of these algorithms is that they rely heavily on symbolic pre-processing and formal calculation in order to improve the efficiency of the problem at hand. Our long term goal is to be able to synthesize automatically a specific solver according to the structure of the problem that has to be managed.

Implementation of the algorithms will be performed within the framework of general purpose software such as `Scilab`, `Maple`, `Mathematica` and will be based on the already existing libraries `ICOS` and `ALIAS`, that are still being developed mostly for internal use.

As a theoretical complexity analysis of interval analysis based solving algorithms is usually extremely difficult, the efficiency of the algorithm are systematically experimentally evaluated through `IcosAlias` or `ALIAS` on various realistic test examples.

Dissemination is also an essential component of our activity as interval analysis based methods are not sufficiently known in the engineering and academic communities.

The study of robotics problems is a focus point of the COPRIN project and will become more so in the future. In this field our objectives are to study and synthesize innovative robotic systems, taking into account the uncertainties that are inherent to such mechatronic device. Experimental work and the development of our own prototypes are strategic for the project as they allow us to validate our theoretical work and to discover new problems that will feed on the long term the theoretical analysis developed by the team members.

2.2. Highlights of the year

As highlights of this year we will insist on our dissemination work:

- in interval analysis: the first release of the `Int4Sci` interval analysis interface for `Scilab`
- the organization of the 12th IFToMM World Congress with over 900 attendees

3. Scientific Foundations

3.1. Scientific Foundations

3.1.1. Interval analysis

We are interested in real-valued system solving ($f(X) = 0$, $f(X) \leq 0$), in optimization problems, and in the proof of the existence of properties (for example it exists X such that $f(X) = 0$ or it exists two values X_1, X_2 such that $f(X_1) > 0$ and $f(X_2) < 0$). There are few restrictions on the f we can deal with as we are able to manage explicit functions using classical mathematical operators (e.g. $\sin(x + y) + \log(\cos(e^x) + y^2)$ or implicit functions (e.g. determining if there are parameter values of a parametrized matrix such that the determinant of the matrix is negative, without calculating the analytical form of the determinant).

Solutions will be searched within a finite domain (called a *box*) which may be either continuous or mixed (i.e. for which some variables must belong to a continuous range while other variables may only have values within a discrete set). An important point is that we aim to find all the solutions within the domain as soon as the computer arithmetic will allow it: in other words we are looking for *certified* solutions. For example, for 0-dimensional system solving, we will provide a box that includes one, and only one, solution together with a numerical approximation of this solution, that may further be refined at will using multi-precision.

The kernel of our methods is the use of *interval analysis* that allows one to manipulate expression whose unknowns have interval values. A basic component of interval analysis is the *interval evaluation* of an expression. Being given an analytical expression F in the unknowns $\{x_1, x_2, ..., x_n\}$ and ranges $\{X_1, X_2, ..., X_n\}$ for these unknowns we are able to compute a range $[A, B]$, called the interval evaluation, such that

$$\forall\{x_1, x_2, ..., x_n\} \in \{X_1, X_2, ..., X_n\} A \leq F(x_1, x_2, ..., x_n) \leq B \tag{1}$$

In other words the interval evaluation provide a lower bound for the minimum of F and an upper bound of its maximum over the box.

For example if $F = x\ sin(x + x^2)$ and $x \in [0.5, 1.6]$, then $F([0.5, 1.6]) = [-1.362037441, 1.6]$, meaning that for any x in [0.5,0.6] we guarantee that $-1.362037441 \leq f(x) \leq 1.6$.

The interval evaluation of an expression has interesting properties:

- it can be implemented in such way that the results are guaranteed with respect to round-off errors i.e. in spite of numerical errors induced by the use of floating point numbers property 1 is still valid
- if $A > 0$ or $B < 0$, then there are now values of the unknowns in their respective ranges that may cancel F
- if $A > 0$ ($B < 0$), then F is positive (negative) for any values of the unknowns in their range

But there is a major drawback of the interval evaluation: there may be an overestimation of $A(B)$ i.e. there may be no value of $x_1, x_2, ..., x_n$ such that $F(x_1, x_2, ..., x_n) = A(B)$. This overestimation occurs because in our calculation each occurrence of a variable is considered as an independent variable and consequently if a variable have multiple occurrences, then an overestimation may occur. Such phenomena can be observed in the previous example where $B = 1.6$ while the real maximum of F is approximately 0.9144. The value of B is obtained as we are using in our calculation the formula $F = xsin(y + z^2)$ with y, z having the same interval value than x.

Fortunately there are methods that allow one to reduce the overestimation and this amount decreases with the width of the ranges. The later remark leads to the use of a branch-and-bound strategy in which for a given box a variable range will be bisected, thereby creating two new boxes that will be stored in a list and processed later on. The algorithm will be completed if all boxes in the list have been processed or if during the process a box generates an answer to the problem at hand (e.g. if we want to prove that $F(X) < 0$, then the algorithm stops as soon it is shown that for a box $\mathcal{B}$ we have $F(\mathcal{B}) \geq 0$).

A generic interval analysis algorithm applies the following steps in sequence on the current box:

1. *exclusion operators*: these operators determine that there is no solution to the problem within a given box
2. *filters*: these operators may reduce the size of the box i.e. decrease the width of the allowed ranges for the variables
3. *existence operators*: they allow one to determine that there is a unique solution within a given box and are usually associated to a numerical scheme that enable to compute this solution in a safe way [20]
4. *bisection*: choose one of the variable and bisect its range for creating two new boxes
5. *storage*: store the new boxes in the list

The scope of the COPRIN project is to address all these steps in order to find the most efficient procedures. Our efforts focuses on mathematical developments (adapting classical theorems to interval analysis, proving interval analysis theorems), on the use of symbolic computation and formal proof (a symbolic pre-processing allows one to automatically adapt the solver to the structure of the problem), on software implementation and on experimental tests (for validation purposes).

3.1.2. Robotics

COPRIN has a long-standing tradition of robotics studies, especially for closed-loop robots. We address first theoretical issues with the purpose of obtaining analytical and theoretical solutions but in many cases only numerical solutions can be considered because of the complexity of the problem. This approach has motivated the use of interval analysis for two reasons:

1. the versatility of interval analysis allows us to address issues that cannot be tackled by any other methods (e.g. singularity analysis)
2. we want to take into account uncertainties (which are inherent to a robotic device) so that we can guarantee that the performance level of the *real* robot will satisfy the same properties than the *theoretical* one, even in the worst case. This is a crucial issue for many applications in robotics (e.g. medical robot [24], [21])

Our field of study in robotics focus on *kinematic* issues such as workspace and singularity analysis, positioning accuracy, trajectory planning [26] and prominently *appropriate design*, i.e. finding the dimensioning of a robot mechanical architecture that guarantee that the real robot will satisfy a given list of requirements.

Our theoretical work must be validated through experiments that are essential for our credibility. A contrario, experiments will feed the theoretical work (quite often COPRIN was the first robotic group to address some theoretical issues that were put into evidence by experiments). Hence COPRIN is working with partners for the development of real robots but also is developing its own prototype (approximately one every 6 years).

In term of application we have focused to now on the development of special machines (machine-tool, ultra-high accuracy positioning device, spatial telescope). Although this activity will be pursued we intend to move toward *service robotics* i.e. robots being closer to human activity. In service robotic we are interested in domotics [28], rehabilitation and medical robots and entertainment. Compared to special machines for which price is not an issue (up to a certain point), cost is an important element for service robotics. Although we plan to develop some simple robotic systems our work focus on a different issue: the management of the robot *modularity*. The mechanical modularity of a robot is obtained by allowing one to change the arrangement of the robot's elements (whose cost may be quite low) so that it is the most appropriate to the task. Many such mechanically modular robots are available (or can be designed at will) but finding the right arrangement of the hardware to fulfill the task requirements in spite of mechanical and control uncertainties is an open problem with no known algorithmic solution and developing such algorithms is our long term aim.

4. Application Domains

4.1. Applications Domains

Keywords: *geometric constraints*, *mechanism theory*, *optimal design*, *robotics*.

While the methods developed in the project may be used for a very large set of application domains (for example we have an activity in control theory and in quantum mechanics), it is clear that the size of the project does not allow all of them to be addressed. Hence we have decided to focus our applicative activities on *mechanism theory* (including *robotics* and especially *service robotics*). In this domain our research focuses on *optimal design* and geometrical modeling of mechanisms, especially for the machine-tool industry, automotive suspensions, virtual reality and medical robotics, which all involve the management of *geometric constraints*. As other domains may exhibit equivalent problems as mechanism theory (e.g. molecular chemistry), they may also be addressed, without constituting a major research axis of the project.

5. Software

5.1. Introduction

Software is an essential part of the research within COPRIN as our researches may be only experimentally validated. Software developments are addressed along various axes:

1. interval arithmetic: although our purpose is not to work in this very specialized area (we generally rely on existing packages) interval arithmetic is an important part of our interval analysis algorithms and we may have to extend the existing packages especially for dealing with multi-precision and arithmetic extensions
2. interval analysis libraries: we daily use two libraries that have been designed in the project and are still under development. A long term work is to develop a generic programming framework that allows for modularity and flexibility, with the objective of being able to test easily new functionalities and to build specific solvers by a simple juxtaposition of existing modules
3. interface to interval analysis: in our opinion interval analysis software must be available within general purpose scientific software (such as `Maple`, `Mathematica`, `Scilab`) and not only as a stand-alone tool. Indeed most end-users will be reluctant to learn a new programming language just to deal with solving problems that are only small elements of a more general problem context. Furthermore interval analysis efficiency may benefit from the functionalities available in the general purpose scientific software.

5.2. Interval analysis libraries

5.2.1. ALIAS

Participants: Jean-Pierre Merlet [correspondant], David Daney, Odile Pourtallier.

The `ALIAS` library (*Algorithms Library of Interval Analysis for Systems*), whose development has started in 1998, is a collection of procedures based on interval analysis for systems solving and optimization.

`ALIAS` is constituted of two parts:

- `ALIAS-C++`: the C++ library (86 000 code lines) which is the core of the algorithms
- `ALIAS-Maple`: the `Maple` interface for `ALIAS-C++` (50 000 code lines). This interface allows one to specify a solving problem within `Maple` and to get the results within the same `Maple` session. The role of this interface is not only to generate automatically the C++ code, but also to perform an analysis of the problem in order to improve the efficiency of the solver. Furthermore, a distributed implementation of the algorithms is available directly within the interface.

In 2007 we have implemented in `ALIAS` two existence operators based on Rouché and Miranda theorems. A filtering operator for equation systems based on the interval Newton method has been implemented: the usual method involves the calculation of $\mathbf{W} = \mathbf{J_0^{-1}} \mathbf{J}(\mathbf{X})$, where $\mathbf{J}(\mathbf{X})$ is the interval evaluation of the Jacobian matrix for the range $\mathbf{X}$ and J_0^{-1} a scalar matrix that is calculated as the inverse of $\mathbf{J}$ computed for the mid-point of $\mathbf{X}$. A drawback of such calculation if that if there are multiple occurrences of the unknowns in $\mathbf{J}$, then there is large overestimation of the elements of $\mathbf{W}$. In the new operator we assume the existence of a procedure that computes the analytical form of $\mathbf{W}$ while trying to reduce the number of multiple occurrences of the variables (such a procedure is provided in `ALIAS-Maple`), thereby allowing us to reduce the overestimation of $\mathbf{W}$ and consequently improving the efficiency of the filtering. To improve the interval evaluation of expressions we have also developed an interval evaluation scheme based on a bicentered form of the expressions.

For polynomials we have implemented a filtering operator based on Weyl theorem, an interval evaluation operator based on a centered form that may allow us to decrease the width of the interval evaluation. For univariate polynomial we have developed a deflation scheme that uses the knowledge of approximate roots to derive a polynomial of degree less than the original one which has the same roots than the remaining roots of the original polynomial.

For ALIAS-Maple our main work has been to modify the source code so that to be compatible with Maple version 9.5 and higher. We have also implemented an interface to all modules that have been implemented in ALIAS-C++.

5.2.2. Int4Sci : a Scilab interface for interval analysis

Keywords: *Scilab*.

Participants: Raphaël Pereira, David Daney [correspondant].

We have decided since 2006 to offer an interface, called Int4Sci, to interval analysis for the scientific software Scilab with as key elements the C++ Bias/Profil interval arithmetic package and the library ALIAS. The first step of this project was to determine the interface method, the Scilab structure for intervals and the installation protocol for Linux. Based on this initial work, a first version of Int4Sci allowing the use of classical interval arithmetic and its extension to infinite values has been implemented. This year we have added several basic interval functionalities such as evaluation of elementary functions, several interval operations and manipulations of intervals and solving of linear interval systems, while working on a full version of the documentation, including an on-line help format and a tutorial. This preliminary version has been tested by COPRIN team members and external collaborators, and the interface has been modified to take into account their feedback. Another major work was to implement a Windows version of the interface by porting Bias/Profil and the ALIAS libraries in Microsoft Visual C/C++.

We plan to fully release this first version of Int4Sci at the end of 2007 after we have dealt with diffusion issues (web site, license, documentation reviewing, installation tests,...).

The next step of this development will be to add functionalities regarding interval polynomials (i.e. polynomials whose coefficients are intervals).

5.2.3. Mathematica Interface to Interval Analysis

Participants: Yves Papegay [correspondant], Jean-Pierre Merlet, David Daney.

In 2006, we have implemented an interface to the ALIAS library for Mathematica, with the aim of providing to the community of Mathematica user a transparent access to the functionalities of ALIAS and of extending the dissemination of our library.

This interface has been extended into an high-level modular interface to ALIAS for prototyping, and testing easily and quickly new combinations of interval analysis algorithms and procedures, with the benefit of using a computational environment that allows for symbolic computation and problem pre-processing [33].

Current investigations addresses symbolic pre-processing of expressions, and symbolic specializations of interval analysis algorithms, especially the ones involving matrices.

6. New Results

6.1. Robotics and mechanism theory

6.1.1. Modeling human postural coordination to improve the control of balance in humanoids

Keywords: *humanoid robot*.

Participant: Nacim Ramdani.

As part of a collaboration with LIRMM and the EDM group (Univ. Montpellier 1), we have addressed the issue of modeling human postural coordination in order to improve the control of balance in humanoid robots.

Recent data in the field of postural coordination shows the existence of self-organized postural states, and transition between them, underlying supra-postural tracking movements. A biomechanical model, capitalizing on stability and optimization criteria, has been introduced which captures the complex postural behaviors observed in humans and can be used to implement efficient balance control principles in humanoids robots. Experimental results on the LIRMM HOAP-3 and the JRL HRP-2 humanoid robots show the relevance of this work [13].

6.1.2. Guaranteed computation of constraints for safe path planning with humanoid robots

Keywords: *humanoid robot*.

Participant: Nacim Ramdani.

Path planning issues are often solved via constrained optimization methods but with constraints which must be satisfied over a whole interval of time or space. The use of fast numerical toolboxes implementing state-of-the-art constrained optimization methods needs to discretize the continuous constraints over a time or space grid. With this approach the obtained solution will satisfy the constraints only for time values corresponding to the time grid. Obviously, some constraints could be violated with catastrophic consequences when dealing with, for instance, the balance of humanoid robots. We have introduced, in a joint work with the DEMAR project-team, a guaranteed discretization method which uses interval analysis to ensure that the constraints are satisfied over the whole time interval. We have numerically analyzed this method by performing a trajectory generation under constraints dedicated to the motion of the HOAP-3 humanoid robot [23].

6.1.3. Design of physiotherapy robot used for ankle rehabilitation

Keywords: *physiotherapy*, *rehabilitation*, *robot design*.

Participants: David Daney [correspondant], Jean-Pierre Merlet, Ioannis Emiris, Christos Syrseloudis.

In collaboration with the University of Athens and the Greek company `Reflexion` COPRIN participates in the development of a new robot to assist a physiotherapist in ankle rehabilitation procedure.

The first step of the collaboration has been to define the type and nature of manipulation that a physiotherapist applies on a patient for rehabilitation. Based on professional interviews, a patient adapted specifications have been proposed and then defined in robotics terms. The second part consists of developing a parametric model of ankle which covers different types of patent morphology. Finally, based on these results, the synthesis of the robot and its architecture were selected. Future work will be to carry out the dimensional design of the robot by using the appropriate design methodologies developed by COPRIN.

Christos Syrseloudis, a PhD student of the University of Athens and a Reflexion engineer, has visited COPRIN in October 2007.

6.1.4. Study of interval representation of serial chain mechanisms

Keywords: *kinematics*, *serial robot*.

Participants: Denny Oetomo, David Daney [correspondant], Jean-Pierre Merlet.

In the field of kinematic studies, interval analysis has been less used for serial robots than for parallel ones, in spite of the high versatility of this methodology. This may be explained by the following major drawback of interval analysis: multiple occurrences of an interval variable in a given expression may lead to a large over-estimate of the interval evaluation of the expression. It just happens that the usual modeling methods of serial robot, where transformations between subsequent links often require redundant representations, lead to expressions involving multiple occurrences of interval variables. We have explored various modeling representations for determining the most suitable one for interval analysis and an application of a 6DOF serial manipulator was performed as an example. The results from this study can significantly contribute to the design of modular robot as stated in the European project `ARES`.

6.1.5. ARES: Assembly of Reconfigurable Endoluminal Surgical System

Keywords: *reconfigurable robot*, *surgical robot*.

Participants: Denny Oetomo, David Daney, Jean-Pierre Merlet [correspondant].

In collaboration with Scuola Superiore Sant'Anna (SSSA) Pisa, ETH Zurich, UC Barcelone, COPRIN participates in the European-Union funded project ARES to establish the first working prototype/proof of concept study of modular robots, swallowed through capsules, that would re-assemble itself into a workable surgical mechanism within the stomach cavity. The deliverables for COPRIN are in the configuration planner of the reconfigurable robots. The problem is to determine a set of possible mechanism topologies that would be guaranteed to satisfy given performance criteria, such as achievable workspace, required forces at the end-effector, in spite of intrinsic uncertainties such as manufacturing tolerances in the modules and clearances in the docking system between the modules. A user interface was completed to graphically display the results of the planner algorithm in relation to the working environment (a model of the human stomach) and to assist in the debugging process of the algorithm.

6.1.6. Singularity of parallel robots

Keywords: *parallel robot*, *singularity*.

Participants: Julien Hubert, Jean-Pierre Merlet [correspondant].

Singularity is an old issue for parallel robots and the COPRIN project is a leading team on this subject. We have proposed last year an interval analysis based algorithm that allows one to detect singularity within a given workspace, even if there are uncertainties in the modeling of the robot [11].

Still remain open issues on this subject:

- singularity should be avoided in general as in the vicinity of such pose the joint forces of the robot may go to infinity, causing a breakdown of the robot. But such situation will not occur for all poses at which the Jacobian matrix loses rank or for any type of external wrench applied on the end-effector. A better knowledge of the "dangerous" singularities is necessary
- the workspace of the robot derived from kinematic constraints such as limited motion of joints may further be reduced if the workspace is separated by singularity regions that cannot be crossed. Hence the influence of singularity on the useful workspace should be clarified
- singularities may also be classified according to the type of infinitesimal motion that occur at such poses but such classification is not determined
- can we classify the singularity not only as function of the pose parameters but also as a function of the robot design parameters ?

All these issues will be addressed within the ANR SIROPA project[1] that we are leading.

6.1.7. Prototype of wire-driven robot

Keywords: *wire-driven parallel robot*.

Participants: Nicolas Chleq, David Daney, Jean-Pierre Merlet [correspondant].

The hardware of this new robot has been completed and planar motions of the robot have been tested. We remind that this robot will feature an original design [25][2], a high mechanical modularity (the location of the wire system may be changed at will and various maximal length changes will be possible) and a very high speed.

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[1] wiki-sop.inria.fr/wiki/bin/view/Coprin/SIROPA
[2] see wiki-sop.inria.fr/wiki/bin/view/Coprin/ROBPACALR for a picture of the robot

Unfortunately, our moving in the new robotic hall combined with unexpected failures in the design of the hall have considerably delayed our experimental work as we have not been able to fully use the robot since July. Still

- the robot motion appear to be satisfactory: laser based measuring device will be used to provide more accurate data on positioning accuracy and trajectory analysis
- the elasticity of the wire seems to play an important role although the literature is very sparse on this subject . We have already implemented a forward kinematics scheme that takes into account this elasticity, assuming that the wires follow Hooke's deformation law. This scheme allows one to compute all solutions of the forward kinematic (currently in about 30s which is quite good as this problem is even more complicated than the forward kinematic of a parallel robot with rigid links) or in real-time if information on the solution is available. A force measurement device will be used to better identify the elastic behavior of the wire. The role of wire elasticity is a joint work with LIRMM within the `RobPacaLr` INRIA Color (see section 8.1.1).

Potential applications of this robot will be domotics (windows washing), entertainment (video game, camera control for movies), virtual reality (haptic device with a large workspace) and medical robotics (e.g. rehabilitation within the 3+3 Med action, see section 8.1.2) to name a few.

6.1.8. Design for workspace of wire-driven robot

Keywords: *robot design*, *wire-driven parallel robot*, *workspace*.

Participants: Marc Gouttefarde, David Daney [correspondant], Jean-Pierre Merlet.

One of the most important issues in wire-driven robot is its wrench-closure workspace, i.e. the set of end-effector poses at which all wrenches in a set (defined by intervals for each wrench component) can be exerted by the end-effector, being given ranges for the wire tensions. In 2006 we have proposed an algorithm for calculating this workspace for a given robot geometry and in 2007 we have extended it to determine all geometries of planar wire-driven robots that have a given wrench-closure workspace [19].

6.2. Algebraic systems and linear algebra

6.2.1. Zero set of uncertain polynomials

Keywords: *uncertain polynomials*.

Participants: David Daney, Jean Pierre Merlet, Odile Pourtallier [correspondant].

The effort for computing the zeros of interval polynomials has been pursued.

We have based our approach on the study of extremal polynomials. The Bartlett's et.al. Edge theorem shows the equivalence of studying a polynomial with n uncertain coefficients to the studying of some polynomials with only one uncertain coefficient, with the drawback that the number of such polynomials exponentially increases with n. We have shown that the number of such polynomials may be further reduced, leading to some methods that remain tractable for reasonable value of n. We have then focused our attention to the solving of polynomials with only one uncertain coefficient by using several exclusion tests (Dedieu and Yakoubsohn, Weyl, Rouché), that have allowed us to significantly reduce the computation time [15].

We have also implemented a deflation method for uncertain or badly conditioned polynomials. Combined with Weyl and Rouché exclusion tests it has proven to be very efficient: for example we have been able to provide certified solutions for Wilkinson polynomial up to degree 19 by using only the `double` arithmetic of C++, while classical numerical algorithms fail to find correct solutions starting at degree 12.

6.2.2. Box approximation of the set of complex eigenvalues of a parametrized matrix

Keywords: *eigenvalues*, *eigenvectors*, *interval matrix*, *linear algebra*.

Participants: David Daney [correspondant], Odile Pourtallier, Arnold Neumaier, Alexandre Goldsztejn.

Being given a system whose behavior is described by a set of implicit equations, interesting information about the properties of the system is provided by the eigenvalues of the Jacobian matrix of equations system.

However, in many cases, this matrix is a function of parameters that may describe, for example, the state of the system or its geometry (e.g. in robotics the Jacobian matrix of the robot is a function of the pose parameters of its end-effector). An interesting problem is to determine the bounds for the eigenvalues of the Jacobian matrix, when ranges for the Jacobian parameters are given (for example to ensure that a given property is satisfied whatever are the values of some uncertain, but bounded, parameters).

Hence we consider a parametrized matrix and we aim at determining an enclosure of all possible complex eigenvalues over given ranges for the parameters. If we calculate the interval evaluation of each component of the matrix we get an interval matrix I_J (i.e. a set of matrices) and the set of matrices J obtained by instantiating each parameter in its range is included in the interval matrix. If a bisection process is used the union K_J of the interval matrices that will be considered in the process will be, in general, such that $J \subseteq K_J \subseteq I_J$.

In 2006, our efforts were concentrated on the calculation of extremal values of real eigenvalues by using interval polynomial solvers and/or regularity test theorems. This year we have extended this algorithm to get the set of all complex eigenvalues, this set being described by a list of two dimensional boxes. Such algorithm may be used, for example, to verify a property on the norm of the eigenvalues of the Jacobian matrix of a robot over a given set of poses for the end-effector or for a set of possible robot dimensions.

A key point of the algorithm is to determine an efficient inner test i.e. a test that allows us to prove that each point of a 2D box in the complex plane is an eigenvalue of one of the matrices in K_J. Although we have started to combine all known algorithms in this field we plan to improve them, especially with the use of symbolic computation, and to illustrate the algorithm capabilities on robotic applications.

6.2.3. Extension of the Hansen-Bliek method and modal intervals theory

Participants: Gilles Chabert, Gilles Trombettoni, Bertrand Neveu.

We have proposed an extension of the Hansen-Bliek method which computes an optimal outer approximation of the solution set of interval linear systems [10]. This extension allows more freedom in the choice of the quantifiers (existential or universal) associated to the coefficients, thus handling a wider variety of problems. A generalization of the LU decomposition based on Kaucher's interval arithmetic has been also given.

We also proposed in [8] a new formulation of the modal intervals theory, with the underlying concept of quantified range, a natural generalization of the range of a function. This new approach allows us to introduce Kaucher's arithmetic with a vivid meaning, and not only as an abstract algebraic extension of the classical interval arithmetic.

6.3. Interval Constraint Programming

6.3.1. Constructive Interval Disjunction

Keywords: *filtering operator*.

Participants: Gilles Trombettoni [correspondant], Gilles Chabert, Pierre Repetto.

We have designed two new filtering operators reducing the search space of systems of (in)equations, based on *constructive disjunction*, as well as a new splitting heuristic [36], [35]. The fist operator (`CID`) is a generic algorithm enforcing constructive disjunction with intervals. The second one (`3BCID`) is a hybrid algorithm mixing constructive disjunction and *shaving*, another technique already used through the algorithm `3B`. Finally, the splitting strategy learns from the CID filtering step the next variable to be split, with no overhead.

Experiments have been conducted on 20 benchmarks and have shown that in most cases `CID` and `3BCID` produce a significant gain in performance of orders over standard strategies. `CID` compares advantageously to the `3B` operator while being simpler to implement. Experiments suggest to fix the CID-related parameter in `3BCID`, offering thus to the user a promising variant of `3B`.

6.3.2. Filtering techniques for interval solvers

Keywords: *constraint propagation*, *domain filtering*, *interval solver*.

Participants: Ignacio Araya, Bertrand Neveu, Gilles Trombettoni [correspondant].

Ignacio Araya is beginning this year a PhD Thesis on filtering techniques for interval solvers. He is studying more particularly which are the best equation forms for improving the performance of consistency propagation algorithms in interval-based solvers. Indeed, adding redundant equations, replacing common expressions by variables can be very fruitful and impressive performance gains can be obtained. For this purpose, he uses symbolic computation tools like Mathematica to handle the equations before solving them with the Ibex solver.

6.3.3. Inter-Block Backtracking

Participants: Bertrand Neveu [correspondant], Gilles Trombettoni, Gilles Chabert.

Based on any obtained decomposition of a system of equations into several subsystems, we have designed an algorithm, called Inter-Block Backtracking (`IBB`), that can construct a total solution by mixing the partial solutions found in the subsystems by an interval-based solver. This year, we have improved `IBB` in several ways:

- `IBB` performs two types of filtering: intra and inter-block. The implementation of the latter is now more incremental, which lowers the associated overhead.
- we have better analyzed which filtering algorithms are well-suited for `IBB` and the interest of 3B-consistency.

A further improvement will be to deliver a unique version of `IBB` including one type of (inter-block) backtracking, one intra-block filtering, and one inter-block filtering.

6.3.4. Distance Equations with Uncertainties

Keywords: *distance constraint*, *uncertainty*.

Participants: Carlos Grandon „ Bertrand Neveu [correspondant], Gilles Chabert.

Carlos Grandon has in his PhD thesis [9] studied systems of distance equations with uncertainties. Uncertainties mean values which are not exactly determined but are bounded by well-known limits. These values are represented as intervals, and frequently come from measurements. The solutions of this problem are no more isolated points but one or several continua of solutions. In a first phase, we proposed a special Branch and Prune algorithm with conditional bisection for computing a rough approximation of each continuum of solutions. A rough approximation (a box) is not enough in all the cases, thus a sharp approximation (a set of boxes) describing continuous solution sets is often needed. We show that this approximation must consider an inner box test in order to detect big parts of the space containing only solutions to the problem. Using inner box tests not only reduces the number of generated boxes but also provides more information about different zones of the space. We proposed and compared some inner box tests for distance equations with uncertainties. When a single solution point belonging to a continuum of solutions is given, an inner box around this point and totally contained inside the continuum of solutions may be very interesting for tolerance issues. For this reason we proposed a strategy for building such a box based on theoretical results of Modal Interval Analysis combined with a well-known technique of Constraint Programming called projection [20]. Finally, the developed techniques have been illustrated in a problem of Robotics in which we solve the direct kinematics of a special class of parallel robot [21].

6.4. Miscellaneous results

6.4.1. Interval uncertainties in game theory

Keywords: *Nash equilibrium*, *Stackelberg equilibrium*, *game theory*.

Participants: Yves Papegay, Odile Pourtallier [correspondant].

Game theory is often used to model decision problems in which the parameters are only known with high uncertainty. This is in particular due to the need of a high level of aggregation in order to obtain tractable models. On the other hand, the classical notions of solutions such as Nash or Stackelberg equilibria are known to be rather unstable with respect to uncertainty. We have started to adapt interval analysis techniques to develop numerical algorithm to compute set of equilibria for uncertain model.

6.4.2. Guaranteed parameters identification in presence of interval uncertainty

Keywords: *parameter identification*.

Participant: Nacim Ramdani.

A crucial problem that occurs when estimating physical parameters from experimental data is the computation of reliable uncertainty bounds for the estimated parameters, while taking into account uncertainty in the model and in the data. When all uncertain quantities are assumed to be bounded unknowns, we have shown that this problem can usually be solved in a guaranteed way by using a set projection algorithm based on interval analysis and constraint propagation, provided that the number of parameters and their uncertainty ranges are not too large.

It is also important to account for any timing errors which is often the case when one deals with pharmacokinetics modeling of drug delivery into living cells: here again, the set projection algorithm may also be used for addressing this issue[12].

6.4.3. Local search for 2D packing problems

Participants: Bertrand Neveu [correspondant], Gilles Trombettoni, Ignacio Araya.

The 2D strip packing problem is a variant of the well-known combinatorial bin packing problem. It consists in placing predefined rectangles in a 2D strip such that no two rectangles overlap, while minimizing the height of the strip.

We have selected this challenging problem to work in collaboration with Maria-Cristina Riff and Xavier Bonnaire from the University of Santa Maria in Valparaiso (Chile) with the financial support from INRIA and CONICYT (Chile). Two incomplete methods have been developed to handle this combinatorial problem.

6.4.3.1. Incremental move for strip-packing

When handling 2D packing problems, numerous incomplete and complete algorithms maintain a so-called bottom-left (BL) property: every rectangle placed in a container cannot be moved left or bottom with the drawback that it is more expensive to maintain all the placed pieces BL when a rectangle is removed.

We have investigated the possibility of violating the BL property. Instead, we propose to maintain the set of “maximal holes”, which allows incremental additions and removals of rectangles with an algorithm that does not require to set any parameter [29], [30], [31].

Experimental results show that the approach is competitive with the best known incomplete algorithms, especially the other metaheuristics (able to escape from local minima).

6.4.3.2. A hyperheuristic for strip-packing

We have developed a second solving method based on hyperheuristics. The hyperheuristic manages a sequence of greedy low-level heuristics, as BLF (Bottom left Fill) or BF (Best Fit), each element of the sequence placing a given number of rectangles. A solution is built by placing the rectangles following the sequence of low-level heuristics. The hyperheuristic performs a hill-climbing algorithm on this sequence by testing different moves (adding, removing, replacing a low-level heuristic).

The first results we obtained on strip packing benchmarksare very competitive with the best approaches.

6.4.4. Symbolic tools for modeling and simulation

Keywords: *accuracy*, *code generation*, *modeling*, *reliability*, *simulation*, *symbolic computation*.

Participant: Yves Papegay.

This activity is the main part of a long-term ongoing collaboration with Airbus whose goal is to directly translate the work of aeronautics engineers into digital simulators to accelerate aircraft design. This project already has applications in the aircraft maker development departments.

Modeling and simulation processes usually begin with using scientific theories for describing physical features with formulae and computation algorithms. Based on these models, numerical codes are then implemented for simulation and visualization of these features. In an industrial context, the large number of parameters and equations involved in models make the whole process very long, complex and expensive, especially as reliable and safe codes are required.

First stage of our work led to a model edition environment, based on symbolic computation tools, that makes it possible to enter the formulae and the algorithms of the models and to validate them numerically on a reduced set of data.

In a second stage, we have addressed the problem of using these models to automatically generate the numerical real-time simulation engines to be plugged in the flight simulator, as well as the technical documentation associated with such simulations, which is indispensable for corporate memory. At the end of 2006, the initial prototype of C code generator for real time simulation was turned into a really powerful, robust and complete tool, extensively tested, accepting a wide class of models as inputs, and a lot of constraints on the generated C code.

In 2007 we focused on the automatic identification of the status parameters of a model and on the automatic initialization of their values for a simulation. We also extended capabilities of the code generator to compute derivatives of the model and hence to perform sensibility analysis.

Our long term objective is to be able to introduce uncertainties in the parameters for obtaining reliable results that may be used for the optimal design of the system.

7. Contracts and Grants with Industry

7.1. Airbus France

Participant: Yves Papegay.

To improve the production of numerical flight simulators from aerodynamics models, Airbus France is interested in methods and tools like those described in section 6.4.4.

In 2007 a one-year long contract succeeded to those signed in 2003 and in 2005 concerning code generation and the impact on sensitivity analysis. For confidentiality reasons, no further details can be given here.

7.2. Amadeus

Participants: Bertrand Neveu, Gilles Trombettoni.

Amadeus is a company that manages flight fares for several airlines with which we have a long-standing collaboration to develop new optimization algorithms based on constraint programming, graph methods for fare quote problems and development of the test suite that is used for software evaluation by Amadeus.

In 2007, we have signed a contract for the development of a software proposing the best prices for an imprecise request that can include hotel and car reservations.

8. Other Grants and Activities

8.1. International and National initiatives

8.1.1. RobPacaLr COLOR

Participants: Jean-Pierre Merlet [correspondant], David Daney, Nacim Ramdani.

A new cable-driven robot has been developed at LIRMM by replacing the rods of an existing 4 d.o.f. parallel prototype by steel cables for the purpose of developing a parallel kinematics crane. Although the COPRIN wire-driven parallel robot has a different mechanical structure and wire type, the two platforms share similarities and we have obtained an INRIA grant for a collaborative work with LIRMM for the studies on the influence of wire deformations, on calibration and appropriate design (finding the geometry of the robot so that the performances of the robot satisfy given requirements).

Both platforms will be used to validate the theoretical results that are jointly developed.

8.1.2. 3+3 Med Roras project

Participants: Jean-Pierre Merlet [correspondant], David Daney.

We have obtained an INRIA grant for a collaborative work with Cassino University, Monastir engineer school and University of Oran for a preliminary work on the development of a wire-driven parallel robot for rehabilitation[3]. A state of the art on rehabilitation and on rehabilitation protocol has been produced. These documents will be used as main inputs to determine if our hardware is appropriate for such task and, more importantly, to develop an appropriate design software that will allow us to calculate the best robot geometry being given a pathology, the patient morphology and the rehabilitation protocol.

8.2. Participation to National and International Conferences

8.2.1. International Conferences

- D. Daney participated to the World Congress of the International Federation for the Theory of Machines and Mechanisms (IFToMM), Besançon, and to IEEE International Conference on Robotics and Automation (ICRA), Rome
- C. Grandón presented a paper at IJCAI 2007, Hyderabad
- J-P. Merlet attended the IEEE Int. Conf, on Robotics and Automation (ICRA), Rome, and presented papers at: 2nd Int. Congress, Design and Modeling of mechanical systems, Monastir, World Congress of the International Federation for the Theory of Machines and Mechanisms (IFToMM), Besançon, Int. Conf. on Intelligent Robots and Systems (IROS), San Diego, 13th International Symposium of Robotics Research, Hiroshima, 1st Int. Rivieran Meeting on ImmoTIC-DomoTIC, Sophia-Antipolis.
- D. Oetomo attended the IEEE Int. Conf. on Robotics and Automation (ICRA), Rome.
- B. Neveu presented papers at the conferences: JFPC 2007, Rocquencourt, and ROADEF 2007, Grenoble, and attended the conferences Constraint Programming CP 2007, Providence (US) and CSCLP 2007, Rocquencourt.
- Y. Papegay has been invited to give a presentation at the Mathematica Paris 2007 Conference, in June, about SymbolicC and the C code generation with Mathematica. He gave a talk at the Wolfram 2007 Technology Conference, Urbana Champaign.
- O. Pourtallier presented papers at: EURO XXII, Praha, ISDG workshop, Rabat and Mathematical Aspects of Computer and Information Sciences, Paris.
- N. Ramdani presented papers at: ECC, Kos, IEEE Int. Conf. on Humanoid Robots, Pittsburgh, IEEE Int. Conf, on Robotics and Automation (ICRA), Rome.
- G. Trombettoni has presented papers at the conferences: Constraint Programming CP'07, Providence (US), French Conference on Constraint Programming JFPC'07, Rocquencourt and Conference on Artificial Intelligence ICTAI'07, Patras (Greece).

[3] see `wiki-sop.inria.fr/wiki/bin/view/Coprin/RORAS`

- B. Neveu and G. Trombettoni visited the research team of Maria-Cristina Riff Rojas at Federico Santa Maria University in Valparaiso, Chile, during 3 weeks, in May 2007, within the INRIA-CONICYT cooperation project, where B. Neveu has been reviewer and jury member of 2 magister theses. Maria-Cristina Riff and Xavier Bonnaire visited in return the COPRIN team in September 2007, during 3 weeks.

8.3. Other Activities

8.3.1. National Activities

- J-P. Merlet is president of IFToMM France and member of the scientific committee of the CNRS Robotics GDR
- N. Ramdani is co-responsible for the working group ”Méthodes Ensemblistes pour l'Automatique” - GDR MACS[4].

8.3.2. INRIA activities

- D. Daney is president of the CUMIR (Comité des Utilisateurs des Moyens Informatiques, Recherche).
- J-P. Merlet is member of the "Bureau du Comité des Projets" and the "Comité de Centre" both from Sophia, of the National Permanent Training Commission, of the Scientific Communication Commission and of INRIA Evaluation Board (CE). As a member of the CE he has been in charge of a working group devoted to the analysis of bibliometric indicator that has produced an analysis report on this subject. He has also participated to the organization of an Intech seminar on service robotics.

8.3.3. European Activities

- J-P. Merlet is member of the scientific committee of the European Conference on Mechanism Science (EUCOMES) and has participated to an EURON summer school on parallel robots.

9. Dissemination

9.1. Leadership within scientific community

A major point of this year has been the organization of the 12th IFToMM World Congress in Besançon in June 2007. This conference is being held every 4 years and this conference was hosted in France for the first time. COPRIN was completely in charge of the overall organization with the help of a local organization managed by M. Dahan from LMARC. Over 1400 papers were submitted and there was approximately 900 attendees from 53 countries.

Another major conference will be organized by COPRIN in 2008:, the IEEE Int. Conf. on Intelligent Robots and Systems (IROS), one of the two major robotics conferences. This conference will be held in September 2008 in Nice and approximately 1000 attendees are expected.

- D. Daney has been webmaster and software manager of IFToMM World Congress
- J-P. Merlet has been General Chair of the IFToMM World Congress and is Associate Editor for ICRA 2008. He is also associate editor of the Mechanism and Machine Theory Journal.
- B. Neveu was member of of the Program Committee of the 13th International Conference on Principles and Practice of Constraint Programming (CP 2007).

[4] http://www.lirmm.fr/ensemble

- Y. Papegay is member of the "commission de spécialistes" number 4 of the University of French Polynesia and a permanent member of the International Organizing Committee of the International Mathematica Symposium conferences serie. He was member of the Program Committee of the Computer Algebra Systems and Their Applications, CASA'2007 conference, and Exhibition Chair of the IFToMM 2007 World Congress.
- O. Pourtallier is secretary of the International Society of Dynamic Games.
- N. Ramdani is member of scientific commission number 61 of University Henri Poincaré Nancy 1 and member of the editorial advisory board of The Open Mechanical Engineering Journal.

9.2. Teaching

- D. Daney gave course in medical robotics, Master of Bio-Medical, Univ. Nice Sophia Antipolis (15h) and in robotics at ISIA Superior Institut of Computer Science and Control (Ecole des Mines de Paris) (8h).
- O. Pourtallier has taught 6 hours on game theory to master OSE, at École des Mines de Paris, Sophia Antipolis, 20 hours on game theory to Mass at UNSA and 6 hours on optimization, to DESS IMAFA at UNSA.
- G. Trombettoni is assistant professor in computer science at IUT R&T (networks and telecoms) of Sophia Antipolis.
- B. Neveu and G. Trombettoni have given lectures on constraint programming in the Computer Science Master and EPU at University of Nice Sophia (20 h).

9.2.1. PhD thesis

- D. Daney was jury member of one PhD defense.
- J-P. Merlet was a reviewer of two PhD defenses.
- B. Neveu was reviewer of one PhD defense and jury member of two PhD defenses.
- N. Ramdani was a reviewer of one PhD defense and jury member of one PhD defense.
- G. Trombettoni was jury member of one PhD defense.

9.3. PhD thesis

Current PhD theses:

1. J. Hubert, Classification of the singularity of parallel robots
2. I. Araya, Filtering techniques for interval solvers

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