



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

*Team DeFI*

*Shape Reconstruction and Identification*

*Futurs*

THEME NUM

*Activity*  
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2007



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# 1. Team

*DeFI is a common team with Ecole Polytechnique created October 1<sup>st</sup>, 2007 and is hosted by the Centre de Mathématiques Appliquées (CMAP) at Ecole Polytechnique.*

## **Head of project-team**

Houssem Haddar [ Research Associate (CR) Inria, HdR ]

## **Administrative assistant**

Fabienne Lancry [ Secretary (SAR) Inria ]

## **Research scientists, Ecole Polytechnique**

Grégoire Allaire [ Professor (PR) Ecole Polytechnique, HdR ]

Olivier Pantz [ Professor (PCC) Ecole Polytechnique ]

## **External Collaborators**

Laurent Bourgeois [ Assistant Professor (AP) ENSTA ]

Antonin Chambolle [ Research Director (DR) CNRS/Ecole Polytechnique, HdR ]

## **PostDoc**

Ozgun Ozdemir [ DGA Grant, from September 1st 2007 till September 1st 2008 ]

## **PhD Students**

Yosra Boukari [ ENIT (Tunisia) ]

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# 2. Overall Objectives

## 2.1. Overall Objectives

The research activity of our team is dedicated to the design, analysis and implementation of efficient numerical methods to solve inverse and/or shape and topological optimization problems in connection with acoustics, electromagnetism, elastodynamics, and waves in general.

Sought practical applications include radar and sonar applications, bio-medical imaging techniques, non-destructive testing, structural design, composite materials, ...

Roughly speaking, the model problem consists in determining information on, or optimizing the geometry (topology) and/or the physical properties of unknown targets from given constraints or measurements, for instance measurements of diffracted waves. In general this kind of problems is non linear. The inverse ones are also severely ill-posed and therefore require special attention from regularization point of view, and non trivial adaptations of classical optimization methods.

Our scientific research interests are three-fold:

- Theoretical understanding and analysis of the forward and inverse mathematical models, including in particular the development of simplified models for adequate asymptotic configurations.
- The design of efficient numerical optimization/inversion methods which are quick and robust with respect to noise. Special attention will be paid to algorithms capable of treating large scale problems (e.g. 3-D problems) and/or suited for real-time imaging.
- Development of prototype softwares for precise applications or tutorial toolboxes.

## 2.2. Highlights

The team has been created October 1<sup>st</sup>, 2007.

## 3. Scientific Foundations

### 3.1. Scientific Foundations

The research activity of our team is dedicated to the design, analysis and implementation of efficient numerical methods to solve inverse and/or shape and topological optimization problems in connection with acoustics, electromagnetism, elastodynamics, and waves in general. We are particularly interested in the development of fast methods that are suited for real-time imaging and/or large scale problems. These goals require to work on both the physical and the mathematical models involved and indeed a solid expertise in related numerical algorithms.

This section intends to give a general overview of our research interests and themes. We choose to present them through the specific example of inverse scattering problems (from inhomogeneities), which will be central in most of foreseen developments. The practical problem would be to identify an inclusion from measurements of diffracted waves that result from the interaction of the sought inclusion with some (incident) waves sent into the probed medium. Typical applications include biomedical imaging where using micro-waves one would like to probe the presence of pathological cells, or imaging of urban infrastructures where using ground penetrating radars (GPR) one is interested in finding the location of buried facilities such as pipelines or waste deposits. This kind of applications requires in particular fast and reliable algorithms.

By “imaging” we shall refer to the inverse problem where the concern is only the location and the shape of the inclusion, while “identification” may also indicate getting informations on the inclusion physical parameters.

Both problems (imaging and identification) are non linear and ill-posed (lack of stability with respect to measurements errors if some careful constrains are not added). Moreover, the unique determination of the geometry or the coefficients is not guaranteed in general if sufficient measurements are not available. As an example, in the case of anisotropic inclusions, one can show that an appropriate set of data uniquely determine the geometry but not the material properties.

These theoretical considerations (uniqueness, stability) are not only important in understanding the mathematical properties of the inverse problem, but also guide the choice of appropriate numerical strategies (which information can be stably reconstructed) and also the design of appropriate regularization techniques. Moreover, uniqueness proofs are in general constructive proofs, i.e. they implicitly contain a numerical algorithm to solve the inverse problem, hence their importance for practical applications. The sampling methods introduced below are one example of such algorithms.

A large part of our research activity is dedicated to numerical methods applied to the first type of inverse problems, where only the geometrical information is sought. In its general setting the inverse problem is very challenging and no method can provide a universal satisfactory solution to it (regarding the balance cost-precision-stability). This is why in the majority of the practically employed algorithms, some simplification of the underlying mathematical model is used, according to the specific configuration of the imaging experiment. The most popular ones are geometric optics (the Kirchhoff approximation) for high frequencies and weak scattering (the Born approximation) for small contrasts or small obstacles. They actually give full satisfaction for a wide range of applications as attested by the large success of existing imaging devices (radar, sonar, echography, X-ray tomography, ...), that rely on one of these approximations.

Generally speaking, the used simplifications result into a linearization of the inverse problem and therefore are usually valid only if the latter is weakly non-linear. The development of these simplified models and the improvement of their efficiency is still a very active research area. With that perspective we are particularly interested in deriving and studying higher order asymptotic models associated with small geometrical parameters such as: small obstacles, thin coatings, wires, periodic media, .... Higher order models usually introduce some non linearity in the inverse problem, but are in principle easier to handle from the numerical point of view than in the case of the exact model.

A larger part of our research activity is dedicated to algorithms that avoid the use of such approximations and that are efficient where classical approaches fail: i.e. roughly speaking when the non linearity of the inverse problem is sufficiently strong. This type of configuration is motivated by the applications mentioned below, and occurs as soon as the geometry of the unknown media generates non negligible multiple scattering effects (multiply-connected and closely spaced obstacles) or when the used frequency is in the so-called resonant region (wave-length comparable to the size of the sought medium). It is therefore much more difficult to deal with and requires new approaches. Our ideas to tackle this problem will be motivated and inspired by recent advances in shape and topological optimization methods and also the introduction of novel classes of imaging algorithms, so-called sampling methods.

The sampling methods are fast imaging solvers adapted to multi-static data (multiple receiver-transmitter pairs) at a fixed frequency. Even if they do not use any linearization of the forward model, they rely on computing the solutions to a set of linear problems of small size, that can be performed in a completely parallel procedure. Our team has already a solid expertise in these methods applied to electromagnetic 3-D problems. The success of such approaches was their ability to provide a relatively quick algorithm for solving 3-D problems without any need for a priori knowledge on the physical parameters of the targets. These algorithms solve only the imaging problem, in the sense that only the geometrical information is provided.

Despite the large efforts already spent in the development of this type of methods, either from the algorithmic point of view or the theoretical one, numerous questions are still open. These attractive new algorithms also suffer from the lack of experimental validations, due to their relatively recent introduction. We also would like to invest on this side by developing collaborations with engineering research groups that have experimental facilities. From the practical point of view the most potential limitation for these methods would be the need of a large amount of data to achieve reasonable accuracy. With that respect we believe that coupling with optimization methods would be very helpful in fixing this point in a satisfactory manner (regarding the balance between cost, precision and the required amount of data).

Among various shape optimization methods, the Level Set method seems to be particularly suited to such coupling. First because it shares similar mechanism as the sampling method by not relying on a surface parametrization of the unknown geometry and on capturing the surface on a cartesian grid. Second, both methods rely on the computation of an indicator function defined on this grid.

The coupling raises more questions on the algorithmic side than on the theoretical one. However, for the latter aspect, an interesting question would be to find the optimal minimal set of data that can be used to make the iterative algorithm rapidly convergent. This question is related to the design of focusing incident waves (which are for instance the basis of applications of the time-reversal principle). In the frequency regime, these incident waves can be constructed from the eigenvalue decomposition of the data operator used by sampling methods. The theoretical and numerical investigations of these aspects are still not completely understood for electromagnetic or elastodynamic problems.

Other topological optimization methods, like the homogenization method or the topological gradient method, can also be used, each one provides particular advantages in specific configurations. It is evident that the development of these methods is very suited to inverse problems and provide substantial advantage compared to classical shape optimization methods based on boundary variation. Their applications to inverse problems has not been fully investigated. The efficiency of these optimization methods can also be increased for adequate asymptotic configurations. For instance small amplitude homogenization method can be used as an efficient relaxation method for the inverse problem in the presence of small contrasts. On the other hand, the topological gradient method has shown to perform well in localizing small inclusions with only one iteration.

For the identification problem, one would like to also have information of the physical properties of the targets. Of course optimization methods is a tool of choice for these problems. However, in some applications only a qualitative information is needed and obtaining it in a cheaper way can be performed using asymptotic theories combined with sampling methods.

A broader perspective of our research themes would be the extension of the above mentioned techniques to time-dependent cases. Taking into account data in time domain is important for many practical applications,

such as imaging in cluttered media, the design of absorbing coatings or also crash worthiness in the case of structural design.

## 4. Application Domains

### 4.1. Radar and GPR applications

Conventional radar imaging techniques (ISAR, GPR, ...) use backscattering data to image targets. The commonly used inversion algorithms are mainly based on the use of weak scattering approximations such as the Born or Kirchhoff approximation leading to very simple linear models, but at the expense of ignoring multiple scattering and polarization effects. The success of such an approach is evident in the wide use of synthetic aperture radar techniques.

However, the use of backscattering data makes 3-D imaging a very challenging problem (it is not even well understood theoretically) and as pointed out by Brett Borden in the context of airborne radar: "In recent years it has become quite apparent that the problems associated with radar target identification efforts will not vanish with the development of more sensitive radar receivers or increased signal-to-noise levels. In addition it has (slowly) been realized that greater amounts of data - or even additional "kinds" of radar data, such as added polarization or greatly extended bandwidth - will all suffer from the same basic limitations affiliated with incorrect model assumptions. Moreover, in the face of these problems it is important to ask how (and if) the complications associated with radar based automatic target recognition can be surmounted." This comment also applies to the more complex GPR problem.

Our research themes will incorporate the development, analysis and testing of several novel methods, such as sampling methods, level set methods or topological gradient methods, for ground penetrating radar application (imaging of urban infrastructures, landmines detection, underground waste deposits monitoring, ...) using multistatic data.

### 4.2. Biomedical imaging

Among emerging medical imaging techniques we are particularly interested in those using low to moderate frequency regimes. These include Microwave Tomography, Electrical Impedance Tomography and also the closely related Optical Tomography technique. They all have the advantage of being potentially safe and relatively cheap modalities and can also be used in complementarity with well established techniques such as X-ray computed tomography or Magnetic Resonance Imaging.

With these modalities tissues are differentiated and, consequentially can be imaged, based on differences in dielectric properties (some recent studies have proved that dielectric properties of biological tissues can be a strong indicator of the tissues functional and pathological conditions, for instance, tissue blood content, ischemia, infarction, hypoxia, malignancies, edema and others). The main challenge for these functionalities is to build a 3-D imaging algorithm capable of treating multi-static measurements to provide real-time images with highest (reasonably) expected resolutions and in a sufficiently robust way.

Another important biomedical application is brain imaging. We are for instance interested in the use of EEG and MEG techniques as complementary tools to MRI. They are applied for instance to localize epileptic centers or active zones (functional imaging). Here the problem is different and consists into performing passive imaging: the epileptic centers act as electrical sources and imaging is performed from measurements of induced currents. Incorporating the structure of the skull is primordial in improving the resolution of the imaging procedure. Doing this in a reasonably quick manner is still an active research area, and the use of asymptotic models would offer a promising solution to fix this issue.



### 4.3. Non destructive testing and parameter identification

One challenging problem in this vast area is the identification and imaging of defaults in anisotropic media. For instance this problem is of great importance in aeronautic constructions due to the growing use of composite materials. It also arises in applications linked with the evaluation of wood quality, like locating knots in timber in order to optimize timber-cutting in sawmills, or evaluating wood integrity before cutting trees. The anisotropy of the propagative media renders the analysis of diffracted waves more complex since one cannot only relies on the use of backscattered waves. Another difficulty comes from the fact that the micro-structure of the media is generally not well known a priori.

Our concern will be focused on the determination of qualitative information on the size of defaults and their physical properties rather than a complete imaging which for anisotropic media is in general impossible. For instance, in the case of homogeneous background, one can link the size of the inclusion and the index of refraction to the first eigenvalue of so-called interior transmission problem. These eigenvalues can be determined from the measured data and a rough localization of the default. Our goal is to extend this kind of idea to the cases where both the propagative media and the inclusion are anisotropic. The generalization to the case of cracks or screens has also to be investigated.

In the context of nuclear waste management many studies are conducted on the possibility of storing waste in a deep geological clay layer. To assess the reliability of such a storage without leakage it is necessary to have a precise knowledge of the porous media parameters (porosity, tortuosity, permeability, etc.). The large range of space and time scales involved in this process requires a high degree of precision as well as tight bounds on the uncertainties. Many physical experiments are conducted *in situ* which are designed for providing data for parameters identification. For example, the determination of the damaged zone (caused by excavation) around the repository area is of paramount importance since microcracks yield drastic changes in the permeability. Level set methods are a tool of choice for characterizing this damaged zone.

## 5. Software

### 5.1. Samplings-2D

**Participant:** Housseem Haddar [correspondant].

This software is written in Fortran 90 and is related to forward and inverse problems for the Helmholtz equation in 2-D. It includes three independent components. The first one solves to scattering problem using integral equation approach and supports piecewise-constant dielectrics and obstacles with impedance boundary conditions. The second one contains various samplings methods to solve the inverse scattering problem (LSM, RGLSM(s), Factorization, MuSiC) for near-field or far-field setting. The third component is a set of post processing functionalities to visualize the results.

### 5.2. Structural Optimization with FreeFem++

**Participants:** Olivier Pantz [correspondant], Grégoire Allaire.

This is a toolbox that contains efficient implementations of shape optimization methods in 2-D using the free finite element software FreeFem++. It supports boundary variation methods, homogenization method, and Level Set method. A web page of this toolbox is available at [http://www.cmap.polytechnique.fr/~allaire/freefem\\_en.html](http://www.cmap.polytechnique.fr/~allaire/freefem_en.html).

### 5.3. RGLSM-3D & LSM-3D

**Participants:** Housseem Haddar [correspondant], M'Barek Fares [CERFACS].

These Fortran 90 codes are dedicated to the solution of the 3-D electromagnetic inverse scattering problem using respectively, RGLSM and LSM. There are parallel versions of these codes that are coupled to the CESC code (solver for electromagnetic scattering problems using integral equation methods) developed at CERFACS. They also support imaging for doubly layered medium.

## 6. New Results

### 6.1. Warning

Since the group has been freshly created, the main research activity of the group members has been accomplished outside the DEFI team.

We refer to the activity report of the optopo group at CMAP for an account of the new results obtained by G. Allaire and O. Pantz. The report is available at <http://www.cmap.polytechnique.fr/~optopo/index.php>.

The following sections contain an account for the research activity of H. Haddar. This part has been duplicated from the activity report of the POEMS group (H. Haddar was a member of POEMS during 2007).

### 6.2. Sampling methods for inverse scattering problems

#### 6.2.1. *The RG-LSM algorithm with an analytic continuation method*

**Participants:** Housseem Haddar, Ozgur Ozdemir.

The RG-LSM algorithm has been introduced by Colton-Haddar as a reformulation of the linear sampling method in the cases where measurements consists of Cauchy data at a given surface. The formulation of the algorithm is based on the reciprocity gap functional on the measurement locations. The advantage of this algorithm was to avoid the need of considering the full inverse scattering problem (requiring for instance the computation of the background Green tensor) and in some sense concentrate only on what happening inside the probed region. The new formulation has also permitted to embed usual sampling method into a more general framework that would be interest in the study of the method.

This method is for instance well suited to imaging of embedded targets when the measurements can be achieved at the interface of the interrogated medium. However, in many practical applications, like imaging of embedded facilities in the soil, these measurements cannot be easily obtained. Only measurement of the scattered field in the air would be feasible. The problem under investigation is to study how accurate would be the coupling of the RG-LSM algorithm with an analytic continuation method that would provide the Cauchy data from the scattered field. In a first approach we considered the simplified case of doubly layered medium where the analytic continuation procedure can be simply achieved with help of longitudinal Fourier transform. The 2-D case have shown that a substantial gain in cpu time can be obtained as compared with applying the linear sampling method with background Green tensor of doubly layered medium being computed. The accuracy is roughly the same. The next steps of this ongoing work will be:

- Consider the case of rough interfaces where the continuation method can be achieved by coupling the Fourier transform with a Taylor series expansion
- Study the stability of the continuation method and quantify the error in terms of the frequency
- Generalize theses results to the electromagnetic problem

#### 6.2.2. *Identification of effective dielectric properties using sampling algorithms*

**Participants:** Housseem Haddar, Ridha Mdimigh.

It is well known that sampling algorithms only provide the geometry of the sought inclusion from multi-static data. However in some special cases we showed that a post processing of this algorithm also provides informations on the physical parameters of the inclusions. The first one is the case where the obstacles size is small compared to the wavelength. It is well known in that case that the center of the inclusion can be uniquely determined from the range of the far field operator associated with the asymptotic expression of the farfields. The MuSiC approach based on projection on the null space of the measurement's matrix or the linear sampling approach based on computing a regularized solutions to a testing equation, can be used to localize these centers. The advantage of the linear sampling method is that the computed solution can be also used to evaluate the Herglotz wave at the sampling point. The latter quantity is shown to be sufficient to determine the effective properties of the inclusions. The advantage of this approach is that it can also generalize to the case of RG-LSM algorithm where instead of the Herglotz wave one evaluates the used approximating operator at the sampling point.

We shall use similar approach to the case of extended targets and anisotropic inclusions.

### 6.2.3. *Transmission Eigenvalues and their application to the identification problem*

**Participant:** Housseem Haddar.

The so called interior transmission problem plays an important role the inverse scattering problem from (anisotropic) inhomogeneities. Solution to this problem associated with singular sources can be used for instance to establish uniqueness of the reconstruction of anisotropic inclusion shape. It is well known also that the injectivity of the far field operator used in sampling method is equivalent to the uniqueness of solution to this problem.

In a first work, in collaboration with F. Cakoni we studied this interior transmission problem in the case of anisotropic Maxwell's equations and with possibly coated parts (that is modeled with an impedance boundary condition). The results extend the one previously established by Haddar to the case where the index norm is less than one and also show that the set of the eigenvalues for this transmission problem is at most discrete. Showing the existence of these eigenvalues is still an open problem due to the fact that the operator involved is not self-adjoint.

The analysis also showed that some lower bounds can be obtained on these eigenvalues in terms of the inhomogeneity shape and the index norm. This was the starting point of the idea of exploiting the eigenvalues to get information on the index of the sought inhomogeneity. More precisely, using the sampling operator with fixed sampling point and by varying the frequency one can localize the presence of eigenvalues when the norm of the solution becomes large. This procedure has been successfully tested in the case of isotropic circles where the eigenvalues can be computed in terms of some equation involving the Bessel functions. The first eigenvalue can be used to get estimate on the index norm of the inclusion. The procedure has been validated in the scalar case (TM and TE modes) and for the case of full aperture. The perspective would be to test this in the case of limited aperture and for anisotropic 3-D cases.

## 6.3. **Electrostatic imaging using conformal mappings**

**Participants:** Yosra Boukari, Housseem Haddar.

In a series of recent papers Akduman, Haddar and Kress have developed a new simple and fast numerical scheme for solving two-dimensional inverse boundary value problems for the Laplace equation that model non-destructive testing and evaluation via electrostatic imaging. In the fashion of a decomposition method, the reconstruction of the boundary shape  $\Gamma_0$  of a perfectly conducting or a nonconducting inclusion within a doubly connected conducting medium  $D \subset \mathbb{R}^2$  from over-determined Cauchy data on the accessible exterior boundary  $\Gamma_1$  is separated into a nonlinear well-posed problem and a linear ill-posed problem. The approach is based on a conformal map  $\Psi : B \rightarrow D$  that takes an annulus  $B$  bounded by two concentric circles onto  $D$ . In the first step, in terms of the given Cauchy data on  $\Gamma_1$ , by successive approximations one has to solve a nonlocal and nonlinear ordinary differential equation for the boundary values  $\Psi|_{C_1}$  of this mapping on the exterior boundary circle of  $B$ . Then in the second step a Cauchy problem for the holomorphic function  $\Psi$  in  $B$  has to be solved via a regularized Laurent expansion to obtain the unknown boundary  $\Gamma_0 = \Psi(C_0)$  as the image of the interior boundary circle  $C_0$ .

In the present work, we are interested in the case where the solution satisfies an impedance boundary condition on  $\Gamma_0$  (which can be seen as an approximation for a transmission problem between two conducting media). In that case the algorithm does not completely decompose the inverse problem into a well-posed nonlinear ordinary differential equation and an ill-posed Cauchy problem. Consequently its analysis and implementation is more involved and more than a straightforward extension of the algorithm for the Dirichlet or Neumann case. The first investigations by Haddar and Kress showed that when the impedance is relatively small the algorithm become unstable. We also noticed that instability is also linked with the size of the interior boundary.

In the present work we showed how the use of conjugate harmonics leads to a stabilization of the problem as in the case of the Neumann boundary conditions. The convergence analysis in the case of concentric circles show convergence for relatively small impedances. However it is not clear at the moment whether none of the algorithm can be convergent for a given setting of the problem or not.

## 6.4. Asymptotic models

### 6.4.1. Interface conditions for time dependent problems and DG methods

**Participant:** Housseem Haddar.

This work is in collaboration with Seun Chun and Jan Hesthaven from Brown University. We established transmission conditions for anisotropic media in the case of the 3-D Maxwell's equation and in the framework of the time dependent problems. The interface conditions are well suited to Discontinuous Galerkin method since the latter implicitly support discontinuities between elements. The interface conditions only results into a modification of the numerical flux used in DG methods. These conditions has been successfully tested in the 1D case up the fourth order where stabilization in time has been applied to the fourth order condition. It is also worth noticing that the expression of these conditions in the anisotropic case are far from being intuitive and cannot be simply deduced from the isotropic one by just replacing constant coefficients with their matrix equivalent.

We are currently addressing the numerical study and implementation of these conditions in the 2-D and 3-D cases. The stability in time for curved geometries is also an open issue due to the presence of curvature corrective terms with the bad signs in the expression of these interface conditions.

### 6.4.2. GIBC for scattering problems from strongly absorbing media

**Participants:** Housseem Haddar, Patrick Joly.

We finalized the theoretical study started with H.M. Nguyen and that was dedicated to the construction and analysis of so-called Generalized Impedance Boundary Conditions (GIBCs) for electromagnetic scattering problems from imperfect conductors. These boundary conditions can be seen as higher order approximations of a perfect conductor condition. We considered the 3-D case with Maxwell equations in a harmonic regime. The construction of GIBCs is based on a scaled asymptotic expansion with respect to the skin depth. The asymptotic expansion is theoretically justified at any order and we gave explicit expressions till the third order. These expressions are used to derive the GIBCs. The associated boundary value problem is analyzed and error estimates are obtained in terms of the skin depth.

## 6.5. The wave equation with fractional order dissipative terms

**Participants:** Housseem Haddar, Jing-Rebecca Li, Denis Matignon.

The dissipative model which describes acoustic waves traveling in a duct with viscothermal losses at the lateral walls is a wave equation with spatially-varying coefficients, which involves fractional-order integrals and derivatives. The main goal of our investigations is to propose an efficient numerical discretization of the coupled model, that avoids for instance the storage of the solution during past steps which would be penalizing for long time simulations. Our approach is based on so-called diffusive representations of the fractional integral where, roughly speaking, the fractional-order time kernel in the integral is represented by its Laplace transform. This allows for efficient time domain discretizations because the value of integral

at each time step can be updated from the value at the previous time step by operations which are local in time, contrary to a naive discretization of the fractional integral. These representations require however the evaluation of an integral over the Laplace variable domain. We propose and analyze two schemes based on the choice of the quadrature rule associated with this integral.

The first one is inspired by the continuous stability analysis of the initial boundary value problem associated with coupled system: wave equation-diffusive representation. The scheme is constructed so that it preserves the energy balance at the discrete level. This is done however at the expense of a loss of accuracy with the simulation time.

The second approach is numerically more efficient and provides uniform control of the accuracy with respect to the simulation time. The idea of this approach is inspired by the work of Greengard and Lin: the convolution integral is split into a local part and a historical part, where for the latter one can exploit the exponential decay of the Laplace kernels to choose quadrature rules that provide uniform error in time. Essentially, the number of quadrature points in the Laplace domain is  $O(-\log(\Delta t))$  where  $\Delta t$  denotes the time step. Thus, if  $M$  is the number of time steps, the numerical scheme we propose requires  $O(M \log(M))$  work and  $O(\log(M))$  memory, compared to  $O(M^2)$  work and  $O(M)$  memory of a naive discretization. Let us notice however that even though more efficient and seems to be numerically stable under the standard CFL, no discrete energy balance has been found for the second scheme.

## 7. Contracts and Grants with Industry

### 7.1. DGA

**Participants:** Laurent Bourgeois, Housseem Haddar, Ozgur Ozdemir.

This contract provides financial support to the Post-Doc of Dr. Ozdemir on coupling RG-LSM methods with a Fourier continuation method to image buried inclusions.

## 8. Other Grants and Activities

### 8.1. National Actions

- Grégoire Allaire is a scientific adviser at CEA Saclay.
- Participation to GdR MOMAS “Modélisation Mathématique et Simulation Numérique liées aux Problèmes de gestion des Déchets Nucléaires”. G. Allaire is the president of the scientific committee of this GdR.

### 8.2. International Initiatives

- The associated team ISIP between the mathematical Department of the University of Delaware and the DEFI team has been accepted by the INRIA committee and starts on January 2008.

### 8.3. Exterior research visitors

- F. Cakoni and D. Colton from the university of Delaware have been visiting the project from September 17 to September 24, 2007.

## 9. Dissemination

### 9.1. Scientific Community Animation

- G. Allaire is member of the editorial board of ESAIM/COCV (Control, optimization, and calculus of variations), Structural and Multidisciplinary Optimization, SIAM/MMS (Multiscale Modeling and Simulation), Annales de la Faculté des Sciences de Toulouse, Discrete and Continuous Dynamical Systems Series B, Computational and Applied Mathematics, Mathematical Models and Methods in Applied Sciences (M3AS).
- G Allaire is member of the Conseil National des Universités (26ème section, mathématiques appliquées), member of the administrative committee of SMAI and president of GAMNI/SMAI (Groupement pour l'Avancement des Méthodes Numériques pour l'Ingénieur).
- H. Haddar has been the responsible of the reviewing process of the WAVES'07 conference at Reading University. He is also member of the Commission de Spécialistes, section 26, of the University of Marseille.

## 9.2. Teaching

G. Allaire

- Course “Analyse Numérique et d’Optimisation”, Ecole Polytechnique (MAP 431, 2ème année)
- Course “Conception optimale des structures”, Ecole Polytechnique (MAP 562, Programme d’Approfondissement SISIM en 3ème année)
- Master M2 , AN&EDP, University of Paris 6. Module "Analyse théorique et numérique des systèmes hyperboliques de lois de conservation" in collaboration with F. Coquel.

H. Haddar

- MASTER M2, AN&EDP, University of Paris 6. Module “Problèmes directs et inverses en diffraction” in collaboration with P. Joly.

O. Pantz

- Course “Analyse Numérique et d’Optimisation”, Ecole Polytechnique (MAP 431, 2ème année)
- Course “Conception optimale des structures”, Ecole Polytechnique (MAP 562, Programme d’Approfondissement SISIM en 3ème année)

## 9.3. Seminars, Conferences, Visits

G. Allaire

- Invited speaker to the 6<sup>th</sup> International Congress of Industrial and Applied Mathematics (ICIAM 2007, Zürich).
- Workshop “Perturbed periodic PDEs, problems with singular boundaries and their numerical aspects”, Cardiff (septembre 2007).
- Journées d’études sur les milieux poreux, Lyon (octobre 2007).
- Colloque bi-annuel du GDR MOMAS, Fréjus (novembre 2007).

Y. Boukari

- Contributed presentation in the third workshop “Tendances dans les Applications Mathématiques en Tunisie, Algérie, Maroc” Alger, April 16-18, 2007.

H. Haddar

- Invited speaker in the workshop *Méthodes pour les problèmes direct et inverse de diffraction : progrès récents*, Université de Pau et des Pays de l’Adour, December 12-14, 2007.

- Invited for a short visit to the Electromagnetic Research Group, Istanbul Technical University, November 14-20, 2007.
- Invited speaker to the AIM workshop, *High-order methods for computational wave propagation and scattering*, Palo Alto, September 9-16, 2007.
- Mini-symposium “Inverse Problems in Electromagnetic Scattering”, AIP 2007 conference, Vancouver, June 25-29, 2007 (presented by F. Cakoni).
- Invited speaker to the Oberwolfach workshop, *Inverse Problems in Wave Scattering*, Oberwolfach, March 4-10, 2007.
- Invited to the Oberwolfach workshop, *Computational Electromagnetics and Acoustics*, Oberwolfach, February 4-10, 2007.
- Invited speaker to the *International Workshop on Integral Equations and Shape Reconstruction*, on the occasion of Prof. Dr. Kress 65th birthday, Goettingen, January 19-21, 2007.

O. Pantz

- Invited speaker to the workshop *Mouvements d’Interfaces, Calcul et Applications (MICA)* IHP, Paris, July 9, 2007.

## 10. Bibliography

### Year Publications

#### Books and Monographs

- [1] G. ALLAIRE. *Conception optimale de structures*, Mathématiques et Applications, vol. 58, Springer, 2007.
- [2] G. ALLAIRE. *Numerical Analysis and Optimization. An Introduction to Mathematical Modelling and Numerical Simulation*, translated by Alan Craig, Oxford University Press, 2007.

#### Articles in refereed journals and book chapters

- [3] G. ALLAIRE, C. CONCA, L. FRIZ, J. ORTEGA. *On Bloch waves for the Stokes equations*, in "DCDS series B", vol. 7, 2007, p. 1–28.
- [4] G. ALLAIRE, G. FACCANONI, S. KOKH. *A strictly hyperbolic equilibrium phase transition model*, in "C. R. Acad. Sci. Paris, Série I", vol. 344, 2007, p. 135–140.
- [5] G. ALLAIRE, F. JOUVE. *Minimum stress optimal design with the level set method*, in "International Journal of Boundary Elements and Applications (accepted)", 2007.
- [6] G. ALLAIRE, R. ORIVE. *Homogenization of periodic non self-adjoint problems with large drift and potential*, in "COCV (accepted)", 2007.
- [7] G. ALLAIRE, A.-L. RAPHAEL. *Homogenization of a convection-diffusion model with reaction in a porous medium*, in "C. R. Acad. Sci. Paris, Série I", vol. 344, 2007, p. 523–528.
- [8] F. CAKONI, H. HADDAR. *A variational approach for the solution of the electromagnetic interior transmission problem for anisotropic media*, in "Inverse Probl. Imaging", vol. 1, n<sup>o</sup> 3, 2007, p. 443–456.

- [9] F. CAKONI, H. HADDAR. *Identification of partially coated anisotropic buried objects using electromagnetic Cauchy data*, in "J. Integral Equations and Applications", vol. 19, n<sup>o</sup> 3, 2007, p. 359–378.
- [10] X. CLAEYS, H. HADDAR. *Scattering from infinite rough tubular surfaces*, in "Math. Methods Appl. Sci.", vol. 30, n<sup>o</sup> 4, 2007, p. 389–414.
- [11] C. FIORALBA, D. COLTON, H. HADDAR. *The computation of lower bounds for the norm of the index of refraction in an anisotropic media*, in "J. Integral Equations and Applications (accepted)", 2007.
- [12] H. HADDAR, J. S. HESTHAVEN. *Editorial: Waves 2005 conference*, in "J. Comput. Appl. Math.", Held at Brown University, Providence, RI, June 20–24, 2005, vol. 204, n<sup>o</sup> 2, 2007, p. 197–198.
- [13] H. HADDAR, P. JOLY, H. M. NGUYEN. *Generalized impedance boundary conditions for scattering problems from strongly absorbing obstacles: the case of Maxwell's equations*, in "M3AS (accepted)", 2007.
- [14] E. MUNOZ, G. ALLAIRE, M. P. BENDSOE. *On two formulations of an optimal insulation problem*, in "SMO", vol. 33, 2007, p. 363–373.
- [15] O. PANTZ. *The modeling of deformable bodies with frictionless (self-)contacts*, in "Archive Rat. Mech. Anal. (accepted)", 2007.
- [16] O. PANTZ, K. TRABELSI. *Simultaneous shape, topology and homogenized properties optimization*, in "SMO", vol. 34, 2007, p. 361–364.
- [17] F. DE GOURNAY, G. ALLAIRE, F. JOUVE. *Shape and topology optimization of the robust compliance via the level set method*, in "COCV (accepted)", 2007.

### **Publications in Conferences and Workshops**

- [18] Y. BOUKARI, H. HADDAR. *Imagerie d'inclusions à faibles impédances par la méthode des applications conformes*, in "TamTam'07 conference proceedings", 2007.
- [19] H. HADDAR. *Imaging damaged parts of buried objects from electromagnetic Cauchy data*, in "Proceedings of the 2007 ACES Conference on Applied Computational Electromagnetics", 2007.
- [20] H. HADDAR, J.-R. LI, D. MATIGNON. *Efficient solution of a wave equation with fractional order dissipative terms*, in "Proceedings of the Waves 2007 conference", 2007.