INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team galaad

Géométrie, Algèbre, Algorithmes

Sophia Antipolis - Méditerranée

THEME SYM

Activity Report

2007

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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable solution of geometric and algebraic problems, which are encountered in fields such as CAGD (Computer Aided Geometric Design), robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry, computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical computation, usually opposed to symbolic computation, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: modeling, computing, analysis in connection with effective algebraic geometry.

3.2. Algebraic Geometric Modeling

We are investigating geometric modeling approaches, based on non-discrete models, mainly of semi-algebraic type. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to handle and solve the underlying non-linear problems.

Effective algebraic geometry is a naturally framework for handling such representations, in which we are developing new methods to solve these non-linear problems. The framework not only provides tools for modeling by also, it makes it possible to exploit the geometric properties of these algebraic varieties, in order to improve this modeling work. To handle and control geometric objects such as parameterised curves and surfaces or their implicit representation, we consider in particular projections techniques. We focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials. These tools allows to change geometric representations, computing an implicit model from a parameterised one. We are interested in dedicated methods to solve these type of problems.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately the algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations. The use of local algebraic representations also raises problems of approximation and reconstruction, on which we are working.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. In addition to the development of symbolic geometric computations that exploit these invariant properties, we are also interested in developing compact representations of shapes, based on algebraic/symbolic descriptions. Our aim is to improve geometric computing performances, by using smaller input data, with better properties of approximation and certified computation.

3.3. Algebraic Geometric Computing

The underlying representation behind the geometric model that we consider are often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases. We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

We are also interested in subdivision methods, which are able to localise efficiently the real roots of polynomial equations. The specificities of these methods are a local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue in analysing these methods is how to obtain good complexity bounds by exploiting the structure of the problem. Many algebraic problems can be reformulated in terms of linear algebra questions. Thus, it is not surprising to see that complexity analysis of our methods leads to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and their implications in solving polynomial equations.

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. The problem of decomposition and factorisation is thus also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.4. Algebraic Geometric Analysis

Analysing a geometric model requires tools for structuring it, which leads first to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structure. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computation are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However in many situations, it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

4. Application Domains

4.1. Shape Design

Keywords: *engineering computer-assisted*, *geometric modeling*.

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and build, by using geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts that we are developping in Computer Aided Geometric Design.

4.2. Shape approximation

Keywords: *approximation*, *engineering*, *reconstruction*.

Many problems encounter in the application of computer sciences started from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or in signal processing. This also appears in computer biology where *Distance geometry* plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tends to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems.

5. Software

5.1. Mathemagix, a free computer algebra environment

Keywords: *algebra*, *compiler*, *fast algorithm*, *hybrid software*, *interpreter*, *matrices*, *multivariate polynomial*, *series*, *univariate polynomial*.

Participants: Grégoire Lecerf, Bernard Mourrain [contact person], Daouda N'Diatta, Olivier Ruatta, Joris van der Hoeven, Julien Wintz.

http://www.mathemagix.org/

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). This should make MATHEMAGIX particularly suitable as a bridge between symbolic computation and numerical analysis.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

This project, supported by the ANR GECKO, aims at structuring collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation. The library SYNAPS developped by GALAAD, is under integration in this framework, as a collection of specialized modules. Other efficient modules on matrices, series, symbolic expressions are provided by our collaborators.

5.2. Synaps, specialised modules for symbolic and numeric computations

Keywords: *algebraic number*, *curves*, *effective algebraic*, *geometry*, *iterative methods*, *linear algebra*, *links symbolic-numeric*, *polynomials*, *resultant*, *solving*, *sparse matrices*, *stability*, *structured matrices*, *subdivision solvers*, *surfaces*.

Participants: Ioannis Emiris, Bernard Mourrain [contact person], Jean-Pascal Pavone, Philippe Trébuchet, Elias Tsigaridas, Julien Wintz.

http://synaps.inria.fr/.

The library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations is evolving as a set of modules integrated in the framework of MATHEMAGIX. These specialised modules can be used as plugins or dynamic binary libraries connected to an external application such as AXEL or to the computer algebra system provided by MATHEMAGIX. These developments are based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

Currently, we are developing of the following components:

- ALGEBRIX: basic arithmetic on vectors, matrices, polynomials in one or several variables, dual/inverse systems, Sturm sequence, univariate resultant, ...
- SUBDIVIX: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms;
- REALROOT: exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; Algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

5.3. Axel, a geometric modeler for algebraic objects

Keywords: *computational algebraic geometry*, *curve*, *implicit equation*, *intersection*, *parameterisation*, *resolution*, *singularity*, *surface*, *topology*.

Participants: Stéphane Chau, Bernard Mourrain, Jean-Pascal Pavone, Julien Wintz [contact person].

http://axel.inria.fr.

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The application contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This package is now distributed as binary packages as well for Linux as for MacOSX. It is hosted at the Inria's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. By the beginning of November the software has been downloaded more than 10000 times.

5.4. Multires, a maple package for multivariate resolution problems

Keywords: *eigenvalues*, *interpolation*, *linear algebra*, *polynomial algorithmic*, *residue*, *resultant*.

Participants: Laurent Busé [contact person], Ioannis Emiris, Bernard Mourrain, Olivier Ruatta, Philippe Trébuchet.

http://www-sop.inria.fr/galaad/logiciels/multires/.

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, and not their effectiveness, which is achieved in a more adapted environment as SYNAPS. It provides methods to build matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay, Jouanolou for projective resultant, Bezout and (sparse) resultant on a toric variety, residual resultant of a complete intersection, functions for computing the degree of residual resultant, algorithms for the geometric decomposition of an algebraic variety. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

6. New Results

6.1. Algebraic Geometric Modeling

6.1.1. Torsion of the symmetric algebra and implicitization

Participants: Laurent Busé, Marc Chardin [Univ. Paris VI], Jean-Pierre Jouanolou [Univ. Strasbourg].

Recently, a method to compute the implicit equation of a parametrized hypersurface has been developed by the authors. In this work, some questions related to this method are adressed. First, we prove that the degree estimate for the stabilization of the MacRae's invariant of a graded part of the symmetric algebra is optimal. Then we show that the extraneous factor that may appear in the process splits into a product of linear forms in the algebraic closure of the base field, each linear form being associated with a non complete intersection base point. Finally, we make a link between this method and a resultant computation for the case of rational plane curves and space surfaces.

6.1.2. On the equations of the moving curves ideal

Participant: Laurent Busé.

Given a parametrization of a plane algebraic curve, some explicit moving curves, that is some explicit generators of a certain associated Rees algebra, are described. The approach is based on a detailed study of the elimination ideal of two homogeneous polynomials in two homogeneous variables that form a regular sequence. Also, a close relation between adjoint linear systems and moving curves, conjectured by David Cox, is studied.

6.1.3. Implicitization of algebraic surfaces parametrized over $\mathbb{P}^1 \times \mathbb{P}^1$

Participants: Laurent Busé, Marc Dohm.

In a recent work, Busé, Chardin, and Jouanolou have developed an implicitization method for algebraic surfaces based on the so-called "approximation complexes". In this work, we have develop this method in another setting, to be precise, for surfaces parametrized by bihomogeneous polynomials, a case which is of interest in numerous applications. We show that the implicit equation of a surface in 3-dimensional projective space parametrized by bi-homogeneous polynomials of bi-degree (d, d), for a given integer $d \geq 1$, can be represented and computed from the linear syzygies of its parametrization if the base points are isolated and form locally a complete intersection. This work has been presented at the ISSAC'2007 conference by Marc Dohm who received the "best student paper" award [18].

6.1.4. Implicitization of canal surfaces

Participants: Marc Dohm, Severinas Zube [Univ. Vilnius, Lithuania].

In surface design, the user often needs to perform rounding or filleting between two intersecting surfaces. Mathematically, the surface used in making the rounding is defined as the envelope of a family of spheres which are tangent to both surfaces. This envelope of spheres is called a canal surface. In this work we present an efficient algorithm for computing the implicit equation of a canal surface generated by a rational family of spheres. By using Laguerre and Lie geometries, we relate the equation of the canal surface to an equation of a dual variety of a certain curve in 5-dimensional projective space. We define a kind of μ-basis for the dual variety to the curve and present a simple algorithm for its computation. The implicit equation of the dual variety and the canal surface are obtained by means of the resultant associated with the μ-basis. This work has been submitted for publication.

6.1.5. Resultant with separated variables

Participants: Laurent Busé, Mohamed Elkadi, André Galligo, Elimane Ba.

We develop adapted resultants for polynomials with separated variables, of type F(x)=G(y). They naturally appear in the applications in CAGD when we write the equations of an intersection problem or of a self-intersection problem. A first paper by Laurent Busé, Mohamed Elkadi and Andre Galligo, was accepted for publication in the journal CAGD. A second paper by Mohamed Elkadi and Andre Galligo, was presented and published at ISSAC'07 [20]. A third paper is in preparation with Elimane Ba for polynomials represented in bases different from the monomial bases.

6.1.6. General classification of (1,2) parametric surfaces in $\mathbb{P}^3$

Participants: Thi Ha Lé, André Galligo.

Patches of parametric real surfaces of low degrees are commonly used in Computer Aided Geometric Design and Geometric Modeling. However the precise description of the geometry of the whole real surface is generally difficult to master, and few complete classifications exist. We study surfaces of bidegree (1,2). We present a classification and a geometric study of parametric surfaces of bidegree (1,2) over the complex field and over the real field by considering a dual scroll. We detect and describe (if it is not void) the trace of self-intersection and singular locus in the system of coordinates attached to the control polygon of a patch (1,2) in the box $[0; 1] \times [0; 1]$. This work is published in [21].

6.2. Algebraic Geometric computing

6.2.1. Solving univariate polynomial equations using continued fraction

Participant: Vikram Sharma.

The efficiency of the continued fraction algorithm for isolating the real roots of a univariate polynomial depends upon the computation of tight lower bounds on the smallest positive root of a polynomial. The known complexity bounds for the algorithm rely on the impractical assumption that it is possible to efficiently compute the floor of the smallest positive root of a polynomial; without this assumption, the worst case bounds are exponential. In the paper entitled "Complexity of Real Root Isolation Using Continued Fractions" [22], we derive the first polynomial worst case bound on the algorithm: for a square-free integer polynomial of degree n and coefficients of bit-length L, the bit-complexity of the continued fraction algorithm is $\widetilde{O}(n^7L^2)$, using a bound by Hong to compute the floor of the smallest positive root of a polynomial; here $\widetilde{O}$ indicates that we are omitting logarithmic factors. This work was presented at ISSAC 2007, Waterloo, Canada.

6.2.2. *Symbolic-numeric methods for solving univariate polynomial equations*

Participants: Vikram Sharma, Bernard Mourrain.

The best known bit-complexity bound for isolating all roots of a univariate integer polynomial of degree n with coefficients of bit-size L is $\widetilde{O}(n^3L)$. The first algorithm to achieve this was proposed by Schönhage; later improvements in *arithmetic complexity* were given by Pan, but the bit-complexity essentially remains unchanged. The drawback of all these algorithms, however, is that their implementations are non-trivial, because they use an extensive array of sophisticated algorithms; and the algorithms that have been implemented remain hardly useful in practice. Independent of all these approaches is the numerical approach based upon Weirstrass's method (also called Aberth's method) and implemented by D. Bini and G. Fiorentino (MPSOLVE), the implementation can be considered to be one of the fastest methods to isolate all the roots, both real and non-real. However, we don't know any complexity bounds on the worst-case behaviour of the algorithm, i.e., we don't know if the algorithm will terminate for all instances. We propose an algorithm for isolating real roots that is simple to implement, comparable in performance to MPSOLVE while guaranteeing the output, and has the same worst-case complexity as Schönhage's algorithm (or at most worst by a factor of L). The algorithm is based upon two ideas:

a) An initial real root isolation phase using only floating-point arithmetic. This works for most of the polynomials.

b) Increasing precision of the computation only if necessary and avoiding exact arithmetic.

The underlying isolating algorithm is based upon Descartes's rule of signs along with the Bernstein representation of a polynomial. A prototype of the algorithm has been implemented in the module SUBDIVIX of SYNAPS.

6.2.3. *Division Algorithms for Bernstein Polynomials*

Participants: Laurent Busé, Ron Goldman [Rice University].

Three division algorithms are presented for univariate Bernstein polynomials: an algorithm for finding the quotient and remainder of two univariate polynomials, an algorithm for calculating the GCD of an arbitrary collection of univariate polynomials, and an algorithm for computing a μ-basis for the syzygy module of an arbitrary collection of univariate polynomials. Division algorithms for multivariate Bernstein polynomials and analogues in the multivariate Bernstein setting of Gröbner bases are also discussed. All these algorithms are based on a simple ring isomorphism that converts each of these problems from the Bernstein basis to an equivalent problem in the monomial basis. This isomorphism allows all the computations to be performed using only the original Bernstein coefficients; no conversion to monomial coefficients is required. This work has been accepted for publication in Computer Aided Geometric Design journal.

6.2.4. *Algebraic methods for solving polynomial systems*

Participants: Mohamed Elkadi, Bernard Mourrain.

Polynomial equations appear in many domains to represent geometric constraints, to formalise relations that connect physical variables, ...Solving such equations is a critical problem which involves algebraic techniques combined with numerical methods. The book [11] is an introduction to the resolution of such polynomial equations. We demonstrate how the geometry of algebraic variety defined by this type of equations, their dimensions, their degrees, their components can be analysed from the properties of the corresponding quotient algebra. We introduce fundamental methods used to study the solutions of these systems of algebraic equations, such as Gröbner bases, resolution via eigenvalues and eigenvectors, resultants computations. Bezoutian, duality, Gorenstein algebra, residues are also studied, from an effective point of view. Some applications and exercises illustrate these methods. The book based on Master courses given at the University of Nice can be useful to undergraduate students and researchers interested by effective algebraic geometry.

6.2.5. Symbolic-numeric methods for solving polynomial systems

Participant: Bernard Mourrain.

In the tutorial paper [16], we first discuss the motivation of doing symbolic-numeric computation, with the aim of developing efficient and certified polynomial solvers. We give a quick overview of fundamental algebraic properties, used to recover the roots of a polynomial system, when we know the multiplicative structure of its quotient algebra. Then, we describe the border basis method, justifying and illustrating the approach on several simple examples. In particular, we show its usefulness in the context of solving polynomial systems, with approximate coefficients. The main results are recalled and we prove a new result on the syzygies, naturally associated with commutation properties. Finally, we recall an algorithm for computing such border bases.

6.2.6. Factorization

Participants: Mohamed Elkadi, André Galligo, Martin Weimann.

In a paper presented and published at SNC '07 [19], André Galligo and Marc van Hoeij (Florida University USA) developed an original geometric approach, based on computation of monodromy actions, to compute factorization of bivariate polynomials with approximate coefficients.

In a paper, submitted at a special issue of the journal of Symbolic Computation, Mohamed Elkadi, André Galligo, Martin Weimann studied the algebraic interpolation problem in toric varieties. This allowed them to extended to the sparse case the algorithm for computing the absolute factorization of bivariate polynomials developed in the PhD thesis of Guillaume Chèze.

6.2.7. Random multivariate polynomials

Participant: André Galligo.

André Galligo has started a study of random multivariate polynomials in order to compute faster frequent special cases of multivariate factorization. In collaboration with Carlos d'Andrea and Martin Sombra from the University of Barcelona (Spain), using adapted estimates on resultants he established that the zeros of certain systems of n "random" polynomials in n variables concentatre on the product of n unit circles. This has been presented at Macis'07 conference.

6.2.8. Solving Toeplitz-block linear systems

Participants: Houssam Khalil, Bernard Mourrain, Michelle Schatzmann [Univ. Lyon I].

Structured matrices appear in various domains, such as scientific computing, signal processing, ...Among well-known structured matrices. Toeplitz and Hankel structures have been intensively studied. So far, few investigations have been pursued for the treatment of multi-level structured matrices.

In a paper submitted for publication, we re-investigate the resolution of Toeplitz systems $T\,u = g$ from a new point of view, by correlating the solution of such problems with syzygies of polynomials or moving lines. We show an explicit connection between the generators of a Toeplitz matrix and the generators of the corresponding module of syzygies. We show that this module is generated by two elements of degree n and the solution of $T\,u = g$ can be reinterpreted as the remainder of an explicit vector depending on g by these two generators.

This approach naturally extends to multivariate problems and we describe the structure of the corresponding generators for Toeplitz-block-Toeplitz matrices.

6.3. Algebraic Geometric Analysis

6.3.1. Vitushkin integral quantities at singularities

Participants: Lionel Alberti, Georges Comte.

Vitushkin invariants are integral quantities measuring the “room” or “complexity” of a real analytic set. For algebraic sets, they can be bounded in terms of the degree of the polynomials defining the set. We look at what happens if we want to localize these quantities at a point. In the complex case, the bounds relate to the multiplicity at the point. But in the real case, we find counter-examples. We then define a subclass of analytic sets that do not have too high dimensional singularities, and then recover a bound for the Vitushkin numbers in terms of the multiplicity by relating the multiplicity to the degree of the generators of the localization of the variety at the point. This work is still in progress and not yet published.

6.3.2. A new method to compute the topology of implicit algebraic curves

Participants: Lionel Alberti, Bernard Mourrain.

We design a new algorithm to compute the topology of a planar algebraic curve defined as the zero set of one square free polynomial in a rectangular domain. The method can be easily generalized to any semi-algebraic simply connected set. We describe two regularity criteria: One to test whether the curve is like a succession of function graphs in one direction, and the other one to test if the topology of the curve is conic at singularities. Then we reduce the original domain to one of these cases by applying a recursive subdivision algorithm. The mathematical tool to deal with the singular points is the topological degree which can be computed from the behavior on the boundary of the domain. The results were published in article [17].

6.3.3. Regularity criteria for the topology of algebraic curves and surfaces

Participants: Lionel Alberti, Bernard Mourrain.

We consider the problem of analysing the shape of an object defined by polynomial equations in a domain of $\mathbb{R}^2$ or $\mathbb{R}^3$. We describe regularity criteria which allow us to certify the topology of the implicit object in a box from information on the boundary of this box. Such criteria are given for planar and space algebraic curves and for algebraic surfaces. These tests can be used in subdivision methods in order to produce a polygonal approximation of the algebraic curves or surfaces, even if it contains singular points. We exploit the representation of polynomials in Bernstein basis to check these criteria and to compute the intersection of edges or facets of the box with these curves or surfaces. Our treatment of singularities exploits results from singularity theory such as an explicit Whitney stratification or the local conic structure around singularities. The results were published in article [12].

6.3.4. Subdivision methods for 2d and 3d implicit curves

Participants: Chen Liang [Univ. of Hong Kong], Bernard Mourrain, Jean-Pascal Pavone.

We describe a subdivision method for handling algebraic implicit curves in 2D and 3D. We use the representation of polynomials in the Bernstein basis associated with a given box, to check if the topology of the curve is determined inside this box by its cintersection with the boundary of this box. Subdivision solvers are involved for computing these points on the faces of the box, and segments joining these points are deduced to get a graph isotopic to the curve. Using envelops of polynomials, we show how this method allows to handle efficiently and accurately implicit curves with large coefficients. We report on implementation aspects and experimentations on 2d curves such as ridge curves or self intersection curves of parameterized surfaces, and on 3D curves such as silhouette curves of implicit surfaces, showing the interesting practical behavior of this approach. The work is published in article [15].

6.3.5. Subdivision tracing algorithm for a 4d implicit curve

Participants: Stéphane Chau, André Galligo.

The intersection of parameterized surfaces problem is one of the major task in Computer Aided Geometric Design (CAGD). In this domain, the surfaces are given by evaluations. So, if we want to deal with this topic, we have to choose a model of approximation at first, and then proceed to their intersection. Many authors use a mesh for each surface, and then intersect the corresponding triangles. However, if we use more complex shape primitives, then the quality of approximation will be better and so the intersection locus. For example, we can use Bézier surface patches of small degree. But if we want to use this kind of representation, we have to intersect efficiently such two polynomial parametrized surfaces. So, we have to study a four dimension implicit curve. We work on a subdivision algorithm to compute the topology of such curve. This approach is efficient and avoids some drawbacks that appear in projection methods which are frequently used. The implementation of this method is integrated in the software AXEL.

6.3.6. Intersection of algebraic surfaces

Participants: Daouda N'Diatta, Bernard Mourrain, Olivier Ruatta [Univ. Limoges].

An algorithm is developped for the computation of the topology of a non reduced space curve defined as the intersection of two implicit algebraic surfaces. It computes a Piecewise Linear Structure (PLS) isotopic to the original space curve. The algorithm is designed to provide the exact result for all inputs. It's a symbolic-numeric algorithm based on subresultant computation. Some algebraic criteria are given to certify the output of the algorithm. The algorithm uses only one projection of the non-reduced space curve augmented with adjency information around some "particular points" of the space curve. An algorithm is given to distinguish these particular points. These algorithms are beeing implemented in the MATHEMAGIX library. Our next goal is the computation of the topology of a non-reduced space curve is one of the essential step of the computation of the topology of an algebraic surface.

6.3.7. Dynamic and generic method for computing an arrangement of implicit curves

Participants: Julien Wintz, Bernard Mourrain.

Arrangements are very important issues in many application fields and have been studied for several years with different points of view. Arrangements are mainly computed using sweep line methods, such as the well known Bentley-Ottman algorithm for computing an arrangement of line segments. These methods are strongly related to the nature of objects and are not well suited for non-trivial objects such as closed curves or surfaces. We propose a new arrangement algorithm [23], which is generic in the sense that it is not related to the nature of the objects to be arranged. The algorithm is dynamic, which means, the arrangement structure is maintained under insertion and deletions of the objects. It is a subdivision method based on a regularity test to ensure that the cell is deemed relevant and on regions computation. This general framework has been first tested for computing an arrangement of implicit parmetric or polygonal curves. A curve is said to be regular in a cell if it is either x-monotonic, y-monotonic or if it contains one and only one singular points and if the number of branches stemming out from this singularity is exactly the number of intersections of the curves with the cell. To compute the number of branches stemming out from a singularity, we use degree theory. To compute the intersection of the curve with the cell border (as well as the topological degree), we use the Bernstein conversion of the polynomials representing the curve. The arrangement is represented by an augmented influence graph on regions obtained by subdivision and merging steps. This algorithm is implemented within the algebraic modeler AXEL for the case of implicit, parametric and piecewise linear curves. Applications of the curve arrangement algorithm are currently investigated before extending the framework to the third dimension.

6.3.8. Tensors decompositions and rank

Participants: Pierre Comon [I3S], Gene Golub [Stanford Univ.], Lek-Heng Lim [Stanford Univ.], Bernard Mourrain, Elias P. Tsigaridas.

The notion of rank which applies for matrices (tensors of order two) and quadratic forms (symmetric tensors of order two) can be extended to higher orders.

In a paper submitted for publication, we study various properties of symmetric tensors in relation to a decomposition into a sum of outer product of vectors. Rank and Symmetric Rank are generically equal, and they always exist in an algebraically closed field. The Generic Rank is generally not maximal, contrary to matrices, and is now known for any values of dimension and order. We are investigating algorithms to compute this decomposition, which are based on algebraic methods related to duality, structured Hankel matrices, eigenvalue and eigenvector computation and polynomial systems solving.

7. Other Grants and Activities

7.1. European actions

7.1.1. ACS

Participants: Lionel Alberti, Laurent Busé, Stéphane Chau, André Galligo, Bernard Mourrain [contact person], Daouda N'Diatta, Jean-Pascal Pavone, Jean-Pierre Técourt, Julien Wintz.

See the ACS project web site.
- Acronym: ACS, number FP6-006413
- Title: Algorithms for Complex Shapes.
- Specific Programme: IST
- RTD (FET Open)
- Start date: started 1st May, 2005 - Duration: 3 years
- Participants:
  Univ. Groningen (Netherlands) [coordinating site]
  ETH Zürich (Switzerland),
  Freie Universität Berlin (Germany),
  INRIA Sophia Antipolis (Galaad & Geometrica),
  MPI Saarbrücken (Germany),
  National Kapodistrian University of Athens (Greece),
  Tel Aviv University (Israel),
  GeometryFactory Sarl.
- Abstract: computing with complex shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension. Certified topology and numerics, applications to shape approximation, shape learning, robust modeling.

7.1.2. AIM@SHAPE

Participants: Lionel Alberti, Laurent Busé, Emmanuel Briand, Stéphane Chau, Mohamed Elkadi, Ioannis Emiris, André Galligo, Thi Ha Lê, Bernard Mourrain [contact person], Jean-Pascal Pavone, Julien Wintz.

See the AIM@SHAPE project web site
- Acronym: aim@shape, number NoE 50766
- Title: AIM@SHAPE, Advanced and Innovative Models And Tools for the development of Semantic-based systems for Handling, Acquiring, and Processing knowledge Embedded in multidimensional digital objects.
- Type of project: network of excellence
- Beginning date: 1st of january 2004 - During: 4 years
- Partners list:
  CNR - Consiglio Nazionale delle Ricerche,
  DISI - Universita di Genova,
  EPFL - Swiss Federal Institute of Technology,
  IGD - Fraunhofer,
  INPG - Institut National Polytechnique de Grenoble,
  INRIA,

CERTH - Center for Research and Technology Hellas,
UNIGE - Université de Genève,
MPII - Max-Planck-Institut für Informatik,
SINTEF,
Technion CGGC,
TUD - Darmstadt University of Technology,
UU - Utrecht University,
WIS - Weizmann Institute of Science.

- Abstract of the project: it is aimed at coordinating research on representing, modeling and processing knowledge related to digital shapes, where by shape it is meant any individual object having a visual appearance which exists in some (two-, three- or higher- dimensional) space (e.g., pictures, sketches, images, 3D objects, videos, 4D animations, etc.).

7.2. Bilateral actions

7.2.1. Associated team CALAMATA

Participants: Ioannis Emiris, Bernard Mourrain [contact person], Nikos Pavlidis, Elias Tsigaridas, George Tzoumas, Michael Vrahatis.

The team of Geometric and Algebraic Algorithms at the National University of Athens, Greece, has been associated with GALAAD since 2003. See its web site.

This bilateral collaboration is entitled CALAMATA (CALculs Algébriques, MATriciels et Applications). The Greek team (http://www.di.uoa.gr/~erga/) is headed by Ioannis Emiris. The focus of this project is the solution of polynomial systems by matrix methods. Our approach leads naturally to problems in structured and sparse matrices. Real root isolation, either of one univariate polynomial or of a polynomial system, is of special interest, especially in applications in geometric modeling, CAGD or nonlinear computational geometry. We are interested in computational geometry, actually, in what concerns curves and surfaces.

In 2007, we had the visit of I.Z. Emiris and G. Tzoumas at Sophia Antipolis to work with us on resultant computation and Voronoi diagrams of conics.

7.2.2. NSF-INRIA collaboration

Participants: Laurent Busé, André Galligo, Mohamed Elkadi, Bernard Mourrain [contact person].

The objective of this collaboration between GALAAD and the Geometric Modeling group at Rice University in Houston, Texas (USA) is to investigate techniques from Effective Algebraic Geometry in order to solve some of the key problems in Geometric Modeling and Computational Biology. The two groups have similar interests and complementary strengths. Effective Algebraic Geometry is the branch of Algebraic Geometry that pursues concrete algorithms rather than abstract proofs. It deals mainly with practical methods for representing polynomial curves and surfaces along with robust techniques for solving systems of polynomial equations. Many applications in Geometric Modeling and Computational Biology require fast robust methods for solving systems of polynomial equations. Here we concentrate our collective efforts on solving standard problems such as implicitization, inversion, intersection, and detection of singularities for rational curves and surfaces. To aid in modeling, we shall also investigate some novel approaches to represent shape.

7.2.3. PAI Procore collaboration

Participants: Laurent Busé, Stéphane Chau, Yi-King Choi [Hong Kong Univ.], André Galligo, Yang Liu [Hong Kong Univ.], Wenping Wang [Hong Kong Univ.], Julien Wintz.

The objective of this collaboration is to conduct research in effective algebra for solving problems in geometric modeling. We investigate the use of implicit models, for compact and efficient shape representation and processing. The application domains are Computer Aided Geometric Design, Robotics, Shape compression, Computer Biology. We focus on algebraic objects of small degree such as quadrics, with the aim to extend the approach to higher degree. In particular, we are interested in the following problems:

- Shape segmentation and representation using quadrics.
- Shape processing using quadrics.
- Collision detection for objects defined by quadric surfaces.

Experimentation and validation will lead to joint open source software implementation, dedicated to quadric manipulations. A package collecting these tools will be produced.

Wenping Wang visited Sophia Antipolis in May and gave a talk in our seminar “Formes & Formules”. Bernard Mourrain and Julien Wintz visited Hong-Kong University for one week from October 21th. This meeting was the occasion to work on modelisation with quadrics including intersection problems, arrangement computation and segmentation by quadrics.

7.2.4. PAI Picasso collaboration

Participants: Laurent Busé, Marc Dohm, Mohamed Elkadi, André Galligo, Bernard Mourrain.

This is a collaboration with the university of Barcelona. The Spanish team is headed by Carlos D’Andrea. The objective of this collaboration is to conduct research in elimination theory and to explore its applications for solving problems in geometric modeling. The following four points will be considered:

- principal case of elimination theory, resultants,
- Effective computation of a resultant system,
- degree and height of resultant systems,
- applications in CAGD.

Laurent Busé, Marc Dohm and André Galligo gave a one week visit to the university of Barcelona from October 8th, October 15th and October 22th respectively.

7.2.5. ECOS-Sud collaboration

Participants: Laurent Busé, Stéphane Chau, Marc Dohm, Mohamed Elkadi, André Galligo, Bernard Mourrain, Julien Wintz.

The first objective of this collaboration with the team of A. Dickenstein at the university of Buenos Aires, Argentina is the development of effective methods for geometric modeling, with a special focus on singularity and numerical stability problems. This includes intersection problems for curves or surfaces, change of representations such as implicitisation via syzygies and moving planes, polytopes analysis and puiseux expansions ...A second objective is the developpement of open tools dedicated to such problems which could be shared by the different groups working on this topic.

This year we had the visit of N. Botbol (Ph.D student, May 1st - 30) and A. Dickenstein (Professor, November 25- December 7) in GALAAD. The visits to Buenos Aires of M. Dohm (Ph.D student) were from October 19 to November 23 and the one of A. Galligo from October 2nd to the 16th.

7.3. National actions

7.3.1. ANR DECOTES, Tensorial decomposition and applications

Years: 2006-2009.
Partners: I3S, CNRS; LTSI, INSERM; GALAAD, INRIA; SBP, Thales communications.

The problem of decomposition of a symmetric or non-symmetric tensor in minimal way is an important problem, which has applications in many domains. It is essential in the process of Blind Identification of Under-Determined Mixtures (UDM), i.e., linear mixtures with more inputs than observable outputs and appear in many application areas, including speech, mobile communications, machine learning, factor analysis with k-way arrays (MWA), biomedical engineering, psychometrics, and chemometrics. The aim of the project DECOTES is to study the key theoretical problems of such decompositions and to devise numerical algorithms dedicated to some selected applications.

With Elias Tsigaridas, at a post-doctoral position from 1st October, we started investigating algebraic methods to compute such a decomposition, in order to extend the approach by Sylvester for binary forms to polynomials with more variables.

7.3.2. ANR GECKO, Geometry and Complexity

Years: 2005-2008
Partners: ALGO, INRIA; GALAAD, UNSA, INRIA; LIX, Ecole Polytechnique; Univ. Paul Sabatier, Toulouse.

The technologic and scientific development of our society raises problems, which after transformation and simplification often lead to systems of polynomial or differential equations and inequalities. The topics of the project GECKO are the study, analysis and implementation of solvers for the resolution of such problems, based on a geometric approach. It involves fundamental operations with univariate and multivariate polynomials (such as Newton process, factorisation, elimination), structured matrices and linear differential equations (non-commutative elimination and integration). One of the objectives is to develop efficient algorithms with good complexity bound, by taking into account the geometric properties of the solutions. These algorithms are implemented in the framework of the open and modular system MATHEMAGIX. See http://gecko.inria.fr/ for more details.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We organize a seminar called “Formes & Formules”. The list of talks is available at http://www-sop.inria.fr/galaad/content/blogcategory/23/162/.

8.1.2. Comittee participations

- A. Galligo is a member of the program comittee of the conference MEGA’07, held in Austria.
- B. Mourrain was chair of the program committee of ISSAC’07, main annual conference on Symbolic Computation (Waterloo, Canada, 29 July-1 August). He was a member of the program committee of SNC’07 (Symbolic Numeric Computation, London, Canada), and of GMP’08 (Geometric Modelisation and Processing, Hangzhou, China, 2008).

8.1.3. Editorial committees

- L. Busé, M. Elakdi and B. Mourrain were guest editors of a special issue of Theoretical Computer Science, for the conference *Computational Algebraic Geometry and Applications*, held in Nice on the occasion of the 60th birthday anniversary of André Galligo. This special issue will be published at the beginning of 2008.
- B. Mourrain is a member of the editorial board of the Journal of Symbolic Computation.
- B. Mourrain (with C. D’Andrea) are guest editors of the special issue of Journal of Symbolic Computation, related to ISSAC’07.

- A. Galligo (with J. Schicho and LM. Pardo) are guest editors of the special issue of Journal of Symbolic Computation, related to MEGA'07.

8.1.4. Organisation of conferences and schools

- B. Mourrain organised a minisymposium "Calcul formel et Géométrie" at the SMAI conference (Praz Sur Arly, France, 5-7 June 2008).
- A. Galligo and B. Mourrain organised the meeting of the ANR GECKO at Sophia Antipolis (November 19 - 21).

8.1.5. PHD thesis commitees

- M. Elkadi, A. Galligo and B. Mourrain were members of the Ph.D. committee Le Thi Ha. B. Mourrain was president of the Jury.

8.1.6. Other comittees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- B. Mourrain is a member of the scientific council of SARIMA.
- A. Galligo is one of the 3 members of the stirring committee of ISSAC.
- A. Galligo was the President du jury of the HDR thesis of Monique Teillaud.

8.1.7. WWW server

- http://www-sop.inria.fr/galaad/.

8.2. Participation at conferences and invitations

- M. Dohm gave a talk at the "Journées Nationales de Calcul Formel", Luminy, France, Jan. 29 - Feb. 2; gave a talk at the "International Symposium on Symbolic and Algebraic Computation", Waterloo, Canada, July 29 - Aug. 1; was invited to give a talk at the algebraic geometry seminar of the University of Barcelona, Spain, Oct. 7 - Oct. 15; was invited to give a talk at the algebraic geometry seminar of the University of Buenos Aires, Oct. 19 - Nov. 22
- L. Alberti gave a talk at the "Workshop on robust shape operations", Sophia-Antipolis, 26 - 28 septembre 2007; at "Pacific Graphics 2007", Maui, HI, USA, Oct. 19 - Nov. 2 and at the "Journées 2007 de l'ANR Gecko", Sophia-Antipolis, 19 - 21 november 2007. He attended the "Journées Nationals de Calcul Formel", Luminy, France, Jan. 29 - Feb. 2.
- L. Busé was invited to give a talk at the commutative algebra and algebraic geometry seminar of the university of Strasbourg, France, april 28-30; was invited to the conference "Non linear computational geometry", May 28-June 2nd, Minneapolis, USA; was invited to the workshop "Syzygies and Modelisation" at Oberwolfach, Germany, November 25th-December 1st.
- S. Chau gave a talk at "Computational methods for algebraic spline surfaces 2007", Strobl, 10 - 14 septembre 2007; at "Workshop on robust shape operations", Sophia-Antipolis, Sept. 26 - 28 and at "Journées 2007 de l'ANR Gecko", Sophia-Antipolis, November 19 - 21.
- Daouda Niang Diatta gave a talk "Computing the topology of a planar and a space space algebraic curve" at the national meeting: Journées Nationales du Calcul Formel which held in Luminy from January 29th to February 2nd at the CIRM. He present the poster "Topology of a non reduced space curve" at the conference ISSAC, Waterloo, Canada. He gave a talk "Computing the topology of a non reduced implicit space curve with certainty" at "Computational methods for algebraic spline surfaces 2007" Strobl, Sept. 10 - 14 2007. He gave a talk "Computing the topology of a non reduced implicit space curve with certainty" at "Journées 2007 de l'ANR Gecko", Sophia-Antipolis, November 19 - 21, 2007.

- A. Galligo was invited to give a talk at the conference “Non linear computational geometry”, May 28-June 2nd, Minneapolis, USA; gave a talk at the “International Symposium on Symbolic and Algebraic Computation”, Waterloo, Canada, July 29 - Aug. 1 and at the Symbolic Numeric Computation conference, London, Canada, July 25-27 ; was invited speaker at the Compass workshop, Strobl, Austria, 10 - 14 september 2007; was invited to give a talk the algebraic geometry seminar of the University of Buenos Aires, and of Montevideo, Uruguay, Oct. 4 - Oct. 16. was invited to give a talk the algebraic geometry seminar of the University of Barcelona, Spain, Oct. 22 - 29 ; was invited to the workshop “Syzygies and Modelisation” (Oberwolfach, Germany, November 25th-December 1st) and gave a talk at the workshop MACIS’07 in Paris Dec. 5-7.
- H. Khalil gave a talk “Toeplitz and Toeplitz-block-Toeplitz matrices and their correlation with syzygies of polynomials” at the 2nd International Conference on Matrix Methods and Operator Equations, July 23 - 28, Moscow, Russia
- B. Mourrain attended the meeting “Journées Nationales de Calcul Formel” (Luminy, France, January 29 - February 1st); gave a talk at the Workshop of the ACS project (Berlin, May 9-11) on “solving polynomial equations by symbolic-numeric methods”; was invited to give a talk on “Subdivision methods to compute the topology of algebraic curves and surfaces” at the conference “Non Linear Computational Geometry”, May 28-June 2nd, Minneapolis, USA; organised a minisymposium “Calcul formel et Géométrie” at the national meeting of SMAI (Praz Sur Arly, France, June 5-7); gave a talk on “An algebraic version of a theorem of Whitney” at the conference MEGA (Strobl, Austria, June 25-29); gave a talk at the minisymposium Approximate algebraic methods for computer-aided geometric design of ICIAM (Zurich, Swiss, 16-18 July); attended the conferences SNC (London, Canada, July 25-27) and ISSAC (Waterloo, Canada, July 29 - August 1st); was invited to give a talk on “Regularity criteria for the topology of semi-algebraic curves and surfaces” at the conference “Mathematic of Surfaces” (Sheffield, England, Sept. 3-6); visited Wenping Wang’s team at the University of Hong Kong (Oct. 22 - 27); attended the conference Pacific Graphics’09 (Maui, USA, Oct. 29 - Nov. 2); was invited to give a tutorial on “subdivision methods to solve non-linear equations” at the SIAM conference on Geometric Design (San Antonio, USA, Nov. 4-8); was invited to the workshop “Syzygies and Modelisation” (Oberwolfach, Germany, November 25th-December 1st).
- Vikram Sharma gave a talk to present the paper "Complexity of Real Root Isolation Using Continued Fractions" at the conference ISSAC (Waterloo, Canada, July 29 - August 1st). He also gave a talk titled "Robust Approximate Zeros in Banach Space" at the Zero Workshop, Seoul, South Korea.
- Julien Wintz gave a talk “Design et architecture du modeleur algébrique géométrique AXEL” at the national meeting: Journées Nationales du Calcul Formel which held in Luminy from January 29th to February 2nd at the CIRM. He has demonstrated an implementation of arrangement computation of implicit curves at the ACS Review Meeting and General Workshop, May 8 - 11, Berlin, Germany. He presented his work on arrangement computation and their implementation within the algebraic geometric modeler AXEL within the european project ACS at the occasion of the ACS Workshop on Robust Shape Operations, September 26-28 in Sophia-Antipolis. He gave a talk “A subdivision arrangement algorithm for semi-algebraic curves: an overview” as a poster paper at the 15th Pacific Conference on Computer Graphics and Applications. The conference held in Maui, Hawaii from October 29th to November 2nd. He gave a talk “The algebraic geometric modeler AXEL: an overview” during the meeting of the ANR Gecko in Sophia-Antipolis on November 20th.

8.3. Formation

8.3.1. Teaching at Universities

- Lionel Alberti, License 2nd year of the University of Nice, algebra course TA (Teacher Assistant), 36h; License 2nd year, game theory TA, 28h.

- Laurent Busé, Master 2nd year of the University of Nice, “Algebraic curves and surfaces for CAGD”, 30 hours.
- Bernard Mourrain, Master 2nd year of the University of Nice, “Algorithms for curves and surfaces”, 20 hours.
- Stéphane Chau, Licence 1st year of the University of Nice, mathematics course, 60h.
- Julien Wintz, IUT 1st year alternated of the University of Nice, Computer architecture, 15h; IUT 1st year alternated, Object Oriented Programming, 45h.

8.3.2. PhD theses in progress

- Lionel Alberti, *Vers une théorie quantitative des singularités*, ED SFA, UNSA.
- Eliman Ba, *Résultants, calculs et applications*,UNSA.
- Stéphane Chau, *Study of singularities used in CAGD*, UNSA.
- Marc Dohm, *Algorithmique des courbes et surfaces algébriques*, UNSA.
- Daouda N'Diatta, *Résultants et sous-résultants et applications*, Univ. Limoges.
- Houssam Khalil, *Matrices structurées en calcul symbolique et numérique*, Univ. Lyon I.
- Julien Wintz, *Algebraic methods for geometric modeling*, INRIA Sophia-Antipolis.

8.3.3. Defended PhD thesis

- Thi Ha Lê, *Classification and intersections of some parametrized surfaces and applications to CAGD*, UNSA.

8.3.4. Internships

See the web page of our interships.

- Redouane Soum *Gestions des modules dans* MATHEMAGIX*, liens avec un solveur par homotopie*, June 15 - September 30.
- Rob van Kruijsdijk, *Quadric surfaces for the approximation of branching structures*, August 15 - November 15.

9. Bibliography

Major publications by the team in recent years

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[7] A. GALLIGO. *Théorème de division et stabilité en géométrie analytique locale*, in "Ann. Inst. Fourier", vol. 29, 1979, p. 107–184.

[8] A. GALLIGO, S. WATT. *A Numerical Absolute Primality Test for Bivariate Polynomials*, in "Proc. Annual ACM Intern. Symp. on Symbolic and Algebraic Computation", 1997, p. 217–224.

[9] B. MOURRAIN. *Algorithmes et Applications en Géométrie Algébrique*, Ph. D. Thesis, Université de Nice Sophia-Antipolis, September 1997.

[10] B. MOURRAIN, V. Y. PAN. *Multivariate Polynomials, Duality and Structured Matrices*, in "J. of Complexity", vol. 16, n^{o} 1, 2000, p. 110-180, ftp://ftp-sop.inria.fr/galaad/mourrain/0004-MP-jocstruct.ps.gz.

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