INRIA

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team Mathfi

Financial Mathematics

Paris - Rocquencourt

THEME NUM

Activity Report

2007

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1. Team

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2. Overall Objectives

2.1. Highlights of the year

Concerning research, a lot of effort has been put this year in the study of Lévy processes in financial modelling with significant contributions in numerical methods for option pricing and control.

Concerning staff, the main evolution in 2007 is the recruiting of Aurélien Alfonsi at ENPC/Cermics in the probability team. He is joining the Mathfi team in September 2007 after spending one academic year as a postdoc in the technical university of Berlin supervised by Alexander Schied. Moreover Marie-Claire Kammerer-Quenez, formely assistant-professor at the University of Marne-la-Vallée has been recruted as a professor at the Universty Paris-Diderot (Paris 7).

2.2. Introduction

MathFi is a joint INRIA project-team with ENPC (CERMICS) and the University of Marne la Vallée, located in Rocquencourt and Marne la Vallée.

The development of increasingly complex financial products requires the use of advanced stochastic and numerical analysis techniques. The scientific skills of the MathFi research team are focused on probabilistic and deterministic numerical methods and their implementation, stochastic analysis, stochastic control. Main applications concern evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, calibration of financial models. Special attention is paid to models with jumps, stochastic volatility models, asymmetry of information. The MathFi project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. Premia web Site: http://www.premia.fr .

3. Scientific Foundations

3.1. Numerical methods for option pricing and hedging

Keywords: *Euler schemes*, *Malliavin calculus*, *Monte-Carlo*, *approximation of SDE*, *finite difference*, *tree methods*.

Participant: all the Mathfi team.

Efficient computations of prices and hedges for derivative products is a major issue for financial institutions.

Monte-Carlo simulations are widely used because of their implementation simplicity and because closed formulas are usually not available. Nevertheless, efficiency relies on difficult mathematical problems such as accurate approximation of functionals of Brownian motion (e.g. for exotic options), use of low discrepancy sequences for nonsmooth functions, quantization methods etc. Speeding up the algorithms is a constant preoccupation in the developement of Monte-Carlo simulations. Another approach is the numerical analysis of the (integro) partial differential equations which arise in finance: parabolic degenerate Kolmogorov equation, Hamilton-Jacobi-Bellman equations, variational and quasi–variational inequalities (see [62]).

This activity in the MathFi team is strongly related to the development of the Premia software.

3.2. Model calibration

A important research direction in mathematical finance is the modeling of the so called *implied volatility smile*, that is, the fact that different traded options on the same underlying have different Black-Scholes implied volatilities. The smile phenomenon clearly indicates that the Black-Scholes model with constant volatility does not provide a satisfactory explanation of the prices observed in the market and has led to the appearance of a large variety of extensions of this model aiming to overcome the above difficulty. Some popular model classes are: the local volatility models (where the stock price volatility is a deterministic function of price level and time), diffusions with stochastic volatility, jump-diffusions, and so on. An essential step in using any such approach is the *model calibration*, that is, the reconstruction of model parameters from the prices of traded options. The main difficulty of the calibration problem comes from the fact that it is an inverse problem to that of option pricing and as such, typically ill-posed.

The calibration problem is yet more complex in the interest rate markets since in this case the empirical data that can be used includes a wider variety of financial products from standard obligations to swaptions (options on swaps). The underlying model may belong to the class of short rate models like Hull-White [60], [52], CIR [54], Vasicek [74] etc. or to the popular class of LIBOR (London Interbank Offered Rates) market models like BGM [53].

The choice of a particular model depends on the financial products available for calibration as well as on the problems in which the result of the calibration will be used.

The calibration problem is of particular interest for MathFi project because due to its high numerical complexity, it is one of the domains of mathematical finance where efficient computational algorithms are most needed.

3.3. Malliavin calculus and applications in finance

Keywords: *Malliavin calculus*, *greek computations*, *sensibility calculus*, *stochastic variations calculus*.
Participants: V. Bally, B. Jourdain, A. Kohatsu-Higa, D. Lamberton, B. Lapeyre, A. Sulem, A. Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [63]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [67], D. Bell [48] D. Ocone [69], B. Øksendal [77]) give expositions of the subject. See also V. Bally [47] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [68].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [64], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

Let us try to give an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods. We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be “smooth” in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequenses of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [57] and [56] and the papers by Bally et al, Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [61].

More recently the Malliavin calculus has been used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P.Imkeller in [61]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [61]).

3.4. Anticipative stochastic calculus and insider trading

Participants: A. Kohatsu-Higa, A. Sulem.

We study controlled stochastic systems whose state is described by anticipative stochastic differential equations. These SDEs can interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals [73]. This methodology is applied for studying utility maximization problems with insiders.

3.5. Stochastic Control and Backward Stochastic Differential equations

Keywords: *BSDE*, *Hamilton-Jacobi-Bellman*, *Stochastic Control*, *free boundary*, *risk-sensitive control*, *singular and impulse control*, *variational and quasi-variational inequalities*.

Participants: V. Bally, J.-Ph. Chancelier, M.C. Kammerer-Quenez, A. Sulem.

Stochastic control consists in the study of dynamical systems subject to random perturbations and which can be controlled in order to optimize some performance criterion. Dynamic programming approach leads to Hamilton-Jacobi-Bellman (HJB) equations for the value function. This equation is of integrodifferential type when the underlying processes admit jumps (see [16]). The theory of viscosity solutions offers a rigourous framework for the study of dynamic programming equations. An alternative approach is the study of optimality conditions (stochastic maximum principle) which leads to backward stochastic differential equations (BSDE). Typical financial applications arise in portfolio optimization, hedging and pricing in incomplete markets, calibration. BSDE's also provide the prices of contingent claims in complete and incomplete markets and are efficient tools to study recursive utilities as introduced by Duffie and Epstein [55].

4. Application Domains

4.1. Application domains

- Option pricing and hedging
- Calibration of financial models
- Modeling of financial asset prices
- Portfolio optimization
- Insurance-reinsurance optimization policy
- Insider modeling, asymmetry of information

5. Software

5.1. Development of the software PREMIA for financial option computations

Keywords: *calibration*, *hedging*, *options*, *pricer*, *pricing*.

Participants: A. Alfonsi, V. Bally, J-Ph.. Chancelier, B. Jourdain, A. Kbaier, A. Kohatsu Higa, A. Kolotaev, B. Lapeyre, J. Lelong, N. Privault, A. Sulem, P. Tankov, X. Wei, X. You, A. Zanette, V. Zherder.

Figure 1.

5.1.1. Description of Premia

PREMIA is a platform dedicated to the development of algorithms and scientific documentation for *option pricing*, *hedging* and *model calibration* (http://www.premia.fr).

This project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is:

1. to be a powerful testing platform for comparing different numerical methods between each other;
2. to build a link between professional financial teams and academic researchers;
3. to provide a useful teaching support for Master and PhD students in mathematical finance.

The development of Premia started in 1999 and 9 are released up to now and registered at the APP agency.

5.1.2. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. Equity derivatives:

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

   For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods Malliavin calculus based methods.

   Dynamic Hedging for Black-Scholes and jump models is available.

   Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. Interest rate derivatives

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. Credit derivatives: CDS, CDO, CDO^2

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. Hybrid products:

   PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. Energy derivatives: swing options

   Mean reverting and jump models ar considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

Premia 9 has been delivered to the consortium members in February 2007. It contains routines for pricing interest rate derivatives in the Libor Market Model, pricing CDOs with various algorithms and pricing equity derivatives using finite Difference for high-dimensional problems. In particular a calibration toolbox for Libor Market model using a database of swaption and cap implied volatility provided by CDC-IXIS has been developped.

5.1.3. Latest features

The release 10 of Premia, developped in 2007, will be delivered in February 2008 to the Consortium. Below is it detailed content:

- **Interest Rate Derivatives:**

  - Calibration of jump diffusion Libor model to caplet price by the Belomestny and Schoenmakers's algorithm [49] (implemented by X. Wei)
  - Pricing Callable Libor Exotics (CLEs) in forward Libor models [72], [70], [71] implemented by J. Yu)

- **Inflation-Indexed Derivatives**
  - F. Mercurio and N. Moreni (2006), "Inflation with a smile", Risk March, Vol. 19(3), 70-75.
  - Pricing of inflation-indexed caplets in market model with stochastic inflation by the algorithm of Mercurio and Moreni, based on the Carr and Madan Fourier transform approach [65] (implemented by X. Wei)
  - Pricing of Inflation-Indexed Derivatives (implemented by X. Wei) [66]

- **Credit Risk Derivatives (CDO, $CD0^2$)**
  - Saddlepoint approximation method for pricing CDOs, J.Comp.Fin. Volume 10, Number 1, Fall 2006, *Yang-Zhang (2006)*
  - Valuing Credit Derivatives Using an Implied Copula Approach, Hull, White (2006)
  - Monte Carlo methods for $CD0^2$.

- **Equity Derivatives**
  - Finite Difference for 3D problems with jumps (collaboration with Prof. Natalini IAC Rome).
  - D. Pommier: Wavelet sparse approach for Heston model.
  - Numerical Quadrature methods for pricing Discrete Monitoring Asian and barrier options (collaboration with Prof. Fusai University of Novara (Italy).
  - E. Dia: Closed Formulas for pricing American, Barrier options and Lookback options in Kou model.
  - A. Sulem and P. Tankov: Utility-based pricing and hedging in models with jumps Indifference pricing with exponential utility. Finite difference for integro-differential HJB equations.
  - B. Jourdain and M. Sbai: Exact Monte Carlo Computation for Asian Options.
  - P. Etoré: Adaptive stratification for Asian options.
  - J. Lelong: Variance Reduction for Mountains Range options.
  - Generalizing the Black-Scholes formula to multivariate contingent claims: *Carmona-Durrleman, J. Comp. Fin. Volume 9, Number 2, Winter 2005/06*
  - Optimal Fourier Inversion in Semy-Analytical Option Pricing, Heston model *R. Lord, C. Kahl, Journal of Computational Finance, Volume 10, Number 4, 2007.*
  - Pricing Variance Swap, Options on Realized Variance in Tempered Stable model: *P. Carr, D. Madan, H. Geman, M. Yor, Finance and Stochastics, 2005, issue 4*
  - Pricing Variance/Volatility Swap, Options on Realized Variance/Volatility in the Heston model
  - Realized Volatility and Variance: Options via Swaps : *P. Carr, R.Lee, Risk, May 2007*
  - Numerical algorithms for backward differential equations.

- **Energy Derivatives**
  - Pricing of Swing options in Lévy models

5.1.4. Publications

A special issue dedicated to Premia will appear in the journal Banques et Marché [14] . This issue contains the following articles:

- Premia: A platform for pricing financial derivatives (B. Lapeyre, A. Sulem, A. Zanette) (2 pages)

- Jump-diffusion models: a practitioner's guide (P. Tankov, K. Voltchokova) (24 pages)

- A review of recent results on approximation of solutions of stochastic differential equations (B. Jourdain, A. Kohatsu-Higa) (18 pages)

- Partial differential equations in finance (Y. Achdou, O. Bokanowski, T. Lelièvre) (36 pages)

- Pricing Parisian barrier options (C. Labart and J. Lelong) (24 pages)

- Libor Market Model in Premia : Bermudan pricer, Stochastic Volatility and Malliavin Calculus (J. Da Fonseca, M. Messaoud) (23 pages)

- Calibration of the LIBOR market model - implementation in Premia (N. Privault, X. Wei) (20 pages)

- An introduction to the multiname modelling in credit risk (A. Alfonsi) (24 pages)

5.1.5. Software organization

The software Premia provides a collection of C/C++ routines and scientific documentation in PDF and HTML. More precisely, Premia is composed of :

- a library designed to describe derivative products, models, pricing methods and which provides basic input/output functionalities.
- a collection of pricing routines: in this way, the routines of Premia can easily be plugged into other financial softwares.
- a scientific documentation system. It is created from hyperlinked PDF files which describe the pricing routines and the general numerical methods involved like Monte Carlo methods, lattice methods, etc.

Premia is available for Windows and Linux operating systems. It provides Excel and Scilab/Nsp interfaces. Reports in PDF can be automatically generated at the end of each computation session.

The size of Premia is 453 Mb, with 10.9 Mb of source code in C.

In 2007, the architecture and interface of Premia have been improved (A. Kolotaev and P. Tankov for Excel and J-Ph. Chancelier and J. Lelong for NSP/Scilab interfaces). J. Lelong has integrated the American Monte Carlo module in the core of Premia making it available both to the Nsp and Excel interfaces. Moreover, he has implementated a C API for matrices and hypermatrices to ease future developpements in Premia.

Antonino Zanette and J. Lelong are taking care of the maintenance of the code and test of the new routines.

5.1.6. Consortium Premia

Premia is developed in interaction with a consortium of financial institutions or departments presently composed of: CALYON, the Crédit Industriel et Commercial, Société Générale, Natexis and IXIS CIB (Corporate & Investment Bank) now unified as Natixis. The participants of the consortium contribute to finance the development of Premia and help to determine the directions in which the project evolves. They have access to the complete software with the source and the documentation. Every year, a new release is delivered to the Consortium members.

Moreover, a restricted version of Premia is available on Premia web site http://www.premia.fr and can be downloaded with a special license for academic and evaluation purposes.

This year the Consortium is being extended to Japan

http://www-csfi.sigmath.es.osaka-u.ac.jp/premia/index.html.

An extension to Hong-Kong is in preparation.

6. New Results

6.1. Numerical methods for option pricing

Participants: A. Alfonsi, B. Jourdain, C. Labart, J. Lelong.

A. Alfonsi and B. Jourdain are working on duality for American options. In [46], they had obtained a Call-Put duality formula for perpetual American options and a theoretical calibration procedure of the volatility function. In [20], they have extended this result to general payoff functions that include Call and Put as particular cases.

J. Lelong and C. Labart are working on the pricing of double barrier Parisian options using Laplace transforms (see [29]. A paper will be soon submitted, the numerical algorithm has already been implemented in PREMIA.

6.2. Discretization of stochastic differential equations

Participants: A. Alfonsi, B. Jourdain, M. Sbai.

A. Alfonsi has started to work on the discretization of the Cox-Ingersoll-Ross process. This SDE presents a singularity that makes its discretization non standard. Many schemes had been proposed in the literature, but most of them were really satisfactory only for small volatility parameters ($\sigma^2 \leq 4a$). In [39], A. Alfonsi proposes a scheme that is accurate for all volatility parameters $\sigma^2 > 0$: a quadratic weak convergence in function of the time-step is shown. Then, using the Ninomiya-Victoir approach, he applies this scheme to the widespread Heston model (for which large volatility parameters are observed) and gets interesting results.

A method allowing exact simulation of the solution of one-dimensional stochastic differential equations has been recently proposed by Beskos and Roberts [51] and improved in a joint work with Papaspiliopoulos [50]. In [26], B. Jourdain and M. Sbai have adapted this method in order to deal with the pricing of arithmetic average Asian options in the Black-Scholes model. Unlike previous Monte-Carlo methods devoted to this problem, this new algorithm does not suffer a biais due to time discretization.

Existing weak convergence results for discretization schemes of SDEs are not adapted to deal with the payoffs of exotic options which involve the whole sample-path of the underlying. M. Sbai and B. Jourdain are trying to analyse the convergence properties for better suited criteria such as the Vasertein distance on the sample-path space.

6.3. Monte Carlo simulations and stochastic algorithms

Keywords: *Monte-Carlo*, *variance reduction*.

Participants: V. Bally, E.H.A. Dia, B. Jourdain, C. Labart, D. Lamberton, B. Lapeyre, J. Lelong.

6.3.1. Variance reduction by adaptive stratified sampling

The stratified sampling technique consists in computing the expectation of a random variable as a sum over strata which form a partition of the state space. The contribution of each stratum is the product of the probability for the random variable to belong to this stratum by the expectation of the random variable conditioned to belong to the stratum. The probabilities are known explicitely whereas the conditional expectations are computed by the Monte Carlo method.

In this approach, the probability of each stratum has to be known analytically and simulation according to the corresponding conditional law must be feasible. It is well-known that allocation of the numbers of random drawings in the strata proportional to their probability achieves variance reduction. But the optimal allocation which involves the conditional variances in the strata is unknown. For fixed strata, P. Etoré (postdoc at ENPC) and B. Jourdain have developped an algorithm aimed at adaptively estimating the optimal allocation of the drawings between the strata when computing the stratified Monte-Carlo estimator of the expectation. For their estimator, they prove a central limit theorem with asymptotic variance equal to the optimal variance and confirm its performance by numerical experiments [40]. With G. Fort and E. Moulines, they are currently trying to develop a stochastic algorithm able to optimize dynamically the boundaries between the strata when these boundaries are hyperplanes orthogonal to a given direction.

6.3.2. Monte-Carlo methods for exotic options in models with jumps

D. Lamberton and his phD student El Hadj Aly DIA's have obtained some results concerning lookback options in jump-diffusion models.

6.3.3. Backward Stochastic Differential equations (BSDEs)

C. Labart has defended her thesis on October, 2007 and arrived immediatly after for a postdoctoral position in Mathfi. She is presently implementing in Premia a numerical algorithm for solving BSDEs, based on Picard's procedure and a sequential Monte Carlo method. She is also working with V. Bally on a slightly different algorithm, using Malliavin calculus techniques, for solving BSDEs.

6.4. Computation of sensitivities and conditional expectations using Malliavin calculus in models with jumps

Keywords: *Malliavin calculus*, *greeks*.

Participants: V. Bally, M.P. Bavouzet, A. Kohatsu-Higa, M. Messaoud, N. Privault, X. Wei, A. Zanette.

V. Bally, M.P. Bavouzet and M. Messaoud are working on Malliavin Calculus for Poisson Point Processes and applications to finance. A paper will appear in Annals of Applied Probability [21]. In [32], N. Privault et X. Wei develop an integration by parts technique for point processes with application to the computation of sensitivities via Monte Carlo simulations in stochastic models with jumps. N. Privault is studying sensitivity analysis and concentration inequalities with application to bounds on option prices.

6.5. Anticipative stochastic calculus in model with jumps

Keywords: *anticipative calculus*, *asymmetry information*, *forward integrals*, *insider*.

Participants: A. Kohatsu-Higa, A. Sulem.

A.Kohatsu-Higa and A.Sulem are working on the extension of their paper on insider trading [9] to models with jumps. Some results have been obtained in [23] where anticipative calculus for Lévy processes and applications to insider trading is studied.

A. Sulem and B. Øksendal propose an anticipative approach for indifference pricing in incomplete markets [45].

6.6. Lower bounds for the density of a functional

Participants: V. Bally, A. Kohatsu Higa.

V. Bally has three projects on this subject in progress, in collaboration with: B. Fernandez and A. Meda from the University of Mexico, in collaboration with L. Caramellino from the University of Roma 3 and finally in collaboration with A. Kohasu-Higa.

6.7. Optimal stopping and American Options

Participants: D. Lamberton, M. Mikou.

Damien Lamberton and Mihail Zervos (previously at King's College, London, now at the London School of Economics) have extended previous results on optimal stopping of one-dimensional diffusions for infinite horizon to finite horizon. In [43], they characterize the value function as the unique solution (in the sense of distributions) of a variational inequality, under minimal regularity assumptions for the payoff function and the coefficients of the diffusion.

D. Lamberton and his PhD student Mohammed Mikou have studied some properties of the exercise boundary of the American put option in an exponential Lévy model (continuity of the exercise boundary, behavior near maturity). A paper has been submitted [41].

6.8. Sparse grids methods for PDEs

Keywords: *adaptive finite elements*, *finite element*, *lattice-based methods*, *sparse grids*.

Participants: Y. Achdou, D. Pommier, A. Sulem.

Recent developments have shown that it may be possible to use deterministic Galerkin methods or grid based methods for elliptic or parabolic problems in dimension d, for $4 \le d \le 10$: these methods are based either on sparse grids [58] or on sparse tensor product approximation spaces [59], [76].

Sparse grids were introduced by Zenger [75] in order to reduce the number of degrees of freedom of discrete methods for partial differential equations. The piecewise-d-linear sparse grid approximation requires only $O\left(h^{-1}|\log h|^{d-1}\right)$ degrees of freedom instead of $O(h^{-d})$ for the standard piecewise linear approximations (h is the mesh step).

D. Pommier has studied these methods for the numerical solution of diffusion or advection-diffusion problems introduced in option pricing. He considers European vanilla contracts in multi factor stochastic volatility models (3 factors) and compares the numerical approximation obtained by Wavelet Galerkin method and Finite Difference method for the parabolic equation and the Integro-Differential Equations in a jump diffusion model. For the integral-operator, he proposes a collocation method adapted to the sparse grid discretization.

6.9. Stochastic control of jump diffusions and applications

Keywords: *jump diffusions*, *stochastic control*.

Participants: M. Arizawa, A. Sulem.

B. Øksendal (Oslo University) and A.Sulem have written a second edition of their book on Stochastic control of Jump diffusions [16] where they added a new chapter on optimal control of stochastic partial differential equations driven by Lévy processes and a new section on optimal stopping with delayed information.

In [35], A. Sulem and B. Øksendal study impulse control problems of jump diffusions with delayed reaction. This means that there is a delay $\delta > 0$ between the time when a decision for intervention is taken and the time when the intervention is actually carried out. They show that under certain conditions this problem can be transformed into a sequence of iterated no-delay optimal stopping problems and there is an explicit relation between the solutions of these two problems.

In [37], A. Sulem and B. Øksendal consider a stochastic differential game in a financial jump diffusion market, where the agent chooses a portfolio which maximizes the utility of her terminal wealth, while the market chooses a scenario (represented by a probability measure) which minimizes this maximal utility. They show that the optimal strategy for the market is to choose an equivalent martingale measure.

Mariko Arizawa is studying homogenization with the Lévy operators. She has established the equivalent relationship between various definitions of viscosity solutions [19].

6.10. Risk measures and applications

Keywords: *BSDE*, *risk measures*.

Participants: M.C Quenez, A. Sulem.

A. Sulem and B. Øksendal have studing risk-indifference pricing in incomplete markets: They define the (seller's) *risk indifference price* p_{risk}^{seller} as the initial payment that makes the *risk* involved for the seller of a contract equal to the risk involved if the contract is not sold, with no initial payment. In the case of jump diffusions models, using stochastic control theory and PDE methods, they find a formula for p_{risk}^{seller} and similarly for p_{risk}^{buyer}. In particular, they prove that $p_{low} \leq p_{risk}^{buyer} \leq p_{risk}^{seller} \leq p_{up}$, where p_{low} and p_{up} are the lower and upper hedging prices, respectively.

M.C. Quenez and Daniel Hernandez-Hernandez (Cimat, Mexico) are studying the problem of characterizing the variance optimal martingale measure $\widetilde{P}$ in a stochastic volatility model. Recall that the variance-optimal martingale measure appears to be a key tool for characterizing the optimal hedging strategy of the mean-variance hedging problem. Laurent and Pham (1999) have solved the problem in terms of classical solutions of PDEs in the particular case where the coefficients of the model do not depend on time and price process (they only depend on Y_t the state process). Quenez and Hernandez-Hernandez consider the general case where the coefficients depend on Y_t, on time t and on price S_t.

M.C. Quenez is studying how some dynamic measures of risk can be induced by Backward Stochastic Differential Equations (BSDEs).

7. Contracts and Grants with Industry

7.1. Consortium Premia

Participants: A. Alfonsi, V. Bally, B. Jourdain, A. Kohatsu-Higa, B. Lapeyre, J. Lelong, N. Privault, A. Sulem, A. Zanette.

The consortium Premia is centered on the development of the pricer software Premia. It is presently composed of the following financial institutions or departments: CALYON, the Crédit Industriel et Commercial, Société Générale, Natexis and IXIS CIB now unifies as Natixis. http://www.premia.fr

Extension to Japan has been realized via a cooperation with the university of Osaka. http://www-csfi.sigmath.es.osaka-u.ac.jp/premia/index.html. Similar procedure is ongoing with Hong Kong City university.

7.2. BNP-Paribas

Participants: D. Pommier, Y. Achdou, A. Sulem.

Cifre agreement BNP-Paribas/INRIA on : "sparse grids for large dimensional financial issues"

7.3. EuroVL

Participants: M. Hamel, N. Privault, A. Sulem.

Cifre agreement between Euro-VL and INRIA on “ Pricing of hybrid financial derivative products on change and interest rate “.

7.4. ANR program

ANR program GCPMF "Grid Computation for Financial Mathematics" (partners : Calyon, Centrale, EDF, ENPC, INRIA, Ixis, Paris 6, Pricing Partner, Summit, Supelec)

Global coordinator: B. Lapeyre

Participation to the proposal “Creditnext” on credit risk derivatives.

7.1. International cooperations

- Part of the European network "Advanced Mathematical Methods for Finance" (AMaMef). This network has received approval from the European Science Foundation (ESF).
- Programme Sakura-Ayame with Osaka University.
- Cooperations with Osaka university and ongoing cooperation with Hong Kong city university for extension of the Premia consortium.
- Collaborations with the Universities of Oslo, Bath, Chicago, Mexico, Osaka, Rome II and III, Tokyo Institute of Technology

8. Dissemination

8.1. Seminar organisation

- B. Jourdain, M.C. Kammerer-Quenez : organization of the seminar on stochastic methods and finance, University of Marne-la-Vallée.
- A. Alfonsi: From Oct. 2007: co-organizer of the working group seminar “ Stochastic methods and finance”.
- M.C. Kammerer-Quenez and A. Kohatsu Higa : members of the organization committee of the Seminaire Bachelier de Mathematiques financieres, Institut Henri Poincaré, Paris.
- B. Lapeyre :

  - Coordination of the ANR program "Grid Computation for Financial Mathematics" (partners : Calyon, Centrale, EDF, ENPC, INRIA, Ixis, Paris 6, Pricing Partner, Summit, Supelec), started in February 2006.

  - Member of the scientific committee of the workshop "New Direction in Monte-Carlo methods" 25-29 june 2007
- A. Sulem:

  - organisation of a course on numerical methods in Finance for professionals, Collège de Polytechnique, October 2007.

  - Co-Organisation (with Peter Imkeller, Esko Valkela and Monique Pontier) of an international Amamef workshop on "Insider models", Toulouse , January 2007
  http://www.lsp.ups-tlse.fr/Fp/Baudoin/amamefindex.html

8.2. Teaching

- A. Alfonsi
  - From Oct. 2007: “Probabilités et Statistiques”, first year course at the Ecole des Ponts.
  - From Oct. 2007: “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.
- V. Bally

  Master 2 of the University Marne la Vallee:

  1. Malliavin Calculus and numerical applications in fiance.

  2. Probabilistic methods for risk analysis.

- B. Jourdain : - Course "Probability theory and statistics", first year ENPC

  - Course "Introduction to probability theory and simulation", first year, Ecole Polytechnique

  - Projects and courses in finance, Majeure de Mathématiques Appliquées, 3rd year, Ecole Polytechnique

- B. Jourdain, B. Lapeyre : course "Monte-Carlo methods in finance", 3rd year ENPC and Master Recherche Mathématiques et Application, university of Marne-la-Vallée

- B. Jourdain, B. Lapeyre, M.C. Kammerer-Quenez, M. Sbai : course "Mathematical methods for finance", 2nd year ENPC.

- C. Labart:

  September to November 2007, tutorials on Markov chains at ENSTA.

- D. Lamberton :

  -Second year of Licence de mathématiques et informatique (multivariate calculus), Université de Marne-la-Vallée.

  -Third year of Licence de mathématiques (differential calculus, differential equations), Université de Marne-la-Vallée.

  - Master course “Calcul stochastique et applications en finance", Université de Marne-la-Vallée.

- B. Lapeyre

  - Course on "Modelisation and Simulation", ENPC, 2nd year.

  - "Exercise in probability", Ecole Polytechnique, 1st year.

  - Course on "Monte-Carlo methods for finance", Master program in Random analysis and systems, University of Marne la Vallée and Ecole des Ponts.

  - Columbia University, "Stochastic algorithms and applications to Monte Carlo methods in finance", March-April 2007, PHD course.

  - Halmstad University, Sweden, Numerical methods for Finance: Monte-Carlo methods, January 2007, Master’s Program in Financial Mathematics.

- J. Lelong
  - Lectures on “Numerical Methods in Finance” at ENSAI (third year course).
  - Lectures on “Numerical Methods in Finance” at Ecole Nationale Supérieure de Techniques Avancées (second year course)
  - lectures on “Monte Carlo methods for American option pricing” at Collège de l’Ecole Polytechnique.
  - Practicals on “Numerical simulation in C++” (second year course, ENSTA).
  - Practicals on Probability and Statistics (first year course ENSTA).
  - Practicals on “Markov Chains”, (second year course, ENSTA).

- M.C. Kammerer-Quenez

  - Courses for undergraduate students in mathematics, Université Marne la Vallée (Calculus, algebra)

  - Course on stochastic processes, graduate program, University of Marne-la-Vallée

  - Introductary course on financial mathematics, ENPC.

  - Graduate course on interest rate models, ENPC (in collaboration with Christophe Michel, CA-LYON)

- A. Sulem

  - Course on numerical methods in finance, Master II MASEF and EDPMAD, University Paris-Dauphine (21 hours)

  - Collège de Polytechnique: Course on numerical methods in stochastic control (October 2007)
- Guest lecturer, Master Program in Financial Mathematics, Halmstad University, Sweden : (20h) (February)

8.3. Internship advising

- B. Jourdain:

  O. Dupont : implementation in the software Premia of algorithms described in the article: "Generalizing the Black-Scholes formula to multivariate contingent claims", Journal of Computational Finance, Vol. 9, No. 2, pp. 43-67, by Carmona et Durrleman.
- B. Lapeyre and R. Keriven:

  Abdelmounaim Abbas-Turki on “Conception of tools inspired from graphic cards for option pricing”.
- J. Lelong:

  Ping Wang, Second year student from ENSTA (Ecole Nationale Sup?rieure de Techniques Avanc?es) on : “An exact simulation method for the Heston model”.
- N. Privault and A. Zanette:

  J. Yu: Implementation in the software Premia of algorithms for algoCallable Libor Exotics in forward Libor interest rate models.
- N. Privault:

  Supervision of X. Wei, research engineer at INRIA for the implementation of algorithms in the calibration of LIBOR interest rate models.
- P. Tankov: Nicolas Delcambre on “Comparison of various option hedging strategies with simulation and market data”.
- A. Zanette and A. Kbaier : Armand Ngoupeyou on Credit risk derivatives

8.4. PhD defences

- Jérôme Lelong

  14 September 2007, ENPC
  Title : Asymptotic properties of stochastic algorithms and pricing of Parisian options.

  Adviser: B. Lapeyre

8.5. PhD advising

- V. Bally

  Since october 2007, S. da Marco, student in Scola Normale Superiore di Pisa, in collaboration with M. Pratelli from the University of Pisa.
- B. Jourdain

  Mohamed Sbai (2nd year)

  "Simulation of stochastic differential equations in finance"

- D. Lamberton

  Mohammed MIKOU (3rd year). American options in exponential Lévy models Allocataire-moniteur at Université de Marne-la-Vallée.

  El Hadj Aly DIA (2nd year). Monte-Carlo methods for exotic options in models with jumps. Allocataire at Université de Marne-la-Vallée.

  Sidi Mohamed OULD ALY (just starting). Exotic options and stochastic volatility models.

  Up-date concerning past Phd students: Vincent Lemaire is now maître de conférences at Université Paris VI. Ahmed Kebaier is now maître de conférences at Université Paris XIII.

- B. Lapeyre

  - Jérôme Lelong, ENPC grant, UMLV (3rd year)

  “Stochastic algorithms and calibration problems in Finance” defended on Septembre 9th 2007 in the Ecole des Ponts.

- M.C. Kammerer-Quenez

  B. Jottreau, UMLV (3rd year)

  “Risk default modeling ”

- N. Privault and A. Sulem

  Mathieu Hamel (2nd year), Universit? Paris-Dauphine (Cifre agreement INRIA/Euro-VL, Filiale Société-Générale)

  “Pricing of hybrid financial derivative products on change and interest rate”

- A. Sulem and Y. Achdou (Paris 6)

  - David Pommier (3d year)

  Cifre agreement INRIA and BNP-Paribas

  “Sparse grid for large dimensional financial issues”.

8.6. Participation to workshops, conferences and invitations

- A. Alfonsi

  - “A Call-Put Duality for Perpetual American Options”, Forschungsseminar Stochastische Analysis und Stochastik der Finanzmärkte, TU Berlin.

  - “A second-order discretization scheme for the CIR process: application to the Heston model”, Groupe de travail Methodes stochastiques et finance.

- Mariko Arizawa

  - "Homogenizations and singular purtubation problems for integro-differential equations and their applications to mathematical finances", "Pauli Symposium" on PDEs on mathematical finance and economics, WPI, Univ. Vienne, November 2007. *

  - "Some homogenization problems of nonlinear PDEs", Seminar on Optimization, INRIA-Rocquencourt, June 2007.

- V. Bally

  - organization of a mini-workshop at the University Marne la Vallée on “lower bounds for the fundamental solutions of PDE problems: analytical and probabilistic approch.” Two mini-courses have been given by V. Vespri (university of Firenze) and S. Polidoro (university of Bolgna) and number of talks given by French probabilits.

  - 9.04-28.04. invited by A. Kohasu-Higa to the Univerity of Osaka. V. Bally gave an introductory curse (21hours) on Malliavin calculus and worked with A. Kohasu-Higa on the lower bounds for the density of diffusion processes under the weak Hormander condition.

- 14.04-18.04. invited by Sergio Polidoro and Andrea Pascucci to the University of Bologna.

Course on Malliavin Calculus.

- 1.07-4.07. invited to the Conference of Roumanian mathematiciens.

talk on Lower bounds for the density of a locally elliptic Ito process.

- 7.09-30.09. invited in the Semester on Stochastic PDEs in the Mittag-Leffler Institute.
talk on Lower bounds for the density of a locally elliptic Ito process in the workshop organized there.

- 25.10.-28.10 invited by M. Pratelli to the University of Pisa.

talk on Malliavin calculus for locally smooth laws and applicaitons to jump type equations. V. Bally also established a Phd co-direction with M. Pratelli for the thesis of S. da Marco, student in Scola Normale Superiore di Pisa.

- B. Jourdain

  - Research Seminar on Stochastic Analysis and Mathematical Finance, Berlin, May 24th : Stochastic flows approach to Dupire's formula.

  - Bachelier seminar, June 15th, Paris : Call-Put duality for perpetual American options and volatility calibration

  - Participation to the Workshop and Mid-Term Conference on Advanced Mathematical Methods for Finance September, 17th-22nd, Vienna

- D. Lamberton

  - 2nd General AMaMeF Conference, Bedlewo (Poland), May 2007. Optimal stopping problems with irregular payoff functions.

  - Workshop and Mid-Term Conference on Advanced Mathematical Methods for Finance, Vienna, September 2007. Optimal stopping problems with irregular payoff functions.

- B. Lapeyre:

  - Boston University, Boston, April 27th, "Adaptive Monte-Carlo Methods in Finance", Department of mathematics and statistics.

  - Invited at Columbia University (March-April)

- J. Lelong

  - Séminaire du CMAP, October 2007, *Pricing Parisian options using numerical inversion of Laplace transforms*.

  - INRIA Sophia Antipolis, June 2007, *New results non truncated stochatic algorithms*.

- D. Pommier

  Conference at ICIAM 2007 - Zurich - Mini Symposium.

- N. Privault

  Seminar on the calibration of the LIBOR market model, 9 january 2007, INRIA Rocquencourt. Talk on "Introduction to the calibration of the LIBOR model - implementation in the Premia software".

  University of Lisbon, 24/05/07, "The LIBOR market model".

  Participation to the presentation of the Premia software at CERMICS, ENPC, Paris: 16/02/07: "Calibration of the LIBOR model and implementation in Premia".

- A. Sulem

  – Invited conference, 2nd Amamef Conférence and Banach Center Conference on "Advances in Mathematics of Finance", Bedlow, Pologne, Mai 2007.

  - Invited plenary conference, SciCADE 07, International Conference on SCIentific Computation And Differential Equations, July 2007, Saint-Malo, France. http://scicade07.irisa.fr/index.php
  - Invited by the Sociedad Espanola de Matematica Aplicada to present Premia in Santiago de Compostella in a "quantitative finance industry forum", July.
  - Presentation of Premia to Commerzbank in London, Reuters, Axa.
  - Autumn 2007: invited in the Semester on Stochastic PDEs in the Mittag-Leffler Institute, Sweden.

- A. Zanette

  - Presentation Premia Citigroup London

  - Bedlewo Amamef Conference, May 2007, Poland

8.7. Miscellaneus

- A. Kohatsu-Higa

  representative of Premia project in Japan with Osaka University

- D. Lamberton

  - "Associate Editor" of *Mathematical Finance* and *ESAIM PS*.

  - in charge of the master programme "Mathématiques et Aplications" (Universities of Marne-la-Vallée, Créteil and Evry, and Ecole Nationale des Ponts et Chaussées).

  - Member of the Steering Committee of the ESF European Network "Amamef" (http://www.iac.rm.cnr.it/amamef/); in charge of the GDR "Méthodes Mathématiques pour la finance", which is the national CNRS group related to the network.

  - Coordinator of an "ACI" "Méthodes d'équations aux dérivées partielles en finance de marché". In this setting, coorganization with H. Berestycki and R. Monneau a workshop on PDE methods in finance in October 2007, see http://cermics.enpc.fr/~monneau/edpmethodsinfinance.html)

- B. Lapeyre
  - President of the Doctoral Department at Ecole des Ponts
  - Global coordinator of the ANR program GCPMF "Grid Computation for Financial Mathematics" (partners : Calyon, Centrale, EDF, ENPC, INRIA, Ixis, Paris 6, Pricing Partner, Summit, Supelec). February 2006 - February 2009.
  - ENPC coordinator of the ANR Program ADAP'MC "Adaptive Monte-Carlo Method", (partners : ENST, Ecole Polytechnique, ENPC, Université Paris-Dauphine)

- J. Lelong

  Participation to the "ARC RARE". Responsible of the chapter dedicated to Finance in the "monograph on rare event analysis using Monte Carlo techniques".

- A. Sulem

  - Vice-President of the Inria Evaluation Board

  - Member of the evaluation committee of the university Paris-Dauphine.

  - Member of the doctoral board of the the university Paris-Dauphine.

  - "First Opponent" of the PhD thesis of Martin Groth, *"Topics in Computational Finance: The Barndorff-Nielsen & Shepard Stochastic Volatility Model"*, June 2007, Oslo University.

9. Bibliography

Major publications by the team in recent years

[1] M. AKIAN, A. SULEM, M. TAKSAR. *Dynamic optimisation of long term growth rate for a portfolio with transaction costs - The logarithmic utility case*, in "Mathematical Finance", vol. 11, n^{o} 2, Avril 2001, p. 153–188.

[2] B. AROUNA. *Adaptative Monte Carlo Method, A Variance Reduction technique*, in "Monte Carlo Methods and Applications", vol. 10, n^{o} 1, 2004.

[3] V. BALLY. *An elementary introduction to Malliavin calculus*, Research Report, n^{o} 4718, Inria, Rocquencourt, February 2003, http://hal.inria.fr/inria-00071868.

[4] V. BALLY, L. CARAMELLINO, A. ZANETTE. *A mixed PDE - Monte Carlo approach for pricing credit default index swaptions*, in "Decision in Economics and Finance", vol. 29, 2006.

[5] V. BALLY, G. PAGÈS, J. PRINTEMS. *First order schemes in the numerical quantization method*, in "Mathematical Finance", vol. 13, n^{o} 1, 2003, p. 1–16.

[6] E. CLÉMENT, D. LAMBERTON, A. KOHATSU-HIGA. *A duality approach for the weak approximation of stochastic differential equations*, in "Annals of Applied Probability", vol. 16, n^{o} 3, August 2006, p. 1124-1154.

[7] E. CLÉMENT, D. LAMBERTON, P. PROTTER. *An analysis of a least squares regression method for american option pricing*, in "Finance and Stochastics", vol. 6, 2002, p. 449–471.

[8] B. JOURDAIN, C. MARTINI. *American prices embedded in European prices*, in "Annales de l'IHP, analyse non linéaire", vol. 18, n^{o} 1, 2001, p. 1-17.

[9] A. KOHATSU-HIGA, A. SULEM. *Utility maximization in an insider influenced market*, in "Mathematical Finance", vol. 16, n^{o} 1, 2006, p. 153–179.

[10] B. LAPEYRE, E. TEMAM. *Competitive Monte-Carlo Methods for the Pricing of Asian Options*, in "Journal of Computational Finance", vol. 5, n^{o} 1, 2001, p. 39-57.

[11] D. LEFÈVRE. *An introduction to Utility Maximization with Partial Observation*, in "Finance", vol. 23, 2002, http://hal.inria.fr/inria-00072440.

[12] M. N'ZI, Y. OUKNINE, A. SULEM. *Regularity and representation of viscosity solutions of Partial differential equations via backward stochastic differential equations*, in "Stochastic processes and their applications", vol. 116, n^{o} 9, 2006, p. 1319–1339.

[13] B. ØKSENDAL, A. SULEM. *Optimal Consumption and Portfolio with both fixed and proportional transaction costs*, in "SIAM J. Control and Optim", vol. 40, n^{o} 6, 2002, p. 1765–1790.

Year Publications

Books and Monographs

[14] *Banques et Marché*, 172 pages, to appear, special issue dedicated to Premia.

[15] D. LAMBERTON, B. LAPEYRE. *Introduction to Stochastic Calculus Applied to Finance*, Financial Mathematical series, Second Edition, Chapman & Hall/CRC, 2007.

[16] B. ØKSENDAL, A. SULEM. *Applied Stochastic Control of Jump Diffusions*, Universitext, Second Edition, Springer Verlag, Berlin, Heidelberg, New York, 2007.

Articles in refereed journals and book chapters

[17] A. ALFONSI. *An introduction to the multiname modelling in credit risk*, in "Banques & Marché", to appear.

[18] A. ALFONSI, B. JOURDAIN. *General Duality for Perpetual American Options*, in "International Journal of Theoretical and Applied Finance", Cermics report 2006-333, to appear.

[19] M. ARIZAWA. *A remark on the definitions of viscosity solutions for the integro-differential equations with Lvy operators*, in "J. Maths. Pures et Appliques", forthcoming.

[20] V. BALLY. *Lower bounds for the density of a locally elliptic Ito process*, in "Annals of Probability", to appear.

[21] V. BALLY, M. BAVOUZET, M. MESSAOUD. *Computations of Greeks using Malliavin Calculus in jump type market models*, in "Annals of Applied Probability", to appear.

[22] J. CHANCELIER, M. MESSAOUD, A. SULEM. *A policy iteration algorithm for fixed point problems with nonexpansive operators*, in "Mathematical Methods of Operations Research", vol. 65, n^o 2, April 2007, p. 239–259.

[23] G. DI NUNNO, A. KOHATSU-HIGA, T. MEYER-BRANDIS, B. ØKSENDAL, F. PROSKE, A. SULEM. *Anticipative stochastic control for Lévy processes with application to insider trading*, in "Handbook of Numerical Analysis", A. BENSOUSSAN, Q. ZHANG (editors), Mathematical Modeling and Numerical Methods in Finance, Elsevier, to appear.

[24] B. JOURDAIN. *Stochastic flows approach to Dupire's formula*, in "Finance and Stochastics", vol. 11, n^o 4, 2007, p. 521-535.

[25] B. JOURDAIN, A. KOHATSU-HIGA. *A review of recent results on approximation of solutions of stochastic differential equations*, in "Banques et Marché", to appear.

[26] B. JOURDAIN, M. SBAI. *Exact retrospective Monte Carlo computation of arithmetic average Asian options*, in "Monte Carlo methods and Applications", vol. 13, n^o 2, 2007, p. 135-171.

[27] B. JOURDAIN, A. ZANETTE. *A Moments and Strike Matching Binomial Algorithm for Pricing American Put Options*, in "Decisions in Economics and Finance", to appear.

[28] D. LAMBERTON, G. PAGÈS. *How fast is the bandit?*, in "Stochastic Analysis and Applications", to appear.

[29] J. LELONG, C. LABART. *Pricing Parisian Options using Laplace transforms*, in "Banques & Marché", to appear.

[30] J. LELONG, A. ZANETTE. *Use of tree methods in finance*, in "Encyclopedia of Quantitative Finance", to appear.

[31] N. PRIVAULT, X. WEI. *Calibration of the LIBOR market model - implementation in Premia*, in "Banque et Marchés", vol. special issue on Premia, to appear.

[32] N. PRIVAULT, X. WEI. *Integration by parts for point processes and Monte Carlo estimation*, in "Journal of Applied Probability", 2007, p. 806-823.

[33] M. QUENEZ. *Backward equations and applications*, in "Encyclopedia of Quantitative Finance", to appear.

[34] A. SULEM, A. ZANETTE. *Premia: An Option pricer*, in "Banques et Marché", to appear.

[35] B. ØKSENDAL, A. SULEM. *Optimal stochastic impulse control with delayed reaction*, in "Applied Mathematics and Optimization", to appear.

Publications in Conferences and Workshops

[36] M. HAMEL. *Get an implied correlation to price equity-interest rates hybrid*, in "Paris International meeting on Finance", AFFI, December 2007.

[37] B. ØKSENDAL, A. SULEM. *A game theoretic approach to martingale measures in incomplete markets*, in "OP&PM Surveys in Applied and Industrial Mathematics, Moscow", Proceedings of the Russian-Scandinavian Symposium "Probability Theory and Applied Probability", Petrozavodsk, Russia, August 26-31, 2006, vol. 14, n^{o} 6, OP&PM Editorial, 2007, p. 985–1082, http://www.math.uio.no/eprint/pure_math/2006/24-06.html.

Internal Reports

[38] M. GAUDENZI, M. LEPELLERE, A. ZANETTE. *The Singular Points Binomial Method for pricing American path-dependent options*, Working paper, Dipartimento di Finanza dell'impresa e dei Mercati Finanziari Universita' di Udine, 2007.

Miscellaneous

[39] A. ALFONSI. *A second-order discretization scheme for the CIR process: application to the Heston model*, submitted, April 2007.

[40] P. ETORÉ, B. JOURDAIN. *Adaptive optimal allocation in stratified sampling methods*, submitted, 2007.

[41] D. LAMBERTON, M. MIKOU. *The optimal exercise price for the American put in exponential L?vy models*, submitted for publication, 2007.

[42] D. LAMBERTON, G. PAGÈS. *A penalized bandit algorithm*, submitted for publication.

[43] D. LAMBERTON, M. ZERVOS. *On the problem of optimally stopping a one-dimensional Ito diffusion*, submitted for publication.

[44] J. LELONG. *Almost sure convergence of randomly truncated stochastic algorithms under verifiable condition*, under revision for Statistics & Probability Letters, 2007, http://hal.archives-ouvertes.fr/hal-00152255.

[45] B. ØKSENDAL, A. SULEM. *An anticipative stochastic calculus approach to pricing in markets driven by Lévy processes*, manuscript, 2007.

References in notes

[46] A. ALFONSI, B. JOURDAIN. *A Call-Put Duality for Perpetual American Options*, Preprint, n^o 307, CREMICS/ENPC, 2006.

[47] V. BALLY. *An elementary introduction to Malliavin calculus*, Research Report, n^o 4718, Inria, Rocquencourt, February 2003, http://hal.inria.fr/inria-00071868.

[48] D. BELL. *The Malliavin Calculus*, Pitman Monographs and Surveys in Pure and Applied Math., n^o 34, Longman and Wiley, 1987.

[49] D. BELOMESTNY, J. SCHOENMAKERS. *A jump-diffusion Libor model and its robust calibration*, Preprint, 2006.

[50] A. BESKOS, O. PAPASPILIOPOULOS, G. O. ROBERTS. *Retrospective exact simulation of diffusion sample paths with applications*, in "Bernoulli", vol. 12, n^o 6, 2006, p. 1077–1098.

[51] A. BESKOS, G. O. ROBERTS. *Exact simulation of diffusions*, in "Ann. Appl. Probab.", vol. 15, n^o 4, 2005, p. 2422–2444.

[52] F. BLACK, E. DERMAN, W. TOY. *A one factor model of interest rates and its application to treasury bond options*, in "Financial Analysts Journal", January-February 1990.

[53] A. BRACE, D. GATAREK, M. MUSIELA. *The Market Model of Interest Rate Dynamics*, in "Mathematical Finance", vol. 7, 1997, p. 127-156.

[54] J. C. COX, J. E. INGERSOLL, S. A. ROSS. *A Theory of the Term Structure of Interest Rate*, in "Econometrica", vol. 53, 1985, p. 363-384.

[55] D. DUFFIE, L. EPSTEIN. *Stochastic differential utility and asset pricing*, in "Econometrica", vol. 60, 1992, p. 353-394.

[56] E. FOURNIÉ, J.-M. LASRY, J. LEBUCHOUX, P.-L. LIONS. *Applications of Malliavin calculus to Monte Carlo methods in Finance, II*, in "Finance & Stochastics", vol. 2, n^o 5, 2001, p. 201-236.

[57] E. FOURNIÉ, J.-M. LASRY, J. LEBUCHOUX, P.-L. LIONS, N. TOUZI. *An application of Malliavin calculus to Monte Carlo methods in Finance*, in "Finance & Stochastics", vol. 4, n^o 3, 1999, p. 391-412.

[58] M. GRIEBEL. *Adaptive sparse grid multilevel methods for elliptic PDEs based on finite differences*, in "Computing", vol. 6, n^{o} 2, 1998, p. 151–179.

[59] M. GRIEBEL, P. OSWALD. *Tensor-product-type subspace splittings and multilevel iterative methods for anisotropic problems*, in "Advances of Computational Mathematics", vol. 4, 1995, p. 171–206.

[60] J. HULL, A. WHITE. *Numerical Procedures for Implementing Term Structure Models I:Single Factor Models*, in "Journal of Derivatives", vol. 2, 1994, p. 7-16.

[61] D. LAMBERTON, B. LAPEYRE, A. SULEM. *Application of Malliavin Calculus to Finance*, in "special issue of Mathematical Finance", January 2003.

[62] B. LAPEYRE, A. SULEM, D. TALAY. *Simulation of Financial Models: Mathematical Foundations and Applications.*, to appear, Cambridge University Press.

[63] P. MALLIAVIN. *Stochastic calculus of variations and hypoelliptic operators*, in "Proc. Inter. Symp. on Stoch. Diff. Equations, Kyoto", Wiley 1978, 1976, p. 195-263.

[64] P. MALLIAVIN, A. THALMAIER. *Stochastic Calculus of variations in Mathematical Finance*, Springer Finance, Springer, 2006.

[65] F. MERCURIO, N. MORENI. *Pricing inflation-indexed options with stochatic volatility*, Preprint, 2005.

[66] N. MORENI. *Pricing Inflation-Indexed Derivatives*, in "Quantitative Finance", vol. 5, n^{o} 3, p. 289-302.

[67] D. NUALART. *The Malliavin Calculus and Related Topics*, Springer–Verlag, 1995.

[68] D. OCONE, I. KARATZAS. *A generalized representation formula with application to optimal portfolios*, in "Stochastics and Stochastic Reports", vol. 34, 1991, p. 187-220.

[69] D. OCONE. *A guide to the stochastic calculus of variations*, in "Stochastic Analysis and Related Topics", H. KOERZLIOGLU, S. ÜSTÜNEL (editors), Lecture Notes in Math.1316, 1987, p. 1-79.

[70] V. PITERBARG. *A practitioner's guide to pricing and hedging callable Libor exotics in forward Libor models*, Working paper, 2003.

[71] V. PITERBARG. *Computing deltas of callable Libor exotics in forward Libor models*, Working paper, 2003.

[72] V. PITERBARG. *Pricing and hedging callable Libor exotics in forward Libor models*, in "The journal of computational finance", vol. 8, n^{o} 2, 2004/2005, p. 65-117.

[73] F. RUSSO, P. VALLOIS. *Stochastic calculus with respect to continuous finite quadratic variation processes*, in "Stochastics and Stochastics Reports", vol. 70, 2000, p. 1–40.

[74] O. VASICEK. *An Equilibrium Characterisation of Term Strucuture*, in "Journal of Financial Economics", vol. 5, 1977, p. 177-188.

[75] C. ZENGER. *Sparse Grids*, in "Parallel Algortihms for PDE, Vieweg, Braunschweig", W. HACKBLUSH (editor), vol. Proc. 6th GAMM Seminar, Kiel, 1991, p. 241-251.

[76] T. VON PETERSDORFF, C. SCHWAB. *Numerical Solution of Parabolic Equations in High Dimensions*, in "Mathematical Modelling and Numerical Analysis", vol. 38, n^{o} 1, 2004, p. 93–128.

[77] B. ØKSENDAL. *An Introduction to Malliavin Calculus with Applications to Economics*, in "Lecture Notes from a course given 1996 at the Norwegian School of Economics and Business Administration (NHH)", NHH Preprint Series, September 1996.