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Team RealOpt

Reformulation based algorithms for Combinatorial Optimization

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THEME NUM

Activity Report

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1. Team

Head of project-team
Francois Vanderbeck [Professor (Université Bordeaux 1)]

Vice-Head of project-team
Arnaud Pêcher [Assistant Professor (Université Bordeaux 1)]

Adminisrative assistant
Brigitte Cournou [Secretary (SAR) Inria]

Research scientist
Philippe Meurdesoif [Assistant Professor (Université Bordeaux 1)]
Sophie Michel [Assistant Professor (Université du Havre)]
Pierre Pesneau [Assistant Professor (Université Bordeaux 1)]

External collaborators
André Raspaud [Professor (Université Bordeaux 1)]
Eric Sopena [Professor (Université Bordeaux 1)]
Annegret Wagler [Research Assistant (University of Magdeburg, Germany)]

PHD Students
Laurent Gely [CIFRE scholarship (SNCF)]
Cédric Joncour [scholarship from the Ministry of Education (Université Bordeaux 1)]
Benoit Vignac [scholarship from Canada (Université de Montréal and Université Bordeaux 1)]

2. Overall Objectives

2.1. Overall Objectives

Nowadays, decision making relies on the support from mathematical models. Quantitative modeling is routinely used in industries and administration, either to design transportation, distribution and production systems or to operate them. Optimization concerns every stage of the decision process: investment budgeting, long term planning, the management of scarce resources, or the planning of day to day operations. In decision aid, many underlying optimization problems are discrete in nature. Their solution is essentially based on enumeration techniques, which is notoriously difficult given the huge size of the solution space. A key to success is the development of finer problem formulations that provide strong approximations and hence help to prune the enumerative solution scheme. One must also avoid the drawback of enumerating what are essentially symmetric solutions. Our project aims to develop tight formulations for combinatorial problems exploiting the complementarity between the latest reformulation techniques, such as Lagrangian and polyhedral approach (columns and cutting planes generation), non-linear programming tools (quadratic programming and semi-definite relaxations) and graph theoretic tools (for induced properties and implicit representations of solutions). Through industrial partnerships, the team targets large scale problems such as those arising in logistics (routing problems), planning and scheduling, in network design and control, and placement problems (cutting and packing problems).

2.2. Highlights

The team was created this year (in March 2007). Sophie Michel got her PhD with us in December 2006 (she started her PhD in January 2004). She got an Assistant Professor position, starting in September 2007, at the University of Le-Havre, in the engineering school in Logistic. Cédric Joncour started his PhD work in September 2007, having been granted a PhD Scholarship from the Ministry of Education. His subject is in line with one of the prospective action of our project: collaboration between graph theory approach and mathematical programming to tackle 2-D packing problems.

3. Scientific Foundations

3.1. Introduction

Keywords: *decomposition approaches*, *graph theory*, *integer programming*, *polyhedral approaches*, *quadratic approaches*.

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are discrete in nature. Binary variables model on/off decision to buy, invest, hire, to send a vehicle, or enforce a precedence. Integer variables model indivisible quantities. And extra variables deal with continuous adjustments. This results in models known as *mixed integer programs* (MIP) where the relations between variables and input parameters are expressed as linear constraints and the goal is set as a linear objective function. MIPs are probably the most widely used modeling tools. They allow a fair description of reality; they are versatile; they can already handle many non-linearities in the cost function and in the constraints; they are also well-suited for global optimization. However useful they may be, these models are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. Commercial solvers (such Ilog-CPLEX or Dash-Optimization's Xpress-mp) are available. But many real-life applications remain beyond the scope of such solvers. The scientific community is actively seeking to extend the capabilities of MIP solvers. Developments made into the context of specific applications, shall become generic tools over time and see their way into commercial software.
The most effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, Lagrangian decomposition methods to produce powerful relaxations, constraint programming to actively reduce the solution domain through logical implications, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidate solutions, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods into one global system. Another key to further progress is the development of finer problem formulations whose relaxation provide stronger approximation that are required for further truncation of enumerative solution schemes. Tighter formulations are also much more likely to yield good quality approximate solutions through rounding techniques. With properly chosen formulations, exact optimization tools can be competitive to construct good approximate solutions within limited computational time when compared to meta-heuristics. Our project brings together researchers with expertise in graph theory (characterization of graph properties, combinatorial algorithms) and mathematical programming (polyhedral approach, Dantzig-Wolfe decomposition, quadratic programming) in the aim of producing better quality formulations and solutions for practical combinatorial problems such as routing, network design, scheduling, packing and placement problems, thanks to the input of novel techniques.

3.2. Polyhedral Approach

Introduced by Edmonds in 1965 [46], the polyhedral approach, once combined with branch-and-bound (in a so-called branch-and-cut algorithm [71]) turned out to be one of the main sources of progress in solving NP-hard combinatorial optimization problems in the 80's-90's. A benchmark problem, in this regard, is the traveling salesman problem [62]. In the early 80's, the best algorithm was able to solve instances with around 300 cities. A recent paper [31] reports that branch-and-cut algorithms are able to solve instances with nearly 25000 cities. Similar significant improvements have been observed for instance for network design problems arising in telecommunication (see Kerivin and Mahjoub, 2005 [61]) or vehicle routing problems in logistic (see Letchford and Salazar-Gonzalez, 2006 [64]).

The goal of this approach is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions, $conv(X)$, where X is the discrete set of solutions to the combinatorial problem on hand. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem and solving the separation problem over the associated polyhedron: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\, x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$ [54]. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Nevertheless, one does not need to know such a description to take advantage of the polyhedral approach. Only a subset of the inequalities can already yield a good approximation of the ideal polytope. Moreover, non exact separation, using heuristic procedures, turns out to be quite efficient for practical purposes.

Polyhedral theory provides ways to derive automatically new inequalities from an initial polyhedral description P of the problem. For instance, it is known [70] that any valid inequalities for an IP can be obtained by iteratively taking linear combinations of existing constraints and rounding their coefficients. Such *general purposes cuts* have only recently made their way as practical tools: for instance, Gomory fractional cuts are now generated by default into commercial MIP solvers. Recent work [37], [50] has consisted in numerically testing the strength of the formulation obtained by application of a single round of such general purpose cuts (called the first-closure): the separation problem being set as an MIP problem which is solved with a commercial MIP solver. Cornuéjols (2006) [38] provides a comparative review of general purpose cuts such as lift-and-project cuts, Gomory mixed integer cuts, mixed integer rounding cuts, split cuts, and intersection cuts, as well as their practical contributions to dual bound improvements. However, the most promising results have often been obtained with so-called *template cuts*, i.e. family of valid inequalities derived in an application specific context: the close form expression of these additional inequalities is a template from which specific cuts are generated dynamically. To prove validity, one can show that such inequalities can be obtained as a special case of general purpose procedures. If it can be shown that the inequalities define so-called *facets* of $conv(X)$, these inequalities are needed for its description. In practice, one needs to develop efficient procedures (exact or heuristic) to separate these inequalities. Then, numerical evaluation can show the impact of the additional inequalities not only on the strength of the resulting dual bound but also in yielding solutions more likely to satisfy integrality restrictions (which is good for primal heuristics).

The connection between polyhedral approach and graph theory is deep. In the polyhedral approach, many facets are related to special classes of graphs. The literature is rich with such examples of facet defining systems described by exhibiting a bijection to a collection of subgraphs of the studied graph: forest polytope (Edmonds, 1961), matching polytope (Edmonds, 1965, [46]) ... There are even results showing that the structure of the polyhedron itself is closely related to the structure of the graph. For instance, Chvátal proved that the adjacency in the stable set polytope of a graph G (i.e., the fact that two solutions satisfy the same facet defining constraint at equality) is characterized as a connectivity property in graph G. Recent works are devoted to give an analogous interpretation of any facet of the stable set polytope. Lipták and Lovász (1999) [65] exhibited one. We gave another one in [5] and used it to generate a new set of facets for the stable set polytope of webs.

3.3. Dantzig-Wolfe decomposition and branch-and-price algorithms

Branch-and-price algorithms are more recent than branch-and-cut algorithms. Although column generation appeared in the 60's as a technique to handle linear programs with a huge number of variables [30], [51], its combination with branch-and-bound to solve integer programs was only developed in the 90's [33] (we did pioneering work on this subject [88]). Branch-and-price proved to be very useful in solving many practical problems that were intractable by other means: crew and fleet assignment problems faced by airline companies [43], [77], vehicle routing problems in public and in fret transport systems [44], cutting stock problems experienced in the paper, textile, or steel industry [73], [79], [81], [87], production planning problems [32], [86], and network design problems [76], [28] are such examples. Branch-and-price has now become the reference method for problems well suited to decomposition and it is making its way in industry: for instance, decision aid software developed by consultant firms like Eurodecision (Paris) or Adopt (Montréal) rely on this approach. The use of this method in the practical context of challenging applications has revealed its limitation. Further developments are required to overcome these difficulties.

Indeed, using column generation in the context of integer programming is not straightforward. The primary challenges revolve around the convergence of the dual bound computations, the enforcement of integrality restrictions and the combination with dynamic cutting plane generation. A first step toward targeting these difficulties was to clarify the underlying Dantzig-Wolfe decomposition principle. While the standard view was to see Dantzig-Wolfe decomposition as the linear programming formulation of the Lagrangian dual [70], we presented it as a reformulation technique that gives rise to an integer master program ([80] does this for the integer case, while [78] extends this view to the mixed integer case). Then, the integrality restrictions of the original formulation translate into integrality restrictions in the reformulated problem (this is the *discretization* approach). Our framework based on the concept of generating sets facilitates the handling of branching decisions (to enforce integrality) and adding cutting planes to the formulation.
Natural applications for the Dantzig-Wolfe approach are problems whose constraint matrix has a block diagonal structure augmented with linking constraints, i.e. a constraint matrix A of the form

$$A = \begin{pmatrix} L & L & ... & L \\ B^1 & 0 & ... & 0 \\ 0 & B^2 & ... & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & ... & B^K \end{pmatrix}, \tag{1}$$

with K blocks B^k, $k = 1, ..., K$. Then, dualizing constraints L decomposes the problem into K sub-problems of smaller size. Moreover, if the K blocks are identical, the column generation reformulation eliminates the symmetry in k and there is only one pricing problem that needs to be solved during the column generation algorithm (the same column being optimal for all K subsystems). Beyond this standard application framework, open questions concern the viability of applying the decomposition principle recursively or to multiple independent subsystems. We develop a first application of a nested decomposition in [81]. A decomposition based on multiple independent subsystems (not necessarily disjoint) should provide tighter bounds according to the theory of Lagrangian decomposition [55]. But, there is typically a large number of dualized constraints linking the different systems and hence many dual prices to adjust (therefore one expects slow convergence). To our knowledge the literature does not report of any column generation approach based on multiple subsystems capturing different combinatorial structures. A recent study however comes close to such multiple decomposition: [42] combines variable redefinition applied to one subsystem and column generation applied to another subsystem. The dual bounds that they obtained are shown to be tighter than with alternative approaches but the computational times are much larger.

3.4. Quadratic approaches

The communication between the non-linear programming and the combinatorial optimization communities is limited although the latter has much to learn from the former. Several of the latest developments in discrete optimization are imports from convex optimization [35], [52], [39]. Quadratic programming (QP), in particular, offers very powerful modeling tools. A *quadratic program* is a formulation in continuous variables whose cost and/or constraint functions are polynomials of degree 2. Several classical constraint types for combinatorial problems are more efficiently modeled with (QP): binary constraints ($x_i \in \{0, 1\}$ can be equivalently set as $x_i^2 = x_i$); sequencing constraints: for instance in scheduling problems, if job j follows job i, denoted by $x_{ij} = 1$, then their completion times must satisfy $C_j \geq C_i + p_j$ where p_j is the processing time of j; this can be modeled as $x_{ij}\,(C_j - C_i - p_j) \geq 0$; transitivity constraints represent requests of the type "if two variables x_i and x_j are equal to 1, a third one x_k should be 1" (their quadratic formulation $x_k \geq x_i x_j$ is stronger than the linear $x_k \geq x_i + x_j - 1$ when one relaxes the integrality restrictions).

Although Quadratic Programming (QP) is NP-Hard, some special cases or relaxations are polynomially solvable. Minimizing a convex quadratic cost over a feasible region described by linear constraints is easy. This method is implemented in commercial MIP solvers. Convex QP with linear constraints and 0-1 variables can then be solved using branch-and-bound and solving the continuous QP relaxation at each node. Also, the *Semi-Definite Programming* (SDP) relaxation of a QP is polynomially solvable. An SDP is an extension of an LP where variables are the components of a matrix that is constrained to be semi-definite. When the objective is not convex, it can be convexified using an augmented relaxation approach [36], [35]: the Hessian is made convex by adding to the objectives a weighted sum of the quadratic binary constraints and the squared norm of equality constraints (optimized weights are obtained by solving an SDP relaxation). When applied to an already convex objective, this approach is still useful to improve the continuous relaxation bound. Solvers are available for SDP [34], but, in their current implementation, they are very sensitive to the conditioning of the matrices.

Even though the numerical solution of QP and SDP remains problematic, recent applications of these techniques to some combinatorial problems have led to major improvements [66], [59], [53], [35]. Generally, an SDP relaxation can be found for any general Integer Program [74]. Starting from the QP reformulation of an IP, a Lagrangian relaxation procedure is applied to yield an SDP: after Lagrangian dualization, the (unconstrained) QP has a solution (in the primal variables) iff some matrix of coefficients is positive semidefinite. The associated SDP bound is always better than classical LP relaxation [63], although sometimes the size of the SDP formulation is problematic. In former studies, we discovered other SDP formulations for Lovász's bound on vertex coloring [67]: a direct quadratization that appears to be intermediate between the ones of [59] and [66]. The SDP formulation obtained by application of the general scheme of [74] is of huge dimension and, because of symmetry, does not bring more than our (compact) SDP formulation. Yet, it can be fruitfully used to compute bounds on generalizations of Vertex Coloring where symmetry does not hold (List Coloring, some problems of Frequency Assignment) [67].

3.5. Graph theory tools

The relationship between graph theory and mathematical programming has led to several famous research advances. Let us cite just a few landmarks. The matching problem (selecting disjoint edges in a graph) is historically the first integer programming problem that could not be solved by linear programming (no compact ideal formulation being known) but for which an efficient (polynomial time) combinatorial algorithm was known [46] (ideal formulations were only known for network flow problems at the time). The combinatorial algorithm lead to a polyhedral description of the matching polytope with exponentially many constraints separable in polynomial time [46]. Another example is the study of perfect graphs. Perfect graphs do not only have nice graph-theoretical properties and behave nicely from an algorithmic point of view, but several characterizations of perfect graphs also form an interface between graph theory, integer programming and semi-definite programming: a graph is perfect iff the convex hull of the incidence vectors of its stable sets is defined by the so-called *clique constraint polytope* (the only constraints needed are those enforcing that at most one node can be selected from each clique, while, for general graph, this polytope defines a relaxation of the stable set polytope). Perfect graphs are also characterized by the fact that their chromatic number (the minimum number of independent stable sets needed to cover the graph) can be computed in polynomial time by semi-definite programming.

Thus, the graph theory tools allow to derive better models/formulations for combinatorial optimization problems (f.i. graph theory characterization of forbidden subgraphs can sometimes be directly expressed as constraints in a mathematical program). Moreover, combinatorial procedure from graph theory can also serve as subroutines in mathematical programming approach, f.i., for cut separation or column generation. In particular, on the issue of symmetries, we believe that progress can come from the complementarity between graph theory and mathematical programming. This is illustrated by the work of Fekete and Schepers (2004) [47] on the 2-dimensional placement problem. In our project, we also play the reverse complementarity, i.e. to use mathematical programming techniques to make progress in graph theory.

3.6. Mathematical programming based heuristics

A heuristic is an algorithm that attempts to build a "good" primal feasible solution to a combinatorial optimization problem with no a priori guarantee on its maximum deviation from optimality. Exact optimization approaches can also serve to build good approximate solutions for complex combinatorial problems, either (i) by truncating an exact algorithm, or (ii) by constructing solution from the relaxation on which the exact approach relies, or (iii) by using exact solvers as subroutines in building heuristic solutions. Point (i) is common practice, even in commercial MIP solvers: one sets a time limit (or another bound on the number of algorithmic steps) or a threshold deviation from optimality. However, the implementation strategies are typically different if the aim is to get quickly good primal solution rather than solving the problem exactly. Point (ii) is key to our project. The chances are that starting from the solution to a stronger relaxation of a problem (i.e. a better formulation), one gets better primal solution in the end. The techniques to build the primal solutions range from greedy constructive procedures, to rounding techniques, using the relaxed solution as a target, or simply exploiting dual information to price choices. Point (iii) is the idea of "MIPPING" questions [48], i.e. to set intermediate questions, such as finding the best solution in the neighborhood of the current solution, as a mathematical program that can be solved with a MIP solver or another combinatorial algorithm. Even though the initial problem might be much too hard for exact methods, the subquestions that arise during its heuristic solution might be within the scope of exact solvers. There too it is important to have a good formulation of the "mipped" question. This points to potential collaboration with other INRIA research teams who develop meta-heuristic approaches (DOLPHIN and TAO).
Heuristics based on exact methods have found a new breath in the recent literature, in part due to the progress of exact commercial solvers. The latest developments are the *Large Scale Neighborhood Search* (an exponential size neighborhood can be explored in a local search procedure, provided that an efficient/polynomial algorithm exists to search it – Ahuja et al. 2002 [29]), the *Relaxation Induced Neighborhood Search* (the components of the LP solution that are close to the best known integer solution are rounded and the residual problem is then solved as a MIP of smaller size – Danna et al. (2005) [41]) and the *feasibility pump algorithm* (the rounded LP solutions defines a target that might be infeasible; the LP is re-optimized with the objective of minimizing the distance to that target; the process iterates in hope of finding good integer solutions – Fischetti et al., 2003, 2005 [49], [48]).

4. Application Domains

4.1. Introduction

Keywords: *environment*, *optimal placement*, *production planning*, *scheduling*, *telecommunications*, *transportation systems*.

Our group has tackled applications in logistic, transportation and routing [15], [69], [68], [27], in production planning [45], [86] and inventory control [68], [27], in network design and traffic routing [2], [13],[19],[28], [40], [58], [60], [75], [76], and in cutting and placement problems [72], [73], [79], [81], [82], [87]. Building on this experience, we plan to find our motivation for algorithmic developments in the study of complex combinatorial problems of industrial relevance. In particular, we are currently involved in two industrial partnerships. With Exeo Solutions (a consultant that has worked for Eco-emballages, Suez, and other main stream group), we study planning and routing problems that arise in waste management [27], [68]. With SNCF, we consider train timetabling problems and their re-optimization after a perturbation in the network [18].

4.2. Transport and logistic

Managerial problems raised by the planning of operations in transport network, the distribution of goods and the associated management of inventories have always been central in Operations Research. The tools of mathematical programming can bring substantial savings given the part of operation and transportation costs in the global cost of logistic. One could think that these major issues should be well resolved by now. This is simply not so. The combinatorial difficulties inherent to these problems require new techniques to increase the size of instances that can be treated. Moreover, new managerial practices and the trend of going from a hierarchical to an integrated optimization yield new problems.
Our experience in this domain includes several industrial studies:

- we optimize simultaneously transportation routes and customer inventories (a problem known as *inventory routing*) with our industrial partner Exeo Solutions [68], [27];
- we have an on-going project with SNCF in the context of the PhD thesis of L. Gely on the operational re-scheduling of a timetable following a perturbation [18].
- for Routing International, Brussels, we combined vehicle routing and planning over a fixed time horizon [69],[15];
- we also work on variants of the traveling salesman problem (TSP): the cumulative TSP arises when cost of an arc is inversely proportional to its position in the circuit (consider for instance a delivery man that is paid more at the beginning of the journey because of the weight he has to carry).

4.3. Telecommunication network design and control

Network design is a wide research domain arising in railway, highway and telecommunication. Applications in telecom are very much studied at the moment, a revival due to the arrival of new transfer technologies such as optical fibers. The aim is to conceive cheap and reliable networks with specific requirements on the topology (which links will be created) or the capacity of the links (which amount of information can be in transit on a link at the same time). Sufficient capacities have to be installed to avoid congestion, several paths may have to link given pairs of nodes to ensure transmission in case of breakdown, all this at the cheapest possible cost.
We work on the design of the network topology, implementing a survivability condition of the form "at least two paths linking each pair of terminals". We extended polyhedral approaches to problem variants with specific requirements. In [2], we deal with the design of so-called SDH/SONET networks, where the links must form cycles of bounded length (see Figure 1); bounded length requirements can also come from re-routing restrictions [58], [40], [13]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes. In previous work, we optimized traffic assignment to minimize such installation cost [76]. We now consider the problem that arises when using new wavelength division multiplexing (WDM) technologies that allow to pack more traffic on optical networks. Several streams can be multiplexed, each of them supported by a different wavelength, in an optical signal [28] (Figure 2 illustrates routing configurations that will be assigned to different wavelengths to handle several thousands of requests). We are also working on the problem of measuring traffic in a network through the placement of markers at minimal cost [19].

4.4. Cutting, placement, and scheduling problems

In cutting stock problems, one has a supply of large pieces of raw material in stock and a set of demands for small "order" pieces. One must satisfy these demands by cutting the required small pieces out of the large pieces from the stock. The objective is primarily to minimize the waste that is counted as the unused part of used pieces of stock material. A solution is given by a set of feasible cutting patterns, i.e. assortments of order pieces that can be cut out of a given large piece of stock material, such that their accumulated production covers the demands. There are many variants of the cutting stock problem. The main ones concerns the number of significant dimensions of the forms (1D, 2D, 3D, or even 1.5D), specific restrictions on the cutting process, the geometrical arrangements of pieces, and the number of cutting stages. There might be secondary objectives related to the balancing of the workload between different cutting machines, the minimization of the number of different cutting patterns used, or the respect of due dates for instances.

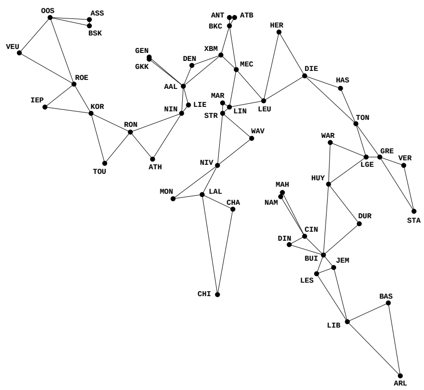

Figure 1. Belgian network with links forming cycles of length bounded by 3.

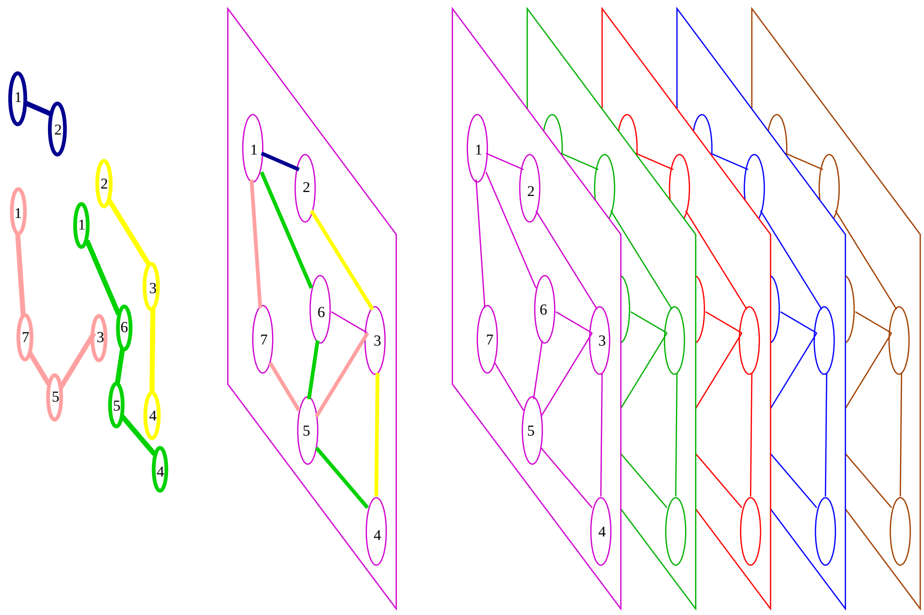

Figure 2. Nested decomposition approach to traffic routing: simple routing patterns and layers of wavelength assignments.

Packing, placement or loading problems can be stated in similar terms. There, one has a set of resources (vehicles, railway cars, machines) that must be packed with items. The objective is to maximize the value of the resulting packing while respecting the capacity of the resources. A solution is given by a set of feasible individual resource packings which together do not pack the same object more than once. Again, many variants exist depending mainly on the number of dimensions in which the capacity of the resources are measured and the specific restrictions on the loading process. Packing problems arise in particular as subproblems in cutting problems since the question of building "good" cutting patterns typically boils down to packing a resource piece with maximum value items. There are many applications of these cutting and packing problems: for instances, the cutting of paper, steel bars, glass, wood, textile, plastic film, optimization of newspaper layout, scheduling parallel machine, line balancing, and more generally scheduling problems with limited resources.
In previous work, we developed specialized algorithms for some variants of the knapsack problem that arise as a subproblem in solving cutting stock problems [82]. Our new results in this domain concern the knapsack problem with setup costs [14]. We also set benchmark results for the 1D cutting stock problem using an exact optimization approach based on branch-and-price [87]. We were first to introduce exact algorithm for the 1D problem with setup minimization (a much harder variant) [79]. We also applied a nested decomposition approach to a 2D multi-cutting-stage variant [81] and considered ways of incorporating industrial side-constraints in an exact approach for the 1D problem [72], [73].

5. Software

5.1. Application specific solvers

The output of our studies on specific applications typically consists in algorithmic developments and codes that implement the application specific solvers or, in some cases, routines that allow to derive theoretical results (f.i. in graph theory). The algorithms produced this year are:

- IRP: A branch-and-price-and-cut code to optimize the planning of collection of waste while properly managing the stock levels [27], [68]. The propotype is currently being experimented on real-life data by our industrial partner Exeo Solutions.
  contact: Sophie Michel
- GRWA: A branch-and-price-and-cut code for routing traffic in a optical telecommunication network [28].
  contact: Benoit Vignac
- LIPARI : A mathematical programming based code for the rescheduling of trains at SNCF.
  contact: Laurent Gely
- An integer programming based code to build facet defining inequalities for the stable set polytope of a claw free graph.
  contact: Arnaud Pêcher

5.2. BapCod : a generic Branch-and-Price Code

contact: FrancoisVanderbeck
Beyond the above application specific algorithms, we are involved in the development of a generic platform for the implementation of the branch-and-price method. Contrary to the cutting plane approach, column generation has not yet made its way into commercial solvers. Despite its successes, the use of a Dantzig-Wolfe decomposition approach is currently restricted to experts. The difficulties inherent to implementing the method and to combining it with other techniques were only overcome in an application specific context. As a consequence, algorithmic ideas have been developed and tested for specific applications only, without convincing arguments for showing their impact across different applications. The challenge is to show that column generation is also a technique that can be automated and make its way into commercial solvers (as cutting plane approach did recently).

We develop the prototype of a generic branch-and-price code, named *BaPCod*, for solving mixed integer programs by column generation. Previous attempts have been limited to offering "*tool-boxes*" to ease the implementation of algorithms combining branch-and-price-and-cut. With these, the user must implement three basic features for its application: the reformulation, the setting-up of the column generation procedure and the branching scheme. Other available codes that offer more by default were developed for a specific class of applications (such as the vehicle routing problem and its variants). Our prototype is a "*black-box*" implementation that does not require user input and is not application specific. The features are
(i) the automation of the Dantzig-Wolfe reformulation process (the user defines a mixed integer programming problem in terms of variables and constraints, identifies subproblems, and can provide the associated solvers if available, but he does not need to explicitly define the reformulation, the explicit form of the columns, their reduced cost, or the Lagrangian bounds) – this automation of the decomposition principle is studied in [83].
(ii) a default column generation procedure with standard initialization and stabilization (it may offer a selection of solvers for the master) – the issue of stabilization is discussed in [1], and
(iii) a default branching scheme – recent progress has been made on the issue of generic branching scheme in [85].

6. New Results

6.1. Network Design and Routing

Participants: Benoit Vignac, Sophie Michel, Pierre Pesneau, Francois Vanderbeck.

6.1.1. Polyhedral approach to survivability problems

In the design of modern networks, the survivability of the network and its performance in everyday use as well as in the case of a breakdown are important issues. A common way to ensure survivability is to create two disjoint paths between each pair of end points (if a link fails, there still exists another path to route the information). Network efficiency then depends on the re-routing strategy. Assuming local re-routing in case of a failure, we impose that each link must belong to cycle of bounded length. The traffic will be rerouted along this cycle [2]. More recently, we considered a second re-routing strategy: we make use of an end-to-end path for re-routing. Then, we must design a network that has disjoint paths of bounded length between each pair of access points. We obtained several results for the special case where there are only one source and one destination in the network [40], [58]. We have extended our study by considering multiple destinations and then by considering both multiple origins and multiple destinations. We developed a polyhedral approach for these problems. Several cutting planes have been introduced along with some proofs of their strength. Based on these cuts, we developed a branch and cut algorithm that proved able to solve real-life instances. This work led to the publication of the paper [13]. We pursue this work with Luis Gouveia and Ridha Mahjoub, testing some directed formulations for these problems. In this scope, Ridha Mahjoub and Pierre Pesneau have visited Luis Gouveia for one week in Lisbon in April 2007.

6.1.2. Nested decomposition approach for telecom network routing problems

Dantzig-Wolfe decomposition is standardly applied to problems whose constraint matrix has a block diagonal structure augmented with linking constraints (see Section 3.3). Then, relaxing the linking constraints allows to decompose the problem into a subproblem associated with each diagonal block. If each of these diagonal block has itself a block diagonal structure, the decomposition principle can be applied recursively. We developed a first application of a nested decomposition in [81]. Our approach to the network design problem of [28] is also a nested decomposition. The problem consists in optimizing bandwidth utilization in optical network by grooming traffic (i.e. packing several requests on a wavelength). Moreover, several streams can be multiplexed on an optical signal, each of them supported by a different wavelength. The technology is called wavelength division multiplexing (WDM). WDM and grooming techniques require expensive opto-electronic systems at nodes: each decomposition or aggregation of traffic requires conversion from the optical to the electrical domain and then back to optical for re-sending. Hence, one must optimize traffic *grooming, routing and* *wavelength assignment* (GRWA) to reduce opto-electronic system installation cost. The GRWA problem is decomposed into a subproblem for each wavelength, each of which is decomposed into basic routing pattern filling subproblems. The latter are simple aggregation of requests for which we can easily compute the cost of installing equipment at nodes. Our master program takes the form of a covering problem where traffic to wavelength assignments are selected so as to cover demands at the cheapest cost. Its Linear Programming (LP) relaxation is solved by column generation. The pricing problem is itself solved by a column generation approach where columns are associated to basic routing patterns.

6.1.3. Branch-and-Price approach to vehicle routing problems

We provide approximate solutions to a problem combining vehicle routing and planning over a fixed time horizon (a problem submitted to us by Routing International, Brussels, a firm that commercializes a software for the planning of pick-ups and deliveries). The instances were originating from the money transport business, involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers. Approximate solutions are constructed using a truncated column generation procedure followed by a rounding heuristic [15], [69]. This mathematical programming based procedure can deal with problems with 50 to 80 customers over 5 working days which is the range of size of most PVRP instances treated in the literature with meta-heuristics. [69] offers a classification of multiple period VRP, while [15] highlights the importance of alternative optimization criteria not accounted for in standard operational models and provide insights on the implementation of a column generation based rounding heuristic.

We also studied a model that integrates the optimization of inventories at customer site and transportation routes with our industrial partner Exeo Solutions that works in the area of waste management. The application combines (i) vehicle routing (minimizing the routing cost of sending vehicles of limited capacity to deliver goods to customers), (ii) planning (optimizing the dates of the visits to customer site over a time horizon), and (iii) inventory management (avoiding stock-out at customer site) to give rise to a problem known as *inventory routing* that is quite complex and was not very well studied in the literature. Previous approaches were mostly heuristics or hierarchical optimization approaches. We developed an exact Branch-and-Price-and-Cut algorithm that we truncate to obtain approximate solution. The approach provides solutions with bounded deviation to optimality for large scale problems (260 customers, 60 time periods, 10 vehicles) [27], [68].

6.1.4. Time-dependent formulation for the vehicle routing problem

We studied a variant of the vehicle routing problem with unit demands using a time dependent formulation approach [26]. Our results arise from the analysis of the relationship between the linear relaxation of a single commodity flow model and the linear relaxation of a pure time dependent formulation (closely related to a formulation for the traveling salesman problem). We introduced a large class of flow bounding constraints that tighten the single commodity flow formulation. These constraints were obtained by projecting the constraints of the time-dependent formulation on the natural design variables. The new inequalities allow to recover in particular the well-known multi-star constraints. Computational results show that instances up to 80 nodes can be easily solved with the time-dependent formulation when the capacity of the vehicle is reasonably small. Our aim is now to study time-dependent formulations for the Traveling Salesman Problem with cumulative cost (that is cost depending on the position of the arcs in the circuit). This case, known as the repairman problem, also arises in some scheduling problems.

6.2. Primal heuristics based on column generation

Participants: Sophie Michel, Benoit Vignac, Francois Vanderbeck.

The decomposition principle can be exploited in greedy, local search or rounding heuristics, even though some of these heuristics are not straightforward to implement in the context of dynamic column generation (variables are only known implicitly and cannot be bounded). The price coordination mechanism of the Dantzig-Wolfe approach brings the global view that may be lacking when local search or constructive heuristics are applied directly to the original problem formulation (these latter approaches are often qualified as myopic). There are examples of decomposition based heuristics in the literature that range from the simplest truncated exact approach to the latest meta-heuristic paradigm. We work at classifying these approaches [27], [68], [84]. In summary, there are four types of methods. (i) The so-called restricted master heuristic: the column generation formulation restricted to a subset of variables is solved as a MIP; the restricted set of columns defining subproblem solutions can either be generated heuristically, or be the columns generated during the master LP solution, or a mixture of both (note that the resulting restricted master problem might be infeasible). (ii) The greedy heuristics consist in generating iteratively the columns that have the smallest ratio of reduced cost per unit of constraint satisfaction (based on dual price estimates), selecting some in the solution and reiterating for the residual master problem. (iii) The rounding heuristics consist in iteratively selecting one or more columns of the LP solution to the master (among those whose valuse is close to integer), round their value, and re-optimize the residual master LP (exactly or approximately) – generating new columns in the process is the key to avoid infeasibility of the above static methods. Variants imply solving residual problems by other heuristic means. (iv) Local search heuristics are often based on removing a few columns from the current solution and re-optimizing the residual master IP (typically with one of the above heuristics). All these heuristics can potentially be applied at any stage of a branch-and-price algorithm. They must integrate in their implementation the standard balance between intensification versus diversification of the search, and can make use of the memory of already explored solutions. All these issues are a matter of putting together efficient implementation strategies (through extensive numerical tuning). Our latest results in this domain can be found in [15], [27], [28], [68], [72], [87].

6.3. Graph coloring and math programming approaches to graph theory

Participants: Philippe Meurdesoif, Arnaud Pêcher, Eric Sopena, André Raspaud, Annegret Wagler.

6.3.1. Circular colorings

Graph coloring is the problem of assigning colors $k \in \{1, ..., K\}$ to the nodes of a graph G defined by its node set N and its edge set E, i.e. $G = (N, E)$, such that any two nodes i and j linked by an edge $e = (i, j) \in E$ have different colors. The chromatic number of a graph G, denoted $\chi(G)$, is the minumun number of colors needed to color the graph. The circular chromatic number of a graph is a well-studied refinement of the chromatic number. Circular-perfect graphs is a superclass of perfect graphs defined by means of this more general coloring concept. The Strong Perfect Graph Conjecture, recently settled by Chudnovsky, Robertson, Seymour and Thomas, provides a characterization of perfect graphs by means of forbidden subgraphs. It is therefore natural to ask for an analogous conjecture for circular-perfect graphs, that is for a characterization of all minimal circular-imperfect graphs. We exhibited in [17] three classes of minimal circular-imperfect graphs, namely, certain partitionable webs, a subclass of planar graphs, and odd wheels and odd antiwheels. As those classes appear to be very different from a structural point of view, we infered that formulating an appropriate conjecture for circular-perfect graphs, as analogue to the Strong Perfect Graph Theorem, is an hopeless task.

The stability number of a graph G, denoted $\alpha(G)$, is the maximal size of a set of pairwise non-adjacent vertices. The clique number of a graph G, denoted $\omega(G)$, is the size of the maximal clique (complete graph) contained in G. An equivalent statement of the strong perfect graph theorem is that minimal imperfect graphs G have $\min\{\alpha(G), \omega(G)\} = 2$. In contrast to this result, it was shown by Pan and Zhu that minimal circular-imperfect graphs G can have arbitrarily large independence number and arbitrarily large clique number. We proved in [21] that claw-free minimal circular-imperfect graphs G have $\min\{\alpha(G), \omega(G)\} \leq 3$.

We also studied in [12] strongly circular-perfect graphs: a circular-perfect graph is strongly circular-perfect if its complement is circular-perfect as well. This family entails perfect graphs, odd holes, and odd antiholes. As main result, we fully characterized the triangle-free strongly circular-perfect graph and proved that, for this graph class, both the stable set problem and the recognition problem can be solved in polynomial time.

6.3.2. A new quadratic bound for graph coloring

D. Cornaz [38] showed from representing color classes in a graph coloration as stars in the complementary graph that the chromatic number of a graph, $\chi(G)$, could be obtained by computing the independance number, $\alpha(D(G))$, of some auxiliary graph $D(G)$. Following this idea, we investigate the lower bound on $\chi(G)$ obtained by approximating $\alpha(G)$ with Lovász' $\theta(G)$ number.

The procedure for constructing $D(G)$ indeed makes use of an order on the vertices so as to break symmetries in the color classes representations. Although the value of $\alpha(D(G))$ does not depend on the order used for the auxiliary graph, we show that when it comes to approximation, things are different: distinct orderings of vertices do result in different lower bounds on $\chi(G)$.

However, it appears that whatever the order is, this (SDP based) bound outperforms (sometimes dramatically) the classical $\theta(\overline{G})$ SDP bound. Sometimes it even dominates the fractional chromatic number, which is known to be the theoretical limit of polynomial-time lower bounding of the chromatic number [56].

Moreover, we have designed heuristics, based on the dual variables of iteratively computed semidefinite programs, to generate an order on the vertices yielding a "good" bound on χ.

6.3.3. Graphs and linear programming

Several characterizations of perfect graphs form an interface between graph theory, integer programming and semi-definite programming: a graph G is perfect iff its *stable set polytope* STAB(G) (the convex hull of the incidence vectors of its sets of pairwise non-adjacent vertices, called stable sets) is equal to the so-called *clique constraint polytope* QSTAB(G) (the only constraints needed are those enforcing that at most one node can be selected from each clique, while, for general graph, this polytope defines a relaxation of the stable set polytope).

For every graph G, the dilation ratio $\min\{t : \text{QSTAB}(G) \subseteq t\ \text{STAB}(G)\}$ of the two polytopes yields the imperfection ratio of G. It is NP-hard to compute and, for most graph classes, it is even unknown whether it is bounded. We bounded in [11] the imperfection ratio for several well-known graph classes.

Clique family inequalities $a\sum_{v\in W} x_v + (a-1)\sum_{v\in W'} x_v \leq a\delta$ form an intriguing class of valid inequalities for the stable set polytopes of all graphs. We proved in [16] that their Chvátal-rank is at most a. This provides an alternative proof for the validity of clique family inequalities, involving only standard rounding arguments.

Providing a complete description of the stable set polytopes of claw-free graphs is a long-standing open problem. Eisenbrand et al. recently achieved a breakthrough for the subclass of quasi-line graphs. As a consequence, every non-trivial facet of their stable set polytope has at most two different, but arbitrarily high left-hand-side coefficients. For claw-free but not quasi-line graphs with stability number at least four, Stauffer conjectured that the same holds true. On the other hand, there are known claw-free graphs with stability number $\alpha(G) = 3$ which induce facets with up to 8 different left hand side coefficients. We proved that the situation is even worse: for every positive integer b, we exhibit a claw-free graph with stability number three inducing a facet with b different left hand side coefficients. This result was achieved using an integer programming reformulation [22].

7. Contracts and Grants with Industry

7.1. Exeo Solutions

Participants: Sophie Michel, Cédric Joncour, Pierre Pesneau, Francois Vanderbeck.

We just completed a research project for Exeo Solutions (we had a 3 year research contract). The *inventory routing* application studied in the thesis of S. Michel [27], [68] combines (i) vehicle routing (minimizing the routing cost of sending vehicles of limited capacity to deliver goods to customers), (ii) planning (optimizing the dates of the visits to customer site over a time horizon), and (iii) inventory management (avoiding stock-out at customer site). We developed a rounding heuristic for the integrated tactical problem based on a column generation reformulation. Central to our approach is a state space relaxation idea that allows to avoid the symmetry in the time period indexing inherent to this cyclic scheduling problem. The approach provides solutions with bounded deviation to optimality for large scale problems (260 customers, 60 time periods, 10 vehicles). A prototype implementation of our algorithm is currently being tested by Exeo Solutions.

7.2. SNCF

Participants: Laurent Gely, Philippe Meurdesoif, Arnaud Pêcher, Pierre Pesneau, Francois Vanderbeck.

We have an on-going contract with SNCF, "Innovation et Recherche", in the context of the PhD thesis of L. Gely (with a CIFRE scholarship). After working on the timetabling problem in the aim of maximizing the throughput (number of trains) that can be handled by a given network [57], we now consider the problem of managing perturbations [18]. Network managers must re-optimize train schedules in the event of a significant unforeseen event that translates into new constraints on the availability of resources. The control parameters are the speed of the trains, their routing and sequencing. The aim is to re-schedule trains to return as quickly as possible to the theoretic timetable and to restrict the consequences of the perturbation to a limited area. The question of formulation is again central to the approach that shall be developed here. The models of the literature are not satisfactory. Continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing). Other standard models based on arc flow in time-space graph blow-up in size. Formulations in time-space graphs have therefore been limited to tackling single line timetabling problems. We are now developing a formulation that strikes a compromise between these two previous models.

8. Dissemination

8.1. Organization of scientific meetings

- Pierre Pesneau : member of the organizing committee of the POC group (Polyèdre et Optimisation Combinatoire) supported by the French Operation Research Society (ROADEF) and the operation research group of the CNRS (GDR RO). *(POC's aim is to promote the polyhedral approach in combinatorial optimization. The group organizes the Polyhedra and Combinatorial Optimization Days yearly, two tutorials, and a doctoral school.)*
- Pierre Pesneau : Member of the scientific committee of INOC 2007, Spa, Belgium.
- Pierre Pesneau : Member of the scientific committee of ICTON - 'Mediterranean Winter' 2007, Sousse, Tunisia.
- F. Vanderbeck : Member of the scientific committee of FRANCORO V / Roadef 2007, Grenoble.
- We organize a work group that meets on a weekly basis to host presentation of research studies or on-going work (see http://realopt.math.cnrs.fr/team/pmwiki.php?n=Project.Welcome?n=Project.Seminars).

8.2. Conferences

- A. Pêcher, P. Pesneau and A. Wagler - "Générer des facettes pour le polytope des stables d'un graphe sans griffes par la programmation entière" - FRANCORO V / Roadef 2007. February 20-23, 2007, Grenoble [22].
- S. Michel and F. Vanderbeck, "Heuristiques basées sur la génération de colonnes", - FRANCORO V / Roadef 2007. February 20-23, 2007, Grenoble.
- L. Gely, Journée Francilliennes de Recherche Opérationnelles (JFRO): Knapsack and Optimization, March 9, 2007.
- P. Pesneau and F. Vanderbeck, Journée Scientifique POC on Facets and combinatoric polyhedra, March 30, 2007, Paris.
- S. Michel, "Inventory Routing Problem", Journée Francilienne de Recherche Opérationnelle (JFRO), April, 2007, Paris.
- L. Gely, "2nd International Seminar on Railway Operations Modelling and Analysis" (RailHannover2007), April 2007.

- Laurent Gély, “Real-time train scheduling at SNCF”, European ARRIVAL meeting on Robust planning and Rescheduling in Railways, April 17-19, 2007, Utrech, The Netherlands [18].
- M. T. Godinho, L. Gouveia, T. L. Magnanti, P. Pesneau, and J. Pires. “On Time-Dependent Models for Unit Demand Vehicle Routing Problems”, International Network Optimization Conference (INOC 2007), April 23-25, 2007, Spa, Belgium (http://www.poms.ucl.ac.be/inoc2007/).
- Ph. Meurdesoif, P. Pesneau, and F. Vanderbeck, “Metter Installation for monitoring network traffic”, International Network Optimization Conference (INOC 2007), April 23-25, 2007, Spa, Belgium.
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- Ph. Meurdesoif, P. Pesneau, and A. Pêcher, JPOC 2007, May 30 - June 1, 2007, Paris.
- S. Michel and F. Vanderbeck, "Inventory Routing Problem solved by Heuristic based on Column Generation", TRISTAN VI, June 10-15, 2007, Thailand (http://tristan.epfl.ch/).
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- A. Pêcher and X. Zhu - "Circular-perfect claw-free graphs" - Eurocomb 2007, September 11 - 15, 2007, Seville, Spain (http://www.congreso.us.es/eurocomb07/).
- Ph. Meurdesoif, Journée Optimisation des Réseaux, (GT du GdR RO et de la ROADEF), Oct 25, 2007, Paris.
- A. Pêcher, Journées Graphes et Algorithmes, Nov 8, 2007, Paris.
- L. Gely, Arrival fall school 2007 on robust network design and delay management, Nov. 14-16, 2007, Sevilla, Spain.
- P. Pesneau, Journée Scientifique POC on Matroïds, Nov 30, 2007, Paris.

External collaborators of the group have also been active in dissiminating their work related to the project (see [24], [23], [20], [25]). In particular, E. Sopena was Invited speaker in CID-2007, Karpacz, Poland.

8.3. Invitations / Workshop

We were invited for short stay abroad or to take part in workshops:

- A. Pêcher and A. Raspaud spent 2 weeks at the University Sun Yat-Sen, Kaohsiung, Taiwan, on the invitation of X. Zhu in February 2007
- P. Pesneau spent one week in Lisbon on the invitation of Louis Gouveia, in April 2007.
- F. Vanderbeck, workshop in Combinatorial Optimization, Jan 7-12, 2007, Aussois, http://www.informatik.uni-koeln.de/aussois2007/ .
- F. Vanderbeck, workshop on Mixed Integer Programming (MIP), July 30 - Aug 2, 2007, Montréal, Canada, (http://www.crm.umontreal.ca/MIP2007/) .

8.4. Visitors

Several international and national colleagues visited us (short visits for scientific exchanges and seminars presentations):

- Ridha Mahjoub (Professor, Laboratoire LAMSADE Université Paris Dauphine) spent 3 days with us, Jan. 24-26, 2007.
- Hervé Bricard (Renault, R & D) visited us on Jan. 26, 2007.
- El-ghazali Talbi (Professor, LIFL, CNRS-INRIA-USTL) visited us on June 13-14, 2007.
- Alain Hertz (Professor, GERAD and Ecole des HEC, Montréal) visited us on July 25, 2007.
- Andrew Miller (Assistant Professor, Dept. of Industrial and Systems Engineering, University of Wisconsin-Madison, USA) visited us on Sept. 3-4, 2007.

8.5. PhD Theses

- Sophie Michel graduated in December 2006 (she started her PhD in January 2004). She did her PhD work on an inventory routing problem in a collaboration with the firm Exeo Solutions [68]. Advisor: F. Vanderbeck.
- Sylvain Coulonge will graduate in December 2007. Advisor: A. Pêcher.
- Cédric Joncour started in September 2007. His doctoral study is on 2-D packing problems. Advisors: A. Pêcher, P. Pesneau, F. Vanderbeck.
- Benoit Vignac (Advisors: B. Jaumard, G. Laporte, F. Vanderbeck) and Laurent Gely (Advisors: P. Pesneau, F. Vanderbeck) pursue their doctoral research.

8.6. Teaching and Administrative Duties

Each member of the team is quite involved in teaching in the thematic specialties of the project, including in the research track of the Masters in applied mathematics or computer science. Moreover, we are largely implied in the organization of the curriculum:

- Arnaud Pêcher is head of studies for the Master of computer science applied to management.
- Philippe Meurdesoif is the project organizer for the operations management specialty of the Master of Applied Mathematics, Statistics and Econometric.
- Pierre Pesneau is head of the professional curriculum of the operations management specialty.
- Francois Vanderbeck is head of the Master of Applied Mathematics, Statistics and Econometrics.

External collaborators of the group are also quite involve locally. In particular, E. Sopena is head of the doctoral school in Mathematics and Computer Science.

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[12] S. COULONGES, A. PÊCHER, A. WAGLER. *Triangle-Free Strongly Circular-Perfect Graphs*, in "accepted in Discrete Mathematics", 2007.

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