INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Tosca

TO Simulate and CAlibrate stochastic processes

Sophia Antipolis - Méditerranée - Nancy - Grand Est

THEME NUM

Activity Report

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1. Team

TOSCA *is located both at INRIA Nancy – Grand Est (Nancy) and INRIA Sophia Antipolis – Méditerranée.*

Head
Denis Talay [DR Inria (Sophia-Antipolis – Méditerranée), HdR]

Vice-heads
Mireille Bossy [CR Inria (Sophia-Antipolis – Méditerranée)]
Madalina Deaconu [CR Inria (Nancy – Grand Est)]

Administrative assistants
Marie-Line Ramfos [AI Inria (Sophia-Antipolis – Méditerranée)]
Hélène Zganic [TR Inria (Nancy – Grand Est), until June 2007]
Laurence Quirot [TR Inria (Nancy – Grand Est), since September 2007]

Staff member (Inria)
Nicolas Champagnat [CR Inria (Sophia-Antipolis – Méditerranée)]
Antoine Lejay [CR Inria (Nancy – Grand Est), HdR]
Etienne Tanré [CR Inria (Sophia-Antipolis – Méditerranée)]

Staff member (Université Nancy 1)
Samuel Herrmann [Assistant Professor, École des Mines de Nancy]
Bernard Roynette [Professor, Université Nancy 1, HdR]
Samy Tindel [Professor, On leave from Université Nancy 1 since September 2007, HdR]
Pierre Vallois [Professor, Université Nancy 1, HdR]

Research scientist (partner)
Pierre-Louis Lions [Professor, Collège de France, HdR]
Sylvain Maire [Assistant Professor, Université de Toulon et du Var, HdR]

On leave from Université de Nice – Sophia Antipolis
Pierre-Emmanuel Jabin [Professor, since September 2007, HdR]

Post-doctoral fellows
Stephania Maroso [Sophia, since August 2007]
Samih Zein [Nancy, since September 2007]

Ph.D. students
Mamadou Cissé [Sophia-Antipolis]
Olivier Collignon [Nancy, CIFRE]
Viet-Dung Doan [Sophia-Antipolis, project-team OASIS and TOSCA]
Anthony Floryszczak [Sophia-Antipolis/ENSMP (CMA), MESR Grant, starting in December]
Angela Ganz [Sophia-Antipolis, CONICYT Grant]
Junbo Huang [Sophia-Antipolis/Hong-Kong]
Jean-François Jabir [Sophia-Antipolis]
Numa Lescot [Paris/Nancy, CIFRE]
Julian Tugaut [Nancy, CIFRE]

Training students
Abdelilah Boualam [Nancy/École des Mines de Nancy, June–August]
Mohammed Zied Ghazai [Nancy/École des Mines de Nancy, June–August]
Najed Ksouri [Nancy/École Polytechnique de Tunis, February–May]
Inès Henchiri [Sophia/École Polytechnique de Tunis, February–June]
Thanh-Hoang Mai [Sophia/École Polytechnique, Palaiseau, April–August]

2. Overall Objectives

2.1. Overall Objectives

The Inria Research team TOSCA is located both at Inria Sophia-Antipolis – Méditerranée and Inria Nancy – Grand Est. It is a follow-up of the OMEGA project which ended in December 2006. The team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, fluid mechanics, biology, chemical kinetics, neuro-sciences and population dynamics.

The problems where stochastic models arise are numerous, and the critical reasons for which stochastic models are used make analyzes and simulations difficult.

The TOSCA team thus aims to develop calibration and simulation methods for stochastic models in cases where **singularities** in the coefficients or **boundary conditions** make them hard to discretize and estimate. For this, we are willing to tackle theoretical and numerical questions which are motivated by real applications.

We are interested in developing **stochastic numerical methods** and **transverse methodologies** that cover several fields of applications, instead of having chosen a particular field of application (e.g., Biology, or Fluid Mechanics, or Chemistry). We justify this way to proceed as follows:

- From a couple of years now, we have attacked singular problems to answer questions coming from economists, meteorologists, biologists and engineers with whom we collaborate within industrial contracts or research programs such as ACI, ANR, GDR. To solve their problems which are so complex that stochastic processes are involved in the modelling, these colleagues need to combine expertise and knowledge in many fields: deterministic computing, computer science, vision, algorithm analysis, etc. We are incompetent in these fields, and therefore we could not pretend to fully treat any of these problems. A contrario, we are requested to bring our expertise in stochastic modelling and simulation to extremely various domains of applications.
- In spite of this diversity, whatever the application is, one has to simulate stochastic processes solutions to equations of the type

$$\begin{cases} X_t(\omega) &= X_0(\omega) + \left(\int_0^t \int_{\mathbb{R}^d} b(X_s, y)\mu_s(dy)ds\right)(\omega) \\ &\quad + \left(\int_0^t \int_{\mathbb{R}^d} \sigma(X_s, y)\mu_s(dy)dZ_s\right)(\omega), \\ \mu_s &= \text{Law of } X_s \text{ for all } s \geq 0, \end{cases} \tag{1}$$

  in order to compute statistics of the laws of functionals of these solutions. In addition, several fields often produce very similar "pathologies" of the model (1) or of the statistics to compute: for example, Pope's Lagrangian stochastic particles in Fluid Mechanics and models in Molecular Dynamics produce the same degeneracy in (1), namely, one has to substitute 'Conditional Law of components of X_s given the other ones' to 'Law of X_s'; as well, when studying chartist strategies in Finance and stochastic resonance in the electrical working of neurons, we encounter close questions on the density functions of the random passage times of processes (X_t) at given thresholds.
- Theory and numerical experiments show that each 'pathology' of the model (1) requires specific analyses and numerical methods. However, they require common abstract tools (Malliavin calculus, propagation of chaos theory, nonlinear PDE analysis, etc.) and common numerical methodologies (stochastic particle systems, Monte Carlo simulations, time discretization of stochastic differential equations, etc.). Thus each application takes benefit from the modelling and numerical knowledge developed for all the other ones.

The TOSCA team is currently studying models in relation with Geophysics, Neuro-sciences, Fluid Mechanics, Chemical Kinetics, Meteorology, Molecular Dynamics, Population Dynamics and Evolution and Finance. We also construct and study stochastic particle systems for Fluid Mechanics, coagulation–fragmentation, stationary nonlinear PDEs, variance reduction techniques for Monte-Carlo computations and numerical methods combining deterministic and stochastic steps to solve nonlinear PDEs in Finance.

3. Scientific Foundations

3.1. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and, try instead to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modelling issues: one has to choose appropriate stochastic models, which requires a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The TOSCA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The TOSCA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The TOSCA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

4.1. Application Domains

TOSCA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Finance, Neurosciences and Biology, Fluid Mechanics and Meteorology, Chemical Kinetics, Diffusions in random media, Transverse problems and Softwares and Numerical experiments.

4.1.1. Finance

For a long time now TOSCA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modelling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.2. Neurosciences and Biology

The interest of TOSCA in biology is developing in three main directions: neurosciences, molecular dynamics and population dynamics. In neurosciences, stochastic methods are developed to analyse stochastic resonance effects and to solve inverse problems. For example, we are studying probabilistic interpretations and Monte Carlo methods for divergence form second-order differential operators with discontinuous coefficients, motivated by the 3D MEG inverse problem. Our research in molecular dynamics focus on the development of Monte Carlo methods for the Poisson-Boltzmann equation which also involves a divergence form operator, and of original algorithms to construct improved simulation techniques for protein folding or interactions. Finally, our interest in population dynamics comes from particular methods, ecology, evolution and genetics. For example, we are studying the emergence of diversity through the phenomenon of evolutionary branching in adaptive dynamics. Some collaborations in biostatistics on cancer problems are also being initiated.

4.1.3. Fluid Mechanics and Meteorology

In Fluid Mechanics we develop probabilistic methods to solve vanishing vorticity problems and to study the behavior of complex flows at the boundary, and their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our studies concern the convergence analysis of these methods on theoretical test cases and the design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluids (the FENE model). A second example concerns Pope's Lagrangian modelling of turbulent flows, motivated by the problem of modelling and computing characteristic properties of the local wind activity in areas where windmills are built. Our goal is to estimate local resources in energy which are subject to meteorological randomness by combining large scale wind models and small scale Monte Carlo techniques, and to simulate management strategies of wind resources.

4.1.4. Chemical Kinetics

The TOSCA team is studying coagulation and fragmentation models, that have numerous areas of applications (polymerization, aerosols, cement industry, copper industry, population dynamics,...). Our current motivation comes from the industrial copper crushers in Chile. We aim to model and calibrate the process of fragmentation of brass particles of copper in industrial crushers, in order to improve their efficiency at a low cost.

4.1.5. Diffusions in random media

A *random medium* is a material with a lot of heterogeneity which can be described only statistically. Typical examples are fissured porous media with rocks of different types, turbulent fluids or unknown or deficient materials in which polymers evolve or waves propagate. These last few years, the TOSCA team has been in relation with the Geophysics community on problems related to underground diffusions, especially those which concern waste transport or oil extraction. We are extending our previous results on the simulation of diffusion processes generated by divergence form operators with discontinuous coefficients. Such an operator appears for example from the Darcy law for the behavior of a fluid in a porous media. We are also developing another class of Monte Carlo methods to simulate diffusion phenomena in discontinuous media.

4.1.6. Transverse problems

Several of the topics of interest of TOSCA do not only concern a single area of application. This is the case of particular methods for the long time simulations of nonlinear McKean-Vlasov PDEs, the problem of simulation of multivalued models, variance reduction techniques or stochastic partial differential equations. For example, multivalued processes have applications in random mechanics or neurosciences, and variance reduction techniques have applications in any situation where Monte Carlo methods are applicable.

4.1.7. Software, numerical experiments

TOSCA is interested in designing algorithms of resolution of specific equations in accordance with the needs of practitioners. We benefit from our strong experience on the programming of probabilistic algorithms on various architectures including intensive computation architectures. In particular, our activity will concern the development of grid computing techniques to solve large dimensional problems in Finance. We are also interested in intensively comparing various Monte Carlo methods on PDEs and in the development of open source libraries for our numerical methods in Fluid Mechanics, Meteorology, MEG or Chemical Kinetics.

5. New Results

5.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Mireille Bossy, Nicolas Champagnat, Madalina Deaconu, Angela Ganz, Inès Henchiri, Samuel Herrmann, Pierre-Emmanuel Jabin, Antoine Lejay, Pierre-Louis Lions, Sylvain Maire, Denis Talay, Etienne Tanré, Samih Zein.

5.1.1. Monte Carlo methods for diffusion processes and PDE related problems

Keywords: *Monte Carlo methods*, *backward stochastic differential equations*, *discontinuous media*, *divergence form operator*, *random walk on spheres/rectangles*, *skew Brownian motion*.

S. Zein started his post-doc in September 2007 on the application of optimization algorithm for the method of random walk on rectangles using importance sampling techniques [52]. The idea is then to look for optimal parameters through an automatic procedure in order to reduce the variance of the Monte Carlo estimator, or to simulate rare events.

A. Lejay and S. Maire have worked on Monte Carlo computations of the first eigenvalue of linear operators like the Laplace operator or the neutron transport operator in a bounded domain [54]. This method relies on a branching mechanism and improves our previous algorithm [84], [82]: The method is more robust, more accurate and provides the first eigenelement of the adjoint operator.

A. Lejay and S. Maire are developing a Monte Carlo method to simulate a diffusion process in discontinuous media. This method relies on a kinetic approximation of the diffusion operator.

All these methods are motivated by problems in physics and finance: estimation of the electrical conductivity in the brain (MEG problem), simulation of diffusions in heterogeneous media, computation of effective coefficients for PDEs at small scales, computation of option prices, ...

5.1.2. Statistical estimators of diffusion processes

In numerous applications, one chooses to model a complex dynamical phenomenon by stochastic differential equations or, more generally, by semimartingales, either because random forces excite a mechanical system, or because time–dependent uncertainties disturb a deterministic trend, or because one aims to reduce the dimension of a large scale system by considering that some components contribute stochastically to the evolution of the system. Examples of applications respectively concern mechanical oscillators submitted to random loading, prices of financial assets, molecular dynamics.

Of course the calibration of the model is a crucial issue. A huge literature deals with the statistics of stochastic processes, particularly of diffusion processes: However, somewhat astonishingly, it seems to us that most of the papers consider that the dimension of the noise is known by the observer. This hypothesis is often questionable: there is no reason to *a priori* fix this dimension when one observes a basket of assets, or a complex mechanical structure in a random environment. Actually we were motivated to study this question by modelling and simulation issues related to the pricing of contracts based on baskets of energy prices within our past collaboration with Gaz de France.

Jointly with J. Jacod (Université Paris 6), A. Lejay and D. Talay have tackled the question of estimating the Brownian dimension of an Itô process from the observation of one trajectory during a finite time interval. More precisely, we aim to build estimators which provide an "explicative Brownian dimension" r_B: a model driven by a r_B Brownian motion satisfyingly fits the information conveyed by the observed path, whereas increasing the Brownian dimension does not allow to fit the data any better. Stated this way, the problem is obviously ill posed, hence our first step consisted in defining a reasonable framework to develop our study. We developed several estimators and obtained partial theoretical results on their convergence. We also studied these estimators numerically owing to simulated observations of various models.

5.1.3. New simulations schemes for the approximation of Feynman-Kac representations

The Feynman-Kac formula is a well-known tool to achieve stochastic representations of the pointwise solution of numerous partial differential equations like diffusion or transport equations. In [79], the authors introduced sequential Monte Carlo algorithms to compute the solution of the Poisson equation with a great accuracy using spectral methods. In [55], S. Maire and E. Tanré have proposed new variants of the Euler scheme and of the walk on spheres method for the Monte Carlo computation of Feynman-Kac representations. We optimize these variants using quantization for both source and boundary terms. Numerical tests are given on basic examples and on Monte Carlo versions of spectral methods for the Poisson equation.

5.1.4. Exact simulation of SDE

Beskos et al. [68], [67] proposed an exact simulation algorithm for one dimensional SDE with constant diffusion coefficient and restrictive assumptions on the drift coefficient. E. Tanré and V. Reutenauer (Calyon) have adapted the results. We extend this algorithm to more general SDEs and we apply it to SDEs with non Lipschitz coefficients (for instance, the CIR model for interest rates). During her internship supervised by E. Tanré, I. Henchiri has performed numerical tests on this subject.

5.1.5. Coagulation-fragmentation equations

Keywords: *Chemical kinetics*, *Smoluchowski equation*.

The phenomenon of coagulation was first described by the Polish physicist Marian von Smoluchowski in 1916, in a paper on the precipitation in colloidal suspensions. It applies and is studied in many domains as: the formation of stars and planets, the behaviour of fuels in combustion engines, the polymerisation phenomenon, etc. Our expertise in this domain is now developed in a new direction.

More precisely, M. Deaconu and E. Tanré are trying to adapt their methodology for a problem arising in the Chilean copper industry. Copper ore is ground in crushers by using steel balls, the aim being to fragment the ore by using the least amount of energy. A stochastic algorithm that evaluates the time spent in the crusher has been already constructed.

A new study is underway, aiming to construct a model which effectively illustrates the phenomenon of fragmentation and accounts for new parameters, such as the position of particles in the crusher, the crusher geometry, the shape, size and number of steel balls used as projectiles and the damage factor. This factor takes into account the effect of low-impact collisions, which may occur before particle fracture. A balance must be found between the model's complexity and its ability to approximate the copper industry crushers. A successful outcome will allow us to optimize crushers efficiency at a minimal cost.

E. Tanré participated to the supervision of the Master internship of Tomás Neumann (Pontificia Universidad Catòlica de Chile) on this subject, defended in June 2007.

5.1.6. On the approximation of equilibrium measure of McKean-Vlasov equations

Keywords: *Equilibrium measure*, *Euler scheme*, *McKean Vlasov equation*, *p-Wasserstein metrics*, *particle systems*.

In her Ph.D. thesis supervised by M. Bossy and D. Talay, A. Ganz studies stochastic particle numerical methods to approximate the limit, when t goes to infinity, of the McKean Vlasov partial differential equation in dimension 1.

$$\frac{\partial V}{\partial t}(x,t) = \frac{1}{2}\frac{\partial^2 V}{\partial x^2}(x,t) - \frac{\partial V}{\partial x}\left(b + \int_{\mathbb{R}} \phi(\cdot, y)\frac{\partial V}{\partial x}(dy)\right)(x,t). \tag{2}$$

We suppose that b satisfies a monotonicity condition, and that ϕ is a non-linear interaction kernel and a small perturbation of b.

Previously, we have used the method introduced in [72] and written $V(x,t)$ as a cumulative distribution function of a nonlinear stochastic process X_t. We have also established the existence and uniqueness of the equilibrium measure of X_t, and obtained an exponential convergence rate to the stationary measure in the 2-Wasserstein distance. We have estimated the convergence rate of $\overline{V}(x,T) = \frac{1}{N}\sum_{i=1}^{N} H(\cdot - X_T^{i,N,\Delta})$ to the distribution function of the equilibrium measure, where $X^{i,N,\Delta}$ is the discretization by Euler scheme for the particle system associated to X_t. We also obtained a bound for the $L^1(\mathbb{R})-$norm of the approximation error.

In 2007 we completed the study of the dimension 1: we established the convergence to the equilibrium measure in the $L^\infty(\mathbb{R})-$norm and obtained a bound for the approximation error in the $L^\infty(\mathbb{R})-$norm. In $L^1(\mathbb{R})$ and $L^\infty(\mathbb{R})-$norms, our approximation converges with a rate proportional to the discretization time step, inversely proportional to $\sqrt{N}$, and with an exponential decay in T.

5.1.7. Evolutionary branching in a jump process of evolution

In [75], under the assumptions of rare mutations and large population, N. Champagnat obtained a microscopic interpretation of a jump process of evolution where the evolution proceeds by a sequence of mutant invasions in the population, in the specific case where only one type can survive at any time. This process, called “trait substitution sequence” (TSS) of adaptive dynamics, is the basis of the theory of adaptive dynamics [85]. This theory provides in particular an interpretation of the phenomenon of evolutionary branching, where the distribution of the phenotypic traits under consideration in the population, initially concentrated around a single value, is driven by selection to divide into two (or more) clusters corresponding to sub-populations with different phenotypes, that stably coexist and are still in competition.

In collaboration with M. Benaïm (Université de Neuchatel, Switzerland) and S. Méléard (École Polytechnique, Palaiseau), we continued the study of a similar scaling limit of an individual-based evolutionary system, in the case where coexistence is possible. We were able to justify the convergence to a generalized TSS under the assumption of symmetric competition, thanks to results from dynamical systems. From this generalized TSS, we were also able to mathematically justify the evolutionary branching criterion proposed by biologists in the limit of small mutation, using results on the asymptotic dynamics of 3-dimensional competitive Lotka-Volterra systems [89]. We are preparing a publication.

5.1.8. Time scales in adaptive dynamics

In collaboration with A. Bovier (WIAS, Berlin), N. Champagnat is studying the different time scales of evolution as a function of the parameters of a similar model as the one of the previous paragraph. Three parameters are involved: the population size, the mutation probability and the size of mutations. When the first parameter goes to infinity and the two others go to zero, three distinct time scales appear. The first one corresponds to directional evolution, the second one to fast evolutionary branching and the last one to slow evolutionary branching. In the first phase, the number of coexisting types is (essentially) constant and in particular cannot increase. The types evolve by maximizing their relative “fitness” according to an ODE known as the "canonical equation of adaptive dynamics" [78]. Once they reach an equilibrium, evolutionary branching occurs on the second time scale similarly as in the previous paragraph. In particular we are able to compute the asymptotic branching time. Finally, on the last time scale, non-local evolutionary branching occurs. Here again, we are able to compute the asymptotic behaviour of the branching using large deviations arguments. All these time scales have been mathematically justified when the initial population is monotype in a simplified model. Some technical difficulties, linked with stability properties of dynamical systems and with convergence of stochastic processes on long time scales, still remain for the generalization of our results to more realistic models and to multitype populations.

5.1.9. The canonical diffusion of adaptive dynamics

N. Champagnat and Amaury Lambert (Laboratoire d'écologie, ENS Paris) have recently obtained [18] the counterpart of the TSS described above when the population size is finite. A diffusion process has also been obtained when the size of the jumps in the TSS vanishes, together with analytical expressions of its coefficients as series. From a biological viewpoint, this diffusion process quantifies the interplay between genetic drift and directional selection, and allows us to give an interpretation of the phenomenon of punctualism, where the evolution of the population is an alternance of long phases of stasis (constant state) and short phases of quick evolution.

We are now working on the numerical computation of the coefficients of the diffusion. They involve the first-order derivatives of the probability of invasion of a mutant with respect to the mutant birth, death and competition rates. We were able to decompose these derivatives into five fundamental components that allow one to determine which mutant type is more likely to invade the population (a problem related to notion of robustness of the population). This numerical study was initiated in [37] and allowed to obtain the convergence of the canonical diffusion to the canonical equation of adaptive dynamics mentioned above when the population size goes to infinity. We also plan to apply our numerical results to a bistable ecological model with two habitats.

5.1.10. Probabilistic interpretation of PDEs of evolution and evolutionary branching

In collaboration with P. Del Moral (INRIA Bordeaux) and L. Miclo (CNRS, Marseille), N. Champagnat and P.-E. Jabin are working on nonlinear Feynman-Kac formulæ giving a probabilistic interpretation of PDEs of the form

$$\partial_t u^\varepsilon = \varepsilon \Delta u^\varepsilon + \frac{1}{\varepsilon}\left(V(x) - \int W(x,y)u^\varepsilon(t,y)dy\right)u^\varepsilon.$$

The Laplacian corresponds to mutations in the phenotype space, and the nonlinear term corresponds to birth and death with interaction. The parameter ε scales the mutation size. Barles and Perthame [63] showed that, in very specific cases, when $\varepsilon \to 0$, u^ε converges to a single Dirac mass which support is characterized by a Hamilton-Jacobi PDE with constraint on its maximum. Using large deviations results, our probabilistic interpretation gives a variational characterization of the solution of this Hamilton-Jacobi equation, that could allow one to extend the results of Barles and Perthame to cases where several phenotypes can coexist in the population, i.e. where evolutionary branching can occur.

5.1.11. Two-types birth and death processes conditioned on non-extinction

In collaboration with P. Diaconis (Stanford University and Université de Nice – Sophia Antipolis) and L. Miclo (CNRS, Marseille), N. Champagnat is studying two-types birth and death processes conditioned on non-extinction. The motivation is to study the competition between two populations (for example a resident one and a mutant one) and to study which population survives when the whole population does not go extinct for a long time. We obtained criterions on the eigenvalues of sub-matrices of the transition matrix that characterizes the cases where one population goes extinct or when both populations survive. In the symmetric cases, where both populations are identical, we obtained a full description of the eigenvalues of the transition matrix in terms of the monotype transition matrix. We next aim at studying small deviations from the symmetric case to identify the largest eigenvalue and the corresponding asymptotic behaviour in the neighbourhood of symmetry.

5.1.12. Large population scalings of individual-based models with spatial and phenotypic interactions

In [19], N. Champagnat and S. Méléard (École Polytechnique, Palaiseau) have studied some large population scalings of individual-based ecological birth-death-mutation-competition models where each particle moves in the (physical) space according to a diffusion reflected at the boundary of a fixed domain. A particular care is put on the spatial interaction range, in order to recover integro-PDE models with local spatial interactions proposed by Desvillettes et al. [77]. Several examples are given, illustrating spatial and phenotypic clustering and the importance of such models, that combine space and evolution, for the study of invasion phenomena.

5.1.13. Systems of differential equations with singular force fields

P.-E. Jabin is mainly working on systems of differential equations with singular force fields. He was recently able to obtain quantitative estimates of compactness for solutions to ODE's with a BV field and a sort of compressibility condition (weaker than bounded divergence). More precisely it is possible to show that solutions to

$$\partial_t X(t,x) = b(X(t,x)), \quad X(0,x) = x,$$

satisfy a uniform bound on a quantity like

$$\sup_r \int_x \log\left(1 + \frac{|X(t,x) - X(t,x+r)|}{|r|}\right),$$

provided that the field is BV and the Jacobian of the flow is bounded. The above quantity was already introduced in a paper by Crippa and DeLellis [76], but with a $W^{1,p}$ condition on the field. This bound also implies well posedness for the system and it improves the well-known result of Ambrosio [60] which required a bounded divergence.

Moreover, in collaboration with Julien Barré (Université de Nice – Sophia Antipolis) and Maxime Hauray (Université Paris 6), we used this idea of quantitative estimates for the study of interacting particles. Considering a number of particles interacting with each other through a potential (electrostatic for example), the question is to obtain the limit when the number of particles goes to infinity. This limit is classically a kinetic equation in the phase space but a rigorous proof was not known whenever the potential was singular. We prove that in a suitable sense, for almost all initial conditions, the trajectories of almost all particles are close to the trajectories predicted by the mean field limit.

5.1.14. Poisson-Boltzmann equation in molecular dynamics

M. Bossy, N. Champagnat, P.-E. Jabin, S. Maire and D. Talay are working in collaboration with T. Malliavin (Institut Pasteur, Paris) on a probabilistic interpretation of the Poisson-Boltzmann equation of molecular dynamics that allows one to compute the electrostatic potential around a bio-molecular assembly (for example a protein). We also aim at developing specific Monte Carlo methods linked to the singularities of the model (operator with a divergence form with discontinuous coefficients, singular source terms, and discontinuous non-linear terms).

5.1.15. Lower bounds for densities of hypo-elliptic diffusions

Motivated by applications to risk analysis in Finance and reliability problems in Random Mechanics, P.-L. Lions and D. Talay are getting precise estimates on lower bounds of certain hypo-elliptic diffusion processes. Lower bounds of Gaussian type are well known for strongly elliptic diffusions. The techniques are very different in the hypo-elliptic case.

5.2. Interacting particle systems in Lagrangian modeling of turbulent flows

Keywords: *confinement of Langevin system*, *discrete mass-transport problem*, *interacting particle system*, *pdf approach*.

Participants: Mireille Bossy, Jean-François Jabir, Pierre-Louis Lions.

This section presents the results obtained by TOSCA on the modeling of turbulent flows. Actually, it could be a part of the previous section on stochastic modeling and applications. We have chosen to present this work in a distinct coherent section.

In the statistical approach of Navier-Stokes equation in $\mathbb{R}^3$, the Eulerian properties of the fluid (such as velocity, pressure and other fundamental quantities) are supposed to depend on possible realizations $\omega \in \Omega$. In the incompressible case, in order to compute the averaged velocity, one needs to model the equation of the Reynolds stress. A direct modeling is for example the so-called k-epsilon turbulence model. An alternative approach [87] consists in describing the flow through a Lagrangian stochastic model whose averaged properties are linked to those of the Eulerian fields by conditional means w.r.t. particle position. In the computational fluid dynamics literature, those models are referred to as stochastic Lagrangian models. For example, the Simplified Langevin model [88], which characterizes particle positions-velocity $(X_t, \mathcal{U}_t)$ in the case of homogeneous turbulent flow, is defined as

$$\begin{cases} dX_t = & \mathcal{U}_t dt, \\ d\mathcal{U}_t = & -\nabla_x \langle \mathcal{P} \rangle (t, X_t) dt - \left(\frac{1}{2} + \frac{3}{4} C_0\right) \langle w \rangle (t, X_t) \left(\mathcal{U}_t - \mathbb{E}\left(\mathcal{U}_t \mid X_t\right)\right) dt \\ & + \sqrt{C_0 \langle w \rangle (t, X_t) k(t, X_t)} dW_t, \end{cases} \tag{3}$$

where C_0, $\langle w \rangle (t,x)$ and $k(t,x)$ are positive real quantities supposed to be known, and W is a Brownian motion. The pressure gradient $\langle \mathcal{P} \rangle (t,x)$ is treated as solution of a Poisson equation in order to satisfy the constant mass-density (i.e. particle positions are uniformly distributed) and divergence free conditions. Moreover in the presence of physical boundaries evaluated at $\partial\mathcal{D}$, the model is submitted to a limit condition of the form: $\mathbb{E}\left(\mathcal{U}_t \mid X_t = x\right) = 0$, for $x \in \partial\mathcal{D}$. The corresponding discrete algorithm is reported in Section 6.1.

5.2.1. *Analysis of the Langevin equations in turbulent modeling*

Keywords: *Stochastic Lagrangian Models*, *Vlasov-Fokker-Planck equations*.

In his Ph.D. thesis supervised by M. Bossy and D. Talay, J.-F. Jabir studies theoretical problems involved by (3) in the case where the pressure gradient is removed while k and $\langle \omega \rangle$ are supposed to be constants.

M. Bossy and J.-F. Jabir study the well-posedness of a simplified version of equation (3) (in particular, without the pressure term) where $\mathcal{U}_t - \mathbb{E}_{\mathbb{P}}\left(\mathcal{U}_t \mid X_t\right)$ is replaced by the kernel $\mathbb{E}_{\mathbb{P}}\left(b(v - \mathcal{U}_t) \mid X_t\right)|_{v=\mathcal{U}_t}$. The conditional expectation involved in (3) implies that this equation is a McKean-Vlasov equation with singular kernels. When b is a bounded continuous function, we proved the well-posedness of the Lagrangian system. In particular, the existence result has been obtained by studying related smoothened interacting particle systems and by proving a propagation of chaos result.
Motivated by the downscaling application (see Section 6.1), we construct and investigate the well-posedness of Langevin system confined within a regular domain $\mathcal{D}$ of $\mathbb{R}^d$ with the imposed boundary condition

$$\mathbb{E}\left(\left(\mathcal{U}_t \cdot n_{\mathcal{D}}(X_t)\right) \mid X_t = x\right) = 0 \text{ for } x \in \partial\mathcal{D} \tag{4}$$

for $n_{\mathcal{D}}$ denoting the outward normal unit vector related to $\mathcal{D}$. The basic idea of our construction is to add a "confinement" term, $-2\left(\mathcal{U}_{t^-} \cdot n_{\mathcal{D}}(X_t)\right) n_{\mathcal{D}}(X_t)$, to the velocity dynamic at each time the particle hits the boundary. Assuming that the law of the Lagrangian system $(X, \mathcal{U})$ admits a trace component along the boundary $\partial\mathcal{D}$ with suitable integrability properties, the law of $(X, \mathcal{U})$ satisfies the specular boundary condition (see [74])

$$\rho(t,x,u) = \rho(t,x,u - 2(u \cdot n_{\mathcal{D}}(x))n_{\mathcal{D}}(x)) \text{ for } (t,x,u) \in (0,T) \times \partial\mathcal{D} \times \mathbb{R}^d \tag{5}$$

and show that (5) implies (4).
When $\mathcal{D} = \mathbb{R}^d \times \mathbb{R}^+$, (strong) existence and uniqueness for the "confined" Langevin system have been established in the simplified case of classical SDE with a constant diffusion thanks to the results of Lachal [81]. Moreover, under suitable conditions on ρ_0 and the drift coefficient we also show that the law of $(X, \mathcal{U})$ satisfies (4) and (5). These results have been also extended for the class of McKean-Vlasov equations with smooth and bounded kernels.
When $\mathcal{D}$ is a bounded regular domain of $\mathbb{R}^d$, in collaboration with P.-L. Lions, we study the Fokker-Planck equation associated to $(X_t, \mathcal{U}_t)$

$$\begin{cases} \partial\rho + (u \cdot \nabla_x \rho) + \nabla_u \left(\rho \left(B(\rho) - \beta u\right)\right) + \frac{\sigma^2}{2} \triangle_u \rho = 0 \text{ on } (0,T] \times \mathcal{D} \times \mathbb{R}^d, \\ B(\rho)(t,x) = \dfrac{\displaystyle\int b(v)\rho(t,x,v)\,dv}{\displaystyle\int \rho(t,x,v)\,dv}, \ \beta > 0, \ \rho_{t=0} = \rho_0 \text{ on } \mathcal{D} \times \mathbb{R}^d, \\ \rho \text{ satisfies (5) on the boundary.} \end{cases}$$

When b is bounded, we establish the well-posedness of this equation using Maxwellian super-solutions and sub-solutions which give upper and lower bounds for the solution.

5.2.2. The position correction step in the Lagrangian simulation of constant mass density fluid

We continue our collaboration with A. Rousseau (MOISE Inria Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand) on this topic (see section 6.1). We combine a "fractional step" method and a "particles in cell" method to simulate the time evolution of the Lagrangian variables of the flow described by equation (3).

The most noticeable problem arises from the mass-density conservation between two time steps within a given cell. The aim is to move the particles in a box such that:

- their density is uniform (i.e. the number of particles is the same in each cell of a considered mesh).
- the "transport cost", which is related to the particles transfer, is minimum.

This problem is not classical at all and is referred in the literature as a problem of discrete optimal transportation, which is known to be nonlinear and numerically very hard to solve in dimension 3.

During summer 2007, A. Rousseau managed a research program at CEMRACS, and the group adapted an algorithm from D. Bertsekas [66], in order to compute the solution of an optimal transport problem that concerns the particles involved in SDM: see [38]. This research program was funded by CEMRACS and INRIA (Service de Formation par la Recherche). Next January, C. Chauvin, from the CEMRACS group, will start a 12-months post-doc position in the TOSCA and MOISE teams.

5.3. Financial Mathematics

Participants: Mireille Bossy, Mamadou Cissé, Nicolas Champagnat, Anthony Floryszczak, Junbo Huang, Thanh-Hoang Mai, Stephania Maroso, Denis Talay, Etienne Tanré, Pierre Vallois.

5.3.1. Modelling of financial techniques

In collaboration with R. Gibson (Zürich University), C. Blanchet and S. Rubenthaler (Université de Nice – Sophia-Antipolis), B. de Saporta (Université Bordeaux 4), D. Talay and E. Tanré elaborate an appropriate mathematical framework to develop the analysis of the financial performances of financial techniques which are often used by the traders. This research is funded by NCCR FINRISK (Switzerland) and is a part of its project "Conceptual Issues in Financial Risk Management".

In the financial industry, there are three main approaches to investment: the fundamental approach, where strategies are based on fundamental economic principles, the technical analysis approach, where strategies are based on past prices behavior, and the mathematical approach where strategies are based on mathematical models and studies. The main advantage of technical analysis is that it avoids model specification, and thus calibration problems, misspecification risks, etc. On the other hand, technical analysis methods have limited theoretical justifications, and therefore none can assert that they are risk-less, or even efficient.

Consider an unstable financial economy. It is impossible to specify and calibrate models which can capture all the sources of instability during a long time interval. Thus it is natural to compare the performances obtained by using erroneously calibrated mathematical models and the performances obtained by technical analysis techniques. To our knowledge, this question has not been investigated in the literature.

We deal with the following model for a financial market, in which two assets are traded continuously. The first one (the bond) is an asset without systematic risk. The second one (the stock) is subject to systematic risk and its instantaneous rate of return changes at independent exponentially distributed random times. The trader does not observe the times of change.

This year we extended our earlier results [70], [71] by taking transaction costs into account. At any time, the trader is allowed to invest all his wealth in the bond or in the stock. This leads to a non-classical stochastic control problem. We have proved that the value function of this control problem is continuous, satisfies a dynamic programming principle, and is the unique viscosity solution of a Hamilton–Jacobi–Bellman (HJB) equation. We developed an algorithm to compute the value function and a sub-optimal trading strategy, and compared its performance to the performance of the moving average strategy. When transactions costs are high, it is difficult for the technical analyst to outperform miscalibrated mathematical strategies. We are now working on the numerical analysis of the HJB equation. We aim to precise the rate of discretization methods.

5.3.2. Optimal dynamic cross pricing of CO2 market

Company strategists in the energy sector find themselves faced with new constraints, such as emission quotas, that can be exchanged through a financial market.

In collaboration with Nadia Maïzi (CMA ENSMP Sophia Antipolis) and Odile Pourtallier (CORPIN INRIA Sophia Antipolis – Méditerranée), M. Bossy and A. Floryszczak propose an approach in order to assess the impact of CO2 tax and quota levels on CO2 market prices. We model the expectation of the yield of an industrial with and without CO2 market, in order to derive the indifference price. This price is established through an optimal dynamic approach leading to the resolution of an Hamilton-Jacobi-Bellman equation. First results, based on simplified tax functions, have been presented in [40].

5.3.3. Artificial boundary conditions for nonlinear PDEs in finance

Under M. Bossy and D. Talay's supervision, M. Cissé studied the problem of artificial boundary conditions for nonlinear PDEs. The motivation of this research concerns American option pricing and the numerical resolution of the variational inequality characterizing prices of American options.

First, we have extended the theorem on existence and uniqueness of solutions of the reflected backward stochastic differential equations (RBSDE) with fixed terminal time to the case of bounded random terminal time. We obtained a probabilistic interpretation of the previous solution as a viscosity solution of variational inequalities with Dirichlet boundary conditions. We obtained a general expression of the localization error in terms of the boundary conditions.

Secondly, we were interested in variational inequalities with Neumann boundary conditions. They can be interpreted as a generalized RBSDEs coupled with a reflected forward stochastic differential equations (RSDE). We used the derivative in the sense of distributions with respect to the initial data of the RSDE and the representation of the gradient of Ma and Zhang [83] to establish a representation theorem for the space derivative of the viscosity solution of the variational inequalities. We apply this result to get a priori estimates on the error induced in numerical simulations by artificial boundary conditions for the PDEs related to American options pricing. The differentiability of the reflected forward SDE in the sense of distributions with respect to the initial data is deduced from results by Bouleau and Hirsch [73] on the absolute continuity of probability measures.

5.3.4. Rate of convergence in the Robbins-Monro algorithm

In his thesis under D. Talay's supervision, J. Huang continues the work on the numerical analysis of HJB equations.

As Karatzas have mentioned in [80], some portfolio optimization problems corresponding to the given HJB equations can also be solved by the martingale method. Then we can expect the martingale method may give a good approximation of boundary conditions for the aforementioned HJB equations. The martingale method is, however, (roughly speaking) a special application of the approach of Lagrange multipliers, which is the basic tool in nonlinear constrained optimization. The Lagrange multiplier often acts as the root of some constraint equation. This implies that in order to approximate the boundary conditions of the HJB equation by using the martingale method one has to solve the constraint equation and find the corresponding root. To this end, we invoke a particular stochastic algorithm firstly proposed by Robbins and Monro and then developed by several other authors.

Generally speaking, for a given equation $h(\theta)=0$, the Robbins-Monro algorithm is like

$$\begin{aligned}\theta_n &= \theta_{n-1}+\gamma_n y_n\\ &= \theta_{n-1}+\gamma_n h(\theta_{n-1})+\gamma_n\eta_n,\end{aligned}$$

where γ_n is the step size, y_n is stochastic and so is η_n. Under some assumptions, people have proved that $\frac{\theta_n-\theta^*}{\sqrt{\gamma_n}}$ is asymptotically normal, where θ^* is the sought-for root. We, however, are interested in the Berry-Esseen bound for the aforementioned term because this kind of bound usually provides a more accurate and neat estimate of the error term in the normal approximation. Thorough analyzes show that $\frac{\theta_n-\theta^*}{\sqrt{\gamma_n}}$ is a sum of two terms: one is a sum of martingale increments and the other is a random variable converging to zero almost surely. Thus our work resides in solving two problems: one is to give the Berry-Esseen bound for a sum of martingale increments and the other is to specify the convergence rate to zero of the second variable mentioned above.

5.3.5. Liquidity risk

P. Protter (Cornell University) and D. Talay are addressing the following question. We have the possibility of trading in a risky asset (which we refer to as a stock) with both liquidity and transaction costs. We further assume that the stock price follows a diffusion, and that the stock is highly liquid. We limit our trading strategies to those which change our holdings only by jumps (i.e., discrete trading strategies), and we begin with $0 and 0 shares, and we end with a liquidated portfolio (that is, we no longer hold any shares of the stock), on or before a predetermined ending time T. The question then is, given the structure of the liquidity and transaction costs, what is the optimal trading strategy which will maximize our gains? This amounts to maximize the value of our risk free savings account. This problem can be solved in this context if it is formulated as a non classical problem in stochastic optimal control. We prove existence and uniqueness results for the related Hamilton–Jacobi–Bellman equation.

5.3.6. Portfolio optimization in incomplete markets

N. Champagnat, T.-H. Mai, S. Maroso, D. Talay and E. Tanré are working on various aspects of portfolio management in incomplete markets within a contract with Natixis (see Section 6.6).

5.3.7. Optimal stopping problems

Keywords: *Optimal stopping*, *h-transform*.

In collaboration with P. Patie (University of Bern, Switzerland), M. Cissé and E. Tanré solve explicitly the optimal stopping problem with random discounting and an additive functional as cost of observations for a regular linear diffusion. This generalizes a result by Beibel and Lerche [64], [65]. The approach relies on a combination of Doob's h-transform, time-changes and martingales techniques. Our results combined with Patie's results [86] allows us to treat a few examples, one of them being the evaluation of a perpetual American type option with payoff defined by

$$P_t = e^{-\int_0^t X_s ds}(X_t - K)^+ \tag{6}$$

where the process X is the instantaneous interest rate modeled by the Vasicek or Ornstein-Uhlenbeck process.

5.3.8. Memory-based persistent counting random walk and applications

In [59], P. Vallois and C. Tapiero (ESSEC) have considered a memory-based persistent counting random walk, based on a Markov memory of the last event. The usefulness to some problems in insurance, finance and risk analysis are discussed.

5.4. Statistical analysis

Participants: Samy Tindel, Pierre Vallois.

The works presented in this Section are developed within the workshop between the probability and statistics teams of Institut Élie Cartan and the group led by B. Bihain, the head of the Laboratoire de Médecine et Thérapie Moléculaire (INSERM, Nancy).

5.4.1. Estimation of uptake kinetics

The articles [35], [46] present some method to estimate parameters in vivo uptake kinetics of photo-sensitizing drugs into cancer cells in presence of random timing errors.

5.4.2. Statistical analysis of genes variations

The article [16] presents a statistical analysis of sequence variations of 17 abundantly expressed genes in a large set of human EST's originating from either normal or cancer samples, which allows to show that cancer EST's have greater variations than normal for 70% of the tested genes.

5.5. Stochastic analysis and applications

Participants: Bernard Roynette, Pierre Vallois.

In this section we present our results on issues which are more abstract than the preceding ones and, at first glance, might appear decorrelated from our applied studies. However most of them are originally motivated by modelling problems, or technical difficulties to overcome in order to analyze in full generality stochastic numerical methods or properties of stochastic models.

5.5.1. Penalization of diffusion processes

Keywords: *Bessel processes*, *Ray-Knight's theorems*, *Sturm-Liouville equation*, *Wiener measure*, *down-crossings*, *enlargement of filtration*, *limiting laws*, *local time*, *maximum*, *minimum*, *normalized exponential weights*, *penalization*, *rate of convergence*.

Jointly with M. Yor (Université Paris 6), P. Vallois and B. Roynette have continued their study of the penalization of diffusion processes [28], [27], [58], [57]. The penalization procedure allows one to modify a given process (for example a Brownian motion or a Bessel processes) in order to get a new path property (for instance boundedness or a different behavior in a neighborhood of 0). This procedure can be very interesting in modelling to go further than the approach based on diffusions.

5.5.2. Approximation schemes for the local time

In [17], [49], P. Vallois and B. Bérard-Bergery (Université Nancy 1) have proposed new approximation schemes related to the local time process of the standard Brownian motion. Some rates of convergence have been exhibited explicitly.

6. Contracts and Grants with Industry

6.1. Collaboration with ADEME: local modeling for the wind velocity

Keywords: *3D-Navier-Stokes equations*, *down-scaling methods*, *particle in mesh models*.

Participants: Mireille Bossy, Jean-François Jabir.

Started in 2005, our joint collaboration with the Laboratoire de Météorologie Dynamique (Université Paris 6, École Polytechnique, École Normale Supérieure) is funded by the French Environment and Energy Management Agency (ADEME) and concerns the modeling and the simulation of local wind energy resources. We collaborate with É. Peirano (ADEME), P. Drobinski and T. Salameh (LMD). The second phase of this collaboration has just begun (October 2007) and include as partners A. Rousseau (MOISE Inria Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand). We investigate a new method for the numerical simulation of the wind at small scales. Thanks to boundary data provided at large scales by the weather forecasting code $MM5$, we propose a Langevin model that rules the behavior of stochastic particles. This model called SDM (Stochastic Downscaling Method) is adapted from previous works introduced by S.B. Pope [87] and has been presented in [42], [36].

This year we have finished a beta version of our *SDM* code and we have started the validation phase of our simulation on a simple meteorological framework using observations from measurements campaigns, namely the campaign FETCH that took place in 1998 in Southern France and runs of the $MM5$ solver with two different resolutions. The first mesh will be coarse, and will be devoted to feed our stochastic model. The second mesh will be finer, and we hope that our simulations will provide comparable results, with a smaller computational cost (CPU time) than the one corresponding to the same resolution, performed with a classical refinement of $MM5$. The very first results are promising.

6.2. ANR GCPMF: Grille de Calcul pour les Mathématiques Financières

Participants: Mireille Bossy, Denis Talay.

We collaborate with the OASIS team within the ANR project entitled “GCPMF” funded by the ANR Research Program “Calcul Intensif et Grilles de Calcul 2005”.

The aim of this ANR program is to highlight the potential of parallel techniques applied to mathematical finance computing on Grid infrastructures. The consortium that conducts this project includes ten participants from academic laboratories in computer science and mathematics, banks, and IT companies.

Financial applications require to solve large size computations, that are so huge that they cannot be tackled by conventional PCs. A typical example addresses time-critical computations required during trading hours, in particular Monte Carlo simulations for option pricing and other derivative products.

A software system called PicsouGrid has been designed and implemented by the OASIS team, which utilizes the ProActive library to parallelize and distribute various option pricing algorithms. Currently, PicsouGrid has been deployed on various grid systems to evaluate its scalability and performance in European option pricing. We also developed several European option pricing algorithms such as standard, barrier, basket options to experiment in PicsouGrid. A part of this work was presented in [69] and [43].

In the second half of 2007, parallel versions of several American option pricing algorithms have been implemented (Longstaff and Schwartz, Ibanez and Zapatero, and Picazo), including those which price options on several assets simultaneously (called basket options). Our work has focused on finding efficient parallelization strategies which can be used for a range of pricing algorithms. The objective is to allow algorithm designers to focus on an efficient serial implementation without concern for the parallelization, and for the model to be used to automatically or semi-automatically provide a load-balanced (for heterogeneous compute resources) parallel implementation. The extended abstract of this work can be found in [39].

6.3. ANR MAEV: Modélisation Aléatoire et Évolution du Vivant (stochastic modelling of the evolution of living systems)

Participant: Nicolas Champagnat.

The general goal of this research project is the development and the analysis of new stochastic models of evolution, which take into account the interactions and the diversity of scales in evolution. The partners (probabilists and evolutionary biologists mainly in Paris, Marseille and Grenoble) are exploring four research directions:

- Evolution at the molecular scale: new models of the evolution of genes taking into account the interactions between sites and the main factors of global changes in the genome (genes duplication, transfer,...).
- Adaptive evolution: macroscopic models of adaptive evolution that are deduced from the microscopic, individual scale and from genes to the organism.
- Shape of random trees: random tree models for molecular evolution or for species evolution, and the mathematical tools to compare them in order to analyse the evolutionary relations between populations or species.
- Coalescence: coalescent processes coding for the evolution of a group of genotypes inside a large population, allowing one to study the polymorphism when dependence between individuals and various scales are taken into account.

A strong interest is brought to algorithmic implementations, numerical analysis and applications to data processing.

The works of N. Champagnat on evolutionary branching, on processes conditioned on non-extinction and on the canonical diffusion of adaptive dynamics belong to this project.

6.4. Contract with Calyon

Participants: Madalina Deaconu, Samuel Herrmann, Antoine Lejay, Denis Talay, Étienne Tanré.

This contract with Calyon started on January 2007 and concerns the efficient simulation of interest rates models through Monte Carlo methods and variance reduction techniques. In particular, we are interested in using and extending the techniques of *exact simulation* and on the techniques of quantification coupled with Monte Carlo simulations.

6.5. Contract with Natixis: adaptative Monte Carlo methods

Participants: Madalina Deaconu, Antoine Lejay, Denis Talay.

The Ph.D. thesis of Numa Lescot started in September 2006 with a CIFRE grant from Natixis. This thesis concerns adaptive Monte Carlo methods in the continuation of the works of O. Bardou [62] and B. Arouna [61], where the importance sampling technique is coupled with stochastic algorithms to perform variance reduction. The aim is to apply this technique together with Malliavin calculus in order to perform variance reduction on the computation of the Greeks.

6.6. Contract with Natixis: portfolio optimization in incomplete markets

Participants: Nicolas Champagnat, Thanh-Hoang Mai, Stephania Maroso, Denis Talay, Etienne Tanré.

This works concerns various aspects of portfolio management in incomplete markets. It is developed in four directions: calibration and estimation of historical probabilities, rupture detection, dynamical portfolio optimization and numerical validation for optimal control. The hedging problem under Gamma constraints and the discrete hedging problem with minimal delay between transactions are also studied in the case of barrier options.

6.7. Natixis Fundation

Participant: Denis Talay.

D. Talay has been elected as the Vice-President of the Fondation d'Entreprise Natixis which aims to contribute to develop research in quantitative finance.

He also serves as a member of the Scientific Committee of the Foundation, jointly with M. Crouhy (President, Natixis), N. El Karoui (École Polytechnique), P-L. Lions (Collège de France), J-P. Laurent (Université Claude Bernard, Lyon).

7. Other Grants and Activities

7.1. Other Grants and Activities

The project-team TOSCA participates to the "Groupe de Recherche GRIP" on stochastic interacting particles and to the European Network AMAMEF on Advanced Mathematical Methods for Finance. D. Talay serves as a member of the scientific committees of these two research networks.

A. Lejay is responsible for the sub-project "Monte Carlo Methods for Discontinuous Media" within the project "Particle Methods" of the "Groupe de Recherche MOMAS" funded by ANDRA, BRGM, CEA, EDF, CNRS and IRSN.

TOSCA participates to the NCCR FINRISK (Financial Risk) forum launched by the Swiss National Science Foundation and managed by the University of Zürich.

8. Dissemination

8.1. Animation of the scientific community

D. Talay serves as an Associate Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Journal of Scientific Computing*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

D. Talay is the President of the French Applied Mathematics Society SMAI.

D. Talay serves as a member of the Committee for junior permanent research positions at Université Bordeaux 1.

M. Bossy serves as a member of the Scientific Committee of the *École Doctorale "Sciences Fondamentales et Appliquées"* of the Université de Nice – Sophia Antipolis.

M. Bossy serves as member of the *Suivi Doctoral* Committee, "Cours et Colloques" Committee and the scientific Committee for the "COopérations LOcales de Recherche" of INRIA Sophia Antipolis – Méditerranée.

N. Champagnat is the administrator, with C. Tran Viet (Université de Lille 1) of the web site of the ANR MAEV.

M. Deaconu serves as member of the Scientific Committee of the MAS group (Probability and Statistics) within SMAI.

M. Deaconu is a permanent reviewer for the *Mathematical Reviews*.

M. Deaconu serves as member of the International Relations Work Group of COST at INRIA.

M. Deaconu serves as member of the *Conseil du laboratoire* of IECN and of the *Commission des spécialistes* of the Departement of Mathematics of the Université Nancy 1.

P.-E. Jabin is member of the hiring committee at the Université de Nice – Sophia Antipolis.

P.-E. Jabin is responsible for the third year of *Licence* in mathematics at the Université de Nice – Sophia Antipolis.

P.-E. Jabin is responsible for the colloquium at the maths department of the Université de Nice – Sophia Antipolis.

A. Lejay serves as a member of the *Commission de Spécialistes* of Université Louis Pasteur in Strasbourg.

A. Lejay serves as a member of the *Commission des moyens informatiques* of INRIA Nancy – Grand Est.

S. Tindel serves as a member of the *Commission de Spécialistes* at the Université Nancy 1.

P. Vallois is the head of the Probability and Statistics group of Institut Élie Cartan.

P. Vallois serves as a member of the council of the UFR STMIA, the *Conseil du laboratoire* and the *Commission de Spécialistes* of the Mathematics Department of Université Nancy 1.

P. Vallois serves as an Associate Editor of *Risk and Decision Analysis*.

P. Vallois serves as expert for the *Ministère de l'Enseignement Supérieur*.

8.2. Diffusion of knowledge and educational works

M. Bossy and N. Champagnat have written an article for the Encyclopedia of Quantitative Finance on *Markov processes and parabolic partial differential equations* [48].

8.3. Animation of workshops

M. Bossy served as a member of the Scientific Committee and Organizing Committee of the *Colloque des Doctorants STIC & SFA*, May 10-11, 2007 Sophia Antipolis.

M. Bossy served as a member of the Scientific Committee of the congress *CANUM 2008*.

S. Herrmann organized a workshop on *Mathematical Finance* at the Institut Élie Cartan.

B. Bihain, the head of the Laboratoire de Médecine et Thérapie Moléculaire (INSERM, Nancy), asked to our group of Probability and Statistics to investigate the proteomic profiling of clinical samples. More precisely, the question involves the differential analysis of the expression levels of a large subset of the proteome of a particular type of clinical specimens to identify those proteins whose change in expression levels might be associated with a given disease process. The statistical tools are Data Analysis, and Support Vectors Machine. P. Vallois, S. Tindel (Université Nancy 1) and J.-M. Monnez (Université Nancy 2) have organized several seminars on these topics. S. Mézières and A. Koudou (Université Nancy 2) also participate to the bio-statistics group. O. Collignon has a Cifre thesis.

D. Talay served as a member of the Scientific Committee of the *Convergences Franco-Maghrébines* workshop (Nice), *Equadiff 2007* Congress (Vienna), and *MCM2007 IMACS* Seminar on Monte Carlo methods (reading). He also co-organized the SMAI-INRIA *Journées EDP-Probabilités* at IHP (Paris).

P. Vallois have organized the *Journées en l'honneur de Bernard Roynette* in Nancy in October. S. Herrmann has also participated to the organization.

P. E. Jabin organized the second edition of the *Cours Poupaud*, in September. He also organized two joint workshops on biology and mathematics between Inria Sophia Antipolis – Méditerranée and the Université de Nice – Sophia Antipolis.

8.4. Teaching

M. Bossy gave a 30h course on *Stochastic calculus and financial mathematics* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis), and a 15h course on *Risk management on energetic financial markets* in the Master *Ingénierie et Gestion de l'Energie* (École des Mines de Paris) at Sophia-Antipolis.

M. Deaconu gave a course on *Stochastic Differential Equations* in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2006–2008.

J.-F. Jabir gave 14h of exercice class on *Stochastic Calculus and Financial Mathematics* in the Master IMAFA at the *Ecole Polytechnique Universitaire*, Sophia-Antipolis.

A. Lejay gave a course on *Financial Mathematics* in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2007–2008.

D. Talay has a part time position of Professor at École Polytechnique. He also teaches *probabilistic numerical methods* at Université Paris 6 (Master degree in Probability), and *numerical methods in finance* at the University of Zürich.

E. Tanré gives a 6h course in the Master IMAFA.

S. Tindel is the organizer of the Financial Mathematics options of the Probability Master at the Élie Cartan Institute (Université Nancy 1).

P. Vallois gives courses in the Mathematical Finance Master 2 program, in Nancy.

8.5. Ph.D. theses and habilitations

Antoine Lejay defended his habilitation entitled *Contributions à la théorie des processus engendrés par des opérateurs sous forme divergence, aux méthodes de Monte Carlo et à la théorie des trajectoires rugueuses* in December at Université Nancy 1.

Sylvain Maire defended his habilitation entitled *Quelques techniques de couplages entre méthodes numériques déterministes et méthodes de Monte Carlo* in December in Université de Toulon et du Var.

8.6. Participation to congresses, conferences, invitations, ...

M. Bossy gave a seminar lecture at the Laboratoire J.-A. Dieudonné, Université de Nice – Sophia-Antipolis.

M. Bossy gave a talk at the *Journée Méthodes Particulaires en Mécanique des Fluides* organized by the Centre National de Recherches Météorologiques and Météo-France, Toulouse.

M. Bossy gave a seminar lecture at the *Groupe de Travail Mécanique des Fluides* de Toulouse, INSA.

N. Champagnat gave talks at the *séminaire commun Dieudonné OMEGA*, Université de Nice in January, at the *Kolloquim on probability theory*, WIAS, Berlin, Germany in May, at the *Workshop on Stochastic Approaches to Evolution*, Götheborg, Sweden in May, at the *MMEE2007 Conference*, University of Sussex, United Kingdom in September, at the *Séminaire de Probabilités*, Université Paul Sabatier, Toulouse in Novembre, at the seminar *Mathématiques et Sciences du Vivant*, INRIA Sophia Antipolis – Méditerranée in December and at the seminar *Modèles de Populations*, INRA, Avignon in December.

M. Cissé spent two weeks (April 9-20, 2007) in the *CIMPA-UNESCO MOROCCO schoool in Stochastic Models in Mathematical Finance*.

M. Cissé gave a seminar lecture at the *Laboratoire d'Études et de Recherches en Statistiques Appliquées au Développement*, Saint-Louis, Senegal.

M. Deaconu, É. Tanré and A. Lejay spent two weeks in Chile in July 2007 to work with R. Rebolledo, C. Mora and S. Torrès and gave several talks at the *First Workshop on renewable energy, energy efficiency and stochastic modelling* at Universidad de Concepción.

M. Deaconu gave a talk at the congress *SMAI 2007* in June.

M. Deaconu and S. Maire have gave a talk at the *ICIAM 2007* conference in Zurich in July.

S. Herrmann organized a mini-symposium at the conference *ICIAM 07* in Zurich in July and at *Equadiff 07* in Vienna in August. He also gave talks at the *Workshop on Random Dynamical Systems* at Bielefeld on November.

P.-E. Jabin gave a talk at the *Meeting of the Phenix GDR*, in September.

P.-E. Jabin gave a seminar at the Université Paris-Dauphine, in December.

J.-F. Jabir gave a talk at the *SMAI 2007* congress in June.

J.-F. Jabir gave a seminar lecture at the Laboratoire J.A. Dieudonné, Universite de Nice – Sophia Antipolis.

J.-F. Jabir gave a seminar lecture at the *séminaire EDP-MOISE*, Grenoble.

A. Lejay gave talks at the at the conference *Stochastic Analysis and Related Fields* in Toulouse in June, at the conference *International Conference on Numerical Analysis and Applied Mathematics (ICNAAM 2007)* in Corfou in September, at the conference *Applications of Stochastic Partial Differential Equations* at the Mittag-Leffler Institute in November, at the *Séminaire de Probabilités de l'Université de Provence* in January 2007, at the *Groupe de travail "Probabilités Numériques et Finance"* at Université Paris 6 in March, at the *Groupe de travail de Probabilités et Statistiques* of Université Paris 5 in October.

D. Talay gave a course at the CIMPA School *Modèles aléatoires en finance mathématiques* in Marrakech. He gave a plenary lecture at the 22nd Biennial Conference on Numerical Analysis in Dundee, at the *Partial Differential Equations an Finance* in Stockholm, during the *Journées en l'honneur de B. Roynette* in Nancy, during the *The practice and theory of stochastic simulation* AIM Workshop in Palo Alto. He gave an invited talk at the *Quantitative Finance Forum FC-2007* in Santiago de Compostela. He co-organised (with M. Hairer, Warwick University) a mini-symposyum during the *SciCADE conference* in Saint-Malo.

S. Tindel gave talks at the *Workshop on Polymers and Applications* in Nice in February, at the *Probability Seminar* at the CINVESTAV, Mexico (Mexico) in April, at the *Workshop on Stochastic Dynamics* in Paris in June, at the *Summer School on Spin Glasses* in Paris in June, and at the *Probability Seminar* of Université Paris 13 (France) in October. S. Tindel has also given a short course at the *Winter School on Stochastic Analysis and Applications* in Valparaíso (Chile) in July.

P. Vallois gave talks at the *Decision and risk analysis conference (Convergence between finance and industry)* in Dallas in May, at the *International conference Skorokhod space, 50 years on* in Kiev in June, *Journées de probabilités* at La Londe-des-Maures in September, at the 9^{th} *colloqium international of mathematics* in Nancy in October.

8.6.1. Invitations

N. Champagnat was invited three days in January at the Université de Neuchatel, Switzerland, two weeks in May at the WIAS in Berlin and one week in November at the Laboratoire d'écologie, Université Paris 6.

A. Lejay and S. Tindel have been invited to participate at the semester on *Stochastic Partial Differential Equations* at the Mittag-Leffler Institute for a month in November and December.

P. Vallois was invited for one week in May at the Polytechnic University of New-York and one week in October-November at Turku (Finland).

Carlos Manuel Mora Gonzàlez (Universidad de Concepción, Chili) has been invited for 10 days in the INRIA in November.

Benoîte de Saporta (Université Bordeaux 4) spent twice 3 days in the INRIA in June and December.

The TOSCA *seminar* organized by N. Champagnat has received the following speakers: Pierre Vallois (IECN and INRIA, Nancy, France), Stephania Maroso (INRIA, Rocquencourt, France), Erwan Faou (IRISA, Rennes, France), Jérôme Lelong (CERMICS, Marne-la-Vallée, France), Joaquin Fontbona (Universidad de Chile, Chili), Magali Kervarec (Université d'Evry, France), Philip Protter (Cornell, USA and Titulaire de la Chaire de Tocqueville-Fulbright 2007-2008, Université Paris-Dauphine, France), Jean Jacod (Université Paris 6, France), Carlos Manuel Mora Gonzàlez (Universidad de Concepción, Chili) and Pierre-Emmanuel Jabin (Université de Nice – Sophia Antipolis and INRIA, Sophia Antipolis – Méditerranée, France).

The joint *Laboratoire J.A. Dieudonné (Université de Nice – Sophia Antipolis)* – TOSCA *seminar* organized by N. Champagnat until June has received the following speakers: Nicolas Champagnat, Mireille Bossy (INRIA, Sophia Antipolis – Méditerranée, France), Nizar Touzi (CMAP, Ecole Polytechnique, Palaiseau, France), Persi Diaconis (Stanford University, USA and Université de Nice – Sophia Antipolis, France), Laurent Miclo (CNRS, Université de Marseille 1, France), Nicolas Rousseau (Université de Nice – Sophia Antipolis, France), Erwan Faou (IRISA, Rennes, France), Jean-François Jabir (INRIA, Sophia Antipolis – Méditerranée, France) and Paavo Salminen (Abo Akademi University, Finland).

The seminar *Probabilités* organized at Nancy by S. Tindel has received the following speakers: Paavo Salminen (Åbo Akademi University, Finland), Marie-José Martinez (INRA Clermont, France), Jean Picard (Université Blaise Pascal, France), Olivier Garet (Université d'Orléans, France), Mathias Rousset (CERMICS, Marne-la-Vallée, France), Andreas Neuenkirch (Goethe Universität, Frankfurt, Germany), Christophe Crambes (Université de Toulouse 3, France), Gilles Hargé (Université d'Evry, France), Shigeyoshi Ogawa (Ritsumeikan University, Japan), Sébastien Darses (Université de Paris 6, France), Szymon Peszat (Polish Academy of Sciences, Cracow, Poland), Dinah Rosenberg (Université de Paris 13, France), Marianne Clausel (Université de Marne-la-Vallée, France), Marta Sanz-Solé (Université de Barcelone, France), Benjamin Jourdain (CERMICS, Marne-la-Vallée, France), Jacques Franchi (Université de Strasbourg, France), Nicolas Fournier (Université de Paris 12, France), Andrey Piatnitski (Narvik Institute of Technology, Norway), Latifa Debbi (Université de Sétif, Algeria), Christophe Leuridan (Université de Grenoble, France), José Alfredo López-Mimbella (CIMAT, Guanajuato, Mexico), Ivan Nourdin (Université Paris 6, France), Philip Protter (Cornell University, USA).

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