INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Team APIS

Analysis of irregular Processes, Images and Signal, applications in biology and medicine

Saclay - Île-de-France

THEME BIO

Activity Report

2008

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2. Overall Objectives

2.1. Introduction

Keywords: *2-microlocal analysis*, *ECG*, *Hölder functions*, *IFS*, *biological systems*, *evolutionary algorithm*, *fractal*, *fractional Brownian motion*, *genetic algorithm*, *harmonic analysis*, *image analysis*, *inverse problem*, *iterated functions system*, *large deviations*, *multifractal analysis*, *optimisation*, *pharmacodynamics*, *signal analysis*, *stable laws*, *texture analysis*, *time-frequency analysis*, *wavelets*.

Biology and medicine constitute a growing source of challenging problems in applied mathematics and computer science. As more and more complex issues are addressed, with measurement devices becoming more accurate and data collections larger, there is an increasing need for new models and methods, as well as for techniques better focused on the needs of patients, medical practitioners and biologists.

Many different approaches may be used to these problems. The one we favour is based on the following ingredients: we consider biological and medical data as *signals or images*, that we *model within a stochastic frame*. Our emphasis when building the models and studying our signals is on *regularity analysis*. Let us briefly develop these points.

1. **Signal and image** processing is ubiquitous in the biological and medical fields, ranging from the analysis of EEG, ECG, and all types of medical images (CT, MRI, ...) to the study of DNA sequences and pharmacodynamics. New or improved data acquisition methods are constantly developed (*e.g.* digital mammography, functional MRI). They provide a wealth of information that calls for advanced processing techniques.

2. **Stochastic models**: most often, a successful signal analysis will use some knowledge based on a model of the data at hand. For instance, automated analysis of R-R intervals in ECG (used to detect various pathologies) is more efficient if one has at least a rough idea of the complex nonlinear interactions between the two, competing, sympathetic and parasympathetic regulations. Modelling is a difficult task in these fields, and one must often introduce randomness in order to account for the too large number of variables involved: stochastic processes are then natural and commonly used tools. In some cases, it is practical to use an approach based on population evolution, where global behaviour emerges as the interaction of many individuals following simple rules. Furthermore, controlling (in an automatic or interactive way) complex stochastic models often necessitates sophisticated stochastic optimisation techniques like evolutionary algorithms.
3. **Regularity analysis**: models obtained this way typically exhibit strong local irregularity, mirroring the one present in actual biological data. As a matter of fact, irregularity is not only omnipresent in these data, but it also often bears discriminant information: for instance, a healthy electrocardiogram is more irregular (in a precise mathematical sense) than a pathological one, with an increase in regularity strongly correlated with the severity of the pathology. Developing methods that allow to measure, analyse and control the local regularity of the data is thus of great importance.

Biological and medical data analysis using stochastic signal analysis is a field where many teams contribute worldwide. See for instance http://www.physionet.org, an open resource for data, problems, tools and methods. Our specificity in this context is the following:

- We focus on a model-based approach, where the models are either parametric stochastic processes or population-based evolutionary processes.
- We mainly analyse the data based on local regularity. The relevance of this approach is testified by the already large number of studies dealing with this aspect (see *e.g. Nature 399, 461 - 465, 1999*). We have an expertise in developing methods for (multi-)fractal signal analysis in a general frame, based on firm theoretical bases and efficient algorithmic developments. In particular, we try to go beyond the way (multi-)fractal analysis is usually performed in these fields, *i.e.* measuring the regularity and using it to classify the data or detect pathologies. Our aim is to build models explaining the *sources* of multifractality. Very few such attempts have been made in biology and medicine. Understanding the mechanisms leading to multifractality is important to correctly interpret its functional purpose or *e.g.* its relation to various pathologies.

APIS also continues developing freeware, most notably FRACLAB (a matlab/scilab toolbox for 1D and 2D signal processing).

Our team has strong collaborations with IrCcyn in Nantes, with French universities: Orsay (LRI), Calais (LIL), Clermont-Ferrand, and with several foreign universities and research centres: University of St-Andrews (Scotland), CRM Montréal (Canada), Impan (Poland), University of California at Riverside, and Acadia University, Canada. The team is involved in the European organisation (former Network of Excellence) EVO*.

We have industrial contracts with Dassault Aviation, we are involved in 4 ANR projects (REVES, COPRIN, OPUS and INCALIN), and in two projects of the System@tic pole (XVISION, EHPOC).

2.2. Highlights of the year

The size of APIS has grown rapidly in 2008 (from 7 members in December 2007 to 14 members in December 2008), including one additionnal senior member, Pierre Bertrand (University professor in delegation since September), and 5 post-docs. In 2008, theoretical progress on fractal processes (sifBm, multistable processes), on the characterisation of irregularity (2-microlocal analysis), and on uncertainty modelling, are the basis of several applications related to medicine (pharmacodynamics, cardiac signals analysis). Additionnally, activities related to agrifood, 3D-tomography and image processing, have been continued in the framework of various applied contracts (INCALIN, REVES, DIGITEO, XVision).

3. Scientific Foundations

3.1. Fractal analysis

Participants: Olivier Barrière, Antoine Echelard, Erick Herbin, Ronan Le Guével, Jacques Lévy Véhel, Evelyne Lutton.

The research of APIS draws on two areas: fractal analysis and artificial evolution. We pursue theoretical developments in these fields motivated by the needs encountered in the modelling and processing of biological and medical data.

We briefly recall below some basic and advanced concepts in fractal analysis and evolutionary computation, and then indicate how these tools are put to use for our applications.

Fractal analysis was developed in view of studying complex irregular objects. Numerous natural phenomena, in particular in physics, biology and medicine, have been shown to exhibit a fractal behaviour. The study of associated models, when available, has lead to significant progress in the understanding and control of these phenomena, for instance in turbulence analysis, non-linear growth, chemical catalysis and wave propagation in irregular media [54], [40].

Our emphasis is on the study of local regularity and multifractal analysis. These areas have both a rich theoretical content and many applications in signal/image analysis [6], [7]. Multifractality occurs when the local regularity exhibits wild temporal variations, so that the mean behaviour bears little information. A multifractal behaviour often emerges as the result of the complex interactions of a large number of elements, each of which acting in a relatively simple way. Such a situation is typical in biology.

Many teams worldwide contribute to local regularity and multifractal analysis, both on the theoretical and applied level. Successful applications include ECG, EEG and DNA sequences analysis, turbulence modelling, road and Internet traffic description and financial data analysis.

It is worthwhile noticing the following dichotomy about multifractal phenomena: "natural" multifractality is always a positive quality that constitutes an efficient answer to some functional constraints. Examples include the organisation of the blood and air flows in the lungs, the geometry of tree branches and many more. In contrast, multifractality of artifacts often constitutes an unwanted complication: for instance, it worsens the behaviour of queues in TCP traffic [1] and makes financial assets management more complex.

As mentioned previously, a precise view on the mechanisms leading to multifractality is important if one wants to understand the purposes it serves and how it is modified in response to external changes or in case of abnormal behaviour. Multifractal models are largely yet to be developed in the biological and medical fields, a gap APIS is trying to fill. Progress is also needed in the definition of new and refined ways of measuring the local regularity, adapted to the characterisation of biological signals. Finally, we address the question of numerical estimation of (multi-)fractal quantities, an area where there is still room for a lot of improvement.

3.2. Evolutionary and population-based approaches

Participants: Olivier Barrière, Jacques Lévy Véhel, Evelyne Lutton.

The transposition of Darwin's theory into computers consists of roughly imitating with programs the capability of a population of living organisms to adapt to its environment with selection/reproduction mechanisms. Since about forty years, various stochastic optimisation methods have been based on this principle. *Artificial Darwinism* or *evolutionary algorithms* is a generic name for these techniques, among which *genetic algorithms* are the most well-known.

The common components of these techniques is *populations* (that represent for example points of a search space) that evolve under the action of stochastic operators. Evolution is usually organised into *generations* and copy in a very simplified way natural genetics. The engine of this evolution is made of *selection* – based on a measurement of the quality of an individual with respect to the problem to be solved – and of *genetic operators*, usually *mutation* and *crossover* or *recombination*, that produce individuals of a new generation.

According to some conditions, that tune the relative importance of the various components of the evolution mechanism, it has been mathematically proved that such a complex stochastic process converges toward what is wished, i.e. a limit distribution concentrated on the global optimum of the considered search space[1].

Artificial Darwinism can be considered as a model of complex natural behaviour: a population of interacting agents obey simple rules and produce a global output, often unpredictable or irregular (sometimes proved to be fractal) – particularly if the selective pressure, i.e. the function that is optimised, is irregular or imprecisely known.

Population-based and evolutionary algorithms are investigated in APIS as models of complex biological process, as a mean to "explain" observed irregularity. We also consider these techniques in a more classical way in other applications (agro-alimentary or medical ones) that is, as powerful stochastic search tools: irregular data and signal analysis often necessitates the control and optimisation of complex models (parameters optimisations, inverse problems resolution). A precise understanding and control of these optimisation algorithms on irregular problems as well as the design of adapted schemes for biology are topics we explore in APIS.

3.3. Local regularity and multifractal analysis of signals and images

Participants: Olivier Barrière, Erick Herbin, Jacques Lévy Véhel.

Studying the local regularity of signals provides relevant tools for their processing. It leads for example to a set of methods for irregular image analysis, including edge detection, texture segmentation, denoising and interpolation [50]. Edges, for instance, are found as points with low local regularity.

Such tools have proved to be efficient for analysing biological data, which are often highly irregular. This irregularity contains relevant information, and precise theoretical notions have to be defined to measure its various aspects. Associated estimation methods must also be developed for practical data analysis.

Specially relevant in this field are the pointwise Hölder exponents and its variants [2]. The pointwise Hölder exponent is a very versatile tool, in the sense that the set of pointwise Hölder functions of continuous functions is quite large (it coincides with the set of lower limits of sequences of continuous functions). We have shown empirically this year that it exhibits an intriguing correlation with the lengths of R-R intervals in ECG logs, a feature yet to be explained by an adequate model. The pointwise Hölder exponent is not stable under integro-differentiation, which sometimes makes its use complicated in signal processing. In addition, it fails to characterize certain aspects of local regularity. A full description is provided by the so-called 2-microlocal analysis, introduced by J.M. Bony. Regularity at each point is specified by two indices, which makes the analysis and estimation tasks more difficult. More precisely, a function f belongs to the 2-microlocal space $C_{x_0}^{s,s'}$, where $s+s'>0, s'<0$, if and only if its $m=[s+s']-$th order derivative exists around x_0, and if there exists $0<\delta<1/4$, a polynomial P with degree lower than $[s]-m$, and a constant C, such that

$$\left|\frac{\partial^m f(x)-P(x)}{|x-x_0|^{[s]-m}}-\frac{\partial^m f(y)-P(y)}{|y-x_0|^{[s]-m}}\right|\leq C|x-y|^{s+s'-m}(|x-y|+|x-x_0|)^{-s'-[s]+m}$$

for all x,y such that $0<|x-x_0|<\delta$, $0<|y-x_0|<\delta$ [44]. These spaces are stable through integro-differentiation, i.e. $f\in C_x^{s,s'}$ if and only if $f'\in C_x^{s-1,s'}$. Knowing to which space f belongs thus allows to predict the evolution of its regularity after derivation, a useful feature if one uses models based on some kind of differential equations. Such models are often encountered in biology and medicine. A lot of work remains to be done in this area, in order to obtain more general characterisations in the deterministic and stochastic frames, to develop robust estimation methods, and to extend the "2-microlocal formalism": this is a tool allowing to detect to which space a function belongs from a computation of the Legendre transform of its *2-microlocal spectrum*. This spectrum provides a wealth of information on the local regularity that remains to be fully understood and exploited.

[1]These theoretical results however do not address the problem of convergence speed which is crucial in algorithmic implementations, EA convergence issues remain an open theoretical questions, nowadays.

Once the local regularity has been defined and measured, it may be used through a multifractal analysis: the general idea is to characterise the "size" of the sets of points with a given regularity. This may be done in a geometrical way, by computing a Hausdorff dimension, or in a statistical fashion, through an estimation of the probability, at a given resolution, to find a point with given regularity. The graphs plotting the dimensions or probabilities as functions of the regularity are called *multifractal spectra* [10], [8], [57]. These subsume a lot of information about the distribution of the regularity, that has proved useful in various situations. A most notable example is the strong correlation reported recently in several works between the narrowing of the multifractal spectrum of ECG and certain pathologies of the heart. Further work in this area that we are pursuing include the definition of multifractal spectra specifically adapted to measure relevant features of biological data, and the development of more robust estimation methods.

3.4. Stochastic processes with prescribed regularity

Participants: Olivier Barrière, Erick Herbin, Jacques Lévy Véhel.

Stochastic modelling is a well-adapted tool in the biological and medical fields because it is generally impossible to describe accurately all the parameters and interactions that come into play. Our objective is to define and study stochastic processes whose *regularity, dependence* and *jump* properties match those of biological data. Strong, long-term dependence has been shown to be present in *e.g.* DNA sequences, among many other data. Such correlations need to be accounted for by specific stochastic models. Jumps are present for instance in epileptic EEG traces, and designing processes whose jump intensity can be tuned precisely is necessary for a faithful modelling.

A simple process that allows to control the local regularity is the *multifractional Brownian motion* (mBm) [41], [42],[9], that generalises the well known fractional Brownian motion (fBm). The local regularity of mBm may be tuned *via* a functional parameter. We have started to check that this is a useful feature for biological and medical data modelling: indeed, most medical images (*e.g.* echographies, mammographies), in particular, are highly textured, with different tissues exhibiting different textures and thus different sets of Hölder exponents. We also apply this kind of model to ECG data.

More complex processes are needed if one wants to also account for jumps in the traces. We are currently building a family of stochastic processes where both the local regularity and the local intensity of jumps may be prescribed at each point. These *multistable multifractional processes* (mmp) [13] are expected to provide relevant models for EEG and other phenomena displaying a time-varying discontinuity structure. Work on mmp in the near future will focus on path synthesis, extension to higher dimensions, parameter estimation and the study of their multifractal structure.

Another area of research that seems promising is that of *set-indexed processes*. A set-indexed process is a process whose parameter is no longer "time" or "location" but may be a compact connected set. This allows for greater flexibility, and should in particular be useful for the modelling of censored data. This situation occurs frequently in biology and medicine, since, for instance, data may not be constantly monitored. We have recently defined set-indexed fractal processes, whose properties we are currently investigating.

3.5. Uncertainty management

Participant: Erick Herbin.

Since a few decades, modelling has gained an increasing part in complex systems design in various fields of industry such as automobile, aeronautics, energy, etc. Nowadays, the challenge of numerical simulation is designing physical systems saving the experimentation steps. Industrial design involves several levels of modelling: from behavioural models in preliminary design to finite-elements models aiming to represent to most sharply physical phenomena. At each of these levels, modelling requires control of uncertainties due to simplifications of models, numerical errors, data imprecisions, variability of surrounding conditions, etc. In a classical view, coping with this variability is achieved from *model registration* by experimentation and fixed *margin* added to the model response.

For a goal of technical and economical performance, it appears judicious to include this margin definition in a more rigorous frame of risk control. In other words, a probabilistic vision of uncertainties should provide decision criteria adapted to management of unpredictability inherent to design issues.

The first requirement analysis in terms of management of uncertainties led to a deployment strategy of reliability methods. Relying on setting up probabilistic decision criterion (translation in probabilistic terms...), it is composed of the three following steps:

1. build a probabilistic description of fluctuations about model's parameters (*Quantification* of uncertainty sources),
2. deduce the implication of these distribution laws on model's response (*Propagation* of uncertainties),
3. and determine the specific influence of each uncertainty source on model's response variability (*Sensitivity Analysis*).

The three following points are prerequisites for deployment of the previous analysis:

- How to capture the distribution law of model's parameters, without directly accessible data ? Their access through another modelling chain may require development of inverse methods for quantification of sources.
- Preferentially, we are interested in generic propagation methods, i. e. which are not specifically attached to the considered model. The usual methods are effective when model complexity allows many solicitation (fast model). However, in the case of finest modelling, we are often facing CPU costly models (aerodynamics, structural mechanics, ...).
- All the various levels of conception, preliminary design or sharp modelling, require registrations by experimentation to reduce model errors. This issue has been present in this frame since a long time and it now involves the definition of a systematic approach, particularly in the statistical uncertainty context.

Moreover, a multi-physical context must be added to these difficulties. The complex system design is most often located at the interface between several disciplines. In that case, modelling relies on a coupling between several models for the various phenomena and design becomes a *multidisciplinary optimisation* problem. In this uncertainty context, the real challenge turns robust optimisation to manage technical and economical risks (risk for non-satisfaction of technical specifications, cost control).

3.6. Evolutionary algorithms for complex problems

Participants: Jacques Lévy Véhel, Evelyne Lutton.

Evolutionary algorithms have been successfully used in optimisation problems related to genome structure and sequence alignments, protein folding, biological data analysis, feature selection, and many other problems related to molecular biology. Evolutionary image and signal processing are also very dynamic research areas. Both topics are well represented in the evolutionary computation domain: many workshops, conferences sessions in all big international conferences (CEC, GECCO, EVO*), special issues of journals, or books, are dedicated to bio-informatics and signal/image processing. However there is still a strong demand in theory, especially regarding the behaviour on "difficult" optimisation landscapes (like the one related to cochlear implants fitting, the HEVEA project), and for the design of new schemes. Having more efficient and more adaptive algorithms opens the way to new applications.
Our research in this domain is organised in the three following directions:

- **The behaviour of simple evolutionary models on irregular functions:**

Our approach to this question is based on regularity analysis. A first analysis, developed in [3] has established quantitative results using a very simplified model of genetic algorithm (the canonical GA). If the function to be optimised f is supposed to be the sampling at resolution $\epsilon = \frac{1}{2^l}$ of a Hölder function (this hypothesis is always valid, even if it happens that the corresponding uni-dimensional underlying function F does not reflect in a simple way the behaviour of the fitness function), we have shown that an adequate tuning of parameters l, p_m (mutation probability), and p_c (crossover probability) tends to facilitate the job of the genetic algorithm, and subsequently improves its performances. This analysis yield also an *a posteriori* validation procedure of the optimisation results of a genetic algorithm (see [4] and [48]).

Then, by considering local Hölder exponents and another simplified model, the (1+1)ES (i.e. a population size of 1 and a mutation only algorithm) on a continuous search space, we have proposed an adaptive mutation with respect to local regularity of the function [53].

Our current work is centered on adaptive operators, in order to build and evaluate genetic operators that adapt to local regularity of the function to be optimised, which is to our mind a key point for applications in biology where irregularity has a discriminant meaning.

- **Cooperation-coevolution and Parisian approach:**

  Within an evolutionary algorithm (EA) it is often possible to formulate the resolution of a problem as a collective task: the searched solution is thus built from the whole evolved population and not anymore as the only best individual of the final population of an EA. The *Parisian approach*, proposed and developed in the FRACTALES and COMPLEX teams since 1999, is based on the capability of EAs to draw a population in quasi-optimal areas of the search space. The idea is thus to build a search landscape and an evolution process where the whole population (or at least a large part of it) represents the searched solution. Individuals collectively correspond to a potential solution to the considered problem. A population is a society that commonly build the searched solution: this is a cooperative co-evolution.

  Theoretical analysis, specification and tuning of such algorithms is of course even more complex than for the the classical evolutionary approaches, and population diversity becomes a crucial factor. Additionally, it is not always simple to split a problem into interconnected sub-problems adapted to a Parisian approach, but the computational gain is very important. This has been made evident in applications like the “fly algorithm” in stereo-vision or in 3D tomographic reconstruction.

  The study of evolution models on functions of controlled regularity can be extended to the Parisian model. Local irregularity has obviously an important influence on the population diversity. These are points we are currently exploring. Our studies are also be based on test-functions specific for cooperation-coevolution that we recently proposed [55].

  We address new applications in biology with Parisian cooperative-coevolution methods (INCALIN project). We also explore hybridisation with non-evolutionary cooperative methods like Ant Colony Optimisation (ACO).

- **Interactive evolution:**

  This research topic is rapidly growing. It is actually a convenient mean to optimise what cannot be expressed using a mathematical formula or an algorithm, and what depends from a subjective judgment. First works in this domain were oriented toward artistic creation[2]. Current applications are various and deal for example with robot movement planning, auditive prosthesis or cochlear implants optimisation (HEVEA project [49]).

  For instance, we developed an interactive multifractal image denoising application (currently available in Fraclab): multifractal denoising is indeed a versatile and efficient technique, but depends on

[2]Our activities on this subject started with an artistic application of fractal shapes design (the ArtiE-Fract software), that is now the basis of a small company, Cétoine (http://cetoine.com). On the other hand, an “evolutionary art” movement is now very active, see http://www.emoware.org/evolutionary_art.asp.

parameters whose setting is not simple. Additionally, the evaluation of a good denoising is strongly dependent on the end-user as well as the application framework. Signal-to-noise ratio is unable to reflect all the subtle components of a human expert appreciation on a denoising result. The artificial evolution framework allows to introduce human evaluation in the algorithmic loop, and to cope with human judgment irregularity (or even inconsistency).

Of course efficient interactions (limited by the user fatigue) and diversity control are important in this framework. This lead us to experiment interactive Parisian model, with an application to data retrieval (for the Novartis-Pharma company) [47].

We do think that interactive evolution has a large application field in biology and medicine especially when sophisticated data models are involved (an example related to cochlear implants optimisation has been developed within the HEVEA project), and we intend to develop methods able to adapt to irregular subjective judgment of one or several end-users.

4. Application Domains

4.1. Agro-alimentary image analysis

Participant: Jacques Lévy Véhel.

In collaboration with IrCcyn, INRA.

Multifractal analysis allows to build efficient algorithms for the classification, segmentation, and denoising of irregular images. It is thus well adapted to the study of highly textured images as the ones encountered in biology and medicine.

A typical application we are currently involved in is made in collaboration with INRA-Nantes within a VANAM project, funded by the Pays-de-Loire region. It consists of analysing enzymatic decomposition of sugar beet pulp. Agro-alimentary industries produce an important quantity of vegetal loss, and current research is oriented toward biological methods for their decomposition and re-use. This necessitates an assessment of the action of various enzymes. We have shown that the regularity of pulp images is highly correlated with the degradation state. This should allow to measure in an easy way the degradation power of a given (combination of) enzyme.

4.2. Modelling some complex agrifood industrial process

Participants: Olivier Barrière, Evelyne Lutton.

The INCALIN project is funded by ANR and associates fundamental laboratories (CREA, INRIA, LIP6), specialised laboratories (INRA) and technical centres (ITFF, INBP) performing research on the food technologies selected (cheese ripening and bread making).

Competitive challenges which agro-alimentary industries are facing are related to quality and sustainability of alimentary products. The aim of the INCALIN project is to build decision support tools for better managing the products quality and, by the way, manufacturing processes. Causal relationships between on the one hand, ingredients, physico-chemical, microbiological characteristics and on the other hand, sensory, nutritional properties, depending on successive process operations are still ill-known in some food technologies (uncertainty of the processes).

The approach developed deals with different type of knowledge in a multimodal context (know-how of operator-experts in terms of formal or informal reasoning, scientific explanation and modelling of the phenomenon, data basis). Among the fragmented knowledge available, the know-how of the operator-experts is probably a key one that should be taken into account. Evolutionary techniques are exploited here to instantiate complex interaction models (Bayesian networks are used in a first experiment) using incomplete expert data sets. Additionally, interactive evolution is considered to take expert judgments into account within an industrial control system.

4.3. Three-dimensional tomographic data processing

Participants: Jean Louchet, Evelyne Lutton, Jean-Marie Rocchisani.

In Nuclear Medicine diagnosis, radioactive substances are administered to patients. The concentration of radioactivity in the body is then estimated from the radiations detected by gamma cameras. In order to get an accurate estimation, a three-dimensional tomography is built from two-dimensional scintigraphic images. Some parasitic effects due to scattering and absorption are then to be corrected. Existing analytical and statistical methods are costly and require heavy computation. We are currently developing a Parisian Evolution Strategy in order to reduce the computing cost of reconstruction without degrading the quality of results. Our approach derives from the Fly algorithm which proved successful on real-time stereo image sequence processing [43], [52], [56].

While this approach had been validated in its principles [51], computation costs were still high due to the complexity of physically modelling random photon trajectories, and the reconstruction results were not quite up to the quality obtained through more classical methods. Following this, we developed an innovative evaluation function based on a specific approach to fitness calculation, called "marginal fitness", giving encouraging results on both simplified synthetic data and real scintigraphic images. The main bottleneck appears when taking into account scattering and absorption in an accurate, realistic way. The current stages of this research concentrate upon building simplified models of these optical effects in order to speed up calculation without hampering the quality of results.

4.4. ECG analysis and modelling

Participants: Olivier Barrière, Jacques Lévy Véhel.

ECG and signals derived from them are an important source of information in the detection of various pathologies, including *e.g.* congestive heart failure and sleep apnea. The fractality of these data has been reported in numerous works over the past years. Several fractal parameters, such as the box dimension, the local regularity and the multifractal spectrum have been found to correlate well with the condition of the heart in certain situations. We participate in this research area in two ways. First, we use refined local regularity characterisations, such as 2-microlocal analysis, and advanced multifractal spectra for a more precise analysis of ECG data. This requires to test current estimation procedures and to develop new ones. Our preliminary studies show that the local regularity of RR intervals, estimated in a parametric way based on a modelling by an mBm, displayed correlations with the amplitude of the signal, a feature that seems to have remained unobserved so far. Second, we have started to build stochastic processes that mimic in a much simplified way some aspects of the sympathetic and parasympathetic systems, and for which we hope it will possible to compute the theoretical local regularity and multifractal spectrum. This may help to elucidate the profound reasons behind the observed multifractality of traces, and how it evolves under abnormal behaviour.

4.5. Pharmacodynamics and patient drug compliance

Participants: Erick Herbin, Jacques Lévy Véhel.

In collaboration with Pierre Emmanuel Lévy Véhel (Paris 6 University), Fahima Nekka (Université de Montréal).

Poor adherence to treatment is a worldwide problem that threatens efficacy of therapy, particularly in the case of chronic diseases. Compliance to pharmacotherapy can range from 5% to 90%. This fact renders clinical tested therapies less effective in ambulatory settings. Increasing the effectiveness of adherence interventions has been placed by the World Health Organisation at the top list of the most urgent needs for the health system. In collaboration with the pharmacy faculty of Montréal University, we shall consider the problem of compliance within the context of multiple dosing. Analysis of multiple dosing drug concentrations, with common deterministic models, is usually based on patient full compliance assumption, i.e., drugs are administered at a fixed dosage. However, the drug concentration-time curve is often influenced by the random drug input generated by patient poor adherence behaviour, inducing erratic therapeutic outcomes. Following work already started in Montréal, we consider stochastic processes induced by taking into account the random drug intake induced by various compliance patterns. Such studies have been made possible by technological progress, such as the “medication event monitoring system”, which allows to obtain data describing the behaviour of patients.

The deterministic model describing the evolution of drug concentration can be considered as a “black box.” The efficiency of the drug is usually an output of this model, when the intake is realized as prescribed. To obtain a robust efficiency of the drug, we need to cope with fluctuations of the behaviour of patients. In that view, we propose:

1. To model the uncertainty of intake with a probability distribution. As in the generic approach which will be studied in EHPOC project, we will face different situations: statistic methods in presence of data, or model approach in presence of qualitative description of the patient behaviour.
2. To measure the influence of this uncertainty on drug efficiency, by propagating uncertainty through the "black box" model. If the level of modelling allows the use of Monte Carlo methods, a direct probabilistic description of the efficiency can be deduced. Then the quantity and schedule of drug intake could be adapted to insure an efficiency at a given confidence level. However, when the complex phenomena involved in the evolution of concentrations is precisely described in the "black box" model, Monte Carlo methods are often unpractical because too expensive in CPU time. The collaborations COPRIN and OPUS will consider reduced models as a statistical model of the complex one. In that case, the uncertain efficiency will be the result of the uncertainty of intake and the error of model.

5. Software

5.1. FRACLAB: a Fractal Matlab/Scilab toolbox

Participants: Olivier Barrière, Christian Choque-Cortez, Antoine Echelard, Jacques Lévy Véhel.

FracLab is a general purpose signal and image processing toolbox based on fractal and multifractal methods. FracLab can be approached from two different perspectives:

- Fractal analysis: A large number of procedures allows to compute various fractal quantities associated with 1D or 2D signals, such as dimensions, Hölder exponents or multifractal spectra.
- Signal processing: Alternatively, one can use FracLab directly to perform many basic tasks in signal processing, including estimation, detection, denoising, modelling, segmentation, classification, and synthesis.

FracLab is not intended to process "fractal" signals (whatever meaning is given to this word), but rather to apply fractal tools to the study of irregular but otherwise arbitrary signals. A graphical interface makes FracLab easy to use and intuitive. In addition, various wavelet-related tools are available in FracLab.

FracLab is a free software. It mainly consists of routines developed in Matlab or C-code interfaced with Matlab and Scilab (a free scientific software package for numerical computations from INRIA). It runs under Linux and Windows environments

The development of FracLab has been continued in 2008 and the new version (2.04) now runs under Linux (32 and 64 bits), Windows (32 and 64 bits) and OSX (Intel) environments, either as a Matlab toolbox or as a standalone executable, which does not require Matlab.

The new website of FracLab is visited by roughly 1500 unique visitors every month, coming from all around the world, mainly USA (17%) and China (12%). We also added interactive demos to show how to get started with FracLab and use it.

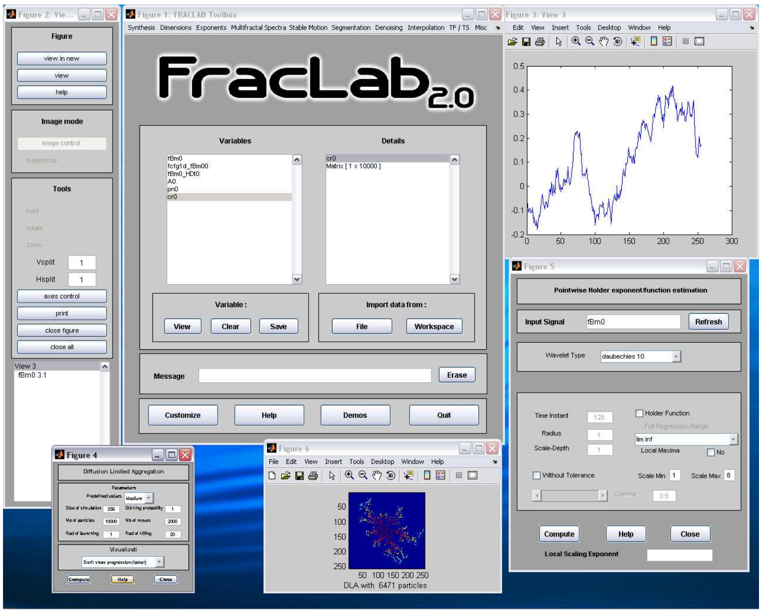

Figure 1. FracLab 2.0 graphical interface

Taking into account all the flavors of FracLab, it has been downloaded 4000 times or so this year. Moreover, FracLab is referenced on more and more websites and forums, bringing a lot of visitors. We are also cited on the official Mathworks website http://www.mathworks.fr/support/solutions/data/1-1C19A.html as a solution to calculate fractal dimensions. The use of FracLab is acknowledged in more than 60 published research papers.

6. New Results

6.1. Fractal strings

Participant: Jacques Lévy Véhel.

In collaboration with Michel Lapidus, John Rock (University of California at Riverside, USA) and Franklin Mendivil (Acadia University, Canada).

We have refined the results obtained previously on two aspects of the generalization of fractal strings:

- We obtained better characterizations of multifractal strings based on the large deviation spectrum, with more accurate results in extreme cases [37].
- We better investigated the behaviour of local fractal strings in the situation where the dimension varies smoothly along the set [39].

In addition, we have proposed an extension of fractal strings in higher dimensions, based on a modification of the definition already used for defining multifractal strings. This extension allows to recover some of the major properties of fractal strings in the self-similar case. The self-affine case is much harder, and requires additional efforts [39].

6.2. Multistable processes

Participant: Jacques Lévy Véhel.

In collaboration with Kenneth Falconer (St Andrews University, Scotland) and Ronan Le Guével (Nantes University).

We have made progress in the study of multistable processes, i.e. processes which are "tangent" at each point to an α-stable process, but where α is a smooth function of time instead of being a real number in the interval $(0,2)$ [36]. We have in particular given a new construction of these processes, based on a series representation [38]. As an example, we describe below a multistable extension of symmetric Lévy motion L_α:

Theorem 1 *(symmetric multistable Lévy motion).* Let $\alpha : \mathbf{R} \to [c,d] \subset (1,2)$ and $b : \mathbf{R} \to \mathbf{R}^+$ be continuously differentiable. Let $(\Gamma_i)_{i\geq 1}$ be a sequence of arrival times of a Poisson process with unit arrival time, $(V_i)_{i\geq 1}$ be a sequence of i.i.d. random variables with distribution $\widehat{m}(dx) = \sum_{j=1}^{+\infty} 2^{-j}\mathbf{1}_{[j-1,j[}(x)dx$ on $\mathbf{R}$, and $(\gamma_i)_{i\geq 1}$ be a sequence of i.i.d. random variables with distribution $P(\gamma_i = 1) = P(\gamma_i = -1) = 1/2$. Assume finally that the three sequences $(\Gamma_i)_{i\geq 1}$, $(V_i)_{i\geq 1}$, and $(\gamma_i)_{i\geq 1}$ are independent and define

$$Y(t) = b(t) C_{\alpha(t)}^{1/\alpha(t)} \sum_{i=1}^{+\infty} \sum_{j=1}^{+\infty} \gamma_i \Gamma_i^{-1/\alpha(t)} 2^{j/\alpha(t)} 1_{[0,t]\cap[j-1,j[}(V_i) \quad (t \in \mathbf{R}_+). \tag{1}$$

then Y is tangent, at all $u \in \mathbf{R}_+$, to the process $Y'_u = b(u) L_{\alpha(u)}$.

We have also started to study the local regularity of such processes, and have obtained preliminary results concerning their local Hölder exponent.

In [36], we have considered the case of stationary processes which admit a tangent process, and designed methods for numerical synthesis of their paths.

Finally, we are also dealing with the estimation problem for these processes. We are studying an estimator of the local regularity and its convergence properties.

6.3. Stationarity and self-similarity characterization of the sifBm

Participant: Erick Herbin.

In collaboration with Ely Merzbach (Bar Ilan University, Israel).

Fractional Brownian motion has been extended by Herbin and Merzbach (2006) for indices which are subsets of a measure metric space. The set-indexed fractional Brownian motion (sifBm) of parameter $H \in (0, 1/2]$ is defined as the zero-mean Gaussian process $\mathbf{B}^H = \{\mathbf{B}^H_U;\ U \in \mathcal{A}\}$ such that

$$\forall U, V \in \mathcal{A}; \quad E[\mathbf{B}^H_U \mathbf{B}^H_V] = \frac{1}{2}\left[m(U)^{2H} + m(V)^{2H} - m(U \bigtriangleup V)^{2H} \right], \tag{2}$$

where $\mathcal{A}$ is a class of subsets of the measure metric space $(\mathcal{T}, d, m)$ satisfying some assumptions.

As one-parameter fractional Brownian motion, the set-indexed fractional Brownian motion satisfies increment stationarity and self-similarity properties.

In [46], a stationarity property was proved for the set-indexed fractional Brownian motion. However, since it only concerns marginal distributions of the whole process, this property is weaker than the classical increment stationarity property for one-parameter fBm, and fails to provide a complete characterization of the sifBm.

In [15], the stationarity definition is strengthened to involve the distribution of the process. It is proved that for any integer n, for all $V \in \mathcal{A}$ and for all increasing sequences $(U_i)_{1\leq i\leq n}$ and $(A_i)_{1\leq i\leq n}$ in $\mathcal{A}$,

$$\forall i,\ m(U_i \setminus V) = m(A_i) \quad \Rightarrow \quad \left(\Delta\mathbf{B}^H_{U_1\setminus V}, \cdots, \Delta\mathbf{B}^H_{U_n\setminus V}\right) \stackrel{(d)}{=} \left(\Delta\mathbf{B}^H_{A_1}, \cdots, \Delta\mathbf{B}^H_{A_n}\right).$$

This so-called $\mathcal{C}_0$-increments m-stationarity property is considered as the good generalization of the increment stationarity property for one-parameter processes since the projection of a stationary process on any flow is one-parameter process with stationary increments.

To consider a self-similarity property for set-indexed processes, we need to assume that the indexing collection is provided with the action of a group G

$$\begin{aligned} G \times \mathcal{A} &\to \mathcal{A} \\ (g, U) &\mapsto g.U \end{aligned}$$

such that

$$\forall U \in \mathcal{A}, \forall g \in G; \quad m(g.U) = \mu(g).m(U),$$

where $\mu : G \to \mathbf{R}_+^*$ is a surjective function.

In [46], the set-indexed fractional Brownian motion is proved to be self-similar, i.e.

$$\left\{\mathbf{B}^H_{g.U};\ U \in \mathcal{A}\right\} \stackrel{(d)}{=} \left\{\mu(g)^H.\mathbf{B}^H_U;\ U \in \mathcal{A}\right\}.$$

Moreover it is proved in [15] that the sifBm is the only Gaussian process satisfying the two properties of increment stationarity and self-similarity.

This characterization provides an important justification for sifBm's definition. Moreover, it leads to natural extension of fractional Brownian motion for multiparameter indices or indices in non-Euclidian spaces (e.g. unit circle of $\mathbf{R}^2$) that satisfies the same kind of fractal properties than one-parameter fBm.

6.4. Stochastic 2-microlocal analysis

Participants: Erick Herbin, Jacques Lévy Véhel.

The frame of 2-microlocal analysis can be used to finely study the regularity of the paths of a stochastic process. In 2005, we proved that the 2-microlocal frontier of a continuous Gaussian process is almost surely a deterministic function that may be evaluated from the incremental covariance.

We have refined our previous results on the 2-microlocal analysis of stochastic processes. More precisely, in [14], we have proved that a lower bound for the 2-microlocal frontier of a non-Gaussian process can be obtained. As an application, if X is a stochastic process defined as a stable integral, i.e.

$$\forall t \in \mathbf{R}_+, \quad X_t = \int_0^t \eta(u)\ dM_u$$

where η is a L^α deterministic function and M is an α-stable random measure ($0 < \alpha < 2$), we can find a lower bound for the 2-microlocal frontier of X and therefore for the local regularity of the sample paths of X.

If M is substituted with a Gaussian measure and η is in L^2, i.e. if X is a Wiener integral, then the result is more accurate: the almost sure frontier of X is completely determined from β's one.

We now plan to obtain accurate 2-microlocal characterizations for certain classes of Lévy processes and for solutions of stochastic differential equations. Extension to differentiable processes will also be studied.

6.5. Uncertainties for fractal stochastic systems, application to RR-intervals analysis

Participants: Erick Herbin, Antoine Echelard, Jacques Lévy Véhel.

Keywords : Stochastic process, fractal, ECG, Holder exponent

A powerful way to deal with uncertainties is to use stochastic processes to model the various sources of errors and variability. We are working since October on the study of uncertainties for fractal stochastic systems.

The PARM (processus autorégulé multifractionnaire), which was introduced by Olivier Barrière in his thesis, is a process whose Hölder regularity depends only of the value of the signal. We are actually trying to model natural signals, such as digital elevation model or ECG, with this process. This requires linking the value of the process to its regularity. In order to achieve this task, numerical experiments are done on the digital elevation models : the points of the image are represented in the "Holder exposant/altitude" plane, with promising first results. One can also hope to classify the signals using this representation. For example, it could allow to classify digital elevation models according to the geological properties of the ground.

We will also be interested in the study of RR intervals, a signal derived from ECG. It is now well documented that ECG and RR intervals have (multi-)fractal properties, and that, statistically, these properties relate well to the condition of the heart. Thus, the "Holder exposant/altitude" representation of RR-intervals could also allow to distinguish between the healthy and pathological patients.

6.6. Uncertainty and heterogeneous coupled model

Participant: Erick Herbin.

In collaboration with Florian de Vuyst (Ecole Centrale Paris), Gilles Fleury and Emmanuel Vasquez (Supelec).

In the numerical simulation context, the uncertainty management involves a propagation step which quantifies the impact of a given probability distribution of inputs onto the response of a given deterministic model. More precisely, the variable of interest $Z \in \mathbf{R}^d$ is linked to uncertain variable $X \in \mathbf{R}^N$ by the relation

$$Z = G(X)$$

where $G : \mathbf{R}^N \to \mathbf{R}^d$ is a given deterministic function.

In a multi-physical context, the model G is constituted by several coupled models. For instance, G can represent aerodynamic phenomena and structural mechanics that interact.

With the goal to estimate the quantity

$$p = P(Z > z_{\min}),$$

Monte-Carlo simulations require a large amount of G model requests, which can lead to impossible computations when mono-physical models are CPU costly. To overcome this obstacle, we can use of reduced-order models. However, the coupling structure between models is approximated in this reduction. This new source of uncertainty must be estimated and taken into account in the constuction of reduced-order models.

A PhD thesis is about to begin to address this problem.

6.7. Uncertainty and numerical simulation

Participant: Erick Herbin.

In collaboration with CEA, Dassault Aviation, EDF, EADS.

A general methodology has been defined to manage uncertainties in the numerical simulation context. An intensive collaboration with R&D entities of industrial compagnies led to a common view of the problem.

The uncertainty management methodology has been applied in robust design context. At the early stage of aircraft design, the models involved are very simplified and the geometric and environmental variables are not completely determined. Then, the prescribed performances of the designed aircraft are uncertain and considered as random variables. In [45], the general methodology was already used to define the global variables of an aircraft satisfying a range performance with a given probability. We have extended this approach to multi-disciplinary optimization used in aircraft design ([21] and [23]).

Following the general methodology, we began the study of uncertainties in CFD models (see [20]).

6.8. Statistics of random processes applied in medicine and health

Participants: Pierre R. Bertrand, Jacques Lévy Véhel.

In collaboration with Alain Chamoux, Véronique Billat, Jean-Marc Bardet, and Antoine Ayache.

The use of the statistics of random processes in biology, medicine and health is an emerging domain. Pierre Bertrand obtained a delegation of one year in the APIS team, INRIA-Saclay from September 1st, 2008, to work on this theme with a special attention on the development of algorithms, new models and new methods in connection with the applications in medicine and health. Three applications domains are considered:

- Heart rhythm in ambulatory, in association with Pr. Alain Chamoux, leader of the service of occupational medicine, Clermont-Ferrand Hospital.
- Heart rhythm in medicine of the sport, in association with Véronique Billat, leader of team U. 902 of INSERM (East Paris University).
- Human balance, former collaboration with Michel Dabonneville (Sport Faculty, Clermont-Ferrand University) and project of new collaborations (Yves Rozenholc, statistician at Medicine Faculty, Paris).

To process data presented in figure 2, the following theoretical tools are considered:

- Statistical estimation of fractal parameters.
- Non parametrical estimation of the spectral density for processes observed at random times, in continuation to the current collaboration with Jean-Marc Bardet (Professor of statistics, University Paris 1) and Antoine Ayache (Professor, University Lille 1).
- Off-line or on-line detection of abrupt changes on the fractal parameters. For example, for the heart rhythm in ambulatory, we work on the development of algorithms allowing to find the correlations between the moments of change on the fractal parameters and falling asleep. This study is done in collaboration with Gilles Teyssière (Aarhus University) and Medhi Fhima (PhD student of Clermont-Ferrand University).

6.9. Modelling complex data in an industrial agrifood process

Participants: Olivier Barrière, Evelyne Lutton.

Keywords: Agrifood, Cheese ripening, Cooperative coevolution, Parisian approach, Genetic Programming, Bayesian Network

This study is part of the INCALIN research project, whose goal is the modelling of agrifood industrial processes, in particular the ripening of the french Camembert cheese. The ripening process is divided into four phases with completely different internal dynamics. The estimation of the current phase is currently based on subjective and robust human expertise, through sight, touch, smell and taste. Correct identification of these phases is an important information for parameter regulation. The expert's knowledge is obviously not limited to these four phases, but they help evaluate the whole dynamics of ripening and detect drift from the standard evolution.

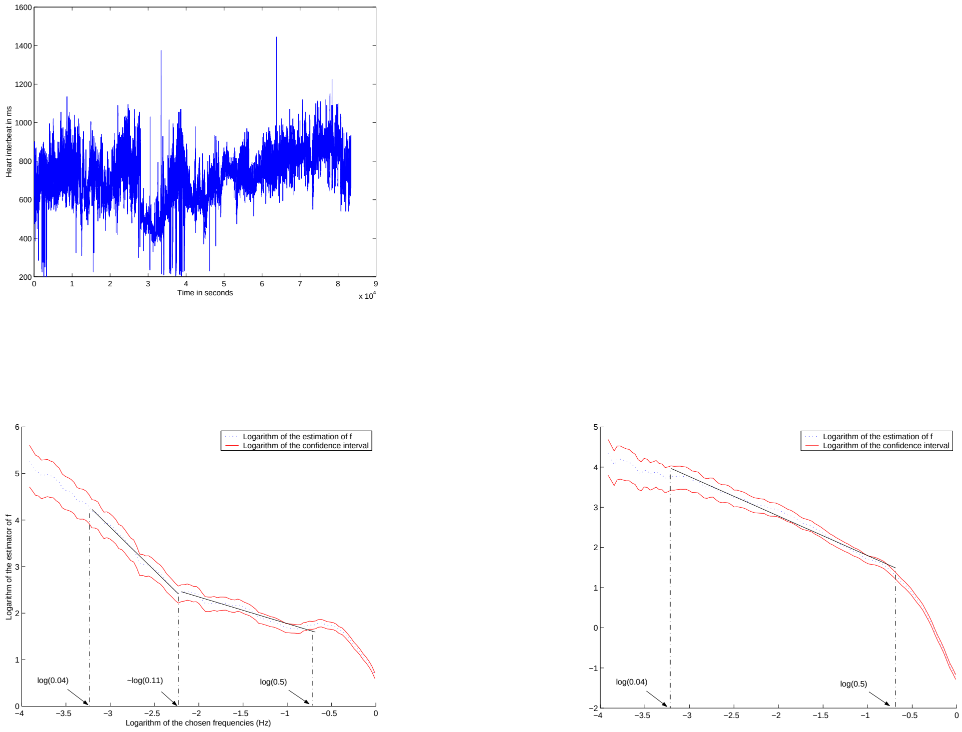

Figure 2. RR intervals for a healthy patient during a period of 24 hours (top) and spectral density estimation during the working hours (bottomleft) and during the sleeping hours (bottom right).

In previous work on cheese ripening modelling, a dynamic Bayesian network has been built, using human expert knowledge, to represent the macroscopic dynamic of each variable. The phase the network is in at time t, plays a determinant role for the prediction of the variables at time $t+1$. Moreover, four relevant variables have been identified, the derivative of pH, la, *Km* and *Ba* at time t, allowing to predict phase at time $t+1$. This leads to a computer-based phase estimation to model the way experts aggregate information from their senses.

Genetic Programming (GP) has been used to search for a convenient formula that links these four derivatives of micro-organisms proportions to the phase at each time step t, without a priori knowledge of the phase at $t-1$. This problem relates to symbolic regression, however, it may be noted that the small number of samples and their irregular distribution make it difficult.

In a first classical GP approach, the phase estimator is searched as a single best "monolithic" function. Although this evolutionary approach already outperforms the previous Bayesian network, it has been decided to split the phase estimation into four combined (and simpler) "phase detectors". The structures searched are then binary output functions (or binarised functions) which characterize one of the four phases. The population is split into four classes such that individuals of class k are good at characterizing phase k. Finally, a global solution is made of at least one individual of each class, in order to be able to classify the sample into one of the four phases via a voting scheme. This way the problem is formulated as a collective task, in the "Parisian approach" style [16].

This approach is more robust than the classical GP approach because it has almost the same recognition rate but with a lower variance. Moreover, it allows to evolve simpler structures during less generations, and yields easier to interpret results. Nevertheless, it often happens that a generation noticeably improves the global fitness, while the following generations are not able to keep it: global fitness is not a monotonically increasing function. In order to avoid this stagnation phenomenon due to over-specialisation of the best individuals, we have experimented a variable-sized population Parisian GP strategy, using adaptive deflating and inflating schemes for the population size [17]. The idea is to group individuals with the same characteristics into "clusters" and remove the most useless ones at the end of every generation while periodically adding "fresh blood" to the population (i.e. new random individuals) if a stagnation criterion is fulfiled. These concurrent mechanisms tend to better maintain low complexity individuals as well as genetic diversity. The deflating scheme allows to obtain the same result as the fixed-size population strategy, but using less fitness evaluations and the deflating-inflating strategy improves the quality of results for the same number of fitness evaluations as the fixed-size strategy.

In parallel to these results, a theoretical study has been carried out to learn the structure of a Bayesian network using an equivalent representation known as the independence model (IM). IM represents data dependencies via a set of independence statements (IS). An IS=$(X, Y|S)$ means that the variable X is independent of Y knowing the set of variables S and is evaluated by χ^2 tests. These ISs are evolving in a Parisian evolutionary scheme and the best of them are selected to build a partial IM. We are currently working on the reconstruction of the structure Bayesian network that best fits a given partial IM.

6.10. Artificial evolution and image processing

Participants: Benoît Kaufmann, Jean Louchet, Evelyne Lutton, Jean-Marie Rocchisani, Franck Vidal, Emmanuel Sapin.

Keywords: Robot vision, stereo analysis, fly algorithm, Cooperative coevolution, Parisian approach.

In this domain, we are using artificial evolution as a general tool to explore the large parameter spaces underlying machine vision problems. The backbone of this research is the Fly algorithm, developed a few years ago in the scope of stereovision. It uses the paradigm of Parisian Evolution which retains the classical mechanisms and operators of artificial evolution but different semantics, where the whole population is considered as a single entity.

Our first topic is further development of the mainstream Fly Algorithm. Previous work in the team has shown it is possible to represent visible objects in a scene using a set of 3-D points (the 'flies'), build and maintain this representation in real time with dynamic scenes even using small computing resources. Boumaza [2002, 2004] developed matching controllers applicable to obstacle avoidance in mobile robotics applications. In order to better explore the potential applications of the Fly technique, we are now moving one step further and laying the first milestones to fly-based SLAM (Self Localisation And Mapping). To this end, E. Sapin (a post-doctoral researcher who joined us in July 2008) and J. Louchet are developing a tool to spatially correlate fly populations derived from different camera positions, in order to derive the camera's self displacement, based on a pseudo-distance between fly populations. This research is partially funded by the XVISION contract.

The second research topic adresses the problem of 3-D image reconstruction from noisy projections in PET/SPECT tomography. J. Louchet, E. Lutton and J.-M. Rocchisani devised a technique using a variation of the original Fly algorithm, which allows the fly population to contentrate into the regions with high concentrations of emissive elements. This research, which had not been funded as yet but already gave some promising results, is now part of the TOMO-EA program with OMTE-DIGITEO, allowing the participation of a post-doctoral researcher, F. Vidal, from December 2008.

Third, we are participating to the REVES project with the Cité des Sciences, Paris. This project aims at giving the general public an interactive, virtual-reality based presentation of the Earth and its satellites. While the other participants are developing the general scenario and custom VR googles, we are involved together with the SIP laboratory of René Descartes University, Paris, in the image processing part. B. Kaufmann joined the team in November 2008 as a post-doctoral researcher to work on the detection and interpretation of user's hand gestures using a real-time evolution technique combined with fast image segmentation operators.

7. Contracts and Grants with Industry

7.1. Contracts and Grants with Industry

The team has contracts with:

- DASSAULT AVIATION on terrain modelling based on mBm and self-regulating processes.
- ANIFRAC project of DIGITEO. The partners involved are ECP and SUPELEC. The goal of the project is the study of a complete chain of uncrertainties in the case where these can be modeled as fractal processes. Two situations are investigated in depth. The first deals with the study of ECG in view of detecting certain pathologies and assessing their evolution. The second considers non-compliance in pharmacodynamics.
- OMTE Digiteo TOMO3D-EA, with CEA-LIST, on the application of the fly algorithm to 3D medical tomography.
- ANR project INCALIN, with INRA, CREA, LIP6, and several agrifood industries on security and quality of industrial food processes (3 years from January 2007).
- System@tic project XVISION, with HGH, NIT SAS, MARTEC, EVITECH companies on the implantation of the fly algorithm and other image processing algorithms adapted to a new CMOS camera (18 months since June 2008).
- EHPOC project of the Pôle de Compétitivité SYSTEM@TIC PARIS-REGION. The industrial partners involved are CEA, Dassault Aviation, EADS, EDF. The goal of the project is the development of a generic methodology to manage uncertainties and its demonstration through industrial cases.
- OPUS project of the Agence Nationale de la Recherche. The industrial partners involved are CEA, Dassault Aviation, EADS, EDF. The goal of the project is the development of a generic software plateform to manage uncertainties.
- ANR project REVES, with Cité des Sciences et de l'Industrie, Univ. Paris V, and two companies: DreamInReal and Laster Technologies, on the use of fly algorithm for augmented reality.

8. Other Grants and Activities

8.1. National initiatives

Our project has collaborations with:

- Nantes University (Anne Philippe), in the frame of the DIGITEO project ANIFRAC and of Ronan Le Guével's thesis. Areas of collaborations include the study of self-regulating processes and the one of multistable processes, in particular from the statistical point of view.
- Lille University (A. Ayache) on the study of various kinds of fractal stochastic processes.
- EHPOC project of the Pôle de Compétitivité SYSTEM@TIC PARIS-REGION. The academic partners involved are ECP (Prof. Florian de Vuyst), INRIA Select (Gilles Celeux).
- OPUS project of the Agence Nationale de la Recherche. The academic partners involved are ECP, Paris VII University (Prof. Josselin Garnier), Grenoble University (Prof. Anestis Antoniadis and Christophe Prudhomme), SUPELEC (Prof. Gilles Fleury and Emmanuel Vasquez). The specific contribution of ECP-SUPELEC is the theoretical development of reduced-order models in a multiphysical context.
- LSIIT, Louis Pasteur University of Strasbourg (P. Collet) on cochlear implants fitting (HEVEA) and fly algorithm,
- Polytech'Tours (Nicolas Monmarche and Mohamed Slimane) on the use of ant colonies models in graphic design and data classification.

An agreement signed in 2006 between INRIA, Cetoine, the Angers University, the Lycee de la mode of Cholet and the e-mode technology plattform, in order to experiment new textile applications of the ArtiE-Fract software, has been prolongated for 2008-2009. An R&D project is currently built among these partners to be presented to the "Pôle Enfant", a Competitive research pole of the "Pays de la Loire" region.

8.2. European initiatives

Our team has collaborations with:

- IMPAN, Warsaw (M. Rams), in the frame of the European SPADE2 project, on several aspects of fractal and multifractal analysis.
- St Andrews University (K. Falconer), for the study of probabilistic aspects of multistable processes. We have also recruited N. Snigireva, who was a Ph.D. student at St Andrews, as a post-doc (as of November 2008).

APIS is involved in an European FP7 project (theme 2, Food, Agriculture, Fisheries and Biotechnologies), on agrifood applications (DREAM : "Design and development of realistic food midels to allow a multidisciplinary and intergrated approach to food quality and nutrition"), with partners of the INCALIN project. This project will start by mid-2009.

8.3. International initiatives

The COMPLEX team collaborates with:

- Ecole Polytechnique-CRM, Montreal (F. Nekka) on pharmacodynamics.
- University of California at Riverside (M. Lapidus) and Acadia University, Canada (F. Mendivil) on multifractal strings.
- The Mexican research institute (CICESE, Fsica Aplicada, Pr Gustavo Olague) under a LAFMI grant.
- Ecole Polytechnique-CRM, Montreal (F. Nekka) on pharmacodynamics.
- Bar Ilan University on theoretical developments around set-indexed fractional Brownian motion (invitations of Erick Herbin in Israël during three months in June 2006, June 2007 and June 2008 and invitation of Prof. Ely Merzbach by APIS project in February 2008).
- University of Nottingham (Gabriela Ochoa, Edmund Burke, Natalio Krasnogor) on cooperative-coevolution.

9. Dissemination

9.1. Organization committees

Evelyne Lutton, Jacques Lévy Véhel and Fahima Nekka, are organisers of the next "Fractals in Engineering" conference, to be held in Montreal.

Evelyne Lutton, Pierre Collet and Marc Schoenauer are involved in the organisation of the next “Evolution Artificielle” conference (Strasbourg, October 2009), and are members of the steering committee of the French association for artificial evolution. Since October 2007, Evelyne Lutton is chairman of this committee.

Evelyne Lutton co-chairs with Hideyuki Takagi the third European Workshop on Interactive Evolution and Humanised Computational Intelligence, EvoInteraction 2009, to be held within the EvoStar event.

Erick Herbin is involved in the organization of the continuing education program "Engager et élaborer une démarche incertitudes", under the labels IMdR (Institut de Maitrise des Risques), SMAI (Société de Mathématiques Appliquées et Industrielles), SFdS (Société Française de Statistiques) and TERATEC.

Erick Herbin is member of the IMdR Work Group "Uncertainty and industry".

Erick Herbin is member of the CNRS Research Group GDR Mascot Num, devoted to stochastic analysis methods for codes and numerical treatment.

Pierre R. Bertrand has been elected to the board of the SFdS (French statistical society), he is chairman of the Bank-Finance-Insurance group (BFA) of the SFdS. He was also involved in the organization of the workshop "Banking fraud, methods and techniques of detection", Bank-Finance-Insurance group of the SFDS (French statistical society), Paris, Institut Henri Poincaré, november 25th 2008.

Pierre R. Bertrand co-organised the session "Wavelet and Statistics", of the first french-tunisian mathematical conference, Djerba, March 16-20 2009. He was also co-organiser of the journée "Fraude bancaire, méthodes et techniques de détection", Paris, November 25th 2008.

Pierre R. Bertrand is involved in the organization of a workshop "Reliability" by SFdS (French statistical society) and SMAI (Société de Mathématiques Appliquées et Industrielles), Paris, Institut Henri Poincaré, spring 2009. He is also involved in the organization of the session "Wavelet and Statistics", of the first French-Tunisian mathematical conference, Djerba, march 16-20 2009.

Pierre R. Bertrand is member and responsible for Clermont-Ferrand of the CNRS Research group GDR "Statistique et santé".

Jean Louchet is member of the AFRIF PhD award committee, and is vice-secretary of the AFRIF.

9.2. Editorial Boards

Jacques Lévy Véhel is an associate Editor of the journal “FRACTALS”.

Evelyne Lutton has been co-editor of a book on Genetic and Evolutionary Image Analysis and Signal Processing, with Stefano Cagnoni and Gustavo Olague.

Evelyne Lutton, Stefano Cagnoni and Gustavo Olague are co-editors of a special issue on Evolutionary Computer Vision of the Evolutionary Computation Journal.

J. Lévy Véhel has acted as an expert for the Canadian CRSNG. He is also an expert for the Canadian MITACS network.

J. Lévy Véhel has been a referee for Fractals, JMNNP, SPA.

Pierre R. Bertrand is associate editor of the *"Journal de la Socité Francaise de Statistique"*. He is in charge of a thematic number *"Statistics, Processes and Health"* of this journal, to be edited in 2009.

Pierre R. Bertrand has been referee for *"Statistic and Probability letter"* and *"Nonlinear Analysis: Modelling and Control"*.

Evelyne Lutton has been referee for IEEE Transactions on Evolutionary Computation, IEEE Signal Processing Letters, JESA, SMC-PartB.

Erick Herbin is reviewer for Mathematical Reviews (AMS).

Erick Herbin has written an expertise report for the ANR call for proposals ARPEGE 2008.

9.3. Other Teaching

- Course "Fractals and Time-frequency analysis" at Ecole Centrale Nantes (Jacques Lévy Véhel, 7h).
- Course "Fractals and wavelets in signal and image processing" at ESIEA (Jacques Lévy Véhel, 15h).
- Course "Artificial Evolution" at ENSTA (Evelyne Lutton, Pierre Collet, Cyril Fonlupt, 21h).
- Probability Course at Ecole Centrale Paris (Erick Herbin, 20h).
- Random Modelling Course at Ecole Centrale Paris (Erick Herbin, 30h).
- Travaux Dirigés on Real and Complex Analysis at Ecole Centrale Paris (Erick Herbin, 10h).
- Erick Herbin is in charge of the Numerical Simulation Program in the Applied Mathematics option of Ecole Centrale Paris.
- Erick Herbin is supervisor of several student's research projects in the field of Mathematics at Ecole Centrale Paris.

9.4. Invited talks and Scientific popularisation

Jacques Lévy Véhel has given an invited lecture at the seminar "Probabilité et Statistique" at Lille University (November 2008), and at the "Image" seminar at Paris University Pierre et Marie Curie(December 2008).

Pierre Bertrand was invited speaker for several seminars held in Paris East University, Grenoble University, Technologic University of Compiègne, Maine University (Le Mans, France, and for the following conferences:

- Computational Mathematics Laboratory (Univ. Monastir, Tunisia), Jan 7, 2008.
- First french-tunisian mathematical conference, Djerba, March 16-20, 2009.
- Models and Images for Porous Media, Medicine Faculty, Paris, January 12-16, 2009.
- Barcelona Conference on Asymptotic Statistics, Barcelona September 2008.
- Rencontres sur la modélisation en physiopathologie, ENS Cachan, 16- 17 June 2008.
- STATDEP 2008 : Statistics for Dependent Data, 4-7 June 2008, ENSAE Paris.
- Journées Techniques Fractales, Orléans, 22-23 May 2008.

9.5. Ph.D. Theses

Jacques Lévy Véhel was a referee on the thesis of Milan Djilas "Interprétation des informations sensorielles des récepteurs du muscle squelettique pour le contrôle externe", (October 2007, University of Montpellier), and of David da Silva: "Caractérisation de la nature multi-échelles des plantes par des outils de géométrie fractale, influence sur l'interception de la lumière" (November 2007, University of Montpellier).

Evelyne Lutton was a referee on the thesis of Mickaël Naud (September 2008, University of Angers)"Contribution au développement d'outils d'aide à la conception assistée par la réalité virtuelle et augmentée : Application au domaine du textile/habillement."

Pierre R. Bertrand was PhD advisor of Imen Kammoun (December 19th 2007, University Paris 1), PhD referee of Olivier Barrière (November 28th 2007, Nantes University).

Jean Louchet was referee for the thesis of Jean-Claude Torrel (21 Dec 2007, Univ. Paris 5), Amir Nakib (5 dec 2007, Univ. Paris 12-Val-de-Marne), Patrice Navy (18 fev 2008, Univ. Paris 6), Sebastien Mahler (20 nov 2008, Univ. du Littoral, Calais). He was Phd advisor of the thesis of Olivier Pauplin (26 nov 2007, Univ. Paris 5) and Renaud Barate (6 nov 2008, Univ . Paris 6).

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