

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# Project-Team CORIDA

# Robust Control Of Infinite Dimensional Systems and Applications

Nancy - Grand Est



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# 2. Overall Objectives

# 2.1. Overall Objectives

CORIDA is a team labeled by INRIA, by CNRS and by University Henri Poincaré, via the Institut Élie Cartan of Nancy (UMR 7502 CNRS-INRIA-UHP-INPL-University of Nancy 2). The main focus of our research is the robust control of systems governed by partial differential equations (called PDE's in the sequel). A special attention is devoted to systems with a hybrid dynamics such as the fluid-structure interactions. The equations modeling these systems couple either partial differential equations of different types or finite dimensional systems and infinite dimensional systems. We mainly consider inputs acting on the boundary or which are localized in a subset of the domain.

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Infinite dimensional systems theory is motivated by the fact that a large number of mathematical models in applied sciences are given by evolution partial differential equations. Typical examples are the transport, heat or wave equations, which are used as mathematical models in a large number of problems in physics, chemistry, biology or finance. In all these cases the corresponding state space is infinite dimensional. The understanding of these systems from the point of view of control theory is an important scientific issue which has received a considerable attention during the last decades. Let us mention here that a basic question like the study of the controllability of infinite dimensional linear systems requires sophisticated techniques such as non harmonic analysis (cf. Russell [75]), multiplier methods (cf. Lions [71]) or micro-local analysis techniques (cf. Bardos–Lebeau–Rauch [60]). Like in the case of finite dimensional systems, the study of controllability should be only the starting point of the study of important and more practical issues like feedback optimal control or robust control. It turns out that most of these questions are open in the case of infinite dimensional systems. More precisely, given an infinite dimensional system one should be able to answer two basic questions:

- 1. Study the existence of a feedback operator with robustness properties.
- 2. Find an algorithm allowing the approximate computation of this feedback operator.

The answer to question 1 above requires the study of infinite dimensional Riccati operators and it is a difficult theoretical question. The answer to question 2 depends on the sense of the word "approximate". In our meaning "approximate" means "convergence", i.e., that we look for approximate feedback operators converging to the exact one when the discretization step tends to zero. From the practical point of view this means that our control laws should give good results if we use a large number of state variables. This fact is no longer a practical limitation of such an approach, at least in some important applications where powerful computers are now available. We intend to develop a methodology applicable to a large class of applications.

# 3. Scientific Foundations

# 3.1. Analysis and control of fluids and of fluid-structure interactions

**Keywords:** Korteweg de Vries equations, Navier-Stokes equations, analysis and control of fluids and fluidstructure interactions, motion of solids in viscous fluids.

The problems we consider are modeled by the Navier-Stokes, Euler or Korteweg de Vries equations (for the fluid) coupled to the equations governing the motion of the solids. One of the main difficulties of this problem comes from the fact that the domain occupied by the fluid is one of the unknowns of the problem. We have thus to tackle a *free boundary problem*.

The control of fluid flows is a major challenge in many applications: aeronautics, pollution issues, regulation of irrigation channels or of the flow in pipelines, etc. All these problems cannot be easily reduced to finite dimensional models so a methodology of analysis and control based on PDE's is an essential issue. In a first approximation the motion of fluid and of the solids can be decoupled. The most used models for an incompressible fluid are given by the Navier-Stokes or by the Euler equations.

The optimal open loop control approach of these models has been developed from both the theoretical and numerical points of view. Controllability issues for the equations modeling the fluid motion are by now well understood (see, for instance, Imanuvilov [67] and the references therein). The feedback control of fluid motion has also been recently investigated by several research teams (see, for instance Barbu [59] and references therein) but this field still contains an important number of open problems (in particular those concerning observers and implementation issues). One of our aims is to develop efficient tools for computing feedback laws for the control of fluid systems.

In real applications the fluid is often surrounded by or it surrounds an elastic structure. In the above situation one has to study fluid-structure interactions. This subject has been intensively studied during the last years, in particular for its applications in noise reduction problems, in lubrication issues or in aeronautics. In this kind of problems, a PDE's system modeling the fluid in a cavity (Laplace equation, wave equation, Stokes, Navier-Stokes or Euler systems) is coupled to the equations modeling the motion of a part of the boundary. The difficulties of this problem are due to several reasons such as the strong nonlinear coupling and the existence of a free boundary. This partially explains the fact that applied mathematicians have only recently tackled these problems from either the numerical or theoretical point of view. One of the main results obtained in our project concerns the global existence of weak solutions in the case of a two-dimensional Navier–Stokes fluid (see [7]). Another important result gives the existence and the uniqueness of strong solutions for two or three-dimensional Navier–Stokes fluid (see [8]). In that case, the solution exists as long as there is no contact between rigid bodies, and for small data in the three-dimensional case.

In [4], we study the large time behavior of solutions of a parabolic equation coupled with an ordinary differential equation. This system is a simplified N-dimensional model for the interactive motion of a rigid body (a ball) immersed in a viscous fluid. After proving the existence and uniqueness of a strong global in time solution, we compute the first term in the asymptotic development of solutions. We prove that the asymptotic profile of the fluid is the heat kernel with an appropriate total mass. The  $L^{\infty}$  estimates we get allow us to describe the asymptotic trajectory of the center of mass of the rigid body as well.

The numerical methods used for computing the solutions of fluid or fluid structure problems in a direct setting (i.e., with given inputs) considerably progressed during the last years. In our project, we have proposed in [6] an original numerical scheme to discretize the equations of motion for the system composed by a viscous incompressible fluid and several rigid bodies. One important characteristic of this scheme is that we use only a fixed mesh for the whole system, and therefore we do not need to remesh at some steps like for instance in the case of ALE methods. We have also developed two codes (in Matlab and in Scilab) based on our numerical scheme.

Another topic of great interest is the control of the interface of two fluids (typically water and air) by using as input the velocity of a moving wall which is a part of the boundary. One of the most popular models for this problem is given by the shallow water equations (Saint Venant equations) which neglect the dispersive effects. The controllability of several important systems governed by this type of equations has received a considerable attention during the last decade. Let us mention here the important work by Coron [62]. If dispersive effects are considered, the relevant model is given by the Korteweg de Vries equation. The first work on the control of this equation goes back to Russell and Zhang (see [76]). An important advance in the study of this problem has been achieved in the work of Rosier [74] where, for the first time, the influence of the length of the channel has been precisely investigated.

# **3.2.** Frequency domain methods for the analysis and control of systems governed by PDE's

**Keywords:** Helmholtz equation, control and stabilization, numerical approximation of LQR problems, timereversal.

We use frequency tools to analyze different types of problems. The first one concerns the control, the optimal control and the stabilization of systems governed by PDE's, and their numerical approximations. The second one concerns time-reversal phenomena, while the last one deals with numerical approximation of high-frequency scattering problems.

#### 3.2.1. Control and stabilization for skew-adjoint systems

The first area concerns theoretical and numerical aspects in the control of a class of PDE's. More precisely, in a semigroup setting, the systems we consider have a skew-adjoint generator. Classical examples are the wave, the Bernoulli-Euler or the Schrödinger equations. Our approach is based on an original characterization of exact controllability of second order conservative systems proposed by K. Liu [72]. This characterization

can be related to the Hautus criterion in the theory of finite dimensional systems (cf. [66]). It provides for time-dependent problems exact controllability criteria **that do not depend on time, but depend on the frequency variable** conjugated to time. Studying the controllability of a given system amounts then to establishing uniform (with respect to frequency) estimates. In other words, the problem of exact controllability for the wave equation, for instance, comes down to a high-frequency analysis for the Helmholtz operator. This frequency approach has been proposed first by K. Liu for bounded control operators (corresponding to internal control problems), and has been recently extended to the case of unbounded control operators (and thus including boundary control problems) by L. Miller [73]. Using the result of Miller, K. Ramdani, T. Takahashi, M. Tucsnak have obtained in [5] a new spectral formulation of the criterion of Liu [72], which is valid for boundary control problems. This frequency test can be seen as an observability condition for packets of eigenvectors of the operator. This frequency test has been successfully applied in [5] to study the exact controllability of the Schrödinger equation, the plate equation and the wave equation in a square. Let us emphasize here that one further important advantage of this frequency approach lies in the fact that it can also be used for the analysis of space semi-discretized control problems (by finite element or finite differences). The estimates to be proved must then be uniform with respect to **both the frequency and the mesh size**.

In the case of finite dimensional systems one of the main applications of frequency domain methods consists in designing robust controllers, in particular of  $H^{\infty}$  type. Obtaining the similar tools for systems governed by PDE's is one of the major challenges in the theory of infinite dimensional systems. The first difficulty which has to be tackled is that, even for very simple PDE systems, no method giving the parametrisation of all stabilizing controllers is available. One of the possible remedies consists in considering known families of stabilizing feedback laws depending on several parameters and in optimizing the  $H^{\infty}$  norm of an appropriate transfer function with respect to this parameters. Such families of feedback laws yielding computationally tractable optimization problems are now available for systems governed by PDE's in one space dimension.

#### 3.2.2. Time-reversal

The second area in which we make use of frequency tools is the analysis of time-reversal for harmonic acoustic waves. This phenomenon described in Fink [63] is a direct consequence of the reversibility of the wave equation in a non dissipative medium. It can be used to **focus an acoustic wave** on a target through a complex and/or unknown medium. To achieve this, the procedure followed is quite simple. First, time-reversal mirrors are used to generate an incident wave that propagates through the medium. Then, the mirrors measure the acoustic field diffracted by the targets, time-reverse it and back-propagate it in the medium. Iterating the scheme, we observe that the incident wave emitted by the mirrors focuses on the scatterers. An alternative and more original focusing technique is based on the so-called D.O.R.T. method [64]. According to this experimental method, the eigenelements of the time-reversal operator contain important information on the propagation medium and on the scatterers contained in it. More precisely, the number of nonzero eigenvalues is exactly the number of scatterers, while each eigenvector corresponds to an incident wave that selectively focuses on each scatterer.

Time-reversal has many applications covering a wide range of fields, among which we can cite **medicine** (kidney stones destruction or medical imaging), **sub-marine communication** and **non destructive testing**. Let us emphasize that in the case of time-harmonic acoustic waves, time-reversal is equivalent to phase conjugation and involves the Helmholtz operator.

In [2], we proposed the first far field model of time reversal in the time-harmonic case.

#### 3.2.3. Numerical approximation of high-frequency scattering problems

This subject deals mainly with the numerical solution of the Helmholtz or Maxwell equations for open region scattering problems. This kind of situation can be met e.g. in radar systems in electromagnetism or in acoustics for the detection of underwater objects like submarines.

Two particular difficulties are considered in this situation

- the wavelength of the incident signal is small compared to the characteristic size of the scatterer,
- the problem is set in an unbounded domain.

These two problematics limit the application range of most common numerical techniques. The aim of this part is to develop new numerical simulation techniques based on microlocal analysis for modeling the propagation of rays. The importance of microlocal techniques in this situation is that it makes possible a local analysis both in the spatial and frequency domain. Therefore, it can be seen as a kind of asymptotic theory of rays which can be combined with numerical approximation techniques like boundary element methods. The resulting method

# 3.3. Observability, controllability and stabilization in the time domain

is called the On-Surface Radiation Condition method.

Keywords: boundary control, coupling mechanism, linear evolution equations, stabilization.

Controllability and observability have been set at the center of control theory by the work of R. Kalman in the 1960's and soon they have been generalized to the infinite-dimensional context. The main early contributors have been D.L. Russell, H. Fattorini, T. Seidman, R. Triggiani, W. Littman and J.-L. Lions. The latter gave the field an enormous impact with his book [70], which is still a main source of inspiration for many researchers. Unlike in classical control theory, for infinite-dimensional systems there are many different (and not equivalent) concepts of controllability and observability. The strongest concepts are called exact controllability and exact observability, respectively. In the case of linear systems exact controllability is important because it guarantees stabilizability and the existence of a linear quadratic optimal control. Dually, exact observability guarantees the existence of an exponentially converging state estimator and the existence of a linear quadratic optimal filter. An important feature of infinite dimensional systems is that, unlike in the finite dimensional case, the conditions for exact observability are no longer independent of time. More precisely, for simple systems like a string equation, we have exact observability only for times which are large enough. For systems governed by other PDE's (like dispersive equations) the exact observability in arbitrarily small time has been only recently established by using new frequency domain techniques. A natural question is to estimate the energy required to drive a system in the desired final state when the control time goes to zero. This is a challenging theoretical issue which is critical for perturbation and approximation problems. In the finite dimensional case this issue has been first investigated in Seidman [78]. In the case of systems governed by linear PDE's some similar estimates have been obtained only very recently (see, for instance Miller [73]). One of the open problems of this field is to give sharp estimates of the observability constants when the control time goes to zero.

Even in the finite-dimensional case, despite the fact that the linear theory is well established, many challenging questions are still open, concerning in particular nonlinear control systems.

In some cases it is appropriate to regard external perturbations as unknown inputs; for these systems the synthesis of observers is a challenging issue, since one cannot take into account the term containing the unknown input into the equations of the observer. While the theory of observability for linear systems with unknown inputs is well established, this is far from being the case in the nonlinear case. A related active field of research is the uniform stabilization of systems with time-varying parameters. The goal in this case is to stabilize a control system with a control strategy independent of some signals appearing in the dynamics, i.e., to stabilize simultaneously a family of time-dependent control systems and to characterize families of control systems that can be simultaneously stabilized.

One of the basic questions in finite- and infinite-dimensional control theory is that of motion planning, i.e., the explicit design of a control law capable of driving a system from an initial state to a prescribed final one. Several techniques, whose suitability depends strongly on the application which is considered, have been and are being developed to tackle such a problem, as for instance the continuation method, flatness, tracking or optimal control. Preliminary to any question regarding motion planning or optimal control is the issue of controllability, which is not, in the general nonlinear case, solved by the verification of a simple algebraic criterion. A further motivation to study nonlinear controllability criteria is given by the fact that techniques developed in the domain of (finite-dimensional) geometric control theory have been recently applied successfully to study the controllability of infinite-dimensional control systems, namely the Navier–Stokes equations (see Agrachev and Sarychev [55]).

# **3.4. Implementation**

#### Keywords: Dicretization, Riccati equation.

This is a transverse research axis since all the research directions presented above have to be validated by giving control algorithms which are aimed to be implemented in real control systems. We stress below some of the main points which are common (from the implementation point of view) to the application of the different methods described in the previous sections.

For many infinite dimensional systems the use of co-located actuators and sensors and of simple proportional feed-back laws gives satisfying results. However, for a large class of systems of interest it is not clear that these feedbacks are efficient, or the use of co-located actuators and sensors is not possible. This is why a more general approach for the design of the feedbacks has to be considered. Among the techniques in finite dimensional systems theory those based on the solutions of infinite dimensional Riccati equation seem the most appropriate for a generalization to infinite dimensional systems. The classical approach is to approximate an LQR problem for a given infinite dimensional system by finite dimensional LQR problems. As it has been already pointed out in the literature this approach should be carefully analyzed since, even for some very simple examples, the sequence of feedbacks operators solving the finite dimensional LQR is not convergent. Roughly speaking this means that by refining the mesh we obtain a closed loop system which is not exponentially stable (even if the corresponding infinite dimensional system is theoretically stabilized). In order to overcome this difficulty, several methods have been proposed in the literature : filtering of high frequencies, multigrid methods or the introduction of a numerical viscosity term. We intend to first apply the numerical viscosity method introduced in Tcheougoué Tebou – Zuazua [80], for optimal and robust control problems.

# 4. Application Domains

# 4.1. Panorama

Keywords: acoustics, aero-acoustics, biology, medicine.

As we already stressed in the previous sections the robust control of infinite dimensional systems is an emerging theory. Our aim is to develop tools applicable to a large class of problems which will be tested on models of increasing complexity. We describe below only the applications in which the members of our team have recently obtained important achievements.

# 4.2. Biology and Medicine

Keywords: inverse problems, modeling in biology, optimal shape.

#### 4.2.1. Medicine

We began this year to study a new class of applications of observability theory. The investigated issues concern inverse problems in Magnetic Resonance Imaging (MRI) of moving bodies with emphasis on cardiac MRI. The main difficulty we tackle is due to the fact that MRI is, comparatively to other cardiac imaging modalities, a slow acquisition technique, implying that the object to be imaged has to be still. This is not the case for the heart where physiological motions, such as heart beat or breathing, are of the same order of magnitude as the acquisition time of an MRI image. Therefore, the assumption of sample stability, commonly used in MRI acquisition, is not respected. The violation of this assumption generally results in flow or motion artifacts. Motion remains a limiting factor in many MRI applications, despite different approaches suggested to reduce or compensate for its effects Welch et al. [81]. Mathematically, the problem can be stated as follows: can we reconstruct a moving image by measuring at each time step a line of its Fourier transform? From a control theoretic point of view this means that we want to identify the state of a dynamical system by using an output which is a small part of its Fourier transform (this part may change during the measurement).

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There are several strategies to overcome these difficulties but most of them are based on respiratory motion suppression with breath-hold. Usually MRI uses ECG information to acquire an image over multiple cardiac cycles by collecting segments of Fourier space data at the same delay in the cycle Lanzer et al. [68], assuming that cardiac position over several ECG cycles is reproducible. Unfortunately, in clinical situations many subjects are unable to hold their breath or maintain stable apnea. Therefore breath-holding acquisition techniques are limited in some clinical situations. Another approach, so called real-time, uses fast, but low resolution sequences to be faster than heart motion. But these sequences are limited in resolution and improper for diagnostic situations, which require small structure depiction as for coronary arteries.

#### 4.2.2. Biology

The observation of nature and of the "perfection" of most of its mechanisms of living beings drives us to search **a principle of optimality** which governs those mechanisms. If a mathematical model exists for describing a biological phenomenon or component of a living being, there is a temptation to quantify the optimality by finding a functional which leads to the optimality principle. The confrontation between the computed optimum and the real one leads us to validate or invalidate the model and/or the choice of the functional. This inverse modeling method consists in finding the mathematical model starting from observations and theirs consequences. If the optimal shape which is issued from the mathematical model is close to the real shape, we have reasons to believe that the full model (equation and functional) is good. If not, one has to reject it and find another one, or improve it.

The mathematical study of these questions strongly uses tools of "shape variation and optimization" as developed in the book [3].

## 4.3. Automotive industry

We applied some modern techniques of automatic control to the command of the cooling of the fuel cells stack; these techniques result in a significant enhancement of the functioning of the cooling circuit. This problem is studied in the framework of the PhD thesis of Fehd Ben Aicha [10] (co-supervised by J.C. Vivalda and M. Sorine, head of the SOSSO2 project).

# 5. Software

# 5.1. SCISPT Scilab toolbox

Keywords: Scilab, sparse matrices.

Participant: Bruno Pinçon [correspondant].

Our aim is to develop Scilab tools for the numerical approximation of PDE's. This task requires powerful sparse matrix primitives, which are not currently available in Scilab. We have thus developed the SCISPT Scilab toolbox, which interfaces the sparse solvers UMFPACK of Tim Davis and TAUCS SNMF of Sivan Toledo. It also provides various utilities to deal with sparse matrices (estimate of the condition number, sparse pattern visualization, etc.). This module has been recently integrated in SCILAB-5.

# 5.2. Simulation of viscous fluid-structure interactions

Keywords: C++, Finite elements, MATLAB, Navier-Sokes equations, method of characteristics.

Participants: Bruno Pinçon, Jean-François Scheid [correspondant], Takéo Takahashi, Jean-Baptiste Tavernier.

We wrote, from September 2006 to September 2008, several codes for the 2D simulation of fluids and of fluidstructure A MATLAB Software has been first developed for the simulation of rigid bodies with an arbitrary shape falling in an incompressible fluid. This MATLAB code uses a number of C++ functions that has been written in order to improve the sparsematrix manipulation system of MATLAB. The interfacing MATLAB/C++ is made through *mexfiles*. This work has been done in collaboration with ALICE team of INRIA and with the use of some of their own numerical tools library. As a result, the computing time has been significantly reduced.

A 2D code has also been developed for the simulation of the fish-like swimming model described in [48]. In this model the deformation of the fish is imposed but its trajectory is not prescribed and should result from the interaction between the deformation and the fluid. The numerical scheme used in this code is an extension of the one developed for rigid-fluid system. This MATLAB code also uses MATLAB/C++ interfacing.

The development of the 3D MATLAB/C++ code has also begun. A 3D Stokes sparse solver for MATLAB is now available. But due to the huge size of the linear systems to be solved, we are not able to perform computations with fine grids/meshes. This should be improved by interfacing another functionality of the numerical library of ALICE team, especially direct linear sparse-solver. A model of turbulence (not necessary in the 2D case) should also be included in order to deal with realistic simulations where the Reynolds number is high.

# **5.3. Biohydrodynamics MATLAB Toolbox (BHT)**

Keywords: Articulated body, MATLAB, Nyström's method, potential fluid equations.

Participants: Alexandre Munnier [correspondant], Bruno Pinçon.

Understanding the locomotion of aquatic animals fascinated the scientific community for a long time. This constant interest has grown from the observation that aquatic mammals and fishes evolved swimming capabilities superior to what has been achieved by naval technology. A better understanding of the biomechanics of swimming may allow one to improve the efficiency, manoeuvrability and stealth of underwater vehicles. During the last fifty years, several mathematical models have been developed. These models make possible the qualitative analysis of swimming propulsion as a continuation of the previously developed quantitative theories. Based on recent mathematical advances, Biohydrodynamics MATLAB Toolbox (BHT) gathers a collection of M-Files for design, simulation and analysis of articulated bodies' motions in fluid. More widely, BHT allows also to perform easily any kind of numeric experiments addressing the motion of solids in ideal fluids (simulations of so-called fluid-structure interaction systems).

This software is available at http://bht.gforge.inria.fr/.

# 6. New Results

# 6.1. Analysis and control of fluids and of fluid-structure interactions

Keywords: Korteweg de Vries equations, Navier-Stokes equations.

**Participants:** Thomas Chambrion, Antoine Henrot, Jean Houot, Alexandre Munnier, Lionel Rosier, Jean-François Scheid, Mario Sigalotti, Takéo Takahashi, Jean-Baptiste Tavernier, Marius Tucsnak, Jean-Claude Vivalda.

The study of the a fluid-structure systems depends a lot of the nature of the fluid considered and in particular on the Reynolds number. We have split the new results obtained in this section according to the viscosity of the fluid. The first part is devoted to the case of a viscous fluid. This is the case which has received more attention from mathematicians in the recent years. In the second part, we have put the results concerning an inviscid fluid. This case is more classical in Fluid Mechanics and could be more interested to understand self-propelled motions which is one the main goal of our work. In the last part, we have finally given some numerical results.

It is worth noting that an important part of this work is done in collaboration with a group from the University of Chile through the associated team ANCIF. The Chilean members of our associated team are Jorge San Martín, Jaime Ortega, Carlos Conca, Patricio Cumsille and Axel Osses.

A review of the results obtained in the viscous fluid case lead to the book chapter [53] written by Jorge San Martín and Marius Tucsnak.

Some of the results, given below, on fluid-structure interaction problems, added to other contributions on control of PDE's constitute the material of the *Habilitation à Diriger des Recherches* of Takéo Takahashi (see [13]), defended on June 23th, 2008.

#### 6.1.1. Incompressible viscous fluids

In reference [79] we gave an existence and uniqueness result in the case of a viscous fluid filling the exterior of an infinite cylinder. The generalization of this result to more general geometries is studied in reference [30]. In this recent work, Cumsille and Takahashi have obtained the well-posedness in the case of a rigid body of arbitrary shape moving in a viscous incompressible fluid.

We continued the work initiated in [77] on the control theoretic interpretation of the motion of aquatic organisms. More precisely, in Sigalotti and Vivalda [49] we consider a finite-dimensional model for the motion of *ciliata*, coupling Newton's laws for the organism with Stokes equations governing the surrounding fluid. We prove that such a system is generically controllable when the space of controlled velocity fields is at least three-dimensional. We also provide a complete characterization of controllable systems in the case in which the organism has a spherical shape. Finally, we offer a complete picture of controllable and non-controllable systems under the additional hypothesis that the organism and the fluid have densities of the same order of magnitude.

In [48], Scheid, Takahashi and Tucsnak have given and analyzed a model for the motion of a fish. The model consists in a solid undergoing an undulatory deformation, which is immersed in a viscous incompressible fluid. The displacement of the "creature" is decomposed into a rigid part and a deformation (undulatory) part. One of the particularities of the work is that no particular constitutive laws for the solid are used but instead the non–rigid part of the deformation is imposed. The advantages of this approach consist in the global character (up to possible contacts) of the obtained strong solutions and in the possibility of using numerical methods inspired by those developed in the rigid-fluid case. In fact, by considering a similar method to the one developed in the article [6], numerical simulations have been performed for the motion of a fish into a viscous incompressible fluid. Recall that the non-rigid part of the deformation is only imposed and the trajectory of the fish is not prescribed. We observe numerically that the fish manages to go ahead and thus a self-propelled motion is reached by the fish. A numerical code has been developed for the 2D case in Scilab with the use of the SCISPT toolbox for sparse solver (see Section 5). Similar techniques have been employed by Scheid in [31] in connection with the level sets method and by Takahashi [18] to extend the topological sensitivity analysis to parabolic and hyperbolic problems.

## 6.1.2. Inviscid fluids

In [35], Houot and Munnier study the well-posedness of the equations modeling the motion of 3 dimensional rigid bodies immersed in a potential fluid flow. Although the existence and uniqueness of smooth solutions is well known in the particular case of a single solid in an infinitely extended fluid, the problem gets more involved in the presence of several bodies or in a bounded fluid cavity. By using shape sensitivity analysis, Houot and Munnier prove that even in this case there exists analytic solutions up to the collision between two solids or between a solid with the fluid boundary. Concerning this problem of collisions, they show that an infinite cylinder, filling with a potential fluid an half space, can collide with the boundary with non zero relative velocity. This result heavily contrasts with what happen in the case of a viscous fluid in which collisions are not possible. They compute the velocity damping of the cylinder when approaching the wall, proving that d'Alembert paradox does not hold in this case. In [41], Munnier extends the results of existence of global strong solutions to the case of deformable bodies (the deformations being prescribed as functions of time) in an incompressible, inviscid fluid whose flow is irrotational. The results in [35] together with the study of the wellposedness in the case of an ideal (not necessarily potential fluid) have been studied in the PhD thesis of Jean Gabriel Houot [11].

In [45], we investigate the motion of a rigid ball surrounded by an incompressible perfect fluid occupying  $R^N$ ,  $N \ge 2$ . We prove the existence, uniqueness, and persistence of the regularity for the solutions of this fluid-structure interaction problem.

The control of solid in a perfect fluid (not necessarily potential), the normal component of the fluid velocity being controlled on a small part of the boundary. For a ball in 2D, it has been shown by Rosier that we may control the position of the ball, the ball being at rest at the beginning and at the end of the control process. The extension to a solid with one axial symmetry (boat) is under investigation with Glass (Paris 6).

Another situation which has been treated is that of a rigid body moving in an inviscid fluid. More precisely, Chambrion and Sigalotti studied the controllability properties of a neutrally buoyant underwater vehicle immersed in an infinite volume of an inviscid incompressible fluid in irrotational motion (see [26]). Due to the potential nature of the flow, the state of the system is fully determined by a finite set of real variables which parameterize the set of configurations (position and attitude) of the moving body and its linear and angular momenta. The dynamics of such momenta, seen from the perspective of the solid, are described by the classical Kirchhoff equations whose control properties have been thoroughly studied (see for instance Leonard et al., Astolfi et al. [69], [58]). In [26] we tackle the less studied problem of controlling and stabilizing the full 12-dimensional nonlinear system describing the evolution of configuration and momenta.

In [37], Takahashi, with Lagoutière (Paris 7 University) and Seguin (Paris 6 University), analyzes a onedimensional fluid-particle interaction model, composed by the Burgers equation for the fluid velocity and an ordinary differential equation which governs the particle movement. The coupling is achieved through a friction term. One of the novelties is to consider entropy weak solutions involving shock waves. The difficulty is the interaction between these shock waves and the particle. It is proved that the Riemann problem with arbitrary data always admits a solution, which is explicitly constructed. Besides, two asymptotic behaviors are described: the long-time behavior and the behavior for large friction coefficients.

#### 6.1.3. Numerical Analysis and Numerical Simulations

Takahashi, with Legendre (Paris Dauphine University), considers in [38] a numerical scheme to compute the motion of a two-dimensional rigid body in a viscous incompressible fluid. This method combines the finite element method and the method of characteristics to solve an Arbitrary Lagrangian Eulerian formulation of the problem. Error estimates are derived for this scheme which imply its convergence.

Since September 2006, an INRIA associated engineer has been recruited to improve the current numerical code, to rewrite it in the Matlab software and finally to develop a 3D software for the fish-like swimming. The improvement of the 2D code is in progress and deals with the Navier-Stokes solver together with the optimization of the Matlab implementation of the numerical code. The Navier-Stokes solver has to allow to tackle realistic situations where the Reynolds number lies between  $10^3$  and  $10^6$ , depending on the nature of the fish-like swimming we consider (trout, salmon, eel, ...). The development of the 3D software has not started yet.

In [27]-[28] we investigate the problem of the detection of a moving obstacle in a perfect fluid occupying a bounded domain in  $R^2$  from the measurement of the velocity of the fluid on a part of the boundary.

# 6.2. Frequency tools for the analysis of PDE's

Participants: Xavier Antoine, Pauline Klein, Bruno Pinçon, Karim Ramdani, Bertrand Thierry.

#### 6.2.1. Observation and Control for operator semigroups

The main topics covered in the monograph [54] are the study of the observation and control operators for systems which can be described by operator semigroups in Hilbert spaces, with emphasis on observability and on controllability properties. The abstract results are supported by a large number of examples coming from partial differential equations. These examples are worked out in detail and they cover to a large extent systems governed by the classical linear partial differential equations. This work is meant to be an elementary introduction in this theory. The first meaning of "elementary" is that the text is aimed to be accessible to

any reader familiar with linear algebra, calculus and which has some basic knowledge on Hilbert spaces and on differential equations. We introduce everything needed on operator semigroups and most of the used background is summarized in the Appendices, often with proofs. The second meaning of "elementary" is that we only cover results for which we can provide complete proofs.

#### 6.2.2. Acoustics

**Time Reversal:** In [22] we propose a mathematical analysis of electromagnetic time reversal by closed mirrors. A mathematical justification of the selective focusing properties of the D.O.R.T. method is given for axially symmetric scatterers.

**Multiple scattering:** In [21], we propose a numerical strategy for computing the solution of two-dimensional time-harmonic scattering problems at high-frequency. The scatterers are assumed to be circular leading therefore to semi-analytical representation formulae of the scattered field through the solution of a large linear system of equations. Taking advantage of the special block Toeplitz structure of the matrix involved in the formulation, an efficient and original numerical method is proposed. Moreover, a specific preconditioned iterative and fast algorithm is given and analyzed for computing the field scattered by general configurations. Several numerical experiments are presented to show the efficiency of the numerical method.

**Photonic crystals:** In [44], we investigate the attenuation of linear scalar waves as they travel through a thick slab of a lossless periodic medium at frequencies for which propagation is prohibited. In particular, we establish rigorously the emergence of a spectral bandgap (namely a frequency interval of exponentially small transmission of energy through a slab of the crystal) when the thickness of the slab tends to infinity. We extend the result to slabs that contain a planar defect for frequencies that do not coincide with guided mode frequencies for the defect. For one-dimensional crystals, we prove that the transmission approaches a nonzero value at guided mode frequencies and give an explicit formula for the transmission. The main analytical tool in the multidimensional case is the Calderón boundary integral projectors for the Helmholtz equation.

#### 6.2.3. Numerical methods for high-frequency scattering problems

The paper [20] presents a full analysis of a new integral preconditioner for the scattering problem by inhomogeneous isotropic media solved through integral equations. It is shown that the method is robust and efficient on some complex computational examples, even for concave scatterers and large wave numbers.

We propose in [32] a new algorithm coupling finite elements with microlocal analysis for reducing the pollution into the finite element method for high-frequency. Three-dimensional computations confirms the gain of our method compared to standard techniques.

Fractional calculus plays a keyrole in computational acoustics. We provide in [19] a review of current developments in this area with computational examples coming from underwater acoustics wave propagations as well as for computing the propagation of solitons in nonlinear media.

In [36], we develop high-frequency solutions to the Helmholtz equation using plane wave finite element basis coupled to high-order artificial boundary conditions. Numerical examples show that high accuracy can be obtained for a small number of degrees of freedom even for large frequencies.

## 6.3. Observability, controllability and stabilization in the time domain

Participants: Fatiha Alabau, Thomas Chambrion, Antoine Henrot, Lionel Rosier, Mario Sigalotti, Marius Tucsnak, Jean-Claude Vivalda.

### 6.3.1. Results on the Schrödinger equation

In [50] we consider the two dimensional Schrödinger and Euler-Bernoulli plate equations. By generalizing results from Ramdani, Takahashi, Tenenbaum and Tucsnak [5], we prove that these systems are exactly observable *in arbitrarily small time*. Moreover, we show that the above results hold even if the observation regions have *arbitrarily small measures*. More precisely, we prove that in the case of homogeneous Neumann boundary conditions with Dirichlet boundary observation, the exact observability property holds for every

observation region with non empty interior. In the case of homogeneous Dirichlet boundary conditions with Neumann boundary observation, we show that the exact observability property holds if and only if the observation region has an open intersection with an edge of each direction. Moreover, we give *explicit estimates for the blow-up rate* of the observability constants as the time and/or the size of the observation region tend to zero. The main ingredients of the proofs are an effective version of a theorem of Beurling and Kahane on non harmonic Fourier series and an estimate for the number of lattice points in the neighborhood of an ellipse.

In [25], [51] we prove the approximate controllability of the bilinear Schrödinger equation in the case in which the uncontrolled Hamiltonian has discrete non-resonant spectrum. The result applies both to bounded or unbounded domains and to the case in which the control potential is bounded or unbounded.

In [39]-[40], we consider the problem of retrieving a potential in Schrödinger equation from the measurement of the state in a small observation region thanks to new Carleman estimates based upon a weak pseudoconvexity condition.

In [47], we investigate the boundary or internal controllability of the cubic Schrödinger equation posed on a bounded interval I. Exact controllability results are established in the energy space  $L^2(I)$  thanks to Bourgain analysis.

In [46], we study the exact boundary controllability of the cubic Schrödinger equation posed on a bounded domain  $\Omega \subset \mathbb{R}^N$  with either the Dirichlet boundary conditions or the Neumann boundary conditions. It is shown that if s > N/2, or  $0 \le s < N/2$  with  $1 \le N < 2 + 2s$ , or s = 0, 1 with N = 2, then the systems are locally controllable in the Sobolev space  $H^s(\Omega)$  around any smooth solution of the cubic Schrödinger equation.

#### 6.3.2. Stabilization

In [16], we study visco-elastic materials for which the feedback law is of memory type when the kernel of the convolution operator is exponentially or polynomially decaying at infinity. We prove stabilization properties with exponential or polynomial decay of the energy for general abstract hyperbolic equations with applications to various models : waves, elasticity, Petrowsky equations.

In [15] we study the case of visco-elastic materials located on a part of the boundary, i.e. the case of boundary memory damping.

In [56], we consider the coupled system for Timoshenko beams, subjected to a single control force. We prove general decay estimates (of polynomial, logarithmic ..., type) depending on the growth properties of the feedback at the origin in case of equal speeds of propagation for the equation of the transverse displacement of the beam and of the rotation angle. In case of different speeds of propagation, we prove polynomial decay for smooth solutions in case of globally Lipschitz nonlinear feedbacks, with linear growth.

Henrot and Cox were interested in a musical problem which strongly use the eigenfrequency of the damped wave equation. It is a question relating to a model for harmonics on stringed instruments which could be set as: is it possible to achieve "Correct Touch" in the pointwise damping of a fixed string? By correct touch, we consider the following. When we place a finger lightly at one of the nodes of the low frequency harmonics, it forces the string to play a note that sounds like a superposition of those normal modes with nodes at the location of the finger. Now, the question is to determine what should be the pressure of the finger in order to best damps the remaining modes. From a mathematical point of view, we consider the wave equation with a damping as  $b\delta_a u_t$  where  $u_t$  is the speed, a the location of the pressure,  $\delta_a$  is the Dirac distribution at a and b the intensity of the pressure. We want to determine, for each a, the b which minimizes the spectral abscissa of the modes not vanishing at a. This involves a precise qualitative analysis of the spectrum of the damped operator in the complex plane. This work is published in [29].

In [43] we investigate the boundary stabilization of a Boussinesq system of KdV-KdV type posed on a bounded domain. We design a two-parameter family of feedback laws for which the solutions issuing from small data are globally defined and exponentially decreasing in the energy space.

#### 6.3.3. Finite dimensional systems

In [14] Sigalotti together with Boscain and Agrachev considered a generalization of Riemannian geometry which naturally arises in quantum control theory (see, for instance, Boscain et al. [61]) and in the study of hypoelliptic operators (Franchi et al. [65]). We obtain in this framework a generalization of the Gauss-Bonnet formula.

In [24] Sigalotti, together with Chaillet, Chitour and Loria, studied the stabilizability of a switched systems for which the dynamics corresponding to some parameters are unstable, under an assumption of permanent excitation. In particular, the paper deals with the case in which the unstable dynamics are given by the double integrator and show that in this case the system is stabilizable provided that it is controllable.

The observability of discrete time systems has been studied by Vivalda in [17]. In [23], we deal with the notion of practical observers for affine systems with unknown inputs. These systems can be written as

$$\dot{x} = f(x) + ug(x)$$
$$y = h(x).$$

We show that a system which is observable with respect to the unknown input can be put into triangular form. Moreover, for this kind of systems, we design a practical observer, that is to say a family of auxiliary dynamical system written as  $\hat{x} = g_{\varepsilon}(\hat{x}, y)$  such that

$$\|\widehat{x}(t) - x(t)\| \le e^{-\lambda(\varepsilon)t} \|\widehat{x}(0) - x(0)\| + \mu(\varepsilon)$$

with  $\lim_{\varepsilon \to +0} \lambda(\varepsilon) = +\infty$  and  $\lim_{\varepsilon \to 0} \mu(\varepsilon) = 0$ .

# 6.4. Biology and Medicine

Participants: Nicolae Cîndea, Antoine Henrot, Yannick Privat, Marius Tucsnak.

#### 6.4.1. Medicine

A quite recent activity of our group consists in proposing new methods for the reconstruction of moving objects via Magnetic Resonance Imaging (MRI). This is the main topic of the Phd thesis of Cîndea, which is directed jointly by Tucsnak and Vuissoz from the MRI laboratory of the University Hospital of Nancy. One of the ideas of this work is to use the theory of observers in order to reconstruct the state from the measurements in the Fourier space. This work requires to solve preliminary modeling and identification problems involving computations that cannot, for the moment, be performed in real time. These problems are tackled by our group by two methods. The first one, presented in [42], reduces the problem to an integral equation which is solved using a regularization procedure. This method is based on the fact that the motion of the moving body has been partially detected by appropriate sensors. The second method uses weaker information on the motion of the body, namely that it is periodic. The aim of our work is to provide a method for cardiac imaging reconstruction at each cardiac or respiratory phase. We propose a method inspired by Zwaan [82]. In his method, heart motion during breath-hold was studied with the assumption that each heartbeat is a rescaled copy of a standard heartbeat. Here, the same assumption is extended to respiratory motion. Similarly to the algorithm presented by Zwaan [82], our method is based on the properties of the Reproducing Kernel Hilbert Spaces (RKHS) [57], which provide a general and rigorous framework for handling interpolation problems, and have been widely used in signal and image processing. This framework is of particular relevance here, as retrospective gating can be reformulated as a scattered data interpolation in a 1D or 2D space. The two methods above are part of the PhD work of Nicolae Cîndea.

## 6.4.2. Biology

In the paper [33], we have considered the case of a dendrite sealed at one end and connected to the soma at the other end. We looked at the shape of a dendrite which minimizes either **the attenuation in time** of the electrical signal or **the attenuation in space** of the signal. In both cases, we were able to prove that the optimal shape is the cylinder (which is closed to the real shape). Then, Privat also considered the case of a dendrite sealed at both ends. For the attenuation in space, the optimal shape is the same, but for the attenuation in time he was able to prove that the problem is ill-posed: it does not exist any solution. Moreover, the minimizing sequences have a strange behavior.

More recently, Y. Privat and A. Henrot have also considered the shape optimization problem for the lung. The questions which are studied are the following:

- 1. is the dichotomic structure of the lung tree and the dimension of each branch optimal?
- 2. is the cylindrical shape of a given branch optimal?

The state equation used for the modelling is the Navier-Stokes system with classical boundary condition. The criterion to be minimized is the dissipated energy. For the first question, Y. Privat (with B. Mauroy) has done some numerical simulations in two and three dimensions. These simulations point out that the problem is indeed a multi-objective optimization problem, since the optimal shape is not the same in the case of inspiration and expiration. For the second question, the authors prove that the cylinder is NOT the optimal shape. This result is interesting both from a practical point of view (think to pipes) and for a theoretical point of view. Actually, there are a very few theoretical results for shape optimization problems in fluid mechanics even if it is a topic extensively studied because of its potential applications. This study has been announced in [34] and the subsequent paper has been submitted (see also the PhD thesis of Yannick Privat [12].

# 7. Contracts and Grants with Industry

# 7.1. Contracts and Grants with Industry

The work in the field of automotive industry described in subsection 4.3 is formalized by a CIFRE contract with Renault. The corresponding PhD Student Fehd BEN AICHA has been employed by Renault since June 2005. He defended his thesis on November 24th, 2008. This thesis proposed control laws and diagnosis in order to optimize the efficiency of the powertrain plant under constraints of autonomy, pollution and cost constraints. The first part of the work concerns the modelling and control of an onboard fuel cell and of an onboard reformer. The second part tackles the classical diesel engine. One point of interest was the regeneration of the particle filters for which several control laws have been proposed.

Matthieu GEIST is preparing his thesis in the framework of a CIFRE agreement with ARCELOR Research. His thesis is co-directed by Olivier PIETQUIN from Supélec and Gabriel FRICOUT from ARCELOR Research. The iron and steel industry can be seen as a huge stochastic, dynamical and partially observable system. The aim of this thesis is to develop a general framework to model as globally and generically as possible a large optimization problem, and to find an optimal solution to this problem. Several approaches are possible, but we focus on reinforcement learning. Notice that the paper [52] has been awarded as one of the top papers of the conference ADVCOMP 2008.

# 8. Other Grants and Activities

# 8.1. National initiatives

#### 8.1.1. Administrative responsibilities

- At INRIA: Tucsnak is member of the Executive Team and of the Project Committee of the INRIA Nancy-Grand Est Research Centre.
- In the Universities and in CNRS committees:
  - Henrot is the head of the Institut Élie Cartan de Nancy (IECN).
  - Tucsnak is member of the Scientific Council of UHP.
  - Alabau is member of CNU, section 26.
  - Our team is part of the GDR entitled "Fluid-Structure Interactions".

#### 8.1.2. National projects

- CPER ("Contrat Plan Etat Région"):
  - Serres, Sigalotti (leader) and Vivalda are in "Stabilité et Commande des Systèmes à Commutations". This is project in the AOC theme, in collaboration with the Automatic Control team at CRAN, is devoted to the stabilization of hybrid systems arising in the domain of DC-DC converters.
  - Scheid, Takahashi (leader), Tavernier and Tucsnak are in the project "Se propulser dans un fluide, analyse, contrôle et visualisation" (AOC theme), in collaboration with the INRIA team, ALICE.
- ANR ("Agence Nationale de la recherche"):
  - Munnier, Scheid, Takahashi and Tucsnak are in the ANR project "MOSICOB" ("ANR Blanche") for three years in collaboration with the University of Paris Sud and with the University of Grenoble.
  - Rosier and Takahashi are in the ANR project "Contrôle d'équations aux dérivées partielles en mécanique des fluides" (in collaboration with the University of Paris 6, the University of Versailles and the University of Nanterre).
- Our team is part of the GDR entitled "Fluid-Structure Interactions".

# 8.2. Associated Team

The CORIDA project is linked with a group of the University of Chile through the associated team ANCIF.

## **8.3. Bilateral agreements**

- 1. A IFCPAR grant with the Tata Institute (Bengalooru, India).
- 2. A "Partenariat Hubert Curien" (PHC) with the IST (Lisbon, Spain).
- 3. A "Partenariat Hubert Curien" (PHC) with the Mathematical Institute of the Academy of Sciences of the Czech Republic (Prague).

# 8.4. Visits of foreign researchers

Evans Harrell (Georgia Tech, Atlanta), Gérard Philippin (U. Laval, Québec), George Weiss (Imperial College, London, England), Ana Leonor Silvestre and Carlos Alves (IST Lisbon, Portugal), M. Kesavan and M. Vaninnathan (India).

# 9. Dissemination

# 9.1. Participation to International Conferences and Various Invitations

# 9.1.1. Organization of Conferences

- Alabau:
  - 6th European-Maghreb Conference on Evolution Equations, CIRM Luminy, co-organized with Ouhabaz (Univ. Bordeaux I) and Monniaux (Univ. Aix Marseille 3).
- Takahashi:
  - "Control of Physical Systems and Partial Differential Equations", June 16-20, 2008, Paris (Institut Henri Poincaré).
  - CEMRACS 2008 "Modelling and Numerical Simulation of Complex Fluids", July 21 August 29, 2008, Marseille (CIRM).
  - "Méthodes et Problèmes de Contrôle en EDP", March 12 March 13, 2008, Nancy (Institut Élie Cartan de Nancy).

#### 9.1.2. Invited conferences

- Alabau:
  - Workshop on Direct, Inverse and Control Problems for PDE's, Cortona, Italy, September 2008;
  - Workshop on PDE's, Rio de Janeiro, Brazil, August 2008.
- Antoine:
  - Workshop on Analysis of Boundary Element Methods, Oberwolfach, Germany, April 2008;
  - Conference on "Advanced Computational Methods in Engineering", Liège, Belgium, May 2008;
  - Workshop on Computational Methods for Quantum, High Frequency and Seismic Waves, Tsinghua University, Beijing, Chine, December 2008.
- Chambrion:
  - International conference on Differential Equations and Topology (dedicated to the centennial Anniversary of L.S. Pontryagin), Moscow, June 17-22, 2008
- Henrot:
  - UAE MathDay, avril 2008, Abu-Dhabi.
  - Free Boundary Problems 08, June 2008, Stockholm, Sweden.
  - Advances in Shape an Topology Optimization, September 2008, Graz, Austria.
- Ramdani
  - Interregional Colloquium of Mathematics, October 2008, Luxembourg.
- Rosier
  - Conference on Control of Physical Systems, Paris, June 2008;
  - Four Generations under One Roof, a workshop in nonlinear analysis, Paris, July 3-4, 2008.
  - Workshop on Partial Differential Equations, Rio de Janeiro, August 26-29, 2008.

- International Conference on the Dispersive and Kinetic Equations, Jinhua (China), 14-16 December, 2008.
- First AMS-SMS meeting, Special Session on Recent Developments in Nonlinear Dispersive Wave Theory, Shanghai, 17-21 December, 2008.
- Takahashi
  - Conference on Vorticity, Rotation and Symmetry-Stabilizing and Destabilizing Fluid Motion, May 19-23, 2008, Marseilles (CIRM);
  - Workshop on Mathematical Fluid Dynamics, Darmstadt, September 8-10, 2008.
- Tucsnak
  - Workshop on Control and Optimization of Partial Differential Equations, Oberwolfach, Germany, Mars 2008;
  - Conference on Inverse Problems, Control and Shape optimization (PICOF), Morocco, April 2008;
  - Conference on Control of Physical Systems, Paris, Juin 2008;
  - Conference of Romanian Mathematicians, Bucarest, September 2008;
  - Workshop on Direct, Inverse and Control Problems for PDE's, Cortona, Italy, September 2008;
  - 6th European-Maghreb Conference on Evolution Equations, CIRM Luminy, November 2008.

# 9.1.3. Invitations

- Alabau
  - Halle University, Germany, April 2008;
  - Monastir University, April 2008.
- Antoine
  - City University of Hong Kong, February 2008.
  - Tsinghua University, Beijing, China, December 2008.
- Takahashi
  - Portugal, November 2008
- Tucsnak
  - Universidad de Chile, Santiago, Chile, December 2008
  - Tata Institute, Bangalore, India, August 2008

#### 9.1.3.1. Seminar talks

- Alabau
  - Bordeaux I University, Analysis Seminar.
- Antoine
  - Metz, LMAM, Partial differential equations seminar.
- Henrot
  - ENSTA Paris: Seminar of the POEMS project-team, April 2008.
  - Lisbon (Portugal), Mathematical Physics Seminar, July 2008.

- Prague (Czech Republic), PDE Seminar, september 2008.
- Besançon, Journée spéciale sur l'optimisation de forme et le contrôle, October 2008.
- Marseille, Analysis Seminar, October 2008.
- Munnier
  - Santiago of Chile, PDE's Seminar of CMM (Center for Mathematical Modeling, University of Chile), January 2008;
  - Chambéry, PDE's Seminar of LAMA (Department of Mathematics of the University of Savoie, France), September 2008;
  - Amiens, PDE's Seminar of LAMFA (Department of Mathematics of the university of Amiens, France), December 2009.
- Ramdani
  - Nancy, Seminar on the control of PDE's;
  - Paris, GdR Ondes, Seminar on Time Reversal.
- Sigalotti
  - Montpellier, ACSIOM seminar.
  - Lyon, PDE's GDR MACS work group.
  - Grenoble, Mathematical physics seminar.
- Tucsnak
  - Laboratoire Jacques-Louis Lions, Université Paris 6, Numerical analysis seminar;
  - Laboratoires de Mathématiques, Université Paris Sud, PDE's and numerical analysis seminar.

## 9.1.4. Participation to conferences

- Bertrand, Klein, Munnier, Pincon, Scheid : National Conference on Numerical Analysis (CANUM08) May 26–30 2008.
- Vivalda: CDC08, Cancun, December 2008.

## 9.1.5. Editorial activities and scientific committee's memberships

- Tucsnak
  - Associated editor of "SIAM Journal on Control" and of "ESAIM COCV".
  - Member of the Scientific Committee of PICOF'09

# 9.2. Teaching activities

Most of the project members are professors or assistant professors so they have an important teaching activity. We mention here only the graduate courses.

- Non linear analysis of PDE's and applications (Alabau);
- Scientific Computing (Henrot);
- Integral equations (Munnier, Pinçon and Ramdani);
- Semigroups and evolution equations in Hilbert spaces (Tucsnak).

# **10. Bibliography**

# Major publications by the team in recent years

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# **Year Publications**

# **Doctoral Dissertations and Habilitation Theses**

- [10] F. BEN AICHA. Modélisation et commande de systèmes de conversion d'énergie pour l'automobile, Ph. D. Thesis, Université Paul Verlaine - Metz, 2008.
- [11] J. HOUOT. Analyse mathématique des mouvements des rigides dans un fluide parfait, Ph. D. Thesis, Université Henri Poincaré Nancy I, 2008.
- [12] Y. PRIVAT. Quelques problèmes d'optimisation de formes en sciences du vivant, Ph. D. Thesis, Université Henri Poincaré Nancy I, 2008.

[13] T. TAKAHASHI. Analyse théorique, analyse numérique et contrôle de systèmes d'interaction fluide-structure et de systèmes de type ondes, Habilitation à Diriger des Recherches, Nancy Université, 2008.

#### **Articles in International Peer-Reviewed Journal**

- [14] A. AGRACHEV, U. BOSCAIN, M. SIGALOTTI. A Gauss-Bonnet-like formula on two-dimensional almost-Riemannian manifolds, in "Discrete Contin. Dyn. Syst.", vol. 20, n<sup>o</sup> 4, 2008, p. 801–822.
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- [26] T. CHAMBRION, M. SIGALOTTI. *Tracking control for an ellipsoidal submarine driven by Kirchhoff's laws*, in "IEEE Trans. Automat. Control", vol. 53, n<sup>o</sup> 1, 2008, p. 339–349.

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