INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

# *Project-Team DeFI*

# *Shape Reconstruction and Identification*

*Saclay - Île-de-France*

THEME NUM

Activity Report

2008

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*DeFI is is a common project team with Ecole Polytechnique hosted by the Centre de Mathématiques Appliquées (CMAP) at Ecole Polytechnique.*

# 1. Team

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# 2. Overall Objectives

## 2.1. Overall Objectives

The research activity of our team is dedicated to the design, analysis and implementation of efficient numerical methods to solve inverse and/or shape and topological optimization problems in connection with acoustics, electromagnetism, elastodynamics, and waves in general.

Sought practical applications include radar and sonar applications, bio-medical imaging techniques, non-destructive testing, structural design, composite materials, ...

Roughly speaking, the model problem consists in determining information on, or optimizing the geometry (topology) and/or the physical properties of unknown targets from given constraints or measurements, for instance measurements of diffracted waves. In general this kind of problems is non linear. The inverse ones are also severely ill-posed and therefore require special attention from regularization point of view, and non trivial adaptations of classical optimization methods.

Our scientific research interests are three-fold:

- Theoretical understanding and analysis of the forward and inverse mathematical models, including in particular the development of simplified models for adequate asymptotic configurations.
- The design of efficient numerical optimization/inversion methods which are quick and robust with respect to noise. Special attention will be paid to algorithms capable of treating large scale problems (e.g. 3-D problems) and/or suited for real-time imaging.
- Development of prototype softwares for precise applications or tutorial toolboxes.

## 2.2. Highlights

Signature of the EADS-X-INRIA Chair : Mathematical Modeling and Numerical Simulation (MMNS) lead by G. allaire (http://www.cmap.polytechnique.fr/mmnschair/home.html)

# 3. Scientific Foundations

## 3.1. Scientific Foundations

The research activity of our team is dedicated to the design, analysis and implementation of efficient numerical methods to solve inverse and/or shape and topological optimization problems in connection with acoustics, electromagnetism, elastodynamics, and waves in general. We are particularly interested in the development of fast methods that are suited for real-time imaging and/or large scale problems. These goals require to work on both the physical and the mathematical models involved and indeed a solid expertise in related numerical algorithms.

This section intends to give a general overview of our research interests and themes. We choose to present them through the specific example of inverse scattering problems (from inhomogeneities), which will be central in most of foreseen developments. The practical problem would be to identify an inclusion from measurements of diffracted waves that result from the interaction of the sought inclusion with some (incident) waves sent into the probed medium. Typical applications include biomedical imaging where using micro-waves one would like to probe the presence of pathological cells, or imaging of urban infrastructures where using ground penetrating radars (GPR) one is interested in finding the location of buried facilities such as pipelines or waste deposits. This kind of applications requires in particular fast and reliable algorithms.

By "imaging" we shall refer to the inverse problem where the concern is only the location and the shape of the inclusion, while "identification" may also indicate getting informations on the inclusion physical parameters.

Both problems (imaging and identification) are non linear and ill-posed (lack of stability with respect to measurements errors if some careful constrains are not added). Moreover, the unique determination of the geometry or the coefficients is not guaranteed in general if sufficient measurements are not available. As an example, in the case of anisotropic inclusions, one can show that an appropriate set of data uniquely determine the geometry but not the material properties.

These theoretical considerations (uniqueness, stability) are not only important in understanding the mathematical properties of the inverse problem, but also guide the choice of appropriate numerical strategies (which information can be stably reconstructed) and also the design of appropriate regularization techniques. Moreover, uniqueness proofs are in general constructive proofs, i.e. they implicitly contain a numerical algorithm to solve the inverse problem, hence their importance for practical applications. The sampling methods introduced below are one example of such algorithms.

A large part of our research activity is dedicated to numerical methods applied to the first type of inverse problems, where only the geometrical information is sought. In its general setting the inverse problem is very challenging and no method can provide a universal satisfactory solution to it (regarding the balance cost-precision-stability). This is why in the majority of the practically employed algorithms, some simplification of the underlying mathematical model is used, according to the specific configuration of the imaging experiment. The most popular ones are geometric optics (the Kirchhoff approximation) for high frequencies and weak scattering (the Born approximation) for small contrasts or small obstacles. They actually give full satisfaction for a wide range of applications as attested by the large success of existing imaging devices (radar, sonar, echography, X-ray tomography, ...), that rely on one of these approximations.

Generally speaking, the used simplifications result into a linearization of the inverse problem and therefore are usually valid only if the latter is weakly non-linear. The development of these simplified models and the improvement of their efficiency is still a very active research area. With that perspective we are particularly interested in deriving and studying higher order asymptotic models associated with small geometrical parameters such as: small obstacles, thin coatings, wires, periodic media, .... Higher order models usually introduce some non linearity in the inverse problem, but are in principle easier to handle from the numerical point of view than in the case of the exact model.

A larger part of our research activity is dedicated to algorithms that avoid the use of such approximations and that are efficient where classical approaches fail: i.e. roughly speaking when the non linearity of the inverse problem is sufficiently strong. This type of configuration is motivated by the applications mentioned below, and occurs as soon as the geometry of the unknown media generates non negligible multiple scattering effects (multiply-connected and closely spaces obstacles) or when the used frequency is in the so-called resonant region (wave-length comparable to the size of the sought medium). It is therefore much more difficult to deal with and requires new approaches. Our ideas to tackle this problem will be motivated and inspired by recent advances in shape and topological optimization methods and also the introduction of novel classes of imaging algorithms, so-called sampling methods.

The sampling methods are fast imaging solvers adapted to muli-static data (multiple receiver-transmitter pairs) at a fixed frequency. Even if they do not use any linearization the forward model, they rely on computing the solutions to a set of linear problems of small size, that can be performed in a completely parallel procedure. Our team has already a solid expertise in these methods applied to electromagnetic 3-D problems. The success of such approaches was their ability to provide a relatively quick algorithm for solving 3-D problems without any need for a priori knowledge on the physical parameters of the targets. These algorithms solve only the imaging problem, in the sense that only the geometrical information is provided.

Despite the large efforts already spent in the development of this type of methods, either from the algorithmic point of view or the theoretical one, numerous questions are still open. These attractive new algorithms also suffer from the lack of experimental validations, due to their relatively recent introduction. We also would like to invest on this side by developing collaborations with engineering research groups that have experimental facilities. From the practical point of view, the most potential limitation of sampling methods would be the need of a large amount of data to achieve a reasonable accuracy. On the other hand, optimization methods do not suffer from this constrain but they require good initial guess to ensure convergence and reduce the number of iterations. Therefore it seems natural to try to combine the two class of methods in order to calibrate the balance between cost and precision.

Among various shape optimization methods, the Level Set method seems to be partic- ularly suited for such a coupling. First, because it shares similar mechanism as sampling methods: the geometry is captured as a level set of an “indicator function” computed on a caretisian grid. Second, because the two methods do not require any a priori knowledge on the topology of the sought geometry. Beyond the choice of a particular method, the main question would be to define in which way the coupling can be achieved. Obvious strategies consist in using one method to pre-process (initialization) or post-process (find the level set) the other. But one can also think of more elaborate ones, where for instance a sampling method can be used to optimize the choice of the incident wave at each iteration step.The latter point is closely related to the design of so clalled “focusing incident waves” (which are for instance the basis of applications of the time-reversal principle). In the frequency regime, these incident waves can be constructed from the eigenvalue decomposition of the data operator used by sampling methods. The theoretical and numerical investigations of these aspects are still not completely understood for electromagnetic or elastodynamic problems.

Other topological optimization methods, like the homogenization method or the topological gradient method, can also be used, each one provides particular advantages in specific configurations. It is evident that the development of these methods is very suited to inverse problems and provide substantial advantage compared to classical shape optimization methods based on boundary variation. Their applications to inverse problems has not been fully investigated. The efficiency of these optimization methods can also be increased for adequate asymptotic configurations. For instance small amplitude homogenization method can be used as an efficient relaxation method for the inverse problem in the presence of small contrasts. On the other hand, the topological gradient method has shown to perform well in localizing small inclusions with only one iteration.

For the identification problem, one would like to also have information of the physical properties of the targets. Of course optimization methods is a tool of choice for these problems. However, in some applications only a qualitative information is needed and obtaining it in a cheaper way can be performed using asymptotic theories combined with sampling methods.

A broader perspective of our research themes would be the extension of the above mentioned techniques to time-dependent cases. Taking into account data in time domain is important for many practical applications, such as imaging in cluttered media, the design of absorbing coatings or also crash worthiness in the case of structural design.

# 4. Application Domains

## 4.1. Radar and GPR applications

Conventional radar imaging techniques (ISAR, GPR, ...) use backscattering data to image targets. The commonly used inversion algorithms are mainly based on the use of weak scattering approximations such as the Born or Kirchhoff approximation leading to very simple linear models, but at the expense of ignoring multiple scattering and polarization effects. The success of such an approach is evident in the wide use of synthetic aperture radar techniques.

However, the use of backscattering data makes 3-D imaging a very challenging problem (it is not even well understood theoretically) and as pointed out by Brett Borden in the context of airborne radar: “In recent years it has become quite apparent that the problems associated with radar target identification efforts will not vanish with the development of more sensitive radar receivers or increased signal-tonoise levels. In addition it has (slowly) been realized that greater amounts of data - or even additional “kinds” of radar data, such as added polarization or greatly extended bandwidth - will all suffer from the same basic limitations affiliated with incorrect model assumptions. Moreover, in the face of these problems it is important to ask how (and if) the complications associated with radar based automatic target recognition can be surmounted.” This comment also applies to the more complex GPR problem.

Our research themes will incorporate the development, analysis and testing of several novel methods, such as sampling methods, level set methods or topological gradient methods, for ground penetrating radar application (imaging of urban infrastructures, landmines detection, underground waste deposits monitoring, ...) using multistatic data.

## 4.2. Biomedical imaging

Among emerging medical imaging techniques we are particularly interested in those using low to moderate frequency regimes. These include Microwave Tomography, Electrical Impedance Tomography and also the closely related Optical Tomography technique. They all have the advantage of being potentially safe and relatively cheap modalities and can also be used in complementarity with well established techniques such as X-ray computed tomography or Magnetic Resonance Imaging.

With these modalities tissues are differentiated and, consequentially can be imaged, based on differences in dielectric properties (some recent studies have proved that dielectric properties of biological tissues can be a strong indicator of the tissues functional and pathological conditions, for instance, tissue blood content, ischemia, infarction, hypoxia, malignancies, edema and others). The main challenge for these functionalities is to built a 3-D imaging algorithm capable of treating multi-static measurements to provide real-time images with highest (reasonably) expected resolutions and in a sufficiently robust way.

Another important biomedical application is brain imaging. We are for instance interested in the use of EEG and MEG techniques as complementary tools to MRI. They are applied for instance to localize epileptic centers or active zones (functional imaging). Here the problem is different and consists into performing passive imaging: the epileptic centers act as electrical sources and imaging is performed from measurements of induced currents. Incorporating the structure of the skull is primordial in improving the resolution of the imaging procedure. Doing this in a reasonably quick manner is still an active research area, and the use of asymptotic models would offer a promising solution to fix this issue.

## 4.3. Non destructive testing and parameter identification

One challenging problem in this vast area is the identification and imaging of defaults in anisotropic media. For instance this problem is of great importance in aeronautic constructions due to the growing use of composite materials. It also arises in applications linked with the evaluation of wood quality, like locating knots in timber in order to optimize timber-cutting in sawmills, or evaluating wood integrity before cutting trees. The anisotropy of the propagative media renders the analysis of diffracted waves more complex since one cannot only relies on the use of backscattered waves. Another difficulty comes from the fact that the micro-structure of the media is generally not well known a priori.

Our concern will be focused on the determination of qualitative information on the size of defaults and their physical properties rather than a complete imaging which for anisotropic media is in general impossible. For instance, in the case of homogeneous background, one can link the size of the inclusion and the index of refraction to the first eigenvalue of so-called interior transmission problem. These eigenvalues can be determined form the measured data and a rough localization of the default. Our goal is to extend this kind of idea to the cases where both the propagative media and the inclusion are anisotropic. The generalization to the case of cracks or screens has also to be investigated.

In the context of nuclear waste management many studies are conducted on the possibility of storing waste in a deep geological clay layer. To assess the reliability of such a storage without leakage it is necessary to have a precise knowledge of the porous media parameters (porosity, tortuosity, permeability, etc.). The large range of space and time scales involved in this process requires a high degree of precision as well as tight bounds on the uncertainties. Many physical experiments are conducted *in situ* which are designed for providing data for parameters identification. For example, the determination of the damaged zone (caused by excavation) around the repository area is of paramount importance since microcracks yield drastic changes in the permeability. Level set methods are a tool of choice for characterizing this damaged zone.

# 5. Software

## 5.1. Samplings-2D

**Participant:** Houssem Haddar [correspondant].

This software is written in Fortran 90 and is related to forward and inverse problems for the Helmholtz equation in 2-D. It includes three independent components. The first one solves to scattering problem using integral equation approach and supports piecewise-constant dielectrics and obstacles with impedance boundary conditions. The second one contains various samplings methods to solve the inverse scattering problem (LSM, RGLSM(s), Factorization, MuSiC) for near-field or far-field setting. The third component is a set of post processing functionalities to visualize the results. A web page of this software is available at `http://www.cmap.polytechnique.fr/~haddar/`.

## 5.2. Structural Optimization with FreeFem++

**Participants:** Olivier Pantz [correspondant], Grégoire Allaire.

This is a toolbox that contains efficient implementations of shape optimization methods in 2-D using the free finite element software `FreeFem++`. It supports boundary variation methods, homogenization method, and Level Set method. A web page of this toolbox is available at `http://www.cmap.polytechnique.fr/~allaire/freefem_en.html`.

## 5.3. RGLSM-3D & LSM-3D

**Participant:** Houssem Haddar [correspondant].

These Fortran 90 codes are dedicated to the solution of the 3-D electromagnetic inverse scattering problem using respectively, RGLSM and LSM. There are parallel versions of these codes that are coupled to the CESC code (solver for electromagnetic scattering problems using integral equation methods) developed at CERFACS by M'Barek Fares. They also support imaging for doubly layered medium.

## 5.4. Toolbox Conformal mappings

**Participants:** Houssem Haddar [correspondant], Yosra Boukari.

This Scilab toolbox is dedicated to the resolution of inverse 2-D electrostatic problems using the conformal mapping method introduced by Akdumann, Kress and Haddar. The toolbox includes the cases of inclusions with Dirichlet, Neumann or impedance boundary conditions.

# 6. New Results

## 6.1. Sampling methods for inverse scattering problems

### *6.1.1. The RG-LSM method applied to urban infrastructure imaging*

**Participants:** Houssem Haddar, Ozgur Ozdemir, Armin Lechleiter.

The RG-LSM algorithm has been introduced by Colton-Haddar in 2005 as a reformulation of the linear sampling method in the cases where measurements consist of Cauchy data at a given surface, by using the concept of reciprocity gap. The main advantage of this algorithm was to avoid the need of computing the background Green tensor (as required by classical sampling methods) as well as the Dirichlet-to-Neumann map for the probed medium (as required by sampling methods for impedance tomography problems). This method is for instance well suited for medical imaging techniques using microwaves (to detect tumors and malignancies characterized by strong variation in dielectric properties). However, in many other practical applications, like imaging of embedded facilities in the soil or mine detection, the required data at the interface cannot be easily obtained and one has only access to measurements of the scattered wave in the air. In order to overcome this limitation we proposed to couple the RG-LSM algorithm with a continuation method that would provide the Cauchy data from the scattered field. We showed that the obtained scheme has the same convergence properties as RG-LSM with exact data and remains competitive with respect to classical approaches. Preliminary numerical results in a 2-D configuration confirmed these conclusions and also gave further insight on the sampling resolution: Due to the ill-posedness of the first step, only the propagative part of the wave is well reconstructed, which may results in poor approximations of the field. However, the second step (RG-LSM) seems not being affected by this error and therefore is the reconstruction of the target. We are currently extending this approach to the case of rough interfaces and would like to apply these techniques to the 3-D electromagnetic case.

### *6.1.2. Identification of effective dielectric properties using sampling algorithms*

**Participants:** Houssem Haddar, Ridha Mdimegh.

It is well known that sampling algorithms only provide the geometry of the sought inclusion from multi-static data. However in some special cases we showed that a post processing of this algorithm also provides informations on the physical parameters of the inclusions. The first one is the case where the obstacles size is mall compared to the wavelength. It is well known in that case that the center of the inclusion can be uniquely determined from the range of the far field operator associated with the asymptotic expression of the farfields. The MuSiC approach based on projection on the null space of the measurement's matrix or the linear sampling approach based on computing a regularized solutions to a testing equation, can be used to localize these centers. The advantage of the linear sampling method is that the computed solution can be also used to evaluate the Herglotz wave at the sampling point. The latter quantity is shown to be sufficient to determine the effective properties of the inclusions. The advantage of this approach is that it can also generalize to the case of RG-LSM algorithm where instead of the Herglotz wave one evaluates the used approximating operator at the sampling point.

We shall use similar approach to the case of extended targets and anisotropic inclusions.

### 6.1.3. Transmission Eigenvalues and their application to the identification of anisotropic inclusions

**Participants:** Anne Cossonnière, Houssem Haddar.

The so-called interior transmission problem plays an important role in the study of inverse scattering problems from (anisotropic) inhomogeneities. Solutions to this problem associated with singular sources can be used for instance to establish uniqueness for the imaging of anisotropic inclusions from muti-static data at a fixed frequency. It is well known also that the injectivity of the far field operator used in sampling methods is equivalent to the uniqueness of solutions to this problem.

In a first work, in collaboration with F. Cakoni we studied this interior transmission problem in the case of anisotropic Maxwell's equations and with possibly coated parts (that is modeled with an impedance boundary condition). The results extend previous work by Haddar where only the case of anisotropies with norm greater than one is studied, and also show that the set of the eigenvalues for this transmission problem is at most discrete. Showing the existence of these eigenvalues is still an open problem due to the fact that the operator involved is not self-adjoint. However, under some assumptions on the material properties, namely when they are sufficiently small or sufficiently large, one can prove existence of at least one eigenvalue. This result was first obtained by Païvaïrinta-Sylvester in the scalar case and we generalized this results to the vectorial and anisotropic case.

The analysis also showed that some lower bounds can be obtained on these eigenvalues in terms of the inhomogeneity shape and the index norm. This was the starting point of the idea of exploiting the eigenvalues to get information on the index of the sought inhomogeneity. More precisely, using the sampling operator with fixed sampling point and by varying the frequency one can localize the presence of eigenvalues when the norm of the solution becomes large. This procedure has been successfully tested in the case of isotropic circles where the eigenvalues can be computed in terms of some equation involving the Bessel functions. The first eigenvalue can be used to get estimate on the index norm of the inclusion. The procedure has been validated in the scalar case (TM and TE modes) and for the case of a full aperture [5]. Moreover, one can indicate the dependance of this eigenvalues with respect to the presence of default inside the inhomgeneity In the framework of the thesis of Anne Cossonnière we are considering the extension of these results to the cases of limited aperture and anisotropic vectorial case.

## 6.2. Electrostatic imaging using conformal mappings

**Participants:** Yosra Boukari, Houssem Haddar.

In a series of recent papers Akduman, Haddar and Kress have developed a new simple and fast numerical scheme for solving two-dimensional inverse boundary value problems for the Laplace equation that model non-destructive testing and evaluation via electrostatic imaging. In the fashion of a decomposition method, the reconstruction of the boundary shape $\Gamma_0$ of a perfectly conducting or a nonconducting inclusion within a doubly connected conducting medium $D \subset \mathbb{R}^2$ from over-determined Cauchy data on the accessible exterior boundary $\Gamma_1$ is separated into a nonlinear well-posed problem and a linear ill-posed problem. The approach is based on a conformal map $\Psi : B \to D$ that takes an annulus $B$ bounded by two concentric circles onto $D$. In the first step, in terms of the given Cauchy data on $\Gamma_1$, by successive approximations one has to solve a nonlocal and nonlinear ordinary differential equation for the boundary values $\Psi|_{C_1}$ of this mapping on the exterior boundary circle of $B$. Then in the second step a Cauchy problem for the holomorphic function $\Psi$ in $B$ has to be solved via a regularized Laurent expansion to obtain the unknown boundary $\Gamma_0 = \Psi(C_0)$ as the image of the interior boundary circle $C_0$.

As a first step toward solving the impedance tomography problem we considered the case where the solution satisfies an impedance boundary condition on $\Gamma_0$ (which can be seen as an approximation for a transmission problem between two conducting media). In that case the algorithm does not completely decompose the inverse problem into a well-posed nonlinear ordinary differential equation and an ill-posed Cauchy problem.

Consequently its analysis and implementation is more involved and require different tools than those used for the Dirichlet or Neumann case. The first investigations by Haddar and Kress showed that when the impedance is relatively small the algorithm becomes unstable. We also noticed that the instability is also linked to the size of the interior boundary. To overcome these difficulties, we proposed to use the Dirichlet to Neumann operator associated with the conjugate harmonics and proved how this leads to a stabilization (and convergence) of the algorithm for small impedances. However, our analysis is still not completely satisfactory as it does not indicate whether the two algorithms (for large and small impedances) are complementary for all configurations or not. This work in ongoing in collaboration with R. Kress (university of Goettingen) and we plan to treat in the very near future the case of Electrical Impedance Tomography (EIT) problem, with realistic electrode models.

## 6.3. Asymptotic models

### *6.3.1. Interface conditions for time dependent problems and DG methods*

**Participant:** Houssem Haddar.

This work is in collaboration with Sehun Chun and Jan Hesthaven from Brown University. We established transmission conditions for anisotropic media in the case of the 3-D Maxwell's equation and in the framework of the time dependent problems. The interface conditions are well suited to Discontinuous Galerkin method since the latter implicitly support discontinuities between elements. The interface conditions only results into a modification of the numerical flux used in DG methods. These conditions has been successfully tested in the 1D case up the fourth order where stabilization in time has been applied to the fourth order condition. It is also worth noticing that the expression of these conditions in the anisotropic case are far from being intuitive and cannot be simply deduced from the isotropic one by just replacing constant coefficients with their matrix equivalent.

We extended the 1-D case to the 2-D and 3-D ones, where stable conditions are designed for curved geometries up tor order 3 and for flat ones up to order 4. These conditions are numerically validated in the 2-D case.

### *6.3.2. Rough Surface Scattering and Generalized Impedance Boundary Conditions*

**Participants:** Houssem Haddar, Armin Lechleiter.

We studied so-called Generalized Impedance Boundary Conditions (GIBCs) in the context of time-harmonic rough surface and rough layer scattering. In such problems one considers scattering objects like an unbounded hypersurface or an inhomogeneous infinitely extended layer. For a variety of interface and boundary conditions including the GBICs, we showed existence and uniqueness of solution for scattering of acoustic or TE/TM polarized electromagnetic waves from such structures. This result is achieved by Rellich identities yielding explicit a-priori bounds on the solution - those also allow to transfer the asymptotic analysis of GBICs for bounded obstacles to the rough surface setting. Currently under investigation is whether these results can be extended to the full electromagnetic rough layer scattering problem.

### *6.3.3. Identification of generalized impedance boundary conditions in inverse scattering problems*

**Participant:** Houssem Haddar.

This is a joint work with Laurent Bourgeois where we address uniqueness and stability issues related to the identification of a medium impedance from the knowledge of far measurements of a scattered wave at a given frequency. We restrict ourselves in these first investigations to the scalar case (the Helmholtz equation), that models either acoustic waves or two dimensional settings of electromagnetic problems. The medium impedance is understood as a "local" operator that links the Cauchy data of the field $u$ on the medium boundary $\Gamma$. More precisely we shall consider the cases where a boundary condition of the form $\partial u/\partial\nu = Zu \quad \text{on} \quad \Gamma$, is satisfied, where $Z$ is a boundary operator and $\nu$ denotes the normal field on $\Gamma$. The exact impedance corresponds to the so-called Dirichlet-to-Neumann (DtN) map that amounts solving the Helmholtz equation inside the medium. Determining this map is "equivalent" to determining the medium physical properties and in general this is a severely ill-posed problem that requires more than a finite number of measurements.

We are interested here in situations where the operator $Z$ is an approximation of the exact DtN map. In general these approximations correspond to asymptotic models associated with configurations that involve a small parameter. These cases include small amplitude roughness, thin coatings, periodic gratings, highly absorbing media, .... The simplest form is the case where $Z$ is a scalar function, which corresponds in general to the lowest order (non trivial) approximations, for instance in the case of very rough surfaces of highly absorbing media (the Leontovich condition). However, for higher order approximations or in other cases the operator $Z$ may involve boundary differential operators. For instance when the medium contains a perfect conductor coated with a thin layer of width $\delta$ then for TM polarization, the approximate boundary conditions of order 1 corresponds to $Z = 1/\delta$ while for the TE polarization it corresponds to $Z = \delta(\partial_{ss} + k^2 n)$ where $s$ denotes the curvilinear abscissa, $k$ the wave number and $n$ is the mean value of the thin coating index with respect to the normal coordinate. Higher order approximations would include curvature terms or even higher order derivatives. This type of conditions will be referred to as Generalized Impedance Boundary Conditions.

We studied in a first investigations the questions of unique identification and stability of the reconstruction of the operator $Z$ from the knowledge of one scattered wave. One easily sees, from the given example, how the identification of the impedance would provide information on some effective properties of the medium (for instance, the thickness of the coating and the normal mean value of its index). Determining these effective properties would be less demanding in terms of measurements and also more stable than solving the inverse problem with the exact DtN map.

## 6.4. Shape and topological optimization methods

### *6.4.1. The Level Set Method*

**Participant:** Grégoire Allaire.

The typical optimization problem in structural design would be to find the strongest structure having the minimal weight for a given exterior load. Classical optimization methods are based on boundary variations by computing the sensitivity of the structure with respect to the parameters that define the geometry. This classical approach, called geometrical approach, has the main drawback of not allowing a change in the shape topology (number of connected components or holes) during iterations, and therefore preserve the one of the initial guess. However, so-called topological optimization methods are those that allow such a change in the topology. These methods have of course a decisive advantage since quite often one has no a priori knowledge on the optimal shape topology.

Our main recent contributions in the framework of topological optimization methods are the extension of the Level Set Method (introduced by Osher and Sethian for front propagation) to solve optimization problems in structural design. Indeed one of the main attractive part of this method is to allow a change in the topology with simple implementation even for complicated objective functions

### *6.4.2. Post-treatment of the homogenization method*

**Participant:** Olivier Pantz.

The aim of shape optimization is to determine the optimal shape (an open set) that minimize a cost-function. We are interested in the cases where the cost function depends on the solution of PDE set on the domain occupied by the shape. In such cases, there is usually no optimal solution: it is often interesting to create many tiny holes in order to decrease the cost-function. In other words, the minimization sequences "converge" toward a generalized (or composite) shape and not toward a genuine shape (i.e. an open set). To be well posed, the minimization problem has to be relaxed by extending the minimization set to composite shapes. The homogenization method consists in computing the minimizer over this extended admissible shapes. The main problem is that the solution thus obtained is composite whereas we are seeking for a genuine shape. Hence, a post-treatment is needed in order to find a genuine shape that cost is close to the optimal one. The most common method used to this purpose consists in adding a penalization of intermediate material density to the cost-function and to resume the optimization process starting with the composite optimal shape. Unfortunately, this method does not allow to control neither the topological and geometrical complexity of the final shape, nor the additional cost. We have proposed a new method, which consists in building a sequence $\Omega_n$ of genuine shape that "converges" toward the optimal shape. As $n$ increases, the shape $\Omega_n$ presents an increasing number of details whereas its cost converges toward the optimal one

### *6.4.3. Simultaneous shape, topology and homogenization optimization*

**Participants:** Grégoire Allaire, Olivier Pantz.

We are currently working on the development of a new shape optimization algorithm performing geometric, topological and shape optimization in a single setting. Combining geometric and topological optimization is now quite common, but this method can be trapped in local minima. Our aim was to take advantages of the fact that homogenization method often leads ot global minima to enhance the geometric/topological method.

### *6.4.4. Numerical simulation of damage evolution*

**Participant:** Grégoire Allaire.

Together with François Jouve and Nicolas Van Goethem we propose a numerical implementation of the Francfort-Marigo model of damage evolution in brittle materials. This quasi-static model is based, at each time step, on the minimization of a total energy which is the sum of an elastic energy and a Griffith energy release rate. Such a minimization is carried over all geometric mixtures of the two, healthy and damaged, elastic phases, respecting an irreversibility constraint. Numerically, we consider a situation where two well separated phases coexist, and model their interface by a level set function that is transported according to the shape derivative of the minimized total energy. In the context of interface variations (Hadamard method) and using a steepest descent agorithm, we compute local minimizers of this quasi-static damage model. Initially, the damaged zone is nucleated by using the so-called topological derivative. We show that, when the damaged phase is very weak, our numerical method is able to predict crack propagation, including kinking and branching.

## 6.5. Hybrid methods for inverse scattering problems

**Participants:** Grégoire Allaire, Houssem Haddar, Olivier Pantz.

It is well admitted that optimization methods offer in general a good accuracy but are penalized by the cost of solving the direct problem and by requiring a large number of iterations due to the ill-posedness of the inverse problem. However, profiting from good initial guess provided by sampling methods these method would become viable. Among optimization methods, the Level Set method seems to be well suited for such coupling since it is based on capturing the support of the inclusion through an indicator function computed on a cartesian grid of probed media. Beyond the choice of an optimization method, our goal would be to develop coupling strategies that uses sampling methods not only as an initialization step but also as a method to optimize the choice of the incident (focusing) wave that serves in computing the increment step. The “notion” of focusing wave has its origin in the well-known time reversal principle and proving this property can be shown by the Factorization method for normal far-field operators. It would be of great interest to investigate the validity of this property for more general situations (absorption, anisotropy). As a first step tward this objective we are experimenting the coupling in the framework of gradient methods with regularistaion of the geometrical derivative by replacing the $L^2$ scalar product with $H^1$ scalar product. The 2-D results in the case of perfect conductors are encouraging.

# 7. Contracts and Grants with Industry

## 7.1. DGA

**Participants:** Houssem Haddar, Ozgur Ozdemir, Armin Lechleiter.

This contract provides financial support to the Post-Docs of Dr. Ozdemir and Dr. Lechleiter on imaging of buried facilities using sampling algorthims.

### 7.2. EADS Foundation

**Participants:** Anne Cossonnière, Houssem Haddar.

A joint proposal DEFI-CERFACS has obtained 100k euros for the development of reduced models associated with scattering problems at moderate frequencies. These models are expected to provide reliable signature of the scatterers that serves the identification problem. The money serves as financial support for the PhD thesis of A. Cossonnière.

## 8. Other Grants and Activities

### 8.1. National Actions

- Grégoire Allaire is a scientific adviser at CEA Saclay.
- Grégoire Allaire is the leader of the EADS-X-INRIA Chair Mathematical Modeling and Numerical Simulation (MMNS): `http://www.cmap.polytechnique.fr/mmnschair/home.html`.

### 8.2. International Initiatives

- Closing of the european network (RTN Marie-Curie) MULTIMAT in september 2008 http://webh01.ua.ac.be/multimat/. A second network (OPTIMAT) is proposed and is under evaluation.
- Associated team Inverse Scattering and Identification Problems (ISIP) between the mathematical Department of the University of Delaware and the DEFI team has been created January 2008 and renewed for 2009 `http://www.cmap.polytechnique.fr/~defi/Prolong-EA-ISIP-09.html`.
- Since 2008, a Stic project DGRST(Tunisie)/INRIA *Méthodes innovantes en imagerie et en contrôle non destructif des structures* between the DeFI team and the LAMSIN has been created and serves with the ENEE associated team ((ENcéphalographie-Epidémiologie-Electronique) as a financial support for our common PhD students.
- Since 2009 a joint proposal with O. Ozdemir from the electromagnetics research group of ITU (Turkey) on “The use of generalized impedance boundary conditions for buried objects imaging and for coatings non destructive testing” has obtained financial support up to 14000 euros from the Turkish National Science Foundation (TUBITAK). The money will serve for PhD students and scientific short visits.

### 8.3. Exterior research visitors

- R. Kress from the university of Goettingen, June 9 to June 15, 2008.
- F. Cakoni and D. Colton from the university of Delaware, August 17 to August 24, 2008.
- M. Khenissi from LAMSIN, University of Monastir, November 30 to December 13, 2008.
- P. Monk from the university of Delaware, December 7 to December 14, 2008.
- F. Ben Hassen from LAMSIN, ENIT, December 16 to December 23, 2008.

## 9. Dissemination

### 9.1. Scientific Community Animation

- G. Allaire is member of the editorial board of ESAIM/COCV (Control, optimization, and calculus of variations), Structural and Multidisciplinary Optimization, SIAM/MMS (Multiscale Modeling and Simulation), Annales de la Faculté des Sciences de Toulouse, Discrete and Continuous Dynamical Systems Series B, Computational and Applied Mathematics, Mathematical Models and Methods in Applied Sciences (M3AS).
- G Allaire is member of the Conseil National des Universités (26ème section, mathématiques appliquées), member of the administrative committee of SMAI and president of GAMNI/SMAI (Groupement pour l'Avancement des Méthodes Numériques pour l'Ingénieur).
- Organisation (jointly with University of Genova) of the International Conference on Inverse Scattering Problems, honoring D. Colton & R. Kress at Sestri Levante (8th-10th May, 2008): http://www.cmap.polytechnique.fr/~haddar/ckconf/welcome.html

## 9.2. Teaching

G. Allaire

- Course “Analyse Numérique et d'Optimisation”, Ecole Polytechnique (MAP 431, 2ème année)
- Course “Conception optimale des structures”, Ecole Polytechnique (MAP 562, Programme d'Approfondissement SISM en 3ème année)
- Master M2 , AN&EDP, University of Paris 6. Module "Analyse théorique et numérique des systèmes hyperboliques de lois de conservation" in collaboration with F. Coquel.

H. Haddar

- MASTER M2, AN&EDP, University of Paris 6. Module “Problèmes directs et inverses en diffraction” in collaboration with P. Joly.
- Course “Analyse Numérique et d'Optimisation”, Ecole Polytechnique (MAP 431, 2ème année)

O. Pantz

- Course “Analyse Numérique et d'Optimisation”, Ecole Polytechnique (MAP 431, 2ème année)
- Course “Conception optimale des structures”, Ecole Polytechnique (MAP 562, Programme d'Approfondissement SISM en 3ème année)

## 9.3. Seminars, Conferences, Visits

G. Allaire

- Franco-Korean Days of Mathematical Analysis and Its Applications, IHP, Paris (14-15 February 2008).
- 4ème Colloque sur les Problèmes Inverses, le Contrôle et l'Optimisation de Formes (PICOF), Marrakech (16-18 April 2008).
- Ecole thématique du GDR CHANT, Roscoff (August 2008).
- Scaling Up and Modeling for Transport and Flow in Porous Medi, Dubrovnik (13-16 October 2008).
- Jahrestagung der DMV, Erlangen (15-19 September 2008).
- Santiago du Chili (December 2008), Tokyo et Kyoto (November 2008), Maryland (November 2008), GDR MOMAS, Lyon (November 2008).

R. Boukari

– 9éme Colloque Africain sur la Recherche en Informatique et en Mathématiques Appliquées, 27-30 octobre 2008, Rabat, Maroc.

H. Haddar

– ICISP conference, Sestri Levante, Italy, 8-10 May 2008.

– IRMAR, University of Rennes, January, 2008.

– Institut Elie Cartan, University of Nancy, January, 2008.

– University of Goettingen, 14-18 January 2008. Mini-course on *Conditions d'impédances géneralisées en électromagnétisme*

A. Lechleiter

– Organized minisymposium at the SIAM Annual Meeting/ SIAM Conference on Imaging Science in San Diego, July 7-11, 2008, together with Nuutti Hyvonen.

– Invited lecturer at the Oberwolfach seminar Mathematics of Photonic Crystals, Oberwolfach, October 12-18, 2008.

– Contributed talk to GDR ONDES, december 5, 2008, ESPCI, Paris.

R. Mdimagh

– 4ème Colloque sur les Problèmes Inverses, le Contrôle et l'Optimisation de Formes (PICOF), Marrakech (16-18 April 2008).

– 9éme Colloque Africain sur la Recherche en Informatique et en Mathématiques Appliquées, 27-30 octobre 2008, Rabat, Maroc.

O. Ozdemir

– University of Marseille, 24-30 March, 2008.

– ICISP conference, Sestri Levante, Italy, 8-10 May 2008.

# 10. Bibliography

## Year Publications

### Articles in International Peer-Reviewed Journal

[1] G. ALLAIRE, F. JOUVE. *Minimum stress optimal design with the level set method*, in "International Journal of Boundary Elements and Applications", to appear, 2008.

[2] G. ALLAIRE, F. DE GOURNAY, F. JOUVE. *Shape and topology optimization of the robust compliance via the level set method*, in "COCV", n$^{o}$ 14, 2008, p. 43–70.

[3] T. ARENS, A. LECHLEITER. *The Linear Sampling Method Revisited*, in "J. Integral Equations and Applications", to appear, 2008.

[4] F. CAKONI, H. HADDAR, M. PIANA. *Editorial: Special Issue on Inverse Scattering Problems dedicated to D. Colton and R. Kress*, in "Inverse Problems and Imaging", to appear, 2008.

[5] C. FIORALBA, D. COLTON, H. HADDAR. *The computation of lower bounds for the norm of the index of refraction in an anisotropic madia*, in "J. Integral Equations and Applications", to appear, 2008.

[6] C. FIORALBA, H. HADDAR. *On the existence of transmission eigenvalues in an inhomogeneous medium*, in "Applicable Analysis", to appear, 2008.

[7] H. HADDAR, P. JOLY, H.-M. NGUYEN. *Generalized impedance boundary conditions for scattering problems from strongly absorbing obstacles: the case of Maxwell's equations*, in "M3AS", n$^{o}$ 10, 2008, p. 1787–1827.

[8] H. HADDAR, J.-R. LI, D. MATIGNON. *Efficient solution of a wave equation with fractional order dissipative terms*, in "J. Comput. Appl. Math.", to appear, 2008.

[9] H. HADDAR, O. OZGUR. *Pre-Processing the Reciprocity Gap Sampling Method for Imaging Buried Objects*, in "IEEE Ant. trans.", to appear, 2008.

[10] A. LECHLEITER. *The Factorization Method is Independent of Transmission Eigenvalues*, in "Inverse Problems and Imaging", to appear, 2008.

[11] A. LECHLEITER, A. RIEDER. *Newton regularization for impedance tomography: convergence by local injectivity*, in "Inverse Problems", vol. 24, 2008.

[12] O. PANTZ. *The modeling of deformable bodies with frictionless (self-)contacts*, in "Archive Rat. Mech. Anal.", vol. 188, n$^{o}$ 2, 2008, p. 183–212.

[13] O. PANTZ, K. TRABELSI. *A post-treatment of the homogenization method for shape optimization*, in "SIAM Journal on Control and Optimization", vol. 47, n$^{o}$ 3, 2008, p. 1380–1398.

## International Peer-Reviewed Conference/Proceedings

[14] G. FACCANONI, S. KOKH, G. ALLAIRE. *Numerical simulation with finite volume of a dynamic liquid-vapour phase transition*, in "Finite volumes for complex applications V, Problems and perspectives", 2008, p. 391–398.

[15] H. HADDAR, R. MDIMAGH. *Identification of small inclusions from multistatic data using the reciprocity gap concept*, in "Proceedings of the CARI'08 conference", 2008, p. 339–343, http://www.cari-info.org/.

## Scientific Books (or Scientific Book chapters)

[16] G. ALLAIRE, S. KABER. *Numerical Linear Algebra*, Texts in Applied Mathematics, vol. 55, Springer, 2008.

## Research Reports

[17] L. BOURGEOIS, H. HADDAR. *Identification of generalized impedance boundary conditions in inverse scattering problems*, Technical report, n$^{o}$ RR-6786, INRIA, 2008, http://hal.inria.fr/inria-00349258/fr/.

[18] F. CAKONI, H. HADDAR. *On the existence of transmission eigenvalues in an inhomogeneous medium*, (will appear as an article in Applicable Analysis), Technical report, n$^{o}$ RR-6779, INRIA, 2008, http://hal.inria.fr/inria-00347840/fr/.