INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team galaad

Géométrie, Algèbre, Algorithmes

Sophia Antipolis - Méditerranée

THEME SYM

Activity Report

2008

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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable solution of geometric and algebraic problems, which are encountered in fields such as CAGD (Computer Aided Geometric Design), robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry and computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology computation, and questions related to algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical computation, usually opposed to symbolic computation, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: modeling, computing and analysis, in connection with effective algebraic geometry.

3.2. Algebraic Geometric Modeling

We are investigating geometric modeling approaches, based on non-discrete models, mainly of semi-algebraic type. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to handle and solve the underlying non-linear problems.

Effective algebraic geometry is a naturally framework for handling such representations, in which we are developing new methods to solve these non-linear problems. The framework not only provides tools for modeling but also, it makes it possible to exploit the geometric properties of these algebraic varieties, in order to improve this modeling work. To handle and control geometric objects such as parameterised curves and surfaces or their implicit representations, we consider in particular projections techniques. We focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials. These tools allows to change geometric representations, computing an implicit model from a parameterised one. We are interested in dedicated methods for solving these type of problems.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately the algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations. The use of local algebraic representations also raises problems of approximation and reconstruction, on which we are working on.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. In addition to the development of symbolic geometric computations that exploit these invariant properties, we are also interested in developing compact representations of shapes, based on algebraic/symbolic descriptions. Our aim is to improve geometric computing performances, by using smaller input data, with better properties of approximation and certified computation.

3.3. Algebraic Geometric Computing

The underlying representation behind the geometric model that we consider are often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases. We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

We are also interested in subdivision methods, which are able to localise efficiently the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue in analysing these methods is how to obtain good complexity bounds by exploiting the structure of the problem. Many algebraic problems can be reformulated in terms of linear algebra questions. Thus, it is not surprising to see that complexity analysis of our methods leads to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and their implications in solving polynomial equations.

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. The problem of decomposition and factorisation is thus also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.4. Algebraic Geometric Analysis

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

4. Application Domains

4.1. Shape design

Keywords: *engineering computer-assisted*, *geometric modeling*.

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, ...Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts that we are developping in Computer Aided Geometric Design.

4.2. Shape approximation

Keywords: *approximation*, *engineering*, *reconstruction*.

Many problems encounter in the application of computer sciences started from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where *Distance geometry* plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tends to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems.

5. Software

5.1. Mathemagix, a free computer algebra environment

Keywords: *algebra*, *compiler*, *fast algorithm*, *hybrid software*, *interpreter*, *matrices*, *multivariate polynomial*, *series*, *univariate polynomial*.

Participants: Grégoire Lecerf, Bernard Mourrain [contact person], Daouda N'Diatta, Olivier Ruatta, Joris van der Hoeven, Philippe Trébuchet, Elias Tsigaridas, Julien Wintz.

http://www.mathemagix.org/

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments are based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It supported by the ANR GECKO and is structuring collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- SUBDIVIX: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms;
- REALROOT: exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; Algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

5.2. Axel, a geometric modeler for algebraic objects

Keywords: *computational algebraic geometry*, *curve*, *implicit equation*, *intersection*, *parameterisation*, *resolution*, *singularity*, *surface*, *topology*.

Participants: Stéphane Chau, Bernard Mourrain, Jean-Pascal Pavone, Julien Wintz [contact person].

http://axel.inria.fr.

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

The application contains data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This package is now distributed as binary packages as well for Linux as for MacOSX. It is hosted at the Inria's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. By the beginning of November the software has been downloaded more than 10000 times.

5.3. Multires, a Maple package for multivariate resolution problems

Keywords: *eigenvalues*, *interpolation*, *linear algebra*, *polynomial algorithmic*, *residue*, *resultant*.

Participants: Laurent Busé [contact person], Ioannis Emiris, Bernard Mourrain, Olivier Ruatta, Philippe Trébuchet.

http://www-sop.inria.fr/galaad/logiciels/multires/.

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, but not their effectiveness, which is achieved in a more adapted environment as SYNAPS. It provides methods for building matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay, Jouanolou for projective resultant, Bezout and (sparse) resultant on a toric variety, residual resultant of a complete intersection, functions for computing the degree of residual resultant, algorithms for the geometric decomposition of an algebraic variety. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residue for a complete intersection of dimension 0.

6. New Results

6.1. Algebraic Geometric Modeling

6.1.1. Implicitization by means of linear and quadratic syzygies

Participants: Laurent Busé, Marc Chardin [Univ. Paris VI], Aron Simis [Univ. Recife].

This work started with the visit of Marc Chardin and Aron Simis by mid-october. Many results have already been obtained regarding the implicitization problem by using the linear syzygies of the parameterization. These syzygies have been preferred to others because they are very simple to compute and because they form the equations of the symmetric algebra of the ideal generated by the parameterization. Many experimental computations shows that one could take advantage of adding the quadratic syzygies to the linear ones, especially because they allow to decrease the regularity degree in which we have to construct the implicitization matrices. The aim of this starting collaboration is to give a general framework for introducing quadratic syzygies in the study of the implicitization problem.

6.1.2. Matrix representations for toric parametrizations

Participants: Nicolas Botbol [Univ. Buenos Aires], Marc Dohm, Alicia Dickenstein [Univ. Buenos Aires].

In this work, we show that a surface in the projective space parametrized over a 2-dimensional toric variety T can be represented by a matrix of linear syzygies if the base points are finite in number and locally form a complete intersection. This constitutes a direct generalization of the corresponding known result over the projective plane. Exploiting the sparse structure of the parametrization, we obtain significantly smaller matrices than in the homogeneous case and the method becomes applicable for parametrizations which previously failed. We also treat in detail the important case where T is a product of two projective lines and give numerous examples.

A paper has been submitted for publication. This research activity is related to the ECOS-Sud collaboration program with the University of Buenos Aires.

6.1.3. On the equations of the moving curve ideal

Participant: Laurent Busé.

Given a parametrization of a rational plane algebraic curve $\mathcal{C}$, some explicit adjoint linear systems on $\mathcal{C}$ are described in terms of determinants. Moreover, some generators of the Rees algebra associated to this parametrization are presented. The main ingredient developed in this work is a detailed study of the elimination ideal of two homogeneous polynomials in two homogeneous variables that form a regular sequence. Finally, a generalization of Abhyankar's Taylor-resultant from the polynomial parameterization to the rational parameterization is given in terms of a certain subresultant of a basis of the syzygies of the parameterization. This work, started in 2007, has been finished and submitted for publication.

6.1.4. Polytopes and implicit equations

Participants: Alicia Dickenstein [Univ. Buenos Aires], Angelos Mantzaflaris, Bernard Mourrain.

Describing the implicit equation defining the image of a polynomial map is practical importance in geometric modeling problems where, for instance, spacial localisation are required. For this purpose, we are interested in tools to improve this change of representation. In work in collaboration with A. Dickenstein conducted through ECOS-Sud collaboration program, we describe the Newton polytope of the implicit equation of parametric surfaces in terms of the Newton polytopes and the singularities of the parametrization.

Manipulating newton polytopes, for example geometric operations such as computation of normals, faces, ridges, extreme or interior points and Minkowski sums naturally arise in toric elimination theory and have important algebraic interpretation. We have implemented these operations in MATHEMAGIX, in the frame of the new module *Polytopix*. Furthermore, we provided a connection to AXEL Modeler via the plugin system in order to visualize polytopes up to dimension three.

6.1.5. Tree reconstructions from scanner point clouds

Participants: Fréderic Boudon [Virtual plants], Christophe Godin [Virtual plants], Bernard Mourrain, Julien Wintz, Wenping Wang [Hong Kong Univ.], Donming Yuan [Hong Kong Univ.].

In this work, we consider the problem of reconstructing complete branching systems faithful to observed tree from 3D laser scans. Since branches may be hidden by leaves, we restrict our approach to trees without leaves (e.g. temperate species observed in winter for example). Our goal is to obtain a compact model that reflects with accuracy the observed branching system. In particular, from the clouds of points obtained from the 3D laser scans, the method should be able to:

- Identify the different plant axes. Ambiguity in axis recognition may come from two main phenomena i) different axes may have parts that are close to each other in the 3D space, ii) parts of the axes may be hidden by other axes.
- Identify axis connections. Connections between axes may also be hidden or partially sampled, making it difficult to guess the correct hierarchy of axes.
- Estimate axis geometry. One key issue of the reconstruction is to recover the axis diameter variation from the partial sampling of the axis surface (each scanned imaged only contains a part of this surface).

We propose a chain of algorithms that is aimed to meet these requirements. A paper describing the approach is in preparation. The work has been partially supported by the PAI Procore collaboration program with the Hong Kong University.

6.1.6. Modeling fibers from Diffusion MRI

Participants: Aurorata Ghosh, Elias P. Tsigaridas, Maxime Descoteaux, Pierre Comon [I3S], Rachid Deriche, Bernard Mourrain.

In Diffusion MRI, the Orientation Distribution Function (ODF) represents a state of the art reconstruction method for detecting fiber bundle crossings and inferring their orientational information. The maxima of the ODF being aligned with the directions of the dominant fiber bundles allows for the identification of the fiber bundles. To be able to correctly estimate these maxima is, therefore, of utmost importance. The ODF is a real antipodally symmetric spherical function.

In [23], we first show how it is possible to represent a real antipodally symmetric spherical function as a constrained polynomial, which is a well studied problem. We then present a simple analytic solution to extract the maxima of the constrained polynomial by solving a system of polynomials. Finally, we work on the ODF as an example and extract its maxima.

6.2. Algebraic Geometric Computing

6.2.1. Univariate solver

Participants: Ioannis Z. Emiris [Univ. Athens], Elias P. Tsigaridas.

In [20], we present exact and complete algorithms, based on precomputed Sturm-Habicht sequences, discriminants and invariants, in order to classify, isolate by means of rational points, and compare the real roots of polynomials of degree ≤ 4. In particular, we establish closed formulas for all isolating points in terms of the input polynomial coefficients. Although these algebraic numbers can be expressed by radicals, this requires roots of complex numbers. This is both inefficient as well as hard to handle for applications in geometric computing and quantifier elimination. We also define rational isolating points between the roots of the quintic. We combine these results with a simple version of Rational Univariate Representation so as to isolate and compute the multiplicity of all common real roots of a bivariate system of rational polynomials of total degree ≤ 2. We present our software within SYNAPS, and perform experiments and comparisons with several public-domain implementations. Our package is 2–10 times faster than numerical methods and exact subdivision-based methods, including software with intrinsic filtering.

6.2.2. *Symbolic-Numeric methods for solving polynomial systems*

Participants: Bernard Mourrain, Philippe Trébuchet [Univ. Paris 6].

Our primal goal is to develop stable approaches for solving polynomial equations with approximate coefficients. We describe and analyze a method for computing border bases of a zero-dimensional ideal I. The criterion used in the computation involves in specific commutation polynomials and leads to an algorithm and an implementation extending the one provided a previous paper presented at the conference ISSAC'05. This general border basis algorithm weakens the monomial ordering requirement for Gröbner bases computations. Up to date, it is the most general setting for representing quotient algebras, embedding into a single formalism Gröbner bases, Macaulay bases and new representation that do not fit into the previous categories. With this formalism, we show how the syzygies of the border basis are generated by commutation relations. We also show that our construction of normal form is stable under small perturbations of the ideal when the number of solutions remains constant. This new feature for a symbolic algorithm has a huge impact on the practical efficiency as it is illustrated by the experiments on classical benchmark polynomial systems. This work is published in [21].

6.2.3. *Perturbation of quotient algebra and flatness criteria*

Participants: Mari-Emi Alonso, Jerome Brachat, Bernard Mourrain.

Solving polynomial equations with approximate coefficients is ubiquitous in many applications. It is also a challenge from an effective algebraic geometry point of view. To tackle this issue, we have developed border basis methods, which are more stable from a numerical point of view.

In order to make this result more precise and to quantify it, we investigate the effect of numerical perturbations of this border basis representation of quotient algebras. One of the motivations is to be able to improve the numerical quality of a quotient representation, after an approximate computation of such a border basis at a given precision. We describe an explicit Newton-type iteration for this purpose. We give effective criteria to check flatness or the stability of a deformation. The connection with Hilbert scheme of points is exploited using explicit equations for this algebraic variety of quotient algebra of a given degree.

A paper describing these results is in preparation.

6.2.4. *Moment matrices and trace for computing with radical ideals*

Participants: Inuit Janovitz-Freireich [North Carolina University], Bernard Mourrain, Lajos Rónyai [Budapest University of Technology and Economics], Agnes Szàntò [North Carolina University].

Let $f_1, ..., f_s \in \mathbb{K}[x_1, ..., x_m]$ be a system of polynomials generating an ideal I, where $\mathbb{K}$ is an arbitrary algebraically closed field. Assume that the factor algebra $\mathcal{A} = \mathbb{K}[x_1, ..., x_m]/I$ is Gorenstein and that we have a bound $\delta > 0$ such that a basis for $\mathcal{A}$ can be computed using multiples of $f_1, ..., f_s$ of degrees at most δ.

Using Sylvester or Macaulay type resultant matrices of $f_1, ..., f_s$ and F, where F is a polynomial of degree δ generalizing the Jacobian, we propose a method to compute moment matrices and, in particular, matrices of traces for $\mathcal{A}$. These matrices of traces in turn allow us to compute a system of multiplication matrices $\{M_{x_i} | i = 1, ..., m\}$ of the radical $\sqrt{I}$, following the approach in previous work by Janovitz-Freireich, Rónyai and Szántó. Additionally, we give bounds for δ for the case where I has finitely many projective roots in $\mathbb{P}^m_{\mathbb{C}}$. Moreover, for an affine zero-dimensional complete intersection, we show how to directly generate the radical of the ideal using Bezoutian matrices associated to the system and its Jacobian. This work is published in [24].

6.2.5. A computational approach to the singularities of a rational plane curves

Participants: Laurent Busé, Carlos D'Andrea [Univ. Barcelona].

The purpose of this collaboration is to give an algorithm that allows us to compute most of the information of the singularities of a rational algebraic curve from one of its proper parameterization. Abhyankar's Taylor resultant provides the δ-invariant of a singular point P, that is to say, the sum of the multiplicity of each singular point which is infinitely near to P. In this collaboration, we are trying to separate this information and get the multiplicity of each singular point, distinct or infinitely near. It is not yet finished and was supported by a PAI Picasso with the University of Barcelona.

6.2.6. Intersecting curves and surfaces using matrix-based representation

Participants: Laurent Busé, André Galligo, Luu Ba Thang.

Luu Ba Thang is a new PhD student in our team. He started to look at matrix-based representations of parameterized surfaces since many techniques has been developed in the team for that purpose. Its main focus will be one the use of this representation to compute the intersection of such a surface with a parameterized space curve.

6.2.7. Determinantal resultants

Participants: Elimane Ba, Laurent Busé.

The aim of this work is to develop the theory of determinantal resultant over an arbitrary commutative ring. Such a theory amounts to study the inertia forms of a determinantal ideal and will offer many effective tools to handle them. We are expecting to obtain many formal properties to compute these resultants, namely base change formula, multplicativity, and so on.

6.2.8. Towards toric absolute factorization

Participants: Mohamed Elkadi, André Galligo, Martin Weimann.

This work presents an algorithmic approach to study and compute the absolute factorization of a bivariate polynomial, taking into account the geometry of its monomials. It is based on algebraic criteria inherited from algebraic interpolation and toric geometry. It has been accepted for publication in the journal of Symbolic Computation, special issue MEGA 2007.

6.2.9. Las Vegas algorithms with modular computations for the absolute factorization of bivariate integer polynomials

Participants: Guillaume Chèze [Univ. Toulouse], André Galligo.

Let $f(X, Y) \in \mathbb{Z}[X, Y]$ be an irreducible polynomial over $\mathbb{Q}$. We study the absolute factorization $f_1 \cdots f_s$ of f. First we give a Las Vegas absolute irreducibility test based on a property of the Newton polygon of f. Then thanks to this test we give a new strategy based on modular computations and LLL which gives the absolue factorization.

We expect to present the results at MEGA 2009.

6.2.10. Algorithms for irreducible decomposition of curves in $\mathbb{C}^n$

Participants: Cristina Bertone, André Galligo.

The aim of this work is constructing an effective method for computing the irreducible components of a curve of the complex n-dimensional space, eventually starting from the existing algorithms for decomposition of curves in the plane $\mathbb{C}^2$, i.e. absolute factorization of polynomials. In the case of curves in $\mathbb{C}^3$, using a generic change of coordinates and information coming from a general planar section of the curve, we can detect the existence of non-trivial irreducible components; we then get some polynomials separating the components on the plane. We would like to lift them to surfaces separating the components of the curve in the 3-dimensional space.

6.2.11. Approximate bivariate factorization using monodromy computations

Participants: André Galligo, Adrien Poteaux.

We study strategies for designing algorithms which factor bivariate approximate polynomials in $\mathbb{C}[x, y]$. The idea is to detect the factors of the polynomial by some analytic continuation process.

At the moment, we are studying the behavior of monodromy groups of generic polynomials. Thus, we have to compute these monodromy groups. Unfortunately, the Maple command `algcurves[monodromy]` is not reliable. Thus, we have finalized the Maple implementation of algorithms described in Adrien Poteaux's PhD thesis, which includes a new symbolic-numeric algorithm to compute numerical approximations of Puiseux series, which are used in a new strategy for computing the monodromy group of a plane algebraic curve. These algorithms enable us to get some monodromy groups of generic polynomials, and we now study the behaviour of such groups.

This work is supported by the ANR GECKO program.

6.2.12. Solving Toeplitz-block linear systems

Participants: Houssam Khalil, Bernard Mourrain, Michelle Schatzmann [Univ. Lyon I].

Structured matrices appear in various domains, such as scientific computing, signal processing, ...Among well-known structured matrices, Toeplitz and Hankel structures have been intensively studied. So far, few investigations have been pursued for the treatment of multi-level structured matrices.

In a paper submitted for publication, we re-investigate the resolution of Toeplitz systems $T\,u = g$ from a new point of view, by correlating the solution of such problems with syzygies of polynomials or moving lines. We show an explicit connection between the generators of a Toeplitz matrix and the generators of the corresponding module of syzygies. We show that this module is generated by two elements of degree n and the solution of $T\,u = g$ can be reinterpreted as the remainder of an explicit vector depending on g by these two generators. This approach naturally extends to multivariate problems and we describe the structure of the corresponding generators for Toeplitz-block-Toeplitz matrices.

These results are described in [14], where algorithms of complexity in $O(n^3/2)$ to solve Band Toeplitz of Band Toeplitz Blocs linear systems are given, as well as a new method for solving standard Toeplitz systems.

6.3. Algebraic Geometric Analysis

6.3.1. Explicit factors of some iterated resultants and discriminants

Participants: Laurent Busé, Bernard Mourrain.

We analyze the result of applying iterative univariate resultant constructions to multivariate polynomials. We consider the input polynomials as generic polynomials of a given degree and exhibit explicit decompositions into irreducible factors of several constructions involving two times iterated univariate resultants and discriminants over the integer universal ring of coefficients of the entry polynomials. Cases involving from two to four generic polynomials and resultants or discriminants in one of their variables are treated. The decompositions into irreducible factors we get are obtained by exploiting fundamental properties of the univariate resultants and discriminants and induction on the degree of the polynomials. As a consequence, each irreducible factor can be separately and explicitly computed in terms of a certain multivariate resultant. With this approach, we also obtain as direct corollaries some results conjectured by Collins and McCallum which correspond to the case of polynomials whose coefficients are themselves generic polynomials in other variables. Finally, a geometric interpretation of the algebraic factorization of the iterated discriminant of a single polynomial is detailed. A paper has been accepted for publication is the Mathematics of Computations journal (to appear in 2009).

6.3.2. On the total order of reducibility of a pencil of algebraic plane curves

Participants: Laurent Busé, Guillaume Chèze [Univ. Toulouse].

The problem of bounding the number of absolutely reducible curves in a pencil of algebraic plane curves is adressed. Unlike most of the previous related works, each reducible curve of the pencil is here counted with its appropriate multiplicity. It is proved that this number of reducible curves, counted with multiplicity, is bounded by $d^2 - 1$ where d is the degree of the pencil. Then, a sharper bound is given by taking into account the Newton's polygon of the pencil. Finally, an effective criterion to determine whether a pencil of curves is composite is proposed.

6.3.3. Complexity bounds on local Betti numbers

Participants: Lionel Alberti, Georges Comte.

We gave a new bound on the Betti numbers of the section of a germ by a generic affine space. The bound is polynomial in the multiplicity of the germ and exponential in the dimension of the ambient space. Contrary to the complex case, those two parameters are not always enough to bound this number of connected components. The result is thus proved under some conditions. We show that those conditions are optimal in a precise sense by giving counter-examples where the multiplicity is constant while the number of connected components diverges to infinity. An article on this work, also described in the last chapter of L. Alberti Ph.D. thesis, is in preparation.

6.3.4. On the distribution of the solutions of systems of polynomial equations

Participants: Carlos d'Andrea, André Galligo, Martin Sombra.

We generalize results on the distribution of roots of univariate polynomials to the sparse multivariate case, using generalized resultants.

6.3.5. Subdivision method for the topology and arrangement of algebraic curves

Participants: Lionel Alberti, Bernard Mourrain, Julien Wintz.

In [16], we describe a new subdivision method to efficiently compute the topology and the arrangement of implicit planar curves. We emphasize that the output topology and arrangement are guaranteed to be correct. Although we focus on the implicit case, the algorithm can also treat parametric or piecewise linear curves without much additional work and no theoretical difficulties.

The method isolates singular points from regular parts and deals with them independently. The topology near singular points is guaranteed through topological degree computation. In either case the topology inside regions is recovered from information on the boundary of a cell of the subdivision. Obtained regions are segmented to provide an efficient insertion operation while dynamically maintaining an arrangement structure.

We use enveloping techniques of the polynomial represented in the Bernstein basis to achieve both efficiency and certification. It is finally shown on examples that this algorithm is able to handle curves defined by high degree polynomials with large coefficients to identify regions of interest and use the resulting structure for either efficient rendering of implicit curves, point localization or boolean operation computation.

6.3.6. Topology of algebraic space curves and surfaces

Participants: Daouda N'Diatta, Bernard Mourrain, Olivier Ruatta [Univ. Limoges].

In [22], a new algorithm is presented for the computation of the topology of a non-reduced space curve defined as the intersection of two implicit algebraic surfaces. It computes a Piecewise Linear Structure (PLS) isotopic to the original space curve. The algorithm is designed to provide the exact result for all inputs. It's a symbolic-numeric algorithm based on subresultant computation. Simple algebraic criteria are given to certify the output of the algorithm. The algorithm uses only one projection of the non-reduced space curve augmented with adjacency information around some "particular points" of the space curve.

We also extend this algorithm to the computation of the topology of an algebraic surface by analysing the polar curve of this surface for a generic direction. A paper on this new algorithms for algebraic surfaces is in preparation.

Both algorithms for computing the topology of an algebraic curve or surface have been implemented with the Mathemagix Computer Algebra System (CAS).

6.3.7. Tensors decompositions and rank

Participants: Jérome Brachat, Pierre Comon [I3S], Gene Golub [Stanford Univ.], Lek-Heng Lim [Stanford Univ.], Bernard Mourrain, Elias P. Tsigaridas.

A symmetric tensor is a higher order generalization of a symmetric matrix. In [19], we study various properties of symmetric tensors in relation to a decomposition into a sum of outer product of vectors. A rank-1 order-k tensor is the outer product of k vectors. Any symmetric tensor can be decomposed into a linear combination of rank-1 tensors, each of them being symmetric or not. The rank of a symmetric tensor is the minimal number of rank-1 tensors that are necessary to reconstruct it.

In a recent work submitted for publication, we present a new algorithm for computing such a decomposition as a sum of rank-1 symmetric tensors, extending the algorithm of Sylvester devised in 1886 for symmetric tensors of dimension 2.

We exploit the fact that every symmetric tensor is equivalently represented by a homogeneous polynomial in n variables of total degree d. Thus the decomposition corresponds to a sum of powers of linear forms.

Our algorithm is based on linear algebra computations of Hankel (and quasi-Hankel) matrices derived from multivariate polynomials and normal form computations, and reformulates Sylvester's approach from the dual point of view.

7. Other Grants and Activities

7.1. European actions

7.1.1. SAGA

SAGA (ShApe, Geometry and Algebra, 2008-2012) is a Marie-Curie Initial Training Network of the call FP7-PEOPLE-2007-1-1-ITN.

The project aims at promoting the interaction between Geometric Modeling and Real Algebraic Geometry and, in general, at strengthening interdisciplinary and inter-sectorial research and development concerning CAD/CAM. Its objective is also to train a new generation of researchers familiar with both academic and industry viewpoints, while supporting the cooperation among the partners and with other interested collaborators in Europe. The partners are:

- SINTEF, Oslo, Norway (Leader);
- University of Oslo, Norway;
- Johannes Kepler Universitaet Linz, Austria;
- Universidad de Cantabria, Santander, Spain;
- Vilniaus Universitetas, Lithuany;

- National and Kapodistrian University of Athens, Greece;
- INRIA Méditerranée, France;
- GraphiTech, Italy;
- Kongsberg SIM GmbH, Austria;
- Missler Software, France;

More information available at http://saga-network.eu/.

7.1.2. Exciting

Exciting – Exact geometry simulation for optimized design of vehicles and Vessels – FP7-CP-SST-2007-RTD-1- 218536 (2008–2011).

This project focuses on computational tools for the optimized design of functional free-form surfaces. Specific applications are ship hulls and propellers in naval engineering and car components, frames, and turbochargers in the automotive and railway transportation industries. The objective is to base the corresponding computational tools on the same exact representation of the geometry. This should lead to huge benefits for the entire chain of design, simulation, optimization, and life cycle management, including a new class of computational tools for fluid dynamics and solid mechanics, simulations for vehicles and vessels based. This seamless integration of CAD and FEM will have direct application in product design, simulation and optimization of core components of vehicles and vessels. The partners are:

- Johannes Kepler University, Linz Autriche (Leader);
- SINTEF, Oslo, Norway;
- Siemens AG, Germany;
- National Technical University of Athens, Greece;
- Hellenic Register of Shipping, Greece;
- University of Technology, Munich Germany;
- INRIA Méditerranée, France;
- VA Tech Hydro, Austria;
- Det Norske Veritas AS, Norway.

More information available at http://exciting-project.eu/.

7.2. Bilateral actions

7.2.1. PAI Procore collaboration

Participants: Laurent Busé, Stéphane Chau, Yi-King Choi [Hong Kong Univ.], André Galligo, Yang Liu [Hong Kong Univ.], Wenping Wang [Hong Kong Univ.], Julien Wintz.

The objective of this collaboration is to conduct research in effective algebra for solving problems in geometric modeling. We investigate the use of implicit models, for compact and efficient shape representation and processing. The application domains are Computer Aided Geometric Design, Robotics, Shape compression, Computer Biology. We focus on algebraic objects of small degree such as quadrics, with the aim to extend the approach to higher degree. In particular, we are interested in the following problems:

- Shape segmentation and representation using quadrics.
- Shape processing using quadrics.
- Collision detection for objects defined by quadric surfaces.

Experimentation and validation will lead to joint open source software implementations, dedicated to quadric manipulations. A package collecting these tools will be produced.

L. Busé, A. Galligo and B. Mourrain visited Hong-Kong University (June 16–22). A workshop on geometric modeling and algebraic geometry was organised at this occasion. Dongming Yuan (Ph.D. student) visited GALAAD during an internship from March 1st to June 30. Wenping Wang (Professor) participated to the SAGA kickoff meeting, November 17–21.

7.2.2. PAI Picasso collaboration

Participants: Laurent Busé, Marc Dohm, Mohamed Elkadi, André Galligo, Bernard Mourrain.

This is a collaboration with the University of Barcelona. The Spanish team is headed by Carlos D'Andrea. The objective of this collaboration is to conduct research in elimination theory and to explore its applications for solving problems in geometric modeling. The following four points will be considered:

- principal case of elimination theory, resultants,
- effective computation of a resultant system,
- degree and height of resultant systems,
- applications in CAGD.

Laurent Busé and André Galligo visit twice the University of Barcelona from April 22th to April 25th and from October 24th to October 28th.

7.2.3. ECOS-Sud collaboration

Participants: Laurent Busé, Stéphane Chau, Marc Dohm, Mohamed Elkadi, André Galligo, Bernard Mourrain, Julien Wintz.

The first objective of this collaboration with the team of A. Dickenstein at the University of Buenos Aires, Argentina is the development of effective methods for geometric modeling, with a special focus on singularity and numerical stability problems. This includes intersection problems for curves or surfaces, change of representations such as implicitisation via syzygies and moving planes, polytopes analysis and Puiseux expansions. A second objective is the development of open tools dedicated to such problems which could be shared by the different groups working on this topic.

This year we had the visit of N. Botbol (Ph.D. student), January 6–20 and October 1st–30 and A. Dickenstein (Professor), October 3–16 in GALAAD and the visits to Buenos Aires of M. Dohm (Ph.D. student), July 11 – August 11; and the one of B. Mourrain, December 8–19.

7.3. National actions

7.3.1. ANR DECOTES, Tensorial decomposition and applications

Years: 2006–2009.
Partners: I3S, CNRS; LTSI, INSERM; GALAAD, INRIA; SBP, Thales communications.

The problem of decomposition of a symmetric or non-symmetric tensor in minimal way is an important problem, which has applications in many domains. It is essential in the process of Blind Identification of Under-Determined Mixtures (UDM), i.e., linear mixtures with more inputs than observable outputs and appear in many application areas, including speech, mobile communications, machine learning, factor analysis with k-way arrays (MWA), biomedical engineering, psychometrics, and chemometrics. The aim of the project DECOTES is to study the key theoretical problems of such decompositions and to devise numerical algorithms dedicated to some selected applications.

With Elias Tsigaridas, at a post-doctoral position, we are investigating algebraic methods to compute such a decomposition in order to extend the Sylvester's approach for binary forms to polynomials with more variables.

7.3.2. ANR GECKO, Geometry and Complexity

Years: 2005–2008

Partners: ALGO, INRIA; GALAAD, UNSA, INRIA; LIX, Ecole Polytechnique; Univ. Paul Sabatier, Toulouse.

The technological and scientific development of our society raises problems, which after transformation and simplification often lead to systems of polynomial or differential equations and inequalities. The topics of the project GECKO are the study, analysis and implementation of solvers for the resolution of such problems, based on a geometric approach. It involves fundamental operations with univariate and multivariate polynomials (such as Newton process, factorisation, elimination), structured matrices and linear differential equations (non-commutative elimination and integration). One of the objectives is to develop efficient algorithms with good complexity bound, by taking into account the geometric properties of the solutions. These algorithms are implemented in the framework of the open and modular system MATHEMAGIX. See http://gecko.inria.fr/ for more details.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We organize a seminar called “Formes & Formules”. The list of talks is archived at http://www-sop.inria.fr/galaad/.

8.1.2. Comitee participations

- Laurent Busé was a member of the Program Committee of the conference ISSAC’2008.

8.1.3. Editorial committees

- L. Busé, M. Elkadi and B. Mourrain were guest editors of a special issue of Theoretical Computer Science [25], for the conference *Computational Algebraic Geometry and Applications*, held in Nice on the occasion of the 60th birthday anniversary of André Galligo.
- B. Mourrain and C. D’Andrea (Univ. Barcelona) are guest editors of the special issue of Journal of Symbolic Computation related to ISSAC’07.
- B. Mourrain is member of the editorial board of the Journal of Symbolic Computation.
- A. Galligo (with J. Schicho and LM. Pardo) are guest editors of the special issue of Journal of Symbolic Computation related to MEGA’07.

8.1.4. Organisation of conferences and schools

- Laurent Busé organized with Teresa Krick, University of Buenos Aires and Chris Peterson Colorado State University, a worshop on *Computational Algebraic Geometry* at the international conference FoCM’2008 at Hong-Kong, 20–22 June. See http://www-sop.inria.fr/galaad/conf/FOCM08/focm.html

8.1.5. Ph.D. thesis committees

- Laurent Busé was a member (as co-advisor) of the PhD committee of Marc Dohm, University of Nice.
- André Galligo was a member of the PhD committee of Stéphane Chau (as advisor), Marc Dohm (co-advisor), Adrien Poteaux (Univ. of Limoges), Julien Wintz;
- Bernard Mourrain was a member of the PhD committee of Lionel Alberti, Stéphane Chau, Houssam Khalil, Richard Leroy (Univ. of Rennes), Julien Wintz.

8.1.6. Other comittees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- A. Galligo is one of the 3 members of the stirring committee of ISSAC.

8.1.7. WWW server

- http://www-sop.inria.fr/galaad/.

8.2. Participation at conferences and invitations

- Elimane Ba participated to the *Journées nationales du Calcul Formel*, Luminy, 20–24 october.
- L. Busé visited the university of Barcelona, Spain, from April 22th to April 25th and from October 24th to October 28th; he was invited and gave a talk at the *Premier colloque franco-maghrébin de Calcul Formel*, Sfax, Tunisia, 23–26 May; he participated to the FOCM'2008 conference, 20–22 june, Hong Kong, China; he was invited and gave a course at the *Journées nationales du Calcul Formel*, Luminy, 20–24 October; he participated and gave a talk to the Kick-off meeting of the SAGA project, Santander, Spain, 17–21 November; he gave a talk and participated to the ending meeting of the ANR GECKO, Paris, 24–28 November.
- M. Dohm participated to the conference Algebraic Geometry, D-modules, Foliations and their interactions. University of Buenos Aires. Jul. 14–26; gave a talk at the conference IV Encuentro Nacional de Algebra, Còrdoba, August 2008.
- Daouda Niang Diatta gave a talk at The International Symposium on Symbolic and Algebraic Computation (ISSAC),in Hagenberg, Austria, at the Research Institute for Symbolic Computation (RISC), July 20–23, 2008; he gave a talk at the seminary *Calculs Algébriques Numériques Symboliques et Optimisation* at Limoges, 17 october; he gave a talk at the *Journées nationales du Calcul Formel*, Luminy, 20–24 october; he gave a talk and participated to the ending meeting of the ANR GECKO, Paris, 24–28 november.
- A. Galligo participated and gave a talk at the *Premier colloque franco-maghrébin de Calcul Formel*, Sfax, Tunisia, 23–26 May; he attended to the ISSAC' 08 conference, Linz, July 2008 where he presented a poster; he participated to the *Journées nationales du Calcul Formel*, Luminy, 20–24 october; he participated to the Kick-off meeting of the EXCITING project, Strobl, 27–29 October 2008; he gave a talk and participated to the ending meeting of the ANR GECKO, Paris, 24–28 november.
- B. Mourrain gave a talk on the computation of the topology of space curve at the *Premier colloque franco-maghrébin de Calcul Formel*, Sfax, Tunisia, 23–26 May; visited Hong Kong University and gave a talk on normal form computation at the FOCM'2008 conference, 20–22 june, Hong Kong, China; gave a talk on subdivision methods for algebraic curves and surfaces at the conference Mathematics methods for Curves and Surfaces, June 26 – July 1st, Tonsberg, Norway; gave a talk on symmetric tensor decomposition at the SIAM Annual Meeting, July 7–11, San Diego; gave an invited talk on stable methods for solving polynomial equations at the conference on Numerical Analysis at the occasion of R. Varga 80th birthday, September 1–5, Kalamata, Greece; participated to the Kick-off meeting of the EXCITING project, October 27– 29, Strobl, Austria; gave a talk on real radical computation at the *Journées nationales du Calcul Formel*, Luminy, 20–24 october; gave talks at the Kick-off meeting of the SAGA project, Castro Urdales, Spain, November 17–21; he gave a talk on perturbations of normal form computation at the ending meeting of the ANR GECKO, Paris, November 24–28.
- Julien Wintz gave a talk on the algebraic-geometric modeler AXEL at the conference EACA, September 10–12, Granada Spain.

- Adrien Poteaux participated and gave a talk at the *Journées nationales du Calcul Formel*, Luminy, 20–24 October and to the Kick-off meeting of the SAGA project, Castro Urdales, Spain, 17–21 November. He also participated to the ending meeting of the ANR GECKO, Paris, 24–28 November.
- Elias Tsigaridas participated to the 24nd European Workshop on Computational Geometry (EuroCG), at Nancy, France, on 18–20 March, 2008. He visited the Department of Informatics and Telematics of Harokopion University of Athens on 27 May 2008, and gave a talk on "Computations with real algebraic numbers and applications"; visitited Department of Applied Mathematics of the University of Crete at Greece, on 29 May 2008, and gave a talk on "Computations with real algebraic numbers and applications". He also participated to the Conference in Numerical Analysis (NumAn 2008), at Kalamata, Greece, at the occasion of R. Varga 80th birthday, September 1–5, and gave a talk about "Using tensors and polynomials systems solvers to follow fibers in the brain"; finally, he invited at the $\mu\Pi\lambda\forall$ seminar of Department of Mathematics of University of Athens, 31 Oct 2008, and gave a talk on "Real solving polynomials and continued fraction. Algorithms and Complexity". He participated to the 3rd Athens Colloquium on Algorithms and Complexity (ACAC), Aug 25–26, Athens, 2008, and gave a talk on "Using polynomials and tensors to detect fibers in the brain". Finally, he participated and presented a poster, "Computing a rational in between", at the Int. Symposium on Symbolic and Algebraic Computation (ISSAC), Linz, Austria, July 20–23, 2008.

8.3. Formation

8.3.1. Teaching at Universities

- Elimane Ba, 2nd year of Deug mathematics, 24 hours; courses at IUT, 40 hours.
- Laurent Busé, Master 2nd year of MDFI, "Algebraic curves and surfaces for CAGD", 3 hours.
- Mohamed Elkadi, Master 2nd year of mathematics, 45 hours; CAPES 96 hours.
- André Galligo, Master 2nd year of mathematics at the University of Nice, 20 hours; courses in Master 1st year of mathematics at the University of Nice, 60 hours.
- Bernard Mourrain, Master 2nd year of the University of Nice, "Algorithms for curves and surfaces", 20 hours. Master MDFI 2nd year of the University of Marseilles, "Calcul Formel", 20 hours.

8.3.2. PhD theses in progress

- Elimane Ba, *Résultants, calculs et applications*, UNSA.
- Cristina Bertone, *Décomposition irréductible de courbes gauches*, Univ. Turino, Italia.
- Jérome Brachat, *Dualité effective pour la résolution d'équations polynomiales*, bourse AMX, UNSA.
- Daouda N'Diatta, *Résultants et sous-résultants et applications*, Univ. Limoges.
- Angelos Mantzaflaris, *Robust algebraic methods for geometric computations*, bourse ITN SAGA, UNSA.
- Luu Ba Thang, *Using matrix-based representations for CAGD*, bourse du gouvernement vietnamien, UNSA.

8.3.3. Defended PhD thesis

- Lionel Alberti, *Quantitive properties of real algebraic singularities*, ED SFA, UNSA.
- Stéphane Chau, *Study of singularities used in CAGD*, ED SFA, UNSA.
- Marc Dohm, *Algorithmique des courbes et surfaces algébriques*, ED SFA, UNSA.
- Houssam Khalil, *Matrices structurées en calcul symbolique et numérique*, Univ. Lyon I.
- Julien Wintz, *Algebraic methods for geometric modeling*, ED STIC, UNSA.

8.3.4. Internships

See the web page of our interships.

- Sadock Chakroun, *Optimisation globale de fonctions polynomiales et application*, Avril 1st - June 30.
- Angelos Mantzaflaris, *Polytopes and polynomials*, May 1st - July 25th.
- Dongming Yuan, *Tree reconstruction from scanned data using quadric approximation*, March 17 - June 30.

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[12] S. CHAU. *Study of singularities used in CAGD*, Ph. D. Thesis, Université Nice Sophia Antipolis, 06 2008.

[13] M. DOHM. *Implicitization of rational algebraic surfaces with syzygy-based methods*, Ph. D. Thesis, Université Nice Sophia Antipolis, 07 2008.

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[16] L. ALBERTI, B. MOURRAIN, J. WINTZ. *Topology and arrangement computation of semi-algebraic planar curves*, in "Computer Aided Geometric Design", vol. 25, 2008, p. 631-651, http://hal.inria.fr/inria-00343110/en/.

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