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Project-Team Geometrica

Geometric Computing

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THEME SYM

Activity Report

2008

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2. Overall Objectives

2.1. Overall Objectives

Geometric computing plays a central role in most engineering activities: geometric modelling, computer aided design and manufacturing, computer graphics and virtual reality, scientific visualization, geographic information systems, molecular biology, fluid mechanics, and robotics are just a few well-known examples. The rapid advances in visualization systems, networking facilities and 3D sensing and imaging make geometric computing both dominant and more demanding concerning effective algorithmic solutions.

Computational geometry emerged as a discipline in the seventies and has met with considerable success in resolving the asymptotic complexity of basic geometric data structures and problems, including convex hulls, triangulations, Voronoi diagrams, geometric arrangements and geometric optimization. However, in the mid-nineties, it was recognized that the applicability in practice of computational geometry techniques was far from satisfactory and a vigorous effort was undertaken to make computational geometry more effective. The PRISME project together with several partners in Europe took a prominent role in this research and in the development of a large library of computational geometry algorithms, CGAL.

GEOMETRICA aims at pursuing further the effort in this direction and at building upon the initial success. Its focus is on effective computational geometry with special emphasis on *curves and surfaces*. This is a challenging research area with a huge number of potential applications in almost all application domains involving geometric computing.

The overall objective of the project is to give effective computational geometry for curves and surfaces solid mathematical and algorithmic foundations, to provide solutions to key problems and to validate theoretical advances through extensive experimental research and the development of software packages that could serve as steps toward a standard for safe and effective geometric computing.

2.2. Highlights

Two members of GEOMETRICAwere this year paper chairs of two major conferences in our field: Pierre Alliez was co-chair of SGP08[37] and Monique Teillaud was chair of SoCG08[38] (http://www.socg.org/2008/).

Through the contract signed between GEOMETRY FACTORY and The MathWorks, the triangulations developed by GEOMETRICA in CGAL will be made available to a large audience through the Matlab numerical platform.

We have obtained the creation of the INRIA «Action de Développement Technologique» CGAL-Mesh. The goal is to enrich CGAL with components devoted to mesh generation, and to favor applications related to medicine and biology.

3. Scientific Foundations

3.1. Mesh generation and geometry processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface[13], 3D mesh generation [9] as well as anisotropic meshes[23]. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e., triangle and tetrahedral meshes[14]. We investigate both greedy approaches based upon Delaunay refinement and filtering[22], and variational approaches based upon energy functionals and associated minimizers.

New methods and tools to process digital geometry in computer graphics and computational science are crucially needed. Geometry processing is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis[47], surface approximation[6] and surface mesh parameterization[21].

3.2. Topological and geometric inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are demanding efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximating data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications [7].

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to understand better the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects [28], [42], [16]. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks [43], [33].

3.3. Data structures and robust geometric computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems [2]. We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results [10].

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations [8] (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed toward extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation [25], [26], [36].

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a given domain with an high quality elements [46].

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and makes new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimension [40], [48].

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situation involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues [3], [4] [19], [34] which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Geometric modeling and shape reconstruction

Keywords: *Geometric modeling*, *geology*, *medical imaging*, *surface reconstruction*.

Modeling 3D shapes is required for all visualization applications where interactivity is a key feature since the observer can change the viewpoint and get an immediate feedback. This interactivity enhances the descriptive power of the medium significantly. For example, visualization of complex molecules helps drug designers to understand their structure. Multimedia applications also involve interactive visualization and include e-commerce (companies can present their products realistically), 3D games, animation and special effects in motion pictures. The uses of geometric modeling also cover the spectrum of engineering, computer-aided design and manufacture applications (CAD/CAM). More and more stages of the industrial development and production pipeline are now performed by simulation, due to the increased performance of numerical simulation packages. Geometric modeling therefore plays an increasingly important role in this area. Another emerging application of geometric modeling with high impact is medical visualization and simulation.

In a broad sense, shape reconstruction consists of creating digital models of real objects from points. Example application areas where such a process is involved are Computer Aided Geometric Design (making a car model from a clay mockup), medical imaging (reconstructing an organ from medical data), geology (modeling underground strata from seismic data), or cultural heritage projects (making models of ancient and or fragile models or places). The availability of accurate and fast scanning devices has also made the reproduction of real objects more effective such that additional fields of applications are coming into reach. The members of GEOMETRICA have a long experience in shape reconstruction and contributed several original methods based upon the Delaunay and Voronoi diagrams.

4.2. Scientific computing

Keywords: *Unstructured meshes*, *finite element method*.

Meshes are the basic tools for scientific computing using finite element methods. Unstructured meshes are used to discretize domains bounded by complex shapes while allowing local refinements. GEOMETRICA contributes to mesh generation of 2D and 3D possibly curved domains. Most of our methods are based upon Delaunay triangulations, Voronoi diagrams and their variants. Anisotropic meshes are also investigated [14], [23]. We investigate in parallel both greedy and variational mesh generation techniques. The greedy algorithms consist of inserting vertices in an initial coarse mesh using the Delaunay refinement paradigm, while the variational algorithms consists of minimizing an energy related to the shape and size of the elements. Our goal is to show the complementarity of these two paradigms. Quadrangle surface meshes are also of interest for reverse engineering and geometry processing applications. Our approach consists of sampling a set of curves on the surface so as to control the final edge alignment, the mesh sizing and the regularity of the quadrangle tiling.

5. Software

5.1. CGAL, the Computational Geometry Algorithms Library

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Manuel Caroli, Olivier Devillers, Samuel Hornus, Pedro Machado Manhães de Castro, Sylvain Pion [contact person], Laurent Saboret, Monique Teillaud, Camille Wormser, Mariette Yvinec.

With the collaboration of Hervé Brönnimann, Frédéric Cazals, Frank Da, Christophe Delage, Andreas Fabri, Julia Flötotto, Philippe Guigue, Menelaos Karavelas, Sébastien Loriot, Abdelkrim Mebarki, Naceur Meskini, Andreas Meyer, Marc Pouget, François Rebufat, Laurent Rineau, and Radu Ursu. http://www.cgal.org

CGAL is a C++ library of geometric algorithms and data structures. Its development has been initially funded and further supported by several European projects (CGAL, GALIA, ECG, ACS, AIM@SHAPE) since 1996. The long term partners of the project are research teams from the following institutes: INRIA Sophia Antipolis - Méditerranée, Max-Planck Institut Saarbrücken, ETH Zürich, Tel Aviv University, together with several others. In 2003, CGAL became an Open Source project (under the LGPL and QPL licenses), and it also became commercialized by GEOMETRY FACTORY, a company *Born of INRIA* founded by Andreas Fabri.

The aim of the CGAL project is to create a platform for geometric computing supporting usage in both industry and academia. The main design goals are genericity, numerical robustness, efficiency and ease of use. These goals are enforced by a review of all submissions managed by an editorial board. As the focus is on fundamental geometric algorithms and data structures, the target application domains are numerous: from geological modeling to medical images, from antenna placement to geographic information systems, etc.

The CGAL library consists of a kernel, a list of algorithmic packages, and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes, circles, spheres, circular arcs...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes : one can choose between Cartesian or homogeneous coordinates, use different types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

A number of packages provide geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d-dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, Boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface.

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview, ...). Partial interfaces with Python, SCILAB and the Ipe drawing editor are now also available.

GEOMETRICA is particularly involved in general maintainance, in the arithmetic issues that arise in the treatment of robustness issues, in the kernel, in triangulation packages and their close applications such as alpha shapes, in meshes... Four researchers of GEOMETRICA are members of the CGAL Editorial Board, whose main responsibilities are the control of the quality of CGAL, making decisions about technical matters, coordinating communication and promotion of CGAL.

CGAL is about 600,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Cygwin...), Visual C++ (Windows), Intel C++... CGAL is released approximately once a year, and a release is downloaded more than 15000 times. Moreover, CGAL is directly available as packages for the Debian, Ubuntu and Fedora Linux distributions.

More numbers about CGAL: there are now 14 editors in the editorial board, with approximately 20 additional developers. The user discussion mailing-list has more than 1000 subscribers with a relatively high traffic of 5-10 mails a day. The announcement mailing-list has more than 3000 subscribers.

6. New Results

6.1. Mesh Generation and Geometry Processing

Keywords: *Isotropic meshing*, *anisotropic meshing*, *level sets*, *mesh optimization*, *tetrahedral meshing*, *triangle meshing*.

6.1.1. Locally uniform anisotropic meshing

Participants: Jean-Daniel Boissonnat, Camille Wormser, Mariette Yvinec.

Anisotropic meshes are triangulations of a given domain in the plane or in higher dimensions, with elements elongated along prescribed directions. Anisotropic triangulations have shown particularly well suited to interpolation of functions or to numerical modeling. Following our previous investigations [14], we propose a new approach to anisotropic mesh generation relying on the notion of locally uniform anisotropic mesh [23]. A locally uniform anisotropic mesh is a mesh designed such that the star around each vertex v coincides with the star that v would have if the metric on the domain was uniform and equal to the metric at v. This definition allows devising a simple refinement algorithm which relies on elementary predicates, and provides, after completion, an anisotropic mesh in dimensions 2 or 3. Prototypes have been implemented for both the 2D and the 3D cases (see Figures 1 and 2).

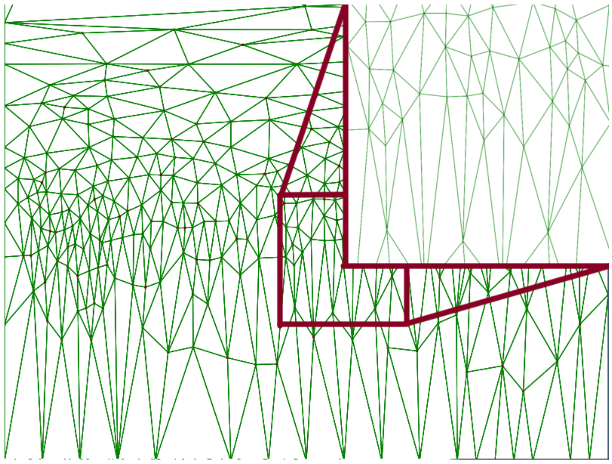

Figure 1.

Anisotropic mesh of a 2D domain with a close-up on the central part.
The red lines separate the zoom from the regular drawing and show the zoomed part.

6.1.2. Mesh Optimization

Participants: Pierre Alliez, Jane Tournois, Camille Wormser.

We are elaborating upon a mesh optimization technique designed to improve the quality of isotropic tetrahedron meshes. Our approach improves over the optimal Delaunay approach introduced by Chen in 2004, which consists of casting the mesh optimization problem as a function interpolation in 4D. In the original approach the optimization is performed by alternating vertex relocations and Delaunay connectivity updates. While the original approach keeps the boundary vertices fixed in order to avoid mesh shrinking, we relocate them in a consistent manner with the interior vertices by reproducing the so-called cospherical property. We also investigate the possibility to approximate the paraboloid instead of interpolating it using a regular triangulation, in order to reduce the number of slivers in the final mesh. At the intuitive level, this amounts to embed a sliver exudation process as part of the mesh optimization. Furthermore, we show how alternating batches of refinement with mesh optimization in a multilevel manner generates mesh with fewer Steiner vertices. Figure 3 shows an optimized mesh compared to a mesh generated by Delaunay refinement. We have not yet published this on-going work.

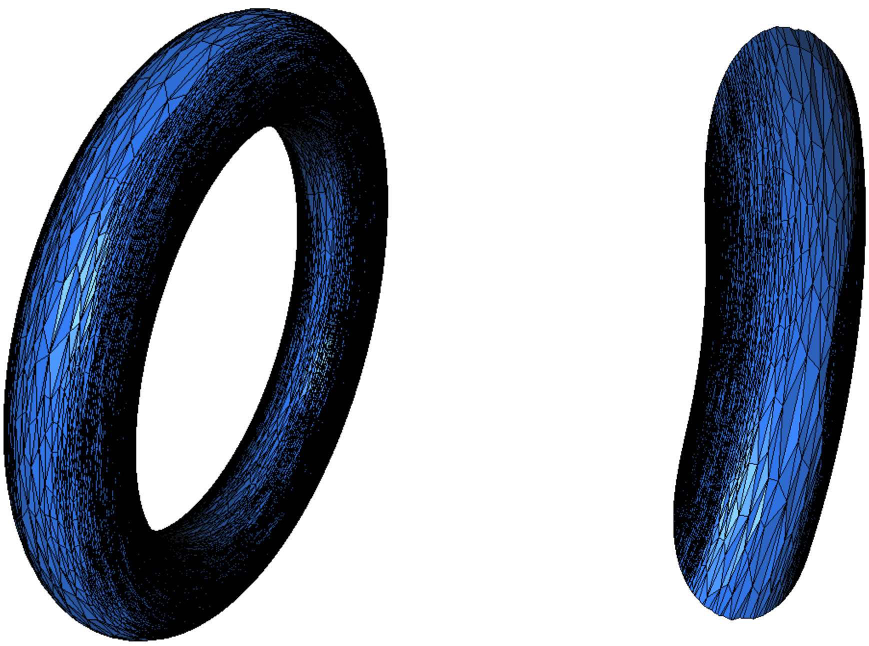

Figure 2.

Anisotropic mesh of the surface of a torus.
This mesh has been generated by Yuanmi Chen during her internship.

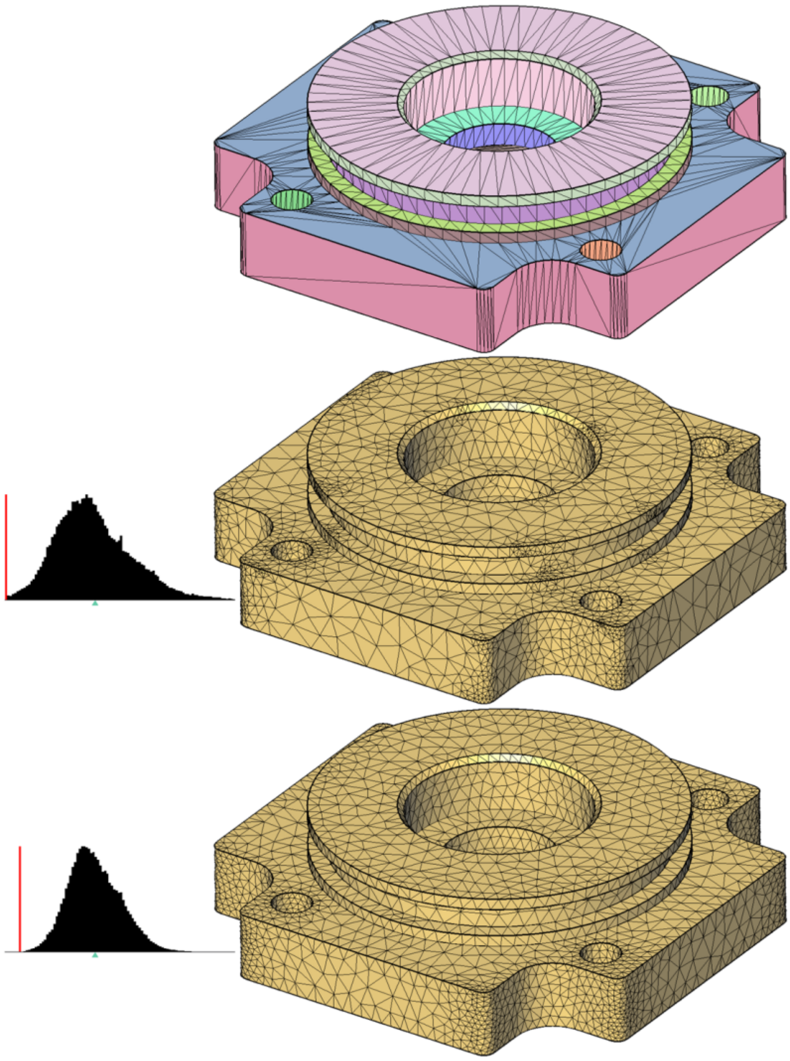

Figure 3. Top: Input polyhedral surface with sharp creases tagged. Middle: Isotropic tetrahedron mesh obtained by Delaunay refinement parameterized so as to control the shape of the elements and the boundary approximation error (no sliver exudation performed). The distribution of dihedral angles is shown to the left. Bottom: Optimized mesh with improved quality (dihedral angles in [11.13;150] degrees).

6.1.3. Conformal Parameterization

Participant: Pierre Alliez.

In collaboration with Patrick Mullen, Yiying Tong and Mathieu Desbrun from Caltech.

We propose a linear-algebra-based conformal parameterization technique to parameterize triangle mesh patches [21]. Unlike previous free-boundary linear methods we do not require point constraints to be added to the linear system, thus reducing distortion. While Laplacian eigenvectors have been proposed as a constraint-free approach to least-distorted maps in the context of manifold learning and graph drawing, we demonstrate that a better conformal parameterization can be found through a generalized eigenvalue problem as it minimizes a weighted conformal energy mostly insensitive to sampling irregularity of the original mesh (see Figure 4). We discuss the similarities and differences between our approach and previous work, and demonstrate numerical advantages of our spectral method on small and large meshes alike.

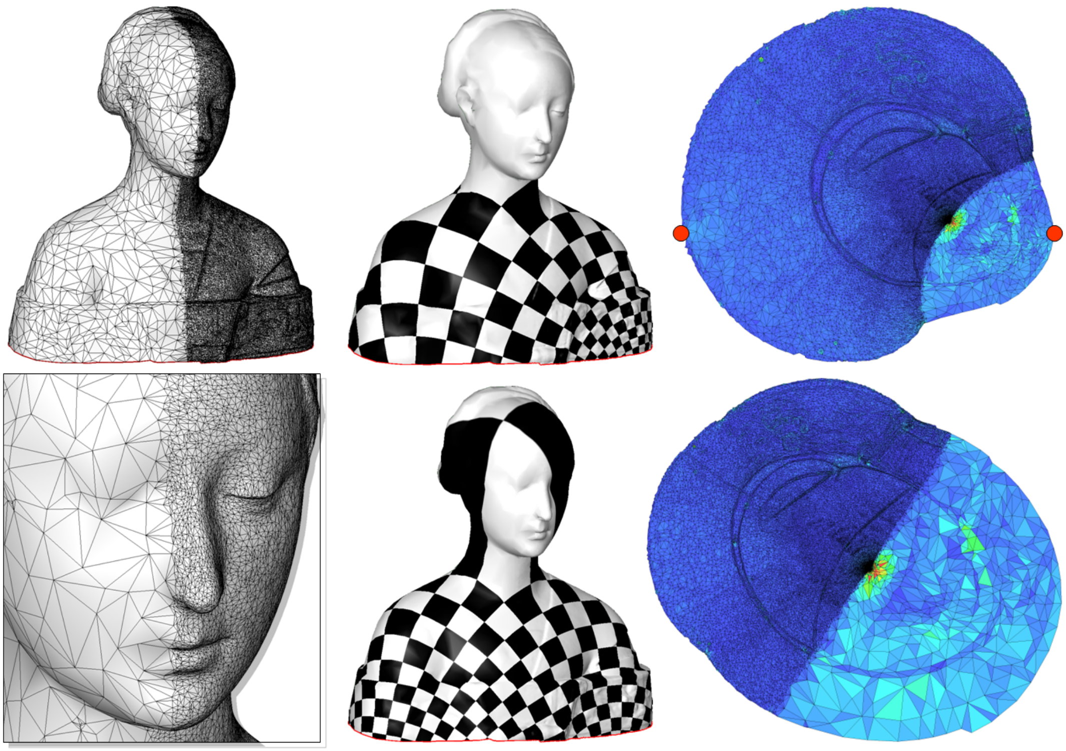

*Figure 4. **Sforza**. On this mesh (50K vertices) with varying sampling rates (left), previous linear methods (top right, least squares conformal maps) fail to capture the symmetry of the mesh in the parameterization (solved in 4s). The two red dots depict the constrained vertices. In contrast, our spectral approach (bottom right) automatically computes a low-distortion conformal map (solved in 5.2s) without any constraints. Middle images depict the sforza mesh with a checkboard texture mapped using the parameterizations shown on the right.*

6.1.4. Principal Component Analysis

Participants: Pierre Alliez, Sylvain Pion, Ankit Gupta.

Principal component analysis is a basic component of many geometric computing and processing algorithms. It is most commonly used on point sets, although applicable to other primitives as well through the computation of covariance matrices. In this work [47] we provide closed form formulas of covariance matrices for sets of 2D and 3D geometric primitives such as segments, circles, triangles, iso rectangles, spheres, tetrahedra and iso cuboids. We also derive covariance matrices for their dimensional variants such as disks, balls, etc. We discuss the flexibility and added value of the present approach by discussing its potential use in applications. Our implementation will be made available through the next release of the CGAL library.

6.2. Topological and geometric inference

Keywords: *Computational topology*, *geometric inference*, *geometric probing*, *implicit surfaces*, *point set surfaces*, *surface learning*, *surface reconstruction*.

6.2.1. Shape reconstruction from unorganized cross-sections

Participants: Jean-Daniel Boissonnat, Pooran Memari.

In this work, we consider the problem of reconstructing 2-dimensional geometric shapes from unorganized 1-dimensional cross-sections and provide theoretical guarantees. We study the problem in its full generality following the approach of Boissonnat and Memari we pursued last year, for the analogous 3D problem. We propose a new variant of this method and provide sampling conditions to guarantee that the output of the algorithm has the same topology as the original object and is close to it (for the Hausdorff distance). Although the problem of reconstructing 3D shapes from point clouds has received a lot of attention, there were no similar results for the problem of reconstructing shapes from planar cross-sections [20] (See Figure 5).

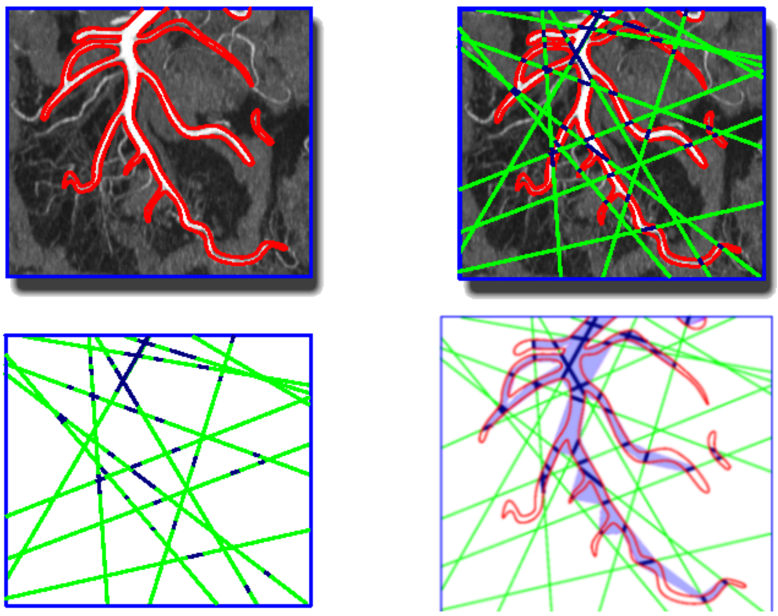

Figure 5.

Top-left: unknown object. Top-right: cross sections.
Bottom-left: input of our algorithm. Bottom-right: reconstructed result (in blue).

6.2.2. A new framework for topological persistence

Participants: Frédéric Chazal, David Cohen-Steiner, Steve Oudot.

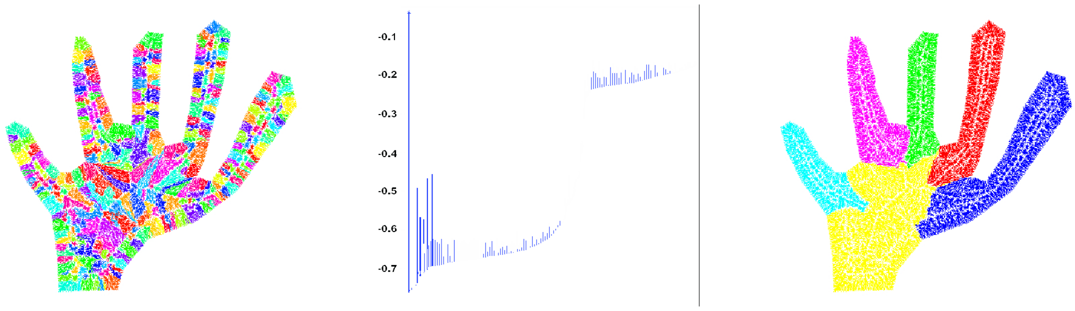

Figure 6. An example of application to shape segmentation on a 2D domain. The segmentation function is the (normalized) diameter of the set of nearest boundary points. The barcode shows six long intervals, corresponding to the palm of the hand and to the five fingers. The results before and after merging non-persistence clusters are shown respectively to the left and to the right of the barcode.

In collaboration with L. Guibas (Stanford University) and M. Glisse (Grenoble).

The concept of topological persistence introduced by H. Edelsbrunner et al. in 2000 is a rather general tool providing an efficient way to encode the qualitative and quantitative behavior of real-valued functions defined over topological spaces. Since its introduction, this encoding, known as the persistence diagram or barcode, has been extensively studied, specifically in topological data analysis where its stability properties allow to infer robust topological information on the studied data sets. Motivated by problems coming from topological data analysis (mainly the one considered in section 6.2.4), we have extended the notions of persistence and persistence diagrams to a larger setting than the one classically considered. We have proven new stability results for the persistence diagrams that lead to new applications in topological and geometric data analysis [42] (see Figure 6).

6.2.3. Persistence based algorithms for topological inference

Participants: Frédéric Chazal, Steve Oudot.

Manifold reconstruction has been extensively studied among the computational geometry community for the last decade or so, especially in two and three dimensions. Recently, significant improvements were made in higher dimensions, leading to new methods to reconstruct large classes of compact subsets of Euclidean space $\mathbb{R}^d$. However, the complexities of these methods scale up exponentially with d, which makes them impractical in medium or high dimensions, even for handling low-dimensional submanifolds.

We have introduced a novel approach [28] that stands in-between reconstruction and topological estimation, and whose complexity scales up with the intrinsic dimension of the data. Specifically, our algorithm combines two paradigms: greedy refinement, and topological persistence. It builds a set of landmarks iteratively, while maintaining nested pairs of complexes, whose images in $\mathbb{R}^d$ lie close to the data, and whose persistent homology eventually coincides with the one of the underlying shape. When the data points are sufficiently densely sampled from a smooth m-submanifold of $\mathbb{R}^d$, our method retrieves the homology of the submanifold in time at most $c(m)n^5$, where n is the size of the input and $c(m)$ is a constant depending solely on m. It can also provably well handle a wide range of compact subsets of $\mathbb{R}^d$, though with higher complexities.

6.2.4. Topological analysis of scalar fields defined over point cloud data

Participants: Frédéric Chazal, Steve Oudot.

In collaboration with L. Guibas and P. Skraba (Stanford University).

Given a real-valued function f defined over some metric space X, is it possible to recover some structural information about f from the sole information of its values at a finite subset L of sample points, whose pairwise distances in X are given? We have provided a positive answer to this question [43]. More precisely, taking advantage of recent advances on the front of stability for persistence diagrams, we have introduced a novel algebraic construction, based on a pair of nested families of simplicial complexes built on top of the point cloud L, from which the persistence diagram of f can be faithfully approximated. We derive from this construction a series of algorithms for the analysis of scalar fields from point cloud data. These algorithms are simple and easy to implement, have reasonable complexities, and come with theoretical guarantees. To illustrate the generality and practicality of the approach, we also obtained experimental results in various applications, ranging from clustering to sensor networks.

6.2.5. Homology inference in the context of sensor networks

Participant: Steve Oudot.

In collaboration with L. Guibas (Stanford University) J. Gao (Stony Brook), and Y. Wang (Stony Brook).

In this work, we investigate the problem of inferring the homology of the domain underlying a sensor network from the sole knowledge of the connectivity between sensors. This problem has recevied a lot of attention in the recent years, and a number of partial solutions have been developed. We propose a complete and provably-good solution to the problem for the special case where the domain underlying the sensors is planar [33].

We first introduce a new feature size for bounded domains in the plane endowed with an intrinsic metric. Given a point x in a domain X, the *systolic feature size* of X at x measures half the length of the shortest loop through x that is not null-homotopic in X. The resort to an intrinsic metric makes the systolic feature size rather insensitive to the local geometry of the domain, in contrast with its predecessors (local feature size, weak feature size, homology feature size). This reduces the number of samples required to capture the topology of X.

Under reasonable sampling conditions, we show that the geodesic Delaunay triangulation $D_X(L)$ of a finite sampling L of a bounded planar domain X is homotopy equivalent to X. Moreover, under similar conditions, $D_X(L)$ is sandwiched between the geodesic witness complex $W_X(L)$ and a relaxed version $W_X^\lambda(L)$. Taking advantage of this fact, we prove that the homology of $D_X(L)$ (and hence the one of X) can be retrieved by computing the persistent homology between $W_X(L)$ and $W_X^\lambda(L)$.

We then investigate further and show that the homology of X can also be recovered from the persistent homology associated with inclusions of type $W_X^\lambda(L) \hookrightarrow W_X^{\lambda'}(L)$, under some conditions on the relaxation parameters $\lambda \leq \lambda'$. Similar results are obtained for Vietoris-Rips complexes as well. Our proofs draw some connections with recent advances on the front of homology inference from point cloud data, but also with several well-known concepts of Riemannian (and even metric) geometry.

On the algorithmic front, we propose algorithms for estimating the systolic feature size of a sampled planar domain X, selecting a landmark set of sufficient density, building its geodesic Delaunay triangulation, and computing the homology of X using geodesic witness complexes or Rips complexes. We also perform some experimental simulations that corroborate our theoretical results.

6.2.6. Extending persistence using Poincaré and Lefschetz duality

Participant: David Cohen-Steiner.

In collaboration with H. Edelsbrunner and J. Harer (Duke University).

Persistent homology has proven to be a useful tool in a variety of contexts, including the recognition and measurement of shape characteristics of surfaces in $\mathbb{R}^3$. In this paper, we extend persistence to essential homology classes (see figure 7 for an example of extended persistence diagram), present an algorithm to calculate it, and describe how it aids our ability to recognize shape features for codimension 1 submanifolds of Euclidean space. The extension derives from Poincaré duality but generalizes to non-manifold spaces. We prove stability for general triangulated spaces and duality as well as symmetry for triangulated manifolds [17].

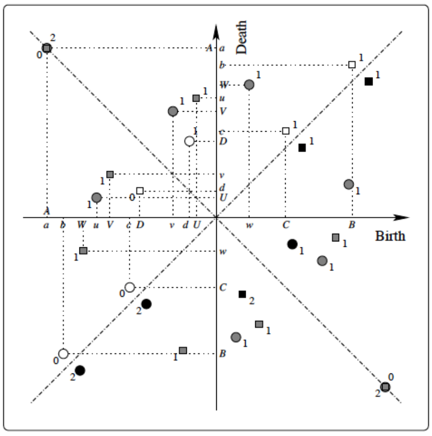

Figure 7. Example of an extended persistence diagram.

6.2.7. Computing geometry aware handle and tunnel loops in 3D models

Participant: David Cohen-Steiner.

In collaboration with T.K. Dey, K. Li (Ohio State University) and J. Sun (Stanford University).

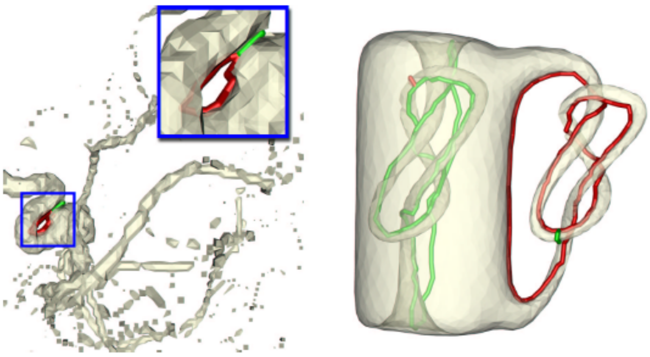

Figure 8. Homology generators selected by our algorithm.

Many applications such as topology repair, model editing, surface parametrization, and feature recognition benefit from computing loops on surfaces that wrap around their "handles" and "tunnels". Computing such loops while optimizing their geometric lengths is difficult. On the other hand, computing such loops without considering geometry is easy but may not be very useful. In this paper, we strike a balance by computing topologically correct loops that are also geometrically relevant (see figure 8 for sample results). Our algorithm is a novel application of the concepts from topological persistence introduced recently in computational topology. The usability of the computed loops is demonstrated with some examples in feature identification and topology simplification [18].

6.3. Data Structures and Robust Geometric Computation

Keywords: *Triangulations*, *Voronoi diagrams*, *compact data structures*, *curves*, *parallel algorithms*, *periodic spaces*, *predicates*, *projective geometry*, *robustness*, *surfaces*.

6.3.1. Parallel Geometric Algorithms for Multi-Core Computers

Participants: Vicente Batista, Sylvain Pion.

In collaboration with David Millman from University of North Carolina at Chapel Hill, and Johannes Singler from Universität Karlsruhe.

Computers with multiple processor cores using shared memory are now ubiquitous. We present several parallel geometric algorithms that specifically target this environment, with the goal of exploiting the additional computing power. The d-dimensional algorithms we describe are (a) spatial sorting of points, as is typically used for preprocessing before using incremental algorithms, (b) kd-tree construction, (c) axis-aligned box intersection computation, and finally (d) bulk insertion of points in Delaunay triangulations for mesh generation algorithms or simply computing Delaunay triangulations. We show experimental results for these algorithms in 3D, using our implementations based on CGAL. This work is a step towards what we hope will become a *parallel mode* for CGAL, where algorithms automatically use the available parallel resources without requiring significant user intervention [41].

6.3.2. Delaunay triangulation of points in higher dimensions

Participants: Jean-Daniel Boissonnat, Olivier Devillers, Samuel Hornus.

We propose a new C++ implementation of the well-known incremental algorithm for the construction of Delaunay triangulations in any dimension. Our implementation follows the exact computing paradigm and is hence robust. An extensive series of comparisons have shown that our implementation outperforms the best available implementations for convex hulls and Delaunay triangulations, and that it can be used for large point sets in spaces of dimensions up to 6 [48].

6.3.3. Delaunay triangulation of moving points

Participants: Pedro Machado Manhães de Castro, Olivier Devillers.

Delaunay triangulations of a set of points is one of the most famous data structures produced by computational geometry. Two main reasons explain this success: -1- computational geometers eventually produce efficient algorithms to compute it, and -2 it actually has effective usage such as meshing for finite elements methods or surface reconstruction from point clouds. For several applications the data are moving and thus the triangulation evolves with time. It arises for example when meshing deformable objects, or in some algorithms relocating the vertices by variational methods. We focused on what we call timestamps moving, that is computing the Delaunay triangulation of a set of moving points at some discrete times. We review existing related work [46], and develop a new approach by evaluating tolerance, allowing points to move inside their tolerance without recomputing the triangulation. Preliminary results in two dimensions are very promising in the context of points placement by the Lloyd iteration [50].

6.3.4. Lower and upper bounds on the number of empty cylinders and ellipsoids.

Participants: Olivier Devillers, Monique Teillaud.

This work has been done in collaboration with Oswin Aicholzer, Franz Aurenhammer, Thomas Hackl, and Birgit Vogtenhuber (Graz)

Given a set $\mathcal{S}$ of n points in three dimensions, we study the complexity of quadrics enclosing the points of $\mathcal{S}$. We prove that the set of empty or enclosing ellipsoids has $\Theta(n^4)$ complexity, the same bound apply to empty or enclosing general cylinders, while for circular cylinders a gap remains between the $\Omega(n^3)$ lower bound and the $O(n^4)$ upper bound. The fact that the upper bound for general quadrics remains tight for general cylinders despite the fact that cylinders have less degrees of freedom than quadrics is quite interesting and a bit surprising.

We also take interest in pairs of empty homothetic ellipsoids with $\Theta(n^6)$ bounds, while the specialized versions yields to $O(n^6)$, $\Omega(n^5)$ for pairs of general homothetic cylinders and $O(n^6)$, $\Omega(n^4)$ for pairs of parallel circular cylinders. This proves that the number of combinatorially distinct Delaunay triangulations obtained by orthogonal projections of $\mathbb{S}$ on a two dimensional plane is $O(n^6)$, $\Omega(n^4)$ [39].

6.3.5. Triangulations in the 3D torus

Participants: Manuel Caroli, Monique Teillaud.

This work was started during the short post-doctoral stay of Nico Kruithof in GEOMETRICA *in 2006.*

CGAL currently provides packages to compute triangulations in $\mathbb{R}^2$ and $\mathbb{R}^3$. Periodic triangulations are widely used in various areas (including mechanics, material engineering, cosmology, astrophysics, biology,...) for simulations. We continued our work on algorithms to compute triangulations in the periodic space $\mathbb{R}^3/\mathbb{Z}^3$, which is topologically equivalent to the 3D hypersurface of a torus in 4D.

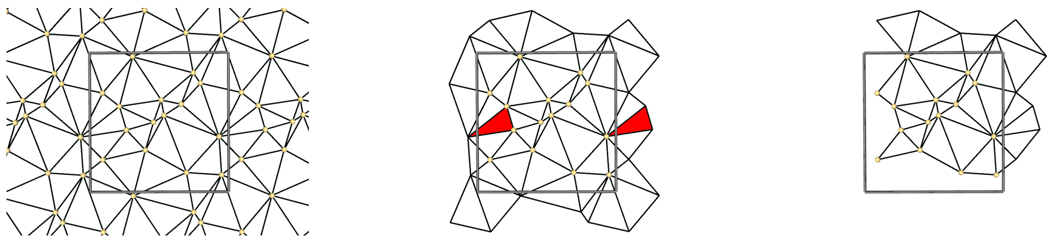

Figure 9. Periodic triangulation, 2D case

We worked on the algorithmic issues arising when trying to efficiently compute Delaunay triangulations in this space. We proved that, whereas it is sometimes necessary to compute in a 27-sheeted covering of the torus, still computing without any duplication is possible when some very realistic conditions on the input data are fulfilled [36]. The algorithm has been implemented [26].

We also studied a potential more generic design for the CGAL 3D triangulation package, that would permit to add functionality to compute triangulations in various spaces [25].

6.3.6. 3D regular complex

Participant: Monique Teillaud.

This work was started during the internship of Antoine Bru in 2007.

Whereas software for storing planar subdivisions are publicly available, as well as software for computing triangulations in 3D (in CGAL and in other software packages), software for storing more general 3D subdivisions are seldom found. Still, there is a strong need in practice for such data structures. Indeed, they could be used to store some Voronoi diagrams. Also, while algebraic issues are usually seen as the bottleneck for computing arrangements of quadric surfaces in 3D, an appropriate data structure is in fact also missing. Arrangements of the simplest quadrics, namely spheres, may have a complicated topology, however, when decomposed in a careful way, the subdivision that is obtained is actually simply a regular complex.

We started to work on the design and implementation of a cellular data structure capable of storing and traversing a manifold regular complex of dimension 3 [35]. This implementation is based on and extending the CGAL Halfedge Data Structure package.

6.3.7. Robust Construction of the Three-dimensional Flow Complex

Participant: Sylvain Pion.

In collaboration with Frédéric Cazals from the ABS *project-team and Aaditya Parameswaran, currently PhD student at Stanford University.*

The Delaunay triangulation and its dual the Voronoi diagram are ubiquitous geometric complexes. From a topological standpoint, the connection has recently been made between these cell complexes and the Morse theory of distance functions. In particular, in the generic setting, algorithms have been proposed to compute the flow complex -the stable and unstable manifolds associated to the critical points of the distance function to a point set. As algorithms ignoring degenerate cases and numerical issues are bound to fail on general inputs, this paper [27] develops the first complete and robust algorithm to compute the flow complex. First, we present complete algorithms for the flow operator, unraveling a delicate interplay between the degenerate cases of Delaunay and those which are flow specific. Second, we sketch how the flow operator unifies the construction of stable and unstable manifolds. Third, we discuss numerical issues related to predicates on cascaded constructions. Finally, we report experimental results with CGAL's filtered kernel, showing that the construction of the flow complex incurs a small overhead w.r.t. the Delaunay triangulation when moderate cascading occurs. These observations provide important insights on the relevance of the flow complex for (surface) reconstruction and medial axis approximation, and should foster flow complex based algorithms. In a broader perspective and to the best of our knowledge, this paper is the first one reporting on the effective implementation of a geometric algorithm featuring cascading.

6.3.8. Computational Geometry vs Computer Network Issues

Participant: Olivier Devillers.

Where does computational geometry appear in computer networks? Or more precisely, can we use knowledge from computational geometry to give a fresh view on some problems in computer networks? We take interest in all kinds of networks, internet, wifi, cell phones, ad-hoc or peer to peer networks, and we try to identify the geometric nature of the problem if it exists. Several sources of geometry may appear, the most obvious ones being what we call the "true" geometry, when the nodes of the network have geometric positions. Geometry can also appear also in the search of *internet coordinates* or other kind of *virtual coordinates* belonging to some space without concrete meaning but allowing the proof of relevant properties, such the existence of simple routing algorithms [12].

Some classical problems in computational geometry such as point location, k-center or clustering are useful to computer networks but with concerns that are different from the usual ones in computational geometry, e.g. load balancing or the ability of decentralized control.

These findings are summarized in an INRIA research report [45].

6.4. Software

Keywords: *arithmetic filters*, *automatic code generator*, *cgal*, *standardization*.

6.4.1. CGAL

A new major release of CGAL, version 3.4, will be available at the beginning of 2009. This release contains the following new package implemented by GEOMETRICA:

- 3D Spherical Kernel, by Pedro Machado Manhães de Castro and Monique Teillaud. This package is an extension of the linear CGAL Kernel. It offers functionalities on spheres, circles, and circular arcs in 3D.

Also, already existing packages have been enriched:

- The Kernel has a new class for circles in 3D, and attached functionalities, by Pedro Machado Manhães de Castro and Monique Teillaud. Also, a few more computations of intersections were added.
- The 2D Circular Kernel now offers several new global functions and more computations of intersections. Pedro Machado Manhães de Castro is now a co-author of the package with Sylvain Pion and Monique Teillaud.

- Principal Component Analysis has been enriched so as to fit (in the least squares sense) most objects from the 2D and 3D kernel objects (segments, balls, spheres, iso cuboids and rectangles, tetrahedra, triangles).

It also contains new packages implemented by our CGAL partners and improvements to some existing packages, the detailed list of which can be found on the CGAL web site. Major general enhancements include the switch to CMake as the building and installing tool instead of our own scripts, and a new portable graphical framework based on Qt4's GraphicsView.

Two courses devoted to the use of CGAL for meshing and for computer graphics have been organized. Pierre Alliez, Efi Fogel, and Andreas Fabri organized a course at SIGGRAPH which gathered 150 attendees with a very positive feedback as the reviewers recommended this course to be offered again [32]. Mariette Yvinec presented a course at Meshing Roundtable [22] on Delaunay refinement and filtering for mesh generation.

In addition, Sylvain Pion and Andreas Fabri presented CGAL at Google, Mountain View, CA, in the Google Tech Talks series, a video of which is available at http://fr.youtube.com/watch?v=3DLfkWWw_Tg.

6.4.2. FPG: A code generator for fast and certified geometric predicates

Participants: Andreas Meyer, Sylvain Pion.

We present [34] a general purpose code analyzer and generator for filtered predicates, which are critical for geometric algorithms. While there already exist such code generators, our contribution is to generate *almost static filters*, a type of filter which could not be generated previously. The generated and safe filtered predicates are almost as fast as their inexact floating point counterparts, in most cases. The tool is also able to parse code closer to the original CGAL C++ code of the predicates, and is able to handle some loops and varying degrees of the input variables.

6.4.3. Standardization of interval arithmetic

Participant: Sylvain Pion.

*In collaboration with Guillaume Melquiond (*PROVAL *project-team). This work is linked to the subject of the internship of Jihun Yu.*

Geometric computations are very sensitive to numerical roundoff errors. There are efficient ways to solve this problem like static filters, but there is also a more general approach using a well-known tool which is interval arithmetic. Since the latter solution is more general and easier to use from a programming point of view, we are pushing for its standardization in the hope to get better support from hardware and compilers in the long term.

In the past years, we have worked on a proposal for standardization of interval arithmetic in C++. This year, a new standardization effort has started as IEEE-1788 to provide a language-independant standard for interval arithmetic, similar in spirit to the IEEE-754 standard for floating-point. This standard would serve as a base for implementations in various languages such as Fortran, Matlab, C++...

In order to get more insight at what could best be done to improve the situation for interval arithmetic support, we have also worked on an implementation of our proposal based on Boost.Interval, using MPFR and CRLIBM as basic building blocks.

This year, we have also submitted a proposal for addition to the C++0x language to support basic floating-point operations with directed rounding-mode. We think that this set of operations is as fundamental as the basic addition of floating-point variables as is done now, and that they deserve the same special compiler support. Compiler knowledge of the rounding modes in particular is key to efficient support of interval arithmetic. Our proposal is described in [49].

7. Contracts and Grants with Industry

7.1. Geometry Factory

The initial development phase of the CGAL library has been made by a European consortium. In order to achieve the transfer and diffusion of CGAL in the industry, a company called GEOMETRY FACTORY has been founded in January 2003 by Andreas Fabri (http://www.geometryfactory.com).

The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORY is a link between the researchers from the computational geometry community and the industrial users.

It offers licenses to interested companies, and provides support. There are contracts in various domains such as CAD/CAM, medical applications, GIS, computer vision...

GEOMETRY FACTORY is keeping close contacts with the original consortium members, and in particular with GEOMETRICA.

In 2008, GEOMETRY FACTORY has new customers for GEOMETRICA packages:
- The MathWorks (USA, will provide 2D and 3D Delaunay in Matlab), - In-Three (USA, Constrained Delaunay, computer graphics), - VirtualWind (USA, Constrained Delaunay, wind simulation), - Polytec (France, Constrained Delaunay, metrology), - Total (France, Surface Mesher), - GE Healthcare (Japan, Surface Mesher), - Saudi Aramco (Saudi Arabia, 2D Delaunay, oil).

This year GEOMETRY FACTORY hired Laurent Rineau, a former PhD student of GEOMETRICA, as software development engineer.

7.2. Thalès Alenia Space

Participants: Jean-Daniel Boissonnat, Trung Nguyen.

In collaboration with Frédéric Falzon and Guillaume Perrin (Thalès Alenia Space).

The goal of this study is to optimize pupil configurations for extended source imaging based on optical interferometry.

The motivation for this work comes from the observation of the Earth from a geostationary orbit (i.e. at a distance of $\sim$ 36000 km) with a resolution of 1 m. A simple calculus shows us that we would need a telescope having a diameter of approximately 20 m for an optical wavelength of $\sim$ 500 nm. Needless to say such an instrument dimension is not adapted to the observation from space and the use of interferometric telescopes (Optical Aperture Synthesis, OAS) is to be considered in this case. We have pursued a geometric approach. The project ended in September 2008.

7.3. Dassault Systèmes

Participants: Frédéric Chazal, Nicolas Montana.

In collaboration with André Lieutier (Dassault Systèmes)

The goal of this study is to develop and implement robust and efficient 3D Boolean operators and surface regularization tools for industrial use.

The motivation of this work comes from machining simulation where the computation of the part of the space swept by a moving tool involves a huge amount of Boolean operations (unions, intersections, differences). Such computations meet two main difficulties (that are both theoretical and technical): First, 3D Boolean operations face robustness issues and second, the output of large sequences of Boolean operations usually consists of very complicated meshes containing many irrelevant topological and geometric features that need to be removed for further processing. In this study, we intend to develop a software based on an original theoretical approach which overcomes these difficulties.

7.4. France-Telecom

Participants: Olivier Devillers, Mariette Yvinec.

In collaboration with Jean-François Morlier (France Telecom).

The goal of this study was to compute an abstract representation of an antenna network for mobile phones using Voronoi diagrams. The project started in January 2007 and ended in March 2008.

8. Other Grants and Activities

8.1. National Actions

8.1.1. ANR Triangles

Participants: Manuel Caroli, Pedro Machado Manhães de Castro, Olivier Devillers, Sylvain Pion, Monique Teillaud.

http://www.inria.fr/sophia/geometrica/collaborations/triangles/

We lead the TRIANGLES project funded by the ANR. The project involves
— the «Laboratoire d'InfoRmatique en Image et Systèmes d'information» (LIRIS), Lyon,
— the «Département d'informatique de l'ENS» and
— the GEOMETRICA team.

Triangulations are essential in many applications, in particular for meshing and shape reconstruction. We want to develop and distribute new results for academic and industrial researchers. The goal of the project is the development of robust and effective algorithms for the manipulation of large sets of points, of mobile sets of points and points in non Euclidean spaces such as periodic spaces (torus, cylinder), projective, oriented projective or hyperbolic spaces. The results obtained will be implemented in the CGAL library and will be applied to computer vision (visual envelopes, camera calibration), fluid dynamics, astronomy, computer graphics and medical applications.

In the GEOMETRICA team, Triangles is co-funding the scholarship of Pedro de Castro (with «Région PACA») and funding traveling and computers. Several meetings have been organized between participants, details can be found on the project's web page.

- Starting date: November 2007

- Duration: 3 years

8.1.2. ANR GAIA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, Arijit Ghosh, David Cohen-Steiner, Samuel Hornus.

The aim of this project is to formalize a collaboration between researchers from computational geometry, machine learning and computer vision to study distortions and in particular Bregman divergences, information theory, statistics, Riemannian geometry, and convex analysis.

The other partners of the project are the Université des Antilles et de la Guyane (R. Nock, coordinator), the Ecole Polytechnique (F. Nielsen) and the Lear project-team (C. Schmid).

- Starting date: November 2007

- Duration: 4 years

8.1.3. ANR Galapagos

Participant: Sylvain Pion.

In this project we wish to apply computerized theorem proving tools to two aspects of geometry. The first aspect concerns computational geometry. The second aspect concerns verifying geometric reasoning steps in usual constructions, such as constructions with rules and compass. Other participants in this contract are the universities of Strasbourg and Poitiers, the ENSIEE in Evry and the Ecole Normale Supérieure in Lyon. The leader of the project is the MARELLE project-team.

- Starting date: November 2007.

- Duration: 3 years.

8.1.4. ANR GeoTopAl

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Sylvain Pion.

This project aims at developing concepts, methods and algorithms for solving problems in the realm of geometric modeling (of complex shapes), reverse engineering and numerical simulation, as well as visualization. The concepts and methods sought after should be rich enough to accommodate a certain mathematical sophistication, while remaining compatible with the constraints inherent to the development of efficient and certified algorithms. (Certification herein refers to a geometric and topological coherence between the input and the output of an algorithm.) Meeting these two goals simultaneously requires a close collaboration between Mathematics —Differential Topology and Geometry, and Computer Science —Computational Geometry and Solid Modeling. Bridging the gap between these disciplines is not traditional and contributes to the main innovative aspect of this project. The goals pursued cover theoretical and applied aspects. On the one hand, the project aims at developing a mathematical theory for geometric and topological approximation. On the other hand, implementation and efficiency issues of algorithms will also be addressed. In particular, algorithms will be validated on a fairly large spectrum of applications involving 3D models in the scope of Digital Geometry Processing.

This project coordinated by GEOMETRICA also involves researchers from the INRIA team-project ABS, CNRS (Grenoble, ENS Paris), and a representative from the industry holding a PAST position (Visiting Professor from Industry) at the university of Grenoble.

- Starting date: 2006.

- Duration: 3 years.

8.1.5. ANR Gyroviz

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Laurent Saboret, Nader Salman, Mariette Yvinec.

The Gyroviz project was selected by the ANR in the framework of the call Audivisual and Multimedia techniques. The project, which was started December 2007 for three years, involves the SME Sofresud (Toulon, coordinator) and IXSEA and research teams from the CEA, INRIA and SupMECA Toulon. The project addresses the challenge of automatic modeling of 3D physical scenes from located frames. The aim of the project is to couple new accurate inertial sensors with an image acquisition device and efficient reconstruction algorithms to obtain an automatic image-based modeling system.

- Starting date: December 2007.

- Duration: 3 years.

8.1.6. DIGITEO project GAS: Geometry Algorithms and Statistics

Participants: Claire Caillerie, Frédéric Chazal, David Cohen-Steiner, Bertrand Michel, Steve Oudot.

The project GAS was selected by the DIGITEO consortium in the framework of the “Domaines d’Intérêt Majeur” call of the Région Île-de-France. The project intends to explore and to develop new research at the crossing of information geometry, computational geometry and statistics. It started in September 2008 for an expected duration of 2 years. The other partners of the project are the Ecole Polytechnique (F. Nielsen) and the SELECT project-team (G. Celeux, P. Massart).

- Starting date: September 2008.

- Duration: 2 years.

8.2. Actions Funded by the EC

8.2.1. STREP FET Open ACS

Participants: Jean-Daniel Boissonnat, Andreas Meyer, Sylvain Pion, Laurent Rineau, Monique Teillaud, Camille Wormser, Mariette Yvinec.

In collaboration with Frédéric Cazals, member of the equipe-projet ABS. *Project web site: http://acs.cs.rug.nl.*

INRIA (project-teams GALAAD, GEOMETRICA and ABS) participates to the IST project ACS.
– Acronym : ACS, IST-006413
– Title : Algorithms for complex shapes with certified topology and numerics.
– Specific program : IST
– STREP (FET Open)
– Starting date : May 1st, 2005 - Duration : 3 years
– Participation mode of INRIA : Participant
– Other participants : Rijksuniversiteit Groningen, Eidgenössische Technische Hochschule Zürich, Freie Universität Berlin, Institut National de Recherche en Informatique et Automatique, Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V., National Kapodistrian University of Athens, Tel Aviv University, GEOMETRY FACTORYSarl
The ACS project aims at advancing the state of the art in computing with complex shapes. Current technology can cope well with curves in the plane and smooth surfaces in three-dimensional space. We want to address a larger class of shapes, including piecewise smooth surfaces, surfaces with singularities, as well as manifolds of codimension larger than one in moderately high dimension.

Increasingly demanding applications require efficient and robust algorithms for complex shapes. Topics that arise and that we address are shape approximation (including meshing and simplification), shape learning (including reconstruction and feature extraction), as well as robust modeling (including Boolean operations). Our work on these topics will be closely intertwined with basic research on shape representations.

A unique and ambitious feature of our approach is the guaranteed quality of all data structures and algorithms we plan to develop. Through certified topology and numerics, we will be able to prove that the output is topologically and numerically consistent, according to specified criteria. A software prototype, dealing with a restricted class of complex shapes, will demonstrate the feasibility of our techniques in practice.

The project web site includes a detailed description of the objectives as well as all results. The project ACS ended in May 2008.

8.2.2. Coordination action FOCUS K3D

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Mariette Yvinec.

Web page: http://www.focusk3d.eu.

FOCUS K3D (ICT-2007-214993) is a Coordination Action of the European Union's 7th Framework Programme.
The other consortium members are: – Istituto di Matematica Applicata e Tecnologie Informatiche Unità Organizzativa di Genova Consiglio Nazionale delle Ricerche (CNR-IMATI-GE), Italy.
– Center for Research and Technology - Thessaly - Laboratory for Information Technology Systems and Services (CERETETH), Greece.
– École Polytechnique Federale de Lausanne - VRlab (EPFL), Switzerland.
– Fraunhofer-Institut für Graphische Datenverarbeitung, Germany.
– Université de Genève - MIRALab, Switzerland.
– SINTEF, Norway.
– Utrecht University, The Netherlands.

The aim of FOCUS K3D is to foster the comprehension, adoption and use of knowledge intensive technologies for coding and sharing 3D media content in application communities by: (i) exploiting the scientific and technological advances in the representation of the semantics of 3D media to increase awareness of the new technologies for intelligent 3D content creation and management; (ii) building user-driven scenarios to evaluate and adapt the technologies so far developed to the requirements of application environments; and (iii) fostering a shift of role of 3D content users, from passive consumers of technologies to active creators.

- Starting date: March 2008.

- Duration: 2 years.

8.3. International initiatives

8.3.1. Associated team Genepi

Participants: Sylvain Pion, Monique Teillaud.

We are involved in an INRIA associated team with Chee Yap (New York University) around the subjects of generic programming and robustness of geometric algorithms. This work includes the specification of algorithms in terms of concepts of geometries. It also includes the interface between algorithms and data structures, as well as collaborations on robustness issues when dealing with curved objects.

8.3.2. Associated team Geotech

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Mariette Yvinec, Camille Wormser.

Starting from 2007 we have established an INRIA associated team with Craig Gotsman, Gill Barequet and Ayellet Tal from Technion, Israel. Our goal is to collaborate on topics commonly referred to as Geometry Processing. More specifically, we exchange ideas and software on surface reconstruction, geometric routing, quadrangle surface tiling and shape matching.

8.3.3. Associated team TGDA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Quentin Mérigot, Steve Oudot.

We are involved in an INRIA associated team with Leo Guibas' group at Stanford University since January 2008. Our collaboration focuses on Topological and Geometric Data Analysis. More specifically, we intend to develop new topological and geometric well founded frameworks and algorithms for the analysis of data sets represented by point clouds in possibly non-Euclidean spaces of any dimension.

8.3.4. Scientific and Technological Cooperation between France and Israel

Participants: Jean-Daniel Boissonnat, David Cohen-Steiner, Mariette Yvinec.

In the framework of the Research Networks Program in Medical and Biological Imaging from the High Council for Scientific and Technological Cooperation between France-Israel, we have obtained a financial support for the following project *Geometric reconstruction of organs from freehand ultrasound*. Our Israelian partner is the Technion-Israel Institute of Technology, located in Haifa.

8.3.5. Partenariat Hubert Curien Amadeus with Austria

Participants: Manuel Caroli, Olivier Devillers, Monique Teillaud [coordinator].

http://www.inria.fr/sophia/geometrica/collaborations/Amadeus/

The PHC has been funding our project *Geometric concepts and* CGAL for two years (2007-2008). It is a cooperation with the Institute for Software Technology at Graz University of Technology. Several visits were organized. One common paper is currently submitted [39].

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Editorial boards of scientific journals

- J-D. Boissonnat is a member of the editorial board of *Discrete and Computational Geometry*, *Algorithmica* and the *International Journal of Computational Geometry and Applications*. He is also on the editorial board of the Springer Verlag book series on Geometry and Computing.
- M. Teillaud is a member of the editorial board of the *International Journal of Computational Geometry and Applications*.
- M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.
- P. Alliez, S. Pion (chair), M. Teillaud and M. Yvinec are members of the CGAL editorial board.
- P. Alliez is a member of the editorial board of *The Visual Computer* and *Computers & Graphics*.

9.1.2. Conference program committees

- Pierre Alliez was paper co-chair of the EUROGRAPHICS Symposium on Geometry Processing (SGP) held in Copenhagen. He was a member of the program committees of EUROGRAPHICS, Pacific Graphics, Shape Modeling International, ACM Symposium on Solid and Physical Modeling, and Symposium on 3D Data Processing, Visualization and Transmission.

- Jean-Daniel Boissonnat was a member of the program committee of SGP'08.

- Monique Teillaud was the program committee chair of SoCG'08, 24th Annual Symposium on Computational Geometry.

- F. Chazal was a member of the program committees of SPM'08 and WALCOM'09.

- S. Oudot was a member of the program committee of SGP'08.

- D. Cohen-Steiner was a member of the program committee of SGP'08.

9.1.3. Ph.D. thesis and HDR committees

— Jean-Daniel Boissonnat was a member of the PhD committee of Camille Wormser (EPI Geometrica), A. Illoul (CNAM), B. Vallet (EPI Alice), G. Dupuy (Université de Pau).
— Olivier Devillers was a member of the PhD committee of Abdelkrim Mebarki.
— Mariette Yvinec was a member of the PhD committee of Camille Wormser.
— Frédéric Chazal was a member of the PhD committee of Pierre Gaillard (CEA).
— Pierre Alliez was reviewer for the PhD of Christopher Dyken (University of Olso), Johan Seland (University of Olso), and Olivier Guillot (Université de La Rochelle).
— David Cohen-Steiner was a member of the PhD committee of Dmitriy Morozov (Duke University).

9.1.4. INRIA committees

- Pierre Alliez is member of the « Comité des cours et colloques » at INRIA Sophia Antipolis - Méditerranée, member of the COST GTAI (Conseil d'orientation scientifique et technologique, groupe de travail actions incitatives) and member of the comission d'animation scientifique.
- Agnès Bessière is member of the « Comité des utilisateurs des moyens informatiques des services du centre de recherche INRIA Sophia Antipolis - Méditerranée » (CUMIS)
- Jean-Daniel Boissonnat chairs the INRIA Evaluation Board.
- Monique Teillaud is a member of the INRIA Evaluation Board, the INRIA Sophia Antipolis - Méditerranée Color (COopérations LOcales de Recherche) Commission, and the INRIA Sophia Antipolis - Méditerranée CSD (Committee for Doctoral Studies).
- Mariette Yvinec is member of the « Comité des utilisateurs des moyens informatiques de recherche de l'INRIA Sophia Antipolis - Méditerranée » (CUMIR)
- Frédéric Chazal is a member of the "Commission scientifique" at INRIA Saclay - Île de France

9.1.5. Other committees

- Jean-Daniel Boissonnat is member of the « Commission de spécialistes » of the Ecole Normale Supérieure de Paris.
- Sylvain Pion is member of the experts group of AFNOR for the standardization of the C++ language within the ISO/WG21 working group.
- Sylvain Pion is member of the IEEE-1788 working group for standardization of interval arithmetic.

9.1.6. Conference organization

- Monique Teillaud organized, with Andreas Fabri from GEOMETRY FACTORY, the CGAL Prospective Workshop on Geometric Computing in Periodic Spaces, October 20, 2008.
Several researchers form various fields (mechanics, material engineering, cosmology, astrophysics, biology) met at this workshop. They all share the fact that they make simulations in periodic domains. Our prototype software computing 3D periodic triangulations [25], [26], [36] was presented to them, and they expressed their needs. http://www.cgal.org/Events/PeriodicSpacesWorkshop/.

- Sylvain Pion organized, with Jean-Paul Rigault from the PULSAR project-team, a one week meeting of WG21, the ISO working group for the standardization of the C++ programming language, in June at INRIA Sophia Antipolis - Méditerranée. There were about 60 attendants, working on finishing the next major revision of the standard, C++0x. At this opportunity, two public talks have been given, the first one by Bjarne Stroustrup, the inventor of C++, and the other one by Lawrence Crowl from Google, the videos can be found at http://www.inria.fr/sophia/geometrica/events/WG21_meeting_june_2008/public_talks.html.

9.1.7. Web site

- Monique Teillaud is maintaining the Computational Geometry Web Pages http://www.computational-geometry.org/, hosted by INRIA. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual Symposium on Computational Geometry.

9.2. Teaching

9.2.1. Teaching responsibilities

- Olivier Devillers is professor «Chargé de cours» at École Polytechnique.
- Pooran Memari, Quentin Mérigot, Jane Tournois and Camille Wormser are monitor fellows and have teaching duties in undergraduate courses at Nice university.
- Sylvain Pion is a member of the admission jury at the Ecole Normale Supérieure.
- Mariette Yvinec is responsible for the course on Geometric Approximation at MPRI.
- Pierre Alliez is responsible for the course "Maillages 3D et Applications" at the Ecole Nationale des Ponts et Chaussées.

9.2.2. Teaching at universities

We give here the details of graduate courses. Web pages of these courses can be found on the web site : http://www.inria.fr/sophia/geometrica/
- **2007-2008** courses (tought in 2008)
- Cours du Diplôme d'Informatique de l'ENS, Introduction au calcul scientifique et ses applications - Calcul géométrique certifié, Monique Teillaud (2h).
- Master STIC 1ère année (Nice), Computational Geometry, 24h (O. Devillers 12h, C. Wormser 12h).
- MPRI (Master Parisien de Recherches Informatiques), Cours de 2ième annee , Objet fondamentaux de la géométrie algorithmique, Introduction à l'approximation géométrique et topologique, Frédéric Chazal (9h).
- ENPC (Ecole Nationale des Ponts et Chaussées, Paris), Maillages 3D et applications, 21h, (P. Alliez, J.-P. Pons, G. Peyré).

- **2008-2009** courses (tought in 2008)

- Conférence de rentrée à l'ENS Cachan, Algorithmique des triangulations et modélisation géométrique, Jean-Daniel Boissonnat (3h).
- Master IFI (Sophia Antipolis), Geometric algorithms, theory and practice, 36h, (P. Alliez, O. Devillers and M. Teillaud).
- MPRI (Master Parisien de Recherches Informatiques), Cours de 2ième annee, Approximation géométrique : maillages et reconstruction, Jean-Daniel Boissonnat, Frédéric Chazal and Mariette Yvinec (24h).

9.2.3. Internships

Internship proposals can be found on the web at http://www.inria.fr/sophia/geometrica/
- Mridul Aanjaneya, *Triangulations of Orbifolds*, IIT Kharagpur.
- Vicente H. F. Batista, *Parallel Delaunay triangulations in* CGAL, Universidade Federal do Rio de Janeiro/COPPE, Brazil.
- Saurabh Chakradeo, *Efficient Intersections and Projectionss for Polyhedral Surfaces*, IIT Bombay.
- Chao Chen, *Total persistence*, RPI New-York. - Yuanmi Chen, *Anisotropic meshes*, ENS Paris.
- Benjamin Galehouse, *d-dimensionl quadtrees for* CGAL, New York University.
- Amit Gupta, *Poisson Surface Reconstruction*, IIT Bombay.
- Biswanath Patel, *Local optimisation of meshes*, IIT Bombay.

9.2.4. Ongoing Ph.D. theses

- Claire Caillerie, *Sélection de modèles pour l'inférence géométrique*, Université Paris XI.
- Manuel Caroli, *New Spaces for Computational Geometry*, université de Nice-Sophia Antipolis.
- Pedro Machado Manhães de Castro, *Triangulating sets of moving points*, université de Nice-Sophia Antipolis.
- Arijit Ghosh, *Computational Information Geometry*, université de Nice-Sophia Antipolis.
- Pooran Memari, *Reconstruction from unorganized cross-sections*, université de Nice-Sophia Antipolis.
- Quentin Mérigot, *Détection de structures géométriques dans les nuages de points*, université de Nice-Sophia Antipolis.
- Nicolas Montana, *Calcul robuste d'enveloppes de solides en mouvement. Application à la simulation de l'enlèvement de matière en usinage*, Université Paris-Sud et Dassault Systèmes.
- Thanh-Trung Nguyen, *Geometric Optimization for the Conception of Telescopes*, université de Nice-Sophia Antipolis.
- Nader Salman, *Reconstruction de surfaces lisses par morceaux*, université de Nice-Sophia Antipolis
- Jane Tournois, *Maillages optimisés*, université de Nice-Sophia Antipolis.

9.2.5. Ph.D. defenses

- Abdelkrim Mebarki – *Structures de données compactes pour la géométrie*, Ph.D. defense, Université de Nice Sophia Antipolis, April 15, 2008 [11].
- Camille Wormser, *Diagrammes de Voronoï généraux et applications*, Ph.D. defense, Université de Nice Sophia Antipolis, December 1, 2008 [12].

9.3. Participation to conferences, seminars, invitations

9.3.1. Invited Talks

- Pierre Alliez "Optimization Techniques for Geometry Processing". EuroCG, Nancy, March 19, 2008.
- Jean-Daniel Boissonnat, "Delaunay Refinement for Manifold Approximation", Seventh conference on Mathematical Methods for Curves and Surfaces, Toensberg, Norway, June 2008.
- Camille Wormser, "Delaunay Refinement for Manifold Approximation", Mini-symposium on Computational Geometry and Topology, SIAM Conference on Discrete Mathematics, June 16-19, 2008.
- Jean-Daniel Boissonnat, "From Segmented Images to Good Quality Meshes". Emerging Trends in Visual Computing, November 18-20, 2008.
- Jean-Daniel Boissonnat, "Anisotropic Mesh Generation". Colloque Approximation, Modélisation Géométrique et Applications. Luminy, 24-28.11.2008.

9.3.2. Conferences and Seminars

Members of the project have presented their published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars or summer schools.
- Pierre Alliez, "Mesh generation and optimization with CGAL", Technion, January 2008.
- Pierre Alliez, "Variational Surface Reconstruction", Tel Aviv, January 2008.
- Sylvain Pion, "Robustesse, efficacité et généricité dans la bibliothèque CGAL", séminaire Performance et Généricité, Laboratoire de Recherche et Développement de l'EPITA, January 2008
- Sylvain Pion, "CGAL: The Open Source Computational Geometry Algorithms Library", Google Tech Talks series, Google, Mountain View, CA, USA, March 2008
- Sylvain Pion, "CGAL: The Open Source Computational Geometry Algorithms Library", Parasol seminar, Texas A&M University, College Station, USA. March 2008
- Pierre Alliez, "Polygon Mesh Processing", Eurographics tutorial, 14-16 April 2008. - Monique Teillaud, "Projective triangulations", New York University. June 2008
- Frédéric Chazal, "Analysis of scalar fields over point cloud data", Stanford University. July 2008
- Olivier Devillers, "Predicates for 3D visibility", Graz computational geometry seminar. November 2008
- Manuel Caroli, "3D periodic triangulations", Graz computational geometry seminar. December 2008

9.3.3. The Geometrica seminar

http://www.inria.fr/sophia/geometrica/
The GEOMETRICA seminar featured presentations from the following visiting scientists:
- Chao Chen (RPI New-York) : Quantifying homology classes.
- Luca Castelli Aleardi (Polytechnique) : Schnyder woods for higher genus triangulated surfaces.
- Hang Si (WIAS Berlin) : Constrained Delaunay triangulations and algorithms.
- Philip Jenke (Tuebingen) : Bayesian surface reconstruction.
- Johannes Singler (Karlsruhe) : The GNU libstdc++ parallel mode.
- Eric Sonnendrucker (Strasbourg) : Modélisation et simulation numérique de tokamaks.
- Patrick Mullen (Caltech) : Variational Eulerian geometry processing of surfaces and foliations.
- Peter Gottschling (TU Dresden) : Generic high performance numerics.
- Cécile Dobrzynski (IMB) : Autour des techniques de remaillage local pour les maillages en tetraèdres.
- Frédéric Alauzet (INRIA) : Modèle continu de maillage et adaptation de maillage anisotrope.
- Mirela Ben Chen (Technion) : Conformal flattening by curvature prescription and metric scaling.
- Takashi Kanai (Tokyo University) : Recent advances of SLIM surfaces.
- Jean-Marie Mirebeau (UPMC) : Maillages bidimensionnels anisotropes optimaux pour les éléments finis.
- Wolfgang Aigner (TU Graz) : Computing the medial axis of 2D curved objects.
- Dominique Chapelle (INRIA) : Extraction de mouvement dans des images par des méthodes d'estimation par filtrage utilisant des systèmes mécaniques distribués.

9.3.4. Scientific visits

- Sylvain Pion visited Texas A&M University in March.
- Olivier Devillers visited EPI-VEGAS in March and June.
- Monique Teillaud visited New York University in June.
- Olivier Devillers (in November), Manuel Caroli and Monique Teillaud (in December) visited the Institute for Software Technology at Graz University of Technology.
- Pierre Alliez visited Technion in January.
- David Cohen-Steiner visited Stanford and Duke universities in July.

The following researchers have been visiting GEOMETRICA
- Patrick Mullen (Caltech), six weeks in June-July.
- Mirela Ben Chen (Technion), one week in July.
- Johannes Singler (Universität Karlsruhe), two weeks in May.
- Oswin Aichholzer, Wolfgang Aigner and Bernhard Kornberger (TU Graz), one week in November.
- Hazel Everett and Sylvain Lazard (Loria), one week in December.

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