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*Project-Team Magique-3D*

*Modélisation avancée en Géophysique 3D*

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# 1. Team

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## 2. Overall Objectives

### 2.1. General setting

The MAGIQUE-3D project is associated to two laboratories of University of Pau (Department of Applied Mathematics - LMA associated with CNRS, and Department of Modeling and Imaging in Geosciences - MIGP, also associated with CNRS). Gathering several researchers of different backgrounds in geophysics, physics, mathematics and scientific computing, the main purpose of this project is to establish a link between progress in high-resolution 3D scientific computing and various fields of geophysics. We wish on the one hand to develop sophisticated modeling tools (by integrating physical aspects of the phenomena under study) and to validate them in a rigorous way, and on the other hand to apply them to real cases of geophysical interest.

A first strength of this project is its intrinsically multi-disciplinary character. Moreover, the topics studied can lead to applications in fields other than seismology or seismic studies in the oil industry, for instance medical tomography, non destructive testing of materials or underwater acoustics.

A second strength of this project is that it is strongly related to the regional and national industrial environment, in particular regarding how to go from theoretical studies of relatively complex media to real applications of the methods developed for real cases encountered in the field. Our main industrial partner is TOTAL, whose main research center is located in Pau. We develop strong collaborations with this petroleum company. The project could also establish links with the industrial valorization unit of University of Pau (ValUPPA), which would ensure close contacts between the researchers of the project and the PME/PMI (small and medium-size companies) of the region of Pau.

Since the MAGIQUE-3D team has been created (January 2005), its overall objectives have been reorganized to answer to numerous questions from its main industrial partner TOTAL. MAGIQUE-3D is now a team-project (from July 2007) and its research program is composed of two main topics that structure the original parts of the activities of the group. The first topic, entitled 'Depth Imaging', is related to modeling of seismic wave propagation in complex geological structures, taking into account underlying physical phenomena. The main part of our research program has been defined jointly by working groups composed of members of MAGIQUE-3D and of our main industrial partner TOTAL. The second topic, that could be given the general title 'Advanced modeling in wave propagation', is related to the realistic numerical simulation of complex three-dimensional geophysical phenomena and its comparison with real data recorded in the field. We also participate in different research programs that allow us to work on other aspects of scientific computing in the context of external collaborations.

### 2.2. Highlights of the year

The MAGIQUE-3D group has been selected to organize the next international congress on Mathematical and Numerical Aspects of Waves Propagation, the so-called WAVES09 conference which will take place from 15 till 19 June 2009 to the Palais Beaumont in Pau. The last edition took place to Reading in Great Britain and gathered more than 230 participants.

## 3. Scientific Foundations

### 3.1. Modeling

The main activities of Magique-3D in modeling concern the derivation and the analysis of models that are based on mathematical physics and are suggested by geophysical problems. In particular, Magique-3D considers equations of interest for the oil industry and focus on the development and the analysis of numerical models which are well-adapted to solve quickly and accurately problems set in very large or unbounded domains as it is generally the case in geophysics.

## 3.2. Depth Imaging

The research program of MAGIQUE-3D for depth imaging is divided into two topics that both deal with the solution of the wave equation in a complex 3D medium but that differ in terms of the methods that are used. The first topic is entitled "One-way and two-way models" and is based on numerical micro-local analysis. From a practical point of view, such investigations are supposed to lead to the solution of approximate wave equations but they have the advantage of giving rise to fast solution methods which, if not accurate in general situation, can provide approximate solutions that can subsequently be used as data to initialize implicit schemes for solving full wave equations. The second topic is entitled "High-performance methods for solving wave equations". It is more classical and deals with the development of methods of resolution that are innovative and based on finite element methods. We essentially focus on the development of mixed hybrid methods which are based on the combination of Lagrange polynomials and plane waves. By using finite element discretization, we can take the topography into account which allows us to outperform existing Reverse Time Migration methods based on finite difference methods.

## 3.3. Wave propagation in porous media

The propagation of waves in porous media is of interest in many geophysics applications. MAGIQUE-3D develops different axes of research on this subject. By using numerical methods like finite differences, finite elements or meshless techniques like the various boundary integral methods (Boundary Integral Method, Indirect Boundary Element Method, Meshless Galerkin Method...) we aim at solving the equations describing porous media. MAGIQUE-3D has started to work on Biot models. The first studies that we have undertaken were devoted to the mathematical analysis of a linear and nonlinear model coupling an elastic wave equation written in the solid structure and a diffusion equation written in the fluid. We currently work on numerical modeling of Biot's model, we develop a numerical method based on the one hand on the use of a three-dimensional spectral-element method for space discretization, which is a finite-element method with an exactly diagonal mass matrix, and on the other hand on an explicit scheme for time discretization.

## 3.4. High Performance methods for solving wave equations

Nowadays the wave equation can be solved with very good accuracy using finite-element methods but the main difficulty that remains is related to the very large amount of computer memory storage that is required, in particular in 3D. Moreover, the solution of the time dependent wave equation with classical finite elements requires the inversion of a very large mass matrix at each time step. To avoid this problem, we develop space-discretization methods such as spectral element methods or Discontinuous Galerkin methods which turn the mass matrix onto an easily invertible (block)-diagonal matrix. We also try to improve the time-discretization methods by using local time stepping schemes which enables us to use a small time step only on the part of the mesh it is needed and to use a coarse time step everywhere else. From a computational point of view, we dedicated a large part of our effort on parallel computing and we have established since the beginning of 2006 a collaboration with the Barcelona Supercomputing Center (BSC, Spain).

## 3.5. Absorbing Boundary Conditions and Perfectly Matched Layers

In most of the problems of wave propagation we have to deal with unbounded domains. It is then helpful to define (artificial) boundaries defining the numerical model under study in order to reduce the computational costs.

Since the innovative work of [56], this question was addressed by using Absorbing Boundary Conditions (ABC) [52] or damping zones (sponge layers) [50]. In both cases, results are often not very satisfactory because spurious phases are reflected inside the computational domain, in particular at grazing incidence in the case of paraxial conditions, and at low frequency in the case of sponge layers. However these conditions are easy to include in numerical schemes and can be constructed such that they preserve the sparsity of the discrete matrix. This justifies MAGIQUE-3D works on the improvement of ABCs.

More recently, Bérenger [49] introduced an innovative condition for Maxwell's equations, which has the property of being perfectly adapted to the model, in the sense that no spurious phase is produced in the domain before discretization and use of a numerical scheme. The resulting model is called a Perfectly Matched Layer (PML). Because of its efficiency, the PML method quickly became very popular in electromagnetics. Next, based on an analogy between Maxwell's equations and linear elasticity written as a first order system in velocity and stress, several authors adapted the PML approach to the propagation of elastic waves in infinite domains (see e.g. [51], [53]). However, for instance in the case of elastodynamics (and even more for porous media) and for shallow-water equations, the classical PMLs are known to be unstable. Moreover the layers do not properly handle grazing rays which gives rise to spurious reflections. This is why MAGIQUE-3D develops new PML models both for elastodynamics, porous media and geophysical fluids.

## 4. Application Domains

### 4.1. Seismic Imaging

The main objective of modern seismic processing is to find the best representation of the subsurface that can fit the data recorded during the seismic acquisition survey. In that context, the seismic wave equation is the most appropriate mathematical model. Numerous research programs and related publications have been devoted to this equation. An acoustic representation is suitable if the waves propagate in a fluid. But the subsurface does not contain fluids only and the acoustic representation is not sufficient in the general case. Indeed the acoustic wave equation does not take some waves into account, for instance shear waves, turning waves or the multiples that are generated after several reflections at the interfaces between the different layers of the geological model. It is then necessary to consider a mathematical model that is more complex and resolution techniques that can model such waves. The elastic or viscoelastic wave equations are then reference models, but they are much more difficult to solve, in particular in the 3D case. Hence, we need to develop new high-performance approximation methods.

Reflection seismics is an indirect measurement technique that consists in recording echoes produced by the propagation of a seismic wave in a geological model. This wave is created artificially during seismic acquisition surveys. These echoes (i.e., reflections) are generated by the heterogeneities of the model. For instance, if the seismic wave propagates from a clay layer to sand, one will observe a sharp reflected signal in the seismic data recorded in the field. One then talks about reflection seismics if the wave is reflected at the interface between the two media, or talks about seismic refraction if the wave is transmitted along the interface. The arrival time of the echo enables one to locate the position of this transition, and the amplitude of the echo gives information on some physical parameters of the two geological media that are in contact. The first petroleum exploration surveys were performed at the beginning of the 1920's and for instance, the Orchard Salt Dome in Texas (USA) was discovered in 1924 by the seismic-reflection method.

### 4.2. Seismology

We already applied our techniques to the study of strong ground motion and associated seismic risk in the Los Angeles basin area. This region consists of a basin of great dimension (more than  $100 \text{ km} \times 100 \text{ km}$ ) which is one of the deepest sedimentary basins in the world (the sedimentary layer has a maximum thickness of 8.5 km underneath Downtown Los Angeles), and also one of the most dangerous in the world because of the amplification and trapping of seismic waves. In the case of a small earthquake in Hollywood (September 9, 2001), well recorded by more than 140 stations of the Southern California seismic network TriNet, we managed for the first time to fit the three components of the displacement vector, most of the previous studies focusing on the vertical component only, and to obtain a good agreement until relatively short periods (2 seconds).



We wish to improve these studies of seismic risk in densely populated areas by considering other regions of the world, for example the Tokyo basin, the area of Kobe or the Mexico City region. We also plan to generalize this type of calculations to the knowledge and modeling of site effects, i.e. of the local amplification of the response of the ground to seismic excitation. The study of such effects is an important observation in urban areas to be able to anticipate the damage to constructions and, if necessary, to plan the organization of search and rescue operations. It is also a significant element of the definition of paraseismic standards. Site effects can be determined experimentally, but that requires the installation of stations for a sufficient period of time to record a few tens of seismic events. Numerical modeling makes it possible to avoid this often long and difficult experimentation, assuming of course that one has good knowledge of the geological structure of the subsurface in the studied area. We thus propose in the MAGIQUE-3D project to use the numerical techniques mentioned above for instance to quantify the effects of topographic variations in the structure.

### 4.3. Non destructive testing, Medical Imaging

The problems of seismic imaging can be related to non destructive testing, in particular medical imaging. For instance, the rheumatologist are now trying to use the propagation of ultrasounds in the body as a noninvasive way to diagnose osteoporosis. Then, the bones can be regarded as elastodynamic or poroelastic media while the muscles and the marrow can be regarded as acoustic media. Hence the computational codes we use for seismic imaging could be applied to such a problem.

### 4.4. Submarine acoustics

The SPEC-FEM software package is able to simulate the propagation of waves in the context of time domain fluid-structure interaction. We have actually started to consider underwater acoustics in collaboration with P. Cristini (MIGP, UPPA) and F. Sturm (Laboratoire de Mécanique des Fluides et d'Acoustique, École Centrale de Lyon, France).

## 5. Software

### 5.1. SPEC-FEM3D

The MAGIQUE-3D project is based (in part) on existing software packages, which are already validated, portable and robust. The SPEC-FEM3D software package, developed by Dimitri Komatitsch and his colleagues in collaboration with Jeroen Tromp and his colleagues at the California Institute of Technology and at Princeton University (USA), and which is still actively maintained by Dimitri Komatitsch and his colleagues, allows the precise modeling of seismic wave propagation in complex three-dimensional geological models. Phenomena such as anisotropy, attenuation (i.e., anelasticity), fluid-solid interfaces, rotation, self-gravitation, as well as crustal and mantle models can be taken into account. The software is written in Fortran95 with MPI message-passing on parallel machines. It won the Gordon Bell Prize for best performance of the Supercomputing'2003 conference. In 2006, Dimitri Komatitsch established a new collaboration with the Barcelona Supercomputing Center (Spain) to work on further optimizing the source code to prepare it for very large runs on future petaflops machines to solve either direct or inverse problems in seismology. Optimizations have focused on improving load balancing, reducing the number of cache misses and switching from blocking to non-blocking MPI communications to improve performance on very large systems. Because of its flexibility and portability, the code has been run successfully on a large number of platforms and is used by more than 150 academic institutions around the world. In November 2008 this software package was again among the six finalists of the prestigious Gordon Bell Prize of the SuperComputing'2008 conference in the USA [34] for a calculation performed in parallel on 150,000 processor cores, reaching a sustained performance level of 0.16 petaflops.

## 5.2. Gar6more2D and Gar6more3D

The softwares Gar6more2D and Gar6more3D compute the solution to the problem of wave propagation in infinite bilayered acoustic/acoustic, acoustic/elastodynamic, acoustic/porous or porous/porous media, respectively in two and three dimensions. They are both written in FORTRAN90 and Gar6more3D has been parallelized using MPI. Gar6more2D has already been used for the validation of poroelastic codes. Both softwares are distributed under a CeCILL license at the url <http://www.spice-rtn.org/library/software/Gar6more2D/> and <http://www.spice-rtn.org/library/software/Gar6more3D/>.

# 6. New Results

## 6.1. Inverse acoustic problems

**Keywords:** *Acoustic scattering problems, Inverse Obstacle problems, Limited aperture, regularized newton method, total variation.*

**Participants:** Chokri Bekkey, H el ene Barucq, Rabia Djellouli.

The determination of the shape of an obstacle from its effects on known acoustic or electromagnetic waves is an important problem in many technologies such as sonar, radar, geophysical exploration, medical imaging and nondestructive testing. This inverse obstacle problem (IOP) is difficult to solve, especially from a numerical viewpoint, because it is ill-posed and nonlinear [54]. Moreover, the precision in the reconstruction of the shape on an obstacle strongly depends on the quality of the given far-field pattern (FFP) measurements: the range of the measurements set and the level of noise in the data. Indeed, the numerical experiments performed in the resonance region (for example [65], [67], [62], [63]), that is for a wavelength that is approximately equal to the diameter of the obstacle, tend to indicate that in practise, and at least for simple shapes, a unique and reasonably good solution of the IOP can be often computed using only one incident wave and *full aperture* far-field data (FFP measured only at a limited range of angles), as long as the aperture is larger than  $\pi$ . For smaller apertures the reconstruction of the shape of an obstacle becomes more difficult and nearly impossible for apertures smaller than  $\pi/4$ .

Given that, and the fact that from a mathematical viewpoint, the FFP can be determined on the entire  $S_1$  from its knowledge on a subset of  $S_1$  because it is an *analytic* function, we propose a solution methodology to extend the range of FFP data when measured in a limited aperture and not on the entire sphere  $S_1$ . Therefore it would be possible to solve numerically the IOP when only limited aperture measurements are available. However, due to the analyticity nature of the FFP, the reconstruction or the extension of the far-field pattern from limited measurements is an inverse problem that is *severely ill-posed* and therefore very challenging from a numerical viewpoint. Indeed preliminary numerical results [43] indicate that the reconstruction of the FFP using the discrete  $L_2$  minimization with the standard Tikhonov regularization is very sensitive to the noise level in the data. The procedure is successful only when the range of measurements is very large which is not realistic for most applications.

We propose a multi-step procedure for extending/reconstructing the FFP from the knowledge of limited measurements. The proposed solution methodology addresses the ill-posedness nature of this inverse problem using a *total variation* of the FFP coefficients as a penalty term. Consequently the new cost function is no longer differentiable. We restore the differentiability to the cost function using a perturbation technique [66] which allows us to apply the Newton algorithm for computing the minimum. The multi-step feature of the proposed method consists in extending the FFP at each step by an  $n$  degrees increment.

We investigate the effect of the frequency regime and the noise level of the performance of the proposed solution methodology. Preliminary results obtained in the case of two-dimensional sound-soft disk-shaped scatterer have been performed. They illustrate the potential of the solution methodology for enriching the FFP measurements for various frequencies and levels of noise.

## 6.2. A new modified equation approach for solving the wave equation

**Keywords:** *High order schemes, acoustic wave equation, discontinuous Galerkin method.*

**Participants:** Cyril Agut, Hélène Barucq, Julien Diaz, Abdelaâziz Ezziani.

In this work, we construct new fourth order schemes in space and time for the wave equation by applying the modified equation technique in an original way. Indeed, most of the works devoted to the solution of the wave equation consider first the space discretization of the system before addressing the question of the time discretization. We intend here to invert the discretization process by applying first the time discretization thanks to the modified equation technique and after to consider the space discretization. After the time discretization, an additional bilaplacian operator, which can not be discretized by the classical finite elements appears. If the acoustic medium is homogeneous or has smooth heterogeneities, the solution is C1 and we therefore we have to consider C1 finite elements (such as the Hermite ones) or Discontinuous Galerkin finite elements (DGFE) whose C1 continuity is enforced through an appropriate penalty term. In a strongly heterogeneous media, the solution is no longer C1 because of the discontinuities of the physical parameters and Hermite elements are not adapted to this problem. DGFE can however be used by imposing the continuity of a suitable physical quantity (corresponding to the classical transmission conditions) instead of the C1 continuity. We have compared the solution obtained by the new method with DGFE to the one obtained with a classical second order method (using also DGFE) in 1D and 2D in a simple homogeneous medium. The results show that the method with the bilaplacian is actually fourth order with the same computational burden as the classical second order method. The numerical tests in heterogeneous media is a work in progress.

## 6.3. An unsplit convolutional Perfectly Matched Layer improved at grazing incidence for the seismic wave equation

**Keywords:** *Elastodynamics, Grazing Incidence, Perfectly Matched Layer.*

**Participants:** Dimitri Komatitsch, Roland Martin, Abdelaâziz Ezziani.

The Perfectly Matched Layer (PML) absorbing technique has become popular in numerical modeling in elastic or poroelastic media because of its efficiency to absorb waves at non-grazing incidence. However, after numerical discretization, at grazing incidence large spurious oscillations are sent back from the PML into the main domain. The PML then becomes less efficient in the case of sources located close to an edge of the truncated physical domain under study, in the case of thin slices or for receivers located at large offset. In a previous article [61], we developed a PML improved at grazing incidence for the elastic wave equation based on an unsplit convolutional formulation for the seismic wave equation written as a first-order system in velocity and stress. This so-called Convolution-PML (CPML) has a cost that is similar in terms of memory storage to that of the classical PML. In [27], we introduce a similar technique for the two-dimensional Biot poroelastic equations and show its efficiency for both non dissipative and dissipative Biot porous models based on a fourth-order staggered finite-difference method used in a thin mesh slice. The results obtained are significantly improved compared to the classical PML.

## 6.4. Higher Order Absorbing Boundary Conditions for the Wave Equation applied to Discontinuous Galerkin Methods

**Keywords:** *Acoustic Wave Equation, Discontinuous Galerkin Method, Optimized Absorbing Boundary Conditions.*

**Participants:** Hélène Barucq, Julien Diaz, Véronique Duprat.

The numerical simulation of waves propagation generally involves boundary conditions which both represent the behavior of waves at infinity and provide a mathematical tool to define a bounded computational domain in which a finite element method can be applied. Most of these conditions are derived from the approximation of the Dirichlet-to-Neumann operator and when they both preserve the sparsity of the finite element matrix and enforce dissipation into the system, they are called absorbing boundary conditions. Most of the approximation procedures are justified into the hyperbolic region which implies that only the propagative waves are absorbed. If the exterior boundary is localized far enough from the source field, the approximation is accurate and the absorbing boundary condition is efficient. However, the objective is to use a computational domain whose size is optimized since the solution of waves problems requires to invert matrices whose order is very large and is proportional to the distance between the source field and the exterior boundary. Hence it is a big deal to derive absorbing boundary conditions which are efficient when the exterior boundary is close to the source field and it is necessary to construct conditions which are efficient not only for propagative waves but both for evanescent and glancing waves. In a recent work [13], new conditions have been derived from the modal analysis of the wave equation set in the neighborhood of a prolate spheroidal boundary. From the numerical analysis of the error, it has been proven [14] that these conditions are efficient for each type of waves and then, they outperform the current absorbing conditions. However, the derivation procedure in [13] is based on the representation of the Helmholtz equation in an elliptic coordinate system reproducing the geometry of the exterior boundary and it is not obvious how to generalize the conditions to an arbitrarily-shaped boundary. Recently, a new condition has been derived [57] from an approximation of the Dirichlet-to-Neumann operator which is valid both for propagative and evanescent waves and it extends the condition which was formerly proposed by Higdon [58]. By using a classical finite element scheme, Hagstrom et al. [57] have shown the improvements induced by the new condition. In this work, we intend to analyze if the new condition can be introduced into a discontinuous Galerkin method which is more accurate to reproduce the propagation of waves into heterogeneous media. To analyze the impact of the new condition on the accuracy of the numerical solution, we also consider the Higdon condition and we are able to compare the efficiency of the two conditions when used into a discontinuous Galerkin approximation.

## 6.5. Simulation of seismic wave propagation in an asteroid based upon a non-blocking MPI spectral-element method

**Keywords:** *Meshing, Parallel Computing.*

**Participants:** Dimitri Komatitsch, Roland Martin, Céline Blitz, Nicolas Le Goff, Philippe Lognonné.

In order to better understand the internal structure of asteroids orbiting in the Solar system, and then the response of such objects to impacts, seismic wave propagation in asteroid 433-Eros is performed numerically based on a spectral-element method at seismic frequencies from 0 Hz to 5 Hz. In the year 2000, the NEAR Shoemaker mission to Eros has provided images of the asteroid surface, which contains numerous fractures that likely extend to its interior. Our goal is to be able to propagate seismic waves resulting from an impact in such models. For that purpose we create and mesh both homogeneous and fractured models with a highly-dispersive regolith layer at the surface using the CUBIT mesh generator developed at Sandia National Laboratories (USA). The unstructured meshes are partitioned using the METIS software package in order to minimize edge-cuts and therefore optimize load balancing in our parallel non-blocking MPI implementation. In [26] we perform actual simulations and show that we can obtain good performance levels and good scaling when we implement overlapping of communications with calculations. Additional calculations are also currently being performed at CINES in Montpellier, France.

## 6.6. Numerical methods combining local time stepping and mixed hybrid elements for the terrestrial migration

**Keywords:** *Local Time Stepping, Migration, Mixed hybrid method.*

**Participants:** Caroline Baldassari, H el ene Barucq, Henri Calandra [Expert Engineer, TOTAL], Bertrand Denel [Research Engineer, TOTAL], Julien Diaz.

We work on the development of a software for the Reverse Time Migration of acoustic waves which combines mixed hybrid elements in space and a local time stepping scheme. The discretization in space allows us to take the topography into account, which usually outperforms finite difference schemes. In order to illustrate this point, we have compared the solution obtained with a Discontinuous Galerkin Finite Element Method (DGFEM) to the one obtained with finite difference methods or with the GSP method and we have shown that DGFEM gives much more accurate results. We have also compared the solution obtained with DGFEM to the one obtained with spectral element methods, which requires the use of quadrilateral (in 2D) or hexahedral (in 3D) cells and is therefore less convenient to deal with a topography. Both solutions present the same order of accuracy, which encourages us to use DGFEM to deal with a topography.

Near the topography (or in very thin layers), the cells of the mesh have to be very small compared to the cells far from the topography. In such configuration, it is not useful to use high order elements in the whole domain and we propose to use second order elements in the fine part of the mesh and high order elements in the coarse part. The numerical experiments we have performed show actually that there is no difference between the solutions obtained with meshes composed only of high-order cells and with meshes composed of second order cells near the topography. Moreover the use of second order cells reduces dramatically the computational burden. All these experiments have been performed with a second order time discretization and the next step is to adapt the order of the time discretization to the one of the space discretization, this will be done during the next year.

## 6.7. Geological horizon propagation into large seismic volumes

**Keywords:** *geological horizon propagation, optimized data access, parallelization.*

**Participants:** Jonathan Gallon, Bruno Jobard, H el ene Barucq.

In the petroleum industry, seismic data is the starting point of many processes for the exploration for oil reservoirs. The dimensions of these *seismic cubes* keep increasing as the regions undergoing exploration keep growing and have increased spatial resolution. We are interested in finding efficient algorithms of *geological horizon propagation* that can handle large 3D and 4D (pre-stack) seismic volumes. Such horizons are used for constructing structural sub-surface models. In order to process larger volumes, our strategy consists in to partition them and simultaneously propagate an initial horizon seed in adjacent sub-blocs. To validate this approach, we take special care to ensure that the individual surfaces connect at the bloc limits and that the overall quality is close to the single-threaded propagation. Further performance improvements are planned by organizing the data along a special layout on disk and using cache hierarchies.

## 6.8. Perfectly Matched Layers for the Shallow Water equations

**Keywords:** *Perfectly Matched Layer, Smith factorization, Water Waves.*

**Participants:** H el ene Barucq, Julien Diaz, Mounir Tlemcani [Assistant Professor, University of Oran, Algeria].

The Perfectly Matched Layer (PML) technique for the numerical absorption of waves, initially introduced about 20 years ago by B erenger [49] in electromagnetism, is now widely used for simulating the propagation of waves in unbounded domains, in particular in time domain acoustics. However, this technique induces strong instabilities when applied to Euler equations [60] or to shallow water equations. Much work has been devoted to the stabilization of the PML for linearized Euler equations [59], [64] and we propose here a stable PML for the linearized shallow water equations with a Coriolis term and a uniform mean flow. The technique follows the one proposed by Nataf [64] for linearized Euler equations and relies on the use of the Smith factorization, which allows us to split the vorticity waves from the advective and entropy waves and to treat these three types of waves separately. Indeed the propagation of vorticity and entropy waves is governed by a classical transport equation and these waves can be easily absorbed by a transparent condition at the end of the layer. To absorb the advective waves, we first have to use the transformation proposed by Hu [59] before applying a classical PML technique to avoid an exponential growth of the waves.

## 6.9. Discontinuous Galerkin methods with plane waves basis for Helmholtz's equation in 3D

**Keywords:** *Discontinuous Galerkin Method, Helmholtz Equation, Plane Waves.*

**Participants:** Mohamed Amara, Rabia Djellouli, Magdalena Grigoroscuta.

We have designed a new mixed-hybrid-type solution methodology to be applied for solving high-frequency Helmholtz problems. The proposed approach distinguishes itself from similar methods by a local approximation of the solution with oscillated finite elements polynomials satisfying the wave equation. A weak continuity of the solution across the element interfaces is enforced using Lagrange multipliers. Note that the discontinuous nature of the approximation at the element-level allows to apply static condensation of primal variable prior to assembly. Therefore, the computational cost of the proposed method is reduced, and is mainly dependent on the total number of Lagrange multipliers degrees of freedom, and by the sparsity pattern of the resulting matrix. Hence, the proposed approach combines the features of standard Galerkin finite elements techniques in terms of implementation complexities, and the oscillating aspect of the shape functions needed for approximating waves in the high frequency regime. Preliminary numerical results obtained in the case of two-dimensional wave guides and scattering problems using lower order element (OP41, OP82) clearly illustrate the computational efficiency of the proposed solution methodology. We have also analyzed mathematically the convergence of the proposed method in the case of OP41 element and have established a priori and a posteriori error estimates. We propose here a new discontinuous Galerkin formulation, based on a local approximation of the solution by plane waves that satisfy the wave equation. In order to enforce a weak continuity across the element interfaces, we introduce Lagrange multipliers. The method is built in a variational formulation framework that leads to a linear system associated with a positive definite Hermitian matrix. This matrix results from using a stabilized-like technique. Therefore, we use a preconditioned conjugate gradient algorithm to solve the system without computing the resulting matrix. We have recently applied a regularized-type procedure to the proposed method to address the loss of the stability due to the violation of the inf/sup condition in the case of higher order elements. Consequently the modified mixed hybrid formulation leads to a linear system associated with a positive definite Hermitian matrix.

## 6.10. Analytical computation of the Green's function of bilayered porous media

**Keywords:** *Cagniard-de Hoop, Green's function, porous media.*

**Participants:** Julien Diaz, Abdelaâziz Ezziani, Nicolas Le Goff.

The Cagniard-de Hoop is particularly well known in the physics and engineering communities for calculating analytical solutions of time-dependent wave propagation problems in stratified media, especially in seismology. However, it had never been applied to the wave propagation in heterogeneous porous media. The computation of analytical solutions in this kind of media is particularly interesting to obtain reference solutions for the validation of numerical codes, but it is also useful for a better understanding of the reflexion/transmission properties of the media. Using the Cagniard-de Hoop technique, we have provided the solution to the problem of wave propagation in bilayered acoustic/poroelastic and poroelastic/poroelastic media both in two and three dimensions. We have implemented these solutions in two computational codes, Gar6more2D (for the 2D problem) and Gar6more3D (for the 3D problem).

## 6.11. Improved numerical approaches for seismic wave equation at a fluid-solid interface in the oil industry

**Keywords:** *Time substepping, fluid-solid coupling, high-order finite elements.*

**Participants:** Ronan Madec, Dimitri Komatitsch, Julien Diaz, Pierre Thore.

In the oil industry, many important oil reservoirs are located offshore and it is therefore of interest to be able to simulate seismic wave propagation in deep offshore geological media and it is crucial to reduce the cost of the calculations in the thick but homogeneous and therefore simple water layer. Explicit numerical methods used to model wave propagation must satisfy a stability condition called the Courant-Friedrichs-Lewy condition (CFL) that depends on the maximum velocity of the medium and on the ratio between the time step and the size of a grid cell. In the case of a deep offshore model, the velocity of P waves in the thick homogeneous water layer located upon the ocean bottom is generally slower than in the solid part of the model located underneath the ocean flow and therefore the local CFL is different within each medium. But to ensure numerical stability, one has to take a small global time step which is imposed by the solid part of the medium, and therefore waste some calculation time in the simpler fluid part of the model. An idea that can be used to overcome this is to implement substepping in time to reduce the computation time by taking a local time step and still honor the CFL condition by increasing the time step in the layer of slower velocity (the water layer). We therefore implemented in our spectral-element code SPECFEM3D the method developed in [55] that ensures the conservation of energy along the interface and we adapted it to a high order finite-element method. Taking advantage of the computation time that we save we can therefore start thinking about computing large and more realistic 3D models of interest for the oil industry in the next few months.

Another issue that we have studied in [18] is how to implement plane wave sources with any incidence angle for our spectral-element time-domain method. We also implement absorbing boundary conditions following the ideas of [48], which is crucial to ensure that no significant spurious waves propagate back into the main domain. We validate the method and check its accuracy for the numerical modeling of seismic wave propagation, in particular in order to compute the response of a free surface with topography under the incidence of a plane wave with different angles by comparing the results obtained to the Method of Fundamental Solutions (MFS), which is a new method to solve the wave equation in the frequency domain.

## 6.12. Construction of new absorbing boundary conditions for the numerical simulation of the scattering by elongated obstacles

**Keywords:** *Dirichlet-to-Neumann operators, local conditions Mathieu and spheroidal functions, performance analysis.*

**Participants:** H el ene Barucq, Rabia Djellouli, Anne-Ga elle Saint-Guirons.

We have constructed new approximate local DtN boundary conditions to be applied on elliptical- or prolate spheroid-shaped exterior boundaries when solving respectively two- or three-dimensional acoustic scattering problems by elongated obstacles such as submarines. These new absorbing conditions are designed to be exact for the first modes. They can be easily incorporated in any finite element parallel code while preserving the local structure of the algebraic system. Unlike the standard approximate local DtN boundary conditions that are restricted to circular- or spherical-shaped boundaries, the proposed conditions are applicable to exterior elliptical-shaped boundaries that are more suitable for surrounding elongated scatterers because they yield to smaller computational domains. The mathematical and numerical analysis of the effect of the frequency and the eccentricity values of the boundary on the accuracy of these conditions, when applied for solving radiating and scattering problems, reveals - in particular- that the new second-order DtN boundary condition retains a good level of accuracy, in the low frequency regime, regardless of the slenderness of the boundary.

## 6.13. New mixed hybrid elements combining Lagrange polynomials and plane waves for the solution of the Helmholtz equation

**Keywords:** *Helmholtz Equation, Lagrange Multipliers, Oscillated Polynomials.*

**Participants:** Mohamed Amara, H el ene Barucq, Rabia Djellouli.

We propose a new mixed-hybrid-type solution methodology to be applied for solving high-frequency Helmholtz problems. The proposed approach distinguishes itself from similar methods by a local approximation of the solution with *oscillated* finite elements polynomials *satisfying* the wave equation. The weak continuity of the solution across the element interfaces is enforced using Lagrange multipliers. A convergence analysis of the method has been performed and it shows the strong stability properties of this new discretization. Numerical results obtained in the case of two-dimensional waveguide problems illustrate the computational efficiency of the proposed solution methodology. Now, we are applying the new finite element method for solving the seismic wave equation.

## 6.14. High Performance Computing for the seismic wave equation at very high resolution

**Keywords:** *High-Performance Computing (HPC), Message-Passing Interface (MPI), parallel computing, seismic wave equation.*

**Participants:** Dimitri Komatitsch, Roland Martin, David Michéa, Nicolas Le Goff, Pieyre Le Loher.

With the very rapid evolution of personal computers, computer clusters, and supercomputers, nowadays the seismic wave equation can be solved with very good accuracy using very precise techniques implemented based on parallel computing in the context of so-called High-Performance Computing (HPC). This has been a central part of our research activity in the last few years and even increasingly more in 2008. In particular with some colleagues from the CINES supercomputing center in Montpellier (France) we have performed some very large scale calculations that are currently being analyzed and published.

Using the high-order finite-element method implemented in our SPECFEM3D software package, which we reviewed in [29], in the context of a collaboration with some colleagues in Taiwan we for instance studied the influence of topography modeled at very high resolution on seismic wave propagation in the region of Taipei in Taiwan [23], [24], [22].

In the context of collaborations with some colleagues in Barcelona (Catalonia, Spain), in San Diego, Caltech and Princeton (USA) and in Japan, we also applied the technique to model seismic wave propagation at very high frequency in the whole Earth [20], [30]. In November 2008 our SPECFEM3D software package was again among the six finalists of the prestigious Gordon Bell Prize of the SuperComputing'2008 conference in the USA [34] for a calculation performed in parallel on 150,000 processor cores, reaching a sustained performance level of 0.16 petaflops.

In the context of a collaboration with Gordon Erlebacher from Florida State University (USA) who visited us for a month in May-June 2008 we ported our modeling algorithm to a NVIDIA graphics video card (Graphical Processing Units – GPU) using the CUDA language on top of a C implementation of our code. This technique is known as General-purpose Processing on Graphical Processing Units (GPGPU) and had never been used before for a high-order finite-element technique, which induces significant technical problems in particular regarding memory accesses. In [21] we used it to improve the speed of our code by a huge factor of 25.

Finally, in [16] we used a finite-element code to model stress redistribution in Island following a large earthquake that occurred there in June 2000 and study how changes in the stress field could have had an influence on the triggering of a second earthquake that occurred in the same region a few days later.

## 7. Contracts and Grants with Industry

### 7.1. Contracts with TOTAL

- Modélisation et simulation numérique pour la migration terrestre par équations d'ondes tridimensionnelles.

Period: 2007 January - 2009 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 145000 euros.



- Résolution de l'équation d'Helmholtz 3D par une méthode de Galerkin discontinue DGM utilisant des bases d'ondes planes.

Period: 2007 January - 2009 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 139000 euros.

- Propagation automatique de Surface nD Filtrage et traitement de la sismique avant stack Period: 2008 January - 2010 december, Management: INRIA Bordeaux Sud-Ouest, Amount: 45000 euros.

## 7.2. Contract with CSUN

In the context of the Associate Team MAGIC.

Period: 2006 January - 2008 December, Total Amount: 46000 USD

# 8. Other Grants and Activities

## 8.1. Depth Imaging Partnership

MAGIQUE-3D maintains active collaborations with TOTAL. In the context of depth imaging and with the collaboration of Henri Calandra from TOTAL, MAGIQUE-3D coordinates research activities dealing with the development of high-performance numerical methods for solving wave equations in complex media. This project involves French academic researchers in mathematics, computing and in geophysics, and is funded by TOTAL. At present time, two PhD students are working in MAGIQUE-3D from 2007 on new numerical imaging methods that are based on the solution of the full wave equation. Two Ph.D students advised by J. Roman and S. Petiton respectively started to work in January 2008 on computing aspects for optimizing the computational performances of our numerical methods.

The different partners plan to work jointly on topics dealing with solving wave equations. We intend to increase the numbers of participants and new research topics are supposed to be considered in 2009 after a kick-off meeting that will be organized in Pau in March 2009.

To our knowledge, our network is the first in the French research community to establish links between industrial and academic researchers in the context of a long-term research program managed by an INRIA team.

## 8.2. ANR Project NUMASIS

MAGIQUE-3D participates in an ANR research program called NUMASIS managed by J.F. Méhaut (INRIA Rhône-Alpes, Grenoble). In this context we naturally collaborate with SCALAPPLIX and RUNTIME from INRIA Bordeaux Sud-Ouest. The main idea of the NUMASIS project is that multiprocessor machines of tomorrow will possess a NUMA architecture introducing multiple levels of hierarchy in computers (multi-modules, multi-core chips, multithreading systems, etc). To use them efficiently, parallel applications must have powerful tools making it possible to guide the distribution of execution and data flows without compromising their portability. The NUMASIS project proposes to evaluate the functionalities provided by current systems, to apprehend their limitations, to design and implement new management mechanisms for processes, data and communications within the basic software (operating system, libraries). The application field selected for NUMASIS is seismology, which appears to us to be representative of current needs in scientific computing.

Numerical modeling of seismic wave propagation in complex geological media is one of the significant research topics in seismology. Various approaches will be studied and their adequacy compared to specificities of NUMA machines will be evaluated. The various calculations will be based on modern numerical algorithms such as spectral elements, high-order finite differences or finite elements applied to realistic 3D models. The NUMASIS project will study problems of parallel algorithms (distribution, scheduling) making it possible to optimize the calculations based on these schemes by using as efficiently as possible the execution frameworks developed for these NUMA architectures.

The NUMASIS project started on January 1, 2006 under reference ANR-05-CIGC-002 and was supposed to end on December 31, 2008 but has been extended for one more year (it will therefore finish at the end of 2009). In the context of this project, MAGIQUE-3D has obtained 84000 euros for three years, which was used mostly to fund the salary of a software engineer (David Michéa, who worked for MAGIQUE-3D on a temporary two-year contract (CDD) from November 1, 2006 to October 15, 2008).

### 8.3. ANR Project AHPI

The endeavour of this project is to develop some methodology for modelling and solving certain inverse problems using tools from harmonic and complex analysis. These problems pertain to deconvolution issues, identification of fractal dimension for Gaussian fields, and free boundary problems for propagation and diffusion phenomena. The target applications concern radar detection, clinical investigation of the human body (e.g. to diagnose osteoporosis from X-rays or epileptic foci from electro/magneto encephalography), seismology, and the computation of free boundaries of plasmas subject to magnetic confinement in a tokamak. Such applications share as a common feature that they can be modeled through measurements of some transform (Fourier, Fourier-Wigner, Riesz) of an initial signal. Its non-local character generates various uncertainty principles that make all of these problems ill-posed. The techniques of harmonic analysis, as developed in each case below, form the thread and the mathematical core of the proposal. They are intended, by and large, to regularize the inverse issues under consideration and to set up constructive algorithms on structured models. These should be used to initialize numerical techniques based on optimization, which are more flexible for modelling but computationally heavy and whose convergence often require a good initial guess. In this context, the development of wavelet analysis in electrical engineering, as well as signal and image processing or singularity detection, during the last twenty years, may serve as an example. However, many other aspects of Fourier analysis are at work in various scientific fields. We believe there is a strong need to develop this interaction that will enrich both Fourier analysis itself and its fields of application, all the more than in France the scientific communities may be more separate than in some other countries.

The project was created in July 2007. A first meeting took place in Pau in October 2007 and a second one in Orléans in September 2008. Collaborations have begun with the Bordeaux team on the use of bandelet formalism for the seismic inversion and MAGIQUE-3D has recruited a post-doc who has started in the team in October 2008.

### 8.4. International collaborations

#### 8.4.1. Visits

- Dimitri Komatitsch, Roland Martin and Nicolas Le Goff have spent one month at the Barcelona Supercomputing Center in May-June 2008;
- Mounir Tlemcani has spent one month in May 2008 in MAGIQUE-3D.
- Magdalena Grigoroscuta has spent one month at CSUN (California State University at Northridge) in June 2008.
- Julien Diaz has spent two weeks at CSUN (California State University at Northridge) in November 2008.
- Chokri Bekky has spent two weeks in MAGIQUE-3D in November 2008.
- Rabia Djellouli has spent one week in MAGIQUE-3D in November 2008.
- Isaac Harari (Professor, Tel Aviv University, Israel) has spent one week in MAGIQUE-3D in November 2008.

#### 8.4.2. Associate team MAGIC

Since January 2006, the team is associated to a team located at CSUN (California State University at Northridge) which is managed by R. Djellouli. Our common program research takes part of the activities we develop in modelisation essentially. Two PhD. students from MAGIQUE-3D are participating to these works and they have the opportunities to be current visitors at CSUN. We are funding both by the DREI from INRIA and CSUN. From 12 till 14 December 2007, we organized a workshop whose program is available at <http://uppa-inria.univ-pau.fr/m3d/WS07/>.

### 8.4.3. New collaborations

- A collaboration started with Charles Dossal (Assistant Professor, Université Bordeaux 1) on the use of curvelets for the seismic inversion. In this framework, Meriem Laleg began her post-doc in october 2008.
- A collaboration with Erwan Faou (CR, Inria Rennes) started on the improvement of local time-stepping method for the wave equation.

## 9. Dissemination

### 9.1. Scientific animation

- We are preparing the organization of the conference Waves 09 which will take place to Pau from 15 till 19 june 2008 (see <http://uppa-inria.univ-pau.fr/m3d/WS07/>). This conference usually gather about 200 participants.

### 9.2. Teaching

#### 9.2.1. Lecture

- Lecture/course to Master students (64 hours) at University of Pau, France, on "Calcul Parallèle et Modélisation en Géophysique" ("Parallel computing and geophysical modeling")
- Lecture/course to Master students (46 hours) at University of Pau, France, on "Propagation d'ondes et imagerie" ("Waves propagation and Imaging")

### 9.3. Participation in Conferences, Workshops and Seminar

Caroline Baldassari

- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *Analysis of the performance of the Interior Penalty Discontinuous Galerkin method*, journée en l'honneur de Gene Golub (CERFACS, Toulouse (France), 29 february 2008, [http://www.cerfacs.fr/algor/Events/Gene\\_Around\\_The\\_World/index.html](http://www.cerfacs.fr/algor/Events/Gene_Around_The_World/index.html))
- *High-order Discontinuous Galerkin Methods for the Reverse Time Migration*, 5th European congress on computational methods in applied sciences and engineering, ECCOMAS 2008 (Venice, Italy, 30 June-4 july 2008, <http://www.iacm-eccomascongress2008.org/>)
- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *Discontinuous Galerkin method for the reverse time migration*, 10th International conference Zaragoza-Pau on applied mathematics and statistics (Residencia universitaria de Jaca, Spain, 15-17 september 2008, <http://pcmap.unizar.es/~jaca2008>)
- H. Barucq, H. Calandra, B. Denel, C. Baldassari and J. Diaz *Discontinuous Galerkin method for the reverse time migration*, journée sur le retournement temporel et méthodes d'imagerie non-itératives pour la caractérisation des milieux et des objets (ESPCI, Paris (France), 5 december 2008)

Hélène Barucq

- H. Barucq, C. Baldassari, H. Calandra, B. Denel and J. Diaz *The reverse time migration technique coupled with finite element methods*, Invited talk in the context of the 5èmes journées du GDR "Etude de la propagation ultrasonore en vue du contrôle non-destructif", 2-6 June 2008, <http://www.lmp.u-bordeaux1.fr/gdr2501/spip.php?rubrique2>.
- H. Barucq, J. Diaz and V. Duprat *High Order Absorbing Boundary Conditions for solving the Wave Equation by Discontinuous Galerkin Methods*, 10th International conference Zaragoza-Pau on applied mathematics and statistics (Residencia universitaria de Jaca, Spain, 15-17 september 2008, <http://pcmap.unizar.es/~jaca2008>)

## Céline Blitz

- C. Blitz, D. Komatitsch and P. Lognonné, American Geophysical Union Annual Meeting, oral presentation, December 15-19, 2008, San Francisco, USA.
- C. Blitz, D. Komatitsch, P. Lognonné, R. Martin and N. Legoff, Modeling the seismic response of 2D models of the asteroid 433 Eros based on the spectral element method, 37th COSPAR (Committee on Space Research) Scientific Assembly, Montreal, Canada, 13-20 July 2008.

## Julien Diaz

- J. Diaz and M. Grote *Arbitrary High Order Local-Time Stepping Scheme for the Wave Equation*, Department of Mathematics Applied Math Seminar, California State University, Northridge, USA, October 17, 2008.
- J. Diaz and P. Joly *Application of Cagniard de Hoop Method to the Analysis of Perfectly Matched Layers*, 3rd IFAC Workshop on Fractional Differentiation and its Applications , FAD'08 (Ankara, Turkey, 5-7 november 2008 <http://www.cankaya.edu.tr/fda08/>)
- H. Barucq, J. Diaz and M. Tlemcani *Perfectly Matched Layers for the Shallow Water equations*, 10th International conference Zaragoza-Pau on applied mathematics and statistics (Residencia universitaria de Jaca, Spain, 15-17 september 2008, <http://pcmap.unizar.es/~jaca2008>)
- C. Agut, J. Diaz and A. Ezziani *A New Modified Equation Approach for solving the Wave Equation*, 10th International conference Zaragoza-Pau on applied mathematics and statistics (Residencia universitaria de Jaca, Spain, 15-17 september 2008, <http://pcmap.unizar.es/~jaca2008>)

## Abdelaâziz Ezziani

- A. Ezziani and P. Joly *Local time stepping and discontinuous Galerkin methods for symmetric rst order hyperbolic systems*, Les premières journées de mathématiques appliquées de Safi, Maroc, 25-27 june, 2008.

## Magdalena Grigoroscuta

- M. Amara, R. Djellouli and M. Grigoroscuta *A new DG formulation for Helmholtz problems using plane waves*, Journée "Gene Golub around the world" (CERFACS, Toulouse, 29th february 2008. <http://www.cerfacs.fr/algor/PastWorkshops/GeneAroundTheWorld2008/index.html>)
- M. Amara, R. Djellouli and M. Grigoroscuta *A discontinuous Galerkin method using plane waves for Helmholtz equation*, Tenth International Conference Zaragoza-Pau on Applied Mathematics and Statistics (Jaca, 15th-17th September 2008. <http://pcmap.unizar.es/~jaca2008/>)

## Dimitri Komatitsch

- D. Komatitsch, Invited talk at the UNESCO ICTP Center in Trieste (Italy), The spectral-element method and three-dimensional seismology for the Ninth Workshop on three-dimensional modeling of seismic wave generation, propagation and their inversion , September 22-26, 2008.
- D. Komatitsch, J. Labarta and D. Michéa, A 21 billion degrees of freedom, 2.5 terabyte simulation of seismic wave propagation in the inner core of the Earth on MareNostrum, keynote presentation in the Proceedings of the 8th. World Congress on Computational Mechanics (WCCM8) and the 5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008), Venice (Italy), 30 June - 4 July 2008
- D. Komatitsch, J. Labarta and D. Michéa, A simulation of seismic wave propagation at high resolution in the inner core of the Earth on 2166 processors of MareNostrum, VECPAR'2008 8th International Meeting on High Performance Computing for Computational Science, Toulouse, France, 24-27 June 2008
- D. Komatitsch, S. Chevrot, J. Labarta and D. Michéa, Invited keynote talk at the EGU meeting in April 2008 in Vienna, Austria: Numerical forward modeling of seismic wave propagation in complex media .

Ronan Madec

- R. Madec, D. Komatitsch, F.J. Sánchez-Sesma, Implementation of plane waves in the Spectral-Element Method and comparison with the Method of Fundamental Solution, Tenth International Conference Zaragoza-Pau on Applied Mathematics and Statistics Jaca, September 15-17th 2008, Spain.
- R. Madec and D. Komatitsch, Implementation of plane waves in the Spectral-Element Method in 2D, Unión Geofísica Mexicana, Oral presentation, 26-30 October, 2008, Puerto Vallarta, Mexico.

David Michéa

- D. Michéa, D. Komatitsch, G. Erlebacher, N. Le Goff and R. Martin, Porting and tuning the SPECFEM free software package for high-performance computing on GPU and Cell BE, invited talk at the 8th RMLL (Rencontres Mondiales du Logiciel Libre) in Mont-de-Marsan, France, July 1-5, 2008.

Roland Martin

- R. Martin, D. Komatitsch and N. Le Goff, An unsplit variational perfectly matched layer technique optimized at grazing incidence: application to the spectral-element method, Proceedings of the 8th. World Congress on Computational Mechanics (WCCM8) and the 5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008), Venice (Italy), 30 June - 4 July 2008
- R. Martin, D. Komatitsch, C. Blitz and N. Le Goff, Simulation of seismic wave propagation in an asteroid based upon an unstructured MPI spectral-element method: blocking and non-blocking communication strategies, VECPAR'2008 8th International Meeting on High Performance Computing for Computational Science, Toulouse, France, 24-27 June 2008.
- R. Martin, D. Komatitsch, An unsplit perfectly matched layer optimized at grazing incidence for the three-dimensional viscoelastic wave equation Tenth International Conference Zaragoza-Pau on Applied Mathematics and Statistics Jaca, September 15-17th 2008, Spain.

Anne-Gaëlle Saint-Guirons

- H. Barucq, R. Djellouli and A.-G. Saint-Guirons *New local DtN-like boundary conditions for exterior elliptical-shaped boundaries*, First workshop on "Solution methodologies for direct and inverse scattering problems: Recent progress and trend" 12<sup>th</sup>-14<sup>th</sup> december 2007, University of Pau, FRANCE.
- H. Barucq, R. Djellouli and A.-G. Saint-Guirons *Analyse de performance de nouvelles conditions approchées locales de type DtN pour des frontières de forme sphéroïdale prolate*, CANUM 2008, 26<sup>th</sup> - 30<sup>th</sup> may 2008, Saint-Jean-de-Monts, FRANCE. <http://smai.emath.fr/canum2008/>
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