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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team POEMS

Wave propagation: Mathematical Analysis and Simulation

Paris - Rocquencourt

THEME NUM

Activity Report

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2. Overall Objectives

2.1. Overall Objectives

The propagation of waves is one of the most common physical phenomena one can meet in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) and to the scale of the universe (electromagnetic waves, gravity waves), to the scale of the atom (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us.

The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in the various domains of physics and engineering science.

The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas justify together a research project in Scientific Computing entirely devoted to this theme.

The project POEMS is an UMR (Unité Mixte de Recherche) between CNRS, ENSTA and INRIA (UMR 2706). The general activity of the project is oriented toward the conception, the analysis, the numerical approximation, and the control of mathematical models for the description of wave propagation in mechanics, physics, and engineering sciences.
Beyond the general objective of contributing to the progress of the scientific knowledge, four goals can be ascribed to the project:

- the development of an expertise relative to various types of waves (acoustic, elastic, electromagnetic, gravity waves, ...) and in particular for their numerical simulation,
- the treatment of complex problems whose simulation is close enough to real life situations and industrial applications,
- the development of original mathematical and numerical techniques,
- the development of computational codes, in particular in collaboration with external partners (scientists from other disciplines, industry, state companies...)

3. Scientific Foundations

3.1. Scientific Foundations

Our activity relies on the existence of mathematical models established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation (or a system of partial differential equations) of the hyperbolic type that are often (but not always) linear for most of the applications we are interested in. The prototype equation is the wave equation:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0,$$

which can be directly applied to acoustic waves but which also constitutes a simplified scalar model for other types of waves (This is why the development of new numerical methods often begins by their application to the wave equation). Of course, taking into account more realistic physics will enrich and complexify the basic models (presence of sources, boundary conditions, coupling of models, integro-differential or non linear terms,...)

It is classical to distinguish between two types of problems associated with these models: the time domain problems and the frequency domain (or time harmonic) problems. In the first case, the time is one of the variables of which the unkonwn solution depends and one has to face an evolution problem. In the second case (which rigorously makes sense only for linear problems), the dependence with respect to time is imposed a priori (via the source term for instance): the solution is supposed to be harmonic in time, proportional to $e^{i\omega t}$, where $\omega > 0$ denotes the pulsation (also commonly, but improperly, called the frequency). Therefore, the time dependence occurs only through this pulsation which is given a priori and plays the rôle of a parameter: the unknown is only a function of space variables. For instance, the wave equation leads to the Helmholtz wave equation (also called the reduced wave equation) :

$$-c^2\Delta u - \omega^2 u = 0.$$

These two types of problems, although deduced from the same physical modelization, have very different mathematical properties and require the development of adapted numerical methods.

However, there is generally one common feature between the two problems: the existence of a dimension characteristic of the physical phenomenon: the wavelength. Intuitively, this dimension is the length along which the searched solution varies substantially. In the case of the propagation of a wave in an heterogeneous medium, it is necessary to speak of several wavelengthes (the wavelength can vary from one medium to another). This quantity has a fundamental influence on the behaviour of the solution and its knowledge will have a great influence on the choice of a numerical method.

Nowadays, the numerical techniques for solving the basic academic and industrial problems are well mastered. A lot of companies have at their disposal computational codes whose limits (in particular in terms of accuracy or robustness) are well known. However, the resolution of complex wave propagation problems close to real applications still poses (essentially open) problems which constitute a real challenge for applied mathematicians. A large part of research in mathematics applied to wave propagation problems is oriented towards the following goals:

- the conception of new numerical methods, more and more accurate and high performing.
- the treatment of more and more complex problems (non local models, non linear models, coupled systems, ...)
- the study of specific phenomena or features such as guided waves, resonances,...
- the development of approximate models in various situations,
- imaging techniques and inverse problems related to wave propagation.

These areas constitute the main fields of interest for the Project POEMS.

4. Application Domains

4.1. Introduction

We are concerned with all application domains where linear wave problems arise: acoustics and elastodynamics (including fluid-structure interactions), electromagnetism and optics, and gravity water waves. We give in the sequel some details on each domain, pointing out our main motivations and collaborations.

4.2. Acoustics

As the acoustic propagation in a fluid at rest can be described by a scalar equation, it is generally considered by applied mathematicians as a simple preliminary step for more complicated (vectorial) models. However, several difficult questions concerning coupling problems have occupied our attention recently.

Aeroacoustics, or more precisely, acoustic propagation in a moving compressible fluid, is for our team a new and very challenging topic, which gives rise to a lot of open questions, from the modelling until the numerical approximation of existing models. Our works in this area are partially supported by EADS (and Airbus). The final objective is to reduce the noise radiated by Airbus planes.

Vibroacoustics, which concerns the interaction between sound propagation ans vibrations of thin structures, also raises up a lot of relevant research subjects. Our collaboration with EADS on this subject, with application to the confort of the cockpits of airplanes, allowed us to develop a new research direction about time domain integral equations.

A particularly attractive application concerns the simulation of musical instruments, whose objectives are both a better understanding of the behavior of existing instruments and an aid for the manufactoring of new instruments. The modeling and simulation of the timpani and of the guitar have been carried out in collaboration with A. Chaigne of ENSTA. We intend to initiate a new collaboration on the piano.

4.3. Electromagnetism

This is a particularly important domain, first because of the very important technological applications but also because the treatment of Maxwell's equations poses new and challenging mathematical questions.

Applied mathematics for electromagnetism during the last ten years have mainly concerned stealth technology, electromagnetic compatibility, design of optoelectronic micro-components or smart materials.

Stealth technology relies in particular on the conception and simulation of new absorbing materials (anisotropic, chiral, non-linear...). The simulation of antennas raises delicate questions related to the complexity of the geometry (in particular the presence of edges and corners). Finally micro and nano optics have seen recently fantastic technological developments, and there is a real need for tools for the numerical simulation in these areas.

Our team has taken a large part in this research in the past few years. In the beginning, our activity was essentially concerned with radar furtivity (supported by the French Army and Aeronautic Companies). Now, it is evolving in new directions thanks to new external (academic and industrial) contacts:

- We have been developing since 2001 a collaboration with ONERA on EM modeling by higher order methods (theses of S. Pernet and M. Duruflé).
- As partners of ONERA, we have been selected by the CEG (a research organism of the French Army) to contribute to the development of a general computational code in electromagnetism. The emphasis is on the hybridization of methods and the possibility of incorporating specific models for slits, screens, wires,...
- Optics is becoming again a major application topic. In the past our contribution to this suject was quite important but remained at a rather academic level. Our recent contacts with the Institut d'Electronique Fondamentale (Orsay) (we have initiated with them a research program about the simulation of micro and nano opto-components) are motivating new research in this field.
- Multiscale modelling is becoming a more and more important issue in this domain. In particular, in collaboration with the LETI(CEA) in Grenoble, we are interested in simulated devices whise some of the geometric characterictics are much smaller than the wavelength.

4.4. Elastodynamics

Wave propagation in solids is with no doubt, among the three fundamental domains that are acoustics, electromagnetism and elastodynamics, the one that poses the most significant difficulties from mathematical and numerical points of view. Our activity on this topic, which unfortunately has been forced to slow down in the middle of the 90's due to the disengagement of French oil companies in matter of research, has seen a most welcomed rebound through new academic and industrial contacts.

The two major application areas of elastodynamics are geophysics and non destructive testing. A more recent interest has also been brought to fluid-stucture interaction problems.

- In geophysics, one is interested in the propagation of elastic waves under ground. Such waves appear as natural phenomena in seisms but they are also used as a tool for the investigation of the subterrain, mainly by the petroleum industry for oil prospecting (seismic methods). This constitutes an important field of application for numerical methods. Our more recent works in this area have been motivated by various research contracts with IFP (French Institute of Petroleum), IFREMER (French Research Institute for the Sea) or SHELL.
- Another important application of elastic waves is non-destructive testing: the principle is typically to use ultra-sounds to detect the presence of a defect (a crack for instance) inside a metallic piece. This topic is the object of an important cooperation with EDF (French Company of Electricity) and CEA Saclay in view on the application to the control of nuclear reactors.

  At a more academic level, we have been interested in other problems in the domain of elastic waves in plates (in view of the application to non-destructive testing) through our participation to the GDR Ultrasons. In this framework, we have developped our resarches on multi-modal methods, exact transparent conditions or shape reconstruction of plates of variable cross section.
- Finally, we have recently been led to the study of fluid-solid interaction problems (coupling of acoustic and elastic waves through interfaces) as they appear in underwater seismics (IFREMER) and stemming from ultra-sound propagation in bones (in contact with the Laboratoire d'Imagerie Paramétrique of Paris VI University).

4.5. Gravity waves

These waves are related to the propagation of the ocean swell. The relevant models are derived from fluid mechanics equations for incompressible and irrotational flows. The applications concern in large part the maritime industry, if particular the questions of the stability of ships, sea keeping problems, wave resistance,... The application we have recently worked on concerns the stabilization of ships and off-shore platforms (contract with DGA).

5. Software

5.1. Advanced software

We are led to develop two types of software. The first category is prototype softwares : various softwares are developed in the framework of specific research contracts (and sometimes sold to the contractor) or during PhD theses. They may be also contributions to already existing softwares developed by other institutions such as CEA, ONERA or EDF. The second category is advanced software which are intended to be developed, enriched and maintained over longer periods. Such sofwares are devoted to help us for our research and/or promote our research. We have chosen to present here only our advanced softwares.

5.2. MELINA

This software has been developed under the leadership of D. Martin for several years in order to offer to the researchers a very efficient tool (in Fortran 77 and object oriented) for easily implementing finite element based original numerical methods for solving partial differential equations. It has specific and original potential in the domain of time harmonic wave problems (integral representations, spectral DtN conditions,...). Nowadays, it is fully functional in various application areas (acoustics and aeroacoustics, elastodynamics, electromagnetism, water waves). It is an open source software with on line documentation available at

http://perso.univ-rennes1.fr/daniel.martin/melina/

The software is regularly used in about 10 research laboratories (in France and abroad) and number of research papers have published results obtained with MELINA (see the Web site). Moreover, every 2 years, a meeting is organized which combines a workshop which teaches new users with presentations by existing users.
During the last four years, apart from various local improvements of the code, new functionalities have been developed:

- Higher order finite elements (up to 10th order),
- Higher order quadrature formulae,
- DtN boundary conditions in 3D.

A new C++ version of the software is under development. We will take advantage of this evolution for extending the class of finite elements (mixed elements, tensor valued elements, ...).

5.3. MONTJOIE

This is a software for the efficient and accurate wave propagation numerical modeling in both time dependent or time harmonic regimes in various domains of application : acoustics, aeroacoustics, elastodynamics and electromagnetism . It is based essentially on the use of quadrilateral/hexaedric conforming meshes and continuous or discontinuous Galerkin approximations, The use of tensor product basis finctions coupled to judicious numerical quadrature techniques leads to important gains in both computing time and memory storage. Various techniques for treating unbounded domains have been incorporated : DtN maps, local absorbing conditions, integral representations and PML's.

We have written an interface for the use of otjher libraries : SELDON, a C++ linear algebra library (interfaced with BLAS and LAPACK) used for iterative linear solvers, MUMPS and UMFPACK for direct linear solvers, ARPACK for eigenvalue computations. The mesh generation is not part of the code. It can be done with Modulef, Gmsh, Ghs3D or Cubit.

This code has been deveploped by Marc Duruflé during his PhD thesis. Some other contributors have brought more specific enrichments to the code.

6. New Results

6.1. Numerical methods for time domain wave propagation

6.1.1. Higher order time discretization of second order hyperbolic problems

Participants: Jerónimo Rodríguez, Patrick Joly.

Our work on the construction of optimal (namely maximizing the allowed CFL number) higher order schemes for second order hyperbolic problems of the form

$$\frac{d^2u}{dt^2} + \mathcal{A}u = 0, \tag{1}$$

(see the activity report of 2007) has been concretized by the publication of one article in Journal of Computational and Applied Mathematics and one invited presentation at the international Workshop on Numerical Methods for Evolution Equations (Heraklion, Crete, September 2008).

6.1.2. Energy preserving schemes for hamiltonian systems

Participants: Patrick Joly, Juliette Chabassier.

A classic way of building stable numerical schemes for PDEs is to preserve, on a discrete level, a continuous conserved quantity, like an energy for instance. Systems of non linear wave equations in Hamiltonian form $\partial_t^2\mathbf{u} - \partial_x \nabla H(\partial_x \mathbf{u}) = 0$ (where the unknown u is valued in $\mathbb{R}^m$) conserve the continuous energy $\mathcal{E}(t) = \int_\Omega \frac{1}{2}(\partial_t \mathbf{u})^2 + H(\partial_x \mathbf{u})$. This property leads, if the hamiltonian H satifies a sort of coercivity property, to a stability result for the solution of the PDE, hence the will to keep an analogous quantity conserved step to step in a discrete way. If this is relatively easy and well-known for scalar equations ($m = 1$, see for instance the works of Furihata), this is less obvious for systems for which existing solutions that we found in the litterature (works by Bilbao, Mickens) seem to be limited to specific hamiltonians. In the general case, we developped an implicit, energy preserving scheme which is also centered in time. The scheme can be written for any value of m. It is second order accurate, unconditionnally stable and its algebraic complexity grows as 2^{m-1}.

6.1.3. Hybrid Meshes for High-Order DGM

Participants: Morgane Bergot, Gary Cohen, Marc Duruflé.

In the frame of her thesis directed by G. Cohen, M. Bergot studied a way to mix high-order hexaedral elements with tetraedra, wedges and pyramids for discontinuous Galerkin methods. Hexaedra and tetraedra have been well studied , and wedge elements are classically obtained by the tensorial product of a triangular element and a 1D element. The main effort is then devoted to the study of pyramidal elements, considering the following elements in the same mesh:

- classical hexaedral elements with Gauss-Lobatto points;
- Hesthaven tetraedra, with Gauss-Lobatto points on the edges;
- wedges resulting from the tensorial product of a triangle with Hesthaven points, and an edge with Gauss-Lobatto points;
- pyramids joining triangular and quadrangular faces in a continous way.

Following Bedrosian, rational polynomial shape functions are used as basis functions on the pyramidal element at any order. A generic formula has been found and basis functions on pyramids have been built at orders 1, 2 and 3. Numerical dispersion and stability have also been studied for the corresponding semi-discrete wave equation at orders 1, 2 and 3.

6.1.4. Representation of solutions to hyperbolic systems

Participant: Jérome Le Rousseau.

We are interested in an original representation formula for the solutions to hyperbolic systems by a multi-product of Fourier integral operators (FIO). Here we mean that an approximation of the solution is given by a finite product of FIOs and the solution is exactly recovered by passing to the limit (in the number of operators in the product). The tools employed to obtain such results are those of microlocal analysis. This representation formula is interesting because each FIO in the product is given in a simple and explicit manner. As a consequence, this representation leads to natural numerical schemes, which can be made computationally efficient with additional approximations. One of the motivation for this work is seismic imaging. This research effort was first conducted for scalar equations. In 2007, we extended this approach to the case of symmetric and symmetrizable systems. This extension is far from being straightforward, in particular as one is then confronted to algebraic operations on matrices, for the phases and amplitudes of the operators, that do not commute. We were also led to introducing a new type of FIOs with a complex matrix phase. For such operators, classical composition theorems do not apply and cannot be directly generalized.

6.1.5. The wave equation with fractional order dissipative terms

Participant: Jing-Rebecca Li.

This is a work in collaboration with Houssem Haddar (INRIA-Saclay DEFI) and Denis Matignon (University of Toulouse, ISAE). We consider a wave equation with fractional order dissipative terms modeling viscothermal losses on the lateral walls of a duct. Diffusive realizations of fractional derivatives are used, first to prove existence and uniqueness results, then to design a numerical scheme which avoids the storage of the entire history of past data. Two schemes are proposed depending on the choice of a quadrature rule in the Laplace domain. The first one mimics the continuous energy balance but suffers from a loss of accuracy in long time simulation. The second one provides uniform control of the accuracy. However, even though the latter is more efficient and numerically stable under the standard CFL condition, no discrete energy balance has been yet found for it. Numerical results of comparisons with a closed-form solutions are generated. A paper containing this work has been submitted and accepted for publication.

6.1.6. Coupling Retarded Potentials and Discontinuous Galerkin Methods for time dependent wave propagation

Participants: Patrick Joly, Jerónimo Rodríguez, Emmanuel Verhille.

This topic is developed in collaboration with T. Abboud (IMACS) and I. Terrasse (EADS) in the framework of the contract ADNUMO with AIRBUS. Let us recall that our objective is to use time-domain integral equations (developed in particular at IMACS and EADS) as a tool for contructing transparent boundary conditions for wave problems in unbounded media. More precisely, we want to couple the use of (possibly high order) discontinuous Galerkin methods for the numerical approximation of the interior equations with a space-time Galerkin approximation of the integral equations that represent the transparent boundary conditions.

This task is a priori difficult due to the very different nature of the two discretization procedures. In particular, the stability of the coupling scheme is a key issue. We have found a solution to this problem by imposing during the construction of the method that the solution of the coupled problem satisfies a discrete energy identity that implies the stability of the method under the usual CFL condition for the interior problem. Moreover, as the discontinuous Galerkin method requires smaller time steps than the discretization of time-domain integral equations, we have included the possibility of using different time steps: the time step for the integral equation is supposed to be a multiple of the time step for the interior equation. This has been solved during the internship of E. Verhille, by adapting the ideas we developed in the past for space-time mesh refinement methods. The well-posedness and the stability of the resulting method has been successfully analyzed.

The method has been first developed for the acoustic wave equation (written as a first order system) then extended to general symmetric hyperbolic systems (in the sense of Friedrichs) with zero order perturbation : this includes in particular linearized Euler equations, namely the model we wish to treat within the ADNUMO project.

The coupling algorithm has been implemented for the wave equation, using the library Sledge++ (http://www.caam.rice.edu/~timwar/NUDG/Software/Sledge++_.html) for the discontinuous Galerkin part and the code Sonate (IMACS) for the integral equation part. The first results on academic problems (see figure 1) are quite encouraging, in particular in terms of accuracy, even though the computational cost of the method remains relatively high.

6.1.7. Coupling finite elements with semi-analytical methods for non destructive testing simulations on elastodynamics

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Joly, Jeronimo Rodriguez.

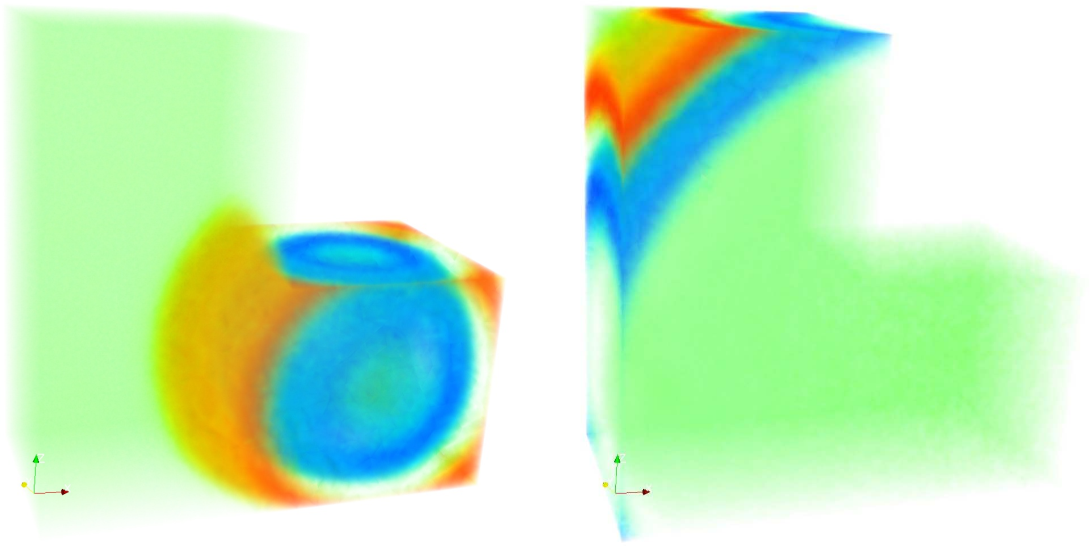

Figure 1. Resolution of the acoustic wave equation in $\mathbb{R}^3$ using a L-shape computational domain. Two snapshots of the pressure field.

This topic is inscribed on the MOHYCAN project funded by the ANR (RNTL) and is a work in collaboration with the laboratories LIST-CEA and SINETICS EDF-Clamart. The MOHYCAN project aims at extending the features of ultrasonic non destructive testing simulation tools to deal with industrial, potentially complex configurations. In such simulations we are interested on computing the signal on a transducer when the media is excited by a punctual source on the interface. On many applications, the computational domain is a simple media (typically piecewise homogeneous media with straight interfaces) that are perturbed by small geometrical details (for example cracks). The main goal of the project is to couple a finite element method solving the full elastodynamic equations in the neighborhood of the complex geometry (ATHENA code, developed by EDF) with a semi-analytical resolution in the regions with simple configurations (where further assumptions allowing to simplify the elastodynamic equations can be made yielding to an important reduction of the computational cost). We have proposed a technique that computes the signal on the transducer on two steps:

- a first contribution coming from the incident field (that does not consider the small defaults) that is computed by the semi-analytical method,
- a second contribution that takes into account the field coming from the scattering by the small obstacles. To do so, we use a finite element method to compute the fields on a small neighborhood of the complex region and then we use the Auld reciprocity principle to propagate the information till the transducer.

The proposed method assumes that the scattered field is out-going with respect to the finite element method computational domain, this means that the coupling is weak (multiple interactions are not considered). The future work will be on the direction of conceiving a stronger coupling.

6.1.8. Fictitious domain methods for wave propagation problems

Participants: Éliane Bécache, Jeronimo Rodriguez.

This is a joint work with Chrysoula Tsogka (Univ. of Crete & FORTH-IACM) and is concerned by the computation of wave scattering on complex media. The fictitious domain method allows to perform most of the computations on uniform meshes keeping a good approximation of the geometry. This method imposes weakly the boundary condition through the introduction of an additional surface unknown (the additional computational effort being thus inexpensive). In [E. Bécache, J. Rodríguez, C. Tsogka, A fictitious domain method with mixed finite elements for elastodynamics, SIAM J. Sci. Comput. (2007)] the authors had introduced a new mixed finite element compatible with the fictitious domain method for the resolution of scattering problems on elastodynamics with Neumann boundary condition. The analysis of this finite element coupled with the fictitious domain method applied to the scalar wave equation with Neumann boundary conditions have been completed and gave rise to the publication [E. Bécache, J. Rodríguez, C. Tsogka, Convergence results of the fictitious domain method for a mixed formulation of the wave equation with a Neumann boundary condition, accepted in M2AN (2008)]. Those results have been presented on several seminars.

6.1.9. Numerical Methods for Vlasov-Maxwell Equations

Participants: Gary Cohen, Alexandre Sinding, Marc Duruflé, Patrick Ciarlet.

There exist a large number of methods for approximating the motion of charged particles. They rely on a suitable discretization of Maxwell's equations, and a suitable discretization of Vlasov's equation. In addition, these discretizations have to be compatible. We report below three possible and complementary approaches.

A. Sinding's thesis, directed by G. Cohen, is devoted to the coupling of higher order hexaedral finite elements for Maxwell's equations with a Particle in Cell (PIC) method for Vlasov's equations. Since continuous approximations seem more fitted to Vlasov'equations, an original implementation of continuous spectral finite elements for solving Maxwell's equations has been constructed. It is based on a mixed formulation of the system. In order to avoid the occurence of spurious modes, a dissipation term is added to the formulation. This penalization term is a good way to get rid of the parasitic modes, by sending them into the complex plane with a negative imaginary part (the parasitic waves becoming evanescent). This solver (high order, continuous fields, numerical dissipation) has given good results in 2D and 3D (Fig.2) and should help in reducing the numerical noise even in the case of discontinuous coefficients and geometries containing corners (which produce singular solutions difficult to approximate by continuous elements) which is a huge drawback of particle in cell methods.

Alternatively, to retain a continuous approximation of the EM fields, one can choose to use some recent mathematical tools. In a follow-up to a theoretical work initiated with Eric Sonnendrücker and Régine Barthelmé (M3AS, 2007), and in collaboration with Simon Labrunie (Nancy Univ), P. Ciarlet aims to revisit existing approaches (see the works of Heintzé *et al* during the 90's) to compute the motion of charged particles, using an $H(curl, div)$ conforming discretization method to compute the electromagnetic fields. The improvements are twofold. First, it is now possible to handle 3D singular geometries, that can generate intense fields, thanks to the use of the Weighted Regularization Method of Costabel-Dauge. Second, a Lagrange multiplier is added to cope with the fact that the charge conservation equation is not, in general, satisfied rigorously at the discrete level. The numerical analysis has been carried out for explicit and implicit discrete time schemes. Also, this study sheds new light on the so-called correction methods, that are very popular among engineers and applied physicists.

Concerning the computation of the trajectories of particles, M. Duruflé proposed an original coupling strategy between Maxwell equations and the motion equations governing the trajectory of particles. With this choice, a discrete energy is conserved, numerical experiments showed that this property was interesting when we were dealing with high-density plasmas, that need to be modeled in devices like POS (Plasma Opening Switch). We have implemented the method in 3-D, and validated it through comparisons with solutions computed by QuickSilver, a code developed in Sandia National Laboratories. This work has been presented in a talk at CANUM 2008, and a paper is in preparation.

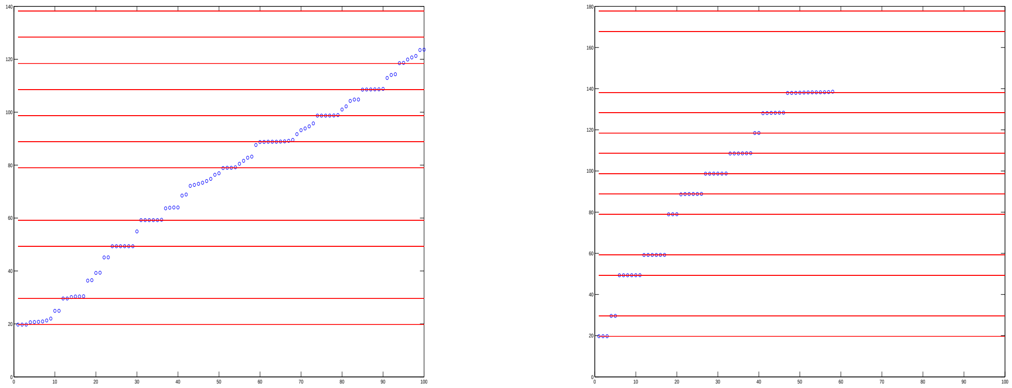

Figure 2. Eigenvalues without dissipation (left) and with dissipation (right) for the 3D cavity. Q5 approximation on a non regular mesh (split tetrahedra). Exact eigenvalues are indicated by red lines.

6.1.10. Modelisation of coupled non linear piano strings

Participants: Patrick Joly, Antoine Chaigne, Juliette Chabassier.

Juliette Chabassier is beginning her PhD on the modelisation and numerical simulation of a grand piano". This topic is developped in collaboration with Antoine Chaigne from ENSTA. Our first contribution concerns the modelisation of non linear piano strings.

The linear wave equation does not describe the complexity of the piano strings vibration enough for physics based sound synthesis. The nonlinear coupling between transversal and longitudinal vibrations has to be taken into account, as does the “geometrically exact” model described by Morse and Ingard. The non linear system derived from this problem fits into the PDE category presented in section 6.1.2: hamiltonian systems. For piano strings, the boundary conditions are not as simple as Dirichlet conditions, because of the bridge vibration. Moreover, most of the notes of a piano are made with several strings, between which the bridge is the coupling point. For now, we modelled this motion by using a damped linear oscillator, as a first elementary model for the coupling with the soundboard of the piano. Finally, the interaction between the strings and a non linear felt hammer has been treated as a unilateral contact problem, following the approach of Leila Rhaouti (a former PhD student of POEMS) for the kettledrum. Our numerical method is based on a variational formulation of the coupled problem, a higher order finite element approximation for the space discretization and energy dissipating schemes, extending the ideas of section 6.1.2, for the time discretization.

6.1.11. Modeling Piezoelectric Sensors

Participants: Gary Cohen, Sébastien Imperiale, Patrick Joly.

S. Imperiale started a thesis on numerical modeling of piezoelectric sensors, supervised by G. Cohen and P. Joly. Non destructive sensors are built up with piezoelectric materials. These materials provide a small displacement when we apply a electric current and vice versa. To simulate their behaviour it is necessary to take into account the coupling between linear elastodynamics equations and Maxwells’ equations. Because of the very large contrast between elastic waves velocity and electromagnetic waves velocity, at the time scale we are interseted in, the appropriate model for the electric part is an electrostatic model. For the numerical computation we should couple an explicit scheme for the mechanical part of the model with an implicit scheme for the electric part. To deal with realistic 3D problems, the spectral mixed finite element method will be used for the space discretization. Finally, to model the periodic structures of the sensors, homogeneization techniques will be used to simplify the full model.

6.2. Time-harmonic diffraction problems

6.2.1. Reduced basis method for harmonic wave propagation problems

Participant: Jeronimo Rodríguez.

This work is in collaboration with Y. Chen (Brown University), J. S. Hesthaven (Brown University) and Y. Maday (Paris VI). We are interested on the accurate, efficient and reliable computation of outputs of interest implicitly depending on a set of parameters through the resolution of a parametrized partial differential equation (PDE). As other model order reduction methods, the reduced basis method is based on the construction of a small dimension ad hoc basis that we use for the resolution of the PDE through a Galerkin approach. This basis is composed by solutions of the PDE for a small set of well selected parameters. When the parameter dependency is affine, an off-line on-line computational strategy can be used. The off-line part of the algorithm, being parameter independent, can be done once and for all. The computational cost of the (parameter dependent) on-line part is very low, providing a good efficiency of the computations.

The algorithm selecting those parameters is based on the use of a posteriori error estimators. On the evaluation of these estimators, the construction of lower bounds for the coercivity and inf-sup stability constants associated to the PDE operator is essential. In [D.B.P. Huynh, G. Rozza, S. Sen, A.T. Patera, A successive constraint linear optimization method for lower bounds of parametric coercivity and inf-sup stability constants, C. R. Acad. Sci. Paris, Ser I. 345, 473-478 (2007)] the authors presented an efficient method, compatible with the off-line on-line computational strategy, where the on-line computation is reduced to minimizing a linear functional under a few linear constraints. These constraints depend on nested sets of parameters obtained iteratively using a greedy algorithm. We have recently improved this method so that it becomes more efficient due to a nice property, namely, that the computed lower bound is monotonically increasing with respect to the size of the nested sets. This method has been tested on a 2D Maxwell (thus non coercive) problem. The new algorithm provides a good approximation of the inf-sup stability constant. The problem we have considered has resonance features for some choices of the parameters that are well captured by the methodology. These new results have been published on [Y. Chen, J.S. Hesthaven, Y. Maday, J. RodrÃguez, A monotonic evaluation of lower bounds for inf-sup stability constants in the frame of reduced basis approximations, C. R. Acad. Sci. Paris, Ser. I (2008)]. Another paper on the same topic has been recently submitted for publication.

6.2.2. Spectral Boundary Elements for Maxwell's Equations

Participants: Gary Cohen, Édouard Demaldent.

E. Demaldent's thesis, directed by G. Cohen and supervised at ONERA by D. Levadoux, deals with numerical simulation issues, and concerns the study of time-harmonic electromagnetic scattering problems. We are mainly interested in integral representation methods and in simulations that need the use of a direct solver. Their range of application is rapidly limited with classical approximation schemes, since they require a large number of unknowns to achieve accurate results. To overcome this problem, we have adapted the "spectral finite element method" to electromagnetic integral equations, then to the hybrid boundary element - finite element method (BE-FEM). The main advantage of our approach is that the H_{div} conforming property (H_{div}-H_{curl} within the BE-FEM) is enforced, meanwhile it can be interpreted as a point-based scheme. This allows a significant increase of the approximation order, that yields to a dramatic decrease of both the number of unknowns and computational costs, while ensuring the accuracy of the result. Another originality of our study lies in the development of high-order anisotropic hexahedral elements, to deal with conducting scatterers coated with a thin layer of material.

6.2.3. Robust computation of eigenmodes in electromagnetism

Participant: Patrick Ciarlet.

This is a twofold work.

First, a collaboration with Annalisa Buffa (CNR, Pavia, Italy), Grace Hechme (former PostDoc) and Erell Jamelot (former PhD student). To overcome the traditional difficulty linked to the apparition of spurious modes of the Maxwell's operator, we have proposed, analyzed and implemented a method based on a saddle-point formulation of the eigenvalue problem. Convincing numerical experiments have been carried out.

Second, a collaboration with François Lefèvre and Stephanie Lohrengel (Reims Univ.) and Serge Nicaise (Valenciennes Univ.). We have extended this method to composite materials, with the help of a generalized Weighted Regularization Method. Numerical results have been obtained recently.
In both cases, the numerical results were benchmarked to those available at the web location http://perso.univ-rennes1.fr/monique.dauge/core/index.html

6.2.4. Time harmonic aeroacoustics

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

We are still working on the finite element approximation of the time harmonic Galbrun's equation for simulating the acoustic scattering and radiation in presence of a mean flow. This is the object of a collaboration with Florence Millot and Sébastien Pernet at CERFACS. Remember that we use an augmented variational formulation to overcome the lack of coerciveness of the original model. The regularizing term requires the evaluation of the vorticity (curl of the main unknown which is the displacement) which solves a time harmonic advection equation. For a low Mach mean flow, we proved that the advection of vortices can be neglected, leading to a simple expression of the regularizing term. To relax this low Mach assumption, we are currently developping a numerical tool to solve the time harmonic advection equation, using a continuous or discontinuous Galerkin approach.

6.2.5. Harmonic wave propagation in locally perturbed infinite periodic media

Participants: Julien Coatleven, Marc Duruflé, Sonia Fliss, Patrick Joly.

Two main domains of applications are concerned :

- the propagation of electromagnetic waves in photonic crystals, a subject that we study in the framework of the ANR Project SimNanoPhot, in collaboration with the Institut d'Electronique Fondamentale (IEF, Orsay University). This project ends at December 2008.
- The propagation of ultrasonic waves in composite materials, in view of their nondestructive testing, a subject developped within a collaboration with EADS.

In such applications, there is a need for efficient numerical methods for computing the propagation of waves inside such structures. In real applications, the media are not really periodic but differ from periodic media only in bounded regions (small with respect to the total size of the propagation domain). In such a case, a natural idea is to reduce the pure numerical computations to these regions and to try to take advantage of the periodic structure of the problem outside to construct artificial (but exact) boundary conditions. That is why we have investigated then the generalization of the DtN approach to periodic media.

This was the subject of the PhD Thesis of Sonia Fliss who should defend the thesis early 2009. In this work, the main principles and theoretical bases of a numerical method for treating the scalar 2D plane problem have been settled rigorously for the case of **absorbing media**. By revisiting the Floquet-Bloch Theory, we propose a method for constructing DtN operators. We show in particular that we can factorize the DtN operator as a product of two operators. The first one is the DtN operator corresponding to a periodic half space problem. We refer the reader to the two previous activity report for more details about the construction of such operator. The second one is the solution of an affine operator equation. The idea is to solve this equation using not physical variables but Floquet-Bloch variables. That means that instead of considering that the operator is defined on $L^2(\mathbb{R})$, we consider that it is defined on $L^2([0, L] \times [0, 2\pi/L])$. The advantage is then to replace the discretization of an infinite set by the discretization of a compact set. Instead of an affine operator equation, we have to solve a set of integral equations with constraints. The difficulties are to know what is the right way to discretize the compact set and how to solve the non-standard integral equation.

For the first difficulty, a hybrid method for the discretisation in space has been studied during the internship of J. Coatléven. The well-posedness and convergence of the discrete formulation has been shown by identifying this problem to a discrete equivalent of the complete 2D problem, and applying methods similar to the continuous case. We have compared this numerical method to the one involving Discontinuous Galerkin Method which presents the advantage of avoiding the enforcement of quasi-periodic boundary conditions in the functional space. This last method was developped using the software Montjoie by Marc Duruflé during his Post Doc. The discretization of the Floquet variable is standard in the case of absorbing media but the numerical analysis of this discretization is not yet done and seems to raise the same questions as for the Fourier variables. For the second difficulty, the resolution of the non standard integral equations needs the inversion of a full matrix whose size is typically the numbers of dof of the discretization of the compact set $[0, L] \times [0, 2\pi/L]$. A use of higher spectral finite elements should allow to reduce the number of dof with the same error of discretization.

The treatment of **non absorbing media** is more complicated and asks new questions. Even the definition of the physical solution is not obvious. We use the Limiting absorption principle. The main theory has been settled, particularly for the locally perturbed periodic waveguides but there is still open questions. A numerical procedure has been developped and has necessited specific developments which corresponds to a numerical limiting absorption principle.

This research offers a number of interesting perspectives and developpments from theoretical, numerical and practical point of view which has justified a new PhD thesis on the subject. The ideas of the PhD thesis of Julien Coatleven are the following. The method allows to compute the solution for a single scatterer in a periodic media via the construction of a DtN map on the boundary of the scatterer. Re-interpreting previous work from the extensive litterature concerning the homogeneous medium, (see for instance the work of M.Grote and C. Kirsch, R. Potthast and al., M. Balabane and V.Tirel) it has become obvious that the method could easily be extended to several scatterers. We have deviced a unified vision for multi-scattering problems, showing that for any situation where one could identify a reference medium and several compact perturbations, the DtN map on the boundary of the union of the scatterers can be factorised in the product of the DtN maps for each scatterer and a propagation operator. The proof uses the property that the scatterer field can be decomposed into a sum of fields scattered by one perturbation, property well-known for the homogeneous problem that we have extended to a general situation. Thus, for a periodic medium, it is only necessary to compute this propagation operator to be able to apply the already existing method to the problem with several scatterers.

Another perspectives of this work is to generalize this study to a 3D problem and to analyse the extension of this method to electromagnetism. Applied to photonic crystals modelization, this work enters in the framework of the collaboration with the laboratory of Fundamental Electronics of Orsay University.

6.2.6. *Modeling of meta-materials in electromagnetism*

Participants: Bonnet-Ben Dhia Anne-Sophie, Patrick Ciarlet.

A collaboration with Carlo Maria Zwölf (former PhD student).
Meta-materials can be seen as particular dielectric media whose dielectric and/or magnetic constant are negative, at least for a certain range of frequency. This type of behaviour can be obtained, for instance, with particular periodic structures. Of special interest is the transmission of an electromagnetic wave between two media with opposite sign dielectric and/or magnetic constants. As a matter of fact, applied mathematicians have to address challenging issues, both from the theoretical and the discretization points of view.

Last year, we studied a simplified scalar model. Surprisingly, this model turns out to be solved efficiently by the most "naive" discretization. Here, the difficulty is to obtain a proof of the convergence of the numerical approximation. This was solved by using Strang's abstract theory for non conforming approximations.

This year, we begun the study of the transmission problem in a 3D electromagnetic setting. Before considering its numerical approximation, we studied some theoretical aspects. In order to achieve well-posedness of this problem, we add to proceed in three steps. First, we proved that the adequate space of electric fields used for the variational formulation is compactly embedded in L^2. Then we provided an equivalent three-field formulation of this problem: the first two fields are respectively the restriction of the electric field to each medium, whereas the third field is equal to the magnetic field in one medium. Finally, we showed that this formulation fits into the coercive plus compact framework.

6.3. Absorbing boundary conditions and absorbing layers

6.3.1. High order Absorbing Boundary Conditions for Anisotropic Models

Participant: Eliane Bécache.

We started recently a new collaboration with Dan Givoli (Technion, Israel) and Tom Hagstrom (Southern Methodist University) on high order ABC for several anisotropic models. We know that some instabilies can happen when using standard PMLs in some anisotropic models. Do we encounter the same kind of difficulties when using high order ABC?

We consider here a very general scalar model, for which there is a general procedure to design high-order ABC, extending previous work done in the isotropic case by Givoli et al, and Hagstrom et al. This model includes in particular the anisotropic scalar wave equation and the convective (dispersive and non-dispersive) wave equation.

We first showed that this construction gives the 'good' condition, in the sense that it is the condition written for outgoing waves (i.e., with the good sign of the group velocity). This can be illustrated with the slowness curves.

Our aim is then to make a deep analysis of these conditions. Is the model well posed? Concerning stability, can we show energy estimates? Can we show the stability using the Kreiss theory? We also intend to analyze the accuracy of these conditions, using a plane wave analysis (reflection coefficients...). We finally intend to implement these conditions and show there efficiency with numerical results, with comparisons with other ABCs and with PMLs. We already got some partial results.

For the anisotropic elastic wave model, we do not know anymore how to design 'good' high order ABC, 'good' in the sense that (i) they are designed for absorbing outgoing waves, (ii) they do not include high order normal derivatives. It is very easy to design a 'good' first-order ABC, for which we can prove well-posedness and stability (decreasing energy). Eliane Becache and Dan Givoli have started to investigate the question of how to deduce from this condition 'good' higher order ones.

6.3.2. Exact bounded PML's with singularly growing absorption

Participants: Eliane Bécache, Andres Prieto.

In a previous work, we analyzed the PML model with a singular damping function. This model has the particularity to be exact, for a layer of finite length. We extended a work done in the frequency domain by Bermudez et al, to the time domain. We considered the model in cylindrical coordinates and obtained some estimates of the solution in the time domain. This year, we explored other methods to obtain analogous results, with the aim of (i) improve the estimates in the time domain (in particular trying to loose one order less of regularity in time), (ii) extend the results to any other coordinates system. A paper is in preparation.

6.3.3. Leaky modes and PML techniques for non-uniform waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Benjamin Goursaud, Christophe Hazard, Andres Prieto.

This topic was initiated in the framework on the ANR SimNanoPHot (with the Institut d'Electronique Fondamentale, Orsay), about the simulation of tapers in integrated optics, or more generally varying cross section open waveguides. Our motivation was to study the possible use of the so-called *leaky modes* in the numerical simulation of such devices. The first part of our work was devoted to the use of PML techniques for the characterization and the numerical computation of leaky modes. On one hand, when we consider an infinite PML surrounding the core of the waveguide, the leaky modes appear as eigenvalues of the transverse component of the Helmholtz operator. The complete spectral analysis of this operator was achieved in 1D, but open questions remains in 2D. On the other hand, the case of a PML of finite width offers a convenient way of computing leaky modes. However, numerical tests have shown a high sensitivity with respect to the discretization. Using the notion of pseudo-spectrum, we have proposed a quantitative interpretation of this instability, which is related to the exponentially increasing spatial behaviour of leaky modes. The remedy consits in reducing the intermediate zone between the core and the PML, which led us to investigate an original application of PML's when the domain surrounded by the PML is not convex.

The second part of our work concerns the use of these leaky modes in a *multimodal method* as it was recently developed at POEMS for closed waveguides. This method consists in coupling a finite element discretization in the longitudinal direction of the guide with a spectral discretisation in the transverse direction, where the latter amounts to decomposing the wave as a series of guided and leaky modes. First numerical results have been recently obtained.

6.3.4. Absorbing layers for one-way wave equations

Participants: Anne-Sophie Bonnet-BenDhia, Christophe Hazard, Patrick Joly, Jérôme Le Rousseau.

This research topic is a starting collaboration with Prof. Alison Malcolm (Massachusetts Institute of Technology, Dept. of Earth, Atmospheric, and Planetary Sciences).

In seismic imaging, the excited wavefield first propagates primarily downwards into the Earth where it reflects from discontinuities in the subsurface and returns to the recording equipment. In this context, one-way wave equations, which describe waves propagating in only one direction with respect to a specific axis (here depth), occur naturally. These one-way equations are commonly used in seismic exploration for underground energy reserves and are also beginning to be applied in global seismology as denser data sets are acquired. We propose to study two aspects of these equations: their extension from smooth to non-smooth material properties and the application of the PML technique to attenuate spurious reflections from computational boundaries. These two aspects are related through both the underlying techniques, which involve the damping of waves, and through the proposed improvements in our ability to locate oil and gas in complex geological structures near (and below) salt bodies,(e.g. Gulf of Mexico). If the material through which waves propagate varies smoothly, implying that the wave equation coefficents do also, solutions of one-way wave equations can be represented through the action of a multi-product of Fourier integral operators. This representation leads directly to numerical implementations, which have been shown to be robust for seismic exploration, even when the materials are not truly smooth. In extreme cases, i.e. large contrast in the coefficients across boundaries and small (few wavelengths) heterogeneities, numerical instabilities are observed, which are poorly understood. We have already begun to develop methods that remove these instabilities involving the modification of the operators in the multi-product and the introduction of a regularization term that amounts to adding an evanescent parabolic (viscous) term. Preliminary comparisons of our numerical results with spectral finite-element methods have been carried out; we plan to perform a more detailed and systematic numerical investigation of such phenomena to direct the more theoretical aspects of this research.

An accurate algorithm to propagate waves through discontinuous structures would be of no use if reflections from the boundaries of the computational domain were the main source of error. For hyperbolic differential equations PMLs provide an efficient answer to this problem, but this has not been well studied for one-way (wave) equations, though we have already demonstrated its potential numerically. We propose to carry out the mathematical analysis to fully understand PMLs for pseudodifferential equations and to apply them to imaging problems. In particular we propose to estimate, both theoretically and numerically, the difference between the exact solution and the solution with PMLs and the decay of energy in the absorbing layers, beginning with the assessment of the smoothness requirements of the considered symbols. Returning to the problem of imaging salt structures, a strategy for imaging salt is to utilize waves that turn, traveling horizontally. In this situation classical one-way methods can be improved upon by choosing a preferred direction in local, curvilinear, coordinates thus dynamically changing the preferred direction. In this case the computational domain is small and thus PMLs are crucial.

6.4. Waveguides, resonances, and scattering theory

6.4.1. Elastic waveguides

Participants: Vahan Baronian, Anne-Sophie Bonnet-Ben Dhia, Eric Lunéville.

In partnership with the CEA, we are developing tools for ultrasonic non-destructive testing in elastic waveguides. A finite element (FE) method has been developed to compute the scattering by an arbitrary local perturbation of the guide. It is based on the modal decomposition of the wave field on the transverse artificial boundaries, taking advantage of the bi-orthogonality relation between the modes. Specific boundary conditions are developed which are perfectly transparent, allowing the FE computation zone to be reduced as small as possible (depending on the number of inhomogeneous and evanescent modes taken into account in the computation).

The perfectly transparent boundary conditions have been implemented previously in the 2D case of Lamb waves. In this case, because an analytic expression of the modes is available, the transparent boundary condition is easy to take into account.

This year, we generalized the method to the case of a 3D waveguide of arbitrary cross section. The main difficulty compared to the 2D case is that we need to compute previously the modes by a FE method, which makes the numerical implementation more intricate.

Time domain simulations are now available : an FFT is first used to determine the frequency spectrum of the incident signal, then the modes, the modal coefficients of the loading and the scattering matrix of the defect are computed at each sampling frequency, and finally, an inverse FFT gives the time dependent signal which could be measured by a transducer.

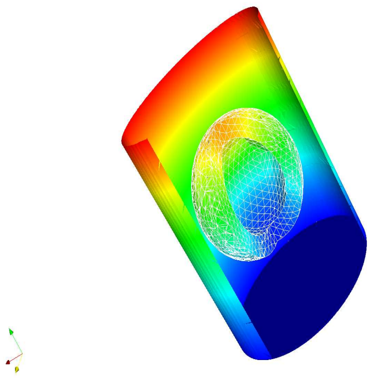

Figure 3. Plot of longitudinal component of displacement field in a homogeneous cylindric waveguide including a spherical cavity

6.4.2. Acoustic propagation in a lined waveguide

Participants: Anne-Sophie Bonnet-Ben Dhia, Jean-François Mercier.

In collaboration with Emmanuel Redon of University of Dijon, we have developed a finite element method for acoustic radiation in an infinite 2D lined guide with a uniform mean flow. In order to bound the domain, transparent boundary conditions are introduced by means of a Dirichlet to Neumann (DtN) operator based on a modal decomposition. This decomposition is easy to carry out in a hard-walled guide. With absorbing lining and in presence of a mean flow, many difficulties occur. The eigenvalue problem is no longer selfadjoint and no orthogonality relation is available to expand on the modes the trace of the acoustic pressure on the artificial boundaries. We show that it is however possible to use a generalized orthogonality relation which is asymptotically satisfied by the high-order modes. This unusual DtN operator has been implemented in the Melina Code.

6.4.3. Scattering by a locally non uniform open waveguide

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard.

We have completed the work in collaboration with Lahcene Chorfi (University of Annaba, Algeria) and Ghania Dakhia (University of Biskra, Algeria) about the scattering of a time-harmonic acoustic wave by a defect located in an open waveguide. Two articles related to this subject are submitted. The first one presents a theoretical analysis of the problem, where the behavior of the scattered wave is controlled by an *modal radiation condition* based on a generalized Fourier transform which diagonalizes the transverse contribution of the Helmholtz operator. The uniqueness of the solution is proved by an original technique which combines a property of the energy flux with an argument of analyticity with respect to the generalized Fourier variable. The existence is then deduced classically from Fredholm's alternative by reformulating the scattering problem as a Lippman–Schwinger equation by means of the Green's function for the layered half-plane. The second article presents the theoretical background on the generalized Fourier transform that was used in the first article. It shows how to extend such a transformation to distributions, with a suitable definition of such distributions. This extension is based essentially on the fact that, as the usual Fourier transform, this transformation has the property to exchange regularity and decay between the physical and spectral variables.

6.4.4. Aharonov-Bohm Effect, Tonomura et al. Experiments and Gaussian Electron Beams.

Participants: Miguel Ballesteros, Ricardo Weder.

The Aharonov-Bohm effect is a fundamental quantum mechanical phenomenon wherein charged particles, like electrons, are physically influenced by the existence of magnetic fields in regions that are inaccessible to the particles. There has been a large controversy, involving over three hundred papers, concerning the existence of the Aharonov-Bohm effect. From the experimental point of view the issue was settled by the fundamental experiments of Tononura *et al.* , who used toroidal magnets to enclose a magnetic flux inside them. In remarkable experiments they were able to superimpose behind the magnet an electron beam that traveled inside the hole of the magnet with another electron beam that traveled outside the magnet and they measured the phase shift produced by the magnetic flux enclosed in the magnet, giving a conclusive evidence of the existence of the Aharonov-Bohm effect. After this experiment the controversy concerns mainly to wich extent the results of Tonomura et al. are predicted by quantum mechanics.

We obtained high-velocity estimates with error bounds for the scattering operator of the Schrödinger equation in three dimensions with electromagnetic potentials in the exterior of bounded obstacles that are handlebodies. A particular case is a finite number of tori. We proved our results with time-dependent methods. We considered high-velocity estimates where the direction of the velocity of the incoming electrons is kept fixed as its absolute value goes to infinity. We gave a method for the reconstruction of the flux of the magnetic field over a cross-section of the torus modulo 2π. Equivalently, we determined modulo 2π the difference in phase for two electrons that travel to infinity, when one goes inside the hole and the other outside it. For this purpose we only needed the high-velocity limit of the scattering operator for one direction of the velocity of the incoming electrons. We have similar results when there are several tori. We also gave a method for the unique reconstruction of the electric potential and the magnetic field outside the handlebodies from the high-velocity limit of the scattering operator. Our results give a rigorous proof that quantum mechnics qualitatively predicts the interference patterns observed by Tonomura *et al.*. These results will be the topic of a future article.

In order to obtain quantitative results we are currently working in the case of a a gaussian electron beam. The aim is to give an analytical, computer assisted, evaluation of the constant that appears in our error bound. We intent to give an approximate solution to the Schrödinger equation for all times and to provide uniform (in time) error bounds between the exact and the approximate solutions, that depend on the variance of the gaussian wave function. We expect that the error bounds grow as the probability of hitting the magnet grows, and also when the variance of the electron beam is small (and hence, by Heisenberg uncertainty principle the dispersion angle is big). If neither of those conditions occur, we expecto to obtain small error bounds for the time evolution and to provide a rigorous quantitative mathematical model of the experiments of Tonomura *et. al.*. This will allow us to make quantitatively precise to which extent quantum mechanics predicts the phase shifts observed by Tonomura *et al*.

6.5. Asymptotic methods and approximate models

6.5.1. Asymptotic models for thin wires

Participants: Xavier Claeys, Patrick Joly.

This is the topic of the PhD thesis of X. Claeys and involves the collaboration of Houssem Haddar (INRIA Saclay) and Francis Collino (CERFACS).

The new results for this year are the following

Numerical aspects During the preceding year, we had implemented and validated a three dimensional version of our method for taking into account thin wires in the simulation of wave propagation. Let us recall that this method combined asymptotic analysis of the Helmholtz equation around a thin wire, a fictitious domain formulation and an enriched Galerkin method. This year, on the basis of our preceding work, we were able to exhibit the link between this 3D version of the enriched Galerkin method and the Holland's scheme applied to the scalar diffraction problem around a wire. This resulted into a new interpretation of Holland's method and led to a theoretical expression of the Holland parameter in a 3D situation (such a result had been obtained in the past but only for 2D problems). In fact, in this new 3D setting the Holland parameter appeared to be a matrix (whereas in a 2D setting this parameter was simply a number). In the end, this led to a systematic manner for computing this Holland matrix, and we numerically validated such a procedure. Moreover, we experimented an enhancement in our procedure that substantially reduced the computational cost for determining this matrix. The performance of Holland's method with such an automatic calibration was comparable to those of the enriched Galerkin method.

Theoretical aspects As regards theoretical studies, we proposed a formalism for asymptotic problems around small obstacles and thin curved wires. Included in this formalism is a general treatment of matching calculus. This formalism led to results of asymptotic analysis of the electromagnetic field around a perfectly conducting thin wire. On the basis of this analysis, we were able to generalize our results on the Holland method to the case of Maxwell's equations.

6.5.2. Multiscale modelling in electromagnetism

Participants: Bérangère Delourme, Patrick Joly.

This topic is developed in collaboration with the CEA-Grenoble (LETI) and H. Haddar (INRIA Saclay-DEFI). We are interested in the scatering of electromagnetic waves by a complex "multi-scale" structure (FIG.4). More precisely, the main difficulty of the problem is the presence of a diffractive ring made of two layers of helicoidal metallic wires regularly and periodically spaced in the azimuthal direction. The distance between two wires is supposed to be small.

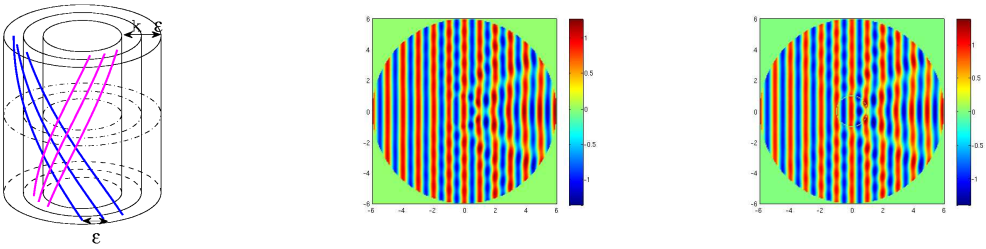

Figure 4. Left: periodic cylinder. Right: exact and approximate solutions

One possible remedy is to replace the use of the exact model by an approximate transmission condition which takes into account the periodicity and the thickness of the ring, that we assume to be both proportional to a single small parameter ε. First, we have constructed such a simplified model in a 2D-case using a two scale asymptotic expansion of the solution with respect to ε : the approach mixes periodic homogenization and matched asymptotic expansions. We have paid a particular attention to the stabilization of the effective transmission condition. Numerical simulations and error estimates have been carried out to validate the accuracy of the model.

We are now working on the 3D-case, which presents new specific difficulties : in particular, we have to treat the double helicity of the wires and to find the appropriate conditions to take into account the extremities of the wires.

6.5.3. Quasi-singularities and electrowetting

Participant: Patrick Ciarlet.

This is a twofold work.

First, there is a collaboration with Samir Kaddouri (former PhD student). Following the PhD thesis of Samir Kaddouri, we undertook the study of quasi-singularities for the 2D-cartesian and 2D-axisymmetric electrostatic model around rounded tips (M3AS, 2007). To complement this work, we are investigating the computation of accurate maps of the values of the electric field, to model corona discharge phenomea around 2D and $2\frac{1}{2}$D tips.

Second, there is a collaboration with Claire Scheid (Oslo Univ., Norway). Handling very small amount of liquid on a solid surface is of great industrial interest. In this field, electrowetting process is now broadly used: one charges a droplet posed on a solid by applying a given voltage between this droplet and a counter-electrode placed beneath the insulator. This allows one to control precisely the wetting of the drop on the solid. For modeling purposes, one has to compute very accurately the shape of the drop near the counter-electrode by solving an electrostatic problem with a piecewise constant electric permittivity. To avoid a numerical locking, this requires the use of *ad hoc* methods. Claire Scheid has been able to solve this problem in 2D configurations, with the help of the Singular Complement Method, which consists of the adjunction of an additional function to the discrete space: this function takes into account the shape of the non-smooth part of the electrostatic potential. We are now considering (more realistic) 3D configurations. In this case, the SCM is not usable numerically, so alternate solutions have to be devised: the generalized WRM (see §6.2.3) is a promising tool.

6.5.4. Asymptotic models for junctions of thin slots

Participants: Patrick Joly, Adrien Semin.

We have continued the work started in 2007. We have considered a network of two slots of same width, and we considered the wave equation, both for the time harmonic case and for the time dependent case. We completely justify the expansion and the error estimates. These results leads to many conferences (GTN Orsay, february 2008; CEMRACS, august 2008), one INRIA Research Report and one publication in ESAIM Proceedings.

6.5.5. *Wave propagation on infinite trees*

Participants: Patrick Joly, Bertrand Maury, Adrien Semin.

Here, we are interested in studying the wave propagation on a 1D tree with many generations, seen as an infinite tree. This research can be seen as a contribution to an emerging topic : the numerical solution of partial differential equations in fractal sets. More precisely, we want to solve a “1 D” wave equation on this tree, this 1D model being obtained, for instance, as an asymptotic model for wave propagation in a domain made of a junction of thin domains (see also section 6.5.4) when the number of domains tends to infinity and the width of the thin domains tends to 0. In our case, we assume that, from a given generation n, the subtrees are self-similar p-adic trees. Our objective is to compute the exact solution for up to the nth generation replacing the self-similar subtrees by transparent conditions. A potential application is the study of the wave propagation in the human lung (collaboration with Bertrand Maury from Orsay University).

We mostly advanced on the adapted functional framework (Sobolev spaces on trees) in order to get a well-posed problem and in particular understand how to give a sense to boundary conditions at "infinity" (for the tree), in the spirit of the previous work by Maury-Salort-Vannier. This implies to understand the notion of trace at infinity. We have shown that the existence of a trace, as well of the nature of the trace space, relies upon the geometrical characteristics of the tree. Our results are closely related on the resolution of a Laplace equation on a self-similar p-adic tree, for which we have been able to construct transparent boundary conditions. The next step is writing transparent conditions for the Helmholtz equation, seen as a perturbation of the Laplace equation at low frequencies (the concept of low frequency being in reference to the size of the remaining subtree : making the frequency tend to 0 is “equivalent” to making the “trucation order” n go to infinity).

6.5.6. *Approximate models in aeroacoustics*

Participants: Anne-Sophie Bonnet-Ben Dhia, Marc Duruflé, Patrick Joly, Lauris Joubert, Ricardo Weder.

This is the subject of the PhD thesis of Lauris Joubert.

We study the propagation of acoustic waves in a duct in the presence of a laminated flow when the width of the duct is small with respect to the wavelength. Last year, under the assumption that the Mach profile of the flow is obtained by a scaling of the Mach profile of a reference flow, we established formally a quasi-1D model limit which appears to be of integro-differential type : it is local in the longitudinal direction and non-local in the transverse one (in scaled coordinates). The well-posedness of the limit problem was analyzed and is directly related with the Mach profile: we have obtained sufficient conditions for the well-posedness of the limit problem but also sufficient conditions for the strong ill-posedness of this problem.

Using perturbation theory, we deduce from the ill-posed case new results on Kelvin-Helmholtz type hydrodynamic instabilities in compressible fluids. An article is in preparation.

In the well-posed case, with Ricardo Weder, we obtained, by combining the use of a Fourier-Laplace transform with complex plane integration techniques, a quasi-explicit representation of the solution of the limit problem. This formula has a double interest. From the theoretical point of view, it has permitted us to establish precise a priori estimates that explain in which sense (in other words, in which functional framework) the problem is well-posed. From the numerical point of view, we expect that the discretization of this formula will lead to an efficient numerical scheme.

We are currently working on the rigorous justification of the approximate model in the well-posed case. The next step of our research on this topic will consist in using the same type of method to design equivalent boundary conditions to take into account the presence of boundary layers in aeroacoustics.

6.6. Imaging and inverse problems

6.6.1. *Quasi-reversibility*

Participants: Laurent Bourgeois, Jérémi Dardé, Eric Lunéville.

We have continued our works on the method of quasi-reversibility to solve second order ill-posed Cauchy problems for an elliptic equation (as they appear in standard inverse problems). This constitutes the subject of the PhD thesis of J. Dardé.

First of all, our efforts have concerned the approximation of the fourth-order problems introduced by the quasi-reversibility approach. As the Morley's element (based on second degree polynomials) does not lead to a sufficient accuracy, we have implemented the Fraeijs De Veubeke's element K.V. 1 (based on third degree polynomials).

Secondly, we have developped a new strategy to identify obstacles in a domain from partial Cauchy data on the boundary of such domain. This strategy uses a coupling between the method of quasi-reversibility and the level set method. The identification of obstacles that are characterized by Dirichlet or Neumann boundary conditions have shown encouraging results by using this method. We aim at providing some various enlighting numerical applications, in particular in situations where standard methods are not applicable. We also aim at providing some theoretical justifications of our strategy.

Finally, a study has been conducted concerning general stability results for ill-posed elliptic problem (which in particular lead to convergence rates for the method of quasi-reversibility), with the help of Carleman estimates. We have improved some logarithmic stability results previously obtained in the sense we have specified the exponent in front of the logarithm as a function of the geometric singularity. We had to consider separately the case of $C^{1,1}$ class domains and the case of $C^{0,1}$ class domains, since the techniques are not the same in both cases. In the $C^{1,1}$ case, we have proved that our stability estimate is nearly optimal, and in the $C^{0,1}$ case, some numerical results show that our stability estimate is not far from optimality.

6.7. Other topics

6.7.1. *"Compressed sensing" technique and application to imaging*

Participant: Jean-François Mercier.

Jean-François Mercier visited the Department of Mathematics at Stanford University from January to August 2008 thanks to a DGA grant and worked with George Papanicolaou and Laurent Demanet on inverse problems and more specifically on methods for imaging. Our goal is to determine an image of good quality from "realistic" measurements. In particular we are interested in the case of a few measures, contiguous and noisy. We focus on the reconstruction of a sampled time signal from measurements of its low frequency Fourier spectrum. We are guided by the "compressed sensing" technology and by the associated theoretical results, originally well suited to recover a time signal from randomly chosen frequency measurements. We have modified and extended this technique to solve our problem. In addition, we have determined the minimum number of measurements this technology requires to determine a nice image with a high probability. Last we have developed a strategy for rebuilding a high frequency signal from low frequencies measurements. This is achieved thanks to the introduction of a high frequency filter.

6.7.2. *Fast solvers for evolution equations*

Participant: Jing-Rebecca Li.

The first part is a work in collaboration with L. Greengard (Courant Institute). We continue the research in the efficient solution of the heat equation, especially in moving and/or unbounded domains. In particular, we considered the numerical evaluation of single and double layer heat potentials in two dimensions on stationary and moving boundaries. One of the principal difficulties in designing high order methods concerns the local behavior of the heat kernel, which is both weakly singular in time and rapidly decaying in space. We show that standard quadrature schemes suffer from a poorly recognized form of inaccuracy, which we refer to as 'geometrically-induced stiffness', but that rules based on product integration of the full heat kernel in time are robust. When combined with previously developed fast algorithms for the evolution of the 'history part' of layer potentials, diffusion processes in complex, moving geometries can be computed accurately and in nearly optimal time.

The second part is a generalization of the work on the heat equation to the evaluation of fractional order integrals and derivatives. We evaluate the fractional integral

$$I^{\alpha}[f](t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \quad \alpha > 0,$$

at time steps $t = \Delta t, 2\Delta t, \cdots$, by making use of the integral representation of the convolution kernel:

$$t^{\alpha-1} = \frac{1}{\Gamma(1-\alpha)} \int_0^{\infty} e^{-\xi\, t}\, \xi^{-\alpha}\, d\xi.$$

We construct an efficient quadrature of this integral representation and use it as a part of a fast time stepping method. The (possible) singularity of f near $\tau = 0$ is taken into account. This new method is particularly well-suited for long time simulations.

6.7.3. Modeling of dendritic solidification

Participant: Jing-Rebecca Li.

This is a work in collaboration with L. Brush (University of Washington) and D. Calhoun (CEA). We use a fast solver for the heat equation in free-space to compute the thermal field in a simulation of crystal growth at low undercoolings. The solver is based on the efficient direct evaluation of the integral representation of the solution to the constant coefficient, free space heat equation with a smooth source term. The computational cost and memory requirements of the new solver are reasonable compared to standard methods and allow one to solve for the thermal field in a computational domain whose size depends only on the size of the growing crystal and not on the extent of the thermal field. This can result in significant computational savings in situations where the crystals grow slowly relative to the expansion of the thermal field and may make parameter regimes typically encountered in experimental situations more accessible to numerical treatment. We demonstrate the method on the problem of anisotropic, dendritic growth at low undercooling in two dimensions.

6.7.4. Functional inequalities and application to control theory

Participants: Bourgeois Laurent, Le Rousseau Jérôme, Léautaud Matthieu.

There are many applications of control theory for PDEs: wave and diffusion phenomena, fluid mechanics, etc. New interactions with biology are expected in particular in cancer therapy. In this framework the therapy acts as a control.

Through a source or boundary term we aim at driving the solution of the PDE to a prescribed state, and at a certain time, when the problem we consider is of evolution type. The work in progress concerns parabolic type equations and systems. The question of the null controllability of linear parabolic partial differential equations with smooth coefficients was first solved in the 1990's by G. Lebeau and L. Robbiano (1995) and by A. Fursikov and O. Yu. Imanuvilov (1996). Our interest here is the extension of such result to different cases that can allow us to get closer to systems that can be faced in applications.

A first interest in the generalization to non smooth coefficients. This research direction was started by J. Le Rousseau with his colleagues in Marseille (LATP, UMR 6632) and is now further pursued in collaboration with L .Robbiano (UVSQ, UMR 8100). An article with L .Robbiano was submitted this year on this subject and a second one is soon to be submitted. The main tool in this research is the derivation of so-called Carleman estimates. We focus our attention on the derivation of local estimates in the region where the coefficient of the principal part of the operator exhibits jumps across interfaces. Through microlocal analysis, and the use of Calderón projectors, we are able to prove such results for arbitrary coefficient jump signs and arbitrary dimension. These two points had remained open for several years.

Such Carleman estimates for elliptic operators allow to prove powerful spectral inequalities that can lead to the null controllability of linear parabolic equations. In the case of parabolic operators these Carleman estimates allow to prove the null controllability of certain classes of non-linear parabolic equations.

In a second research focus, in the framework of Matthieu Léautaud's PhD thesis, which has started this fall, we aim to prove such estimates and further spectral inequalities for systems of coupled parabolic equations. For systems, many questions remain open. The goal is to obtain spectral inequalities for such systems of the type described above with one "observation" on only one component of the solution. For systems, a controllability result exists, by Benabdallah *et al.*, with the control acting on only one of the components of the solution. This however requires the observation/control region to meet the coupling region of the system. Our research aims to obtain spectral inequalities for systems that are not self-adjoint, and to relax this condition on the observation region.

6.7.5. Linear elasticity

Participant: Patrick Ciarlet.

This is a collaboration with Philippe Ciarlet (City University of Hong Kong).

Following a number of theoretical studies (carried out with the help of C. Amrouche (CRAS, 2007), G. Geymonat and F. Krasucki (CRAS, 2007)), we are considering the numerical approximation of linearized elasticity problems, via the St-Venant approach. Within this framework, one builds conforming finite element subspaces of the (square integrable symmetric) stress tensors. In this sense, this approach is different from the mixed approaches, whose aim is to compute approximations of both the displacement and stress.

This problem has been solved for the planar case. The stress tensors are approximated with the help of discrete, piecewise constant, symmetric tensor fields, defined on triangles, and one solves a discrete optimization problem. Higher degree approximations, and extensions to quadrilaterals and to 3D configurations are currently investigated.

7. Contracts and Grants with Industry

7.1. Contract POEMS-CEA-LIST

Participants: Gary Cohen, Édouard Demaldent.

G. Cohen perticipates to Projet CASSIS headed by the LIST laboratory of CEA and funded by the EADS Fundation which started in June 2008. This project aims to simulate elastic waves in thin layered anisotropic media for non-destructive testing. In collaboration with E. Demaldent, who will start a post-doc at Inria in the beginning of 2009, G. Cohen must provide a code based on spectral element methods to model these waves.

7.2. Contract POEMS-ONERA-CE Gramat

Participants: Gary Cohen, Marc Duruflé, Morgane Bergot.

In collaboration with ONERA-DEMR, G. Cohen participates with M. Bergot to the FEMGD project funded by CEG (Centre d'Études de Gramat), which started in 2004. This project is devoted to the construction of a software using spectral discontinuous Galerkin methods for Maxwell's equations. This project came to an end in December 2008.

7.3. Contract POEMS-CE Gramat

Participants: Gary Cohen, Alexandre Sinding.

In collaboration with ONERA-DEMR, G. Cohen participates with A. Sinding to the NADEGE project funded by CEG (Centre d'Études de Gramat), which started in September 2008. This project is devoted to the construction of a software based on FEMGD for solving Vlasov-Maxwell's equations by a PIC method.

7.4. Contract POEMS-DGA

Participants: Laurent Bourgeois, Eric Luneville.

This contract concerns the identification of buried objects using the linear sampling method.

7.5. Contract POEMS-EADS

Participants: Sonia Fliss, Patrick Joly.

This contract is about the numerical simulation of elastic wave propagation in composite materials (periodic structures with a defect) in the time harmonic regime.

7.6. Contract POEMS-Airbus

Participants: Patrick Joly, Jeronimo Rodríguez.

This contract (Project ADNUMO) is about the hybridation of time domain numerical techniques in aeroacoustics (Linearized Euler equations).

8. Other Grants and Activities

8.1. National Cooperations

- SimNanoPhot : project of the ANR in collaboration with IEF (Institut d'Electronique Fondamentale) of the University of Orsay. It concerns the modelization of micro and nano-structures in optics.
- GDR Ultrasons: this GDR, which regroups more than regroup 15 academic and industrial research laboratories in Acoustics and Applied Mathematics working on nondestructive testing. It has been renoveled this year with the participation of Great Britain.
- ANR (RNTL) project *MOHYCAN: MOdélisation HYbride et Couplage semi-ANalytique pour la simulation du CND.*
  Topic: *On the coupling of the finite element code ATHENA with the semi-analitic code CIVA. Non-destructif testing.* Collaborators: CEA-LIST (main contact), EDF and CEDRAT.

8.2. International Cooperations

- The Project is involved in a STIC project with the LAMSIN of ENIT (Tunis) with A. Ben Abda and N. Gmati.
- The GDR Ultrasons has been extended to a collaboration with United Kingdom.

- The research project, “Improved Imaging in Non-Smooth Media”, is a starting collaboration with Prof. Alison Malcolm (Massachusetts Institute of Technology, Dept. of Earth, Atmospheric, and Planetary Sciences). It has obtained the support of the MIT-France fund (MIT-France Seed Fund for Collaborative Research) for the years 2008–2009.

8.3. Visiting researchers

- Peter Monk, Professor at University of Delaware, USA.
- Dan Givoli, Professor at Technion, Israel.
- Ricardo Weder, Professor at University of Mexico, Mexico.
- Alison Malcolm, Professor at MIT, USA.

9. Dissemination

9.1. Various academic responsibilities

- A. S. Bonnet-Ben Dhia is the Head of the Electromagnetism Group at CERFACS (Toulouse)
- A. S. Bonnet-Ben Dhia is in charge of the relations between l’ENSTA and the Master “Dynamique des Structures et des Systèmes Couplés (Responsible : Etienne Balmes)”.
- P. Ciarlet is an editor of DEA (Differential Equations and Applications) since July 2008
- P. Joly is a member of the scientific committee of CEA-DAM.
- P. Joly is a member of the Commission de Spécialistes of the University Paris VII.
- P. Joly is a member of the Hiring Committee of Ecole Polytechnique in Applied Mathematics.
- P. Joly is a member of the Post Docs Commission of INRIA Rocquencourt.
- P. Joly is a member of the Scientific Committee of the Seminar in Applied Mathematics of College de France (P. L. Lions).
- P. Joly is an editor of the journal Mathematical Modeling and Numerical Analysis.
- P. Joly is a member of the Book Series Scientific Computing of Springer Verlag.
- P. Joly is an expert for the MRIS (Mission pour l’Innovation et la Recherche Scientifique) of DGA (Direction Générale de l’Armement)
- M. Lenoir is a member of the Commission de Spécialistes of CNAM.
- M. Lenoir is in charge of Master of Modelling and Simulation at INSTN.
- J. Le Rousseau is coordinator of the CoNum project, “Numerical control, application to biology”, Projet ANR JeuneS chercheurs.
- J. Le Rousseau is elected member of CNU (26ème section)
- J. R. Li is one of the guest editors of the special issue of the Journal of Computational mathematics that will follow the WAVES2007 Conference.
- E. Lunéville is the Head of UMA (Unité de Mathématiques Appliquées) at ENSTA.
- The Project organizes the monthly Seminar Poems (Coordinators: X. Claeys, J. Dardé, J. Chabassier)

9.2. Teaching

- Eliane Bécache
  - *Introduction à la théorie et l’approximation de l’équation des ondes*, 3ème année ENSTA-Paris et Master 2 UVSQ

  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
- Laurent Bourgeois
  - *Outils élémentaires pour les EDP*, ENSTA, Paris
  - *Contrôle optimal des EDP*, ENSTA, Paris
- Anne-Sophie Bonnet-Ben Dhia
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles. MA102*, Cours de Tronc Commun de 1ère année à l'ENSTA
  - *Propagation d'ondes*, Cours commun au Master de Dynamique des Structures et des Systèmes Couplés et à l'Option de Mécanique (filière VO) de l'Ecole Centrale de Paris
  - *Propagation dans les guides d'ondes. C7-3*, Cours de 3ème année à l'ENSTA. En collaboration avec Eric Lunéville.
  - *Théorie spectrale des opérateurs autoadjoints et application aux guides optiques. MAE21*, Cours de 2ème année à l'ENSTA. En collaboration avec Christophe Hazard et Jean-François Mercier.
- Juliette Chabassier
  - *Introduction au calcul scientifique*, MA 261, Cours de remise à niveau AST de l'ENSTA, Paris
- Patrick Ciarlet
  - *Introduction à MATLAB*, ENSTA (1st year)
  - *La méthode des éléments finis*, ENSTA (2nd year)
  - *Calcul distribué, un point de vue théorique*, ENSTA (3rd year), et Master "Modélisation et Simulation" (2nd year)
  - *L'equation de Maxwell et leur discrétisation*, ENSTA (3rd year), et Master "Modélisation et Simulation" (2nd year)
- Julien Coatleven
  - *Méthode des éléments finis*, ENSTA, Paris
- Gary Cohen
  - *Cours de Master II: Méthodes numériques pour les équations des ondes*, Université de Paris-Dauphine October-December 2006
- Bérangère Delourme
  - *Travaux dirigés d'Analyse*, Licence 1ère année, Université Paris Dauphine
- Marc Duruflé
  - *Introduction à la discrétisation des EDP*, ENSTA, Paris
- Sonia Fliss
  - *Fonctions à variables complexes*, ENSTA, Paris
  - *Introduction à la discrétisation des EDP*, ENSTA, Paris
- Christophe Hazard
  - *Outils élémentaires d'analyse pour les EDP*, 1ère année, ENSTA Université Paris XI
  - *Théorie spectrale des opérateurs auto-adjoints et applications aux guides optiques*, 3ème année, ENSTA

- Sebastien Impériale
  - Travaux dirigés d' Algèbre et d' Analyse, Licence 1ère année, Université de Versailles
- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA, Paris
  - *Outils élémentaires d'analyse pour les EDP*, ENSTA, Paris
  - *Problèmes directs et inverses en théorie de la diffraction*, Master 2 Paris 6, Parcours Analyse Numérique et EDP
  - *Méthodes volumiques et couches PML pour les problèmes de propagation d'ondes en régime transitoire*, Collège Polytechnique.
- Eric Lunéville
  - *Introduction à MATLAB*, ENSTA, Paris
  - *Contrôle optimal des EDP*, ENSTA, Paris
- Jean-François Mercier
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, Travaux dirigés de 1ère année à l'ENSTA
  - *Variable complexe*, Travaux dirigés de 2ème année à l'ENSTA
  - *Ondes dans les milieux continus*, Cours de 3ème année à l'ENSTA
- Jeronimo Rodríguez
  - *Optimisation*. ENSTA (Paris).
  - *Introduction à la discrétisation des EDPs*. ENSTA (Paris).
  - *Introduction à la simulation numérique*. ENSTA (Paris).
  - *La méthode des élements finis*. ENSTA (Paris).
- Adrien Semin
  - *Algorithmique et programmation*, Licence 3ème année, Université Paris XI
  - *Schémas numériques pour les EDO*, Licence 2ème année, Université Paris XI
  - *Simulations numériques en Matlab*, Licence 3ème année, Université Paris XI

9.3. Participation in Conferences, Workshops and Seminars

- Eliane Bécache
  - *Some contributions to Perfectly Matched Layers*, WCCM8 / ECCOMAS 2008, June 30 –July 5, 2008, Venice, Italy, jul, 2008.
- Anne Sophie Bonnet-Ben Dhia
  - *Balade mathématique: De la fibre optique au pot d'échappement, suivez le guide d'ondes*. Conférence de vulgarisation, Lycée Victor Duruy, Paris., mai 2008.
  - *Etude de formulations éléments finis en présence d'une interface entre un diélectrique et un métamatériau*, NUMELEC, déc 2008.
  - *Finite element computation of leaky modes in stratified waveguides*, 5èmes journées du GDR étude de la propagation ultrasonore en vue du contrôle non-destructif, Anglet, june., 2008.
  - *A finite element method for time harmonic acoustics in arbitrary flows*. Acoustics'08, jul, 2008.

- Laurent Bourgeois
  - *Stabilité conditionnelle pour les problèmes elliptiques mal posés : influence de la régularité du bord*, Premier congrès de la SM2A, Rabat (Maroc), 6-8/02/2008, 2008.
  - *The linear sampling method in a waveguide : a formulation based on modes*, Conférence ICIPE 2008, VVF Dourdan, Dourdan (France), 15-19/6/2008, 2008.
- Patrick Ciarlet
  - Numerical Analysis Seminar at IRMAR, Rennes Univ., Mar. 2008
  - Numerical Analysis Seminar at the Mathematics Institute, Zürich Univ., Apr. 2008
  - PDEs and Applications Seminar at the Elie Cartan Institute, Nancy Univ., Jun. 2008
  - PDEs Seminar at the Jean Kuntzmann Laboratory, Grenoble Univ., Nov. 2008
- Xavier Claeys
  - *Asymptotics and numerical analysis for wave diffraction by thin wires*, Seminaire du GMM a l'INSA de Toulouse, nov, 2008.
- M. Duruflé
  - *Eléments finis d'ordre élevé*, CANUM 2008.
- Sonia Fliss
  - *Transparent boundary conditions, Wave propagation and Periodic media*, Séminaire du Cermics (ENPC), 2008.
  - *Wave propagation in locally perturbed periodic media*, Conférence "EDP et applications 2008", Hammamet (Tunisie), 2008.
- Patrick Joly
  - *Couplage entre potentiels retardés et méthodes de Galerkin discontinues. Applications en acoustique.* Inauguration de la Chaire Modélisation Mathématique et Simulation Numérique, Ecole Polytechnique, Octobre 2008, oct 2008.
  - *Numerical Methods for Wave Propagation in locally perturbed Periodic Media*, Mathematical Topics in Electromagnetic Fields and Wave Propagation, Karsruhe (Allemagne), apr, 2008.
  - *Higher order explicit time stepping and optimal CFL condition for second order hyperbolic problems second order hyperbolic problems.* Numerical Methods for Evolution Equations, Heraklion (Crète) Evolution Equations, Heraklion (Crete), sep, 2008.
- Jing-Rebecca Li
  - *A fast time stepping method for evaluating fractional integrals*, Fractional Differentiation and its Applications, Ankara, Turkey, Nov, 2008.
  - *Fast computation of time convolutions: the heat equation and fractional order integrals*, Numerical Methods Seminar, University Paris 6, Dec, 2008.
- Jean-François Mercier
  - *Acquisition compressée et application à l'imagerie.* Seminaire POEMS, Nov, 2008.
- Jeronimo Rodriguez
  - *Coupling Discontinuous Galerkin Methods and Retarded Potentials for Time Dependent Wave Propagation Problems*, 5th European Congress in Computational Methods in Applied Sciences and Engineering, jul, 2008

  - *A Posteriori Error Estimation in the Frame of Reduced Basis Approximations*, Séminaire du Département de Génie Mathématique et Modélisation, INSA, Toulouse, dec, 2008
  - *Reduced Basis Output Bounds for Harmonic Wave Propagation Problems*, Séminaire interne du CERFACS, Toulouse, feb, 2008.
  - *La méthode de domaines fictifs pour la diffraction d'ondes avec de conditions aux limites de Neumann*, Journée Thématique "Frontières immergées et applications", LATP, Marseille, nov 2008.
- Adrien Semin
  - *Propagation of acoustic waves in junction of thin slots*, CEMRACS 2008, aug, 2008.
  - *Propagation d' ondes acoustiques dans des jonctions de fentes minces*, Groupe de Travail Numérique - Université d'Orsay, feb, 2008.
- Ricardo Weder
  - *Estimations de grande vitesse pour l' opérateur de dispersion et effet d'Aharonov-Bohm.* Université de Paris Sud, Dec. 2008. Université de Rennes 1, Nov. 2008.
  - *Le problème de scattering inverse à énergie fixe pour l'équation des Schrödinger avec des potentiels électromagnétiques*, Université de Paris Diderot, Nov. 2008.
  - *Revêtement Electromagnétique d'Invisibilité*, Séminaire POEMS, Oct. 2008. CMAP, Ecole Polytechnique, Nov. 2008.

10. Bibliography

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Doctoral Dissertations and Habilitation Theses

[1] X. CLAEYS. *Analyse asymptotique et numérique de la diffraction d'ondese par dese fils minces*, Ph. D. Thesis, 2008.

[2] C. M. ZWÖLF. *Méthodes variationnelles pour la modélisation des problèmes de transmission d'onde électromagnétique entre diélectrique et méta-matériau.*, Ph. D. Thesis, 2008.

Articles in International Peer-Reviewed Journal

[3] F. ASSOUS, P. CIARLET, E. GARCIA. *A characterization of singular electromagnetic fields by an inductive approach*, in "Numerical Analysis and Modeling", vol. 5, 2008, p. 491–515.

[4] M. BALLESTEROS, R. WEDER. *High-velocity estimates for the scattering operator and Aharanov-Bohm effect in tree dimensions*, in "Communication in Mathematics Physics", to appear.

[5] V. BARONIAN, A.-S. BONNET-BENDHIA, E. LUNÉVILLE. *Transparent boundary conditions for the harmonic diffraction problem in an elastic waveguide*, in "J. Comput. Appl. Math", to appear.

[6] P. BENNER, J.-R. LI, T. PENZL. *Numerical solution of large-scale Lyapunov equations, Riccati equations, and linear-quadratic optimal control problems*, in "Numerical Linear Algebra with Applications", vol. 15, 2008, p. 755–777.

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[8] A.-S. BONNET-BENDHIA, P. CIARLET, C. M. ZWÖLF. *A new compactness result for electromagnetic waves. Application to the transmission problem between dielectrics and metamaterials*, in "Math. Models Meth. App. Sci.", vol. 18, 2008, p. 1605–1631.

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[10] L. BOURGEOIS, E. LUNÉVILLE. *The Linear Sampling Method in a waveguide : a modal formulation*, in "Inverse Problems", vol. 24, 2008, 015018.

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[13] Y. CHEN, J. S. HESTHAVEN, Y. MADAY, J. RODRÍGUEZ. *A Monotonic Evaluation of Lower Bounds for Inf-Sup Stability Constants in the Frame of Reduced Basis Approximations*, in "C. R. Acad. Sci. Paris, Ser. I", to appear, 6 .

[14] P. CIARLET, P. CIARLET. *Direct computation of stresses in planar linearized elasticity*, in "Math. Models Meth. App. Sci.", to appear.

[15] P. CIARLET, P. CIARLET. *A new approach for approximating linear elasticity problems*, in "C. R. Acad. Sci. Paris, Ser. I", vol. 346, 2008, p. 351–356.

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