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Project-Team Tosca

TO Simulate and CAlibrate stochastic processes

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TOSCA *is located both at INRIA Nancy – Grand Est (Nancy) and INRIA Sophia Antipolis – Méditerranée.*

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2. Overall Objectives

2.1. Overall Objectives

The Inria Research team TOSCA is located both at Inria Sophia-Antipolis – Méditerranée and Inria Nancy – Grand Est. It is a follow-up of the OMEGA project which ended in December 2006. The team develops and analyzes stochastic models and probabilistic numerical methods. The present fields of applications are in finance, fluid mechanics, biology, chemical kinetics, neuro-sciences and population dynamics.

The problems where stochastic models arise are numerous, and the critical reasons for which stochastic models are used make analyzes and simulations difficult.

The TOSCA team thus aims to develop calibration and simulation methods for stochastic models in cases where **singularities** in the coefficients or **boundary conditions** make them hard to discretize and estimate. For this, we are willing to tackle theoretical and numerical questions which are motivated by real applications.

We are interested in developing **stochastic numerical methods** and **transverse methodologies** that cover several fields of applications, instead of having chosen a particular field of application (e.g., Biology, or Fluid Mechanics, or Chemistry). We justify this way to proceed as follows:

- From a couple of years now, we have attacked singular problems to answer questions coming from economists, meteorologists, biologists and engineers with whom we collaborate within industrial contracts or research programs such as ACI, ANR, GDR. To solve their problems which are so complex that stochastic processes are involved in the modelling, these colleagues need to combine expertise and knowledge in many fields: deterministic computing, computer science, vision, algorithm analysis, etc. We are incompetent in these fields, and therefore we could not pretend to fully treat any of these problems. A contrario, we are requested to bring our expertise in stochastic modelling and simulation to extremely various domains of applications.
- In spite of this diversity, whatever the application is, one has to simulate stochastic processes solutions to equations of the type

$$\begin{cases} X_t(\omega) &= X_0(\omega) + \left(\int_0^t \int_{\mathbb{R}^d} b(X_s,y)\mu_s(dy)ds\right)(\omega) \\ &\quad + \left(\int_0^t \int_{\mathbb{R}^d} \sigma(X_s,y)\mu_s(dy)dZ_s\right)(\omega), \\ \mu_s &= \text{Law of } X_s \text{ for all } s \geq 0, \end{cases} \tag{1}$$

  in order to compute statistics of the laws of functionals of these solutions. In addition, several fields often produce very similar "pathologies" of the model (1) or of the statistics to compute: for example, Pope's Lagrangian stochastic particles in Fluid Mechanics and models in Molecular Dynamics produce the same degeneracy in (1), namely, one has to substitute 'Conditional Law of components of X_s given the other ones' to 'Law of X_s'; as well, when studying chartist strategies in Finance and stochastic resonance in the electrical working of neurons, we encounter close questions on the density functions of the random passage times of processes (X_t) at given thresholds.
- Theory and numerical experiments show that each 'pathology' of the model (1) requires specific analysis and numerical methods. However, they require common abstract tools (Malliavin calculus, propagation of chaos theory, nonlinear PDE analysis, etc.) and common numerical methodologies (stochastic particle systems, Monte Carlo simulations, time discretization of stochastic differential equations, etc.). Thus each application takes benefit from the modelling and numerical knowledge developed for all the other ones.

The TOSCA team is currently studying models in relation with Geophysics, Neuro-sciences, Fluid Mechanics, Chemical Kinetics, Meteorology, Molecular Dynamics, Population Dynamics and Evolution and Finance. We also construct and study stochastic particle systems for Fluid Mechanics, coagulation–fragmentation, stationary nonlinear PDEs, variance reduction techniques for Monte-Carlo computations and numerical methods combining deterministic and stochastic steps to solve nonlinear PDEs in Finance.

3. Scientific Foundations

3.1. Scientific Foundations

Most often physicists, economists, biologists, engineers need a stochastic model because they cannot describe the physical, economical, biological, etc., experiment under consideration with deterministic systems, either because of its complexity and/or its dimension or because precise measurements are impossible. Then they renounce to get the description of the state of the system at future times given its initial conditions and try instead to get a statistical description of the evolution of the system. For example, they desire to compute occurrence probabilities for critical events such as overstepping of given thresholds by financial losses or neuronal electrical potentials, or to compute the mean value of the time of occurrence of interesting events such as the fragmentation up to a very low size of a large proportion of a given population of particles. By nature such problems lead to complex modelling issues: one has to choose appropriate stochastic models, which require a thorough knowledge of their qualitative properties, and then one has to calibrate them, which requires specific statistical methods to face the lack of data or the inaccuracy of these data. In addition, having chosen a family of models and computed the desired statistics, one has to evaluate the sensitivity of the results to the unavoidable model specifications. The TOSCA team, in collaboration with specialists of the relevant fields, develops theoretical studies of stochastic models, calibration procedures, and sensitivity analysis methods.

In view of the complexity of the experiments, and thus of the stochastic models, one cannot expect to use closed form solutions of simple equations in order to compute the desired statistics. Often one even has no other representation than the probabilistic definition (e.g., this is the case when one is interested in the quantiles of the probability law of the possible losses of financial portfolios). Consequently the practitioners need Monte Carlo methods combined with simulations of stochastic models. As the models cannot be simulated exactly, they also need approximation methods which can be efficiently used on computers. The TOSCA team develops mathematical studies and numerical experiments in order to determine the global accuracy and the global efficiency of such algorithms.

The simulation of stochastic processes is not motivated by stochastic models only. The stochastic differential calculus allows one to represent solutions of certain deterministic partial differential equations in terms of probability distributions of functionals of appropriate stochastic processes. For example, elliptic and parabolic linear equations are related to classical stochastic differential equations, whereas nonlinear equations such as the Burgers and the Navier–Stokes equations are related to McKean stochastic differential equations describing the asymptotic behavior of stochastic particle systems. In view of such probabilistic representations one can get numerical approximations by using discretization methods of the stochastic differential systems under consideration. These methods may be more efficient than deterministic methods when the space dimension of the PDE is large or when the viscosity is small. The TOSCA team develops new probabilistic representations in order to propose probabilistic numerical methods for equations such as conservation law equations, kinetic equations, nonlinear Fokker–Planck equations.

4. Application Domains

4.1. Application Domains

TOSCA is interested in developing stochastic models and probabilistic numerical methods. Our present motivations come from Finance, Neurosciences and Biology, Fluid Mechanics and Meteorology, Chemical Kinetics, Diffusions in random media, Transverse problems and Softwares and Numerical experiments.

4.1.1. Finance

For a long time now TOSCA has collaborated with researchers and practitioners in various financial institutions and insurance companies. We are particularly interested in calibration problems, risk analysis (especially model risk analysis), optimal portfolio management, Monte Carlo methods for option pricing and risk analysis, asset and liabilities management. We also work on the partial differential equations related to financial issues, for example the stochastic control Hamilton–Jacobi–Bellman equations. We study existence, uniqueness, qualitative properties and appropriate deterministic or probabilistic numerical methods. At the time being we pay a special attention to the financial consequences induced by modelling errors and calibration errors on hedging strategies and portfolio management strategies.

4.1.2. Neurosciences and Biology

The interest of TOSCA in biology is developing in three main directions: neurosciences, molecular dynamics and population dynamics. In neurosciences, stochastic methods are developed to analyze stochastic resonance effects and to solve inverse problems. For example, we are studying probabilistic interpretations and Monte Carlo methods for divergence form second-order differential operators with discontinuous coefficients, motivated by the 3D MEG inverse problem. Our research in molecular dynamics focus on the development of Monte Carlo methods for the Poisson-Boltzmann equation which also involves a divergence form operator, and of original algorithms to construct improved simulation techniques for protein folding or interactions. Finally, our interest in population dynamics comes from particular methods, ecology, evolution and genetics. For example, we are studying the emergence of diversity through the phenomenon of evolutionary branching in adaptive dynamics. Some collaborations in biostatistics on cancer problems are also being initiated.

4.1.3. Fluid Mechanics and Meteorology

In Fluid Mechanics we develop probabilistic methods to solve vanishing vorticity problems and to study the behavior of complex flows at the boundary, and their interaction with the boundary. We elaborate and analyze stochastic particle algorithms. Our studies concern the convergence analysis of these methods on theoretical test cases and the design of original schemes for applicative cases. A first example concerns the micro-macro model of polymeric fluids (the FENE model). A second example concerns Pope's Lagrangian modelling of turbulent flows, motivated by the problem of modelling and computing characteristic properties of the local wind activity in areas where windmills are built. Our goal is to estimate local resources in energy which are subject to meteorological randomness by combining large scale wind models and small scale Monte Carlo techniques, and to simulate management strategies of wind resources.

4.1.4. Chemical Kinetics

The TOSCA team is studying coagulation and fragmentation models, that have numerous areas of applications (polymerization, aerosols, cement industry, copper industry, population dynamics,...). Our current motivation comes from the industrial copper crushers in Chile. We aim to model and calibrate the process of fragmentation of brass particles of copper in industrial crushers, in order to improve their efficiency at a low cost.

4.1.5. Diffusions in random media

A *random medium* is a material with a lot of heterogeneity which can be described only statistically. Typical examples are fissured porous media with rocks of different types, turbulent fluids or unknown or deficient materials in which polymers evolve or waves propagate. These last few years, the TOSCA team has been in relation with the Geophysics community on problems related to underground diffusions, especially those which concern waste transport or oil extraction. We are extending our previous results on the simulation of diffusion processes generated by divergence form operators with discontinuous coefficients. Such an operator appears for example from the Darcy law for the behavior of a fluid in a porous media. We are also developing another class of Monte Carlo methods to simulate diffusion phenomena in discontinuous media.

4.1.6. Transverse problems

Several of the topics of interest of TOSCA do not only concern a single area of application. This is the case of particular methods for the long time simulations of nonlinear McKean-Vlasov PDEs, the problem of simulation of multivalued models, variance reduction techniques or stochastic partial differential equations. For example, multivalued processes have applications in random mechanics or neurosciences, and variance reduction techniques have applications in any situation where Monte Carlo methods are applicable.

4.1.7. Software, numerical experiments

TOSCA is interested in designing algorithms of resolution of specific equations in accordance with the needs of practitioners. We benefit from our strong experience on the programming of probabilistic algorithms on various architectures including intensive computation architectures. In particular, our activity will concern the development of grid computing techniques to solve large dimensional problems in Finance. We are also interested in intensively comparing various Monte Carlo methods on PDEs and in the development of open source libraries for our numerical methods in Fluid Mechanics, Meteorology, MEG or Chemical Kinetics.

5. New Results

5.1. Probabilistic numerical methods, stochastic modelling and applications

Participants: Xavier Aubert, Mireille Bossy, Nicolas Champagnat, Madalina Deaconu, Angela Ganz, Samuel Herrmann, Pierre-Emmanuel Jabin, Antoine Lejay, Pierre-Louis Lions, Sylvain Maire, Sylvain Rubenthaler, Denis Talay, Etienne Tanré, Julian Tugaut, Samih Zein.

5.1.1. Monte Carlo methods for diffusion processes and PDE related problems

Keywords: *Green function*, *Monte Carlo method*, *diffusion processes*, *discontinuous media*, *geophysics*, *particle method*.

A. Lejay and S. Maire continued the numerical study of the simulation of processes with discontinuous coefficients in some simple cases by comparing two approaches: the one comes from the application of the results obtained for one-dimensional processes and the second one uses a kinetic approximation. In the last approach, we use a transport equation to approach the second-order partial differential equation.

A. Lejay and S. Maire have also worked on Monte Carlo computations of the first eigenvalue of linear operators like the Laplace operator or the neutron transport operator in a bounded domain [32]. This method relies on a branching mechanism and provides the first eigenelement of the adjoint operator.

A. Lejay, within the Groupement MOMAS has followed the collaboration with S. Huberson (Université de Poitiers) in order to compare results obtained by Monte Carlo methods and results obtained by particle systems. In addition, since both techniques rely on getting a good approximation of the Green function, they were working on using the results developed in the Monte Carlo methods for processes with discontinuous coefficients in order to improve the numerical schemes for particles methods.

In his post-doc from September 2007 to March 2008, S. Zein studied the application of optimization algorithm for the method of random walk on rectangles by using importance sampling techniques [57].

5.1.2. Study of particular self-stabilizing diffusions

Keywords: *Laplace method*, *exit time*, *interacting particle system*, *large deviations*, *self-stabilization*.

The well-known Kramers-Eyring law characterizes the time needed by classical diffusions to exit from some bounded domain. This description was extended by S. Herrmann, P. Imkeller and D. Peithmann [27] to particular self-stabilizing diffusions using the large deviation theory. These diffusions represent typically the motion of a particle subject to three sources of forcing. Firstly, it wanders in a landscape whose geometry is determined by a convex potential. Secondly, its trajectories are perturbed by Brownian noise of small amplitude. The third source of forcing can be thought of as self-stabilization: the particle is attracted by its own distribution. The convexity of the potential is essential in this previous study. S. Herrmann and J. Tugaut investigate non-convex situations namely symmetric double-well potentials.

5.1.3. Coagulation-fragmentation equations

Keywords: *Chemical kinetics*, *Smoluchowski equation*.

The phenomenon of coagulation was first described by the Polish physicist Marian von Smoluchowski in 1916, in a paper on the precipitation in colloidal suspensions. It applies and is studied in many domains as: the formation of stars and planets, the behavior of fuels in combustion engines, the polymerisation phenomenon, etc. Our expertise in this domain is developed in a new direction.

In [74] E. Tanré and his chilean collaborators propose a stochastic model for mineral grinding based on the probabilistic interpretation of Smoluchowski's equation of fragmentation, also known as the *comminution equation*. The model allows us to construct an algorithm which simulates the dynamics of individual mineral particle fragmentation, yielding to a Monte Carlo approximation of solutions to the comminution equation. As a result, several interesting variables depending on the size of particles can be simulated as well, which is particularly interesting for designing energy consumption optimization analysis.

A new study is underway, aiming to construct a model which effectively illustrates the phenomenon of fragmentation and accounts for new parameters, such as the position of particles in the crusher, the crusher geometry, the shape, size and number of steel balls used as projectiles and the damage factor. This factor takes into account the effect of low-impact collisions, which may occur before particle fracture. A balance must be found between the model's complexity and its ability to approximate the copper industry crushers.

A successful outcome will allow us to optimize crushers efficiency at a minimal cost. The people involved in this study are M. Deaconu and E. Tanré and their chilean collaborators.

5.1.4. Stochastic Resonance for neuronal models

Gilles Lebeau (University of Nice), S. Rubenthaler, D. Talay and E. Tanré have continued the work on the modelling of stochastic resonance effects in the neuronal activity [96], [97]. We study whether the leaky integrate-and-fire (LIF) model for neuron membrane potentials exhibits the following stochastic resonance property: assuming that the input is periodic of frequency f, an increase in the noise makes the density of the law of the first hitting time of the threshold (who generates a spike) exhibiting modes at a distance close to $1/f$.

The main tools are issued from the theory of PDE. In particular, we use a spectral analysis of the infinitesimal generator of a non homogeneous Markov process derived from the LIF model by transformations localizing the study in specific critical spatial regions and time intervals (where the process is close to the threshold).

5.1.5. Asymptotic development in propagation of chaos for various particle algorithms

P. Del Moral, F. Patras and S. Rubenthaler have worked on the propagation of chaos in Feynman-Kac particle systems in discrete time [58]. Here, a Feynman-Kac particle system is an interacting particle system whose empirical measure approximates a target law which is a Feynman-Kac measure. The propagation of chaos is the following property: the law of q particles in the population of N particles converges towards the q tensor product of the target law. We have obtained a development of the discrepancy between the two previous laws in powers of N, up to any order. This development relies on representations of the interaction history of the particles and uses rather involved combinatorics on trees.

In 2008, the authors have applied the same ideas to Feynman-Kac systems in continuous time. They also obtained a similar development for Nanbu and Bird systems, using different techniques. These last systems are particle systems used to approximate the solution of the mollified Boltzmann equation [105]. These developments in propagation of chaos provide a simple way to prove already known convergence results of particle systems (almost sure, in the L^q norm or of central limit type). This work also provides new results on the convergence of U-statistics of such systems.

5.1.6. New simulation schemes for the approximation of Feynman-Kac representations

The Feynman-Kac formula is a well-known tool to achieve stochastic representations of the pointwise solution of numerous partial differential equations like diffusion or transport equations. In a joint work with E. Gobet [104], S. Maire has introduced sequential Monte Carlo algorithms to compute the solution of the Poisson equation with a great accuracy using spectral methods. In [33], S. Maire and E. Tanré have proposed new variants of the Euler scheme and of the walk on spheres method for the Monte Carlo computation of Feynman-Kac representations. These new schemes have been optimized using quantization for both source and boundary terms. They have also introduced stochastic spectral formulations with very good properties in terms of conditioning. In [69], they have studied these formulations in detail focusing on the error bounds that can be reached in the case of linear approximations.

5.1.7. Exponential scheme for the Schrödinger equation

Keywords: *Schrödinger equation*, *exponential scheme*, *quantum mechanics*.

M. Deaconu and A. Lejay, in collaboration with C. Mora (Universidad de Concepción, Chile), are studying the numerical solution of the stochastic Schrödinger equation in some particular situations. In order to handle this they are looking for exponential schemes.

5.1.8. Well posedness for ODE's with singular fields

New quantitative methods have recently been developed to obtain existence and uniqueness of solutions to ODE's with singular force fields. They were used by Crippa and DeLellis for $W^{1,p}$ fields with $p > 1$ but this has been improved to SBV by P-E Jabin [62].

With additional structure, it is possible to require less than a full derivative. In the simplest Hamiltonian case (particle in a potential), a work in preparation by N. Champagnat and P-E Jabin decreases this requirement to essentially $H^{3/4}$.

5.1.9. Mean field limits

The limit of a large system of interacting particles is naturally described by a kinetic equation. This is well understood in the case of regular interactions (Lipschitz) since the works of Dobrushin, Braun and Hepp, Neunzert, Spohn. However, the case of singular interactions (electrostatic for example) is still a crucial open problem.

In such situations, it turns out that it is possible to control the flow and its limit for "almost all" initial configuration (work in preparation by J. Barré, M. Hauray, P-E Jabin). However, in very large dimensions, determining the correct notion and interpretation of "almost all" in the previous statement is tricky, still leaving some open questions.

5.1.10. Blow-up estimates for Keller-Segel models

The Keller-Segel system (also known as Patlak-Keller-Segel) is a classical model to describe chemotaxis phenomena, *i.e.* the movement of cells or bacteria under the influence of a chemical signal.

When the chemical agent is itself produced by the cells, then blow-up of the solutions can occur in finite time. P-E Jabin and M. Rigot [63] propose a direct method using only the energy estimate to obtain the correct rate of blow-up of the solution.

5.1.11. Derivation of a model of particles fragmentation in a fluid

Modelling and analyzing the interaction between particles and fluids is particularly crucial to describe fragmentation processes. P.-E. Jabin, in collaboration with J. Soler [64], followed an approach which simplifies the model by assuming that the particles are constituted by spheres jointed by springs. Then the aim is to deduce the terms appearing in the Navier–Stokes-type equations for the fluid and the counterpart influence in the Boltznmann system for the particles. The resulting coupled system is analyzed by means of a refined averaging lemma.

5.1.12. Mathematical Models of Immune Competition Related to Cancer Dynamics

There are many approaches to the competition between the immune system and tumor cells at an early stage. The one followed by P.-E. Jabin in collaboration with I. Brazzoli and E. De Angelis in [53], considers simple description for immune and endothelial cells but takes into consideration the level of aggression of the tumor cells. Looking at macroscopic quantities, the asymptotic behavior is very classical with convergence back to a healthy state or a blow-up of the tumor cells. However in the first case, what really happens is that there are fewer and fewer tumor cells but they are more and more aggressive (and after correct rescaling one finds a sort of travelling wave structure).

5.1.13. Approximation of equilibrium distributions of some stochastic systems with McKean-Vlasov interactions

Keywords: *Equilibrium measure*, *Euler scheme*, *McKean Vlasov equation*, *p-Wasserstein metrics*, *particle systems*.

In her Ph.D. thesis supervised by M. Bossy and D. Talay and defended in October [14], A. Ganz studied the numerical approximation of the equilibrium measure associated to a nonlinear McKean stochastic differential equation. We considered the situation of nonlinear drift coefficient which can be decomposed into a linear part having ergodic properties, and a nonlinear Lipschitz perturbation part (in the sense of McKean).

In previous works, we have established the existence and the uniqueness of the equilibrium measure associated to our stochastic system, as well as the exponential convergence rate to the equilibrium measure. Moreover, using the propagation of chaos property of the particle system, and the Euler discretization scheme to approximate the SDE, we analyzed the approximation of the integral of any test function w.r.t. the equilibrium measure.

This year, considering the one-dimensional case, we studied the numerical approximations of the density and the cumulative distribution function of the equilibrium measure. We used the algorithm proposed by Bossy and Talay [98] and we have obtained the optimal rate of convergence of this approximation in $L^1(\mathbb{R})$ and $L^\infty(\mathbb{R})$ norms. This result generalizes the techniques developed in by M. Bossy [95] to the ergodic case: here all the rate of convergence estimations must be uniform w.r.t. to the integration time of the simulated particle system.

5.1.14. Evolutionary branching in the evolutionary dynamics of stochastic individual-based population models

We consider a stochastic birth-death-mutation-competition model of evolution, where the population is described at the level of individuals and where each individual is characterized by a phenotypic trait. The simulation of such models often exhibits the phenomenon of evolutionary branching, where the distribution of the phenotypic traits in the population, initially concentrated around a single value, is driven by selection to states where the population is sub-divided into two (or more) clusters with different phenotypes, that stably coexist and are still in competition. The biological theory of adaptive dynamics [108], studies various interpretations and implications of this phenomenon. It also proposes a heuristic criterion for evolutionary branching in the asymptotics of large population, rare mutations and small mutational effects.

In a work in collaboration with S. Méléard [56], N. Champagnat has obtained a full mathematical justification of this criterion. They applied the three previous asymptotics on the microscopic model in a specific order: first, they obtained a simplified model, called the polymorphic evolution sequence (PES), by correctly combining the limits of large population and rare mutations; second, they proved that, in the limit of small mutations, the PES exhibits the evolutionary branching pattern under explicit conditions on the parameters of the model. This work generalizes [100] and makes use of precise asymptotic results on dynamical systems, particularly on 3-dimensional competitive Lotka-Volterra systems [116].

Another direction to tackle this problem is followed by N. Champagnat in collaboration with A. Bovier (Univ. Bonn). They examine the simultaneous combination of the three limits. On the one hand, this requires a much more precise analysis of branching processes and convergence of jump processes to deterministic dynamical systems than in [100]. On the other hand, it avoids some technical difficulties related to high-dimensional Lotka-Volterra dynamical systems, and it allows them to precisely describe the timescales of adaptive dynamics. Three time scales have been identified: the timescale of directional evolution; the timescale of evolutionary branching and a longer timescale for a new phenomenon, which can be described as a discontinuous evolutionary branching. This is still work in progress.

5.1.15. The canonical diffusion of adaptive dynamics

N. Champagnat and Amaury Lambert (Laboratoire d'écologie, ENS Paris) have recently obtained [101] the counterpart of the PES described in the previous section when the population size is finite. A diffusion process has been obtained when the size of the jumps in the PES vanishes, together with analytical expressions of its coefficients as series. From a biological viewpoint, this diffusion process quantifies the interplay between genetic drift and directional selection.

We are now working on the numerical computation of the coefficients of the diffusion. They involve the first-order derivatives of the probability of invasion of a mutant with respect to the mutant birth, death and competition rates. We were able to decompose these derivatives into five fundamental components that allow one to determine which mutant type is more likely to invade the population (a problem related to notion of robustness of the population). The numerical study of the coefficients of the diffusion was done in [42] and allowed us to obtain the convergence of the canonical diffusion to a model known as the "canonical equation of adaptive dynamics". The next step is to apply our numerical results to bistable ecological models, for example with two habitats.

5.1.16. Hamilton Jacobi equations with constraints in population dynamics

P.-E. Jabin and N. Champagnat are studying scaling limits of PDE models of evolving populations in order to describe the phenomenon of evolutionary branching, already mentioned above, from a deterministic viewpoint. Such a study requires a correct scaling of the size of mutations and of time. This scaling has been numerically studied by P.-E. Jabin and other collaborators [103] for a model with two resources. However, the mathematical analysis of this scaling in very difficult, even in the case of a single resource. This problem is related to Hamilton-Jacobi PDEs with a constraint on the maximum, for which existence and uniqueness questions are still unsolved in general [87]. We are considering a simplified model with a specific structure of the interaction coefficients allowing us to obtain existence and regularity results of the scaling limit, either in the case of a single resource or in the case of two resources and a convex interaction. An important point is that the solution of the Hamilton-Jacobi PDE is only of bounded variation with respect to time. Uniqueness questions are still under study. They are related to a precise characterization of the dynamics at time points where there are jumps in the time derivative of the solution of the Hamilton-Jacobi PDE or where evolutionary branching occurs.

5.1.17. Poisson-Boltzmann equation in molecular dynamics

M. Bossy, N. Champagnat, P.-E. Jabin, S. Maire and D. Talay are working in collaboration with T. Malliavin (Institut Pasteur, Paris) on a probabilistic interpretation of the Poisson-Boltzmann equation of molecular dynamics that allows one to compute the electrostatic potential around a bio-molecular assembly (for example a protein). This is an elliptic, 3-dimensional, divergence-form PDE with singular source term and discontinuous, piecewise-constant coefficients. We are developing stochastic numerical methods to solve this equation. This problem involves three main themes.

1. A probabilistic part, where we need to characterize the stochastic processes corresponding to divergence-form operators with a discontinuous, piecewise-constant coefficient. We have generalized the existing results on similar problems, all restricted to the one-dimensional case, to a multidimensional setting. This stochastic process is solution to a SDE with a term involving the local time of a one-dimensional auxiliary process. We have extended Itô's and Feynman-Kac's formulas to such processes.

2. A discretization part, where one must propose, implement and compare different methods of discretizations of the previous stochastic process. These methods should generalize the existing methods in dimension 1. They should also involve an efficient simulation algorithm of Brownian paths, such as walk on spheres method. The subject of X. Aubert's internship was to test and extend existing methods proposed by Mascagni [107]. An article is being written on these first two points.
3. An algorithmic geometry part, where one wants to take advantage of the precise geometry of the problem: the discontinuity of the parameters of the PDE occurs at the boundary of the molecule, represented as a union of spheres; in this situation, it is particularly crucial to determine as efficiently as possible the closest sphere from a given point. This is related to the construction of Voronoi diagrams of spheres in dimension 3. This question is not fully understood from a theoretical point of view and no robust implementation is known yet. We have started a collaboration with the Project-Team GEOMETRICA of INRIA Sophia Antipolis – Méditerranée on this specific problem.

5.1.18. A probabilistic approach for high dimensional least squares approximations

In collaboration with E. Faou (IRISA) and C. Chipot (eDAM, Univ. Nancy 1), N. Champagnat has worked on another problem related to molecular dynamics. We considered least-squares problems set on high-dimensional spaces, in situations where none of the classical methods can be implemented (high dimension and bad conditioning). We propose an approximation of the solution, obtained by a Monte Carlo approximation of the average of the solution of random small least-squares problems drawn as sub-systems of the original system. We give conditions to ensure the convergence and the consistency of the method, and analyze the cost of the algorithm in some specific situations. Our algorithms generalize and improve the statistical analysis of distributed multipoles (SADM) algorithm, introduced by Chipot *et al* [102], used to compute the atomic multipoles from the quantum mechanical electrostatic potential mapped on a grid of points surrounding a given bio-molecule.

5.1.19. Statistical estimators of diffusion processes

Keywords: *Skew Brownian motion*, *martingale limit theorems*, *maximum likelihood stochastic processes*, *statistical estimation*.

A. Lejay in collaboration with S. Torres (Universidad de Valparaíso, Chile) and E. Mordecki (Universidad de la República, Uruguay), has studied the asymptotic behavior of the likelihood ratio for the Skew Brownian motion. This is a non-standard case, since the process is not ergodic. They have seen that it is possible to characterize the limiting distribution of the maximum likelihood and then to test if a diffusion is a Brownian motion or a Skew Brownian motion.

A. Lejay, in collaboration with R. Rebolledo and M. Arenas (Pontificia Universidad Catolica, Chile), has studied a parametric estimator for the coefficients of a one dimensional diffusion whose passage positions at successive levels are known. For this, they have used the scale function and some theory of estimation that relies on limit theorems for martingales.

5.1.20. Lower bounds for densities of hypo-elliptic diffusions

Motivated by applications to risk analysis in Finance and reliability problems in Random Mechanics, P.-L. Lions and D. Talay are getting precise estimates on lower bounds of certain hypo-elliptic diffusion processes. Lower bounds of Gaussian type are well known for strongly elliptic diffusions. The techniques are very different in the hypo-elliptic case.

5.2. Interacting particle systems in Lagrangian modeling of turbulent flows

Keywords: *confinement of Langevin system*, *discrete mass-transport problem*, *interacting particle system*, *pdf methods*.

Participants: Mireille Bossy, Claire Chauvin, Jean-François Jabir, Pierre-Louis Lions.

This section presents the results obtained by TOSCA on the modeling of turbulent flows. Actually, it could be a part of the previous section on stochastic modeling and applications. We have chosen to present this work in a distinct coherent section.

In the statistical approach of turbulent flow, the Eulerian properties of the fluid (such as velocity, pressure and other fundamental quantities) are supposed to be random fields. The evolution of the corresponding averaged quantities are governed by the unclosed Reynolds Navier Stokes equation. Once the unclosed terms are modeled, the resulting equations can be simulated by some PDEs resolution methods.

An alternative approach proposed by Pope [111] consists in describing the flow through a stochastic particle model of the position, velocity and other relevant property of a generic fluid particle. In the computational fluid dynamics literature, those models are referred to as Lagrangian stochastic models. For example, the Simplified Langevin Model [112], which characterizes particle positions-velocity $(X_t, \mathcal{U}_t)_{t\geq 0}$ in the case of homogeneous turbulent flow, is defined as

$$\begin{cases} dX_t = & \mathcal{U}_t dt, \\ d\mathcal{U}_t = & -\nabla_x \langle \mathcal{P} \rangle (t, X_t)dt - \left(\frac{1}{2} + \frac{3}{4}C_0\right) \frac{\varepsilon(t, X_t)}{k(t, X_t)} \left(\mathcal{U}_t - \mathbb{E}\left(\mathcal{U}_t \mid X_t\right)\right) dt \\ & + \sqrt{C_0 \varepsilon(t, X_t)} dW_t, \end{cases} \tag{2}$$

where C_0, $\varepsilon(t,x)$ and $k(t,x)$ are scalar quantities supposed to be known, and W is a Brownian motion. Furthermore, the averaged pressure $\langle \mathcal{P} \rangle (t,x)$ must solve a Poisson equation in order to ensure constant mass density (i.e. particle positions are uniformly distributed) and divergence free conditions. In addition, assuming boundary conditions at $\partial\mathcal{D}$, the model is submitted to a limit condition of the form: $\mathbb{E}\left(\mathcal{U}_t \mid X_t = x\right) = V(t,x)$, for $x \in \partial\mathcal{D}$.

An application of the stochastic Lagrangian approach to the downscaling problem in meteorology is presented in Section 6.1.

5.2.1. *Mathematical analysis of the Lagrangian stochastic models and their confinement*

Keywords: *McKean–Vlasov equation of conditional type*, *McKean–Vlasov–Fokker–Planck equations*, *specular boundary conditions*.

In his Ph.D. thesis [15] supervised by M. Bossy and D. Talay and defended in October 2008, J.-F. Jabir studied theoretical problems involved by Lagrangian stochastic models in simplified cases where, in particular, the pressure term is neglected. These results provide the first theoretical study on the Lagrangian stochastic models.

• We considered a simplified Lagrangian system:

$$\begin{cases} X_t &= X_0 + \int_0^t \mathcal{U}_s \, ds \\ \mathcal{U}_t &= \mathcal{U}_0 + \int_0^t B(s, X_s, \mathcal{U}_s) ds + \int_0^t \sigma(s, \mathcal{U}_s) \, ds, \end{cases} \tag{3}$$

where $B(s,x,u) = \mathbb{E}[b(\mathcal{U}_s, u) \mid X_s = x]$ and b is a bounded interaction kernel. Due to the conditional expectation and the partially degenerate diffusion, (3) provides a special case of nonlinear equations in the sense of McKean. When the kernel b is continuous and σ is elliptic, bounded and Hölder continuous, we proved the well-posedness of the Lagrangian system (3), and a propagation of chaos result for a related particle system [45].

• Motivated by the downscaling application of the Lagrangian stochastic models (see Section 6.1), we also constructed a Lagrangian system confined within a regular domain $\mathcal{D}$ of $\mathbb{R}^d$, and satisfying the mean no-permeability boundary condition:

$$\mathbb{E}\left((\mathcal{U}_t \cdot n_{\mathcal{D}}(X_t)) \mid X_t = x\right) = 0 \text{ for } x \in \partial\mathcal{D}, \tag{4}$$

where $n_{\mathcal{D}}$ is the outward normal unit vector related to $\mathcal{D}$. The basic idea of our construction is to add a confinement term, $-2\,(\mathcal{U}_{t^-} \cdot n_{\mathcal{D}}(X_t))\,n_{\mathcal{D}}(X_t)$, to the velocity dynamic of (3) at each time X hits the boundary. Under suitable hypotheses, this term ensures that the distribution $\rho_t(x,u)$ of $(X_t, \mathcal{U}_t)$ admits a trace $\gamma(\rho)(t,x,u)$ for $x \in \partial\mathcal{D}$ satisfying the specular boundary condition

$$\gamma(\rho)(t,x,u) = \gamma(\rho)(t,x,u - 2(u \cdot n_{\mathcal{D}}(x))n_{\mathcal{D}}(x)) \text{ for } (t,x,u) \in (0,T) \times \partial\mathcal{D} \times \mathbb{R}^d. \tag{5}$$

>From (5), we show that (4) holds. In the case $\sigma = 1$ and $\mathcal{D} = \mathbb{R}^{d-1} \times (0, +\infty)$, we established the well-posed for the "confined" Lagrangian system, as well as the convergence of a particle approximation. Pursuing these works, M. Bossy and J.-F. Jabir are studying the construction of Lagrangian dynamics submitted to non homogeneous Dirichlet boundary conditions.
• We continue our collaboration with P.-L. Lions for the construction of confined Lagrangian systems in general bounded domains $\mathcal{D}$. Introducing the notions of Maxwellian super-solutions and sub-solutions, we established the well-posedness of a weak solution to the McKean–Vlasov–Fokker–Planck equation associated to a confined system satisfying (4). We are now studying the hitting time of $\partial\mathcal{D}$ by the Brownian primitive, from which we expect to obtain a general construction of the confined systems satisfying (4).

5.2.2. Stochastic Lagrangian simulation for turbulent flows

We continue our collaboration with A. Rousseau (MOISE INRIA Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand) on this topic (see section 6.1). The post-doctoral position of Claire Chauvin, in collaboration with the Project-Team MOISE (INRIA-Grenoble), was devoted to the improvement of the Stochastic Lagrangian simulation methodology and the continuation of the software development of Stochastic Downscaling Method (SDM), initiated in 2006 in TOSCA. SDM refines the characteristics of the wind (velocity and variability) inside a mesh given by a coarse Numerical Weather Prediction (NWP) Model. The wind velocity is simulated by fluid particles, living inside a domain $\mathcal{D}$, whose evolution is governed by some Stochastic Differential Equations. Such a model raises several original problems, requiring a strong interaction between mathematics, scientific computation and physics. In 2008, the improvements made in SDM are the following:

- **The confinement of the particles, and the fractional step algorithm**: On the one hand, a joint work of M. Bossy and J.-F. Jabir [15], about the theoretical modelling of the confinement, has brought answers on the way to characterize the confinement of the particles. On the other hand, the internship of O. Aboura, in 2006, directed by M. Bossy and D. Talay, has given some keys to characterize the statistics of the flow, in case of a linear Lagrangian model for the velocity. These two advancements, together with the understanding of the numerical behavior of SDM, have led to the conception and the development of an adapted discrete reflexion scheme [38].
- **The Splitting Scheme**: The simulation of Equation (2) would require the computation of the pressure gradient, which is very costly in general and almost prohibitive in the SDM context. Instead, we solve (2) without the pressure term, and then correct the particle position and velocity by simulating the effects of the pressure on the flow :
    1. **The constant mass density constraint**: This constraint was already identified to be solved by an optimal transportation problem (see [113]), for which the discrete version is expressed as a combinatorial optimization problem. C. Chauvin customized the so-called Auction Algorithm [90] in the context of particle methods. The papers [22], [38] present several algorithmic implementations and benchmark their performance in the SDM situation.

2. **The incompressibility condition**: It is usually solved by a Poisson equation in CFD. This condition asks for a specific treatment in the case of a downscaling method. The aspects of the incompressiblity condition are now identified in our SDM software [38], and the next step will be to study and implement a numerical solution for this problem.

5.3. Financial Mathematics

Participants: Blandine Bérard-Bergery, Mireille Bossy, Mamadou Cissé, Nicolas Champagnat, Samuel Herrmann, Junbo Huang, Nicolas Grima, Stefania Maroso, Christophe Profeta, Bernard Roynette, Denis Talay, Xiaolu Tan, Etienne Tanré, Pierre Vallois.

5.3.1. Exact simulation of SDE

Beskos et al. [92], [91] proposed an exact simulation algorithm for one-dimensional SDEs with constant diffusion coefficient and restrictive assumptions on the drift coefficient. In [75], E. Tanré and V. Reutenauer (Calyon) have adapted the results and extended the algorithm to more general SDEs. We apply it to SDEs with non Lipschitz coefficients (for instance, the CIR model for interest rates). Furthermore, we apply Malliavin calculus to simulate exactly the Greeks, that is the derivatives of the prices with respect to initial condition.

5.3.2. Impulse control with delay

This work is part of the contract with NATIXIS of Section 6.8.

It is known that the replication of a barrier option needs a lot of transactions when the asset is close to the barrier before maturity, because the “Gamma” of the option explodes. In practice, the portfolio’s re-allocations can only be done at discrete times (typically once a day at fixed time), and this may imply a big loss. The practitioners usually cover this risk by selling the option at a higher price than the theoretical price.

A first approach to solve this problem, followed by S. Maroso, E. Tanré and N. Champagnat, consists in studying the problem of super-replication of the option under Gamma constraints, generalizing to the case of barrier options previous works by Soner and Touzi [114].

N. Champagnat, S. Maroso, D. Talay, X. Tan And E. Tanré have explored another direction to solve this problem. Instead of using a strategy imposing a single portfolio reallocation everyday at fixed time, we propose a more flexible strategy, with a constraint of minimum delay (for example one day) between two transactions. This problem translates into a stochastic impulse control problem with a delay constraint. We have implemented a numerical finite difference method to solve the corresponding Hamilton-Jacobi-Bellman PDE in order to compare the gain of each strategy. The theoretical validation of this method is under progress. We are also considering two other approaches of the problem: reduction of the dimension and a hybrid method for the numerical resolution (probabilistic and finite differences).

5.3.3. Local and Global risk minimization

During his internship under the supervision of D. Talay, E. Tanré and N. Champagnat, X. Tan has studied the links between local and global minimization of risk in discrete and continuous models, within the contract with NATIXIS of Section 6.8. In particular, they obtained convergence results of the discrete strategies to the continuous strategies when the time-step converges to 0, in the two cases of global and local quadratic risk minimization.

5.3.4. Numerical approximation for super-replication problem

S. Maroso, in collaboration with O. Bokanowski (Université Paris 7), B. Bruder (Société Générale) and H. Zidani (ENSTA), has studied in [54] a super-replication problem of European options with gamma constraints. The initially unbounded control problem is set back to a problem involving a viscosity PDE solution with a set of bounded controls. Then a numerical approach is introduced, unconditionally stable with respect to the mesh steps. A generalized finite difference scheme is used since basic finite differences cannot work in our case. Numerical tests illustrate the validity of our approach.

5.3.5. Convergence results for Howard algorithm

S. Maroso, in collaboration with O. Bokanowski (Université Paris 7) and H. Zidani (ENSTA), has proved in [51] some convergence results of Howard's algorithm for the resolution of the problem

$$\min_{a \in A}(B^a x - b^a) = 0$$

where B^a is a $N \times N$ matrix, b^a is a vector in $\mathbb{R}^N$, and A is a compact set. We show a global super-linear convergence result, under a monotonicity assumption on the matrices B^a . In the particular case of an obstacle problem of the form $\min(Ax - b, x - g) = 0$ where A is an $N \times N$ matrix satisfying a monotonicity assumption, we show the convergence of Howard's algorithm in no more than N iterations, instead of the usual 2^N bound. Still in the case of obstacle problem, we establish the equivalence between Howard's algorithm and a primal-dual active set algorithm. We also propose an Howard-type algorithm for a "double-obstacle" problem of the form $\max(\min(Ax - b, x - g), x - h) = 0$. We finally illustrate the algorithms in the discretization of nonlinear PDEs arising in the context of mathematical finance (American option, and Merton's portfolio problem), and for the double-obstacle problem.

We are now trying to obtain similar results for the infinite dimensional case.

5.3.6. Rate of convergence in the Robbins-Monro algorithm

Under the supervision of D. Talay, J. Huang continued the study on the convergence of the Robbins-Monro (RM) algorithm.

By using the smoothing inequality for characteristic functions, we provided a Berry-Esseen type rate of convergence in the central limit theorem for martingales. This result can be applied to investigate the convergence rate of the RM algorithm. We thus established a Berry-Esseen bound for the RM algorithm as simple as follows

$$\theta_n = \theta_{n-1} + \gamma_n h(\theta_{n-1}) + \gamma_n \eta_n,$$

where h is assumed to be a smooth function with bounded derivatives and η_n is a martingale difference. In order to solve problems appearing in mathematical finance, this result needs to be extended to cover more general h functions. This is the object of current works.

5.3.7. Memory-based persistent counting random walk and applications

In [36], [86], P. Vallois and C. Tapiero (ESSEC) have considered a memory-based persistent counting random walk, based on a Markov memory of the last event. The usefulness for some problems in insurance, finance and risk analysis are discussed. In [61], P. Vallois and S. Herrmann study the link between the persistent random walk and the telegraph equation.

5.3.8. Around the Black-Scholes formula

Keywords: *Cameron-Martin relationship*, *last passage times*, *reflection principle*, *time-inversion*.

Jointly with D. Madan (University of Maryland) and M. Yor (Université Paris 6), B. Roynette proved that the celebrated Black-Scholes formula can be expressed as a cumulative function of the last passage time of Brownian motion [37], [68].

5.3.9. Modelling of financial techniques: rupture in volatility

We carry on studying the performances of several technical analysis techniques. We focus on strategies which are the most popular among the technical traders community. First, we dealt with the moving average indicator [94], [93]. This indicator is adapted to detect rupture on the drift term.

This year, we have studied the Average True Range (ATR). The ATR is a technical analysis rule based on a mobile average of the True Range Indicator (that is the difference between the maximum and the minimum of an asset price during one day). It is considered by analysts as a mean to detect ruptures of volatility: an increase of the value of the ATR corresponds to an increase of the volatility. As any technical analysis rule, the ATR is not model-dependent and does not need calibration. On the other hand, in a given model with good calibration, the ATR is sub-optimal compared with a mathematically designed tool.

We have studied the sensitivity of Brownian range with respect to the volatility and observed that the sensitivity of the range is higher than the squared increments one: up to 10 values of increments per days would be necessary to bring as much information as a single value of the range. This indicates that adding the range in a detection tool can bring significant improvement of the results.

Then, as using the range only shows good results, we continued by assuming that a model based tool, using the same information, could improve the detection. Using Monte-Carlo approximations of laws of distribution of geometric Brownian ranges, we created a filter based on ranges maximum likelihood, and state transition probability. Filtering is model-dependent, but state detections are reduced to 2 to 4 days. The detection tools are tested by simulating their mean utility, for strategies in $\{0, 1\}$ and a logarithmic utility function. Compared with a fixed strategy (the trader invest all his/her wealth in the stock), both detection tools increase the utility, but the increase with the filter is 15 % bigger than with ATR detection.

These results show that the range of a process can bring significant improvement in discrete time problems concerning volatility. It also shows that empirical indicators can be based on strong mathematical properties. This enables one to replace the model-independent empirical indicator by a model-dependent, more efficient detection tools, such as filtering.

The people involved in this study are N. Grima and E. Tanré.

5.3.10. Modelling of financial techniques: resistance and support levels

We have also considered a classical pattern of charts studied via technical analysis. In a phase of consolidation, a price does not have either ascending trend, nor decreasing trend. The price moves between two barrier levels: the upper one is called resistance, the lower one is called support. When the price bounces three times on the support, price will likely not go down the bottom barrier. We can expect a rise and that the price will go up through the upper barrier. This belief will have an influence on trader's behavior, and therefore on price. However, this kind of rule has no mathematical justification.

B. Bérard-Bergery, C. Profeta and E. Tanré have proposed a mathematical model derived from the Black and Scholes model which presents this kind of behavior. The trajectory makes a random number of downcrossings between two fixed levels before reaching an upper level and leaving. Then, we evaluate the optimum portfolio strategy in the case of a logarithmic utility function. The efficiency of the strategy is compared with the classical one.

5.3.11. Optimal stopping problems

Keywords: *Optimal stopping*, *h-transform*.

In collaboration with P. Patie (University of Bern, Switzerland), M. Cissé and E. Tanré solve explicitly the optimal stopping problem with random discounting and an additive functional as cost of observations for a regular linear diffusion. This generalizes a result by Beibel and Lerche [88], [89]. The approach relies on a combination of Doob's h-transform, time-changes and martingales techniques. Our results combined with Patie's results [109] allows us to treat a few examples, one of them being the evaluation of a perpetual American type option with payoff defined by

$$P_t = e^{-\int_0^t X_s ds} (X_t - K)^+ \tag{6}$$

where the process X is the instantaneous interest rate modeled by the Vasicek or Ornstein-Uhlenbeck process.

5.3.12. Artificial boundary conditions for nonlinear PDEs in finance

Under M. Bossy and D. Talay's supervision, M. Cissé studied the problem of artificial boundary conditions for nonlinear PDEs. The motivation of this research concerns American option pricing and the numerical resolution of the variational inequality characterizing prices of American options.

First, we have extended the theorem on existence and uniqueness of solutions of the reflected backward stochastic differential equations (RBSDE) with fixed terminal time to the case of bounded random terminal time. We obtained a probabilistic interpretation of the previous solution as a viscosity solution of variational inequalities with Dirichlet boundary conditions. We obtained a general expression of the localization error in terms of the boundary conditions.

Secondly, we were interested in variational inequalities with Neumann boundary conditions. They can be interpreted as a generalized RBSDEs coupled with a reflected forward stochastic differential equations (RSDE). We used the derivative in the sense of distributions with respect to the initial data of the RSDE and the representation of the gradient of Ma and Zhang [106] to establish a representation theorem for the space derivative of the viscosity solution of the variational inequalities. We apply this result to get a priori estimates on the error induced in numerical simulations by artificial boundary conditions for the PDEs related to American options pricing. The differentiability of the reflected forward SDE in the sense of distributions with respect to the initial data is deduced from results by Bouleau and Hirsch [99] on the absolute continuity of probability measures.

5.3.13. Liquidity risk

P. Protter (Cornell University) and D. Talay are addressing the following question. We have the possibility of trading in a risky asset (which we refer to as a stock) with both liquidity and transaction costs. We further assume that the stock price follows a diffusion, and that the stock is highly liquid. We limit our trading strategies to those which change our holdings only by jumps (i.e., discrete trading strategies), and we begin with \$0 and 0 shares, and we end with a liquidated portfolio (that is, we no longer hold any shares of the stock), on or before a predetermined ending time T. The question then is, given the structure of the liquidity and transaction costs, what is the optimal trading strategy which will maximize our gains? This amounts to maximize the value of our risk free savings account. This problem can be solved in this context if it is formulated as a non classical problem in stochastic optimal control. We prove existence and uniqueness results for the related Hamilton–Jacobi–Bellman equation.

5.4. Statistical analysis

Participants: Olivier Collignon, Aurélien Deya, Maha Ghribi, Samy Tindel, Pierre Vallois.

The works presented in this Section are developed within the workshop between the probability and statistics teams of Institut Élie Cartan and the group led by B. Bihain, the head of the Laboratoire de Médecine et Thérapie Moléculaire (INSERM, Nancy).

5.4.1. Algorithm for wavelet denoising. Application to the analysis of brain signals for epileptic patients

S. Tindel in collaboration with C. Lacaux (IECN), A. Muller (IECN) and R. Ranta (CRAN) studied the probabilistic analysis of the *peeling* algorithm for wavelet denoising with application to the analysis of brain signals for epileptic patients. The application part of the project is a further collaboration with a team at the *Centre Alexis Vautrin* (Nancy) leaded by F. Guillemin.

5.4.2. Cancer detection via fluorescence techniques

M. Ghribi and S. Tindel, in collaboration with W. Blondel (CRAN) and E. Pery (CRAN) considered the following problem. For hollow organs such as bladder or skin, we explore the possibility of cancer detection via fluorescence techniques, which have the advantage of being non-invasive. From a mathematical point of view, this amounts to a supervised classification problem for light spectrums. The difficulty is then that we deal with a small sample (120 bladder sites) of high dimensional data (each spectrum is composed of around 1000 points). In this context, we try to use efficiently a mixture of discrimination techniques based on regularized discriminant analysis, logistic regression and support vector machines, in order to (i) perform a variable selection procedure and (ii) separate the cancer from the normal spectrums accurately.

5.5. Stochastic analysis and applications

Participants: Blandine Bérard-Bergery, Bernard Roynette, Pierre Vallois.

In this section we present our results on issues which are more abstract than the preceding ones and, at first glance, might appear decorrelated from our applied studies. However most of them are originally motivated by modelling problems, or technical difficulties to overcome in order to analyze in full generality stochastic numerical methods or properties of stochastic models.

5.5.1. Penalization of diffusion processes

Keywords: *Bessel processes*, *Ray-Knight's theorems*, *Sturm-Liouville equation*, *Wiener measure*, *down-crossings*, *enlargement of filtration*, *limiting laws*, *local time*, *maximum*, *minimum*, *normalized exponential weights*, *penalization*, *rate of convergence*.

Jointly with M. Yor (Université Paris 6), B. Roynette and P. Vallois have continued their study of the penalization of diffusion processes [82], [81], [35], [80], [78], [71], [79], [84], [25], [85]. The penalization procedure allows one to modify a given process (for example a Brownian motion or a Bessel processes) in order to get a new path property (for instance boundedness or a different behavior in a neighborhood of 0). This procedure can be very interesting in modelling to go further than the approach based on diffusions.

5.5.2. Generalized Gamma convoluted variables

Keywords: *Dirichlet processes*, *Stieltjes transform*, *Wiener-Gamma representation*.

The articles [77], [29] present some study of the GGC variables and their Wiener-Gamma representation. Some examples concerning the excursions of Bessel processes are pointed out.

5.5.3. Approximation schemes for the local time

In [48], B. Bérard-Bergery and P. Vallois have proposed new approximation schemes related to the local time process of the standard Brownian motion. Some rates of convergence have been exhibited explicitly.

6. Contracts and Grants with Industry

6.1. Collaboration with ADEME: local modeling for the wind velocity

Participants: Mireille Bossy, Claire Chauvin.

Started in 2005, our joint collaboration with the Laboratoire de Météorologie Dynamique (Université Paris 6, École Polytechnique, École Normale Supérieure) is funded by the French Environment and Energy Management Agency (ADEME) and concerns the modeling and the simulation of local wind energy resources. We collaborate with É. Peirano (ADEME), P. Drobinski and T. Salameh (LMD). The second phase of this collaboration has begun in October 2007 and includes as partners A. Rousseau (MOISE Inria Grenoble – Rhône-Alpes) and F. Bernardin (CETE Clermont-Ferrand). We investigate a new method for the numerical simulation of the wind at small scales (see Section 5.2.2). Thanks to boundary data provided at large scales by the weather forecasting code *MM5*, we propose a Langevin model that rules the behavior of stochastic particles. This model called *SDM* (Stochastic Downscaling Method) is adapted from previous works introduced by S.B. Pope [110] and has been presented in [38], [40].

A new version of our *SDM* code has been developed, which answers to the first physical requirements of the model, with a really low computational cost [44]. The phase of meteorological validation, based on simulations of a simple meteorological framework using observations from measurement campaigns, was successful and published in [18]. It underlined the high potential of our Lagrangian Stochastic Model to simulate the wind variability.

6.2. ANR GCPMF: Grille de Calcul pour les Mathématiques Financières

Participants: Mireille Bossy, Denis Talay, Viet-Dung Doan.

We collaborate with the OASIS team within the ANR project entitled "GCPMF" funded by the ANR Research Program "Calcul Intensif et Grilles de Calcul 2005".

The aim of this ANR program is to highlight the potential of parallel techniques applied to mathematical finance computing on Grid infrastructures. The consortium that conducts this project includes ten participants from academic laboratories in computer science and mathematics, banks, and IT companies.

Financial applications require to solve large size computations, that are so huge that they cannot be tackled by conventional PCs. A typical example addresses time-critical computations required during trading hours, in particular Monte Carlo simulations for option pricing and other derivative products. A software system called PicsouGrid has been designed and implemented by the OASIS team, which makes use of the ProActive library to parallelize and distribute various option pricing algorithms [115].

This year, in collaboration with Francoise Baude and Abhijeet Gaikwad from OASIS, we have continued the design and implementation of a Grid software. Besides parallel algorithms for the pricing of European options by Monte Carlo Methods, we have pursued our specific effort to integrate some parallel algorithms of American pricing methods. We concentrated our efforts on the particularly intensive problem of pricing high-dimensional Bermudan–American options. We aim to address this problem in the context of grid computing, relying on the ProActive Java distributed computing platform. We parallelized the Classification–Monte Carlo algorithm, which relies on classification techniques from the machine learning domain, for pricing Bermudan-American options. In [46] and [39], we evaluate the performance of two machine learning techniques, Boosting and Support Vector Machines, and compare the numerical results with respect to accuracy, speed up and their applicability in the grid settings. Furthermore, the paper also contributes to the numerical experiments on a high-dimensional Bermudan-American option with 40 underlying assets.

6.3. ANR MAEV: Modélisation Aléatoire et Évolution du Vivant (stochastic modelling of the evolution of living systems)

Participant: Nicolas Champagnat.

The general goal of this research project is the development and the analysis of new stochastic models of evolution, which take into account the interactions and the diversity of scales in evolution. The partners (probabilists and evolutionary biologists mainly in Paris, Marseille and Grenoble) are exploring four research directions:

- Evolution at the molecular scale: new models of the evolution of genes taking into account the interactions between sites and the main factors of global changes in the genome (genes duplication, transfer,...).
- Adaptive evolution: macroscopic models of adaptive evolution that are deduced from the microscopic, individual scale and from genes to the organism.
- Shape of random trees: random tree models for molecular evolution or for species evolution, and the mathematical tools to compare them in order to analyze the evolutionary relations between populations or species.
- Coalescence: coalescent processes coding for the evolution of a group of genotypes inside a large population, allowing one to study the polymorphism when dependence between individuals and various scales are taken into account.

A strong interest is brought to algorithmic implementations, numerical analysis and applications to data processing.

The works of N. Champagnat on evolutionary branching and on the canonical diffusion of adaptive dynamics are part of this project.

6.4. ANR MODECOL: Using mathematical MODeling to improve ECOLogical services of prairial ecosystems

Participant: Nicolas Champagnat.

This research project has been accepted recently. Our research activity is just starting. It is part of the SYSCOMM ANR program (Complex Systems and Mathematical Modeling).

The general goal of this project is to develop computational modeling of terrestrial plant community ecology via the simulation of a prairie (e.g. interactive terrestrial plant populations) in relation with environmental data. This project is focused on:

- developing an original tool-box that takes advantage of complementary mathematical disciplines to assess ecological problems. Our project proposes the coupling of Partial Differential Equation to approximate environmental conditions and Individual-Based stochastic Models to integrate the population dynamic of prairies. This construction will be companion by application of methodologies of different mathematical fields such as Markovian processes, genetic algorithms and data assimilation. These simulations will be extensively processed thanks to the use of new tools in distributed computing and webcomputing.
- building this development on a strong synergy between partners in ecology and in computational and mathematical modelling. This dialog may ensure a total adaptation of these complex tools to the specificity of ecological problems and to the application of such models for solving concrete problems in ecosystem management.

An application is proposed as part of this project that stands at the boundary between agronomical and ecological sciences. It concerns the recent European Common Agricultural Policy which requires setting up herbal strips around intensive cereal fields for purificating water from extra nitrate and pesticides. This application will be developed directly with the end-users. We aimed at providing software in prairies engineering that could be used as a support for decision making concerning this new policy.

6.5. ARC POPEYE

Participant: Nicolas Champagnat.

The ARC Popeye (POPulations, gamE, theorY and Evolution) is coordinated by E. Altman and A. Jean-Marie (MAESTRO, INRIA Sophia Antipolis – Méditerranée). It gathers members of INRIA, INRA and Universities from Sophia Antipolis, Avignon, Montpellier and Grenoble. The ARC Popeye focuses on the behavior of large complex systems that involve interactions among one or more populations. By population we mean a large set of individuals, that may be modeled as individual agents, but that we will often model as consisting of a continuum of non-atomic agents. The project brings together researchers from different disciplines: computer science and network engineering, applied mathematics, economics and biology. This interdisciplinary collaborative research aims at developing new theoretical tools as well as at their applications to dynamic and spatial aspects of populations that arise in various disciplines, with a particular focus on biology and networking.

6.6. Contract with Calyon

Keywords: *Monte Carlo methods*, *Quantization*, *variance reduction*.

Participants: Aymen Bergaoui, Madalina Deaconu, Antoine Lejay, Denis Talay, Étienne Tanré, Yiqing Wang.

This contract with Calyon started on January 2007 and concerns the efficient simulation of interest rates models through Monte Carlo methods and variance reduction techniques.

We have followed our collaboration with the team *interest rates and hybrid derivatives quantitative research* in the investment bank Calyon and we have continued to develop a variance reduction method that uses functional quantization in order to construct a good control variate.

This method has been the subject of intensive numerical tests and has proved to be both easy to implement, robust and efficient.

6.7. Contract with Natixis: adaptive Monte Carlo methods

Participants: Madalina Deaconu, Antoine Lejay, Numa Lescot, Denis Talay.

The Ph.D. thesis of Numa Lescot started in September 2006 with a CIFRE grant from Natixis. This thesis concerns adaptive Monte Carlo methods where the importance sampling technique is coupled with stochastic algorithms to perform variance reduction. The aim is to apply this technique together with Malliavin calculus in order to perform variance reduction on the computation of the Greeks.

6.8. Contract with Natixis: portfolio optimization in incomplete markets

Participants: Nicolas Champagnat, Stefania Maroso, Denis Talay, Xiaolu Tan, Etienne Tanré.

This work concerns various aspects of portfolio management in incomplete markets. It is developed in four directions: calibration and estimation of historical probabilities, rupture detection, dynamical portfolio optimization and numerical validation for optimal control. The hedging problem under Gamma constraints and the discrete hedging problem with minimal delay between transactions are also studied in the case of barrier options.

7. Other Grants and Activities

7.1. NCCR FINRISK

TOSCA participates to the NCCR FINRISK (Financial Risk) forum launched by the Swiss National Science Foundation and managed by the University of Zürich.

7.2. Natixis Fundation

Participant: Denis Talay.

D. Talay is the Vice-President of the Fondation d'Entreprise Natixis which aims to contribute to develop research in quantitative finance.

He also serves as a member of the Scientific Committee of the Foundation, jointly with M. Crouhy (President, Natixis), N. El Karoui (École Polytechnique), P-L. Lions (Collège de France), J-P. Laurent (Université Claude Bernard, Lyon).

8. Dissemination

8.1. Animation of the scientific community

M. Bossy serves as a member of the Scientific Committee of the *École Doctorale "Sciences Fondamentales et Appliquées"* of the Université de Nice – Sophia Antipolis.

M. Bossy is responsible for the *Suivi Doctoral* Committee of INRIA Sophia Antipolis – Méditerranée. She is also responsible for the *Formation par la recherche* at INRIA Sophia Antipolis – Méditerranée.

M. Bossy serves as a member of the *Cours et Colloques* Committee of INRIA Sophia Antipolis – Méditerranée and serves as a member of the *Commission Consultative Paritaire Scientifique de INRIA* since November.

N. Champagnat is elected member of the *Comité de Centre* and is representant of researchers at the *Comité des Projets* of INRIA Sophia Antipolis – Méditerranée.

M. Deaconu served as member of the Scientific Committee of the MAS group (Probability and Statistics) within SMAI until August 2008.

M. Deaconu is a permanent reviewer for the *Mathematical Reviews*.

M. Deaconu serves as member of the *Conseil du laboratoire* of IECN and of the *Commission des spécialistes* (25-26 Sections in Pure and Applied Mathematics) at the Université Nancy 1.

P.-E. Jabin is member of the hiring committee at the Université de Nice – Sophia Antipolis. He is responsible for the third year of *Licence* in mathematics at the Université de Nice – Sophia Antipolis. He is also responsible for the colloquium at the maths department of the Université de Nice – Sophia Antipolis.

A. Lejay serves as a member of the *Commission de Spécialistes* of Université Louis Pasteur in Strasbourg.

A. Lejay serves as a member of the *Commission des moyens informatiques* of INRIA Nancy – Grand Est.

S. Maire is member of the *Commission de Spécialistes* (25-26 Sections in Pure and Applied Mathematics) at the Université du Sud Toulon-Var.

D. Talay serves as an Associate Editor of: *Stochastic Processes and their Applications*, *Annals of Applied Probability*, *ESAIM Probability and Statistics*, *Stochastics and Dynamics*, *SIAM Journal on Numerical Analysis*, *Mathematics of Computation*, *Journal of Scientific Computing*, *Monte Carlo Methods and Applications*, *Oxford IMA Journal of Numerical Analysis*, *Stochastic Environmental Research and Risk Assessment*.

D. Talay is the President of the French Applied Mathematics Society SMAI.

D. Talay serves as a member of the Committee for junior permanent research positions at Université Bordeaux 1.

D. Talay has been a member of the international evaluation committee for mathematics departments and laboratories in Portugal.

S. Tindel serves as a member of the *Commission de Spécialistes* (25-26 Sections in Pure and Applied Mathematics) at the Université Nancy 1.

P. Vallois is the head of the Probability and Statistics group of Institut Élie Cartan.

P. Vallois serves as a member of the council of the UFR STMIA, the *Conseil du laboratoire* and the *Commission de Spécialistes* of the Mathematics Department of Université Nancy 1.

P. Vallois serves as an Associate Editor of *Risk and Decision Analysis*.

P. Vallois serves as expert for the *Ministère de l'Enseignement Supérieur*.

8.2. Animation of workshops

M. Bossy served as a member of the Scientific Committee of the congress *CANUM 2008*.

M. Bossy served as a member of the Scientific Committee and Jury member of the *V Grid Plugtests* on *Super Quant Monte-Carlo Challenge* October 20-24, 2008 Sophia Antipolis.

N. Champagnat served as a member of the organizing comity of the Summer School of the ANR MAEV *Evolutionary Biology and Probabilistic Models*, held in La Londe les Maures in September.

M. Deaconu in collaboration with T. Lelièvre (Cermics, ENPC) organized the minisymposia *Modèles hybrides* at the congress *CANUM 2008* at Saint Jean de Monts.

S. Ferrigno, A. Muller, J.M. Monnez, S. Tindel and P. Vallois will organize the *Journée apprentissage* in Nancy in December 2008.

S. Herrmann organized a workshop on *Mathematical Finance* at the Institut Élie Cartan.

P.-E. Jabin co-organized a conference on *Bioinformatics Modeling in Biology and Medicine* in Nice in October.

D. Talay served as a member of the scientific committee of the *5th Colloquium on Backward Stochastic Differential Equations* in June, the *European Summer School in Financial Mathematics* in September.

B. Bihain, the head of the Laboratoire de Médecine et Thérapie Moléculaire (INSERM, Nancy), asked to our group of Probability and Statistics to investigate the proteomic profiling of clinical samples. More precisely, the question involves the differential analysis of the expression levels of a large subset of the proteome of a particular type of clinical specimens to identify those proteins whose change in expression levels might be associated with a given disease process. The statistical tools are Data Analysis, and Support Vectors Machine. P. Vallois, S. Tindel (Université Nancy 1) and J.-M. Monnez (Université Nancy 2) have organized several seminars on these topics. S. Mézières and A. Koudou (Université Nancy 2) also participate to the bio-statistics group. O. Collignon has a Cifre thesis.

8.3. Teaching

M. Bossy gave a 20h course on *Continuous Probabilistic Models with Applications in Finance* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis), and a 15h course on *Risk management on energetic financial markets* in the Master *Ingénierie et Gestion de l'Energie* (École des Mines de Paris) at Sophia-Antipolis. M. Bossy also gave a 12h course on *Particle Methods* at the Master M2 *Probabilité et Applictions* at Université Paris 6.

N. Champagnat gave a 15h course and 8h of exercice classes on *Continuous Probabilistic Models with Applications in Finance* in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis).

M. Deaconu gave a course on *Stochastic Differential Equations* in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2007–2008.

A. Lejay gave a course on *Financial Mathematics* in the Master 2 Recherche of Applied Mathematics at Université Nancy 1 for 2007–2008.

D. Talay has a part time position of Professor at École Polytechnique. He also teaches *probabilistic numerical methods* at Université Paris 6 (Master degree in Probability), and *numerical methods in finance* at the University of Zürich.

E. Tanré gave a 9 hours course in the Master IMAFA (*Informatique et Mathématiques Appliquées à la Finance et à l'Assurance*, *Ecole Polytechnique Universitaire*, Nice – Sophia Antipolis).

8.4. Ph.D. theses and habilitations

Blandine Bérard-Bergery defended her Ph.D. thesis entitled *Approximation du temps local et intégration par régularisation* in October 2007 at Université Nancy 1.

Mamadou Cissé defended his Ph.D. thesis entitled *Méthodes probabiliste pour les conditions au bord artificielles d'équation aux dérivées partielles non linéaires en finance. Problème d'arrêt optimal pour une diffusion régulière* in July at INRIA Sophia Antipolis – Méditerranée.

Madalina Deaconu defended his habilitation entitled *Processus stochastiques associés aux équations d'évolution linéaires ou non-linéires et méthodes numériques probabilistes* in May at Université Nancy 1.

Angela Ganz defended her Ph.D. thesis entitled *Approximations des distributions d'équilibre de certains systèmes stochastiques avec interactions McKean-Vlasov* in October at INRIA Sophia Antipolis – Méditerranée.

Jean-François Jabir defended his Ph.D. thesis entitled *Modèles stochastiques lagrangiens de type McKean–Vlasov conditionnel et leur confinement* in October at INRIA Sophia Antipolis – Méditerranée.

8.5. Participation to congresses, conferences, invitations, ...

M. Bossy gave talks at the *Seminar del Centro de Modelamiento matematico* Santiago, Chili, in April 2008, at the *Stochastic Analysis Seminar at the Mathematical Institute* University of Oxford in June 2008, at the Fifth European Congress of Mathematics, 14 - 18 July 2008, Amsterdam and at the *Séminaire du CMAP* at Ecole Polytechnique in November 2008.

N.Champagnat gave talks at the conference *Inhomogeneous random systems* at IHP in Paris in January, at the *Séminaire Applications des Mathématiques* at ENS Cachan (Ker Lann) in March, at the *Séminaire de probabilités* of the Univeristé Bordeaux 1 in April, at the *2nd Canada France Congress* in Montreal in June, at the *Séminaire du LATP* of the University of Provence (Marseille) in June, at the Workshop *Applications of spatio-temporal dynamical systems in biology* at the University of Nice – Sophia Antipolis in June, at the *Journées MAS de la SMAI* at the University of Rennes I in August, at the conference on *Bioinformatics Modeling in Biology and Medicine* in Université de Nice – Sophia Antipolis in October, at the *Séminaire Bio-Maths* at the University Lyon I in November and at the *Séminaire Mathématiques, Evolution, Génome* at the Université de Provence (Marseille) in December.

C. Chauvin gave a talk at the *1ère journée INRIA/LJAD d'Analyse Numérique et Calcul Scientifique* in INRIA Sophia Antipolis – Méditerranée in October.

S. Herrmann gave a seminar lecture in Rennes, in November, and presented a poster at the *International Conference on Stochastic Resonance 2008*, in Perugia in August.

J.Huang gave a talk at the *Journées MAS de la SMAI 2008*, Rennes in August.

P.-E. Jabin gave talks at the *8th International Conference on Operations Research* in Cuba in February, at the *franco-canadian conference* in Montreal in June, at the *International conference on hyperbolic problems* in l'Aquila in July. He gave seminars at Paris VI, ENS Lyon and Zürich University. He finally gave summer school courses in Granada in April, in Porto Ercole in June, in l'Aquila in July and in Les Houches in August.

J.-F. Jabir gave a talk at the *Journée des Jeunes Probabilistes et Statisticiens* in April 2008 at Aussois.

A. Lejay has given talks in the Workshop on *Numerics and Stochastics* at Helsinki in August, at the *Journées MAS de la SMAI* at Rennes in September, at the *Symposium on Optimal Quantization and Application to Mathematical Finance* at Université Paris 6 in September. He also gave three seminars at University of Heidelberg, at the seminar of the *Chaire Risques Financiers* at École Polytechnique, and at Université Marne-la-Vallée.

S. Maire has given talks at the seminar *ADAP'MC : Méthodes de Monte Carlo adaptatives* at IHP in Paris in January, at the Workshop *Numerical methods in molecular simulation* held at the HIM in Bonn in April and at the *MCQMC 2008* conference in Montréal in July.

S. Maroso gave a seminar lecture at the OSIRIS department, EDF Clamart, in March 2008.

S. Rubenthaler has given an invited talk at the *joint conference SSC-SFDS* in Ottawa in May, and talks at the working group *Population Monte-Carlo*, part of the *Sequential Monte-Carlo program, SAMSI*, Research Triangle Park (North Carolina, USA) in September, at the *Statistical Seminar* of Duke University, Durham (North Carolina, USA) in October, at the working group *Population Monte-Carlo*, part of the *Sequential Monte-Carlo program, SAMSI*, Research Triangle Park (North Carolina, USA) in October and at the *Probability and Statistics Seminar*, Imperial College, London in November. He was also discussion session leader of the opening workshop of the *Sequential Monte-Carlo Program, SAMSI*, Research Triangle Park (North Carolina, USA) in September.

D. Talay gave talks at the Oberwolfach *Conference on Stochastic Analysis in Finance and Insurance* in January, at the University of Toulouse in February, at Cornell University in April, at the Colloque Numérique Suisse in April (plenary talk), at Warwick University in May, at the *5th Colloquium on Backward Stochastic Differential Equations* in June, at the University of Orléans in July (conference in the honor of D. Lépingle), at the Maths-AmSud and Stic-AmSud meeting in Lima in November (opening conference) and at the Workshop on Random Dynamical Systems at Bielefeld University in November.

E. Tanré gave talks at the *Journées MAS-SMAI* held in Rennes in August and at the *Groupe de travail MATH-BIO* in Paris 6 in November. He also gave two seminars at the Pontificia Universidad Catòlica de Chile in September.

S. Tindel gave talks at the *Séminaire de Probabilités* at the Université de Strasbourg in January, at the *Seminar on Stochastic Analysis* at Ascona (Switzerland) in May, at the *Journées Fractionnaires Parisiennes* at Paris in June, at the *Probability Seminar* at Frankfurt (Germany) in December, at the *Journée Apprentissage* at Nancy in December, at the *Séminaire de Probabilités* of Université de Rennes in December.

J. Tugaut presented a poster at the *Colloque Jeunes Probabilistes* in Aussois, in March and gave a seminar lecture in Nancy.

P. Vallois gave talks at the *9-th colloqium international of mathematics* in Nancy in October, at the *Séminaire de probabilités* at Clermont-Ferrand on January 2008, at the *Groupe de travail : aspects fractals* at Paris 6 on January 2008, at the *Sixth Seminar on Stochastic Analysis, Random Fields and Applications* at Ascona on May 2008, at the *Colloque : Calcul stochastique en l'honneur de D. Lepingle* at Orléans in July 2008, at the *Journées de probabilités* at Lille in September 2008, and at the *Séminaire de probabilités* at Rennes in November 2008.

8.5.1. Invitations

M. Bossy was invited two weeks, in April, at the Universidad de Chile, Santiago.

N. Champagnat spent one week in Rennes at IRISA within IPSO project-team in March and one week in Berlin at the Weierstrass Institute (WIAS) in April.

S. Herrmann spent 3 weeks in Berlin at the Humboldt University in November.

A. Lejay has been visiting Osaka University from October for 6 months withing the "sabbatical program" at INRIA. He also spent two weeks in Chile (Santiago and Valparaíso) in August.

S. Maroso was invited for three times one week at ENSTA in Paris in June, July and October 2008, for collaboration with H. Zidani and O. Bokanowski.

S. Rubenthaler spent two weeks at HEC in Montréal in May and 2 months and a half at SAMSI (Statistical and Applied Mathematical Sciences Institute, a partnership of Duke University, North Carolina State University, the University of North Carolina at Chapel Hill, and the National Institute of Statistical Sciences) between August and October.

D. Talay spent one week at Cornell University in April.

E. Tanré spent a week in the University of Bern (Switzerland) in April. He also spent two weeks in Chile at the Pontificia Universidad Catòlica de Chile in September.

P. Vallois was invited one week on October 2008 at Monastir (CMCU agreements).

Idir Ouassou (ENSA Marrakech, Maroco) has been visiting the TOSCA project-team in Sophia-Antipolis for one month in May.

Pierre Patie (University of Bern) has been visiting the TOSCA project-team in Sophia-Antipolis for one month in January, with a grant given by NCCR-FINRISK (Switzerland).

Soledad Torres (Universidad de Valparaíso) has been visiting the TOSCA project-team in Nancy for 2 months in January and February.

The TOSCA *seminar* organized by N. Champagnat has received the following speakers: Mariko Arisawa (Mathfi, INRIA Rocquencourt), Pierre Patie (IMSV, University of Bern), Benjamin Bruder (SGAM, Paris), Giambattista Giacomin (Université Paris 7), Bernard Roynette (IECN, Université Nancy I and INRIA Lorraine), Samuel Herrmann (IECN, Université Nancy I and INRIA Lorraine), Nawaf Bou-Rabee (Mathematics Department, Free University Berlin), Marc Yor (LPMA, Université Paris 6), Lorenzo Zambotti (Université Paris 6) and Pierre-Louis Lions (Collège de France).

The seminar *Probabilités* organized at Nancy by S. Tindel has received the following speakers: Aurélie Muller (IECN, Nancy), Aline Kurtzmann (Oxford University, UK), Anton Thalmaier (Université du Luxembourg), Bernard Roynette (IECN, Nancy), Pierre-Yves Louis (Universität Potsdam, Germany), Albrecht Boettcher (TU Chemnitz, Germany), Vincent Bansaye (Université de Paris 6), Eva Locherbach (Université de Paris 12), Philippe Chassaing (IECN, Nancy), Nicola Kistler (ENS Lyon), Nicolas Broutin (INRIA Rocquencourt), Jérôme Lelong (INRIA Rocquencourt), Marta Sanz-Solé (Universitat de Barcelona, Catalogne), Olivier Zindy (WIAS Berlin, Germany), Delphine Féral (Universität Bonn, Germany), Soledad Torres (Universidad de Valparaíso, Chili), Blandine Bérard-Bergery (IECN, Nancy), Grégory Miermont (Université de Paris Sud), Jean Bertoin (Université de Paris 6), Matthieu Marouby (Université de Toulouse 3), Lucas Gérin (IECN, Nancy), Marc Wouts (Université de Paris 10), Frédéric Bertrand (Université de Strasbourg), Arnaud Gloter (Université de Marne-la-Vallée), Xiang-Dong Li (Université de Toulouse 3), Anatoli Yambartsev (University of Sao Paulo, Brésil), Jorge León (CINVESTAV, Mexico), Armelle Guillou (Université de Strasbourg), Eugène Pechersky (IITP, Russian Academy of Science), Jean-Baptiste Goueré (Université d'Orléans), Vincent Rivoirard (Université d'Orsay), Pierre Calka (Université de Paris 5), Julian Tugaut (IECN, Nancy) and Joseph Najnudel (Universität Zürich, Switzerland).

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[11] B. BERARD BERGERY. *Approximation du temps local et intégration par régularisation*, Ph. D. Thesis, Université Henri Poincaré, Nancy I, October 2007.

[12] M. CISSÉ. *Méthodes probabiliste pour les conditions au bord artificielles d'équation aux dérivées partielles non linéaires en finance. Problème d'arrêt optimal pour une diffusion régulière*, Ph. D. Thesis, Université de Nice / INRIA Sophia-Antipolis, July 2008.

[13] M. DEACONU. *Processus stochastiques associés aux équations d'évolution linéaires ou non-linéaires et méthodes numériques probabilistes*, Habilitation à Diriger les Recherches, Ph. D. Thesis, Université Henri Poincaré, Nancy I, May 2008.

[14] A. GANZ. *Approximations des distributions d'équilibre de certains systèmes stochastiques avec interactions McKean-Vlasov*, Ph. D. Thesis, Université de Nice / INRIA Sophia-Antipolis, October 2008.

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