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Project-Team apics

Analysis and Problems of Inverse type in Control and Signal processing

Sophia Antipolis - Méditerranée

Theme : Modeling, Optimization, and Control of Dynamic Systems

Activity Report

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2. Overall Objectives

2.1. Overall Objectives

The Team aims at designing and developing constructive methods in modeling, identification and control of dynamical, resonant and diffusive systems.

2.1.1. Research Themes

- Function theory and approximation theory in the complex domain, with applications to frequency identification of linear systems and inverse boundary problems for the Laplace and Beltrami operators:
  - System and circuit theory with applications to the modeling of analog microwave devices. Development of dedicated software for the synthesis of such devices.
  - Inverse potential problems in 2-D and 3-D and harmonic analysis with applications to non-destructive control (from magneto/electro-encephalography in medical engineering or plasma confinement in tokamaks for nuclear fusion).
- Control and structure analysis of non-linear systems with applications to orbit transfer of satellites.

2.1.2. International and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLim (Limoges), CEA-IRFM (Cadarache).
- Exchanges with UST (Villeneuve d'Asq), University Bordeaux-I (Talence), University of Orléans (MAPMO), University of Pau (EPI Inria Magique-3D), University Marseille-I (CMI), CWI (the Netherlands), SISSA (Italy), the Universities of Illinois (Urbana-Champaign USA), California at San Diego and Santa Barbara (USA), Michigan at East-Lansing (USA), Vanderbilt University (Nashville USA), Texas A&M (College Station USA), ISIB (CNR Padova, Italy), Beer Sheva (Israel), RMC (Kingston, Canada), University of Erlangen (Germany), Leeds (UK), Maastricht University (The Netherlands), Cork University (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien (Austria), TFH-Berlin (Germany), CINVESTAV (Mexico), ENIT (Tunis), KTH (Stockholm).
- The project is involved in the ANR projects AHPI (Math., coordinator) and Filipix (Telecom.), in a EMS21-RTG NSF program (with Vanderbilt University, Nashville, USA), in an NSF Grant with Vanderbilt University and the MIT, in an EPSRC Grant with Leeds University (UK), in a Inria-Tunisian Universities program (STIC, with LAMSIN-ENIT, Tunis).

3. Scientific Foundations

3.1. Identification and approximation

Identification typically consists in approximating experimental data by the prediction of a model belonging to some model class. It consists therefore of two steps, namely the choice of a suitable model class and the determination of a model in the class that fits best with the data. The ability to solve this approximation problem, often non-trivial and ill-posed, impinges on the effectiveness of a method.

Particular attention is payed within the team to the class of stable linear time-invariant systems, in particular resonant ones, and in isotropically diffusive systems, with techniques that dwell on functional and harmonic analysis. In fact one often restricts to a smaller class —*e.g.* rational models of suitable degree (resonant systems, see section 4.3) or other structural constraints— and this leads us to split the identification problem in two consecutive steps:

1. Seek a stable but infinite (numerically: high) dimensional model to fit the data. Mathematically speaking, this step consists in reconstructing a function analytic in the right half-plane or in the unit disk (the transfer function), from its values on an interval of the imaginary axis or of the unit circle (the band-width). We will embed this classical ill-posed issue (*i.e.* the inverse Cauchy problem for the Laplace equation) into a family of well-posed extremal problems, that may be viewed as a regularization scheme of Tikhonov-type. These problems are infinite-dimensional but convex (see section 3.1.1).

2. Approximate the above model by a lower order one reflecting further known properties of the physical system. This step aims at reducing the complexity while bringing physical significance to the design parameters. It typically consists of a rational or meromorphic approximation procedure with prescribed number of poles in certain classes of analytic functions. Rational approximation in the complex domain is a classical but difficult non-convex problem, for which few effective methods exist. In relation to system theory, two specific difficulties superimpose on the classical situation, namely one must control the region where the poles of the approximants lie in order to ensure the stability of the model, and one has to handle matrix-valued functions when the system has several inputs and outputs, in which case the number of poles must be replaced by the McMillan degree (see section 3.1.2).

When identifying elliptic (Laplace, Beltrami) partial differential equations from boundary data, point 1. above can be recast as an inverse boundary-value problem with (overdetermined Dirichlet-Neumann) data on part of the boundary of a plane domain (recover a function, analytic in a domain, from incomplete boundary data). As such, it arises naturally in higher dimensions when analytic functions get replaced by gradients of harmonic functions (see section 4.2). Motivated by free boundary problems in plasma control and questions of source recovery arising in magneto/electro-encephalography, we aim at generalizing this approach to the real Beltrami equation in dimension 2 (section 6.3.3) and to the Laplace equation in dimension 3 (section 6.3.1).

Step 2. above, i.e., meromorphic approximation with prescribed number of poles—is used to approach other inverse problems beyond harmonic identification. In fact, the way the singularities of the approximant (*i.e.* its poles) relate to the singularities of the approximated function is an all-pervasive theme in approximation theory: for appropriate classes of functions, the location of the poles of the approximant can be used as an estimator of the singularities of the approximated function (see section 6.3.2).

We provide further details on the two steps mentioned above in the sub-paragraphs to come.

3.1.1. Analytic approximation of incomplete boundary data

Keywords: *Beltrami equations*, *Hardy spaces*, *extremal problems*, *harmonic functions*, *inverse problems*.

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Given a planar domain D, the problem is to recover an analytic function from its values on a subset of the boundary of D. It is convenient to normalize D and apply in each particular case a conformal transformation to meet a “normalized” domain. In the simply connected case, which is that of the half-plane, we fix D to be the unit disk, so that its boundary is the unit circle T. We denote by H^p the Hardy space of exponent p which is the closure of polynomials in the L^p-norm on the circle if $1 \leq p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which make it possible to speak of (traces of) analytic functions on the boundary.

A standard extremal problem on the disk is [68]:

(P_0) Let $1 \leq p \leq \infty$ and $f \in L^p(T)$; find a function $g \in H^p$ such that $g - f$ is of minimal norm in $L^p(T)$.

When seeking an analytic function in D which approximately matches some measured values f on a sub-arc K of T, the following generalization of (P_0) naturally arises:

(P) Let $1 \leq p \leq \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \leq M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capsulizing the expected behavior of the model off K, while M is the admissible error with respect to this expectation. The value of p reflects the type of stability which is sought and how much one wants to smoothen the data.

To fix terminology we generically refer to (P) as a *bounded extremal problem*. The solution to this convex infinite-dimensional optimization problem can be obtained upon iteratively solving spectral equations for appropriate Hankel and Toeplitz operators, that involve a Lagrange parameter, and whose right hand-side is given by the solution to (P_0) for some weighted concatenation of f and ψ. Constructive aspects are described in [43], [45], [87], for $p=2$, $p=\infty$, and $1<p<\infty$, while the situation $p=1$ is essentially open.

Various modifications of (P) have been studied in order to meet specific needs. For instance when dealing with loss-less transfer functions (see section 4.3), one may want to express the constraint on $T\setminus K$ in a pointwise manner: $|g-\psi|\leq M$ a.e. on $T\setminus K$, see [29], [47] for $p=2$ and $\psi=0$.

The above-mentioned problems can be stated on an annular geometry rather than a disk. For $p=2$ the solution proceeds much along the same lines [23]. When K is the outer boundary, (P) regularizes a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from overdetermined Dirichlet-Neumann data on the outer boundary (see sections 4.2 and 6.3). Interestingly perhaps, it becomes a tool to approach Bernoulli type problems for the Laplacian, where overdetermined observations are made on the outer boundary and we *seek the inner boundary* knowing it is a level curve of the flux (see section 6.3.3). Here, the Lagrange parameter indicates which deformation should be applied on the inner contour in order to improve the fit to the data.

Continuing effort is currently payed by the team to carry over bounded extremal problems and their solution to more general settings.

Such generalizations are twofold: on the one hand Apics considers 2-D diffusion equations with variable conductivity, on the other hand it investigates the ordinary Laplacian in $\mathbf{R}^n$. The targeted applications are the determination of free boundaries in plasma control and source detection in electro/magneto-encephalography (EEG/MEG, see section 6.3.2).

An isotropic diffusion equation in dimension 2 can be recast as a so-called real Beltrami equation [73]. This way analytic functions get replaced by "generalized" ones in problems (P_0) and (P). Hardy spaces of solutions, which are more general than Sobolev ones and allow one to handle L^p boundary conditions, have been introduced when $1<p<\infty$ [46]. The expansions of solutions needed to constructively handle such problems have been preliminary studied in [64], [65]. The goal is to solve the analog of (P) in this context to approach Bernoulli-type problems (see section 6.3.1).

At present, bounded extremal problems for the n-D Laplacian are considered on half-spaces or balls. Following [88], Hardy spaces are defined as gradients of harmonic functions satisfying L^p growth conditions on inner hyperplanes or spheres. From the constructive viewpoint, when $p=2$, spherical harmonics offer a reasonable substitute to Fourier expansions [13]. Only very recently were we able to define operators of Hankel type whose singular values connect to the solution of (P_0) in BMO norms. The L^p problem also makes contact with some nonlinear PDE's, namely to the p-Laplacian. The goal is here to solve the analog of (P) on spherical shells to approach inverse diffusion problems across a conductor layer.

3.1.2. Meromorphic and rational approximation

Keywords: *critical point theory*, *meromorphic approximation*, *orthogonal polynomials*, *rational approximation*.

Participants: Laurent Baratchart, José Grimm, Martine Olivi, Edward Saff, Herbert Stahl [TFH Berlin].

Let as before D designate the unit disk, T the unit circle. We further put R_N for the set of rational functions with at most N poles in D, which allows us to define the meromorphic functions in $L^p(T)$ as the traces of functions in H^p+R_N.

A natural generalization of problem (P_0) is

> (P_N) Let $1\leq p\leq\infty$, $N\geq 0$ an integer, and $f\in L^p(T)$; find a function $g_N\in H^p+R_N$ such that g_N-f is of minimal norm in $L^p(T)$.

Problem (P_N) aims, on the one hand, at solving inverse potential problems from overdetermined Dirichlet-Neumann data, namely to recover approximate solutions of the inhomogeneous Laplace equation $\Delta u = \mu$, with μ some (unknown) distribution, which will be discretized by the process as a linear combination of N Dirac masses. On the other hand, it is used to perform the second step of the identification scheme described in section 3.1, namely rational approximation with a prescribed number of poles to a function analytic in the right half-plane, when one maps the latter conformally to the complement of D and solve (P_N) for the transformed function on T.

Only for $p = \infty$ and continuous f is it known how to solve (P_N) in closed form. The unique solution is given by the AAK theory, that allows one to express g_N in terms of the singular vectors of the Hankel operator with symbol f. The continuity of g_N as a function of f only holds for stronger norms than uniform, [85].

The case $p = 2$ is of special importance. In particular when $f \in \overline{H}^2$, the Hardy space of exponent 2 of the *complement* of D in the complex plane (by definition, $h(z)$ belongs to $\overline{H}^p$ if, and only if $h(1/z)$ belongs to H^p), then (P_N) reduces to rational approximation. Moreover, it turns out that the associated solution $g_N \in R_N$ has no pole outside D, hence it is a *stable* rational approximant to f. However, in contrast with the situation when $p = \infty$, this approximant may *not* be unique.

The former Miaou project (predecessor of Apics) has designed an adapted steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; it seems today the only procedure meeting this property. Roughly speaking, it is a gradient algorithm that proceeds recursively with respect to the order N of the approximant, in a compact region of the parameter space [40]. Although it has proved rather effective in all applications carried out so far (see sections 4.2, 4.3), it is not known whether the absolute *minimum* can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as done by the Endymion software section 5.5 and RARL2 software, section 5.2).

In order to establish convergence results of the algorithm to the global minimum, Apics has undergone a long-haul study of the number and nature of critical points, in which tools from differential topology and operator theory team up with classical approximation theory. The main discovery is that the nature of the critical points (*e.g. local minima*, saddles...) depends on the decrease of the interpolation error to f as N increases [48]. Based on this, sufficient conditions have been developed for a local *minimum* to be unique. This technique requires strong error estimates that are often difficult to obtain, and most of the time only hold for N large. Examples where uniqueness or asymptotic uniqueness has been proved this way include transfer functions of relaxation systems (i.e., Markov functions) [49], the exponential function, and meromorphic functions [8]. The case where f is the Cauchy integral on a hyperbolic geodesic arc of a Dini-continuous function which does not vanish "too much" has been recently answered in the positive, see section 6.7. An analog to AAK theory has been carried out for $2 \le p < \infty$ [9]. Although not computationally as powerful, it has better continuity properties and stresses a continuous link between rational approximation in H^2 and meromorphic approximation in the uniform norm, allowing one to use, in either context, techniques available from the other[1].

A common feature to all these problems is that critical point equations express non-Hermitian orthogonality relations for the denominator of the approximant. This is used in an essential manner to assess the behavior of the poles of the approximants to functions with branched singularities which is of particular interest for inverse source problems (*cf.* sections 6.3.2, 6.7).

In higher dimensions, the analog of problem (P_N) is the approximation of a vector field with gradients of potentials generated by N point masses instead of meromorphic functions. The issue is by no means understood at present, and is a major endeavor of future research problems.

Certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus. Such questions have become over years an increasingly significant part of the team's activity (see sections 4.3, 6.6, 6.7, and 6.10). When translated over to the circle, a prototypical formulation consists in approximating the modulus of a given function by the modulus of a rational function of degree n.

[1] When $1 \le p < 2$, problem (P_N) is still fairly open.

When $p = 2$ this problem can be reduced to a series of standard rational approximation problems, but usually one needs to solve it for $p = \infty$. The case where $|f|$ is a piecewise constant function with values 0 and 1 can also be approached via classical Zolotarev problems [86], that can be solved more or less explicitly when the pass-band consists of a single arc. A constructive solution in the case where $|f|$ is a piecewise constant function with values 0 and 1 on several arcs (multiband filters) is one recent achievement of the team. Though the modulus of the response is the first concern in filter design, the variation of the phase must nevertheless remain under control to avoid unacceptable distortion of the signal. This is an important issue, currently under investigation within the team under contract with the CNES, see section 6.10.

From the point of view of design, rational approximants are indeed useful only if they can be translated into physical parameter values for the device to be built. This is where system theory enters the scene, as the correspondence between the frequency response (i.e., the transfer-function) and the linear differential equations that generate this response (i.e., the state-space representation), which is the object of the so-called *realization* process. Since filters have to be considered as dual modes cavities, the realization issue must indeed be tackled in a 2×2 matrix-valued context that adds to the complexity. A fair share of the team's research in this direction is concerned with finding realizations meeting certain constraints (imposed by the technology in use) for a transfer-function that was obtained with the above-described techniques (see section 6.8).

3.1.3. Behavior of poles of meromorphic approximants and inverse problems for the Laplacian

Keywords: *discretization of potentials*, *free boundary inverse problems*, *meromorphic and rational approximation*, *orthogonal polynomials*, *singularity detection*.

Participants: Laurent Baratchart, Edward Saff, Herbert Stahl [TFH Berlin], Maxim Yattselev.

We refer here to the behavior of the poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. If one normalizes the contour to be the unit circle T, we are back to the framework of section 3.1.2 and to problem (P_N); the invariance of the problem under conformal mapping was established in [6]. The research so far has focused on functions whose singular set inside the contour is zero or one-dimensional.

Generally speaking, the behavior of poles is particularly important in meromorphic approximation to obtain error rates as the degree goes large and also to tackle constructive issues like uniqueness. However, the original motivation of Apics is to consider this issue in connection with the approximation of the solution to a Dirichlet-Neumann problem, so as to extract information on the singularities. The general theme is thus *how do the singularities of the approximant reflect those of the approximated function?* The approach to inverse problem for the 2-D Laplacian that we outline here is attractive when the singularities are zero- or one-dimensional (see section 4.2). It can be used as a computationally cheap preliminary step to obtain the initial guess of a more precise but heavier numerical optimization.

For sufficiently smooth cracks, or pointwise sources recovery, the approach in question is in fact equivalent to the meromorphic approximation of a function with branch points, and we were able to prove [4], [6] that the poles of the approximants accumulate in a neighborhood of the geodesic hyperbolic arc that links the endpoints of the crack, or the sources [44]. Moreover the asymptotic density of the poles turns out to be the equilibrium distribution on the geodesic arc of the Green potential and it charges the end points, that are thus well localized if one is able to compute sufficiently many zeros (this is where the method could fail). The case of more general cracks, as well as situations with three or more sources, requires the analysis of the situation where the number of branch points is finite but arbitrary, see section 6.7). This are outstanding open questions for applications to inverse problems (see section 6.3), as also the problem of a general singularity, that may be two dimensional.

Results of this type open new perspectives in non-destructive control, in that they link issues of current interest in approximation theory (the behavior of zeroes of non-Hermitian orthogonal polynomials) to some classical inverse problems for which a dual approach is thereby proposed: to approximate the boundary conditions by true solutions of the equations, rather than the equation itself (by discretization).

Let us point out that the problem of approximating, by a rational or meromorphic function, in the L^p sense on the boundary of a domain, the Cauchy transform of a real measure, localized inside the domain, can be viewed as an optimal discretization problem for a logarithmic potential according to a criterion involving a Sobolev norm. This formulation can be generalized to higher dimensions, even if the computational power of complex analysis is then no longer available, and this makes for a long-term research project with a wide range of applications. It is interesting to mention that the case of sources in dimension three in a spherical or ellipsoidal geometry, can be attacked with the above 2-D techniques as applied to planar sections (see section 6.3).

3.1.4. Matrix-valued rational approximation

Keywords: *inner matrix*, *rational approximation*, *realization theory*, *reproducing kernel space*.

Participants: Laurent Baratchart, Martine Olivi, José Grimm, Jean-Paul Marmorat, Bernard Hanzon, Ralf Peeters [Univ. Maastricht].

Matrix-valued approximation is necessary for handling systems with several inputs and outputs, and it generates substantial additional difficulties with respect to scalar approximation, theoretically as well as algorithmically. In the matrix case, the McMillan degree (i.e., the degree of a minimal realization in the System-Theoretic sense) generalizes the degree.

The problem we want to consider reads: *Let $\mathcal{F} \in (H^2)^{m\times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n which is nearest possible to $\mathcal{F}$ in $(H^2)^{m\times l}$*. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The approximation algorithm designed in the scalar case generalizes to the matrix-valued situation [67]. The first difficulty consists here in the parametrization of transfer matrices of given McMillan degree n, and the inner matrices (i.e., matrix-valued functions that are analytic in the unit disk and unitary on the circle) of degree n enter the picture in an essential manner: they play the role of the denominator in a fractional representation of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization).

The set of inner matrices of given degree has the structure of a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (parametrization valid in a neighborhood of a point), and we must handle changes of charts in the course of the algorithm. Such parametrization can be obtained from interpolation theory and Schur type algorithms, the parameters being interpolation vectors or matrices [33], [10], [11]. Some of these parametrizations have a particular interest for computation of realizations [10], [11], involved in the estimation of physical quantities for the synthesis of resonant filters. Two rational approximation codes (see sections 5.2 and 5.5) have been developed in the team.

Problems relative to multiple local minima naturally arise in the matrix-valued case as well, but deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The already investigated case of rational functions of the sought degree (the consistency problem) was solved using rather heavy machinery [7], and that of matrix-valued Markov functions, that are the first example beyond rational function has made progress only recently [39].

Let us stress that the algorithms mentioned above are first to handle rational approximation in the matrix case in a way that converges to local minima, while meeting stability constraints on the approximant.

3.2. Structure and control of non-linear systems

3.2.1. Feedback control and optimal control

Keywords: *control*, *control Lyapunov functions*, *feedback*, *optimal control*, *stabilization*.

Participants: Jean-Baptiste Pomet, Ludovic Rifford.

Using the terminology of the beginning of section 3.1, the class of models considered here is the one of finite dimensional nonlinear control systems; we focus on control rather than identification. In many cases, a linear control based on the linear approximation around a nominal point or trajectory is sufficient. However, there are important instances where it is not, either because the magnitude of the control is limited or because the linear approximation is not controllable, or else in control problems like path planning, that are not local in nature.

State feedback stabilization consists in designing a control law which is a function of the state and makes a given point (or trajectory) asymptotically stable for the closed–loop system. That function of the state must bear some regularity, at least enough to allow the closed-loop system to make sense; continuous or smooth feedback would be ideal, but one may also be content with discontinuous feedback if robustness properties are not defeated. One can consider this as a weak version of the optimal control problem which is to find a control that minimizes a given criterion (for instance the time to reach a prescribed state). Optimal control generally leads to a rather irregular dependence on the initial state; in contrast, stabilization is a *qualitative* objective (i.e., to reach a given state asymptotically) which is more flexible and allows one to impose a lot more regularity.

Lyapunov functions are a well-known tool to study the stability of non-controlled dynamic systems. For a control system, a *Control Lyapunov Function* is a Lyapunov function for the closed-loop system where the feedback is chosen appropriately. It can be expressed by a differential inequality called the "Artstein (in)equation" [36], reminiscent of the Hamilton-Jacobi-Bellmann equation but largely under-determined. One can easily deduce a continuous stabilizing feedback control from the knowledge of a control Lyapunov function; also, even when such a control is known beforehand, obtaining a control Lyapunov function can still be very useful to deal with robustness issues. Moreover, if one has to deal with a problem where it is important to optimize a criterion, and if the optimal solution is hard to compute, one can look for a control Lyapunov function which comes "close" (in the sense of the criterion) to the solution of the optimization problem but leads to a control which is easier to work with.

These constructions were exploited in a joint collaborative research conducted with Thales Alenia Space (Cannes), where minimizing a certain cost is very important (fuel consumption / transfer time) while at the same time a feedback law is preferred because of robustness and ease of implementation (see section 4.4).

3.2.2. Optimal transportation

Keywords: *control*, *control Lyapunov functions*, *feedback*, *optimal control*, *stabilization*.

Participants: Ahed Hindawi, Jean-Baptiste Pomet, Ludovic Rifford.

The study of optimal mass transport problems in the Euclidean or Riemannian setting has a long history which goes back to the pioneering works [84], [74], and was more recently revised and revitalized by [59], [83]. It is the problem of finding the cheapest transformation that moves a given initial measure to a given final one, where the cost between points is a (squared) Euclidean or Riemannian distance.

There has been, quite newly, a lot of interest in the same transportation problems with a cost coming from optimal control, i.e. from minimizing an integral quadratic cost, among trajectories that are subject to differential constraints coming from a control system. The case of controllable affine control systems without drift (in which case the cost is the sub-Riemannian distance) is studied in [35], [32] and [21].

This is a new topic in the team, starting with the PhD of A. Hindawi, whose goal is to tackle the problem of systems with drift. The optimal transport problem in this setting borrows methods from control and at the same time helps understanding optimal control because it is a more regular problem.

3.2.3. Transformations and equivalences of non-linear systems and models

Keywords: *classification*, *non-linear control*, *non-linear feedback*, *non-linear identification*.

Participants: Laurent Baratchart, Jean-Baptiste Pomet.

Here we study certain transformations of models of control systems, or more accurately of equivalence classes modulo such transformations. The interested reader can find a richer overview in the first chapter of the HDR Thesis [12]. The motivations are two-fold:

- From the point of view of control, a command satisfying specific objectives on the transformed system can be used to control the original system including the transformation in the controller.
- From the point of view of identification and modeling, the interest is either to derive qualitative invariants to support the choice of a non-linear model given the observations, or to contribute to a classification of non-linear models which is missing sorely today. This is a prerequisite for a general theory of non-linear identification; indeed, the success of the linear model in control and identification is due to the deep understanding one has of it.

A *static feedback* transformation is a (non-singular) re-parametrization of the control depending on the state, together with a change of coordinates in the state space. A *dynamic feedback* transformation consists of a dynamic extension (adding new states, and assigning them a new dynamics) followed by a state feedback on the augmented system. Dynamic equivalence is obviously more general than static equivalence. Let us now stress two specific problems that we are tackling.

Dynamic Equivalence. Very few invariants are known. Any insight on this problem is relevant to the above questions. Some results [24] are accounted for in section 6.14.

A special equivalence class is the one containing linear controllable systems. It turns out that a system is in this class —*i.e.* is dynamic linearizable— if and only if there is a formula that gives the general solution by applying a nonlinear differential operator to a certain number of arbitrary functions of time; such a formula is often called a *(Monge) parametrization* and the order of the differential operator the order of the parametrization. Existence of such a parametrization has been emphasized over the last years as very important and useful in control, see [66]; this property (with additional requirements on the parametrization) is also called flatness.

An important question remains open: how can one algorithmically decide whether a given system has this property or not, i.e., is dynamic linearizable or not? The mathematical difficulty is that no a priori bound is known on the order of the above mentioned differential operator giving the parametrization. Within the team, results on low dimensional systems have been obtained [1], see also [37]; the above mentioned difficulty is not solved for these systems but results are given with *priori* prescribed bounds on this order.

From the differential algebraic point of view, the module of differentials of a controllable system is free and finitely generated over the ring of differential polynomials in d/dt with coefficients in the ring of functions on the system's trajectories; the above question is the one of finding out whether there exists a basis consisting of closed differential forms. Expressed in this way, it looks like an extension of the classical Frobenius integrability theorem to the case where coefficients are differential operators. Of course, some non classical conditions have to be added to the classical stability by exterior differentiation, and the problem is open. In [38], a partial answer to this problem was given, but in a framework where infinitely many variables are allowed and a finiteness criterion is still missing. The goal is to obtain a formal and implementable algorithm to decide whether or not a given system is flat around a regular point.

Topological Equivalence. Compared to static equivalence, dynamic equivalence is more general, hence might offer some more robust "qualitative" invariants; another way to enlarge equivalence classes is to look for equivalence modulo possibly non-differentiable transformations.

In the case of dynamical systems without control, the Hartman-Grobman theorem states that every system is locally equivalent via a transformation that is solely bi-continuous, to a linear system in a neighborhood of a non-degenerate equilibrium. A similar result control systems would say, typically, that outside a "meager" class of models (for instance, those whose linear approximation is non-controllable), and locally around nominal values of the state and the control, no qualitative phenomenon can distinguish a non-linear system from a linear one, all non-linear phenomena being thus either of global nature or singularities.

In [41], we proved a "Hartman Grobman Theorem for control systems", under weak regularity conditions, but it is too abstract to be relevant to the above considerations on qualitative phenomena: linearization is performed by functional non-causal transformations, whose structure is not well understood; it however acquires a concrete meaning when the inputs are themselves generated by a finite-dimensional dynamics.

A stronger Hartman Grobman Theorem for control systems —where transformations are homeomorphisms in the state-control space— cannot hold, in fact; this is proved in [15], commented in section 6.13: almost all topologically linearizable control systems are differentiably linearizable. In general (equivalence between nonlinear systems), topological invariants are still to be investigated.

4. Application Domains

4.1. Introduction

The bottom line of the team's activity is two-fold, namely function theory and optimization in the frequency domain on the one hand, and the control of certain systems governed by differential equations on the other hand. Therefore one can distinguish between two main families of applications: one dealing with the design and identification of diffusive and resonant systems (these are inverse problems), and one dealing with the control of certain mechanical systems. For applications of the first type, approximation techniques as described in section 3.1.1 allow one to deconvolve linear equations, analyticity being the result of either the use of Fourier transforms or the harmonic character of the equation itself. Applications of the second type mostly concern the control of systems that are "poorly" controllable, for instance low thrust satellites. We describe all these below in more detail.

4.2. Geometric inverse problems for elliptic partial differential equations

Participants: Laurent Baratchart, Yannick Fischer, José Grimm, Juliette Leblond, Ana-Maria Nicu, Jonathan R. Partington, Stéphane Rigat [Univ. Aix-Marseille I], Emmanuel Russ [Univ. Aix-Marseille III], Edward Saff, Meriem Zghal.

We are mainly concerned with classical inverse problems like the one of localizing defaults (as cracks, pointwise sources or occlusions) in a two or three dimensional domain from boundary data (which may correspond to thermal, electrical, or magnetic measurements), of a solution to Laplace or to some conductivity equation in the domain. These defaults can be expressed as a lack of analyticity of the solution of the associated Dirichlet-Neumann problem that may be approached, in balls, using techniques of best rational or meromorphic approximation on the boundary of the object (see section 3.1).

Indeed, it turns out that traces of the boundary data on 2-D cross sections (disks) coincide with analytic functions in the slicing plane, that has branched singularities inside the disk [5]. These singularities are related to the actual location of the sources (namely, they reach in turn a maximum in modulus when the plane contains one of the sources). Hence, we are back to the 2-D framework where approximately recovering these singularities can be performed using best rational approximation.

In this connection, the realistic case where data are available on part of the boundary only offers a typical opportunity to apply the analytic extension techniques (see section 3.1.1) to Cauchy type issues, a somewhat different kind of inverse problems in which the team is strongly interested.

The approach proposed here consists in recovering, from measured data on a subset K of the boundary ∂D of a domain D of R^2 or R^3, say the values F_K on K of some function F, the subset $\gamma \subset D$ of its singularities (typically, a crack or a discrete set of pointwise sources), provided that F is an analytic function in $D \setminus \gamma$.

- The analytic approximation techniques (section 3.1.1) first allow us to extend F from the given data F_K to all of ∂D, if $K \neq \partial D$, which is a Cauchy type issue for which our algorithms provide robust solutions, in plane domains (see [23] for 2D annular domains, and [13] for 3D spherical situations, also discussed in section 6.3). Note that identification schemes for an unknown Robin coefficient together with stability properties have been obtained in the same way [62].
- From these extended data on the whole boundary, one can obtain information on the presence and the location of γ, using rational or meromorphic approximation on the boundary (section 3.1). This may be viewed as a discretization of γ by the poles of the approximants [4].

This is the case in dimension 2, using classical links between analyticity and harmonicity [2], but also in dimension 3, at least in spherical or ellipsoidal domains, working on 2-D plane sections, [5], [78].

The two above steps are shown in [20] to provide a robust way of locating sources from incomplete boundary data in a 2-D situation with several annular layers. Numerical experiments have already yielded excellent results in 3-D situations and we are now on the way to process real experimental magneto-encephalographic data, see also sections 5.7, 6.3.2. The PhD theses of A.-M. Nicu and M. Zghal are concerned with these applications, in collaboration with the Odyssée team of Inria Sophia Antipolis, and with neuroscience teams in partner-hospitals (hosp. Timone, Marseille).

Such methods are currently being generalized to problems with variable conductivity governed by a 2-D Beltrami equation, see [46], [64], [65]. The application we have in mind is to plasma confinement for thermonuclear fusion in a Tokamak, more precisely with the extrapolation of magnetic data on the boundary of the chamber from the outer boundary of the plasma, which is a level curve for the poloidal flux solving the original div-grad equation. Solving this inverse problem of Bernoulli type is of importance to determine the appropriate boundary conditions to be applied to the chamber in order to shape the plasma [53]. These issues are the topics of the PhD theses of S. Chaabi and Y. Fischer, and of a joint collaboration with the CEA-IRFM (Cadarache), the Laboratoire J.-A. Dieudonné at the Univ. of Nice-SA, and the CMI-LATP at the Univ. of Marseille I (see section 6.3.3), see [64], [65].

Inverse potential problems are also naturally encountered in magnetization issues that arise in nondestructive control. A particular application, which the object of a joint NSF-supported project with Vanderbilt University and MIT, is to geophysics where the remanent magnetization a rock is to be analyzed using a squid-magnetometer in order to analyze the history of the object; specifically, the analysis of Martian rocks is conducted at MIT, for instance to understand if inversions of the magnetic field took place there. Mathematically speaking, the problem is to recover the (3-D valued) magnetization m from measurements of the vector potential:

$$\int_{\Omega} \frac{\operatorname{div} m(x')\, dx'}{|x-x'|},$$

outside the volume Ω of the object.

4.3. Identification and design of resonant systems: hyperfrequency filter identification

Participants: Laurent Baratchart, Stéphane Bila [XLim, Limoges], José Grimm, Jean-Paul Marmorat, Fabien Seyfert.

One of the best training grounds for the research of the team in function theory is the identification and design of physical systems for which the linearity assumption works well in the considered range of frequency, and whose specifications are made in the frequency domain. Resonant systems, either acoustic or electromagnetic based, are prototypical devices of common use in telecommunications.

In the domain of space telecommunications (satellite transmissions), constraints specific to onboard technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study (of the Helmholtz equation) states that essentially only a discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be seen as being decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that there are 16 quantities that should be optimized. Quantities like the diameter and length of the cavities, or the width of the 11 slits are fixed in the design phase.

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all the cavities have the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since the screws are conductors, they act more or less as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of the iris is to the contrary of a screw: no condition is imposed where there is a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 Ghz.

Near the resonance frequency, a good approximation of the Maxwell equations is given by the solution of a second order differential equation. One obtains thus an electrical model for our filter as a sequence of electrically-coupled resonant circuits, and each circuit will be modeled by two resonators, one per mode, whose resonance frequency represents the frequency of a mode, and whose resistance represent the electric losses (current on the surface).

In this way, the filter can be seen as a quadripole, with two ports, when plugged on a resistor at one end and fed with some potential at the other end. We are then interested in the power which is transmitted and reflected. This leads to defining a scattering matrix S, that can be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The filter is rational of order 4 times the number of cavities (that is 16 in the example), and the key step consists in expressing the components of the equivalent electrical circuit as a function of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the design, particularly the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (i.e., the underlying system may not have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the identification strategy is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80Mhz in the example).
- solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. section 3.1.1). This provides us with a scattering matrix of order roughly 1/4 of the number of data points.
- Approximating this scattering matrix by a rational transfer-function of fixed degree (8 in this example) via the Endymion or RARL2 software (cf. section 3.1.4).
- A realization of the transfer function is thus obtained, and some additional symmetry constraints are imposed.
- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).

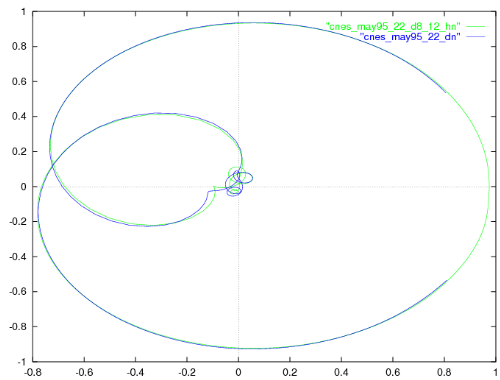

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}

The final approximation is of high quality. This can be interpreted as a validation of the linearity hypothesis for the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, useful for the synthesis of repeating devices.

The team investigates today the design of output multiplexors (OMUX) where several filters of the previous type are coupled on a common guide. In fact, it has undergone a rather general analysis of the question “How does an OMUX work?” With the help of numerical simulations and Schur analysis, general principles are being worked out to take into account:

- the coupling between each channel and the “Tee” that connects it to the manifold,
- the coupling between two consecutive channels.

The model is obtained upon chaining the corresponding scattering matrices, and mixes up rational elements and complex exponentials (because of the delays) hence constitutes an extension of the previous framework. Its study is being conducted under contract with Thales Alenia Space (Toulouse) (see sections 7.1).

4.4. Spatial mechanics

Participants: Alex Bombrun [Univ. of Heidelberg], José Grimm, Jean-Baptiste Pomet.

Generally speaking, aerospace engineering requires sophisticated control techniques for which optimization is often crucial, due to the extreme functioning conditions. The use of satellites in telecommunication networks motivates a lot of research in the area of signal and image processing; see for instance section 4.3 for an illustration. Of course, this requires that satellites be adequately controlled, both in position and orientation (attitude). This problem and similar ones continue to motivate research in control. The team has been working for six years on control problems in orbital transfer with low-thrust engines, including four years under contract with Thales Alenia Space (formerly Alcatel Space) in Cannes.

Technically, the reason for using these (ionic) low thrust engines, rather than chemical engines that deliver a much higher thrust, is that they require much less "fuel"; this is decisive because the total mass is limited by the capacity of the launchers: less fuel means more payload, while fuel represents today an impressive part of the total mass.

From the control point of view, the low thrust makes the transfer problem delicate. In principle of course, the control law leading to the right orbit in minimum time exists, but it is quite heavy to obtain numerically and the computation is non-robust against many unmodelled phenomena. Considerable progress on the approximation of such a law by a feedback has been carried out using *ad hoc* Lyapunov functions.These approximate surprisingly well time-optimal trajectories. The easy implementation of such control laws makes them attractive as compared to genuine optimal control. Here the $n-1$ first integrals are an easy means to build control Lyapunov functions since any function of these first integrals can be made monotone decreasing by a suitable control. See [54] and the references therein.

5. Software

5.1. The Tralics Translator

Participant: José Grimm.

The development of the LaTeX to XML translator, named Tralics, was continued (see section 6.1). A new version was sent to the APP in February 2007, its IDDN number is IDDN.FR.001.510030.001.S.P.2002.000.31235. Binary versions are available for Linux, Windows and MacOS X. Its web page is http://www-sop.inria.fr/apics/tralics. It is now licensed under the CeCILL license version two, see http://www.cecill.info. Latest release is version 2.13.6, dated 24-11-2009.

5.2. RARL2

Participants: Jean-Paul Marmorat, Martine Olivi [corresponding participant].

RARL2 (Réalisation interne et Approximation Rationnelle L2) is a software for rational approximation (see section 3.1.4) http://www-sop.inria.fr/apics/RARL2/rarl2-eng.html.

This software takes as input a stable transfer function of a discrete time system represented by

- either its internal realization,
- or its first N Fourier coefficients,
- or discretized values on the circle.

It computes a local best approximant which is *stable, of prescribed McMillan degree*, in the L^2 norm.

It is akin to the arl2 function of Endymion (see section 5.5) from which it differs mainly in the way systems are represented: a polynomial representation is used in Endymion, while RARL2 uses realizations, this being very interesting in certain cases. It is implemented in Matlab. This software handles *multi-variable* systems (with several inputs and several outputs), and uses a parametrization that has the following advantages

- it incorporates the stability requirement in a built-in manner,
- it allows the use of differential tools,
- it is well-conditioned, and computationally cheap.

An iterative research strategy on the degree of the local minima, similar in principle to that of arl2, increases the chance of obtaining the absolute minimum (see section 6.4) by generating, in a structured manner, several initial conditions.

RARL2 performs the rational approximation step in our applications to filter identification (section 4.3) as well as sources or cracks recovery (section 4.2). It was released to the universities of Delft, Maastricht, Cork and Brussels. The parametrization embodied in RARL2 was recently used for a multi-objective control synthesis problem provided by ESTEC-ESA, The Netherlands (section 6.4). An extension of the software to the case of triple poles approximants is now available. It gives nice results in the source recovery problem (section 6.3.2). It is used by FindSources3D (see 5.7).

5.3. RGC

Participants: Fabien Seyfert [corresponding participant], Jean-Paul Marmorat.

The identification of filters modeled by an electrical circuit that was developed by the team (see section 4.3) led us to compute the electrical parameters of the underlying filter. This means finding a particular realization (A, B, C, D) of the model given by the rational approximation step. This 4-tuple must satisfy constraints that come from the geometry of the equivalent electrical network and translate into some of the coefficients in (A, B, C, D) being zero. Among the different geometries of coupling, there is one called "the arrow form" [51] which is of particular interest since it is unique for a given transfer function and also easily computed. The computation of this realization is the first step of RGC. Subsequently, if the target realization is not in arrow form, one can nevertheless show that it can be deduced from the arrow-form by a complex- orthogonal change of basis. In this case, RGC starts a local optimization procedure that reduces the distance between the arrow form and the target, using successive orthogonal transformations. This optimization problem on the group of orthogonal matrices is non-convex and has a lot of local and global minima. In fact, there is not always uniqueness of the realization of the filter in the given geometry. Moreover, it is often interesting to know all the solutions of the problem, because the designer cannot be sure, in many cases, which one is being handled, and also because the assumptions on the reciprocal influence of the resonant modes may not be equally well satisfied for all such solutions, hence some of them should be preferred for the design. Today, apart from the particular case where the arrow form is the desired form (this happens frequently up to degree 6) the RGC software gives no guarantee to obtain a single realization that satisfies the prescribed constraints. The software Dedale-HF (see 5.6), which is the successor of RGC, solves in a guaranteed manner this constraint realization problem.

5.4. PRESTO-HF

Participant: Fabien Seyfert.

PRESTO-HF: a toolbox dedicated to lowpass parameter identification for microwave filters http://www-sop.inria.fr/apics/personnel/Fabien.Seyfert/Presto_web_page/presto_pres.html. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single shot:

- determination of delay components, that are caused by the access devices (automatic reference plane adjustment),

- automatic determination of an analytic completion, bounded in modulus for each channel,
- rational approximation of fixed McMillan degree,
- determination of a constrained realization.

For the matrix-valued rational approximation step, Presto-HF relies either on hyperion (Unix or Linux only) or RARL2 (platform independent), two rational approximation engines developed within the team. Constrained realizations are computed by the RGC software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following strong assumption: far off the passband, one can reasonably expect a good approximation of the rational components of S_{11} and S_{22} by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox is currently used by Thales Alenia Space in Toulouse and a license agreement has been recently negotiated with Thales airborne systems. XLim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements are currently being considered with the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

5.5. Endymion

Participant: José Grimm.

The development of *Endymion*, http://www-sop.inria.fr/apics/endymion/index.html has been stabilized. It is a software licensed under the CeCILL license version two, see http://www.cecill.info. It has been registered under the number IDDN.FR.001.310002.000.S.P.2009.000.10000 at the the APP It was developed on Linux, but works as well on MacOS. The core of the system is formed by a library that handles numbers (short integers, arbitrary size rational numbers, floating point numbers, quadruple and octuple precision floating point numbers, arbitrary precision real numbers, complex numbers), polynomials, matrices, etc. Specific data structures for the rational approximation algorithm *arl2* and the bounded extremal problem *bep* are also available. One can mention for instance splines, Fourier series, Schur matrices, etc. These data structures are manipulated by dedicated algorithms (matrix inversion, roots of polynomials, a gradient-based algorithm for minimizing ψ, Newton method for finding a critical point of ψ, etc), and input-output functions that allow one to save data on disk, restore them, plot them, etc. The software is interactive: there is a symbolic interpreter based upon a Lisp interpreter. For instance the coefficient of z^2 in P can be obtained via Lisp syntax `(getcoef P 2)` or modified via the symbolic syntax `P[2]++`.

5.6. Dedale-HF

Participant: Fabien Seyfert.

Dedale-HF is a software meant to solve exhaustively the coupling matrix synthesis problem in reasonable time for the users of the filtering community. For a given coupling topology the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of a given filter characteristics (see section 6.8). Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements (see Figure 3).

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to a particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. problem of the user, i.e., the one corresponding to a user-specified filter characteristics. The reference files are computed off line using Groebner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such a continuation technique combined with an efficient implementation of the integrator produces a drastic reduction of the computational time, say, by a factor of 20.

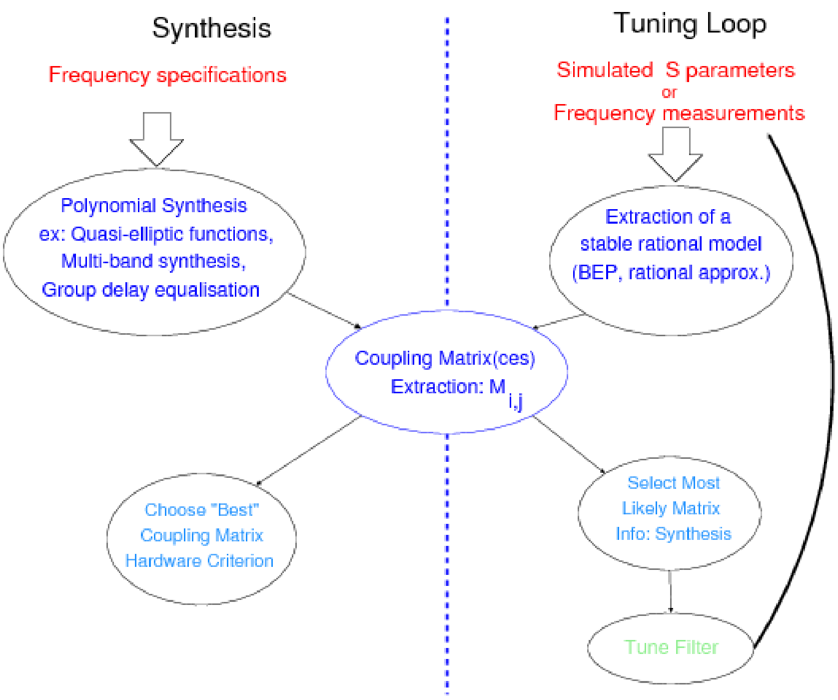

Figure 3. Overall view of the design and tuning process of a microwave filter

Access to the database and integrator code is done via the web on http://www-sop.inria.fr/apics/Dedale/WebPages. The software is free of charge for academic research purposes: a registration is however needed in order to access full functionality. Up to now 90 users have registered among the world (mainly: Europe, U.S.A, Canada and China) and 4000 reference files have been downloaded.

As mentioned in 6.8 an extension of this software that handles symmetrical networks is under construction.

5.7. FindSources3D

Participants: Rania Bassila, Maureen Clerc [EPI Odyssée], Juliette Leblond [corresponding participant], Jean-Paul Marmorat.

FindSources3D is a software dedicated to source recovery for the inverse EEG problem, in 3-layer spherical settings, from pointwise data (see http://www-sop.inria.fr/apics/FindSources3D/). Through the algorithm described in section 4.2, it makes use of RARL2 (section 5.2) for the rational approximation step in plane sections. The data transmission preliminary step (“cortical mapping”) is solved using boundary element methods through the software OpenMEEG (its CorticalMapping features) developed by the Odyssée Team (see http://www-sop.inria.fr/odyssee/software/OpenMEEG/). A first release of FindSources3D is now available, which will be demonstrated and distributed within the medical teams we are in contact with (see figures 4, 5).

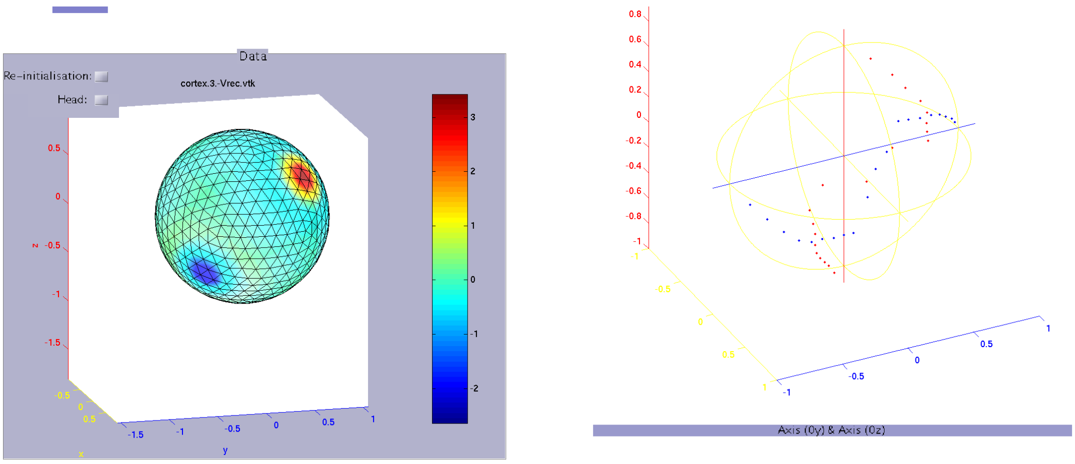

Figure 4. Potential on a sphere for 2 sources, and associated triple poles in 2 different section directions.

6. New Results

6.1. Tralics: a LaTeX to XML Translator

Participant: José Grimm.

The major use of Tralics remains the production of the RaWeb (Scientific Annex to the Annual Activity Report of Inria), [22]. The input is a LaTeX file, converted by Tralics into an XML file; this file is converted to another XML file, conforming to a new DTD, via *xslrproc*; this new file is converted to HTML or XSL-FO via the use of style sheets; the XSL-FO file formatted into Pdf by pdfTeX, thanks to the *xmltex* package that teaches TeX the subtleties of XML and utf-8 encoding, and two packages for the XSL-FO and MathML commands. This process is completely automatic: it suffices to call *make*; another possibility consists in sending a tar file with the sources to *iRAbot* (the Inria RaWeb Robot). Once authors have finished writing their contributions, all files are sent to a common place, and processed by a tool named RalyX. Since 2002, a lot of people have worked on these tools, for instance M.P. Durollet, J. Grimm, C. Rossi, B. Marmol, A.M. Vercoustre, A. Benveniste, I. Vatton, J.-P. Verjus, J.-C. Le Moal, L. Pierron.

Other applications of Tralics consist in putting scientific papers on the Web; for instance Cedram (http://www.cedram.org (Centre de diffusion de revues académiques mathématiques), that publishes the *Journal de théorie des nombres de Bordeaux*, uses Tralics for the abstracts and plans to translate full papers; on the other hand the Connexions project of the Rice University is a environment for collaboratively developing, freely sharing, and rapidly publishing scholarly content on the Web, it uses it as LaTeX importer.

The main philosophy of Tralics is to have the same parser as TeX, but the same semantics as LaTeX. This means that commands defined in the eTeX binary, including `\chardef`, `\catcode`, `\csname`, `detokenize` etc., behave in Tralics. On the other hand, the semantics of most LaTeX commands are preserved; some command were re-implemented this year in order to take into account certain details (including local and global float placement instructions).

A minor versions have been released this year, namely 2.15.6 in November. The documentation consists in some technical reports [72] [71], [70], [69], they are regularly updated, especially the HTML version (produced by Tralics). Some new packages were added to the system (graphicx, xkeyval, color), and for efficiency reasons, part of the code is implemented in the C++ kernel. The referencing system was completely rewritten, so that for instance the XML document contains the same equation numbers as the PostScript version (in the RaWeb case, equation numbers are computed by the XML-to-HTML style sheet). The RaWeb preprocessor was removed: all commands specific to the Activity Report are now defined in package files, the kernel containing some primitives that can check the validity of some arguments versus a keyword list defined in the configuration file. One can add or removes entry types and fields in the bibliography.

6.2. Proving Bourbaki with Coq

Participant: José Grimm.

This is a new research theme. Our objective is to use the proof assistant Coq in order to formally prove a great number of theorems in Algebra. We started with the first book (Theory of sets, [58]) of the series "Elements of Mathematics". The first chapter describes Formal Mathematics, and we have shown that it is possible to interpret it in the Coq language. Note that Bourbaki expresses $\forall$ in terms of $\exists$, which is not possible in Coq, and states that if $\forall x, R$ is false, then $\exists x, \neg R$. This is a non-constructive statement. Moreover, this implies a general version of the axiom of choice (if for all x there is an y satisfying $P(x,y)$, then there is a mapping f such that $P(x, f(x))$ holds for all x). We use some ideas of Carlos Simpson (University of Nice), and decide that a set is a type, and that $X \in Y$ is true if and only if there is a representative of X of type Y (this is non-constructive, since "representative" is only defined through axioms).

The second chapter of Bourbaki covers the theory of sets proper. It defines ordered pairs, correspondences, union, intersection and product of a family of sets, as well as equivalence relations. Its implementation in Coq corresponds to 300 definitions and 1300 lemmas or theorems. It is described in [30]. The third chapter of Bourbaki covers the theory of ordered sets, well-ordered sets, equipotent sets, cardinals, natural integers, and infinite sets; its implementation in Coq is described in in [31]. All results of the book been proved in Coq (230 definitions and 1200 lemmas), except inverse limits, direct limits and structures, which will be considered later; moreover there are more than one hundred exercises, most of them are non-trivial, and solving them will take some time.

Finite cardinals satisfy an induction principle (this is a special case of transfinite induction); This is the same induction principle as that of natural integers in Coq, so that these two notions are isomorphic. This means that every theorem of the Coq library about natural integers translates directly into a theorem about finite cardinals. This allows us to prove theorems like: The number of increasing (resp. strictly increasing) mappings of a set with p elements into a set with n elements is the number of subsets of p elements of a set with $p+n$ (resp. n) elements.

We use the following 4 axioms. Let's denote by E the type of sets. We assume existence of a function R, of type $\forall x : E, x \to E$, such that, if $x : E$, then for all $a : x$ and $b : x$, $Ra = Rb$ implies $a = b$. The relation $a \in b$ is defined by $\exists c : b, Rc = a$. The first axioms says that if a and b are sets, then $\forall u, u \in a \Leftrightarrow u \in b$ implies $a = b$. The empty set $\varnothing$ is inductively defined as a type without constructor. We assume existence of a function C, of type $\forall t : E, (t \to P) \to Nt \to t$ (the first argument is a property p, and the second is a proof q that the type t is non-empty). The axiom of choice says that, if there exists x such that $p(x)$, then $C(p, q)$ satisfies p (This corresponds to Bourbaki's axioms scheme S5 that says that $\tau_x(p)$ satisfies p in such a case). We assume existence of a function I of type $\forall x : E, (x \to E) \to E$. This means that, if f is a function such that $f(x)$ is a set for all x, then $I(f)$ is a set. The third axiom says $y \in I(f)$ if and only if there exists $a : x$ such that $f(a) = y$. It implies existence of union of sets, but this Scheme of Substitution is slightly more general than Bourbaki's Scheme of Selection and Union, since it implies in particular the axioms of the set of two elements. The final axiom says that for any property P, if P is not false then it is true.

6.3. Inverse Problems for 2-D and 3-D elliptic operators

Participants: Laurent Baratchart, Aline Bonami [Univ. Orléans], Maureen Clerc [EPI Odyssée], Yannick Fischer, Sandrine Grellier [Univ. Orléans], Mohamed Jaoua, Juliette Leblond, Jean-Paul Marmorat, Ana-Maria Nicu, Théo Papadopoulo [EPI Odyssée], Jonathan R. Partington, Stéphane Rigat [Univ. Aix-Marseille I], Emmanuel Russ [Univ. Aix-Marseille III], Edward Saff, Meriem Zghal.

6.3.1. 3-D boundary value problems for Laplace equation

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to treat incomplete experimental data is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG since the latter involves propagating the initial conditions from the boundary to the center of the domain where the singularities (i.e., the sources) are sought. Here, the domain is typically made of several homogeneous layers of different conductivities.

Such problems offer an opportunity to state and solve extremal problems for harmonic fields for which an analog of the Toeplitz operator approach to bounded extremal problems [43] has been obtained. Still, a best approximation on the subset of a general vector field by a harmonic gradient under a L^2 norm constraint on the complementary subset can be computed by an inverse spectral equation for some Toeplitz operator. Constructive and numerical aspects of the procedure (harmonic 3-D projection, Kelvin and Riesz transformation, spherical harmonics) and encouraging results have been obtained on numerically simulated data[13]. Issues of robust interpolation on the sphere from incomplete pointwise data are also under study (splines, spherical harmonics, spherical wavelets, spherical Laplace operator, ...), in order to improve numerical accuracy of our reconstruction schemes.

The analogous problem in L^p, $p \neq 2$, is considerably more difficult. A collaborative work is going on, in the framework of the ANR project AHPI, aiming mainly at the case $p = \infty$. It was obtained that the BMO distance between a bounded vector field on the sphere and a bounded harmonic gradient is within a constant of the norm of a Hankel-like operator, acting on L^2 divergence-free vector fields with values in L^2 gradients. Estimating the constant requires solving further extremal problems in L^1 on the best approximation of a gradient by a divergence free vector field. This issue is currently being studied in L^p where it leads to analyze particular solutions to the the p-Laplacian on the sphere.

6.3.2. Sources recovery in 3-D domains, application to MEEG inverse problems

The problem of sources recovery can be handled in 3-D balls by using best rational approximation on 2-D cross sections (disks) from traces of the boundary data on the corresponding circles (see section 4.2).

The team started to consider more realistic geometries for the 3-D domain under consideration. A possibility is to parametrize it in such a way that its planar cross-sections are quadrature domains or R-domains. In this framework, best rational approximation can still be performed in order to recover the singularities of solutions to Laplace equations, but complexity issues are delicate. The preliminary case of an ellipsoid, which requires the preliminary computation of an explicit basis of ellipsoidal harmonics, has been studied in [78] and is one of the topics of the PhD thesis of M. Zghal.

In 3-D, epileptic regions in the cortex are often represented by pointwise sources that have to be localized from measurements on the scalp of a potential satisfying a Laplace equation (EEG, electroencephalography). A breakthrough was made which makes it possible now to proceed via best rational approximation on a sequence of 2-D disks along the inner sphere [5].

A dedicated numerical software “FindSources3D” (see section 5.7) has been developed, in collaboration with the team Odyssée.

Further, it appears that in the rational approximation step of these schemes, *multiple* poles possess a nice behaviour with respect to the branched singularities (see figures 4, 5). This is due to the very basic physical assumptions on the model (for EEG data, one should consider *triple* poles). Though numerically observed, there is no mathematical justification why these multiple poles have such strong accumulation properties, which remains an intriguing observation. This is the topic of [63].

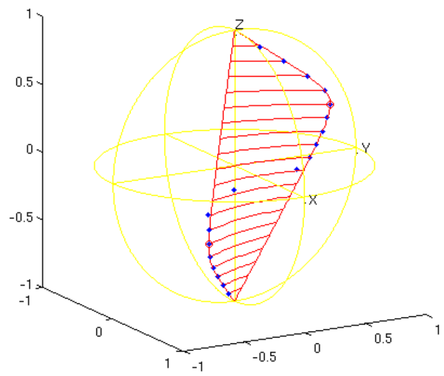

Figure 5. Localization of 2 sources (red circles) by a single triple pole in plane sections (blue points).

Also, magnetic data from MEG (magneto-encephalography) will soon become available, which should enhance sources recovery.

This approach should also become interesting for geophysical issues, concerning the discretization of the gravitational potential by means of pointwise masses. This is another recent topic of A.-M. Nicu’s PhD thesis and of our present collaboration with LAMSIN-ENIT, hence the reason why she also made a long working stay there (Univ. El Manar, Tunis, Tunisia, April-June).

Magnetic sources localization by analytic and rational approximation on plane sections is currently analyzed from experimental SQUID data, from Vanderbilt University Physics Dept. We have started analyzing the kernel of the magnetization operator, which is the Riesz potential of the divergence. The natural assumptions to handle magnetizations that are piecewise constant (allowing for characteristic elements embedded in the slab to be analyzed) has led us to study Riesz transforms and Hodge decompositions of functions of bounded variation, namely functions whose distributional derivatives are signed measures. The kernel, it has been found, can be described in terms of measures whose balayage on the boundary of the object vanishes. The constructive characterization of those is a rather difficult problem, but the restriction to more specific classes, like piecewise constant or unidirectional magnetizations that are of common use in the field, seems better suited to the purpose of algorithmically recovering m, up to a divergence-free term. The role of the extrapolation techniques initiated by the project team, using bounded extremal problems, should be important in this connection. This research sheds light on the connections between inverse current problems (aiming at the inversion of the Biot-Savart operator) and inverse magnetization problems (aiming at the inversion of the potential of a divergence).

6.3.3. 2-D boundary value problems for conductivity equations, application to plasma control

In collaboration with the CMI-LATP (University Marseille I) and in the framework of the ANR AHPI, the team considers 2-D diffusion processes with variable conductivity. In particular its complexified version, the so-called *real Beltrami equation*, was investigated. In the case of a smooth domain, and for a smooth conductivity, we analyzed the Dirichlet problem for solutions in Sobolev and then in Hardy classes [46].

Their traces merely lie in L^p $(1 < p < \infty)$ of the boundary, a space which is suitable for identification from pointwise measurements. Again these traces turn out to be dense on strict subsets of the boundary. This allows us to state Cauchy problems as bounded extremal issues in L^p classes of generalized analytic functions, in a reminiscent manner of what was done for analytic functions as discussed in section 3.1.1. Recently, dual formulations were obtained and some multiplicative (fibered) structure for the solution was obtained based on old work by Bers and Nirenberg on pseudo-analytic functions. An article is being written on these topics.

The case of a conductivity that is merely in L^∞, which is important for inverse conductivity problems, is under examination (PhD thesis of S. Chaabi). There, it is still unknown whether solutions exist for all p.

The application that initially motivated this work comes from free boundary problems in plasma confinement (in tokamaks) for thermonuclear fusion. This work was started in collaboration with the Laboratoire J. Dieudonné (University of Nice) and is now the topic of a collaboration with two teams of physicists from the CEA-IRFM (Cadarache).

In the transversal section of a tokamak (which is a disk if the vessel is idealized into a torus), the so-called poloidal flux is subject to some conductivity outside the plasma volume for some simple explicit smooth conductivity function, while the boundary of the plasma (in the Tore Supra Tokamak) is a level line of this flux [53]. Related magnetic measurements are available on the chamber, which furnish incomplete boundary data from which one wants to recover the inner (plasma) boundary. This free boundary problem (of Bernoulli type) can be handled through the solutions of a family of bounded extremal problems in generalized Hardy classes of solutions to real Beltrami equations, in the annular framework. Such approximation problems also allow us to approach a somewhat dual extrapolation issue, raised by colleagues from the CEA for the purpose of numerical simulation. It consists in recovering magnetic quantities on the outer boundary (the chamber) from an initial guess of what the inner boundary (plasma) is.

In the particular case at hand, it is possible to explicitly compute a basis of solutions (Bessel functions) that help the computations, see [64], [65]. However, many other choices are possible, which are under study. This is the topic of the PhD thesis of Y. Fischer.

In the most recent tokamaks, like Jet or ITER, an interesting feature of the level curves of the poloidal flux is the occurrence of a cusp (a saddle point of the poloidal flux, called an X point), and it is desirable to shape the plasma according to a level line passing through this X point for physical reasons relating to the efficiency of the energy transfer. This will be the topic of future studies.

6.4. Interpolation and parametrizations of transfer functions

Participants: Martine Olivi, Bernard Hanzon, Ralf Peeters [Univ. Maastricht].

Our work of the past ten years on balanced realizations of lossless systems, Schur parameters, canonical forms and applications were the topic of a semi-plenary session, given by R. Peeters at the conference Sysid09 [28]. Our last results on subdiagonal pivot structure for input-normal pairs and associated canonical forms were also presented at this conference [27]. These forms generalise to the MIMO case the well-known Hessenberg form in discrete-time and Schwarz-Ober form in continuous-time. Their use for model reduction purposes seems to be relevant and is currently under investigation.
For the class of lossless discrete-time systems, subdiagonal forms can be computed from a specific backward recursive Schur algorithm. In continuous-time, the relevant recursive algorithm in connection with these forms involves a boundary interpolation problem. We got a parametrization of the (subdiagonal) Ober-Schwarz canonical form (SISO) in terms of boundary interpolation values (angular derivatives). These results were presented at the ERNSI meeting (poster). Boundary interpolation of matrix lossless functions and its applications to the parametrization of filter banks leading to orthogonal wavelets is under study (see section 6.5).

6.5. Approximation and parametrization of wavelets

Participants: Martine Olivi, Bernard Hanzon, Ralf Peeters [Univ. Maastricht], Jean-Paul Marmorat, Vikentiy Mikheev, Jérémie Giraud-Telme.

An application of our rational approximation methods to orthogonal wavelets has been investigated. The problem is to implement wavelets in analog circuits in view of medical signal processing applications. A dedicated method has been developed [75] based on an L^2-approximation of the wavelet by the impulse response of a stable causal low order filter. However, this method fails to find an accurate and sufficiently small order approximation in some difficult cases (Daubechies db7 and db3). The idea was to use the software RARL2 to perform a model reduction on an accurate high order (100-200) approximation. However, an admissibility condition for wavelets is that the integral of a wavelet equals zero, which means that it has one vanishing moment. The low order approximation is still required to have an integral zero, otherwise undesired bias will show up when the wavelet is used in an application. We thus had to adapt a version of the RARL2 software to address this constraint. Since we are dealing with a quadratic optimization problem under a linear constraint, this can be solved analytically. We could thus reformulate the problem of L^2-approximation subject to this constraint into an optimization problem over the class of lossless systems. This could be handled by the software with only minor changes and we were able to perform an accurate approximation of order 8 for db7 (Figure 6). This way to address a linear or a convex constraint could be used for other purposes, for example to impose passivity.

In close connection, we investigate the possibility to parametrize wavelets with (more) vanishing moments using interpolation theory at the boundary. A very useful and concise description of the class of filter banks leading to orthogonal wavelets is by means of its associated *lossless* polyphase filter [89]. A vanishing moment condition can be expressed as a boundary interpolation condition for the lossless polyphase filter. We thus exploited our previous works on the parametrization of lossless matrix functions with interpolation conditions. We got explicit parametrizations of 2×2 polyphase matrices of arbitrary order n with (up to) 3 vanishing moments built in, in terms of angular derivative (positive) parameters. However, the conditions were cleverly handled in an unusual recursive fashion that we still do not completely understand. These results have been presented at the ANR-AHPI meeting.

6.6. Schur rational approximation

Participants: Laurent Baratchart, Stanislas Kupin [Univ. Bordeaux 1].

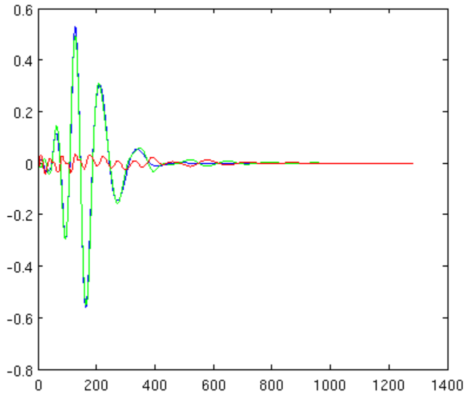

Figure 6. Daubechies wavelet db7 and its rational approximation (degree 8)

Passive devices play an important role in many application areas: telecommunication, chemical process control, economy, biomedical processes. Network simulation software packages (as ADS or SPICE) require passive models for their components. However, identifying a passive model from band limited frequency data is still an open and challenging problem. Schur rational approximation is a new way to approach this problem and has been the subject of [80]. In this work, a parametrization of all strictly Schur rational functions of degree n is constructed from a multipoint Schur algorithm, the parameters being both the interpolation values and interpolation points. Examples are computed by an L^2 norm optimization process and the results are validated by comparison with the unconstrained L^2 rational approximation. Over the last two years, the results of [76] on the hyperbolic convergence of the classical Schur algorithm were generalized to the case of the multipoint Schur algorithm, which is a more delicate situation because we allow for the interpolation points to approach the boundary circle. Orthogonal rational functions and a recent generalization of Geronimus theorem were used [79], combined with Hilbertian techniques from reproducing kernel spaces and new quantitative versions of the Beurling theorem, to obtain an analog of the Szegö theorem where the interpolation points tend to the boundary, provided the approximated function is continuous and less than 1 in a neighborhood of the accumulation set of the interpolation points. This generalized the results in [60] and was of novel type since the n-th orthogonal rational function inverts the Szegö function modulo the Poisson kernel. This yields a rather unexpected theorem on the behaviour of certain orthogonal polynomials with varying weight. This year we obtained new bounds for orthogonal rational functions, based on $\overline{\partial}$-estimates for the squared modulus of the Szegö function, that yield new results even in the case of polynomials since they yield information even in cases where the measure has vanishing density provided it is Sobolev-$W^{1-1/p,p}$-smooth on the circle for some $p > 1$. An article is being written that summarizes these results.

The research has been pursued on a major open issue, namely how to choose the interpolation points with respect to the approximated Schur function so as to yield the best convergence possible. We also started analyzing the consequences of our work for the representation of certain non-stationary stochastic processes. In this connection the case of vanishing densities and singular components is under investigation.

6.7. Rational and meromorphic approximation

Participants: Laurent Baratchart, Herbert Stahl [TFH Berlin], Maxim Yattselev.

The results of [4] and [6] were extensively used over the last years to prove the convergence in capacity of L^p-best meromorphic approximants on the circle (*i.e.* solutions to problem (P_N) of section 3.1.2) when $p \geq 2$, for those functions f that can be written as Cauchy transforms of complex measures supported on a hyperbolic geodesic arc $\mathcal{G}$ [18], [17], [25], [26]. A rational function can also be added to f without modifying the results, which is useful for applications to inverse sources problems. Some mild conditions (bounded variation of the argument and power-thickness of the total variation) were required on the measure. Here, we recall that convergence in capacity means that the (logarithmic) capacity of the set where the error is greater than ε goes to 0 for each fixed $\varepsilon > 0$. This convergence can be quantified, namely it is geometric with pointwise rate $\exp\{-1/C - G\}$ where C is the capacity of the condensor $(T, \mathcal{G})$ and G the Green potential of the equilibrium measure. The results can be adapted to somewhat general interpolation schemes [18], [17]. From this work it follows that the counting measures of the poles of the approximants converge, in the weak-* sense, to the Green equilibrium distribution on $\mathcal{H}$. In particular the poles cluster to the endpoints of the arc, which is of fundamental use in the team's approach to source detection (see section 6.3.2).

This year the weak-* convergence of the poles of best H^2 rational approximants to Cauchy integrals over general symmetric contours for the Green Potential, and not merely geodesic arcs, were established using the reflected symmetry of the poles and the interpolation nodes of such approximants across the circle, and analyzing the location of continua of minima weighted through a discretization of the weight and a limiting process. This warrants the use of rational approximation to functions with arbitrarily many branchpoints in source detection. A paper is currently being written on this topic.

The technique we just described only yields convergence in capacity and n-th root asymptotics. To obtain strong asymptotics, additional assumptions must be made on the approximated function. Last year, we proved strong asymptotics of multipoint Padé interpolants, in appropriate interpolation nodes, to Cauchy integrals over arbitrary analytic arcs, when the density of the measure with respect to a positive power of the equilibrium distribution on the arc is Dini-smooth. In addition, the density may in fact vanish in finitely many points like a small fractional power of the distance to these point [16]. Moreover, the polar singularities of the function, if any, are asymptotically reproduced by the approximants with their multiplicities. This is important for inverse problem of mixed type, like those mentioned in section 6.3.2, where monopolar and dipolar sources are handled simultaneously.

This year we proved under appropriate smoothness assumptions that the result still holds without restrictions on the density, that is, the power of the equilibrium distribution with respect to which we compute its derivative needs no longer be positive (in the language of orthogonal polynomials, this means we can handle arbitrary Jacobi weights). The lower the power the smoother the density should be. Typically, if the power is zero (so that we only consider the density with respect to arclength on the arc), a fraction of a derivative is sufficient (i.e. the density should belong to a $W^{1-1/p,p}$-class on the arc for some $p > 2$. When the power gets negative, $C^{k,\alpha}$-classes of Hölder-smoothness for the k-th derivative are required, where k is related to the integer part of the Jacobi exponents and α to their fractional part. This time however, the density is not allowed to vanish.

This result more or less settles the issue of convergence of multipoint Padé approximants to Cauchy integrals over arcs, because it asserts that uniform convergence holds, under mild assumptions on the density, when the interpolation points are chosen in some appropriate manner (symmetric with respect to a weighted equilibrium potential adapted to the contour), and because we also proved that whenever a convergent interpolation scheme exists to a Cauchy integral with smooth density on an arc, with interpolation keeping off the arc, then the arc must be analytic.

The technique of proof uses a $\overline{\partial}$-generalization, over varying contours, of the Riemann-Hilbert approach to the asymptotics of orthogonal polynomials as adapted to the segment in [77] This provides us with precise (Plancherel-Rotach type) asymptotics for the non-Hermitian orthogonal polynomials which is the denominator of the approximant. Asymptotics for the latter are even obtained on the arc where the measure of orthogonality is supported. In the case of non-positive Jacobi powers, $\overline{\partial}$-estimates and Muckenhoupt weights are also needed. A paper has been written and submitted to report on this research [50].

We have pursued this line of research for functions defined as Cauchy integrals over union of (possibly intersecting) arcs, and obtained convergence results over regular threefolds. The current goal is to understand which systems of arcs can be construed as critical configurations for weighted potential problems, and whether the above analysis can be extended beyond arcs to 2-D singular sets.

In another collection, the results of [9] have been carried over for analytic approximation to the matrix case in [14]. The surprising fact was that not every matrix valued function generates a vectorial Hankel operator meeting the AAK theorem when $p < \infty$. This led us to the generalization of the latter based on Hankel operators with matrix argument.

6.8. Circuit realisations of filter responses: determination of canonical forms and exhaustive computations of constrained realisations

Participants: Smain Amari, Jean Charles Faugère [EPI SALSA, INRIA Rocquencourt], Stéphane Bila [XLim, Limoges], Fabien Seyfert.

Groomed by industrial users like Thales Alenia-Space, we made some progress in the analysis of the realizations of 2x2 lossless scattering systems whose scattering response $(S_{i,j})$ satisfies the so-called *auto-reciprocal* condition $S_{1,1} = S_{2,2}$. It was shown that auto-reciprocal inner responses admit a canonical circuit realisation of the form of Fig. 7. The length difference $(m - l)$ of the two antennas of Fig. 7 is equal to the Cauchy index on the imaginary axes of the filter function to be realised. Surprisingly enough this form appears to be central in the new modal framework S.Amari is currently developing on dual mode filters ([34]). It was shown that the classical folded form can be advantageously replaced by the latter yielding a design procedure with nearly no tuning required (all the physical dimensions of the filter can be computed exactly from the circuit parameters): a paper has been published on this topic [19]. In future work, we will focus on the practical implementation of this analysis within the software Dedale-HF 5.6.

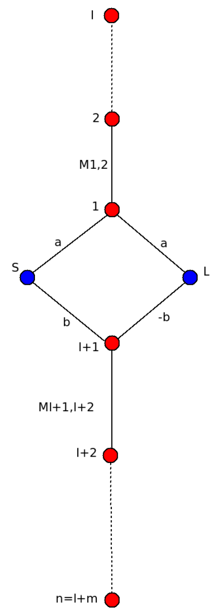

Figure 7. Canonical realisation for auto-reciprocal responses - upper antenna contains l resonators and lower one m

We also made some progress on the problem of circuit realisations with mixed type (inductive or capacitive) coupling elements. An algebraic formulation of the synthesis problem of circuits with mixed type elements has been obtained which relies on a set of two matricial equations. As opposed to the classical low pass case with frequency independent couplings the unknown is no longer a similarity transform but a general non-singular

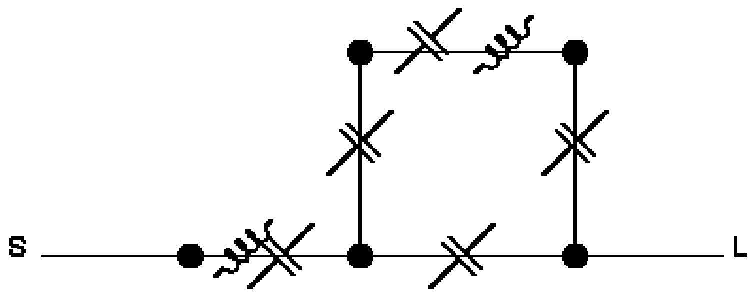

Figure 8. 5th order circuit with two resonant couplings that allows the synthesis of asymmetrical responses with up two 3 transmission zeros

matrix acting on two coupling matrices: the capacitive and the inductive one. First results were obtained in this field allowing the exact synthesis of filters with resonating coupling elements, see 8. Applications of this technique to synthesise extremely compact filters, with sharp responses, is being studied in collaboration with the Royal Military College (Canada). Note however, that the filter orders for which this synthesis is computationally tractable, for the moment, is modest (no more than 5 or 6). Further developments, focusing in particular on an efficient algebraic formulation of the problem, are needed in order to convince engineers of its relevancy when compared to generic local optimisation techniques. The state of our work was presented at Rome (European Microwave Conference) and Toulouse (CNES-ESA filter workshop) while a publication is currently being reviewed.

6.9. Synthesis of compact multiplexers and the polynomial structure of $n \times n$ inner matrices

Participants: Martine Olivi, Fabien Seyfert, Stéphane Bila [Xlim], Hussein Ezzedin [Xlim], Amine Rouini.

The objective of our work in the ANR Filipix is the derivation of efficient algorithms for the synthesis of microwave multiplexers. In our opinion, an efficient frequency design process calls for the understanding of the structure of $n \times n$ loss-less reciprocal rational functions for $n > 2$. Whereas the case $n = 2$ is completely understood and a keystone of filter synthesis, very little seems to be known about the polynomial structure of such matrices when they involve more than 2 ports.

We therefore started with the analysis of the 3×3 case typically of practical use in the manufacturing of diplexers. Based on recent results obtained on minimal degree reciprocal Darlington synthesis [42] we derived a polynomial model for 3×3 reciprocal inner rational matrices with given MacMillan degree. The latter writes as follow:

$$S = \frac{1}{q} \begin{pmatrix} p & p_2 & p_1 \\ \cdots & (-p_2^2 p^* + p_1^2 q)/rr^* & -(p_1 p_2 p^* + p_1^* p_2^* q)/rr^* \\ \cdots & \cdots & ((p_2^*)^2 q - (p_1^*)^2 p^*)/rr^* \end{pmatrix}$$

where we define

$$rr^* = p_1 p_1^* + p_2 p_2^*$$

and the following divisibility conditions must hold

$$\left\{\begin{array}{l}(-p_2^2p^* + p_1^2q)/rr^* \\ (p_1p_2p^* + p_1^*p_2^*q)/rr^* \\ ((p_2^*)^2q - (p_1^*)^2p^*)/rr^*\end{array}\right.$$

If the polynomials p and r of degree $k \leq n-1$ are given together with a condition at infinity on S, then one can show that there exist 2^k (3×3) inner extensions (of MacMillan degree n) of $S_{1,1}$. Their computation involves mainly linear algebra. During his internship, Amine Rouini designed and programmed a procedure which makes effective this extension process. From a practical synthesis point of view the extension process that starts with the polynomials p_1 and p_2 is more relevant but remains technically problematic: some issues concerning the stability of the derived polynomial q are still unsolved for the moment. The study of particular forms of the polynomial model in connexion with some special circuit topologies used for the implementation of the diplexer are also currently under investigation.

6.10. The Zolotarev problem and multi-band filter design

Participants: Vincent Lunot [Thales Alenia Space, Cannes], Fabien Seyfert.

The theoretical developments took place over the last two years, while deepenings of the numerical aspects were carried out in 2007. This study was conducted under contract with the CNES and Thales-Alenia-Space (Toulouse), and was part of V. Lunot's doctoral work [80]. The problem goes as follows. On introducing the ratio of the transmission and reflexion entries of a scattering matrix, the design of a multi-band filter response (see section 4.3) reduces to the following optimization problem of Zolotarev type [86]:

$$\text{letting: } E_{n,m}(K,K') = \{p \in P_m(K), q \in P_n(K') \text{ such that } \forall x \in I\,,\ \left|\frac{p(x)}{q(x)}\right| \leq 1\},$$

$$\text{solve: } \max_{(p,q)\in E_{m,n}(K,K')} \min_{x\in J} \left|\frac{p(x)}{q(x)}\right| \tag{1}$$

where $I = \bigcup I_i$ (resp. $J = \bigcup J_i$) is a finite union of compact intervals I_i of the real line corresponding to the pass-bands (resp. stop-bands), and $P_m(K)$ stands for the set of polynomials of degree less than m with coefficients in the field K. Depending on the physical symmetry of the filter, it is interesting to solve problem (1) either for $K = K' = \mathbf{R}$ ("real" problem) or $K = \mathbf{C}, K' = \mathbf{R}$ ("mixed" problem), or else $K = K' = \mathbf{C}$ ("complex" problem). The "real" Zolotarev problem can be decomposed into a sequence of concave maximization problems, whose solution we were able to characterize in terms of an alternation property. Based on this, a Remez-like algorithm has been derived in the polynomial case (i.e., when the denominator q of the scattering matrix is fixed), which allows for the computation of a dual-band response (see Figure 10) according to the frequency specifications (see Figure 9 for an example from the spacecraft SPOT5 (CNES)). We have designed an algorithm in the rational case which, unlike linear programming, avoids sampling over all frequencies. This raises the issue of the "generic normality" (*i.e.* the maximum degree) of the approximant with respect to the geometry of the intervals. This question remains open. The design of efficient procedures to tackle the "mixed" and "complex" cases remains a challenge. The software *easyFF* was registered at the APP under the number IDDN.FR.001.150004.000.S.P.2009.000.3150. A license agreement is beeing worked for a permanent distribution of this software to our academic partners: Xlim and the Royal Military College of Canada. Applications of the Remez algorithm to filter synthesis are described in [52], [82]. An article on the general approach based on linear programming has been published [81].

6.11. Synthesis and Tuning of broad band microwave filters

Participants: Smain Amari, Magued Bekheit [RMC, Kingston, Canada], Fabien Seyfert.

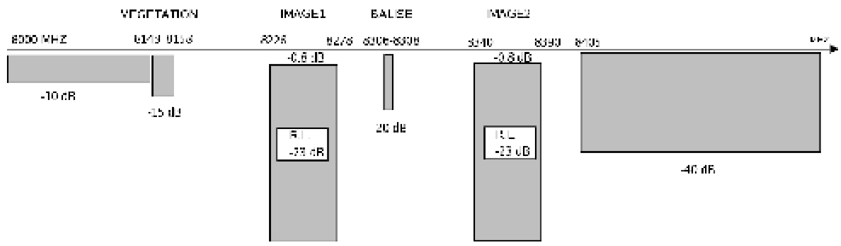

Figure 9. SPOT5 specifications

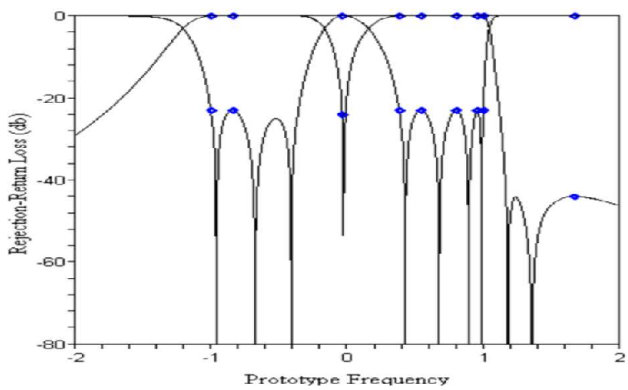

Figure 10. 7^{th} order dual-band response and its critical points

Some important results have been obtained in order to handle tuning and synthesis of broad band filters. One of the major problems when dealing with wide band filters is the break down of the classical low pass model which relies on a narrow band assumption. We showed however that there exists a unifying “low pass formalism” which is valid in the narrow band as well as in the wide band situations. The latter relies on the following remark. Let S be any inner, real, symmetric ($S^t = S$), rational matrix, which is identity at infinity and has MacMillan degree n. Then the rational matrix S_r defined by:

$$S_r(-is^2) = \frac{I\left(\frac{is+1}{is-1}\right) + S(s)}{I + S(s)\left(\frac{is+1}{is-1}\right)} \tag{2}$$

is again an inner, complex, symmetric matrix, which is identity at infinity, and has MacMillan degree n. It can be shown that S is entirely characterised by the knowledge of its reduced “complex” version S_r. Measurements of S on two conjugate frequency bands are mapped to measurements of S_r on a single band, which up to the use of a linear frequency transformation can be cast to the normalized band $[-1, 1]$. Usual techniques used to recover rational models from low pass responses measured on a single frequency interval can therefore be used to recover high pass responses via the use of the generalized reduced response S_r. Implementation attempts of the latter in the Presto-Hf software were started and encouraging results where obtained for the tuning of an ultra-wide band filter realized with suspended strip lines. Figure 11 shows data and their rational approximation of this 10^{th} order filter (reduced order 5) with a bandwidth ratio of approximately 10% (in collaboration with the university of Ulm, Germany).

Concerning the synthesis of the response of such filters we had already shown that the latter amounts to a Zolotarev problem with a non-polynomial weight (with a square root singularity). For fixed transmission zeros we were however able to derive explicit formulas for the optimal (in the Chebychev sense) filter function F_n:

$$F_n(w) = \cosh\left(\cosh^{-1}\left(\frac{\mathcal{T}'(w)}{w}\right) + \sum_{k=1}^{n-1}\cosh^{-1}\left(f_k(w)\right)\right),\ f_k(w) = \frac{\mathcal{T}(w) - 1/\mathcal{T}(z_k)}{1 - \mathcal{T}(w)/\mathcal{T}(z_k)} \tag{3}$$

where $\mathcal{T}$ is a suitable parabolic frequency transformation and the $z_k's$ are prescribed transmission zeros. Recurrence formulas for the practical computation of F_N have been derived and implemented as part of the Dedale-HF software package. As for the realization of such responses, first results were obtained with resonant coupling elements 6.8

In collaboration with the RMC and possibly with XLim and ST-Microelectronics (Tours) our goal is now to test the validity of our unified approach on real examples. Data collection campaigns obtained during tuning phases are scheduled. Joint publications about the topic are also in progress.

6.12. Frequency approximation and OMUX design

Participants: Laurent Baratchart, Jean-Paul Marmorat, Fabien Seyfert, Damien Pacaud [Thales Alenia Space, Toulouse].

An OMUX (Output MUltipleXor) can be modeled in the frequency domain through scattering matrices of filters, like those described in section 4.3, connected in parallel onto a common guide. The problem of designing an OMUX with specified performance in a given frequency range naturally translates into a set of constraints on the values of the scattering matrices and of the phase shift introduced by the guide in the considered bandwidth.

An OMUX simulator on a Matlab platform was designed last year and checked against a number of designs proposed by Thales Alenia Space (a.k.a. TAS). Under the terms of a contract with TAS (2007), it has been used to design a dedicated software to optimize OMUXes whose second release to TAS has taken place this year.

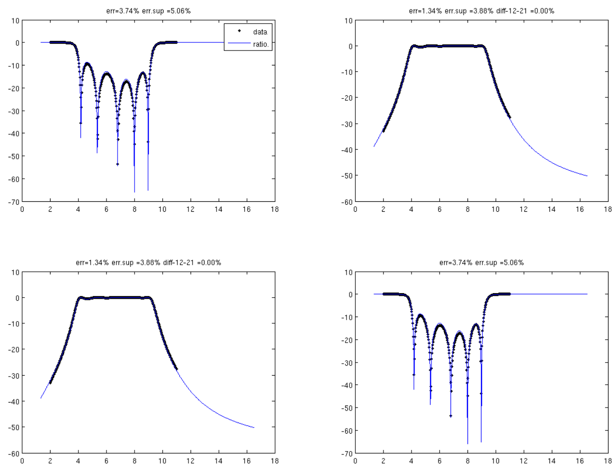

Figure 11. Bode diagramm of the response of an ultra-wide band filter and its rational approximation of degree 10 - Frequencies are in GHz

The software proceeds by adding channels recursively, applying to the new channel the above short-circuit and reflection-in-the bandwidth rules. This yields an initial guess for the global "optimizer" which seems to regularly outperform those currently used by TAS. More extensive tests are being conducted. A natural sequel should consist of the study of the so-called "manifold-peaks" that may impede a design based on ideal assumptions of losslesness.

A new problem was brought in by Damien Pacaud (TAS) concerning a de-embedding problem one encounters while tuning T-junction diplexers. Let S be the measured scattering matrix of a diplexer composed of a junction with response T and two filtering devices with response A and B as plotted on figure 12. The de-embedding question is the following: given S and T, is it possible to derive A and B ? Although the question may appear classical very little seems to be known about it in the literature and among measurements specialists.

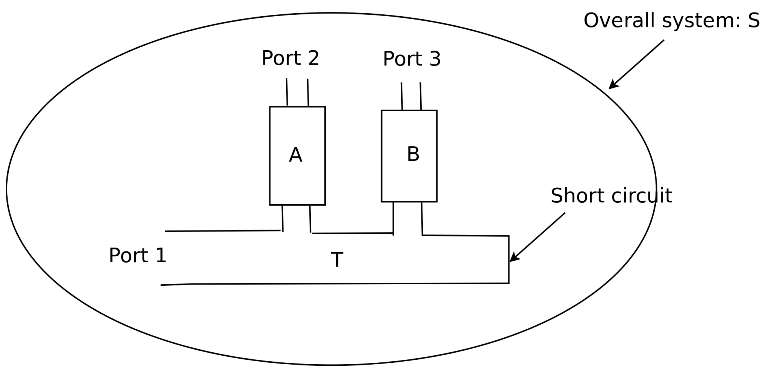

Figure 12. Diplexer made of a junction T and two filtering devices A and B

Using algebraic elimination techniques we derived the following interesting relation:

$$S_{1,1} - S_{1,2} * S_{1,3}/S_{2,3} = T_{1,1} - T_{1,2} * T_{1,3}/T_{2,3}$$

which shows that the reflexion term $S_{1,1}$ can be deduced, from the remaining measurements (independently from A and B). As a consequence of this redundancy we showed that de-embedding problem, in its current statement, is ill posed. Even if additional loss-less conditions are made, the following statement holds: for every phase parameter of the transmission $A_{1,2}$ there exists a unique set of measurements A and B that are compatible with S and T. Moreover closed form expressions exists for the derivation A and B.

In order to overcome the ill posed character of the de-embedding problem we currently study approaches where several measuring campaigns are made, for example while varying the short circuit position of the junction T. Generalisation of our preliminary results to general multiplexers are also of great interest.

6.13. Local topological linearization of control systems

Participants: Jean-Baptiste Pomet, Laurent Baratchart.

The article [15] states the following result: if a system is locally equivalent to a controllable linear system via a bi-continuous transformation –a local homeomorphism in the state-control space– it is *also* equivalent to this same controllable linear system via a transformation that bears a triangular structure, inducing a diffeomophism on state spaces that is as smooth as the system itself (real analytic for a real analytic system), and a transformation on inputs that bears this smoothness but whose inverse may not, at some singularities. This basically says that topological linearization is almost as stringent as smooth linearization.

This negative answer to the question raised in the last paragraph of section 3.2.3 calls for the following question, which is important for modeling control systems: are there local "qualitative" differences between the behavior of a non-linear system and that of its linear approximation when the latter is controllable? It would also be interesting to know whether, for equivalence between arbitrary systems (not assuming that one of them is linear controllable), the gap between topological and smooth equivalence is still negligible.

6.14. Necessary conditions for dynamic equivalence

Participant: Jean-Baptiste Pomet.

If two control systems on manifolds of the same dimension are dynamic equivalent (see section 3.2.3), we prove in [24] that either they are static equivalent –*i.e.* equivalent via a classical diffeomorphism– or they are both ruled; for systems of different dimensions, the one of higher dimension must be ruled. A ruled system is one whose equations define at each point in the state manifold, a ruled submanifold of the tangent space. It was already known that a differentially flat system must be ruled; this is a particular case of the present result, in which one of the systems is "trivial" (i.e., linear controllable).

This is an important contribution because it is difficult in general to prove that two systems are *not* dynamic equivalent. No general general necessary condition (or obstruction) was known; that condition is also the only general obstruction known for flatness.

6.15. Average control systems

Participants: Alex Bombrun [Univ. of Heidelberg], Jean-Baptiste Pomet.

In the terminology of [54], [55], a *Kepler control system* is a system in dimension n whose drift has $n-1$ first integral and compact trajectories and where the control is "small" in the sense that we are interested in asymptotic properties as the bound on the control tends to zero. It is the case in low thrust orbital transfer, see section 4.4, for negative energy, *i.e.* in the so-called elliptic domain.

For this class of systems, a notion of *average control system* is introduced in [54], [55]. Using averaging techniques in this context is rather natural, since the free system produces a fast periodic motion and the *small* control a slow one; averaging is a widespread tool in perturbations of integrable Hamiltonian systems, and the small control is in some sense a "perturbation". In some recent literature, one proceeds as follows: the control is pre-assigned, for instance to time optimal control via Pontryagin's Maximum Principle or else to some feedback designed beforehand. Then, averaging is performed on the resulting ordinary differential equation, whose limit behavior is analyzed when the control magnitude tends to zero.

The novelty of [54] (see also [56]) is to average *before* assigning the control, hence getting a *control system* that describes the limit behavior better. For that reason, the average control system is a convenient tool when comparing different control strategies.

It allowed us to answer an open question stated in [61] on the minimum transfer-time between two elliptic orbit when the thrust magnitude tends to zero, see [57].

Under some controllability conditions that are trivially satisfied in the case at hand, we proved that the average system is one where the velocity set has nonempty interior, i.e. all velocity directions are allowed at any point, and the constraint is convex; mathematically this yields a Finsler structure (in the same way as a controllable system without drift with a quadratic constraint on the control yields a sub-Riemannian structure). An article is in progress, reporting on these results [55].

7. Contracts and Grants with Industry

7.1. Contracts CNES-IRCOM-INRIA

Contract (reference Inria: 2470, CNES: 60465/00) involving CNES, XLim and Inria, whose objective is to work out a software package for identification and design of microwave devices. The work at Inria concerns the design of multiband filters with constraints on the group delay. The problem is to control the logarithmic derivative of the modulus of a rational function, while meeting specifications on its modulus.

8. Other Grants and Activities

8.1. Scientific Committees

L. Baratchart is a member of the editorial board of *Computational Methods and Function Theory* and *Complex Analysis and Operator Theory*.

8.2. ANR project "AHPI"

AHPI (Analyse Harmonique et Problèmes Inverses), is a "Projet blanc" in Mathematics involving Inria-Sophia (L. Baratchart coordinator), the Université de Provence (LATP, Aix-Marseille), the Université Bordeaux I (LATN), the Université d'Orléans (MAPMO), Inria-Bordeaux and the Université de Pau (Magique 3D). It aims at developing Harmonic Analysis techniques to approach inverse problems in seismology, Electro-encephalography, tomography and nondestructive control.

8.3. ANR project "Filipix"

Filipix (FILtering for Innovative Payload with Improved fleXibility) is a "Projet Thématique en Télécommunications", involving Inria-Sophia (Apics), XLim, Thales Alenia Space (Centre de Toulouse, coordinator).

8.4. Other national or international actions

NSF EMS21 RTG is a students exchange program with Vanderbilt University (Nashville, USA).

NSF CMG collaborative research grant DMS/0934630, “Imaging magnetization distributions in geological samples”, with Vanderbilt University and the MIT (USA).

EPSRC research grant EP/F020341/1 (Operator theory in function spaces on finitely-connected domains), with Leeds University (UK) and the University Lyon I, 2007-2009.

A program **Inria-Tunisian Universities** (STIC) links Apics to the LAMSIN-ENIT (Tunis).

Apics is linked with the CEA-IRFM (Cadarache), through a grant with the **Région PACA**, for the thesis of Y. Fischer.

Apics is part of the regional working group SBPI (Signal, Noise, Inverse Problems), with teams from Observatoire de la Côte d'Azur and Géoazur (CNRS) http://www-sop.inria.fr/apics/sbpi.

8.5. The Apics Seminar

The following scientists gave a talk at the team's seminar:

- *Slim Chabaane*, Univ. Sfax (Tunisie), Quelques estimations logarithmiques de type optimal dans les espaces de Hardy Sobolev $H^{k,\infty}$.
- *Camilla Colombo*, Univ. Glasgow, Méthodes d'optimisation et de contrôle pour des missions d'interception d'astéroïdes.
- *Karine Dadourian*, Ecole Centrale Marseille, Approximation non-linéaire multiéchelles. Application à la compression d'images.
- *Blaise Faugeras*, LJAD-Univ. Nice SA (Nice), Identification de l'équilibre du plasma dans un Tokamak en temps réel.
- *Moncef Mahjoub*, LAMSIN-ENIT (Tunis), Complétion de données dans un domaine annulaire. Application à la résolution de quelques problèmes inverses.
- *Jean Baptiste Pomet*, Équivalence et linéarisation des systèmes de contrôle.
- *Laurent Praly*, CAS, Mines Paris-Tech, Nonlinear Observer Design with an Appropriate Riemannian Metric.
- *Pierre Rouchon*, CAS, Mines Paris-Tech, Feedback generation of quantum Fock states by discrete QND measures.
- *Ed Saff*, Axis-supported External Fields on the Sphere.
- *Meriem Zghal*, Problème inverse d'identification de sources en EEG: Résolution et étude de la stabilité.

9. Dissemination

9.1. Teaching

Courses

- L. Baratchart, Mathematics, Vanderbilt University (Nahville, TN, USA), since August.
- Y. Fischer, Mathématiques pour l'ingénieur, section Mathématiques Appliquées et Modélisation, 3rd year, École Polytechnique Univ. Nice-Sophia Antipolis (EPU).
- J. Leblond, Centre Montessori, collège, Mouans-Sartoux, until June.
- A.-M. Nicu, Mathématiques, section Génie des Eaux, 3rd year, EPU, since September.
- M. Olivi, Mathématiques pour l'ingénieur (Fourier analysis and integration), section Mathématiques Appliquées et Modélisation, 3rd year, EPU.

- M. Zghal, Mathématiques, sections Mathématiques Appliquées et Modélisation, Génie des Eaux, 3rd year, EPU.

Ph.D. Students

- Slah Chaabi, « Problèmes extrémaux pour l´équation de Beltrami réelle 2-dimensionnelle et application à la détermination de frontières libres », co-advised, Univ. Aix-Marseille I.
- Yannick Fischer, « Problèmes inverses pour l'équation de Beltrami et extrapolation de quantités magnétiques dans un Tokamak », Univ. Nice-Sophia Antipolis.
- Ahed Hindawi « Transport optimal en contrôle », Univ. Nice-Sophia Antipolis.
- Ana-Maria Nicu, « Inverse potential problems for MEG/EEG », Univ. Nice-Sophia Antipolis.
- Meriem Zghal, « Constructive aspects of some inverse problems (Cauchy, sources) for Laplace equation in ellipsoidal domains », co-advised, Univ. Tunis El Manar (Tunisia).

HDR

- Jean-Baptiste Pomet, « Equivalence et linéarisation des systèmes de contrôle », defended in october [12].

Committees

- J. Leblond was a member of the PhD defense committees of R. Mdimegh, Univ. Tunis El Manar (Tunisie), and of H. Meftahi, Univ. Lille I.

9.2. Community service

L. Baratchart is Inria's representative at the « conseil scientifique » of the Univ. Provence (Aix-Marseille). He was a member of the "Commissions de spécialistes" of the Univ. of Lille and Bordeaux.

J. Grimm is a representative at the « comité de centre » (Research Center INRIA-Sophia).

J. Leblond is a member of the « Commission d'Évaluation » (CE) of INRIA[2]. She is a member of the « Commission d'Animation Scientifique » (CAS) of the Research Center.

M. Olivi is a member of the CSD (Comité de Suivi Doctoral) of the Research Center.

J.-B. Pomet is a representative at the « comité technique paritaire » (CTP) of INRIA.

F. Seyfert is a member of the CDL (Comité de Développement Logiciel) of the Research Center.

9.3. Conferences and workshops

L. Baratchart was a plenary speaker at the Conference "Time series analysis and system identification", Vienna (Austria). He gave a communication at the Conference "Computational Methods in Function Theory" (CMFT, Ankara), and was a "colloquium speaker" at Univ. of Michigan (East Lansing, USA), and at Univ. of Mississipi (Oxford, USA). He participated to the Colloque "Aspects géométriques des EDP", CIRM, Luminy (Mars), together with Y. Fischer.

Y. Fischer gave a presentation at the First Franco-Tunisian Conference on Mathematics, CFTM1, Djerba (Tunisia, Mars), and at the Congress SMAI 2009, La Colle sur Loup (France, May). He was also invited to give a talk at the meeting of the ANR project AHPI, Sophia-Antipolis (April), at a working group of Labo. J.-A. Dieudonné, Univ. Nice Sophia-Antipolis (Nice, May), and at the "Séminaires Croisés", INRIA-Sophia, session "Simulations numériques pour ITER et la Fusion", October. He participated to the Summer School of the Large Scale Initiative "Fusion", IRMA, Strasbourg (September).

[2] Thus, of several recruitment or hiring committees of Inria researchers, or evaluation seminars of Inria teams, and participates to working groups.

J. Leblond was an invited speaker at the Colloque “Aspects géométriques des EDP”, CIRM, Luminy (Mars). She gave a plenary talk at the Conference on Distributed Parameter Systems (CDPS), IFAC, Toulouse (July), and at the Workshop “Operator Theory and Applications”, ICMS, Edinburgh (Scotland, Sept.). She was invited to give a talk at the Seminars of the team Algorithms, INRIA-Rocquencourt (January) and of the team Analyse et Géométrie, LATP-CMI, Univ Aix-Marseille I (April). She organized at INRIA-Sophia-Antipolis the meeting of the ANR project AHPI (together with E. Russ and S. Sorres, April), and the session “Simulations numériques pour ITER et la Fusion” of the “Séminaires Croisés” (October).

A.-M. Nicu participated to the Summer School “Traitement du signal et des images”, Peyresq (France, July) and to the School CEA-EDF-INRIA, “Eléctrophysiologie cardiaque et cérébrale”, INRIA-Rocquencourt (November).

M. Olivi attented the Sysid Meeting in St-Malo (France). She gave a presentation at the Groupe de Travail Identification of the GDR MACS (Paris, France) and at the Department of Knowledge Engineering, University of Maastricht. She presented a poster at the 2009 ERNSI Meeting in Stift Vorau (Austria).

F. Seyfert was invited to give a talk on “Broad band filter synthesis” at the European Microwave Conference in Roma (Italy). He also gave a talk at the CNES-ESA International Workshop on Microwave Filters in Toulouse (France).

M. Zghal presented a poster at the Congress SMAI 2009, La Colle sur Loup (France, May), for which she got a price of the best conference posters.

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