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Project-Team galaad

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Theme : Algorithms, Certification, and Cryptography

Activity Report

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Galaad is a joint project with Laboratoire J.A. Dieudonné U.M.R. C.N.R.S. n^O 6621, University of Nice Sophia-Antipolis.

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2. Overall Objectives

2.1. Overall Objectives

Our research program is articulated around effective algebraic geometry and its applications. The objective is to develop algorithmic methods for effective and reliable solution of geometric and algebraic problems, which are encountered in fields such as CAGD (Computer Aided Geometric Design), robotics, computer vision, molecular biology, etc. We focus on the analysis of these methods from the point of view of complexity as well as qualitative aspects, combining symbolic and numerical computation.

Geometry is one of the key topics of our activity, which includes effective algebraic geometry, differential geometry and computational geometry of semi-algebraic sets. More specifically, we are interested in problems of small dimensions such as intersection, singularity, topology analysis, and computation with algebraic curves and surfaces.

These geometric investigations lead to algebraic questions, and particularly to the resolution of polynomial equations. We are involved in the design and analysis of new methods of effective algebraic geometry. Their developments and applications are central and often critical in practical problems.

Approximate numerical computation, usually opposed to symbolic computation, and the problems of certification are also at the heart of our approach. We intend to explore these bonds between geometry, algebra and analysis, which are currently making important strides. These objectives are both theoretical and practical. Recent developments enable us to control, check, and certify results when the data are known to a limited precision.

Finally our work is implemented in software developments. We pay attention to problems of genericity, modularity, effectiveness, suitable for the writing of algebraic and geometrical codes. The implementation and validation of these tools form another important component of our activity.

3. Scientific Foundations

3.1. Introduction

Our scientific activity is defined according to three broad topics: modeling, computing and analysis, in connection with effective algebraic geometry.

3.2. Algebraic Geometric Modeling

We are investigating geometric modeling approaches, based on non-discrete models, mainly of semi-algebraic type. Such non-linear models are able to capture efficiently complexes shapes, using few data. However, they required specific methods to handle and solve the underlying non-linear problems.

Effective algebraic geometry is a naturally framework for handling such representations, in which we are developing new methods to solve these non-linear problems. The framework not only provides tools for modeling but also, it makes it possible to exploit the geometric properties of these algebraic varieties, in order to improve this modeling work. To handle and control geometric objects such as parameterised curves and surfaces or their implicit representations, we consider in particular projections techniques. We focus on new formulations of resultants allowing us to produce solvers from linear algebra routines, and adapted to the solutions we want to compute. Among these formulations, we study in particular *residual* and *toric* resultant theory. The latter approach relates the generic properties of the solutions of polynomial equations, to the geometry of the Newton polytope associated with the polynomials. These tools allows to change geometric representations, computing an implicit model from a parameterised one. We are interested in dedicated methods for solving these type of problems.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately the algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations. The use of local algebraic representations also raises problems of approximation and reconstruction, on which we are working on.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. In addition to the development of symbolic geometric computations that exploit these invariant properties, we are also interested in developing compact representations of shapes, based on algebraic/symbolic descriptions. Our aim is to improve geometric computing performances, by using smaller input data, with better properties of approximation and certified computation.

3.3. Algebraic Geometric Computing

The underlying representation behind the geometric model that we consider are often of algebraic type. Computing with such models raise algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring, defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases. We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems. Our focuses are effective computation of the algebraic residue, interpolation problems, and the relation between coefficients and roots in the case of multivariate polynomials.

We are also interested in subdivision methods, which are able to localise efficiently the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue in analysing these methods is how to obtain good complexity bounds by exploiting the structure of the problem. Many algebraic problems can be reformulated in terms of linear algebra questions. Thus, it is not surprising to see that complexity analysis of our methods leads to the theory of structured matrices. Indeed, the matrices resulting from polynomial problems, such as matrices of resultants or Bezoutians, are structured. Their rows and columns are naturally indexed by monomials, and their structures generalize the Toeplitz matrices to the multivariate case. We are interested in exploiting these properties and their implications in solving polynomial equations.

When solving a system of polynomials equations, a first treatment is to transform it into several simpler subsystems when possible. The problem of decomposition and factorisation is thus also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. For instance, the (difficult) problem of approximate factorization, the computation of perturbations of the data, which enables us to break up the problem, is studied. More generally, we are working on the problem of decomposing a variety into irreducible components.

3.4. Algebraic Geometric Analysis

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many context, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate these objects while controlling the quality of approximation. Subdivision techniques based on the algebraic formulation of our problems are exploited in order to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimising these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the developments of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship to the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encounter in the application of computer sciences started from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where *Distance geometry* plays a significant role, for example, in the reconstruction from NMR experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tends to be more and more easily used yield large set of data points from which one has to recover compact geometric model. We are working in collaboration with groups in agronomy on the problems of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which use the same (bspline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. Software

5.1. Mathemagix, a free computer algebra environment

Participants: Bernard Mourrain [contact person], Daouda N'Diatta, Elias Tsigaridas, Angelos Mantzaflaris.

http://www.mathemagix.org/

MATHEMAGIX is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments are based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (*template*) and the control of their instantiations.

The language of the interpreter is imperative, strongly typed and high level. A compiler of this language is available. A special effort has been put on the embeding of existing libraries written in other languages like C or C++. An interesting feature is that this extension mechanism supports template types, which automatically induce generic types inside Mathemagix. Connections with GMP, MPFR for extended arithmetic, LAPACK for numerical linear algebra are currently available in this framework.

The project aims at building a bridge between symbolic computation and numerical analysis. It is structuring collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

- REALROOT: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables; continued fraction expansion of roots of univariate polynomials; Bernstein basis representation of univariate and multivariate polynomials and related algorithms; exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.
- SHAPE: tools to manipulate curves and surfaces of different types including parameterised, implicit with different type of coefficients; Algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library SYNAPS (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler AXEL.

Collaborators: Grégoire Lecerf, Olivier Ruatta, Joris van der Hoeven and Philippe Trébuchet.

5.2. Axel, a geometric modeler for algebraic objects

Participants: Angelos Mantzaflaris, Bernard Mourrain [contact person], Gang Xu.

http://axel.inria.fr.

We are developing a software called AXEL (Algebraic Software-Components for gEometric modeLing) dedicated to algebraic methods for curves and surfaces. Many algorithms in geometric modeling require a combination of geometric and algebraic tools. Aiming at the development of reliable and efficient implementations, AXEL provides a framework for such combination of tools, involving symbolic and numeric computations.

Applications contain data structures and functionalities related to algebraic models used in geometric modeling, such as polynomial parameterisation, B-Spline, implicit curves and surfaces. It provides algorithms for the treatment of such geometric objects, such as tools for computing intersection points of curves or surfaces, detecting and computing self-intersection points of parameterized surfaces, implicitization, for computing the topology of implicit curves, for meshing implicit (singular) surfaces, etc.

This package is now distributed as binary packages for Linux as well as for MacOSX. It is hosted at the INRIA's gforge (http://gforge.inria.fr) and referenced by many leading software websites such as http://apple.com. The first version of the software has been downloaded more than 15000 times, since it is available.

Collaborators: Stéphane Chau, Jean-Pascal Pavone and Julien Wintz.

5.3. Multires, a Maple package for multivariate resolution problems

Participants: Laurent Busé [contact person], Bernard Mourrain.

http://www-sop.inria.fr/galaad/software/multires/.

The Maple package MULTIRES contains a set of routines related to the resolution of polynomial equations. The prime objective is to illustrate various algorithms on multivariate polynomials, but not their effectiveness, which is achieved in a more adapted environment such as MATHEMAGIX. It provides methods for building matrices whose determinants are multiples of resultants on certain varieties, and solvers depending on these formulations, and based on eigenvalues and eigenvectors computation. It contains the computations of Bezoutians in several variables, the formulation of Macaulay, Jouanolou for projective resultants, Bezout and (sparse) resultants on a toric variety, residual resultants of complete intersections, functions for computing the degree of residual resultants, algorithms for the geometric decomposition of algebraic varieties. Furthermore, there are tools related to the duality of polynomials, particularly the computation of residues for complete intersections of dimension 0.

Collaborators: Ioannis Emiris, Olivier Ruatta and Philippe Trébuchet.

5.4. diffalg, a Maple package for differential algebra

Participant: Evelyne Hubert [contact person].

The Maple package *diffalg* is a collection of routines to handle systems of polynomial differential equations and inequations. The functionalities include differential elimination, expansion of the solutions into formal power series and analysis of singular solutions. The underlying theory and terminology belongs to differential algebra.

Collaborators: François Boulier et François Lemaire from Univerity of Lille.

5.5. AIDA, a Maple package for algebraic invariants and their differential algebra

Participant: Evelyne Hubert [contact person].

http://www-sop.inria.fr/members/Evelyne.Hubert/aida/.

The Maple AIDA package is a collection of routines to explore algebra of differential invariants: computation of generating sets of invariants, rewritings, syzygies, and their differential analogues. The package builds on the Maple libraries Groebner, Vessiot and diffalg.

6. New Results

6.1. Algebraic Geometric Modeling

6.1.1. Elimination and nonlinear equations of Rees Algebra

Participant: Laurent Busé.

A new approach is established to computing the image of a rational map, whereby the use of approximation complexes is complemented with a detailed analysis of the torsion of the symmetric algebra in certain degrees. In the case the map is everywhere defined this analysis provides free resolutions of graded parts of the Rees algebra of the base ideal in degrees where it does not coincide with the corresponding symmetric algebra. A surprising fact is that the torsion in those degrees only contributes to the first free module in the resolution of the symmetric algebra modulo torsion. An additional point is that this contribution – which of course corresponds to non linear equations of the Rees algebra – can be described in these degrees in terms of non Koszul syzygies via certain upgrading maps in the vein of the ones introduced earlier by J. Herzog, A. Simis and W. Vasconcelos. As a measure of the reach of this torsion analysis we could say that, in the case of a general everywhere defined map, half of the degrees where the torsion does not vanish are understood.

This work is in collaboration with Marc Chardin, Univ. Paris VI, and Aron Simis, Univ. Recife. A preprint version of it, submitted for publication, is available at http://hal.inria.fr/inria-00431783/en/.

6.1.2. Multihomogeneous resultant formulae for systems with scaled support

Participant: Angelos Mantzaflaris.

Constructive methods for matrices of multihomogeneous resultants for unmixed systems have been studied by Weyman, Zelevinsky, Sturmfels, Dickenstein and Emiris. We generalize these constructions to *mixed* systems, whose Newton polytopes are scaled copies of one polytope, thus taking a step towards systems with arbitrary supports. First, we specify matrices whose determinant equals the resultant and characterize the systems that admit such formulae. Bézout-type determinantal formulae do not exist, but we describe all possible Sylvester-type and hybrid formulae. We establish tight bounds for the corresponding degree vectors, as well as precise domains where these concentrate; the latter are new even for the unmixed case. Second, to specify resultant matrices explicitly in the general case we make use of multiplication tables and strong duality theory. The encountered matrices are classified; these include a new type of Sylvester-type matrix as well as Bézout-type matrices, which we call partial Bezoutians. Our public-domain Maple implementation includes efficient storage of complexes in memory, and construction of resultant matrices.

It is done in collaboration with Ioannis Emiris (NKUA) and is published in [23].

6.1.3. Convolution surfaces based on polygonal curve skeletons

Participant: Evelyne Hubert.

For its application to the reconstruction of tree branches within the project PlantScan3D, we reviewed and generalized Convolution Surfaces. The technique is used in Computer Graphics to generate smooth 3D models around polygonal line serving as skeletons. Convolution surfaces are defined as level set of a function obtained by integrating a kernel function along this skeleton. To allow interactive modeling, the technique has relied on closed form formulae for integration obtained through symbolic computation software.

We provided new qualitative results and generalizations on the topic. On the one hand we provide the relationship between the level set and the thickness around the skeleton elements. On the other hand we obtained the closed form formulae for all the kernels in the most commonly used families - power inverse and Cauchy - through recurrences. We also exhibit recurrences to include polynomial weights on the skeleton. This allows to have a varying shape along the skeleton, the coefficients of the polynomial weight acting as controls.

Other types of skeletons are under study such as polygons and arcs of circles. Helices and super-helices appeared in the modelisation of hair and could also be of use for modeling tree branches as those can present torsion. We also plan to investigate kernels with compact supports, which are given by piecewise polynomials. Those allow locality and provide piecewise polynomial representations of the shape to be approximated.

This work is done in collaboration with Marie-Paul Cani, INRIA Grenoble. A preprint version of this work is available at http://hal.inria.fr/inria-00429358/en/.

6.1.4. Detail-preserving axial deformation using curve pairs

Participant: Gang Xu.

Traditional axial deformation is simple and intuitive for users to modify the shape of objects. However, unexpected twist of the object may be obtained. The use of a curve-pair allows the local coordinate frame to be controlled intuitively. However, some important geometric details may be lost and changed in the deformation process. In this work, we present a detail-preserving axial deformation algorithm based on Laplacian coordinates. Instead of embedding the absolute coordinates into deformation space in the traditional axial deformation, we transform the Laplacian coordinates at each vertex according to the transformation of local frames at the closest points on the axial curve. Then the deformed mesh is reconstructed by solving a linear system that describes the reconstruction of the local details in least squares sense. By associating a complex 3D object to a curve-pair, the object can be stretched, bend, twisted intuitively through manipulating the curve-pair, and can also be edited by means of view-dependent sketching. This method combines the advantages of axial deformation and Laplacian mesh editing. Experimental results are presented to show the effectiveness of the proposed method.

This work was done in collaboration with Wenbing Ge, Peking University, Kin-Chuen Hui, The Chinese University of Hong Kong, Guoping Wang, Peking University, and the results are published in [25].

6.1.5. Detail-preserving sculpting deformation

Participant: Gang Xu.

Sculpting deformation is a powerful tool to modify the shape of objects intuitively. However, the detail preserving problem has not been considered in sculpting deformation. In the deformation of a source object by pressing a primitive object against it, the source object is deformed while geometric details of the object should be maintained. In order to address this problem, we present a detail preserving sculpting deformation algorithm by using Laplacian coordinates. Based on the property of Laplacian coordinate, we propose two feature invariants to encode the Laplacian coordinate. Instead of mapping the source mesh to the primitive mesh, we map the smooth version of source mesh to the primitive mesh and use the Laplacian coordinates to encode the geometric details. When the smooth version of the source mesh is deformed, the Laplacian coordinates of the deformed mesh are computed for each vertex firstly and then the deformed mesh is reconstructed by solving a linear system that satisfies the reconstruction of the local details in least squares sense. Several examples are presented to show the effectiveness of the proposed approach.

This work was done in collaboration with Wenbing Ge, Peking University, Kin-Chuen Hui, The Chinese University of Hong Kong, Guoping Wang, Peking University, and the results are published in [26].

6.1.6. Tree reconstructions from scanner point clouds

Participant: Bernard Mourrain.

In this work, we present a reconstruction pipeline for recovering branching structure of trees from laser scanned data points. The process is made up of two main blocks: segmentation and reconstruction. Based on a variational k-means clustering algorithm, cylindrical components and ramified regions of data points are identified and located. An adjacency graph is then built from neighborhood information of components. Simple heuristics allow us to extract a skeleton structure and identify branches from the graph. Finally, a B-spline model is computed to give a compact and accurate reconstruction of the branching system.

This work was done in collaboration with Fréderic Boudon, Virtual plants, Christophe Godin, Virtual plants, Julien Wintz, Wenping Wang, Hong Kong Univ., Donming Yuan, Hong Kong Univ. and the results are published in [29].

6.2. Algebraic Geometric Computing

6.2.1. The Hilbert scheme of points and its link with border basis

Participants: Jérôme Brachat, Bernard Mourrain.

We give new explicit representations of the Hilbert scheme of μ points in $\mathbb{P}^r$ as a projective subvariety of a Grassmanniann variety. This new explicit description of the Hilbert scheme is simpler than the existing ones and global. It involves equations of degree 2. We show how these equations are deduced from the commutation relations characterizing border bases. Next, we consider infinitesimal perturbations of an input system of equations on this Hilbert scheme and describe its tangent space. We propose an effective criterion to test if it is a flat deformation, that is if the perturbed system remains on the Hilbert scheme of the initial equations. In particular, this criterion involves in particular formal reduction with respect to border bases.

A preprint version of this work, done in collaboration with Mariemi Alonso, Departmento de Algebra, UCM and submitted for publication, is available at http://hal.inria.fr/inria-00433127.

6.2.2. Modular Las Vegas algorithms for polynomial absolute factorization

Participants: André Galligo, Cristina Bertone.

Let $f(X,Y) \in \mathbb{Z}[X,Y]$ be an irreducible polynomial over $\mathbb{Q}[X,Y]$. We study the absolute factorization $f_1 \cdots f_s$ of f. First we give a Las Vegas absolute irreducibility test based on a property of the Newton polygon of f. Then thanks to this test we give a new strategy based on modular computations and LLL which gives the absolute factorization.

These results, in collaboration with Guillaume Chèze, were presented at MEGA 2009 and are described in http://hal.inria.fr/inria-00436063/en/.

6.2.3. Algorithms for irreducible decomposition of curves in $\mathbb{C}^N$

Participants: André Galligo, Cristina Bertone.

The aim of this work is to construct an effective method for computing the irreducible components of a curve of the affine N-dimensional space $\mathbb{C}^N$, eventually starting from the existing algorithms for decomposition of curves in the plane $\mathbb{C}^2$, i.e. algorithms for the absolute factorization of polynomials. We developed a strategy using the modular techniques of section 6.2.2 allowing us to compute: the number of irreducible components of a curve, their degrees, multiplicities, the Hilbert function of the "simple" components and the algebraic extensions for the non-rational components.

6.2.4. Subdivision methods for solving polynomial equations

Participant: Bernard Mourrain.

This works presents a new algorithm for solving a system of polynomials in a domain of $\mathbb{R}^n$. It can be seen as an improvement of the Interval Projected Polyhedron algorithm proposed by Sherbrooke and Patrikalakis. It uses a powerful reduction strategy based on univariate root finder using Bernstein basis representation and Descarte's rule. We analyse the behavior of the method, from a theoretical point of view, show that for simple roots, it has a local quadratic convergence speed and give new bounds for the complexity of approximating its real roots in a box of $\mathbb{R}^n$. The improvement of our approach, compared with classical subdivision methods, is illustrated on geometric modeling applications such as computing intersection points of implicit curves, self-intersection points of rational curves, and on the classical parallel robot benchmark problem. An implementation of this algorithm is available in the module `realroot` of MATHEMAGIX project.

This work, done in collaboration with Jean-Pascal Pavone, is published in [20].

6.2.5. Multivariate continued fraction solvers for polynomial equations

Participants: Angelos Mantzaflaris, Bernard Mourrain, Elias Tsigaridas.

We present a new algorithm for isolating the real roots of a system of multivariate polynomials, given in the monomial basis. It is inspired by existing subdivision methods in the Bernstein basis; it can be seen as generalization of the univariate continued fraction algorithm or alternatively as an analog of Bernstein subdivision in the monomial basis. The representation of the subdivided domains is done through homographies, which allows us to use only integer arithmetic and to treat efficiently unbounded regions. We use univariate bounding functions, projection and preconditioning techniques to reduce the domain of search. The resulting boxes have optimized rational coordinates, corresponding to the first terms of the continued fraction expansion of the real roots. An extension of Vincent's theorem to multivariate polynomials is proved and used for the termination of the algorithm. New complexity bounds are provided for a simplified version of the algorithm. Our C++ implementation in done as part of `realroot` module of the MATHEMAGIX system.

This work has been published in [28].

6.2.6. Computing nearest GCD with certification

Participants: André Galligo, Bernard Mourrain.

A bisection method, based on exclusion and inclusion tests, is used to address the nearest univariate GCD problem formulated as a bivariate real minimization problem of a rational fraction.

The paper [22] presents an algorithm, a first implementation and complexity analysis relying on Smale's α-theory. We report its behavior on an illustrative example.

This work was done in collaboration with Guillaume Chèze, Univ. Toulouse, Jean-Claude Yakoubsohn, Univ. Toulouse.

6.2.7. Continuation and monodromy on random Riemann surfaces

Participants: André Galligo, Adrien Poteaux.

The main motivation is to analyze and develop further factorization algorithms for bivariate polynomials in $\mathbb{C}[x, y]$, which proceed by continuation methods.

We consider a Riemann surface X defined by a polynomial $f(x, y)$ of degree d, the coefficients of which are chosen randomly. In this way we can suppose that X is smooth, that the discriminant $\delta(x)$ of f has $d(d-1)$ simple roots and also that $\delta(0) \neq 0$, that is to say that the corresponding fiber has d distinct points $\{y_1, ..., y_d\}$. When we lift a loop $0 \in \gamma \subset \mathbb{C} - \Delta$ by a continuation method, we get d paths in X connecting $\{y_1, ..., y_d\}$, hence defining a permutation of that set. This is called monodromy.

We present experimentations in Maple to get statistics on the distribution of transpositions corresponding to the loops turning around each point of Δ. Multiplying families of "neighbor" transpositions, we construct permutations then subgroups of the symmetric group. This allows us to establish and study experimentally some conjectures on the distribution of these transpositions then on transitivity of the generated subgroups.

These results provide interesting insights on the structure of such Riemann surfaces; hence of their union. They are used to develop fast algorithms for absolute multivariate polynomial factorization, under some genericity hypothesis: we assume that the factors behave like random polynomials the coefficients of which follow uniform distributions.

This work is published in [24].

6.2.8. Curve/surface intersection problem by means of matrix representations

Participants: Laurent Busé, Thang Luu Ba, Bernard Mourrain.

We introduce matrix representations of plane algebraic curves and space algebraic surfaces for Computer Aided Geometric Design (CAGD). The idea of using matrix representations in CAGD is quite old. The novelty of our contribution is to enable non square matrices, extension which is motivated by recent research in this topic. We show how to manipulate these representations by proposing a dedicated algorithm to address the curve/surface intersection problem by means of numerical linear algebra techniques.

This work has been published in the proceedings of the SNC'09 conference [27].

6.2.9. Bernstein Bezoutian and some intersection problems

Participants: Elimane Ba, Mohamed Elkadi.

We study the Bézier curve-surface and Bézier surface-surface intersection problems avoiding the well-know unstable conversion between Bernstein basis and power basis. These varieties are given by parameterizations in Bernstein bases and all computations are performed in that form. We construct an adapted resultant for generic Bernstein polynomial systems with a special shape appearing in the intersection problems. This work has been submitted to the journal CAGD.

6.2.10. Isotopic triangulation of a real algebraic surface

Participant: Bernard Mourrain.

We present a new algorithm for computing the topology of a real algebraic surface S in a ball B, even in singular cases. We use algorithms for 2D and 3D algebraic curves and show how one can compute a topological complex equivalent to S, and even a simplicial complex isotopic to S by exploiting properties of the contour curve of S. The correctness proof of the algorithm is based on results from stratification theory. We construct an explicit Whitney stratification of S, by resultant computation. Using Thom's isotopy lemma, we show how to deduce the topology of S from a finite number of characteristic points on the surface. An analysis of the complexity of the algorithm and effectiveness issues are also studied.

This work, done in collaboration with Lionel Alberti and Jean-Pierre Técourt is published in [12].

6.2.11. Towards toric absolute factorization

Participants: André Galligo, Mohamed Elkadi.

We present an algorithmic approach to study and compute the absolute factorization of a bivariate polynomial, taking into account the geometry of its monomials. It is based on algebraic criteria inherited from algebraic interpolation and toric geometry.

This work, done in collaboration with Martin Weimann, Univ. Barcelona, is published in [17].

6.3. Algebraic Geometric Analysis

6.3.1. A sparse flat extension theorem for moment matrices

Participant: Bernard Mourrain.

We prove a generalization of the flat extension theorem of Curto and Fialkow for truncated moment matrices. It applies to moment matrices indexed by an arbitrary set of monomials and its border, assuming that this set is connected to 1. When formulated in a basis-free setting, this gives an equivalent result for truncated Hankel operators. Applications to real radical computation and tensor decomposition are considered.

This work was done in collaboration with Monique Laurent, CWI, Nertherland, and published in [19].

6.3.2. Differential invariants of a Lie group action: syzygies on a generating set

Participant: Evelyne Hubert.

Given a group action, known by its infinitesimal generators, we exhibit a complete set of syzygies on a generating set of differential invariants. For that we elaborate on the reinterpretation of Cartan's moving frame by Fels and Olver (1999). This provides constructive tools for exploring algebras of differential invariants.

This research on the algebraic structure of differential invariants was initiated with a view towards differential elimination. Yet differential invariants are essential means of characterizing a form independently form a group action (such as the traditional rigid motions).

This work has been published in [18].

6.3.3. On the total order of reducibility of a pencil of algebraic plane curves

Participant: Laurent Busé.

In this work, the problem of bounding the number of reducible curves in a pencil of algebraic plane curves is addressed. Unlike most of the previous related works, each reducible curve of the pencil is here counted with its appropriate multiplicity. It is proved that this number of reducible curves, counted with multiplicity, is bounded by $d^2 - 1$ where d is the degree of the pencil. Then, a sharper bound is given by taking into account the Newton's polygon of the pencil.

A preprint version of this work, done in collaboration with Guillaume Chèze, Univ. Toulouse and submitted for publication, is available at http://hal.archives-ouvertes.fr/hal-00348561/en/.

6.3.4. Noether's forms for the study of non-composite rational functions and their spectrum

Participant: Laurent Busé.

In this work, the spectrum and the decomposability of a multivariate rational function are studied by means of the effective Noether's irreducibility theorem given by Ruppert. With this approach, some new effective results are obtained. In particular, we show that the reduction modulo p of the spectrum of a given integer multivariate rational function r coincides with the spectrum of the reduction of r modulo p for p a prime integer greater or equal to an explicit bound. This bound is given in terms of the degree, the height and the number of variables of r. With the same strategy, we also study the decomposability of r modulo p. Some similar explicit results are also provided for the case of polynomials with coefficients in a polynomial ring.

A preprint version of this work, done in collaboration with Guillaume Chèze, Univ. Toulouse and Salah Najib, Univ. Lille and submitted for publication, is available at http://hal.inria.fr/inria-00395839/en/.

6.3.5. *Analysis of intersection of quadrics through signature sequences*

Participant: Bernard Mourrain.

We present an efficient method for classifying the morphology of the intersection curve of two quadrics (QSIC) in PR3, 3D real projective space; here, the term morphology is used in a broad sense to mean the shape, topological, and algebraic properties of a QSIC, including singularity, reducibility, the number of connected components, and the degree of each irreducible component, etc. There are in total 35 different QSIC morphologies with non-degenerate quadric pencils. For each of these 35 QSIC morphologies, through a detailed study of the eigenvalue curve and the index function jump we establish a characterizing algebraic condition expressed in terms of the Segre characteristics and the signature sequence of a quadric pencil. We show how to compute a signature sequence with rational arithmetic in order to determine the morphology of the intersection curve of any two given quadrics. Two immediate applications of our results are the robust topological classification of QSIC in computing B-rep surface representation in solid modeling and the derivation of algebraic conditions for collision detection of quadric primitives

This work is done in collaboration with Changhe Tu, Shandong University, Wenping Wang, Hong Kong University, Jiaye, Wang, Shandong University and is published in [21].

6.3.6. *On the distribution of the solutions of systems of polynomial equations*

Participant: André Galligo.

In a work done in collaboration with Carlos D'Andrea, Univ. Barcelona and Martin Sombra, Univ. Bordeaux, we generalize a celebrated result, due to P. Erdös and P. Turán, on the distribution of roots of univariate polynomials to the sparse multivariate case.

In relation with probabilistic algorithms for monodromy computation, we consider $K \geq 2$ independent copies of the random walk on the symmetric group S_N starting from the identity and generated by the products of either independent uniform transpositions or independent uniform successive transpositions. At any time $n \in \mathbb{N}$, let G_n be the subgroup of S_N generated by the K positions of the chains. In the uniform transposition model, we prove that there is a cut-off phenomenon at time $N \ln(N)/(2K)$ for the non-existence of fixed point of G_n and for the transitivity of G_n, thus showing that these properties occur before the chains have reach equilibrium. In the uniform successive transposition model, a transition for the non-existence of fixed point of G_n appears at time of order $N^{1+\frac{2}{K}}$ (at least for $K \geq 3$), but there is no cut-off phenomenon. In the latter model, we recover a cut-off phenomenon for the non-existence of fixed point at a time proportional to N by allowing the number K to be proportional to $\ln(N)$. The main tools of the proofs are spectral analysis and coupling techniques.

A preprint of this work, done in collaboration with Laurent Miclo, Univ. Toulouse and submitted for publication, is available at http://hal.archives-ouvertes.fr/hal-00384188/en/.

7. Other Grants and Activities

7.1. European actions

7.1.1. *SAGA*

SAGA (ShApe, Geometry and Algebra, 2008-2012) is a Marie-Curie Initial Training Network of the call FP7-PEOPLE-2007-1-1-ITN.

The project aims at promoting the interaction between Geometric Modeling and Real Algebraic Geometry and, in general, at strengthening interdisciplinary and inter-sectorial research and development concerning CAD/CAM. Its objective is also to train a new generation of researchers familiar with both academic and industry viewpoints, while supporting the cooperation among the partners and with other interested collaborators in Europe. The partners are:

- SINTEF, Oslo, Norway (Leader);
- University of Oslo, Norway;
- Johannes Kepler Universitaet Linz, Austria;
- Universidad de Cantabria, Santander, Spain;
- Vilniaus Universitetas, Lithuany;
- National and Kapodistrian University of Athens, Greece;
- INRIA Méditerranée, France;
- GraphiTech, Italy;
- Kongsberg SIM GmbH, Austria;
- Missler Software, France;

More information available at http://saga-network.eu/.

The Ph.D. thesis of Angelos Mantzaflaris on Robust algebraic methods for geometric computations is supported by the Marie-Curie ITN SAGA.

We hosted Johan Seland as an experienced research fellow of the Marie-Curie ITN SAGA between March and May. The collaboration concerned the use of GPU for rendering algebraic surfaces.

7.1.2. Exciting

Exciting – Exact geometry simulation for optimized design of vehicles and Vessels – FP7-CP-SST-2007-RTD-1- 218536 (2008–2011).

This project focuses on computational tools for the optimized design of functional free-form surfaces. Specific applications are ship hulls and propellers in naval engineering and car components, frames, and turbochargers in the automotive and railway transportation industries. The objective is to base the corresponding computational tools on the same exact representation of the geometry. This should lead to huge benefits for the entire chain of design, simulation, optimization, and life cycle management, including a new class of computational tools for fluid dynamics and solid mechanics, simulations for vehicles and vessels based. This seamless integration of CAD and FEM will have direct applications in product design, simulation and optimization of core components of vehicles and vessels. The partners are:

- Johannes Kepler University, Linz Autriche (Leader);
- SINTEF, Oslo, Norway;
- Siemens AG, Germany;
- National Technical University of Athens, Greece;
- Hellenic Register of Shipping, Greece;
- University of Technology, Munich Germany;
- INRIA Méditerranée, France;
- VA Tech Hydro, Austria;
- Det Norske Veritas AS, Norway.

More information available at http://exciting-project.eu/.

7.2. Bilateral actions

7.2.1. PAI STAR South Corea collaboration

Participants: Laurent Busé, André Galligo, Gang Xu, Evelyne Hubert, Angelos Mantzaflaris, Bernard Mourrain.

The objective of this collaboration is to conduct research in algebraic techniques for solving geometric modeling problems. More specially, we are interested in developing efficient and robust methods to solve non-linear constraints which appear in geometric computation. These methods will be used in applications such as shape design and reconstruction, for solving interpolation or approximation problems. A typical area in which we will apply our methods is ship design. Experimentation and validation will lead to open source software implementation.

Collaborators from Seoul National University: Park Sung Ha, Tae-Wan Kim, Sharma Rajiv, Hur Seok.

A. Galligo and B. Mourrain visited Seoul National University (July 27 - Aug. 2).

7.2.2. ECOS-Sud collaboration

Participants: Jérôme Brachat, Laurent Busé, Mohamed Elkadi, André Galligo, Bernard Mourrain.

The first objective of this collaboration with the team of A. Dickenstein at the University of Buenos Aires, Argentina is the development of effective methods for geometric modeling, with a special focus on singularity and numerical stability problems. This includes intersection problems for curves or surfaces, change of representations such as implicitization via syzygies and moving planes, polytopes analysis and Puiseux expansions. A second objective is the development of open tools dedicated to such problems which could be shared by the different groups working on this topic.

This year we had the visit of N. Botbol (Ph.D. student), May 15 - June 14, A. Dickenstein (Professor), June 1-15. J. Brachat (Ph.D. student), gave a visit to Buenos Aires, November 23 – December 17.

7.3. National actions

7.3.1. ANR DECOTES, Tensorial decomposition and applications

Years: 2006–2009.
Partners: I3S, CNRS; LTSI, INSERM; GALAAD, INRIA; SBP, Thales communications.

The problem of decomposition of a symmetric or non-symmetric tensor in minimal way is an important problem, which has applications in many domains. It is essential in the process of Blind Identification of Under-Determined Mixtures (UDM), i.e., linear mixtures with more inputs than observable outputs and appear in many application areas, including speech, mobile communications, machine learning, factor analysis with k-way arrays (MWA), biomedical engineering, psychometrics, and chemometrics. The aim of the project DECOTES is to study the key theoretical problems of such decompositions and to devise numerical algorithms dedicated to some selected applications.

With Elias Tsigaridas, within the framework of a post-doctoral position, we are investigating algebraic methods to compute such a decomposition in order to extend the Sylvester's approach for binary forms to polynomials with more variables.

8. Dissemination

8.1. Animation of the scientific community

8.1.1. Seminar organization

We organize a seminar called "Formes & Formules". The list of talks is archived at http://www-sop.inria.fr/teams/galaad/.

8.1.2. Comittee participations

- Bernard Mourrain was a member of the program committee of the IEEE International Conference on Shape Modeling and Applications (SMI'09), Tsinghua University, Beijing, China, June 26-28, 2009; the conference Algebraic Geometry and Geometric Modeling July 21-26, Lijiang, China; the International Workshop on Symbolic-Numeric Computation (SNC'09) Kyoto, Japan, August 3-5; the 2009 SIAM/ACM Joint Conference on Geometric and Physical Modeling, San Francisco, USA, October 5-8.

8.1.3. Editorial committees

- Evelyne Hubert and Bernard Mourrain are members of the editorial board of the Journal of Symbolic Computation.
- B. Mourrain (with I. Kostireas and V.Y. Pan) are guest editors of the special issue of Theoretical Computer Science related to SNC'09.

8.1.4. Organisation of conferences and schools

- Bernard Mourrain coorganized (with P. Comon and C. Simpson) the workshop on Tensors and interpolation, June 10-12, Nice, France

8.1.5. Ph.D. thesis committees

- Bernard Mourrain was a member of the PhD committee of D. N'Diatta (Univ. Limoges), and of G. Tzoumas (National Kapodistrian Univ. of Athens, Greece).

8.1.6. Other comittees

- L. Busé is an elected member of the administrative council of the SMF (the French Mathematical Society).
- A. Galligo is one of the 3 members of the steering committee of ISSAC.
- Evelyne Hubert served as grant referee for NSA, Fulbright (USA), EPSRC (UK), NSERC (Canada), ANR (France). She is a member of the advisory board of MEGA.
- Bernard Mourrain served as grant referee for ANR (France). He is also a member of the advisory board of MEGA. He is chair of the local INRIA Committee for Courses and Conferences.

8.1.7. WWW server

- http://www-sop.inria.fr/teams/galaad. The website of the team, supported by the Joomla CMS, was migrated to the latest Joomla version.
- http://www-sop.inria.fr/teams/galaad/wiki. We launched a MediaWiki platform where the members of the project collect content related to their research using this fast and collaborative interface.

8.2. Participation at conferences and invitations

- Elimane Ba participated to the *Spring school "Local Algebra"* in Nice (March 30 - April 3).
- Cristina Bertone gave a talk at the conference MEGA (Barcelona, Spain, June 15-19). She participated to the Workshop "Geometrical and Combinatorial aspects of Commutative Algebra" (Villafranca Messina, Italy, September 16-18).
- Laurent Busé was invited to give a course at the two week (July 13-24) *Summer School on Computational Mathematics: Applied Computational Algebraic Geometric Modelling*, Soria, Spain. He was invited to Paris VI University for the periods: January 7-10, April 8-10, November 24-26. He also attended the administrative councils of SMF in Paris: May 6, October 9-11.

- Daouda Niang Diatta was invited at the seminar of the project-team SALSA INRIA/LIP6 University of Paris 6 (March 27) and he gave a talk on The Computation of the Topology of the Intersection of two Parametric Surfaces. He was invited at the seminar on Real Algebraic Geometry of the University of Rennes 1 (June 3) and gave a talk on Isotopic Meshing of an Implicit Algebraic Curve. He participated to the CIMPA meeting at Yaoundé Cameroun (August 24 - September 4) and gave a talk on Topology Computation.
- André Galligo participated to the conferences *MEGA* (June 15-19) in Barcelona and SMAI in Villeneuve-Loubet (May 25-29). He presented a paper in the *3rd International Workshop on Symbolic-Numeric Computation (SNC), Kyoto, Japan* (August 3-5) SNC'09, (Kyoto) and gave a talk at the Workshop on Geometrical and Combinatorial aspects of Commmutative Algebra (September 16-18) in Messina, Italy.
- Evelyne Hubert was invited to participate and give a talk at the *Differential Equations Workshop* (February 6-8) at RISC, Austria organized by F. Winkler and E. Shemyakova as an action of the European project Science. She participated to the two work meeting (March 19-20, October 14-15) for the PlantScan3D project in Montpellier. She was invited to present her reseach at the *International Conference on Mathematics Mechanization - In honor of Professor Wen-Tsun Wu's Ninetieth Birthday* (May 11-13) and visit the *Key Laboratory of Mathematics Mechanization* in Beijing, China (May 4-8). She was a guest lecturer at the *Discrete Systems and Special Functions* workshop held at the Newton Institute, Campbridge, UK (June 29 - July 3). She delivered a seminar talk (September 21) in the project-team Algorithms in Rocquencourt, France.
- Angelos Mantzaflaris attended the two week (July 13-24) *Summer School on Computational Mathematics: Applied Computational Algebraic Geometric Modelling, Soria, Spain* and gave a talk there. He presented a paper in the *34th International Symposium on Symbolic and Algebraic Computation (ISSAC), KIAS, Seoul, Korea* (July 28-31) as well as in the *3rd International Workshop on Symbolic-Numeric Computation (SNC), Kyoto, Japan* (August 3-5). He delivered a talk at the *4th Athens Colloquium on Algorithms and Complexity (ACAC), Athens, Greece* (August 20-21) and the *International Conference on Modern Mathematical Methods in Science and Technology (M3ST), Poros, Greece* (September 3-5). He presented his work at *Algorithms Project's Seminar*, Paris - Rocquencourt, France (September 28).
- Bernard Mourrain gave a talk at the conferences *MEGA* (June 15-19) in Barcelone, participated to the annual meeting of SMAI in Villeneuve-Loubet (May 25-29), gave a talk at the conference *Algebraic Geometry and Geometric Modeling*, July 21-26, 2009, Lijiang, China, participated to the work meeting (March 19-20), for the PlantScan3D project in Montpellier, participated to the *3rd International Workshop on Symbolic-Numeric Computation* (SNC'09), Kyoto, Japan, August 3-5; participated to the meetings of the EU project Exciting at Athens, Greece, March 22-24 and at Gershing, Munich, Germany, October 12-14; gave an invited talk at the Workshop on *Complexity of Numerical Computation*, Fields Institute, Toronto, Canada, October 20-24; gave an invited talk at the conference for the 60th birthday of Michelle Schatzman, Lyon, December 8-9.
- Elias Tsigaridas visited Dept. of Informatics, University of Athens and was invited to give talk on 31 October 2008, about "Real solving polynomials and continued fraction. Algorithms and Complexity" at the $\mu\Pi\lambda\forall$ seminar. He was invited to give a talk at the *DIAMANT seminar day, at CWI, Netherlands* on 13 March 2009, about "Algebraic algorithms and (some) applications". He was invited to give a talk at Dept. of Mathematics, University of Athens on 30 April 2009, in "Some problems in computational geometry". He was invited at RISC institute at Linz, Austria and gave two talks, on at the *RISC forum* about "(exact) Algebraic algorithms and (some) applications", 11 May 2009, and one on "Real solving in 1D, 2D, and beyond" at *RISC, Computer Algebra seminar*, on 14 May 2009. He was invited to give a talk on "Using tensors and polynomial system solvers to follow fibers in the brain" at *ECP seminar, Ecole Centrale Paris*, on 26 May 2009. He participated at the *Workshop on tensors and interpolation* held at Nice, and gave a talk on "Symmetric tensor decomposition". He participated at MEGA 2009 conference at Barcelona, Spain,

and gave a talk “On the complexity of complex root isolation”, on 16 June 2009. He participated at *Panhellenic Logic Symposium* at Patras, Greece and gave a talk on “Quantifier elimination for small degree polynomials” on 16 July 2009. He was invited at the Dept. of Computer Science at Århus, Denmark, and gave a talk “Computational Mathematics Seminar, Dept. of Computer Science”, on 23 July 2009, at the *Computational Mathematics Seminar*. Finaly, we partipated at the *European Conference on Signal Processing (EUSIPCO)*, at Glasgow, Scotland, and gave a talk, “Symmetric Tensor Decomposition”, on 25 August 2009.

- Gang Xu gave a talk at the conference *Algebraic Geometry and Geometric Modeling*, July 21-26, 2009, Lijiang, China, about “Parametrization of computational domain for isogeometric analysis”. He gave two talks at the *IEEE International Conference on CAD/Graphics 2009*, August 19-21, Yellow Mountain, China, about “Direct manipulation of RDMS free form deformation” and “Detail-preserving sculpting deformation”. Finally, he gave a talk at the meeting of the EU project Exciting at Gershing, Munich, Germany, October 12-14, entitled “Analysis-aware optimal parametrization of computational domain in isogeometric analysis”.

8.3. Formation

8.3.1. Teaching at Universities

- Elimane Ba gave 40 hours of tutorials to the L2 SM and 24 hours to the L1 MP at the UNSA.
- Laurent Busé gave an invited course of 8 hours at the "Summer School on Computational Mathematics", SORIA, Spain 13-24 July.
- Mohamed Elkadi gave a course of 18 hours at L1 info and 38 hours of TD; 25 hours of TDs at L1 SV; 30 hours at Master2 math; 84 hours of TDS for CAPES and 7 hours of help to students.
- André Galligo gave course and tutorials in Master 1 *Curves and surfaces* and *Applied Algebra* at UNSA. He also gave tutorials to the L1 SVT, L1 SVT and L2 Sc Eco, UNSA.
- Evelyne Hubert participated to the jury *Modeling-Computer Algebra* for the *Agregation de Mathématiques* (July).
- Bernard Mourrain gave an introductive course on *Algebraic geometry and Geometric Modeling* at ENS Cachan, September 10, 2009 and at Univ. Limoges, Master II, Septembre 28.

8.3.2. PhD theses in progress

- Elimane Ba, *Résultants, calculs et applications*, UNSA.
- Cristina Bertone, *Décomposition irréductible de courbes gauches*, Univ. Turino, Italia.
- Jérôme Brachat, *Dualité effective pour la résolution d’équations polynomiales*, bourse AMX, UNSA.
- Angelos Mantzaflaris, *Robust algebraic methods for geometric computations*, bourse ITN SAGA, UNSA.
- Thang Luu Ba, *Using matrix-based representations for CAGD*, bourse du gouvernement vietnamien and INRIA, UNSA.

8.3.3. Defended PhD thesis

- Daouda N’Diatta, *Résultants et sous-résultants et applications*, Univ. Limoges [11].

8.3.4. Internships

See the web page of our internships.

- Christopher Goyet, *Ordre total de réductibilité d’un pinceau de courbes algébriques planes*, March 1st - June 30, M2 of maths at UNSA.
- Julie Puig, *Objects dynamiques dans* AXEL, July 15 - September 15.

9. Bibliography

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Doctoral Dissertations and Habilitation Theses

[11] D. DIATTA. *Calcul effectif de la topologie de courbes et surfaces algébriques réelles*, Université de Limoges, 09 2009, http://tel.archives-ouvertes.fr/tel-00438817/en/, Ph. D. Thesis.

Articles in International Peer-Reviewed Journal

[12] L. ALBERTI, B. MOURRAIN, J.-P. TÉCOURT. *Isotopic triangulation of a real algebraic surface*, in "Journal of Symbolic Computation", vol. 44, n^{o} 9, 2009, p. 1291-1310, http://hal.inria.fr/inria-00433141/en/.

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