INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team Geometrica

Geometric Computing

Saclay - Île-de-France, Sophia Antipolis - Méditerranée

Theme : Algorithms, Certification, and Cryptography

Activity Report

2009

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2. Overall Objectives

2.1. Overall Objectives

Geometric computing plays a central role in most engineering activities: geometric modelling, computer aided design and manufacturing, computer graphics and virtual reality, scientific visualization, geographic information systems, molecular biology, fluid mechanics, and robotics are just a few well-known examples. The rapid advances in visualization systems, networking facilities and 3D sensing and imaging make geometric computing both dominant and more demanding concerning effective algorithmic solutions.

Computational geometry emerged as a discipline in the seventies and has met with considerable success in resolving the asymptotic complexity of basic geometric data structures and problems, including convex hulls, triangulations, Voronoi diagrams, geometric arrangements and geometric optimization. However, in the mid-nineties, it was recognized that the applicability in practice of computational geometry techniques was far from satisfactory and a vigorous effort was undertaken to make computational geometry more effective. The PRISME project together with several partners in Europe took a prominent role in this research and in the development of a large library of computational geometry algorithms, CGAL.

GEOMETRICA aims at pursuing further the effort in this direction and at building upon the initial success. Its focus is on effective computational geometry with special emphasis on *curves and surfaces*. This is a challenging research area with a huge number of potential applications in almost all application domains involving geometric computing.

The overall objective of the project is to give effective computational geometry for curves and surfaces solid mathematical and algorithmic foundations, to provide solutions to key problems and to validate theoretical advances through extensive experimental research and the development of software packages that could serve as steps toward a standard for reliable and effective geometric computing.

2.2. Highlights

2009 has been rich in new contributions from GEOMETRICA to CGAL, with the following new components: 3D periodic triangulations, 3D mesh generation, surface reconstruction from point sets, point set processing and an AABB tree data structure for efficient intersection and distance computations.

Pierre Alliez became an associate editor of the ACM Transactions on Graphics. Monique Teillaud became an editor of CGTA, Computational Geometry: Theory and Applications. She was elected a member of the Computational Geometry Steering Committee.

3. Scientific Foundations

3.1. Mesh Generation and Geometry Processing

Meshes are becoming commonplace in a number of applications ranging from engineering to multimedia through biomedecine and geology. For rendering, the quality of a mesh refers to its approximation properties. For numerical simulation, a mesh is not only required to faithfully approximate the domain of simulation, but also to satisfy size as well as shape constraints. The elaboration of algorithms for automatic mesh generation is a notoriously difficult task as it involves numerous geometric components: Complex data structures and algorithms, surface approximation, robustness as well as scalability issues. The recent trend to reconstruct domain boundaries from measurements adds even further hurdles. Armed with our experience on triangulations and algorithms, and with components from the CGAL library, we aim at devising robust algorithms for 2D, surface, 3D mesh generation [9] as well as anisotropic meshes. Our research in mesh generation primarily focuses on the generation of simplicial meshes, i.e., triangular and tetrahedral meshes. We investigate both greedy approaches based upon Delaunay refinement and filtering, and variational approaches based upon energy functionals and associated minimizers [22], [13].

New methods and tools to process digital geometry in computer graphics and computational science are crucially needed. Geometry processing is motivated by the fact that previous attempts to adapt common signal processing methods have led to limited success: Shapes are not just another signal but a new challenge to face due to distinctive properties of complex shapes such as topology, metric, lack of global parameterization, non-uniform sampling and irregular discretization. Our research in geometry processing ranges from surface reconstruction to surface remeshing through curvature estimation, principal component analysis, surface approximation[6] and surface mesh parameterization.

3.2. Topological and Geometric Inference

Due to the fast evolution of data acquisition devices and computational power, scientists in many areas are demanding efficient algorithmic tools for analyzing, manipulating and visualizing more and more complex shapes or complex systems from approximating data. Many of the existing algorithmic solutions which come with little theoretical guarantee provide unsatisfactory and/or unpredictable results. Since these algorithms take as input discrete geometric data, it is mandatory to develop concepts that are rich enough to robustly and correctly approximate continuous shapes and their geometric properties by discrete models. Ensuring the correctness of geometric estimations and approximations on discrete data is a sensitive problem in many applications [7].

Data sets being often represented as point sets in high dimensional spaces, there is a considerable interest in analyzing and processing data in such spaces. Although these point sets usually live in high dimensional spaces, one often expects them to be located around unknown, possibly non linear, low dimensional shapes. These shapes are usually assumed to be smooth submanifolds or more generally compact subsets of the ambient space. It is then desirable to infer topological (dimension, Betti numbers,...) and geometric characteristics (singularities, volume, curvature,...) of these shapes from the data. The hope is that this information will help to better understand the underlying complex systems from which the data are generated. In spite of recent promising results, many problems still remain open and to be addressed, need a tight collaboration between mathematicians and computer scientists. In this context our goal is to contribute to the development of new mathematically well founded and algorithmically efficient geometric tools for data analysis and processing of complex geometric objects. Our main targeted areas of application include machine learning, data mining, statistical analysis, and sensor networks.

3.3. Data Structures and Robust Geometric Computation

GEOMETRICA has a large expertise of algorithms and data structures for geometric problems [2]. We are pursuing efforts to design efficient algorithms from a theoretical point of view, but we also put efforts in the effective implementation of these results [10].

In the past years, we made significant contributions to algorithms for computing Delaunay triangulations [8] (which are used by meshes in the above paragraph). We are still working on the practical efficiency of existing algorithms to compute or to exploit classical Euclidean triangulations in 2 and 3 dimensions, but the current focus of our research is more aimed toward extending the triangulation efforts in several new directions of research.

One of these directions is the triangulation of non Euclidean spaces such as periodic or projective spaces, with various potential applications ranging from astronomy to granular material simulation.

Another direction is the triangulation of moving points, with potential applications to fluid dynamics where the points represent some particles of some evolving physical material, and to variational methods devised to optimize point placement for meshing a domain with a high quality elements [24].

Increasing the dimension of space is also a stimulating direction of research, as triangulating points in medium dimension (say 4 to 15) has potential applications and makes new challenges to trade exponential complexity of the problem in the dimension for the possibility to reach effective and practical results in reasonably small dimensions [26].

On the complexity analysis side, we pursue efforts to obtain complexity analysis in some practical situation involving randomized or stochastic hypotheses. On the algorithm design side, we are looking for new paradigms to exploit parallelism on modern multicore hardware architectures.

Finally, all this work is done while keeping in mind concerns related to effective implementation of our work, practical efficiency and robustness issues [3], [4] which have become a background task of all different works made by GEOMETRICA.

4. Application Domains

4.1. Geometric Modeling and Shape Reconstruction

Modeling 3D shapes is required for all visualization applications where interactivity is a key feature since the observer can change the viewpoint and get an immediate feedback. This interactivity enhances the descriptive power of the medium significantly. For example, visualization of complex molecules helps drug designers to understand their structure. Multimedia applications also involve interactive visualization and include e-commerce (companies can present their products realistically), 3D games, animation and special effects in motion pictures. The uses of geometric modeling also cover the spectrum of engineering, computer-aided design and manufacture applications (CAD/CAM). More and more stages of the industrial development and production pipeline are now performed by simulation, due to the increased performance of numerical simulation packages. Geometric modeling therefore plays an increasingly important role in this area. Another emerging application of geometric modeling with high impact is medical visualization and simulation.

In a broad sense, shape reconstruction consists of creating digital models of real objects from points. Example application areas where such a process is involved are Computer Aided Geometric Design (making a car model from a clay mockup), medical imaging (reconstructing an organ from medical data), geology (modeling underground strata from seismic data), or cultural heritage projects (making models of ancient and or fragile models or places). The availability of accurate and fast scanning devices has also made the reproduction of real objects more effective such that additional fields of applications are coming into reach. The members of GEOMETRICA have a long experience in shape reconstruction and contributed several original methods based upon the Delaunay and Voronoi diagrams.

4.2. Scientific Computing

Meshes are the basic tools for scientific computing using finite element methods. Unstructured meshes are used to discretize domains bounded by complex shapes while allowing local refinements. GEOMETRICA contributes to mesh generation of 2D and 3D possibly curved domains. Most of our methods are based upon Delaunay triangulations, Voronoi diagrams and their variants. Anisotropic meshes are also investigated. We investigate in parallel both greedy and variational mesh generation techniques. The greedy algorithms consist of inserting vertices in an initial coarse mesh using the Delaunay refinement paradigm, while the variational algorithms consists of minimizing an energy related to the shape and size of the elements. Our goal is to show the complementarity of these two paradigms. Quadrangle surface meshes are also of interest for reverse engineering and geometry processing applications. Our approach consists of sampling a set of curves on the surface so as to control the final edge alignment, the mesh sizing and the regularity of the quadrangle tiling.

5. Software

5.1. CGAL, the Computational Geometry Algorithms Library

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Manuel Caroli, Olivier Devillers, Michael Hemmer, Samuel Hornus, Pedro Machado Manhães de Castro, Sylvain Pion, Laurent Saboret, Stéphane Tayeb, Monique Teillaud, Mariette Yvinec.

With the collaboration of Hervé Brönnimann, Frédéric Cazals, Frank Da, Christophe Delage, Andreas Fabri, Julia Flötotto, Philippe Guigue, Menelaos Karavelas, Sébastien Loriot, Abdelkrim Mebarki, Naceur Meskini, Andreas Meyer, Marc Pouget, François Rebufat, Laurent Rineau, Radu Ursu, and Camille Wormser. http://www.cgal.org

CGAL is a C++ library of geometric algorithms and data structures. Its development has been initially funded and further supported by several European projects (CGAL, GALIA, ECG, ACS, AIM@SHAPE) since 1996. The long term partners of the project are research teams from the following institutes: INRIA Sophia Antipolis - Méditerranée, Max-Planck Institut Saarbrücken, ETH Zürich, Tel Aviv University, together with several others. In 2003, CGAL became an Open Source project (under the LGPL and QPL licenses), and it also became commercialized by GEOMETRY FACTORY, a company *Born of INRIA* founded by Andreas Fabri.

The aim of the CGAL project is to create a platform for geometric computing supporting usage in both industry and academia. The main design goals are genericity, numerical robustness, efficiency and ease of use. These goals are enforced by a review of all submissions managed by an editorial board. As the focus is on fundamental geometric algorithms and data structures, the target application domains are numerous: from geological modeling to medical images, from antenna placement to geographic information systems, etc.

The CGAL library consists of a kernel, a list of algorithmic packages, and a support library. The kernel is made of classes that represent elementary geometric objects (points, vectors, lines, segments, planes, simplices, isothetic boxes, circles, spheres, circular arcs...), as well as affine transformations and a number of predicates and geometric constructions over these objects. These classes exist in dimensions 2 and 3 (static dimension) and d (dynamic dimension). Using the template mechanism, each class can be instantiated following several representation modes : one can choose between Cartesian or homogeneous coordinates, use different types to store the coordinates, and use reference counting or not. The kernel also provides some robustness features using some specifically-devised arithmetic (interval arithmetic, multi-precision arithmetic, static filters...).

A number of packages provide geometric data structures as well as algorithms. The data structures are polygons, polyhedra, triangulations, planar maps, arrangements and various search structures (segment trees, d-dimensional trees...). Algorithms are provided to compute convex hulls, Voronoi diagrams, Boolean operations on polygons, solve certain optimization problems (linear, quadratic, generalized of linear type). Through class and function templates, these algorithms can be used either with the kernel objects or with user-defined geometric classes provided they match a documented interface.

Finally, the support library provides random generators, and interfacing code with other libraries, tools, or file formats (ASCII files, QT or LEDA Windows, OpenGL, Open Inventor, Postscript, Geomview...). Partial interfaces with Python, SCILAB and the Ipe drawing editor are now also available.

GEOMETRICA is particularly involved in general maintainance, in the arithmetic issues that arise in the treatment of robustness issues, in the kernel, in triangulation packages and their close applications such as alpha shapes, in meshes... Four researchers of GEOMETRICA are members of the CGAL Editorial Board, whose main responsibilities are the control of the quality of CGAL, making decisions about technical matters, coordinating communication and promotion of CGAL.

CGAL is about 700,000 lines of code and supports various platforms: GCC (Linux, Mac OS X, Cygwin...), Visual C++ (Windows), Intel C++... A new version of CGAL is released approximately twice a year, and it is downloaded about 10000 times a year. Moreover, CGAL is directly available as packages for the Debian, Ubuntu and Fedora Linux distributions.

More numbers about CGAL: there are now 14 editors in the editorial board, with approximately 20 additional developers. The user discussion mailing-list has more than 1000 subscribers with a relatively high traffic of 5-10 mails a day. The announcement mailing-list has more than 3000 subscribers.

5.2. CULT, the Computational Unified Library for Topology

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, Arijit Ghosh, Marc Glisse, Samuel Hornus, Steve Oudot, Primoz Skraba.

With the collaboration of Gunnar Carlsson Vin de Silva, Leonidas Guibas, Dmitriy Morowov, and Michael Vejdemo-Johansson. http://gforge.inria.fr/projects/cult

Since April 2009, GEOMETRICA has been part of a joint effort with the Geometric Computation and Applied Topology groups at Stanford University to develop the successor of the PLEX library, now considered a standard in computational topology. The new library, named CULT (Computational Unified Library for Topology), will offer a vast set of tools that reflects the state of the art in the field. It should be able to handle very large and high-dimensional data sets within reasonable amount of time and space, while keeping enough flexibility for people to be able to build their own algorithms upon it. Potential interactions with other libraries like CGAL will also be investigated.

Making the library effective will require a lot of work on designing new data structures whose complexities scale up reasonably with the size and dimensionality of the input data. Usually, people have to deal with complexes containing dozens of millions of simplices. Such complexes do not fit into main memory, at least with a naive data representation. On the other hand, brute-force compression of the data results in prohibitively slow access times. A trade-off between space and time needs to be found, which will be one of the research topics of GEOMETRICA for 2010.

6. New Results

6.1. Mesh Generation and Geometry Processing

6.1.1. Mesh Optimization

Participants: Pierre Alliez, Jane Tournois.

In collaboration with Camille Wormser from ETH Zürich and Mathieu Desbrun from Caltech.

We propose a practical approach to isotropic tetrahedron meshing of 3D domains bounded by piecewise smooth surfaces. Building upon recent theoretical and practical advances, our algorithm interleaves Delaunay refinement and mesh optimization to generate quality meshes that satisfy a set of user-defined criteria. This interleaving is shown to be more conservative in number of Steiner point insertions than refinement alone, and to produce higher quality meshes than optimization alone (see Figure 1). A careful treatment of boundaries and their features is presented, offering a versatile framework for designing quality isotropic tetrahedron meshes [22].

6.1.2. Sliver Removal

Participants: Pierre Alliez, Jane Tournois.

In collaboration with Rahul Srinivasan, IIT Bombay.

Isotropic tetrahedron meshes generated by Delaunay refinement algorithms are known to contain a majority of well-shaped tetrahedra, as well as spurious sliver tetrahedra. As the slivers hamper stability of numerical simulations we aim at removing them while keeping the Delaunay property of the triangulation for simplicity. The solution which explicitly perturbs the slivers through random vertex relocation and Delaunay connectivity update is very effective but slow. In this paper we present a perturbation algorithm which favors deterministic over random perturbations. The added value is an improved efficiency and effectiveness. Our experimental study applies the proposed algorithm to meshes obtained by Delaunay refinement as well as to carefully optimized meshes [35].

6.1.3. Feature Preserving Delaunay Mesh Generation from 3D Multi-Material Images

Participants: Jean-Daniel Boissonnat, Dobrina Boltcheva, Mariette Yvinec.

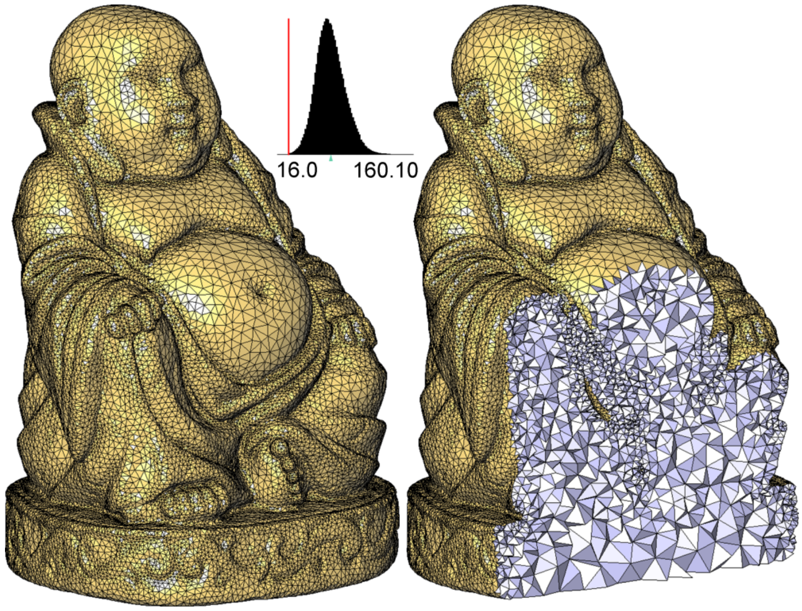

Figure 1. Isotropic tetrahedron mesh generated by interleaving Delaunay refinement and optimization. The distribution of tetrahedron dihedral angles is shown (here no angle is lower than 16 degrees).

Generating realistic geometric models from 3D segmented images is an important task in many biomedical applications. Segmented 3D images impose particular challenges for meshing algorithms because they contain multi-material junctions forming features such as surface patches, edges and corners. The resulting meshes should preserve these features to ensure the visual quality and the mechanical validity of the models. We present a feature preserving Delaunay refinement algorithm which can be used to generate high-quality tetrahedral meshes from segmented images. See Figure 2. The idea is to explicitly sample corners and edges from the input image and to constrain the Delaunay refinement algorithm to preserve these features in addition to the surface patches. Our experimental results on segmented medical images have shown that, within a few seconds, the algorithm outputs a tetrahedral mesh in which each material is represented as a consistent submesh without gaps and overlaps. The optimization property of the Delaunay triangulation makes these meshes suitable for the purpose of realistic visualization or finite element simulations [27], [15].

6.1.4. Discrete Critical Values: a General Framework for Silhouettes Computations

Participants: Frédéric Chazal, Nicolas Montana.

This work has been done in collaboration with A. Lieutier (Dassault Systèmes).

Many shapes resulting from important geometric operations in industrial applications such as Minkowski sums or volume swept by a moving object can be seen as the projection of higher dimensional objects. When such a higher dimensional object is a smooth manifold, the boundary of the projected shape can be computed from the critical points of the projection. In this work, using the notion of polyhedral chains introduced by Whitney, we introduce a new general framework to define an analogous of the set of critical points of piecewise linear maps defined over discrete objects that can be easily computed. We illustrate our results by showing how they can be used to compute Minkowski sums of polyhedra and volumes swept by moving polyhedra [19], [61].

6.2. Data Structures and Robust Geometric Computation

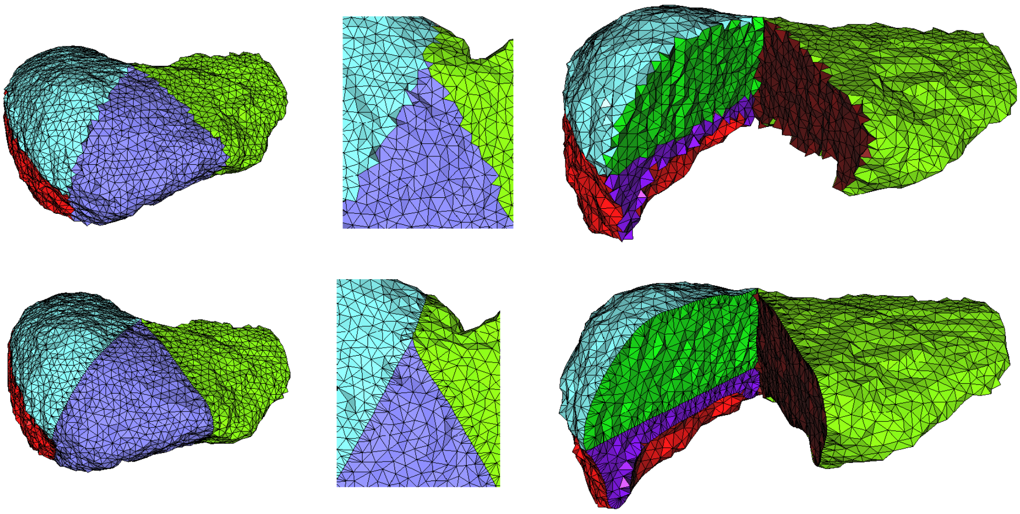

Figure 2. Meshes generated from a segmented liver image representing 4 anatomical liver regions. The first raw shows meshes obtained with the usual Delaunay refinement algorithm. The second raw shows meshes generated with our feature preserving extension. The 3rd column shows some internal interfaces between the anatomical regions.

6.2.1. Parallel Geometric Algorithms for Multi-Core Computers

Participant: Sylvain Pion.

In collaboration with Vicente Batista (former INRIA intern), David Millman from University of North Carolina at Chapel Hill, and Johannes Singler from Universität Karlsruhe.

Computers with multiple processor cores using shared memory are now ubiquitous. We present several parallel geometric algorithms that specifically target this environment, with the goal of exploiting the additional computing power. The d-dimensional algorithms we describe are (a) spatial sorting of points, as is typically used for preprocessing before using incremental algorithms, (b) kd-tree construction, (c) axis-aligned box intersection computation, and finally (d) bulk insertion of points in Delaunay triangulations for mesh generation algorithms or simply computing Delaunay triangulations. We show experimental results for these algorithms in 3D, using our implementations based on CGAL. This work is a step towards what we hope will become a *parallel mode* for CGAL, where algorithms automatically use the available parallel resources without requiring significant user intervention [25].

6.2.2. Delaunay Triangulation of Points in Higher Dimensions

Participants: Jean-Daniel Boissonnat, Olivier Devillers, Samuel Hornus.

We propose a new C++ implementation of the well-known incremental algorithm for the construction of Delaunay triangulations in any dimension. Our implementation follows the exact computing paradigm and is hence robust. An extensive series of comparisons have shown that our implementation outperforms the best available implementations for convex hulls and Delaunay triangulations, and that it can be used for large point sets in spaces of dimensions up to 6 [26].

To circumvent prohibitive memory usage, we also propose a modification of the algorithm that uses and stores only the Delaunay graph (the edges of the full triangulation). We show that a careful implementation of the modified algorithm performs only 6 to 8 times slower than the original algorithm while drastically reducing memory usage in dimension 4 or above.

6.2.3. Filtering Relocations on a Delaunay Triangulation

Participants: Pierre Alliez, Pedro Machado Manhães de Castro, Olivier Devillers, Jane Tournois.

Updating a Delaunay triangulation with moving vertices is a bottleneck in several domains of application. Rebuilding the whole triangulation from scratch is surprisingly a very viable option compared to relocating the vertices. This can be explained by several recent advances in efficient construction of Delaunay triangulations. However, when all points move with a small magnitude, or when only a fraction of the vertices move, rebuilding is no longer the best option. This paper considers the problem of efficiently updating a Delaunay triangulation when its vertices are moving under small perturbations. The main contribution is a set of filters based upon the concept of vertex tolerances. Experiments show that filtering relocations is faster than rebuilding the whole triangulation from scratch under certain conditions [24].

6.2.4. Point Location Strategies in Delaunay Triangulation

Participants: Pedro Machado Manhães de Castro, Olivier Devillers.

Point location in spatial subdivision is one of the most well-known problem in computational geometry. In the case of triangulations of $\mathbb{R}^d$, we revisit the problem to exploit a possible coherence between the query points.

For a single query, walking in the triangulation is a classical strategy with good practical behavior and expected complexity $O(n^{\frac{1}{d}})$ if the points are evenly distributed. For a batch of query points, the main idea is to use previous queries to improve the current one; we compare various strategies that have an influence on the constant hidden in the big-O notation.

Still to improve a query, close to another one, we show how the Delaunay hierarchy can be used to answer a query q with a $O(\log \#(pq))$ randomized expected complexity, where $\#(.)$ indicates the number of simplexes crossed by the line pq, and p is a previously located query. The data structure has $O(n \log n)$ construction complexity and $O(n)$ memory complexity [60].

6.2.5. Efficient Static and Dynamic Proximity Queries using Locality-Sensitive Hashing

Participant: Steve Oudot.

This work has been carried out in collaboration with David Arthur and Aneesh Sharma, both from Stanford University.

The approximate Nearest Neighbor search problem (NN) asks to pre-process a given set of points P in such a way that, given any query point q, one can retrieve a point in P that is *approximately* closest to q. Of particular interest is the case of points lying in high dimensions, which has seen rapid developments since the introduction of the Locality-Sensitive Hashing (LSH) data structure by Indyk and Motwani. Combined with a space decomposition by Har-Peled, the LSH data structure can answer approximate NN queries in sub-linear time using polynomial (in both d and n) space. Unfortunately, it is not known whether Har-Peled's space decomposition can be maintained efficiently under point insertions and deletions, so the above solution only works in a static setting.

In this work we present a variant of Har-Peled's decomposition, based on random semi-regular grids, which can achieve the same query time with the added advantage that it can be maintained efficiently even under adversarial point insertions and deletions. The outcome is a new data structure to answer approximate NN queries efficiently in dynamic settings.

Another related problem known as Reverse Nearest Neighbor search (RNN) is to find the *influence set* of a given query point q, i.e. the subset of points of P that have q as their nearest neighbor. Although this problem finds many practical applications, very little is known about its complexity. In particular, no algorithm is known to solve it in high dimensions in sub-linear time using sub-exponential space. In this work we show how to pre-process the data points, so that Har-Peled's space decomposition combined with modified LSH data structures can solve an approximate variant of the RNN problem efficiently, using polynomial space. The query time of our approach is bounded by two terms: the first one is sub-linear in the size of P and corresponds roughly to the incompressible time needed to locate the query point in the data structure; the second one is proportional to the size of the output, which is a set of points as opposed to a single point for (approximate)

NN queries. An interesting feature of our RNN solution is that it is flexible enough to be applied indifferently in monochromatic or bichromatic settings [50].

6.2.6. Are Extreme Points Robust to Perturbations ?

Participant: Olivier Devillers.

This work has been done in collaboration with Dominique Attali (GipsaLab) and Xavier Goaoc (Loria).

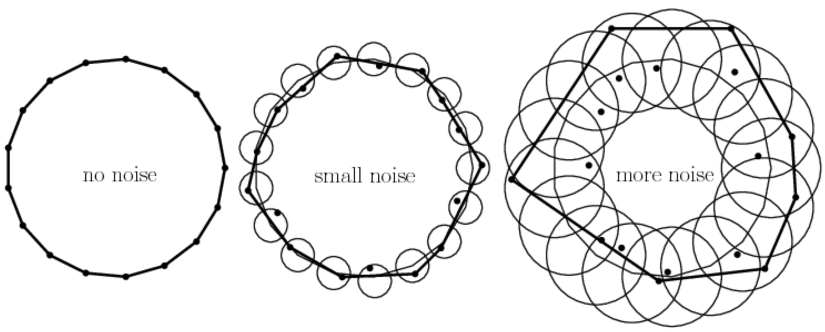

Figure 3. Noise reduce the complexity

Assume that X' is a noisy version of a point set X in convex position (all points are vertices of their convex hull). How many extreme points does X' have? (See Figure 3).

We consider the case where X is an (ϵ, κ)-sample of a sphere in $\mathbb{R}^d$ and the noise is random and uniform: X' is obtained by replacing each point $x \in X$ by a point chosen randomly uniformly in some region $R(x)$ of size δ around x. We give upper and lower bounds on the expected number of extreme points in X' when $R(x)$ is a ball (in arbitrary dimension) or an axis-parallel square (in the plane). Our bounds depends on the size n of X and δ, and are tight up to a polylogarithmic factor. These results naturally extend in various directions (more general point sets, other regions $R(x)$, more general distributions...).

We also present experimental results that show that our bounds for random noise provide good estimators of the behavior of snap-rounding, where X' is obtained by rounding each point of X to the nearest point on a grid of step δ [51].

6.2.7. Deletion in Two Dimensional Delaunay Triangulation: Asymptotic Complexity is Pointless

Participant: Olivier Devillers.

The theoretical complexity of vertex removal in a Delaunay triangulation is often given in terms of the degree d of the removed point with usual results $O(d)$, $O(d \log d)$, or $O(d^2)$. In fact the asymptotic complexity is of poor interest since d is usually quite small. In this paper, we carefully design code for small degrees $3 \le d \le 7$, it improves the global behavior of the removal for random points by a factor of 2 [58].

The new method is implemented and will be submitted for inclusion into CGAL.

6.2.8. Triangulation on the Sphere

Participants: Manuel Caroli, Pedro Machado Manhães de Castro, Monique Teillaud.

This work has been done in collaboration with Sébastien Loriot (ABS), Camille Wormser (ETH Zürich) and Olivier Rouiller (École Centrale Lille).

We propose two ways to compute the Delaunay triangulation of points on a sphere, or of *rounded* points close to a sphere, both based on the classic incremental algorithm initially designed for the plane. (See Figure 4.) We use the so-called space of circles as mathematical background for this work. We present a fully robust implementation built upon existing generic algorithms provided by the CGAL library. The efficiency of the implementation is established by benchmarks [54].

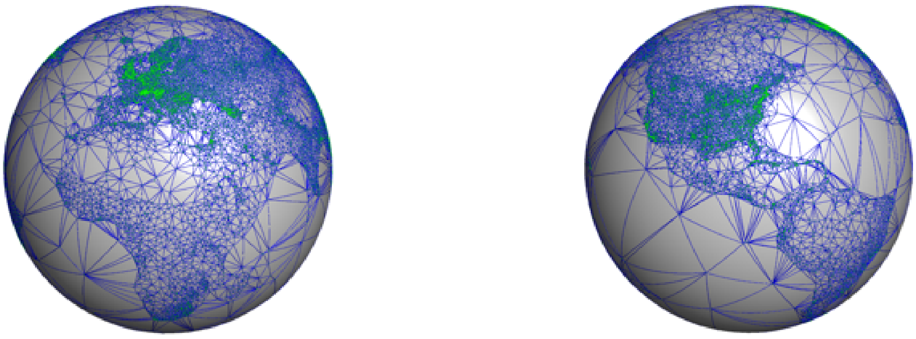

Figure 4. Delaunay triangulation of 20950 weather stations all around the world

6.3. Topological and Geometric Inference

6.3.1. Proximity of Persistence Modules and their Diagrams

Participants: Frédéric Chazal, David Cohen-Steiner, Marc Glisse, Steve Oudot.

This work has been done in collaboration with L. J. Guibas (Stanford University).

Topological persistence has proven to be a key concept for the study of real-valued functions defined over topological spaces. Its validity relies on the fundamental property that the persistence diagrams of nearby functions are close. However, existing stability results are restricted to the case of continuous functions defined over triangulable spaces. In this work, we present new stability results that do not suffer from the above restrictions. Furthermore, by working at an algebraic level directly, we make it possible to compare the persistence diagrams of functions defined over different spaces, thus enabling a variety of new applications of the concept of persistence [29].

6.3.2. Analysis of Scalar Fields over Point Cloud Data and Clustering

Participants: Frédéric Chazal, Steve Oudot, Primoz Skraba.

This work has been done in collaboration with L. J. Guibas (Stanford University).

Given a real-valued function f defined over some metric space X, is it possible to recover some structural information about f from the sole information of its values at a finite set L of sample points on X, whose pairwise distances in X are given? We provide a positive answer to this question. More precisely, taking advantage of recent advances on the front of stability for persistence diagrams, we introduce a novel algebraic construction, based on a pair of nested families of simplicial complexes built on top of the point cloud L, from which the persistence diagram of f can be faithfully approximated. We derive from this construction a series of algorithms for the analysis of scalar fields from point cloud data and for the clustering of point cloud data sets [30], [57].

6.3.3. Gromov-Hausdorff Stable Signatures for Shapes using Persistence

Participants: Frédéric Chazal, David Cohen-Steiner, Steve Oudot.

This work has been done in collaboration with L. J. Guibas and F. Mémoli (Stanford University).

We introduce a family of signatures for finite metric spaces, possibly endowed with real valued functions, based on the persistence diagrams of suitable filtrations built on top of these spaces. We prove the stability of our signatures under Gromov-Hausdorff perturbations of the spaces. We also extend these results to metric spaces equipped with measures. Our signatures are well-suited for the study of unstructured point cloud data, which we illustrate through an application in shape classification [16].

6.3.4. Stability of Curvature Measures

Participants: Frédéric Chazal, David Cohen-Steiner.

This work has been done in collaboration with A. Lieutier (Dassault Systèmes) and B. Thibert (Grenoble University).

We address the problem of curvature estimation from sampled compact sets. The main contribution is a stability result: we show that the Gaussian, mean or anisotropic curvature measures of the offset of a compact set K with positive μ-reach can be estimated by the same curvature measures of the offset of a compact set K' close to K in the Hausdorff sense. We show how these curvature measures can be computed for finite unions of balls. The curvature measures of the offset of a compact set with positive μ-reach can thus be approximated by the curvature measures of the offset of a point-cloud sample. These results can also be interpreted as a framework for an effective and robust notion of curvature [18].

6.3.5. Persistent Homology for Images, Kernels, and Cokernels

Participant: David Cohen-Steiner.

This work has been done in collaboration with H. Edelsbrunner, J. Harer, and D. Morozov (Duke University).

We extend the notion of persistent homology to sequences of images, kernels, and cokernels of maps induced by inclusion in a filtration of pairs of spaces. Specifically, we note that persistence in this context is well defined, we prove that the persistence diagrams are stable, and we explain how to compute them. We also show that image persistence diagrams allow to deal with the case where both the function and its domain are corrupted by noise, whereas classical persistence can only cope with function noise. In particular, we provide an efficient way to estimate the persistent homology of a function known only at a finite set of points [31].

6.3.6. Robust Voronoi-based Feature and Curvature Estimation

Participant: Quentin Mérigot.

This work has been done in collaboration with L. Guibas and M. Ovsjanikov (Stanford).

We present an efficient and robust method for extracting principal curvatures, sharp features and normal directions of a piecewise smooth surface from a point cloud sampling, with theoretical guarantees. Our method is integral and uses convolved covariance matrices of Voronoi cells of the point cloud which makes it provably robust in the presence of noise. We show analytically that our method recovers correct principal curvature directions in smooth parts of the shape, and correct feature directions and feature angles at the sharp edges of a piecewise smooth surface, with the error bounded by the Hausdorff distance between the point cloud and the underlying surface [32].

6.3.7. Numerical Methods for Surface Reconstruction from Point Sets

Participant: Pierre Alliez.

The increasing amount of uncertainty in measurement point sets has motivated a number of approximating surface reconstruction approaches which compute an implicit function such that one of its isosurfaces approximate well the input points. In this article [36] we discuss two numerical reconstruction methods. The first method computes an implicit function through solving for the Poisson problem from points which are enriched with oriented normals. The second method computes an implicit function through solving for a generalized eigenvalue problem and does not require orienting the data points beforehand. These approaches are positioned with respect to recent approaches geared toward an increased level of robustness to noise, sparse sampling and outliers.

6.3.8. Manifold Reconstruction using the Tangential Complex

Participants: Arijit Ghosh, Jean-Daniel Boissonnat.

We give a provably correct algorithm to reconstruct a k-dimensional manifold embedded in d-dimensional Euclidean space [52]. Input to our algorithm is a point sample coming from an unknown manifold. Unlike previous methods, we do not construct any subdivision of the embedding d-dimensional space. As a result, the running time of our algorithm depends only linearly on the extrinsic dimension d while it depends quadratically on the size of the input sample, and exponentially on the intrinsic dimension k. To the best of our knowledge, this is the first certified algorithm for manifold reconstruction whose complexity depends linearly on the ambient dimension. We also prove that for a dense enough sample the output of our algorithm is isotopic to the manifold and a close geometric approximation of the manifold.

6.3.9. Geometric Tomography with Guarantees

Participants: Pooran Memari, Jean-Daniel Boissonnat.

In collaboration with Omid Amini, CNRS-DMA, ENS.

We consider the problem of reconstructing an embedded compact 3-manifold (with boundary) in $\mathbb{R}^3$ from its cross-sections with a given set of cutting planes having arbitrary orientations. Under appropriate sampling conditions that are satisfied when the set of cutting planes is dense enough, we prove that the algorithm presented by Liu et al., as well as a related algorithm based on the Voronoi diagram of the cross-sections, preserves the homotopy type of the object [49]. Using the homotopy equivalence, we also show that the reconstructed object is homeomorphic to the original object. To the best of our knowledge, this is the first time that 3D shape reconstruction from cross-sections comes with such theoretical guarantees.

6.3.10. Surface Reconstruction from Multi-View Stereo

Participants: Nader Salman, Mariette Yvinec.

We describe an original method to reconstruct a 3D scene from a sequence of images. See Figure 5. Our approach uses both the dense 3D point cloud extracted by multi-view stereovision and the calibrated images. It combines depth-maps construction in the image planes with surface reconstruction through restricted Delaunay triangulation. The method may handle very large scale outdoor scenes. Its accuracy has been tested on numerous scenes including the dense multi-view benchmark proposed by Strecha et al. Our results compare favorably with the current state of the art [34], [33].

6.3.11. Manifold Reconstruction in Arbitrary Dimensions using Witness Complexes

Participants: Jean-Daniel Boissonnat, Steve Oudot.

This work has been done in collaboration with Leonidas J. Guibas from Stanford University.

It is a now well-established fact that the witness complex is closely related to the restricted Delaunay triangulation in low dimensions. Specifically, it has been proved that the witness complex coincides with the restricted Delaunay triangulation on curves, and is still a subset of it on surfaces, under mild sampling assumptions. Unfortunately, these results do not extend to higher-dimensional manifolds, even under stronger sampling conditions. In this work, we show how the sets of witnesses and landmarks can be enriched, so that the nice relations that exist between both complexes still hold on higher-dimensional manifolds. We also use our structural results to devise an algorithm that reconstructs manifolds of any arbitrary dimension or co-dimension at different scales. The algorithm combines a farthest-point refinement scheme with a vertex pumping strategy. It is very simple conceptually, and it does not require the input point sample to be sparse. Its time complexity is bounded by $c(d)n^2$, where n is the size of the input and $c(d)$ is a constant depending solely on the dimension d of the ambient space.

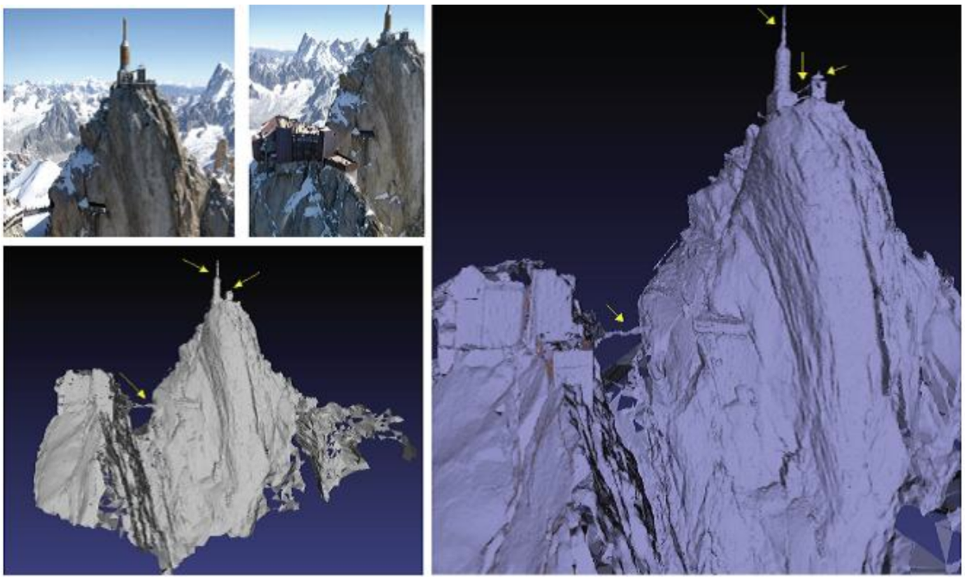

Figure 5. On the left: two images of the Aiguille du midi dataset (©B.Vallet/IMAGINE); below: our low resolution reconstruction (270 000 triangles). On the right: our high resolution reconstruction (1 000 000 triangles). Note how most details pointed by yellow arrows are recovered in both resolutions.

6.4. Software

6.4.1. CGAL

A major new release of CGAL, version 3.5, has been been made available in October 2009. Besides improvements to existing packages, this release contains the following new packages implemented by GEOMETRICA:
— *3D Mesh Generation* [45]. This package generates 3 dimensional meshes. It computes isotropic simplicial meshes for domains or multidomains provided that a domain descriptor, able to answer queries from a few different types on the domain, is given. In the current version, the package generates meshes for domains described through implicit functions, 3D images or polyhedral boundaries. The output is a 3D mesh of the domain volume and conformal surface meshes for all the boundary and subdividing surfaces.
— *Surface Reconstruction from Point Sets* [40]. This package implements an implicit surface reconstruction method: Poisson Surface Reconstruction. The input is an unorganized point set with oriented normals.
— *3D Periodic Triangulations* [43]. This package allows to build and handle triangulations of point sets in the three dimensional flat torus. Triangulations are built incrementally and can be modified by insertion or removal of vertices. They offer point location facilities.
— *Point Set Processing* [41]. This package implements a set of algorithms for analysis, processing, and normal estimation and orientation of point sets.
— *AABB tree* [42]. This package implements a hierarchy of axis-aligned bounding boxes (an AABB tree) for efficient intersection and distance computations between 3D queries and sets of bounded 3D geometric objects.
— *CGAL_ipelets* [44]. Object that eases the writing of Ipe's plugins that use CGAL. Plugins for several CGAL 2D algorithms are provided as demo.

Also, an important extension of the 3D Spherical Kernel, for operations on circular arcs lying on a common sphere, was released in CGAL 3.5 [46].

The new release also contains new packages implemented by our CGAL partners and improvements to some existing packages: a detailed list can be found on the CGAL web site.

CGAL has been presented as a demo paper at ACM SIGSPATIAL GIS 2009, November 4-6 in Seattle. A course at SIGGRAPH Asia will be provided December 2009 in Yokohama, Japan, by A. Fabri and P. Alliez.

This year, two one-week CGAL developers meetings have been organized: in January in Dagstuhl and in September at ETH Zürich. Sylvain Pion, Monique Teillaud, Mariette Yvinec, Samuel Hornus, Stéphane Tayeb, Manuel Caroli and Pedro Machado Manhães de Castro have participated to these meetings.

6.4.2. Standardization of Interval Arithmetic

Participant: Sylvain Pion.

*In collaboration with Guillaume Melquiond (*PROVAL *project-team).*

Geometric computations are very sensitive to numerical roundoff errors. There are efficient ways to solve this problem like static filters, but there is also a more general approach using a well-known tool which is interval arithmetic. Since the latter solution is more general and easier to use from a programming point of view, we are pushing for its standardization in the hope to get better support from hardware and compilers in the long term.

In the past years, we have worked on a proposal for standardization of interval arithmetic in C++. Since 2009, a new standardization effort has started as IEEE-1788 to provide a language-independent standard for interval arithmetic, similar in spirit to the IEEE-754 standard for floating-point. This standard would serve as a base for implementations in various languages such as Fortran, Matlab, C++...

This year, we have submitted a revision of a proposal for addition to the C++0x language to support basic floating-point operations with directed rounding-mode [62], which we have presented to the C++ committee at the WG21 meetings in Summit (NJ) in March and Frankfurt in July. We think that this set of operations is as fundamental as the basic addition of floating-point variables as it is done now, and that they deserve the same special compiler support.

7. Contracts and Grants with Industry

7.1. Geometry Factory

The initial development phase of the CGAL library has been made by a European consortium. In order to achieve the transfer and diffusion of CGAL in the industry, a company called GEOMETRY FACTORY has been founded in January 2003 by Andreas Fabri (http://www.geometryfactory.com).

The goal of this company is to pursue the development of the library and to offer services in connection with CGAL (maintenance, support, teaching, advice). GEOMETRY FACTORY is a link between the researchers from the computational geometry community and the industrial users.

It offers licenses to interested companies, and provides support. There are contracts in various domains such as CAD/CAM, medical applications, GIS, computer vision...

GEOMETRY FACTORY is keeping close contacts with the original consortium members, and in particular with GEOMETRICA.

In 2009, GEOMETRY FACTORY had the following new customers for CGAL packages developed by GEOMETRICA:
— Aranz (New Zealand, constrained Delaunay triangulation)
— Evolute (Austria, principal curvature estimation, architecture)
— Golaem (France, constrained Delaunay triangulation, animation)
— Schlumberger (USA, Delaunay triangulation, oil and gas)
— Tenet (UK, Delaunay triangulation, GIS).

Moreover, research licenses (in-house research usage for all of CGAL) have been purchased by:
— École Nationale des Ponts et Chaussées (France)
— SolidWorks (USA, CAD/CAM)
— Dassault Systèmes (France)
— Navteq (USA, GIS)
— ZIB (Germany)
— KU Leuven (Belgium)
— BRGM (France, geophysics).

7.2. Dassault Systèmes

Participants: Frédéric Chazal, Nicolas Montana.

In collaboration with André Lieutier (Dassault Systèmes)

The goal of this study is to develop and implement robust and efficient 3D Boolean operators and surface regularization tools for industrial use.

The motivation of this work comes from machining simulation where the computation of the part of the space swept by a moving tool involves a huge amount of Boolean operations (unions, intersections, differences). Such computations meet two main difficulties (that are both theoretical and technical): First, 3D Boolean operations face robustness issues and second, the output of large sequences of Boolean operations usually consists of very complicated meshes containing many irrelevant topological and geometric features that need to be removed for further processing. In this study, we intend to develop a software based on an original theoretical approach which overcomes these difficulties.

8. Other Grants and Activities

8.1. National Actions

8.1.1. ADT CGAL-Mesh

Participants: Pierre Alliez, Mariette Yvinec, Jean-Daniel Boissonnat, Stéphane Tayeb, Jane Tournois, Dobrina Boltcheva.

CGAL-Mesh is a two-year INRIA technological development action started March 2009. Building upon components from CGAL, we have started implementing generic and robust mesh generation algorithms for surfaces, 3D domains as well as time-varying 3D domains. We primarily target applications which involve data acquired from the physical world: geology, medicine, 3D cartography and reverse engineering. We wish to establish for the whole duration of the action a close collaboration with industrial and academic partners so as to maximize the impact of the platform for a number of applications and research experiments.

- Starting date: March 2009

- Duration: 2 years

8.1.2. ANR Triangles

Participants: Manuel Caroli, Pedro Machado Manhães de Castro, Olivier Devillers, Sylvain Pion, Monique Teillaud.

Web site: http://www.inria.fr/sophia/geometrica/collaborations/triangles/

We lead the TRIANGLES project funded by the ANR. The project involves
— the «Laboratoire d'InfoRmatique en Image et Systèmes d'information» (LIRIS), Lyon,
— the «Département d'informatique de l'ENS» and
— the GEOMETRICA team.

Triangulations are essential in many applications, in particular for meshing and shape reconstruction. We want to develop and distribute new results for academic and industrial researchers. The goal of the project is the development of robust and effective algorithms for the manipulation of large sets of points, of moving sets of points and points in non Euclidean spaces such as periodic spaces (torus, cylinder), projective, oriented projective or hyperbolic spaces. The results obtained will be implemented in the CGAL library and will be applied to computer vision (visual envelopes, camera calibration), fluid dynamics, astronomy, computer graphics and medical applications.

In the GEOMETRICA team, Triangles is co-funding the scholarship of Pedro de Castro (with «Région PACA») and funding travel expenses and computers. Several meetings have been organized between participants, details can be found on the project's web page.

- Starting date: November 2007

- Duration: 3 years

8.1.3. ANR GAIA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, Arijit Ghosh, David Cohen-Steiner, Samuel Hornus.

The aim of this project is to formalize a collaboration between researchers from computational geometry, machine learning and computer vision to study distortions and in particular Bregman divergences, information theory, statistics, Riemannian geometry, and convex analysis.

The other partners of the project are the Université des Antilles et de la Guyane (R. Nock, coordinator), the Ecole Polytechnique (F. Nielsen) and the Lear project-team (C. Schmid).

- Starting date: November 2007

- Duration: 4 years

8.1.4. ANR Galapagos

Participant: Sylvain Pion.

In this project, we wish to apply computerized theorem proving tools to two aspects of geometry. One aspect concerns computational geometry. The second aspect is focused on verifying geometric reasoning steps in usual constructions, such as constructions with rules and compass. Other participants in this contract are the universities of Strasbourg and Poitiers, the ENSIEE in Evry and the Ecole Normale Supérieure in Lyon. The leader of the project is the MARELLE project-team.

- Starting date: November 2007.

- Duration: 3 years.

8.1.5. ANR GIGA

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Mariette Yvinec, Steve Oudot, Marc Glisse, Primoz Skraba.

GIGA stands for Geometric Inference and Geometric Approximation. GIGA aims at designing mathematical models and algorithms for analyzing, representing and manipulating discretized versions of continuous shapes without losing their topological and geometric properties. By shapes, we mean sub-manifolds or compact subsets of, possibly high dimensional, Riemannian manifolds. This research project is divided into tasks which have Geometric Inference and Geometric Approximation as a common thread. Shapes can be represented in three ways: a physical representation (known only through measurements), a mathematical representation (abstract and continuous), and a computerized representation (inherently discrete). The GIGA project aims at studying the transitions from one type to the other, as well as the associated discrete data structures.

Some tasks are motivated by problems coming from data analysis, which can be found when studying data sets in high dimensional spaces. They are dedicated to the development of mathematically well-founded models and tools for the robust estimation of topological and geometric properties of data sets sampled around an unknown compact set in Euclidean spaces or around Riemannian manifolds.

Some tasks are motivated by problems coming from data generation, which can be found when studying data sets in lower dimensional spaces (Euclidean spaces of dimension 2 or 3). The proposed research activities aim at leveraging some concepts from computational geometry and harmonic forms to provide novel algorithms for generating discrete data structures either from mathematical representations (possibly deriving from an inference process) or from raw, unprocessed discrete data. We target both isotropic and anisotropic meshes, and simplicial as well as quadrangle and hexahedron meshes.

This project coordinated by GEOMETRICA also involves researchers from the INRIA team-project ABS, CNRS (Grenoble), and a representative from the industry holding a PAST position (Visiting Professor from Industry) at the university of Grenoble.

- Starting date: October 2009.

- Duration: 4 years.

8.1.6. ANR Gyroviz

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Laurent Saboret, Nader Salman, Mariette Yvinec.

The Gyroviz project was selected by the ANR in the framework of the call Audivisual and Multimedia techniques. The project, which was launched in December 2007 for three years, involves the SME Sofresud (Toulon, coordinator) and IXSEA and research teams from the CEA, INRIA and SupMECA Toulon. The project addresses the challenge of automatic modeling of 3D physical scenes from located frames. The aim of the project is to couple new accurate inertial sensors with an image acquisition device and efficient reconstruction algorithms to obtain an automatic image-based modeling system.

- Starting date: December 2007.

- Duration: 3 years.

8.1.7. DIGITEO project GAS: Geometry Algorithms and Statistics

Participants: Claire Caillerie, Frédéric Chazal, David Cohen-Steiner, Bertrand Michel, Steve Oudot.

The project GAS was selected by the DIGITEO consortium in the framework of the "Domaines d'Intérêt Majeur" call of the Région Île-de-France. The project intends to explore and to develop new research at the crossing of information geometry, computational geometry and statistics. It started in September 2008 for an expected duration of 2 years. The other partners of the project are the Ecole Polytechnique (F. Nielsen) and the SELECT project-team (G. Celeux, P. Massart).

- Starting date: September 2008.

- Duration: 2 years.

8.1.8. DIGITEO Chair C3TTA: Cell Complexes in Computational Topology: Theory and Applications

Participants: Claire Caillerie, Frédéric Chazal, David Cohen-Steiner, Steve Oudot, Primoz Skraba.

The primary purpose of this project is to bring about a close collaboration between the chair holder Dr Vin de Silva and Digiteo teams working on the development of topological and geometric methods in Computer Science. The research program is motivated by problems coming from the increasing need of studying and analyzing the (often huge) data sets that are now available in many scientific and economic domains. Indeed, due to the improvements of measurement devices and data storage tools, the available data about complex shapes or complex systems are growing very fast. These data being often represented as point clouds in high dimensional (or even infinite dimensional) spaces there is a considerable interest in analyzing and processing data in such spaces. Despite the high dimensionality of the ambiant space, one often expects them to be located around an unknown, possibly non linear, low dimensional shape. It is then appealing to infer and analyse topological and geometric characteristics of that shape from the data. The hope is that this information will help to process more efficiently the data and to better understand the underlying complex systems from which the data are generated. In the last few years, topological and geometric approaches to obtain such information have encountered an increasing interest. The goal of this project is to bring together the complementary expertises in computational topology and geometry of the involved Digiteo teams and in applied geometry and algebraic topology of V. de Silva to develop new topological approaches to the previous mentioned domain. The project intends to develop both the theoretical and practical sides of this subject. The other partners of the project are the Ecole Polytechnique (L. Castelli-Aleardi and F. Nielsen) and the CEA (E. Goubault).

- Starting date: January 2009.

- Duration: 3 years.

8.2. Actions Funded by the EC

8.2.1. Coordination action FOCUS K3D

Participants: Pierre Alliez, Jean-Daniel Boissonnat, Mariette Yvinec.

Web page: http://www.focusk3d.eu/.

FOCUS K3D (ICT-2007-214993) is a Coordination Action of the European Union's 7th Framework Programme. The other consortium members are:
– Istituto di Matematica Applicata e Tecnologie Informatiche - Unità Organizzativa di Genova - Consiglio Nazionale delle Ricerche (CNR-IMATI-GE), Italy.
– Center for Research and Technology - Thessaly - Laboratory for Information Technology Systems and Services (CERETETH), Greece.
– École Polytechnique Federale de Lausanne - VRlab (EPFL), Switzerland.
– Fraunhofer-Institut für Graphische Datenverarbeitung, Germany.
– Université de Genève - MIRALab, Switzerland.
– SINTEF, Norway.
– Utrecht University, The Netherlands.
The aim of FOCUS K3D is to foster the comprehension, adoption and use of knowledge intensive technologies for coding and sharing 3D media content in application communities by: (i) exploiting the scientific and technological advances in the representation of the semantics of 3D media to increase awareness of the new technologies for intelligent 3D content creation and management; (ii) building user-driven scenarios to evaluate and adapt the technologies so far developed to the requirements of application environments; and (iii) fostering a shift of role of 3D content users, from passive consumers of technologies to active creators.

- Starting date: March 2008.

- Duration: 2 years.

8.3. International initiatives

8.3.1. Associate team TGDA

Participants: Jean-Daniel Boissonnat, Frédéric Chazal, David Cohen-Steiner, Quentin Mérigot, Steve Oudot.

We are involved in an INRIA associated team with the group of Prof. Leonidas Guibas at Stanford University since January 2008. Our collaboration focuses on Topological and Geometric Data Analysis. More precisely, our aim is to develop new topological and geometric frameworks and algorithms for the analysis of data sets represented by point clouds in possibly high-dimensional or non-Euclidean spaces. In 2008 and 2009, exchanges between the two groups were numerous, both at junior level and at senior level. We hope to keep up to the same rate in 2010. Among the scientific outcomes of this collaboration are a new stability theory for topological persistence, a new analysis method for scalar fields defined over sampled Riemannian manifolds, and a clustering algorithm based on persistence.

8.3.2. Associate team DDGM

Participants: Pierre Alliez, Jane Tournois, David Cohen-Steiner.

We are involved in an INRIA associate team with Prof. Desbrun's group at Caltech since January 2009. Our goal is to collaborate on topics commonly referred to as Geometry Processing. This year we have exchanged on mesh optimization, surface reconstruction and quadrangle surface tiling. In addition to Prof. Desbrun three students from Caltech were involved in the collaboration. We applied for renewal of the associate team as we plan to intensify our collaborations.

8.3.3. Associate team OrbiCG

Participants: Manuel Caroli, Monique Teillaud.

The associate team OrbiCG started this year. It is a joint project with two institutes of the University of Groningen: the Institute of Mathematics and Computing Science led by Gert Vegter, and Rien van de Weijgaert from the Kapteyn Astronomical Institute. This research was originally motivated by the needs of astronomers in Groningen who study the evolution of the large scale mass distribution in our universe by running dynamical simulations on periodic 3D data. Our goal is to extend the traditional focus of computational geometry on the Euclidean space $\mathbb{R}^d$ ("urbi") to encompass various spaces ("orbi"), in particular orbit spaces of the Euclidean space, of the hyperbolic space, and of the sphere.

Several members of GEOMETRICA participated to the workshop "Subdivide and Tile: Triangulating spaces for understanding the world" organized by the OrbiCG partners: M. Caroli, F. Chazal, M. Hemmer, Q. Mérigot, and M. Teillaud, as well as J. Bernauer from ABS. Several Dutch members of the Associate Team were also present: Jakob van Bethlehem, Patrick Bos, Amit Chattopadhyay, Bernard Jones, Erwin Platen, Pratyush Pranav, Fatma Senguler-Ciftci, Esra Tigrak, Gert Vegter, and Rien van de Weijgaert.

9. Dissemination

9.1. Animation of the scientific community

9.1.1. Editorial boards of scientific journals

— J-D. Boissonnat is a member of the editorial board of *Discrete and Computational Geometry*, *Algorithmica*, the *International Journal of Computational Geometry and Applications* and the *Journal of Computational Geometry*. He is also on the editorial board of the Springer Verlag book series on Geometry and Computing.
— M. Teillaud is a member of the editorial board of CGTA, *Computational Geometry: Theory and Applications* and of IJCGA, the *International Journal of Computational Geometry and Applications*.
She was a guest editor for the special issue of DCG, *Discrete and Computational Geometry*, dedicated to the 24th Annual Symposium on Computational Geometry [48].
— M. Yvinec is a member of the editorial board of *Journal of Discrete Algorithms*.
— P. Alliez is an associate editor of *The Visual Computer* since 2006 and of the *ACM Transactions on Graphics* since September 2009.
— P. Alliez, S. Pion (chair), M. Teillaud and M. Yvinec are members of the CGAL editorial board.

9.1.2. Conference program committees

— M. Yvinec was a member of the program committee of the Symposium on Computational Geometry, SoCG'09.
— S. Pion was a member of the videos/multimedia program committee of SoCG'09.
— M. Teillaud is a member of the Computational Geometry Steering Committee, which coordinates the annual Symposium on Computational Geometry (SoCG).
— P. Alliez was short paper co-chair of EUROGRAPHICS 2009. He was a member of the program committees of EUROGRAPHICS Symposium on Geometry Processing (SGP), Pacific Graphics, SIAM/ACM Joint Conference on Geometric and Physical Modeling and IMA Mathematics of Surfaces XIII conference.
— J.-D. Boissonnat was a member of the program committee of EUROGRAPHICS Symposium on Geometry Processing and of the Brasilian-French conference Colibri 2009.
— D. Cohen-Steiner was a member of the program committee of SGP'09.

— F. Chazal was a member of the program committee of WALCOM'09.

9.1.3. Ph.D. thesis and HDR committees

— P. Alliez was reviewer for the PhD of Patrick Labatut (ENS Paris), and member of the PhD committees of Jean-Marie Favreau (University of Clermont-Ferrand) and Jane Tournois (University of Nice-Sophia Antipolis).
— J.-D. Boissonnat was a member of the HDR committee of P. Alliez.
— D. Cohen-Steiner was a member of the PhD committee of Q. Mérigot (University of Nice-Sophia Antipolis).
— F. Chazal was a member of the HDR committee of L. Biard (Univ. Grenoble) and of the PhD committees of P. Gaillard (Univ. Compiègne and CEA) and Q. Mérigot (University of Nice-Sophia Antipolis).
— O. Devillers was reviewer of the PhD committee of Clément Jamin (LIRIS, Lyon) and the one of Julien Dardenne (Creatis, Lyon) and member of the PhD committee of Tristan Rousillon (LIRIS, Lyon).
— M. Yvinec was reviewer for the Phd of Ehsan Aganj (Ecole National des Ponts et Chaussées) and member of the HDR committe of Guillaume Caumon (Centre de Recherches Pétrographiques et Géochimiques, Ecole Nationale Supérieure de Géologie).

9.1.4. INRIA committees

— P. Alliez is member of the COST GTAI (conseil d'orientation scientifique et technologique, groupe de travail actions incitatives), of the commission d'animation scientifique (CAS) and of the comité de suivi doctoral (CSD).
— J.-D. Boissonnat chaired the INRIA Evaluation Board until july 2009.
— J.-D. Boissonnat has been a member of the DR2 recruitment committee of INRIA and of the CR2 recruitment commitee of INRIA Rhône-Alpes.
— M. Teillaud is a member of the INRIA Evaluation Board, the INRIA Sophia Antipolis - Méditerranée CDT (Committee for Technologic Development), and Color (LOcal Research COoperations) Commission. She was a member of the CSD (Committee for Doctoral Studies) until September.
— M. Yvinec was member of the « Comité des utilisateurs des moyens informatiques de recherche de l'INRIA Sophia Antipolis - Méditerranée » (CUMIR) until Frebruary 2009.
— F. Chazal is a member of the "Commission scientifique" at INRIA Saclay - Île de France.

9.1.5. Other committees

— J.-D. Boissonnat is a member of the « Commission de spécialistes » of the Ecole Normale Supérieure de Paris.
— S. Pion is a member of the experts group of AFNOR for the standardization of the C++ language within the ISO/WG21 working group.
— S. Pion is a member of the IEEE-1788 working group for standardization of interval arithmetic.

9.1.6. Conference organization

— M. Teillaud co-organized the workshop "Subdivide and Tile: Triangulating spaces for understanding the world", with Gert Vegter and Rien van de Weijgaert. The workshop proposal was reviewed and accepted by the Lorentz Center in Leiden, and was held from 16 Nov 2009 through 20 Nov 2009. The workshop was supported by the Lorentz Center and by the INRIA Associate Team OrbiCG. 46 participants from many countries (Australia, Austria, Germany, Spain, United Kingdom, United States, not to mention the Netherlands and France) attended the workshop.
The purpose of this workshop was to bring together researchers with different areas of expertise to exchange knowledge on various aspects of geometric computing in a variety of different spaces. The multidisciplinarity of the participants was an important aspect of the workshop. Participants came from areas like mathematics, computational geometry, software engineering, astronomy, molecular biology, physics, chemistry, fluid dynamics.
The list of participants, the program, as well as the slides of talks, are available from the workshop web site http://www.lorentzcenter.nl/lc/web/2009/357/info.php3?wsid=357.

— M. Teillaud was invited to co-organize the Dagstuhl Seminar on Computational Geometry, with Pankaj K. Agarwal (Duke University) and Helmut Alt (Freie Universität Berlin). 42 participants from the international community attended, among whom most gave talks. The list can be found on the Seminar web page http://www.dagstuhl.de/no_cache/en/program/calendar/semhp/?semnr=2009111 (some of the slides are also available on the site) and in the proceedings [47].

— In the framework of the European Corordinated action FOCUS K3D, Pierre Alliez and Mariette Yvinec co-organized the "Workshop on anatomical model" with Michela Spagnuolo from the CNR-IMATI institute in Genova. The workshop was held at INRIA Sophia Antipolis on june 16-17th, featured 15 talks and gathered more than 35 persons. The goal of the workshop was to bring together interdisciplinary people to discuss recent results and research directions in the field of modelling anatomical objects for diagnosis, therapy planning, surgery simulation or legal medicine applications, with an emphasis on knowledge technologies applied to computer-aided medicine.

— S. Oudot organized the workshop *Recent Advances on Topological and Geometric Data Analysis* for the associated team TGDA. The workshop was held at the Institut des Systèmes Complexes (Paris, 5ème), from July 8th to July 10th 2009. Its goal was to present the latest results obtained in the area of computational topology, both within the associated team TGDA and outside. It gathered about 40 scientists, including some people from other scientific communities who share an interest in topological approaches, like machine learning and wireless sensor networks. See the webpage for more details: http://geometrica.saclay.inria.fr/workshops/TGDA_07_2009/workshop.html

9.1.7. Web site

M. Teillaud is maintaining the Computational Geometry Web Pages http://www.computational-geometry.org/, hosted by INRIA. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual Symposium on Computational Geometry.

9.2. Teaching

9.2.1. Teaching responsibilities

— S. Pion was a member of the admission jury at the Ecole Normale Supérieure.
— M. Teillaud is a member of the jury of the Agrégation de Mathématiques, whose report can be found at http://agreg.org/Rapports/rapport2009.pdf.

9.2.2. Teaching at universities

We give here the details of graduate courses. Web pages of these courses can be found on the web site : http://www.inria.fr/sophia/geometrica/

2008-2009 courses (taught in 2009)
— Master IFI 1ère année (Nice), *Computational Geometry*, O. Devillers (12h).
— Ecole Nationale des Ponts et Chaussées, *3D Meshes and Applications*, P. Alliez (15h).

2009-2010 courses (taught in 2009)
— Master IFI (Sophia Antipolis), *Geometric algorithms, theory and practice*, P. Alliez, O. Devillers and M. Teillaud (36h).
— J.-D. Boissonnat has a chair at Tsinghua University (Beijing), where he gave a course entitled *Computational Geometry for Curves and Surfaces*, 12h.
— MPRI (Master Parisien de Recherches Informatiques), second year courses, *Algorithmes pour l'approximation géométrique*, J.-D. Boissonnat, F. Chazal and M. Yvinec (24h).
— EFREI (École française d'électronique et d'informatique), *Geometric Algorithms*, P. Alliez and N. Salman (45h).
— M. Teillaud gave a conference entitled "Surfaces and Triangles" to students of Classes Préparatoires of Lycée Masséna and CIV, in the framework of their preparation to the TIPE (Travail d'initiative personnelle encadré).

9.2.3. Internships

Internship proposals can be found on the web at http://www.inria.fr/sophia/geometrica/
— Laurent Caraffa, *3D convex hull optimization for* CGAL, M1 Université de Nice.
— Hugo Feree, *Probing Implicit Surfaces for Surface Mesh Generation*, L3 ENS Lyon.
— Vissarion Fisikopoulos, *Meshing of Periodic Surfaces*, Master Athens.
— Olivier Rouiller, *Triangulations on the Sphere*, École Centrale Lille.
— Rahul Srinivasan, *Perturbing Slivers in Delaunay Meshes*, IIT Bombay.
— Alexandre Bos, *Persistence-based signatures for shapes*, ENS Cachan.
— Clément Maria, *Structures compactes pour le graphe de Delaunay*, L3 ENS Cachan.

9.2.4. Ongoing Ph.D. theses

— Claire Caillerie, *Sélection de modèles pour l'inférence géométrique*, Université Paris XI.
— Manuel Caroli, *New Spaces for Computational Geometry*, Université de Nice-Sophia Antipolis.
— Pedro Machado Manhães de Castro, *Triangulating sets of moving points*, Université de Nice-Sophia Antipolis.
— Arijit Ghosh, *Computational Information Geometry*, Université de Nice-Sophia Antipolis.
— Pooran Memari, *Reconstruction from unorganized cross-sections*, Université de Nice-Sophia Antipolis.
— Nicolas Montana, *Calcul robuste d'enveloppes de solides en mouvement. Application à la simulation de l'enlèvement de matière en usinage*, Université Paris-Sud et Dassault Systèmes.
— Nader Salman, *Reconstruction de surfaces lisses par morceaux*, Université de Nice-Sophia Antipolis.
— Bertrand Pellenard, *Resurfacing*, Université de Nice-Sophia Antipolis.

9.2.5. Ph.D. and HDR defenses

— Pierre Alliez, *Variational Approaches to Digital Geometry Processing*, HDR defense, Université de Nice-Sophia Antipolis, June 2nd.
— Jane Tournois, *Optimisation de maillages*, Université de Nice-Sophia Antipolis, November 4th.
— Quentin Mérigot, *Détection de structures géométriques dans les nuages de points*, Université de Nice-Sophia Antipolis, December 10th.

9.3. Participation to conferences, seminars, invitations

9.3.1. Invited Talks

Invited talks at conferences :
— P. Alliez, "Méthodes numériques pour la reconstruction de surfaces à partir de nuages de points", Annual Conference of the "Société Mathématique de France", Montpellier, France.
— J.-D. Boissonnat, "Manifold Reconstruction of Smooth Manifolds in 3 and Higher Dimensional Spaces", Sibgrapi 2009, Rio de Janeiro, Brasil.
— F. Chazal, "A Geometric (and Partial) Introduction to Dimensionality Reduction", conference Chimiométrie 2009, Paris, France.

Participation to invitation-only workshops :
— P. Alliez, Workshop on Computational Mathematics of Discrete Surfaces, Banff, Canada.
— F. Chazal, Workshop on Data Analysis using Computational Topology and Geometric Statistics, Banff, Canada.
— F. Chazal, "Geometric Inference for Probability Distribution", Summer School on Theory and Practice of Computational Learning, Chicago, USA.
— J.-D. Boissonnat, "Meshing Manifolds embedded in High Dimensional Spaces with Light Scaffolding", Dagstuhl Seminar on Computational Geometry, Dagstuhl, Germany, March 2009.
— M. Teillaud, "Triangulating the 3D Periodic Space", Dagstuhl Seminar on Computational Geometry, Dagstuhl, Germany, March 2009.

9.3.2. Conferences and Seminars

Members of the project have presented their published articles at conferences. The reader can refer to the bibliography to obtain the corresponding list. We list below all other talks given in seminars, summer schools and other workshops.
— S. Pion, "Parallel Multi-Core Geometric Algorithms in CGAL", Workshop on Massively Multiprocessor and Multicore Computers, Rocquencourt, France, February 2009.
— S. Pion, "CGAL: The Computational Geometry Algorithms Library", Universität Karlsruhe, Germany, July 2009.
— S. Pion, "Robustesse des algorithmes géométriques: des outils arithmétiques à l'interface utilisateur dans CGAL", Rencontres Arithmétique de l'Informatique Mathématique, RAIM'09, Lyon, France, October 2009.
— M. Teillaud, "CGAL", Seminar at the Institut de Mathématiques de Toulouse, France, June 2009.
— M. Teillaud, "CGAL", Colloquium series of the Master program Game and Media Technology, University of Utrecht, the Netherlands, November 2009.
— M. Teillaud, "Computing 3D Periodic Triangulations", Mittagsseminar of the Institute of Theoretical Computer Science, ETH Zürich, Switzerland, September 2009.
— O. Devillers, "Succinct representation of planar graphs", Journées Combinatoire et Algorithmes du Littoral Méditerranéen, Marseille, France, October 2009.
— P. de Castro, "Filtering Relocations on a Delaunay Triangulation", Centro de Informatica, Universidade Federal de Pernambuco, Brasil, August 2009.
— M. Caroli, "Computing periodic triangulations", Workshop Subdivide and Tile: Triangulating spaces for understanding the world, Lorentz Center, Leiden, The Netherlands, 16-20 November 2009.
— M. Caroli, "Triangulating the 3D Periodic Space", Seminar of EPI VEGAS, INRIA Nancy – Grand Est, June 2009.
— F. Chazal, "Geometric inference", Workshop Subdivide and tile: Triangulating spaces for understanding the world (invited 2 hours tutorial), Lorentz Center, Leiden, The Netherlands.
— F. Chazal, "Geometric inference for probability distributions", Seminar at the Laboratoire de Statistique Théorique et Appliquée de Paris 6, November 2009.
— D. Cohen-Steiner, "Geometric Inference for Probability Distributions", seminar at ISTA Vienna, Austria.
— S. Oudot, Fundamentals of Communications and Networking, INRIA - Bell Labs workshop, June 1-3 2009, Murray Hill, USA.

Moreover, several members of the team participated to the Journées de Géométrie Algorithmique, which took place at Le Rouge Gazon, Saint Maurice sur Moselle in January 2009. V. de Silva gave a course entitled *Topology in the 21st century: nonlinear statistics, sensor networks, and other applications*, while F. Chazal gave an *Introduction aux méthodes de réduction de la dimension*. S. Oudot, P. Skraba, Q. Mérigot, N. Salman, P. Memari and S. Hornus also gave presentations.

9.3.3. The Geometrica seminar

http://www.inria.fr/sophia/geometrica/
The GEOMETRICA seminar featured presentations from the following visiting scientists:
— Maks Ovsjanikov (Stanford) : Detection of intrinsic symmetries of shapes.
— Marc Glisse (LIS) : Persistance homologique et simplification.
— Gael Guennebaud : Defining surfaces from point sets.
— David Cazier (U. Strasbourg) : Cartes combinatoires et extensions.
— Guillaume Damiand (LIRIS) : Utilisation des cartes pour le traitement d'images et la modélisation géométrique.
— Mathieu Brédif (IGN) : Recalage de modèles polyédriques de bâtiments prenant en compte la topologie.
— Claire Mathieu (Brown U.) : Partitionnement avec données bruitées.
— Rien van de Weygaert (Groningen) : Morphology and structure of the cosmic web.
— Ulrich Bauer (U. Goettingen) : Topological simplification of discrete Morse functions on surfaces.
— Max Wardetzky (U. Goettingen) : How Geometry can help simulating elastic materials.

9.3.4. Scientific visits

— O. Devillers visited EPI-VEGAS in July.
— S. Pion visited Karlsruhe University in July.
— S. Oudot visited Stanford University in July.
— M. Teillaud and M. Caroli visited the University of Groningen in November.
— P. Alliez visited Caltech in November-December.
— J.-D. Boissonnat visited the Pontificia Universidade Catolica (PUC) and the Instituto Nacional de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro.
— J.-D. Boissonnat visited Tsinghua University (Beijing) in december 2009.

The following researchers have been visiting GEOMETRICA.
— Mikael Vejdemo Johansson (Stanford), two weeks in April.
— Mathieu Desbrun (Caltech) in April and June-July.
— Patrick Mullen (Caltech), in June-July.
— Dmitriy Morozov (Stanford), two weeks in July.
— Gert Vegter (Institute of Mathematics and Computing Science, Groningen) and Rien van de Weijgaert (Kapteyn Astronomical Institute, Groningen), two weeks in November.
— Dominique Attali (GipsaLab) and Xavier Goaoc (EPI-VEGAS), one week in November.
— Hazel Everett and Sylvain Lazard (EPI-VEGAS), one week in December.

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