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INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Project-Team geostat

Geometry and Statistics in acquisition data

Bordeaux - Sud-Ouest

Theme : Stochastic Methods and Models

Activity Report

2009

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The team has been created on January the 1^{st}, 2008 and became an INRIA project (first phase) on November the 1^{st}, 2009 (until October 31, 2010). Second phase (INRIA project team) to iccur before October 31, 2010.

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2. Overall Objectives

2.1. Overall objectives

singularity-exponent A measure of the local irregularity in a complex signal. Singularity exponents can be evaluated in many different ways. GEOSTAT focuses on a microcanonical formulation.

MMF Microcanonical Multiscale Formalism. The formalism used and developped in GEOSTAT in the analysis of complex signals and systems.

Fundamentally, GEOSTAT explores new methods for analyzing and understanding complex signals in different applicative domains through the theoretical advances of the MMF, and the framework of **reconstructible systems**. The underlying theoretical results that motivates the use of the methodologies developped in GEOSTAT, and their application to different types of signals, is found in advances around *renormalization* methods in Physics, and the emergence of the notion of **universality class**. Derived from ideas in Statistical Physics, the methods developped in GEOSTAT offer new ways to relate and evaluate quantitatively the *local irregularity* in complex signals and systems, the statistical concepts of *information content* and *most informative subset*. That latter notion is developed through the notion of *transition front* and *Most Singular Manifold*. As a result, GEOSTAT is aimed at providing *radically new approaches* to the study of signals acquired from different complex systems (their analysis, their classification, the study of their dynamical properties etc.). The common characteristic of these signals, as required by *universality classes*, being the existence of a *multiscale organization* of the systems. That point will be explained in this document. For instance, the classical notion of *edge* or *border*, which is of multiscale nature, and whose importance is well known in Computer Vision and Image Processing, receives, through the MMF, profound and rigorous new definitions. Used in conjunction with appropriate *reconstruction formula*, the MMF is capable of generalizing in a consistent manner the notion of *edge* so that the generalized definition is adequate to the case of chaotic data. The description is analogous to the modelling of states far from equilibrium, that is to say, there is no stationarity assumption. From this formalism we derive methods able to determine geometrically the most informative part in a signal, which also defines its global properties. In this way, the MMF allows the reconstruction, at any prescribed quality threshold, of a signal from its most informative subset, and is able to quantitatively evaluate key features in complex signals (unavailable with classical methods in Image or Signal Processing). We are then able to define, in an unprecedented rigorous and precise manner, the notion of *transition front* in a signal, which is much more complex than previously expected and, most importantly, related to multiscale notions encountered in the study of non-linearity. For instance, we give new insights to the computation of dynamical properties in complex signals, in particular in signals for which the classical tools for analyzing dynamics give poor results (such as, for example, correlation methods or optical flow for determining motion in turbulent datasets).

Given that brief account, the problematics in GEOSTAT can be summarized at first glance in the following items:

- the accurate determination in any n-dimensional complex signal of *singularity exponents* **at every point in the signal domain** (as opposed to global and non-localized exponents in other classical approaches). This accurate determination is an extremely complex problem, related to properties associated to non-linearity and non-equilibrium states in analogies coming from Statistical Physics [3]. The **singularity exponents** give information about local power-law transitions around a point, they are related to a generalized notion of *information content* and *transition front*, and are defined from analogies observed in the behaviour of intensive variables around critical points in complex systems [66].
- The geometrical determination and organization of *singular manifolds* associated to various transition fronts in complex signals, the study of their geometrical arrangement, and the relation of that arrangement with statistical properties or other global quantities associated to the signal.
- The study of the relationships between the dynamics in the signal and the distributions of singularity exponents.
- The study of the relationships between the distributions of singularity exponents and other quantities associated to *predictibility* in complex signals and systems, such as cascading properties, large deviations and Lyapunov exponents.
- The ability to compute *optimal wavelets* and relate such wavelets to the geometric arrangement of singular manifolds and cascading properties.
- The translation of *recognition*, *analysis* and *classification problems* in complex signals to simpler and more accurate determinations involving new operators acting on singular manifolds using the framework of reconstructible systems.
- In the applicative domain, GEOSTAT will focus its research activities to the study of three main classes of signals: remote sensing satellite acquisitions in Oceanography (study of different phenomena -i.e. geostrophic or non-geostrophic- complex oceanic dynamics, mixing phenomena, ocean/climate interaction), Speech processing (analysis, recognition, classification), signals in Astronomy (multi-dimensional implementation of the MMF, analysis of solar data, atmospheric perturbation of acquisitions with optical devices, interstellar medium).

3. Scientific Foundations

3.1. Scale invariance, singularity exponents and universality classes

In the past decades, the emergence of *renormalization methods* in Physics has led to new ideas and powerful multiscale methods to tackle problems where classical approaches failed. These concepts encompass a wide range of physical systems and they underline fundamental ideas [41]:

- *collective behaviour* of microscopic degrees of freedom, revealed by the study of *fluctuations* and *statistical correlations*,
- critical phenomena observed at any scale, associated to critical divergence of mascroscopic variables described by *scale laws* and *singularity exponents*,
- scale homogeneity and scale separation are replaced by the more general concept of scale invariance, and, as a consequence, the definition of **hierarchical geometric structures** that correlate the different scales in a system,
- the fundamental concept of **universality class**.

In the typical example of critical transitions [66], [79], [24], critical divergences are naturally associated to intensive variables, they indicate dramatical changes in the physical state of the system, and they are characterized by the values of **singularity exponents**. One observes divergence phenomena in correlation lengths and the presence of statistical fluctuations at all the scales. These phase transitions occur at a certain temperature, called the *critical temperature* θ_c and the description of a physical quantity X around the *reduced temperature* $\tau = \theta - \theta_c$ is governed by the value of the singularity exponent:

$$h = \lim_{\tau \to 0} \frac{\log |X(\tau)|}{\log |\tau|} \tag{1}$$

hence

$$X(\theta) \;=\; \alpha(\theta - \theta_c)^h \;+\; o\left((\theta - \theta_c)^h\right) \tag{2}$$

Scale invariance is a natural consequence of this phenomenon and it implies that the functionnal dependency of variables remains unchanged under scale change. Fluctuations propagate through all scales in the same way, and microscopic dynamics become irrelevant. To properly characterize a critical point, it is necessary to determine as precisely as possible the value of the exponent, and one of the main difficulties is to determine the appropriate model to compute the exponents. The importance of that determination, which moreover makes one of the important scientific motivations in GEOSTAT, comes from the fact that an important number of relations between the exponents have their origin in thermodynamical considerations, *they go beyond the consideration of a particular system*. In this context, it becomes natural to search for such singularity behaviours in a wide range of acquired datasets. Recently, it has been understood that systems characterized by same singularity exponents form an *universal class*, as explained by renormalization techniques: in the vicinity of a critical point, the microscopic dynamics vanish to leave the place to macroscopic components precisely determined by the exponents [66].

Singularity exponents are **universal**, they do not depend on the microscopic description of the system, but only of global properties such as its dimension and the range of the interaction. In Statistical Physics, *universality* appears in the following manner: very different systems have the same exponents, a notion that can be explained by *renormalization*: near a critical point fluctuations and disturbances occur at all scales, and thus a correct description is given by a scale invariant theory. This modern meaning of universality was introduced in L. Kadanoff *et al.* [38], and has found applications in other fields outside Physics, such as distributed systems and multi agents systems [52]. In GEOSTAT the notion of *universality class* forms the cornestone that allows the determination of fine dynamical properties of signals and systems associated to scale invariant phenomena with the help of generalized *singularity exponents*. In other words, the singularity exponents (and the geometric notions they are associated with) give information and radically new methods for the analysis of signals associated to acquisition of different physical systems. An example of a typical study that motivates the determination of *singularity exponents* and the existence of a common *universality class* encompassing different physical systems can be found in [5], in which a common multiscale signature is found, in a microcanonical sense and with the use of the MMF, in six signals of different nature (stock markets, currency exchanges, temperature time series in the ocean, Sea Surface Temperature satellite maps, high-Reynolds velocity fields in 2D turbulence, and natural images from Hans van Hateren's web database): the singularity spectra of these signals coincide, a result compatible with the existence of a common *universality class* to which the systems belong. For a general reference on the subject, see also [41].

The determination of a singularity exponent at every point of a signal's domain is a subject of interest [17], [59], [60], [57], [37], [63], [74], [28]. The notion of reconstruction was introduced in [68], then verified in the case of certain meteorological datasets [34], in econometric time series [75] and most importantly in oceanographic datasets [70], a domain which is an area of intense research between GEOSTAT partners. The study of ocean dynamics is of particular interest in the thematics developed in GEOSTAT, because a great amount of work has been devoted to establish a relationship between a super-geometric structure defined from the singularity exponents and a *proxy* stream function of the oceanic flow, based on the works in ocean dynamics of [16]. These results are presently extented in many directions, notably for the determination of energy exchanges between the ocean and the atmosphere. Moreover, the potential of the geometric super-structures to recover dynamic variables from the data has been explored in the meteorological framework to determine rain precipitation in thermal infrared Météosat data [69]. These last examples in Oceanography and Meteorology underline a key aspect in GEOSTAT: our goal is to link specific information in a complex signal to physical descriptions meaningful in terms of geometry and statistics: for example, domains related to GEOSTAT include multiplicative cascading, singularity analysis, transition front detection (see [22], [36], works of D. Schertzer, A. Arneodo *et al.*). Note also that GEOSTAT is not intended to be another project on multifractal analysis.

However the real world shows that the sole consideration of singularity exponents is not sufficient to assess the dynamical properties. One has to study the continuum of their organization. This is the entry point for the consideration of geometric structures: the realization of critical manifolds is a research theme under investigation in various contexts since many years, and the resolution of turbulent dynamics is not assessed by a single geometrical interface because, in a famous paper, Parisi and Frisch [51] have established the relationships between the spectrum of singularity exponents associated to structure functions and the geometrical hierarchy predicted in *Fully Developped Turbulence* (FDT). Henceforth, the different geometrical super-structures organize themselves according to the dynamics to produce the singularity exponents observed in signals. From thereon, the interest for singularity exponents has grown firmly, and a great deal of research has been devoted to relate them to the statistical content, in various contexts (see [18] and the references therein).

In many different branches of applied sciences, sensors are delivering datasets that reproduce, at increasing spatial and temporal resolutions, the overwhelming complexity of natural phenomena. Here are some examples:

- the gigantic increase (in terms of spatial, temporal and spectral resolutions) of satellite data,
- the considerable increase of bandwith for the network delivery of high quality speech or image data,
- new imagery techniques in astronomy,
- new techniques in microscopy for biologic imagery,

These examples are not exhaustive, we mention them only to underline the new kind of research problems they raise to classical methodologies in statistical modeling and processing: the ever increasing complexity present in these datasets is not merely a quantitative hurdle that will be solved by considerations bound to mere computing power or hardware progress; they relentlessly lead to question the very pertinence of these classical methodologies, and they ask for new methodologies able *to cope with the apparition of processes and phenomena which were not, up to now, accessible in the datasets*. Most interestingly, these progresses in the quality of acquired datasets match the advances of new concepts from Statistical Physics, such as for instance in the generic power of *renormalization methods* [41] for the characterization of multiscale phenomena around critical points in universality classes, and multiresolution analysis. This point of view forms the distinctive approach in GEOSTAT. For instance, new types of high resolution data in satellite imagery (in various spectral domains, even microwave for altimetry with the coming advent of SWOT data in 2012) challenge classical segmentation and matching methods; turbulence phenomena directly available in oceanographic datasets raise new methodologies for the assessment of the dynamics of the oceans. The same remarks apply in astronomy (e.g. the MUSE project) and speech processing.

GEOSTAT theoretical advances are focused, for one part, on the definition of *reconstruction formula* which allow the restitution of a signal (spatial or temporal predictibility) from knowledge about that signal on certain subsets known as *geometric super-structures*, of statistical importance (hence the name of the GEOSTAT proposal: **Geo**metry and **Stat**istics in acquisition data). These *geometric super-structures* are, of course, derived from *singularity exponents*: they convey the most significant information (in a statistical sense) across the scales in the system. This proves, at least empirically in an effective way, the signifiance of singularity exponents (as they are defined in a microcanonical way in the MMF). The *information content* associated to these geometric sets, their relationships with the class of reconstructible systems unveils the definition of new methods for recognition, classification and statistical modeling. This is a key aspect in GEOSTAT, which is justified by the physical signification of singularity exponents in universality classes: for instance, the reconstruction formula permits the characterization of fine phenomena in turbulent signals (see, for instance, figure 1). In GEOSTAT we are studying the introduction, in classification and recognition methods, of fine multiscale parameters derived from the MMF in complex signals such as speech or astronomical data. The theory of reproducing kernel Hilbert spaces, developped within the framework of reconstructible systems and the MMF, also has a strong potential in many domains such as approximation theory, recognition and classification ([32], [80], [43]). In GEOSTAT, reproducing kernels will be studied in conjunction with the MMF for recognition and classification, and serve as the basis for new reconstruction formula.

In Image Processing and Computer Vision, a fundamental notion is found in the concept of *edge* or *border* [31], which is the building block of many other attributes, static or dynamic. A major advance in GEOSTAT is to redefine this fundamental notion [48], [50], [71], [72], [74], [73], [3], to give it, through the MMF, a considerable extension well adapted to the case of multiscale data, and relate it to the notion of information content and transition front in a such a way that it finds a natural link with the notion of critical points in Statistical Physics. As a consequence, the notion of *transition front* in turbulent data can be quantitatively and precisely defined, and the evaluation of the dynamics in complex images associated to turbulent signals can be done with algorithms completely different, in nature, to those classically used in Image Processing, such as the optical flow or correlation methods and neural networks. These latter methods, which work quite well for rigid or elastically deformable objects, raise serious difficulties in the case of geophysical signals such as oceanographic images. These hurdles constitute an important topic of research and they are investigated, among other topics, for example by the **FLUMINANCE** project at INRIA, which has developped sophisticated methods to analyze motion in turbulent data -different in nature to those proposed in GEOSTAT-. In Oceanography, it is a very difficult problem to assess the motion of turbulent and coherent structures such as vortices or thermal fronts from satellite maps. A major advance in GEOSTAT is to use the MMF to obtain dynamical attributes using only one single snapshot in temporal sequences (see, among others [70]). Moreover the evaluation of dynamical properties using the MMF shows accurate numerical stability with satellite data, which are plagued by many artifacts related to the acquisition process.

3.2. The Microcanonical Multiscale Formalism

In the study of fully developped turbulence, the *multifractal formalism*, which is called *canonical formalism* in this document (to make a precise distinction with the *microcanonical formalism* developed in GEOSTAT) has been established with solid foundations [18]. From all aspects, and specially from a dynamical systems perspective, the *microcanonical formalism* studied in GEOSTAT is completely different by nature, most notably in the situation far from the statistical equilibrium, which is the most common case in all complex signals, and where the moments of significant variables are not easily obtained. The MMF can be seen as an extension of the *canonical* formalism, in the sense that the latter lies on purely statistical descriptions.

To dwelve into the realm of a description outside statistical equilibrium, a fundamental concept is given by the notion of singularity exponent [76], [70], [68], [82], [34], [35], [69], [3]. Singularity exponents can be computed in many different ways. In particular, it is a notion that is dependent of the formalism used (canonical

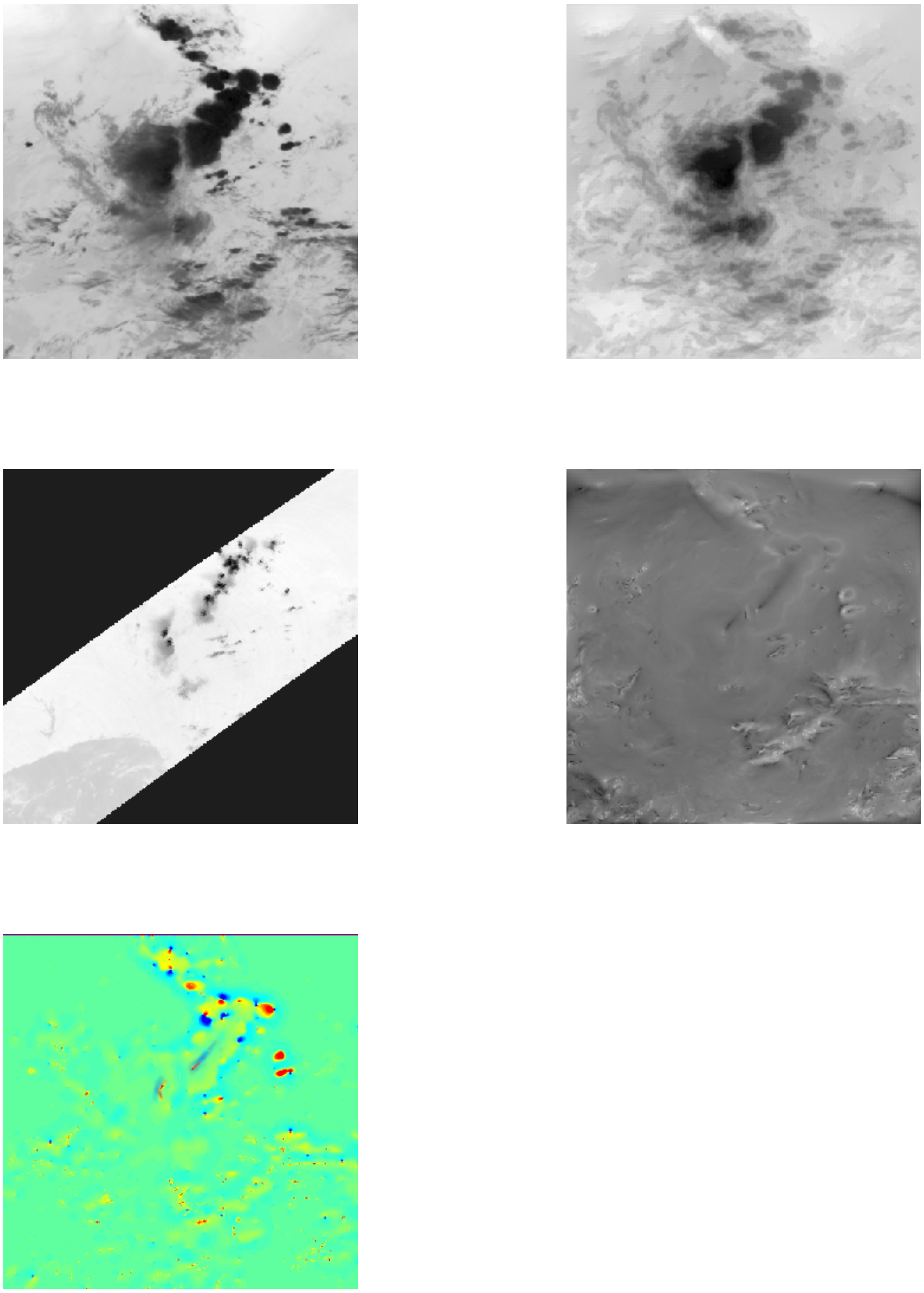

Figure 1. ***Top:*** *excerpt from a MétéoSat infrared image acquired on 31-07-1998 at 16.00 GMT above West Africa (left) and reduced signal (right).* ***Middle:*** *Left: corresponding image acquired in the hyper-frequency range by the TRMM satellite, displaying ice crystals at the top of clouds, associated to precipitation. Ice crystals correspond to darkest pixels. Right: modulus of the multiscale source field, computed within the framework of reconstructible systems, shown in logarithmic scale. Zeros and poles are respectively associated to black and white pixels.* ***Bottom:*** *divergence of the velocity field estimated by classical optical flow. In blue: positive divergences, in red: negative divergences. There is a clear correspondance between divergence's extrema and zeros and poles of the source field. The source field is computed from one single acquisition, as opposed to optical flow. This example shows some of the MMF's potential.*

or microcanonical). This distinction is of fundamental importance. In the so-called *canonical* formalism, the exponents are obtained through a family of expected values associated to operators $\mathbb{T}_{\mathbf{r}}$ depending on the scale $\mathbf{r}$:

$$\langle|\mathbb{T}_{\mathbf{r}}\mathbf{s}|^p\rangle \;=\; \alpha_p\,\mathbf{r}^{\tau_p} \;+\; o\left(\mathbf{r}^{\tau_p}\right) \tag{3}$$

and the coefficients τ_p are then related to the Legendre spectrum of the *canonical* multiplicative cascade. In the **microcanonical formalism**, the singularity exponents are not computed through the values of moments, but depend instead on the spatial location of a point in the signal domain:

$$\mathbb{T}_{\mathbf{r}}\mathbf{s}(\mathbf{x}) \;=\; \alpha(\mathbf{x})\,\mathbf{r}^{h(\mathbf{x})} \;+\; o\left(\mathbf{r}^{h(\mathbf{x})}\right)\;(\mathbf{r}\to 0) \tag{4}$$

As soon as coordinates (spatial or temporal, or depending on the characteristics of the signal's domain) are introduced, **geometric super-structures** can be defined; they are naturally associated to the singularity exponents, the multiscale hierarchy, and can also be related to the singularity spectrum defined in the canonical formalism [53]. A fundamental aspect in GEOSTAT is to study their relationships to the dynamics of the underlying complex system. One of these geometric entities, the so-called *most singular manifold* plays a central role in the framework of **reconstructible systems**, because of its signification as a set of *transition fronts* [3], [33]. The MMF, through the reconstruction formula, therefore provides a theoretical background for the study of complex signals of different types (turbulence, geophysical fluids, oceanographic, meteorologic and astronomic data, speech signal and biological datasets). Hence the scientific and methodological justification for the creation of an INRIA project on these themes. As an example we show, in figure 2 the singularity exponents computed over a meteorological sequence, and figure 4 shows the application of the MMF on ocean colour data with the help of the **FluidExponents** software.

3.3. Framework of reconstructible systems and motion analysis

The main strength of the MMF consists in its ability to compute accurately the value of a singularity exponent around any point $\mathbf{x}$ in the domain of a complex signal $\mathbf{s}$. The following $\mathbb{T}_{\mathbf{r}}\mathbf{s}(\mathbf{x})$, which is one of the functionals used in implementing the MMF (precisely, in this case, a measure in the signal domain) is defined through the density of a generalized gradient:

$$\mathbb{T}_{\mathbf{r}}\mathbf{s}(\mathbf{x}) = \int_{\mathcal{B}_{\mathbf{r}}(\mathbf{x})} \|\nabla\mathbf{s}\|(\mathbf{y})d\mathbf{y} \tag{5}$$

and whose singularity exponents $h(\mathbf{x})$ derived from the behaviour described in equation (4) can be computed by appropriate wavelet projections ($\mathcal{B}_{\mathbf{r}}(\mathbf{x})$: ball of radius $\mathbf{r}$ centered at point $\mathbf{x}$). A multiscale hierarchy directly related to *information content* (and, in the case of turbulence, to *cascading properties*) is defined from the distribution of the singularity exponents $h(\mathbf{x})$. That distribution being bounded from below for physical signals, a specific geometric super-structure, the *Most Singular Manifold* is the geometrical set associated to the lowest value $h_\infty = \text{Min}\ h(\mathbf{x})$:

$$\mathcal{F}_\infty = \{\mathbf{x},\ h(\mathbf{x}) = h_\infty\}. \tag{6}$$

In the framework of reconstructible systems ([82], [3], [68], [76]) the set $\mathcal{F}_\infty$ is shown to correspond to the statistically most informative part in the signal, and, consequently, an operator can be defined to recover the whole signal from its restriction to the *Most Singular Manifold* $\mathcal{F}_\infty$:

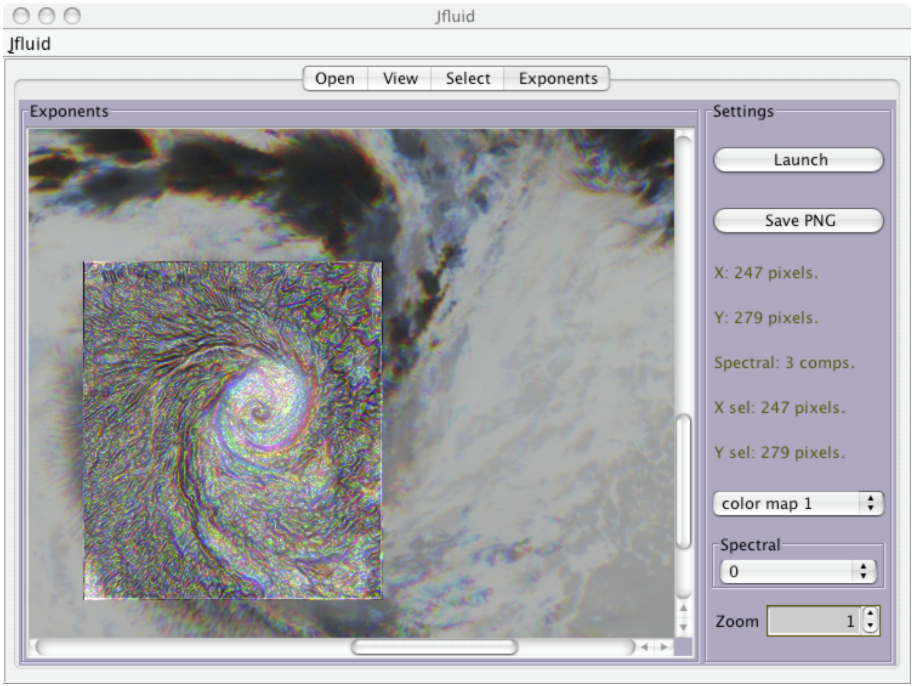

Figure 2. Computation of the singularity exponents on a meteorological sequence. Exponents are computed in three consecutive images, mapped onto RGB channels. Note the distribution of the exponents in the different channels near the vortex's center.

$$\mathbf{s}(\mathbf{x}) = \mathcal{L}(\mathbf{s}_{|\mathcal{F}_\infty})(\mathbf{x}) \tag{7}$$

and the operator $\mathcal{L}$ can be completely specified (usually in Fourier space) from physical considerations about processes [82], [3], [68]. The framework of reconstructible systems opens the way to a whole area of research, for instance for the problem of motion analysis in oceanographic acquisition datasets [70], [67], [54]. In contrast to conservation methods in Image Processing (optical flow, correlation methods etc.) the reconstruction formula permits the determination of the dynamics from *one single acquisition* in a temporal image sequence. See figure 3.

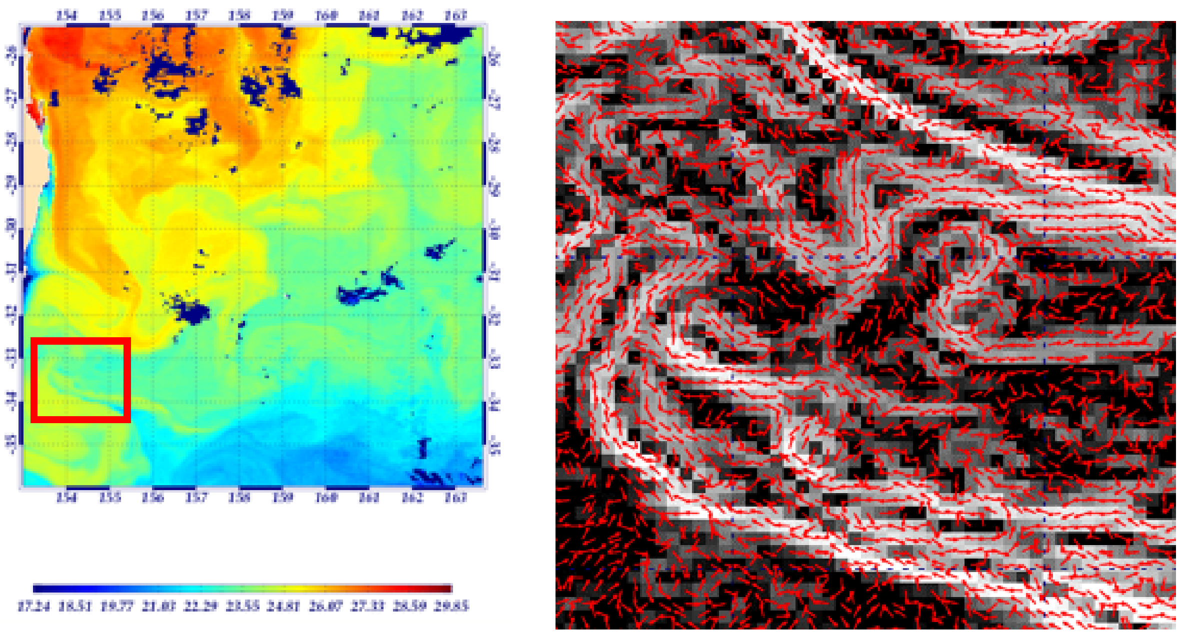

Figure 3. ***Left:*** *excerpt from a Sea Surface Temperature (SST) image acquired by Modis. The image is in false colors and the value of a pixel records the temperature, in Celsius degrees, of the sea surface's upper layer. The image shows the coherent structures and turbulent aspects of the oceanic flow. In red is an excerpt, specifically chosen, containing important turbulent motion (the turbulent character can be evaluated, for instance, from the values of Lyapunov exponents).* ***Right****: application of the MMF, optimal wavelets and reconstruction formula lead to a proper determination of the motion field using only one image in the sequence. The background records the value of singularity exponents. The vector field is depicted in red in the foreground, renormalized to unitary vectors. The proper determination of turbulent motion in real acquisitions like this one shows one of the strong potential of the MMF.*

3.4. Optimal wavelets

The notion of *optimal wavelet* has been introduced by O. Pont (who is currently holding a post-doc position in GEOSTAT) [53],[4]. Optimal wavelets have the fascinating potential of unlocking the signal's microcanonical cascading properties through simple wavelet decomposition. In other words, the turbulent properties of the signal become apparent from the optimal wavelet decomposition. Nevertheless the proper determination of an optimal wavelet associated to a given signal is a very difficult problem, and GEOSTAT is developping important research in this direction. The subject has a strong potential in speech analysis and oceanic motion determination (for instance: descending motion information through an optimal wavelet across the scales to obtain oceanic motion at high spatial resolution from knowledge at lower spatial resolutions). Optimal wavelets are associated to the notion of *microcanonical cascade*: they try to catch the cascading properties in the microcanonical sense, around any point in the spatial domain of the signal. They raise research problem that encompass both theoretical and empirical approaches. From a theoretical standpoint, we need to determine wich signals are describable as microcanonical cascades or, more precisely, whether they always have an optimal wavelet. It seems that natural systems, which are compliant with Parisi-Frisch statistical-geometrical duality do have this property. In the practical realm, they open new approaches in the forecasting of temporal series.

3.5. Physical approaches

The notion of *predictibility*, associated to turbulent phenomena, and their acquisitions by sensors, is the subject of intense research in relation to complex systems [25], [40], [26], [41]; it opens new directions of research in the processing of signals displaying multiscale properties [18].

It is a wide field of research possessing a strong potential for deriving new methods in the analysis of multiscale complex signals. These works show the limitations inherent to classical methods, such as ordinary Lyapunov exponents, to characterize unpredictibility in datasets related to the acquisitions of stochastic signals highly intermittent such as those we are studying. In GEOSTAT, certain theoretical developments around the notion of *predictibility* in the presence of coherent structures are being studied, along with the new approaches able to characterize, in complex signals having multiscale properties, geometric subsets related to the statistical information content. The MMF is not the sole approach, of course, so that GEOSTAT will also investigate new characterizations of intermittency which tend to localize spatially geometrical subsets in the signal domain, associated to information content.

Although approximation methods using techniques inherited from dynamical systems theory are in use in the analysis of some turbulent signals, some recent works [25] insist on the limitations brought by classical Lyapunov exponents w.r.t. predictibility: a Lyapunov exponent is a global quantity measuring an average divergence rate. In the general case, there are some fluctuations in finite time, which play an important role in predictibility, which lead to the consideration of *large deviations*. A major goal in GEOSTAT is to characterize, in a single spatial realisation of the signal the singularity exponents, and to relate that information to others tools in the analysis of non-linear systems such as multiplicative cascading, Lyapunov exponents in finite time and large deviations.

The domain of applicability of the MMF is restricted to those signals having certain multiscale geometric structures, and they form a large set of natural signals; a celebrated example is found in turbulent signals, in which a hierarchy of geometric structures is related to the Legendre spectrum, according to the famous paper of Paris and Frisch [51]. Without trying to delimitate precisely the range domain of the theory, we can mention a broad class of natural signals which fall within its scope: geophysical fluids in the fully developped turbulence regime. In this type of intermittent systems, some geomeric structures dominate the general organization of the temporal dynamics. In the classical approach to fully developped turbulence ([23], [49]), statistical tools are used in first instance. The MMF tends to localize, in the signal domain, characteristics that goes beyond a purely statistical description attainable through the moments (structure functions) of variables. Of course, fully developped turbulence is given here as a standard example, and GEOSTAT is interested in a broader class of complex signals [20], [72].

The determination of geometrical "super-structures" is not new in itself: [18], [46], [17]. However, in the GEOSTAT proposal, the statistical signifiance of the geometric super-structures is emphasized by the type of singularity exponents computed in the MMF. GEOSTAT is looking for stable, robust algorithms and methods for the determination of the exponents and the geometric structures associated with them. GEOSTAT proposes a new type of approach to temporal evolution in complex signals, the understanding of complex dynamics in relation to reconstruction formula, and the combination of these new techniques with existing methodologies in classification and statistical modeling. The study of reconstructible systems from a theoretical point of view, the derivation of fast real-time algorithms and their integration, the use for the automatic processing of

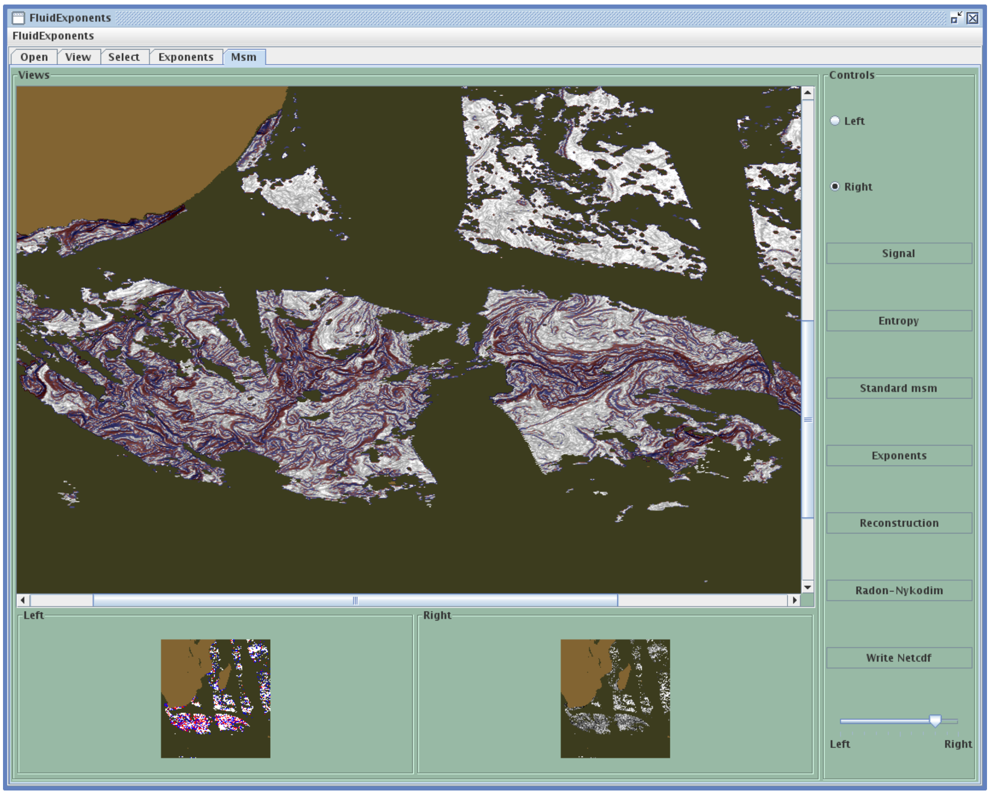

Figure 4. Visualization of the singularity exponents, computed on an ocean colour image, in the vicinity of South Africa. Green areas correspond to missing data (clouds, among others). This image displays, at least qualitatively, how the singularity exponents bring information on the oceanic flow, because chlorophyll concentration acts as a passive tracer.

the immense datasets of satellite data, and the study of the MMF's ability to automatically derive high-level semantic properties in complex signals are also important lines of research in GEOSTAT.

GEOSTAT corresponds to a new phase in the analysis of complex signals, with the systematical use of paradigms coming from Statistical Physics and renormalization techniques about the emergence of multiscale properties and singularity exponents, which allow not only to determine complexity inside signals but also to localize geometrically, inside these signals, the information content related to the emergence of complexity.

3.6. International context

GEOSTAT is easily situated in the landscape of other methodologies developped for more than 20 years, for example what we call the *canonical formalism* in this document, and which is also known under the name *multifractal formalism*. These approaches are fundamentally based on statistical assumptions whose origins can be traced back since the seminal works by Kolmogorov in 1941 [39]. Some of the fundamental steps in the development of purely statistical approaches to turbulence are:

- the numerous studies demonstrating the multifractal character of turbulent flows, for instance [19],
- the relationships between fully developped turbulence and multiplicative cascades (the bibliogaphy on this subject is immense, we only cite [23], [30], [49]),
- the relationships between multiplicative cascades and multifractal geometric entities [51], [63], [64],
- the first approach combining statistics and geometry: multiscale skeletons and WTMM [46],
- fundamental concepts on singularity analysis and their relationships with turbulent flows [27], [65].

The study of signals in which a hierarchy of geometric structures is evidenced goes far beyond the single domain of fully developped turbulence; it is fundamentally related to signals and systems (universality classes in critical systems) in which the values of singularity exponents give access to global properties of the signal: there is a comon attractor leading the dynamics; note that one important point is the correct evaluation of these exponents. This defines a wide area of research in itself, and justifies the approach undertaken in GEOSTAT; henceforth, geometric structures related to singularity exponents have been studied in geophysical signals of different types [58], [29], in the study of clouds [21], [56], pluviometry [61], and in Oceanography [70], to take only few examples. More generally, the geometrical super-structures computed in the framework of the MMF, and their physical signifiance in the analysis of dynamics, clears the path for new research in many domains were complex signals are available: natural images [48], [72], time series (including biological data) [37], [44], trafic in networks [55], econometric data [47], [62], [75], [77], [5].

3.7. Some examples

Since the emergence of satellite Earth Observation data in the domains of Oceanography, Meteorology and more generally Geophysics, increasing amounts of datasets are accumulating in the archives, increasing the need of tools to analyze them. Without pretending to solve the problem of automatic analysis of these signals, the immensity of these databases requires that the fundamental low-level processes of analysis are adapted to the nature of the data. Moreover, the usefulness of these data is no more to prove: in the domain of Oceanography, for instance, it is well acknowledged that satelite datasets have revolutionned the field, by providing intantaneous portrayals of the ocean at various spatial and temporal scales [45]. In other domains, such as astronomical observation, the same observation also applies. To better exploit the existing instruments and the future devices in preparation, one has to extract pertinent information and go back to the physical parameters associated to the acquired objects.

These datasets, although not easily analyzed, often display common characteristics: they are of **multiscale** nature. The importance of this remark is at the centre of developments, both in Computer Science [42], [81], and in renormalization techniques in Physics [41]. A wide variety of signals can be contemplated from this kind of approach, a remark at the basis of the GEOSTAT proposal, which allows us to propose such a widenend framework of application domains, instead of focusing on a particular class of geophysical signals. Researchers involved in the GEOSTAT proposal are currently collaborating with researchers coming from different horizons, such as Astronomy, Signal Processing, Oceanography, Speech, Statistical Modeling. with the goal of obtaining results in a wide class of information systems.

The MMF is being applied by researchers in GEOSTAT to certain geophysical data, either of meteorological or oceanographic nature.

In Oceanogaphy, the first results obtained are related to the evaluation of certain characteristics of ocean dynamics such as the geostrophic stream function in combined altimetry and Sea Surface Temperature (SST) datasets. The precise determination of the stream function is of utter importance in the study of ocean dynamics, for instance in the study of large scale phenomena associated to oceanographic currents such as the gulf stream and global circulation. The MMF provides a coherent and well appropriated framework, because the geometric super-structures associated to singularity exponents receive a physical interpretation: they are associated to the multiplicative cascade (work in progress) and, in the fully developped turbulence regime, they are advected by the oceanic flow. From this one can deduce the fact that some of these geometric super-structures, the *most singular manifolds* are composed of instantaneous streamlines so that, in the geostrophic approximation, the stream function is proportional to a *reduced signal* obtained by anistropic diffusion along these manifolds. These works, some of them portrayed on cover of *Physical Review Letters* [70]form an intense area of research in GEOSTAT.

In the meteorological domain, the MMF is used to characterize convection areas in Météosat thermal infrared datasets, and consequently to locate precipitation areas in convective clouds. Thermal infrared data are particularly well suited to undergo analysis through the MMF because temperature is an intensive physical variable transported and advected by the turbulent flow (atmosphere is mostly adiabatic at the scale of an analysis). The notion of *source field*, an object naturally associated, in the framework of reconstructible systems, to geometric super-structures and singularity exponents is able to provide a separation between the advective and convective parts in the turbulent signal, and to determine precipitation areas in convective towers [34], [35], [33], [69], [78].

4. Application Domains

4.1. Panorama

We define the following lines of research for the next **8 years** in GEOSTAT:

- Theoretical developments around the MMF, in relation with reconstructible systems, predictibility, multiscale cascades and optimal wavelets.
- New methods from the MMF for the analysis of ocean dynamics in satelite acquisitions.
- Non-linear and multiscale approaches to Speech Processing.
- Development of Signal Processig methodologies in astronomical imaging.
- MMF and multiscale methods in the analysis of astronomical datasets.

These lines of research are firmly defined in terms of theoretical and applicative developments. GEOSTAT has defined an accurate network of collaborations for achieving these goals. We describe these lines of research in the following subsections.

The research in GEOSTAT is centered on theoretical and applicative developments in relation to specific applicative domains (Speech Processing, remote sensing in Oceanography, Astronomy). Consequently, the core of GEOSTAT primary collaborations are of a stronger importance. For that reason, we will propose, in 2010, an *associate team* partnership status (which is formally defined at INRIA) to our core collaborations to provide a solid collaborational foundation to GEOSTAT.

4.2. Theoretical developments

Participants: Hussein Yahia, Khalid Daoudi, Oriol Pont, Annick Lesne, Antonio Turiel, Véronique Garçon, Sylvie Rocques, Régine-André Obrecht, Christine Provost, Vahid Khanagha, Joel Sudre, Alex Potamianos, Petros Maragos, Ioanis Klasinas, Reda Jourani.

The previous sections have defined precisely the starting point of GEOSTAT's scientific themes. In GEOSTAT we are primarily interested in the determination and the study of geometric super-structures, accessible through a single realization of a physical system, and not through stationary averages. A valuable approach consists in introducing the exponents inside a specific measure attached to a signal, and this point of view has produced many interesting results already published by researchers involved in GEOSTAT. Moreover, the MMF raises complicated numerical problems.

A quite important point is the possibility of studying multiplicative cascading in the framework of the MMF. Consequently, the relation between a particular and important geometric super-structure, the so-called **Most Singular Manifold** (MSM) and the signal is deterministic, a result which allows the derivation of a **reconstruction formula**: a signal can be fully reconstructed from its restriction to the MSM, hence the statistical signifiance of the MSM as the *most informative transition front*, on top of its dynamical properties. This is the framework of **reconstructible systems**. This notion of reconstruction is at the core of many GEOSTAT thematics, because it is the starting point for the definition of different *reduced signals*; these reduced signals, when compared with appropriate tools (such as, for instance, the Radon-Nykodim derivative in the study of convection) to the original signal, allow the study of the dynamic in complex signals. The reconstruction itself consists in the diffusion, using an *universal propagator*, of the gradient in Fourier space. In figure 5, we show some examples of reconstruction on a MétéoSat image.

The theoretical objectives in GEOSTAT are:

- To study, from a theoretical point of view, the notions developped for complex signals about predictibility (extensions of Lyapunov exponents, in particular in finite time, various notions of entropy, large deviations); establish the relationships between these notions and those developped in the framework of reconstructible systems such as singularity exponents, and to the dynamics of the multiscale signals as well.
- To enhance recognition techniques, statistical modeling techniques, classification (reproducing kernels, SVMs) of signals having multiscale properties in the framework of reconstructible systems and the MMF.
- Develop and implement news tools for predictibility in complex multiscale signals.
- Describe the geometric super-structures from a methodological and mathematical point of view, in particular understand how they form and how to compute them efficiently in signals. Understand their relationships with the dynamics. Study of optimal wavelet bases to understand the relationships with the multiplicative cascade.
- geometric super-structures and the dynamics of complex multiscale signals: how do these geometric sets record information about the past in a complex multiscale signal, and how to use the MMF to get temporal information in an acquisition. Study of the relationships with other tools in the analysis of complex dynamics.
- More generally, what properties of a complex system are related to the emergence of geometric structures ? And, in return, how to act on these systems to control their response ?
- Develop algorithms and numerical methods to extract and use the geometrical super-structures.

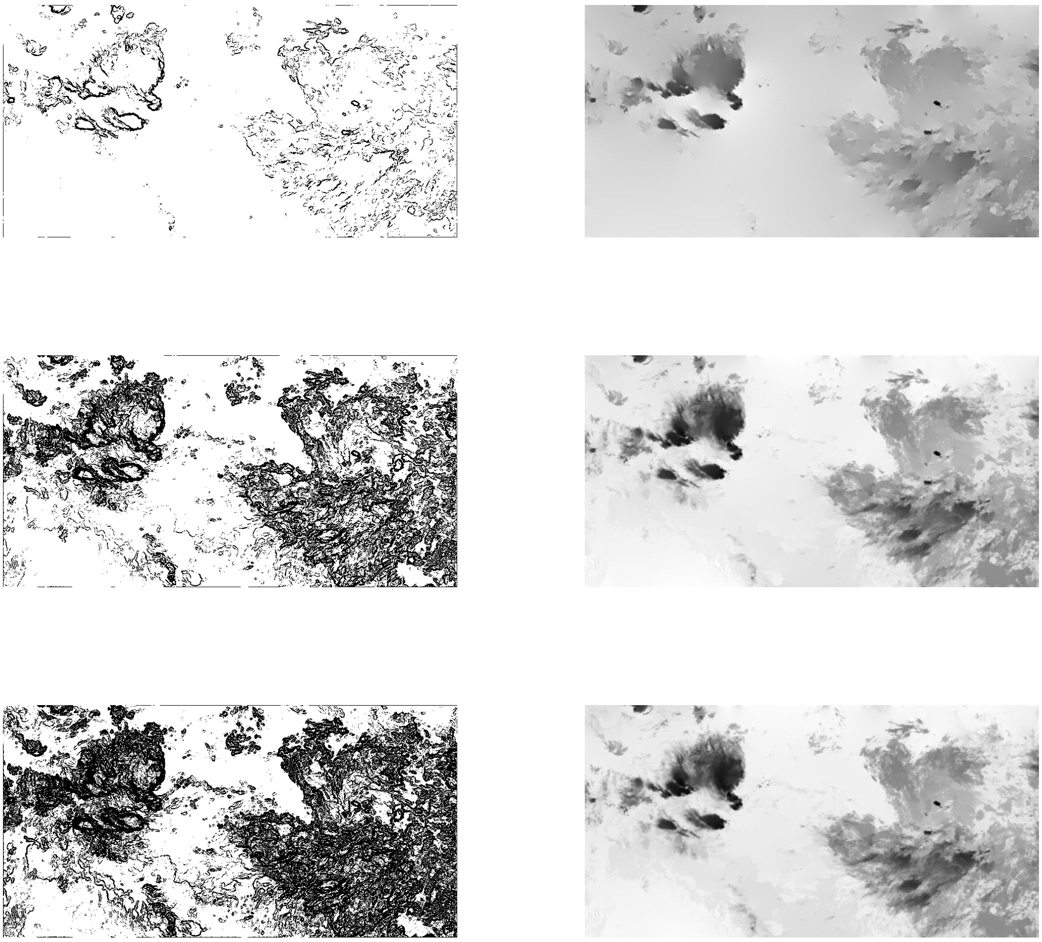

Figure 5. Reconstruction of a MétéoSat image from geometric super-structures. The various geometric structures are associated to different thresholds in the distribution of singularity exponents: $h_\theta = -0.2$, $h_\theta = 0.0$, *and* $h_\theta = 0.1$ *(top to bottom).* ***Images on the left:*** *geometric super-structures; densities:* 6.01%, 28.69% *and* 42.40%. ***Images on the right:*** *reconstructions; PSNRs= 20.06 dB, 30.01 dB and 34.91 dB.*

- Study the apparition of geometrical sets associated to the emergence of complexity in new acquisitions datasets.
- Development of the theory and applications of *optimal wavelets*.

4.3. Ocean dynamics

Participants: Hussein Yahia, Oriol Pont, Annick Lesne, Antonio Turiel, Véronique Garçon, Christine Provost, Joel Sudre.

In Oceanography, the main objective is to bring the MMF as a radically new and powerful tool to study ocean dynamics in satellite acquisition datasets. The use of the Microcanonical Multiscale Formalism in Oceanography and Earth Observation emerges naturally from a first set of results, exposed for instance in [16], which schows that the geometric structures derived from singularity exponents are advected by the ocean flow.

- Obtention of the quasi-geostrophic stream function from acquisition data. Validation.
- Study of the relationships between ocean dynamics and the geometric super-structures derived from singularity exponents, particularly w.r.t. the evaluation of intantaneous oceanic currents. More generally, relate the geometric super-structures in the MMF to ocean dynamics.
- Geometric super-structures in ocean colour data (marine biogeochesmistry, patchiness and sub-mesoscale structure of α cholorophyl fields, CDOM (colored dissolved organic material), OPC (organic particular carbon), dispersion of tracer fields during in-situ experiments with iron fertilization or chemical pollutants, study of algae blooms with the MMF, dynamics of eastern coastal upwellings.
- MMF and mixing phenomena in ocean colour data. Evaluation of turbulence in the oceans.
- Combination of information given by the MMF from merged SST (Sea Surface Temperature) and altimetry, comparison with geometric super-structures computed at the same temporal frequency on ocean colour data. Interpretation in terms of dynamic biological activity, source and sink determination in the tracer equation conservation.
- Use of operational products derived from GEOSTAT; energy transfer between the ocean and the atmosphere. Ocean/climate interaction: more than 50% of vertical transfers between the ocean and the atmosphere are associated to sub-mesoscale (< 10 km). Presently, this spatial resolution is out of altmetric range. The importance of the MMF lies in the fact that there is no synoptic acquisition of vector fields at sub-mesoscale resolution. The first experimental satellite to acquire the dynamics at these scales will be SWOT, to be launched in 2012 if budget prevision goes as planned. SWOT will only be experimental, in test phase during 3 to 6 months only.
- MMF and climate change, through the study of ocean-atmosphere heat transfers and greenhouse gazes sources and sinks.

4.4. Speech signal

Participants: Hussein Yahia, Khalid Daoudi, Oriol Pont, Vahid Khanagha, Antonio Turiel, Régine-André Obrecht, Alex Potamianos, Petros Maragos, Ioanis Klasinas, Reda Jourani.

Some specialists in speech signal analysis have been interested in the use of Lyapunov exponents and other considerations from deterministic chaotic systems applied to such signals, to measure their chaotic behaviour and adapt the analysis accordingly.

First, researches in GEOSTAT focus on demonstrating the turbulent character of speeh signal, because of its formation processes, and, as we noted earlier, classical Lyapunov exponents are insufficient for deriving entirely this character. Then, GEOSTAT will focus in developping, for speech signal analysis, the necessary tools in the Microcanonical Multiscale Formalism that will take into account efficiently the non-linear aspects of these signals and adapt the recognition and classification methods accordingly. Hence the first step consists in understanding the signification of singularity exponents for speech signals, and to adapt the MMF for deriving meaningful geometric super-structures from these signals. Speech signals need a complete re-writing of some procedures used in the MMF, for instance by taking into account finite variable window sizes associated to the computation of singularity exponents and the observation of source field. Then, a vast research programme will consist in understanding and adapting the framework of reconstructible systems, and to derive the methods for analysis, classification and recognition. The study of speech signal will be the occasion, in GEOSTAT, of understanding subtle aspects specific to 1D-signals, to derive generic models for temporal series, and to study temporal evolution in a simplified case.

4.5. The MMF and the analysis of astronomical datasets

Participants: Hussein Yahia, Khalid Daoudi, Oriol Pont, Sylvie Roques, Antonio Turiel, Michel Rieutord, Thierry Roufier, Emmanuel Berné.

In the domain of Astronomy, impressive progress have been achieved these recent years by considering, at a very fundamental level, the acquisition process by the devices, and by driving the image analysis accordingly. Nowadays, algorithms must be able to merge information of different nature, and take into account the ever increasing number of spectral bands available, and use this dimensionality in the merging processes. Moreover signals in radioastronomy are now multifield and multiscale, they measure synchronously many different vectorial measures at increasing spatial resolution. In GEOSTAT, we will study data coming from the MUSE intrument (Multi Unit Spectroscopic Explorer), a 3D spectro-imager operating in the visible domain, which will become one of the instruments in the Very Large Telescope (VLT). We propose the development of methods that will take into account the specific aspects of the instrument, for instance the large dimensionality of voxels cubes produced ((300×300 pixels and 4096 channels), the variability of the impusional response both spatially and spectrally, and the combination of multiple pauses. GEOSTAT will also define research lines in the following areas:

- Interstellar medium turbulence.
- Study of the solar magnetic field.
- Solar granulation.
- Atmospheric perturbations in ground acquisition of optical devices (adaptative optics).

5. Software

5.1. FluidExponents

Participants: Hussein Yahia [correspondant], Qinqin Long, Antonio Turiel [ICM-CSIC Barcelona].

Since 2004, we are developping a software plaform, called **FluidExponents** which fully implements the MMF. **FluidExponents** serves different purposes and share interesting characteristics:

- **FluidExponents** is entirely written Java, and it implements all core MMF functionalities,
- it comes in two separate entities: a set of Java libraries for development, and a user interface for interacting with the constructs defined from the MMF,
- it is composed of around 100 000 lines of Java code,
- it serves as the basis development platform in GEOSTAT.

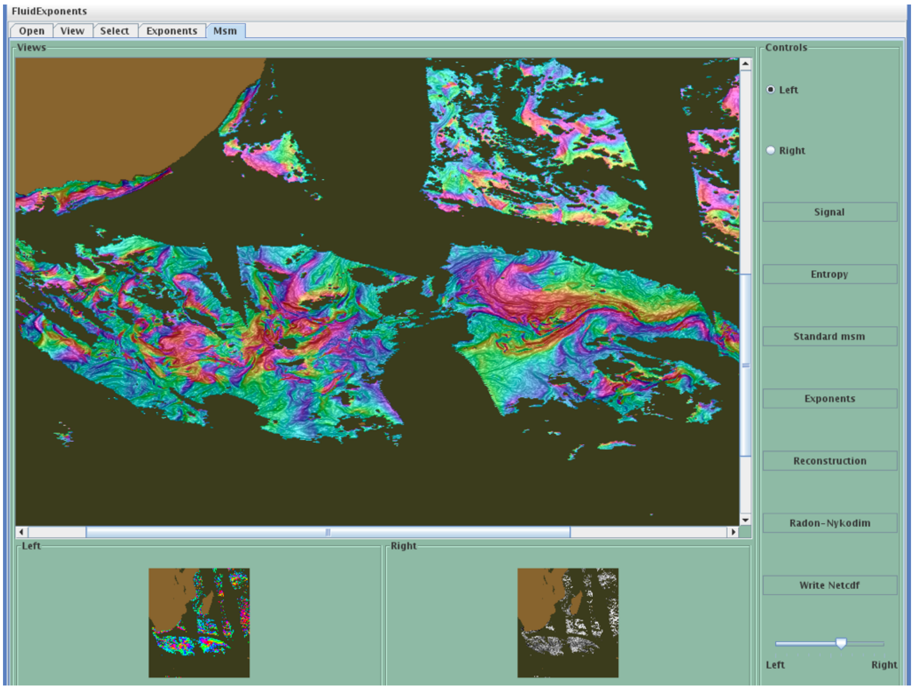

Figure 6. A session of the ***FluidExponents*** *software, using the user interface to compute MMF constructs on a typical geophysical dataset in the Netcdf format.*

The use of **FluidExponents** in GEOSTAT is systematic: all new research results (coming from PhDs for instance) are implemented within it. Functionalities that have a strong potential are implemented in the standard version of **FluidExponents**.

We show, in figure 6, a session of **FluidExponents** using the user interface.

In GEOSTAT, we have the strong intention of consideraly developping the **FluidEponents platform**, for dissemination and research:

- A lighter version of **FluidExponents** is developped to be diffused in the scientific community, for disseminating the implementation and use of the MMF (mainly in the geophysics community),
- We are currently in the process of making a deposit of **FluidExponents**.

We are currently in the process of understanding and specification, with our spanish partner, of a commercial implementation and use of **FluidExponents**, through the possibilities offered by INRIA on the creation of start-ups.

6. New Results

6.1. Optimal wavelets and ocean dynamics

Participants: Hussein Yahia, Oriol Pont, Véronique Garçon, Joel Sudre, Claire Pottier [CNES].

In collaboration with the LEGOS team and Claire Pottier from CNES, we are developping the use of optimal wavelets in conjunction with the formalim of reconstructible systems and the MMF to obtain high spatial resolution maps of the ocean dynamics of SST (Sea Surface Temperature) data. The method develops by an estimation of the cascading properties associated to the presence of a set of hierarchical geometric structures associated to the flow. A determination of these cascading properties is achieved through an approximation of an optimal wavelet. Then, ocean dynamics information derived from low spatial resolution altimetric data is transferred through the scales to determine an oriented geostrophic stream function.

6.2. Speech signal and the MMF

Participants: Oriol Pont, Khalid Daoudi, Hussein Yahia, Régine Obrecht, Vahid Khanagha.

In this line of research, the pertinence of non-linear methods and use of the MMF for Speech Processing are investigated. First results seek to establish the intermittent and multiscale properties of the speech signal, which is the prerequisite to any attempt at using multiscale and non-linear approaches for studying these types of signals. Preliminary results clearly show that the speech signal has intermittent and multiscale properties, and that the singular exponents, together with the most singular manifolds they determine in these 1D signals, convey key information in phonemes.

6.3. Reconstructible systems in Image Processing

Participants: Oriol Pont, Khalid Daoudi, Antonio Turiel, Hussein Yahia, Dehbia Idres.

The formalism of reconstructible systems permits the reconstruction of signals knowing their most singular manifold and the distribution of the gradient along this set. We are evaluating the performance of the reconstruction given the type of singular manifold given as input, and in comparison with classical edge detectors. The goal is to provide a rigourous notion of edge that would apply to any type of signal, and that would generalize appropriatly in the case of turbulent data.

7. Contracts and Grants with Industry

7.1. Hiresubcolor

Participants: Hussein Yahia, Oriol Pont, Véronique Garçon, Joel Sudre, Antonio Turiel, Christine Provost [LOCEAN].

3-year contract, CNES-NASA funding, started mid 2008. Title: *HIRESUBCOLOR : Multiscale methods for the evaluation of high resolution ocean surface velocities and subsurface dynamics from ocean color, SST and altimetry*. Project leader: H. Yahia.

7.2. RTRA-STAE

Participants: Khalid Daoudi, Hussein Yahia, Véronique Garçon, Sylvie Roques, Régine Obrecht.

Proposal, started end 2008, whose title is: *Approches géométriques et multiéchelles pour la prédictibilité et l'analyse de données complexes astrophysiques et géophysiques satellitaires*.

Participating labs: INRIA, CNES-LEGOS, LATT, IRIT. Project leader: K. Daoudi.

8. Dissemination

8.1. Animation de la Communauté scientifique

- Lecture given by H. Yahia (LATT Laboratory). October 28, 2008.
- Participation to the GODAE Meeting, Nice, October 2008 (H. Yahia, J. Sudre).
- H. Yahia was a member of the Jury of Ms. V. Nieves's PhD thesis, Barcelona, Spain. November 2008.
- H. Yahia has presented the results of the Hiresubcolor contract to the CNES, at Toulouse. March, 3, 2009.
- H. Yahia and K. Daoudi have presented the GEOSTAT to the CRTS, Rabat, Marocco. March 2009.
- H. Yahia was a member of the Jury of Mr. O. Pont's PhD thesis, Barcelona, Spain. April 2009.
- K. Daoudi has organized the RTRA-STAE meeting at Toulouse. May 2009.
- H. Yahia, K. Daoudi, O. Pont, A. Lesne, J. Sudre and A. Turiel have given presentations to the RTRA Meeting. May 2009.
- GEOSTAT results have been presented at the OSTST meeting, Seattle. June 2009.
- GEOSTAT has invited Prof. N. Bouguila, Concordia University, during 1 week, for a presentation and common research subjects. June 2009.
- H. Yahia has given a presentation at the University of Heidelberg (team of Prof. C. Garbe), during the participation of a gathering for a european ITN proposal. August 2009.
- H. Yahia has given a presentation to the Bordeaux Mathematical Institute (IMB). October 1st, 2009.
- H. Yahia has given a presentation to the Jean Rond D'Alembert Institute, Paris 6 University. November 5, 2009.
- O. Pont has presented his PhD. Barcelona, Spain. April 2009.
- K. Daoudi has been participating to the Conference IEEE International Workshop on Multimedia Signal Processing (MMSP), Oct. 2009, Rio de Janeiro, Brazil.
- K. Daoudi has been participating to the Conference 13th Asia Conference on Knowledge Discovery and Data Mining (PAKDD), Apr. 2009, Bangkok, Thailand.
- K. Daoudi has been participating to the Conference IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Apr. 2009, Taipei, Taiwan.
- K. Daoudi has been participating to the Conference IEEE workshop on Machine Learning for Signal Processing (MLSP), Oct. 2008, Cancun, Mexico.

8.2. Teaching

- H. Yahia has given Java and Computer Graphics lectures at the Pole Leonard de Vinci, from december 2008 to January 2009.

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[14] K. DAOUDI, J. LOURADOUR. *A comparison between sequence kernels for SVM speaker verification*, in "IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)", 2009.

Scientific Books (or Scientific Book chapters)

[15] K. DAOUDI. *Iterated function systems and some generalizations: local regularity analysis and multifractal modeling of signals*, in "Scaling, Fractals and Wavelets", Wiley & Son Publishers, 2009.

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